Probabilistic Assessment of Handling Qualities Characteristics in Preliminary Aircraft Design

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Abstract

A method is introduced and demonstrated which uses parametric stability derivative data (in the form of regression equations) and probabilistic analysis techniques to evaluate the impact of uncertainty on the handling qualities characteristics of a family of aircraft alternatives. While the method is based on the use of elementary design parameters familiar to the configuration designer, it enables the computation of responses more familiar to the stability and control engineer. This connection is intended to bring about a more complete accounting of stability and handling quality characteristics in aircraft design, based on engineering analysis instead of historical data. Another key advantage of the method is that it allows for the quantification of analysis imprecision and information quantity/quality trades through fidelity uncertainty models. The metrics for these quantifications are the cumulative distribution function and probability sensitivity derivatives. The method is exemplified through the investigation of the longitudinal handling qualities trends for a defined High Speed Civil Transport design space, in the presence of fidelity uncertainty in the stability derivatives.

Introduction

This paper describes techniques developed for evaluating aircraft stability and handling qualities. These techniques are part of a larger, overall design methodology under development by the authors. The core focus of the overall method is evaluating aircraft system feasibility and viability in a multidisciplinary and probabilistic fashion. A simplified view of the new approach is shown inFigure 1, and Ref. [1] provides a comprehensive description of key elements of, and the rationale behind, the method. In this setting, the desire to reduce design cycle time and to improve the quality of information available during conceptual design motivate the need for multidisciplinary analysis. As a consequence, the method begins with the building of parametric relationships of disciplinary metrics as a function of elementary design variables (e.g. configuration geometry), based on engineering analysis. These relationships are called *metamodels*. For example, response surface equations (RSEs) representing drag polars may be formed by using a aerodynamic analysis code with geometric variables as inputs.[2] These RSE's, which capture the individual discipline physics for a class of aircraft, are then integrated into a sizing/synthesis code, which sizes the vehicle for a given mission. After uncertainty models are established for the operation of the vehicle (e.g. random variables for fuel cost, utilization rate, etc.), standard Monte Carlo simulation methods or Fast Probability Integration (FPI)[3] techniques may then be used to determine the system feasibility and viability via the construction of cumulative probability distributions (CDFs) for key system constraints and objectives. The ultimate objective of these probabilistic feasibility and viability investigations is to find *robust solutions*, which are solutions that *maximize the probability* of achieving success while satisfying imposed constraints.[4]

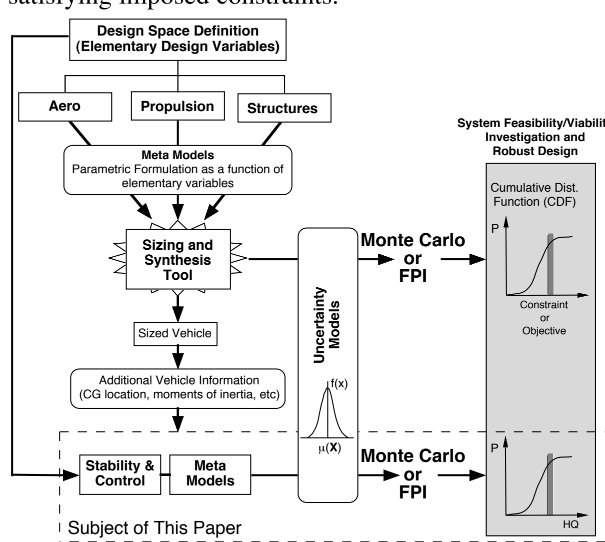

Figure 1: Representation of Proposed Multidisciplinary Aircraft Design Method

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AIAA 98-0492, 36th Aerospace Sciences Meeting & Exhibit

Under the motivation of this new approach to design, it is desirable to be able to relate (on a quantitative basis) issues involving aircraft handling qualities (HQ) and the benefits/costs of control augmentation systems directly to the vehicle configuration design parameters. A major problem in creating this linkage has to do with information, both in terms of *quantity and quality*. Useful stability and control analyses require detailed force (i.e. lift and drag) and moment (i.e. pitching, rolling, yawing) information for each point in the configuration/flight condition design space, at as high a fidelity level as possible. However, as the fidelity increases, computational efficiency (critical in a design setting) decreases. In the past, "fall-back" approaches to addressing HQ in an iterative design environment included using 1st-order relationships to estimate the stability and control derivatives (often with some correction factors via wind tunnel tests) or assigning tail volume coefficients consistent with historical data for similar aircraft. The study of a new and unique aircraft, however, renders both these approaches ineffective, and their use may lead to erroneous results. The use of the aforementioned metamodels offer an intriguing opportunity to attack this information quantity/quality problem.

Since stability and HQ analysis requires information after sizing, such as center of gravity positions and moments of inertia, the stability and control discipline is not yet incorporated directly into sizing as the other disciplines. Rather, stability and control analysis occurs *after* the vehicle is sized, when the needed information is available, as shown in Figure 1. Metamodels for stability derivatives are then generated as functions of not only geometric variables but also variables that result from the sizing process. These models are then operated on directly through a Monte Carlo simulation or FPI, with handling qualities metrics and stability requirements serving as the response random variables to be examined. Uncertainty models in this case (to be introduced in this paper) capture the error in these derivative estimates. *The overall advantage to this framework is that it enables the configuration designer to compute the probability of a given configuration meeting handling qualities criteria and to determine which portion of the uncertainty model has the greatest impact.*

The use of metamodels for modeling aircraft stability derivative models was introduced in Ref. [5]. Metamodels are computationally efficient models which approximate complex physical phenomena. These metamodels are obtained experimentally (or based on computer codes which represent the fundamental relations), and they can be constructed in such a way as to meet the desires of the designer, in terms of accuracy versus efficiency. Metamodel accuracy can be increased, but at the expense of more experimental (or simulation) data. A variety of types of metamodels exist, including regression polynomials, neural networks, fuzzy inference systems, or any other type of input-output mapping. Depending on particular circumstances, designers must investigate the cost/benefits of experimentally obtained metamodels versus "reduced order" analysis.

While metamodels address information quantity, the question of quality remains. Information quality stems from the fidelity of the analysis employed. The required fidelity is a function of the particular application under study and the nature of the desired results. For example, a supersonic commercial transport, such as the proposed High Speed Civil Transport (HSCT), is subject to significant types of flow effects (such as cross flow, vortex bursting, etc.) as well as aeroelastic effects associated with its high degree of flexibility.[6] Thus, a significant amount of fidelity in aerodynamic modeling is required to produce models useful to the configuration and control system designers. In fact, a combined Computational Fluid Dynamics (CFD)/Finite Element Method (FEM) approach to computing forces and moments accounting for structural flexibility and aerodynamic non-linearities is not out of the question.

The introduction of fidelity uncertainty highlights the aforementioned trade between information quantity and quality. Developing a method for evaluating the tradeoff between aircraft stability/control information quantity and quality is a key focus of this paper. Specifically, this tradeoff takes the form of investigating the sensitivity of aircraft HQ and stability responses to uncertainty in the stability and control derivatives. Naturally, this investigation must employ probabilistic methods. The uncertainty in the derivatives is assumed to be from two sources. First, the use of relatively low fidelity aerodynamic analysis to represent the behavior of a highly flexible, non-linear vehicle such as an HSCT will clearly produce inaccuracies. Second, the use of metamodels to create rapid approximations to the analysis adds additional error, though this error can be minimized through careful use of statistical techniques. Thus, both the traditional sensitivity (i.e. change in objective per unit change in input variable) as well as the sensitivity of the objective variance (i.e. the change in objective variance per unit change in input variable mean or variance) are important. In the HSCT example, if the variance of the lift curve slope uncertainty distribution is found to significantly increase the objective variability, the designer would recognize a need to increase the fidelity of the lift curve slope estimate. Ref. [5] showed that metamodels could be used to estimate the probability that any single aircraft within a collection of alternatives would be stable, including uncertainty in the pitch stability slope.

The inclusion of stability augmentation strategies, while not part of this paper, is a topic of current research by the authors. Related work in this area of aircraft model uncertainty was reported in Ref. 7. The emphasis there was on *control system synthesis* only, so the uncertainties models described were not clearly tied to any rationale related to the analysis or to vehicle design. However, the presentation of "probabilistic root locus" diagrams proved insightful. In terms of the effort to combine *aircraft synthesis* with control system design, the work described in Refs. [8, 9, 10, 11] are examples of formulations for combining closed loop control design with configuration design, though none proposed a probabilistic representation or employed the usefulness of the metamodel concept, both of which are introduced in this paper.

Problem Formulation

Review of Metamodel Generation through RSM

A systematic method was proposed and demonstrated in Ref. [5] for rapidly generating linearized dynamic models from fairly complex vehicle geometries. The method makes use of the Response Surface Methodology (RSM) combined with a vortex lattice based aerodynamic prediction program to create regression equations which functionally relate longitudinal stability derivatives to configuration geometry. RSM is a well established technique used in many fields, especially experimental applications, for forming relationships between a series of inputs and outputs when that relationship is otherwise unavailable or too expensive to create. The RSM comprises a group of statistical techniques for empirical model building and exploitation. In general, the RSM starts with a generic model of the relationship to be formed. Such a model takes the form of Eq. (1), where R is the true response, f is the modeled effects, and ε is the combined random and experimental error. The related Design of Experiments (DOE) is used to determine how many and which data points are needed to regress the RSM model for a certain level of statistical significance. For computer simulations (as opposed to physical experiments), the error parameter is assumed to have zero experimental error. Though computer simulations themselves are imprecise models of natural phenomena, they are *assumed* perfectly repeatable in this study. This is a fundamental difference as compared to real experimental designs and does slightly alter the structure of the DOE matrices used.

$$R = f + \varepsilon \tag{1}$$

For the application problem in this paper ,a second degree model is proposed , as in Eq. (2):

$$R = b_o + \sum_{i=1}^{k} b_i x_i + \sum_{i=1}^{k} b_{ii} x_i^2 + \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} b_{ij} x_i x_j + \varepsilon \tag{2}$$

where, b_i are regression coefficients for the first degree terms, b_{ii} are coefficients for the pure quadratic terms, b_{ij} are coefficients for the cross-product terms (second order interactions), and b_o is the intercept term. The components of Eq. (2) are now further defined. The $\mathbf{x_i}$ terms are the "main effects", the $\mathbf{x_i}^2$ terms are the "quadratic effects", and the $\mathbf{x_i x_j}$ are the "second-order interaction terms". If a 2nd-order polynomial is found to be inadequate, other forms are possible, such as exponential or logarithmic, through a transformation of both the independent and dependent variables. RSM is often used for *optimization*, thus the attractiveness of the low order polynomial representation. When using RSM to simply *predict* outputs from combinations of inputs, the model which yields the best fit should be used. Figure 2 and the paragraph that follows summarize the steps required in executing the RSM for building metamodels.

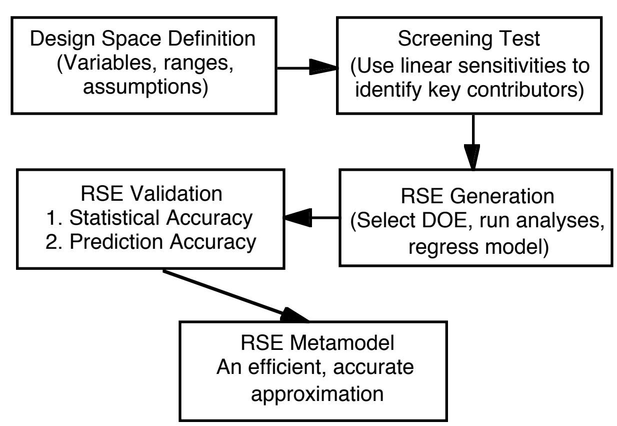

Figure 2: Flowchart for Metamodels Construction through RSM

Metamodel Generation Summary:

1) A *design space* is established (by selection of design variables and their ranges) and a corresponding set of simulations are performed based on the selected experimental design.

2) A *Pareto analysis is performed through a screening test* for each response to determine the relative importance of each design variable on the selected response.

3) Once a set of important variables is identified for each response, a new experimental design is selected following to *generate a relationship between the design variables and the responses in the form of Response Surface Equations (RSEs)*. Since these RSEs represent the analysis outputs in an *approximate* manner, validation is required, both to determine the quality of the regression (statistical verification) and the prediction accuracy.

The fact that all engineering analyses contains error, compounded with the fact that metamodels of such analyses will introduce additional inaccuracy, motivates the need for modeling and assessing fidelity uncertainty.

Fidelity Uncertainty and Probabilistic Sensitivities

Fidelity uncertainty arises when models are used to predict reality. This usually is due to limitations of computer analyses in accurately predicting physical phenomena. While methods of dealing with uncertainty in the aerospace disciplines are numerous (most notably in control system design, well established methods for dealing with parameter uncertainty in aircraft synthesis and design are only beginning to emerge.[1,4] *In the setting of vehicle design, mitigation of fidelity uncertainty introduces a tradeoff between desired accuracy in the model versus the cost to achieve this level of accuracy.* A notional depiction of this tradeoff is depicted in Figure 3.

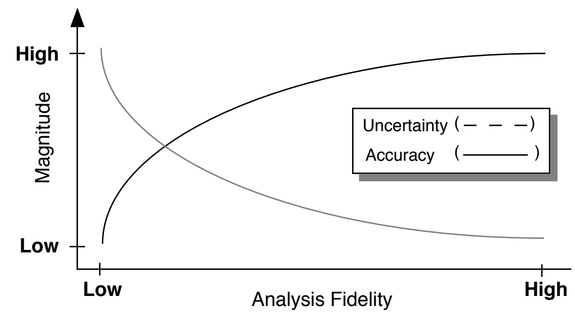

Figure 3: Continuum of Analysis Fidelity

It is constructive to expand on this concept of fidelity uncertainty through a specific area of its occurrence. A descriptive discretization of the continuum of analysis fidelity levels in the modeling of aerodynamics for performance and stability control purposes is given in Table I. Performance aerodynamics are defined as those quantities needed for vehicle sizing, such as drag polars. S&C aerodynamics are those quantities needed for static and dynamic stability analyses, such as stability and control force and moment derivatives. The fidelity of the analysis used can have wide ranging effects on the accuracy of the response including, but not limited to: flexibility effects on the static stability, aeroelastic effects on dynamic derivatives and responses, nonlinear aerodynamics associated with elevated angle of attack, stall, and other unsteady effects.

Modeling Fidelity Uncertainty

Fidelity uncertainty is best represented through a probability density function (PDF). However, choosing the appropriate density function for modeling fidelity

Table I: Analysis Continuum Discretization - Aerodynamics for Performance and S&C

Fidelity / Reqmt.	*Empirical (low fidelity)*	*Linear, Steady, Rigid (low-medium fidelity)*	*Non-Linear, Steady, Static Flexible (medium fidelity)*	*Non-Linear, Unsteady, Static and Dynamic Flexibility (high fidelity)*
Performance Aero	Drag Polars derived from similar aircraft	Drag Polars from linear aerodynamic analysis	Non-linear effects; Loads estimation	Aero for transonic, maneuver analysis, etc.
S&C Aero	Estimates of S&C derivatives via historical analogy	Dynamic model in the linear regime at moderate incidence; Assume instantaneous responses	High incidence non-linear corrections, static flexibility corrections to S&C derivatives	Estimates for highly flexible, highly maneuverable, high AOA aircraft; Transients modeled

uncertainty depends on the magnitude and the "structure" of the uncertainty. For example, the uncertainty structure may be unknown initially, but as knowledge about the phenomena under study and the underlying assumptions of the analyses used to predict the phenomena increase, more "structure" may be built into the PDF. This structure may take the form of a mean shift, added skewness, etc. A notional Gaussian PDF with minimal structure (a) and moderate structure (b) is shown in Figure 4.

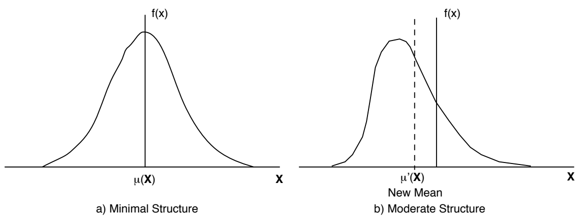

Figure 4: Fidelity Uncertainty PDFs

Many analyses codes used in conceptual design are conservative,[12] but this may change as higher fidelity tools are used along the design timeline. A notional example of the evolution of fidelity uncertainty models with time in predicting drag is shown in Figure 5.

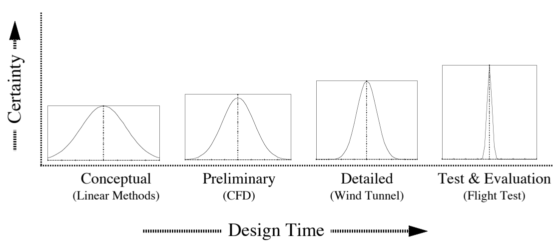

Figure 5: Notional Fidelity Evolution for Drag Prediction

A key aspect of design has always been sensitivity analysis. Whether for trade studies or for gradient calculations for optimization, the effect on an objective of an incremental change in a underlying variable is always useful information. In this paper, the concept of sensitivity analysis is extended to a probabilistic nature. As the structure of the fidelity uncertainty changes (i.e. the parameters defining the random variable change), it is important for the designer to know the impact of those changes. A metric for evaluating this impact is the probability sensitivity derivative (PSDs).[3]

Probabilistic sensitivities measure the change in the probability of a selected response with respect to the change in the parameter of a distribution (e.g. mean, variance). This is in contrast to a deterministic sensitivity, which measures the change in magnitude of the selected response with respect to a perturbation of a parameter. Both sensitivities are critical in design. However, the probabilistic sensitivities allow for the *assessment of the effect of changes in the fidelity uncertainty models* and the determination of whether the uncertainty models are overly conservative (or not conservative enough). If Z is an objective function, and if X is a set of design variables, and μ and σ are the mean and standard deviation of a PDF, then expressions for the two types of sensitivities are given in Eqs. (3) and (4).

$$\frac{\partial Z}{\partial X_i} \quad \text{(deterministic)} \tag{3}$$

$$\frac{\partial P\left(Z \le z_o\right)}{\partial\left(\mu(X_i)\right)},\ \frac{\partial P\left(Z \le z_o\right)}{\partial\left(\sigma(X_i)\right)} \quad \text{(probabilistic)} \tag{4}$$

The sensitivities also give an indication of the ranked importance of the design variables to the response mean and variability. The use of these sensitivities can be extremely important in directing a design organization as to *where to invest resources to improve analysis capability*. For example, if the response of interest is very highly sensitive to uncertainty due to flexibility of the aircraft structure, investments can be made to reduce this uncertainty through increased fidelity in the structural analysis

Now that the important tools and techniques for performing probabilistic analysis have been presented, the specifics of the application problem to be solved are covered next.

<u>Longitudinal Handling Qualities</u>

Handling Qualities (HQs) refers to the "goodness" of an aircraft's flying characteristics as perceived by the pilot and as represented through quantitative metrics. The longitudinal flying characteristics of the aircraft are considered key in determining the overall quality of the handling of the aircraft. This is due to the importance of pitch control, used as both a primary axis control and as glide path control as described in Mil-Standard 1797A (Ref. 13). Specifically, the short term pitch response is extremely useful to investigate. Ref. 13 recommends the use of metrics incorporating the relationship between the damping ratio of the short period oscillation (ζ_{sp}), the undamped natural frequency of the short period oscillation (ω_{sp}), and the change in steady state normal acceleration per unit change in angle of attack for an incremental pitch control deflection at constant airspeed and Mach number. The control anticipation parameter (CAP) metric is the recommended metric of choice for short term pitch response. The equation for CAP will be developed in the next section.

Results: HSCT Application

The HSCT is envisioned to be a next generation supersonic commercial transport, economically competitive with current subsonic transports while also being environmentally acceptable (in terms of noise and emissions). The unique shape, mission, aerodynamics, and variety of configurations (3 surface, canard-wing, wing-tail) of an HSCT limit the use of classical or historically-based estimates of static and dynamic airframe characteristics. The following example problem demonstrates the new method proposed here for investigating the impact of fidelity uncertainty on key HQ and stability metrics. This will be accomplished through the use of RSM, uncertainty models, and probabilistic analysis techniques.

Description of HSCT-specific S&C Derivative RSEs

An initial implementation of the Response Surface Methodology has been completed for four key longitudinal derivatives in Ref. 5. The four derivatives are listed in Table II.

Table II: Stability Derivative Examined in Ref. 5

Derivative	Expression
Pitch Stability	$C_{M\alpha}$
Pitch Damping	C_{Mq}
Elevator Control Power	$C_{M\delta e}$
Lift Curve Slope	$C_{L\alpha}$

The High Angle of Attack Stability and Control *(HASC95)* program is used to estimate the longitudinal forces, moments, and associated aerodynamic derivatives.[14] *HASC95* is comprised of a generalized vortex lattice method for force and moment calculations, a semi-empirical strake/wing vortex analysis, and a two dimensional, unsteady, separated flow analogy routine for analyzing smooth forebody shapes. The vortex and forebody analysis portions were not used in the generation of the results of this paper. The authors feel further validation work is required to understand how these modules perform within *HASC95*. The *HASC95* code is currently used at both NASA (Langley Research Center) and the U.S. Air Force Wright Laboratory (Flight Control Division) and is especially suited for unique configurations (such as an HSCT) or for investigations in flight regimes where non-linear behavior is expected. This program is the "simulation engine" used to generate the regression data for the model building exercise

The RSE generation process of Figure 2 was followed. A series of HSCT geometric design variables and ranges were identified. These variables and their ranges are shown in Figure 6. The center of gravity (CG) position is included as a variable because it is assumed that the CG can be manipulated independent of geometry via fuel management. The significance of CG as a variable becomes more pronounced if statically unstable designs are allowed, as a rearward CG minimizes trim drag, but also increases the control authority for stabilization.

The subset of important variables for each of the four responses were obtained via a linear screening test, as described earlier. With these subsets, a second order polynomial model of the form of Eq. (2) was used to form the four separate RSEs. Three of these metamodels are shown in Figure 7 in the form of prediction profiles. These profiles graphically show the relationships between the effects and response. Note that these relationships include linear effects, interactions between design variables, and second order effects. This particular "snapshot" of the design space happens to show linear relationships. If some of the design variables are moved away from their midpoints (as indicated by the vertical lines), curvature may result. For clarity of presentation, the variable ranges have been normalized to "-1" and "1".

The trends displayed by these metamodels agree with what one might expect. Since the positions of the wing and CG (variables XWING and CG) most affect the moment reference point, they show significant impact (as seen by their slope) on the pitching moment derivatives. Both the statistical accuracy (i.e. how well the RSE predicts points used in the regression) and predictive accuracy (i.e. how well the RSE predicts arbitrary points in the design space) of these RSEs were found to be quite good.[5] The regression coefficients for each response, expanded from the notation in Eq. (2), are listed in Appendix A.

Variable	Minimum (-1)	Mid-Point (0)	Maximum (1)	Remarks
X1	1.54	1.615	1.69	normalized by wing semi-span
Y1	0.44	0.51	0.58	normalized by wing semi-span
X2	2.10	2.23	2.36	normalized by wing semi-span
X3	2.40	2.49	2.58	normalized by wing semi-span
X4	2.19	2.275	2.36	normalized by wing semi-span
X5	2.19	2.345	2.50	normalized by wing semi-span
XWING	26%	28%	31%	wing position, % fuselage length
SW	8500	9000	9500	wing ref. area, square feet
XTAIL	82%	84.7%	87.4%	Tail position, % fuselage length
ST	875	922.5	970	Horiz. tail ref. area, square ft.
XHT1	0.95	1.18	1.20	normalized by HT semi-span
XHT3	1.90	2.00	2.10	normalized by HT semi-span
CG	56%	57.5%	59%	CG, %fuselage

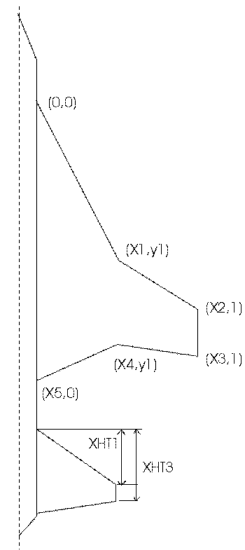

Figure 6: Wing and Tail Geometry Definition[5]

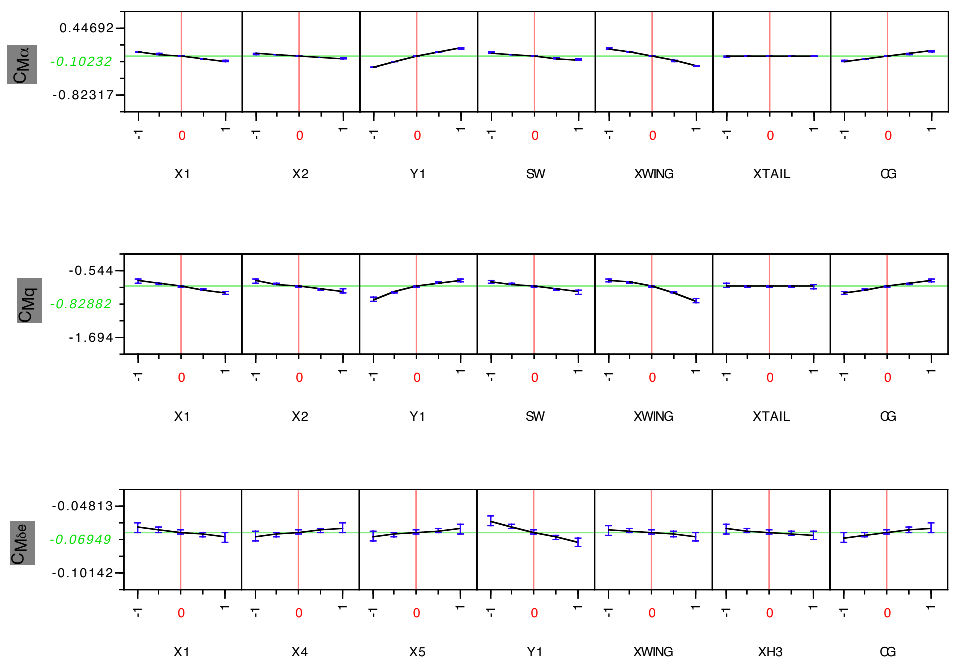

Figure 7: RSE - Prediction Profiles for $C_{M\alpha}$, $C_{L\alpha}$, C_{Mq} (Ref.[5])

At this point in the process, relationships have been established between the aircraft geometric design variables and the stability derivatives. These relationships, in the form of RSEs, are valid only within the select design space The next step is to use these models in calculating measures of the quality of the aircraft design, and the effect of uncertainty on such measures.

Defining Fidelity Uncertainty Models

In this paper, it is assumed that there is little information concerning *HASC95* to warrant adding structure to the uncertainty PDFs, except for the lift curve slope. The vortex lift, present in delta wings like the HSCT, was not modeled. Thus, the lift curve slope prediction and thus its PDF will likely be skewed to a lower value. The fidelity uncertainty in the other derivative predictions are represented by normal distributions (see Table III).

Table III: Fidelity Uncertainty Models

Derivative	Probability Density Function
$C_{M\alpha}$	Normal (μ = 0, σ=.1)
C_{Mq}	Normal (μ = 0, σ=.1)
$C_{M\delta e}$	Normal (μ = 0, σ=.1)
$C_{L\alpha}$	Weibull (loc.=-0.3, scale = 0.3, slope = 2)

Thus, for each analysis used in generating a CDF, the derivative values used to construct the dynamic equations are equal to the RSE prediction (e.g. $(\bullet)_{nom}$) plus the fidelity uncertainty (i.e. $(\bullet)_{unc}$), as exemplified in Eq. (5) for $C_{M\alpha}$:

$$C_{M\alpha} = \left(C_{M\alpha}\right)_{nom} + \left(C_{M\alpha}\right)_{unc} \tag{5}$$

<u>Computing CAP and the "Distance to Level 1"</u>

Using the metamodels for the stability derivatives and the conditions described in Table IV, a two state dynamic model is constructed using the standard short period mode approximation to compute the damping ratio (ζ_{sp}) and natural frequency (ω_{nsp}). The well known relationships are displayed in Eqs. (6-14). In the equations, M is pitching moment, Z is the force along the body-fixed z-axis, and L is the lift force. If a full order dynamic model is desired, then metamodels for the appropriate additional derivatives would be required. Depending on the aircraft under study, and the flight conditions of interest, this may be advisable. Further, such additional information would be yet another way to shrink the magnitude of the fidelity uncertainty.

Table IV: Flight Condition & HSCT Parameters

Parameter	Value
Mach (M)	0.3
Altitude (h)	5000 ft
Velocity (u_o)	329.08 ft/s
Dynamic Pressure (q)	110.92 slug/(ft•s^2)
Wing Area (S)	*variable*
Mean Aero. Chord (c)	*variable*
Weight (W)	428,000 lbs.
Mass (m)	13,292 slugs

$$M_\alpha = u_0 M_w = \frac{(C_{M_\alpha})qS\bar{c}}{I_{yy}} \tag{6}$$

$$M_q = \frac{(C_{M_q})\bar{c}^2 qS}{2u_0 I_{yy}} \tag{7}$$

$$M_{\delta e} = (C_{M\delta e})qS\bar{c} / I_{yy} \tag{8}$$

$$Z_\alpha = u_0 Z_w = \frac{-(C_{L_\alpha})qS}{mu_0} \tag{9}$$

$$Z_{\delta e} = -(C_{L\delta e})qS / m \tag{10}$$

$$\omega_{n_{sp}} = \sqrt{M_q \frac{Z_\alpha}{u_0} - M_\alpha} \tag{11}$$

$$\xi_{sp} = \frac{-(M_q + \frac{Z\alpha}{u_0})}{2\omega_{n_{sp}}} \tag{12}$$

$$(n/\alpha) = \frac{u_0[-Z_w + (Z_{\delta e} - M_w / M_{\delta e})]}{g} \tag{13}$$

$$CAP = \frac{\omega_{n_{sp}}^2}{(n_z/\alpha)} \tag{14}$$

Ref. 13 defines longitudinal handling qualities requirements based on CAP and ζ_{sp}. These requirements for a Class III aircraft (large, heavy transport) in a Category C (terminal) flight phase are represented by Figure 8. Overlayed on this figure are the 79 design alternatives in the design space which were used to form the stability derivative RSEs described above. In the short period approximation, the damping is inversely proportional to the natural frequency, ω_{nsp}. As ω_{nsp} (and thus CAP) increases, damping decreases. This is illustrated in the trends in Figure 8 (where the inverse relationship becomes a linear one in the log-log axis system). Only stable designs are shown (since the unstable designs have a divergent root, and thus negative damping).

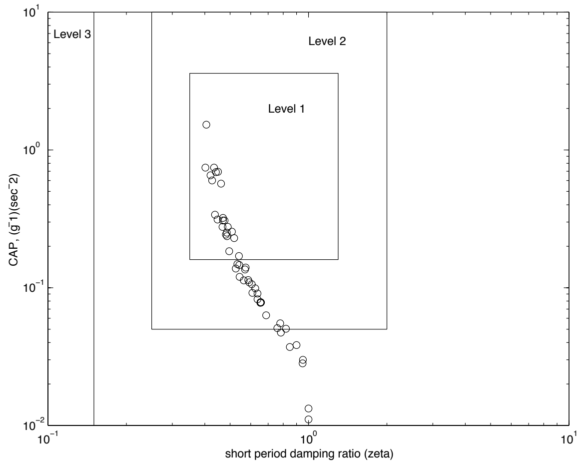

Figure 8: Loglog CAP plot, with each 'O' denoting individual (stable) aircraft

Figure 9 displays the same data points as Figure 8, but in a Cartesian coordinate plot. The fact that the data points are confined to a relatively small area indicates that the design space under consideration contains HSCT alternatives which have relatively low CAP and ζ_{sp} values. Delta wing / arrow wing transports like the HSCT are known to have mediocre to poor longitudinal, open loop stability and HQ characteristics; thus, few aircraft in the space would be expected in Level 1 designs. In any case, the goal in this paper is not to stabilize and improve the HQs, but to demonstrate how one constructs, examines, and interprets probabilistic representations of aircraft handling qualities.

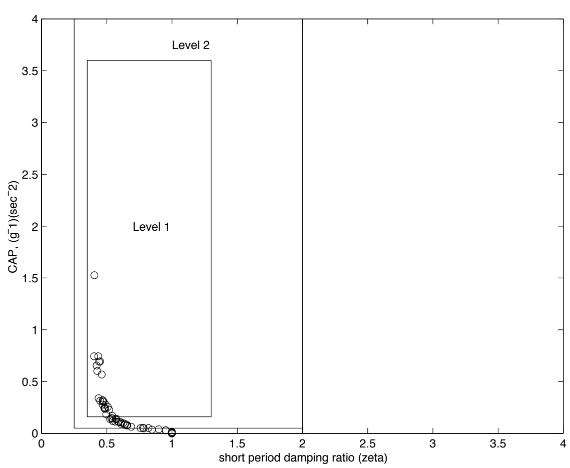

Figure 9: Cartesian CAP plot, with each 'O' denoting individual (stable) aircraft

The HQ levels, as spelled out in Mil-Std. 1797, are a function of the CAP and the ζ_{sp} (see Figure 8). Thus, the results of an uncertainty investigation would result in a joint probability distribution, with both CAP and ζ_{sp} as random variables. One way to interpret this resulting distribution is to define a continuous random variable which is a function of both CAP and ζ_{sp}. Such a random variable is defined as follows:.

$$\mathbf{Z} = \textit{Distance from } (CAP, \zeta_{sp}) \textit{ to Level 1 Boundary}$$

Calculation of **Z** is illustrated in Figure 10. The rectangle defining Level 1 serves as a boundary, which is then used to measure distances from points in the design space to Level 1. Values less than one (i.e. inside the boundary) are negative, while values outside the boundary are positive. Values outside the boundary are computed by using a region discretization (as shown in Figure 10) to determine the minimum distance from the point to the boundary.

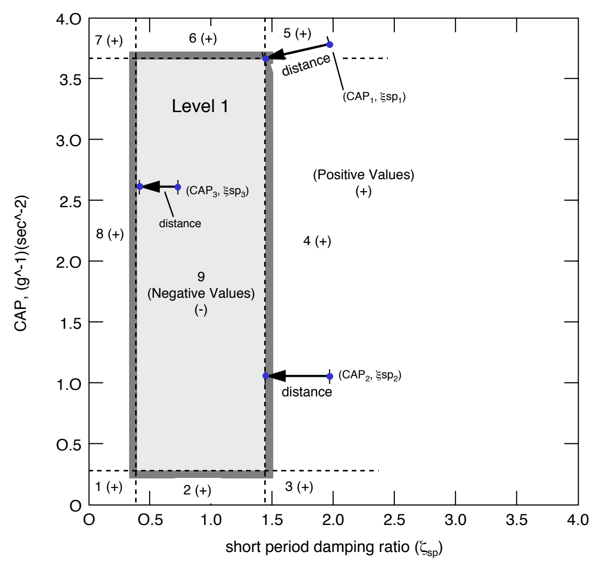

Figure 10 : Computation of Random Variable Z - "Distance to Level 1 Boundary"

Computing CDFs and Sensitivities

The variability of **Z** can result from two types of investigations. One type, called a *design space feasibility search*, assigns uniform distributions to all the design parameters, and thenconstructs a CDF for **Z** by sampling many times over entire design space. The resulting CDF is used to determine the probability of meeting Level 1 requirements.

A design space feasibility search was carried out for the HSCT using the seven design variables contained in the RSEs. The uniform distribution used ranges from -1 to 1 and is assigned to each of the seven elementary variables contained in the RSEs. This distribution is illustrated in Figure 11. Recall that -1 and 1 are used since the RSEs were formed using normalized values.

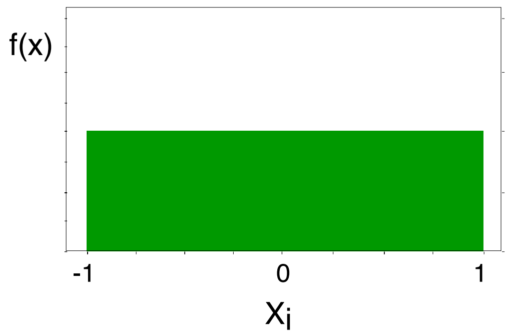

Figure 11 : Uniform Distribution for Design Variables

The efficiency of evaluating the RSEs, using the short period approximations (Eqs. (6-14), and computing the "Distance to Level 1" random variable **Z**, make the use of a standard Monte Carlo simulation viable, and so it is used here to generate the CDFs. However, if the underlying analyses grows in complexity, the advantages offered by the FPI technique will be needed. Issues related to the advantages/disadvantages of Monte Carlo and FPI are discussed at length in Ref. [15].

Figure 12 displays the computed CDF for **Z**. Naturally, the constraint line indicating the boundary lies at zero on the CDF. Thus, in reading the figure, one can see that there is about a 0.27 probability that the a vehicle from the design space will achieve Level 1 response. The sharp rise in the CDF at about P=0.8 represent the statically unstable designs, showing that about 20% of the design space consists of unstable bare airframes. This is important information, since augmentation design must address both HQ and stability. The unstable designs were all arbitrarily assigned distance values of 1.45, to emphasize and make this point clear on the CDF.

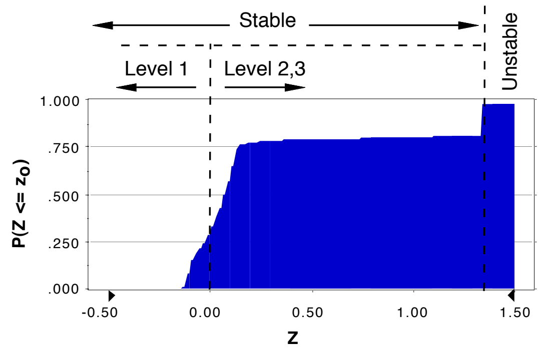

Figure 12 : CDF for Z: - "Distance to Level 1" Design Space Feasibility Search

Figure 13 displays the probabilistic sensitivities for this analysis, evaluated at the mean value on the CDF. Since there is no uncertainty here, these sensitivities should correspond to the "deterministic" sensitivities displayed in Figure 7, and Figure 13 verifies this result. The three most significant variables are once again Y1, XWING, and CG. The fact that the movement of the leading edge wing kink location is of slightly greater importance than the CG and wing position is an interesting result.

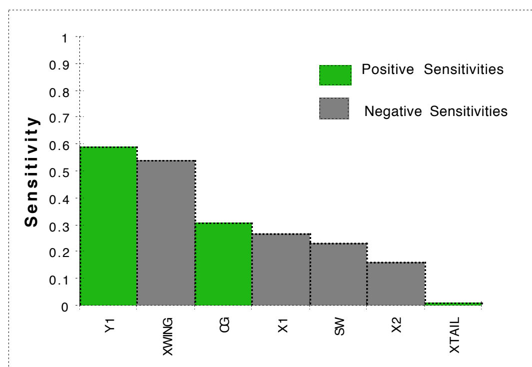

Figure 13: Sensitivities for Design Space Feasibility Search

The second type of analysis incorporates the fidelity uncertainty models introduced earlier, and thus termed *Uncertainty Assessment*. As Table III indicates, normal distributions are assigned to represent the variability due to inaccuracies in predicting $C_{M\alpha}$, $C_{L\alpha}$, and C_{Mq}. A Weibull distribution is used to model the lift curve slope uncertainty, as shown in Figure 14. The skewness indicates the expectation that the lift will be underpredicted. Figure 15 contains the CDF, where the variability is now due to the fidelity uncertainty in the stability derivatives *in addition to* the uniform distributions on the design parameters.

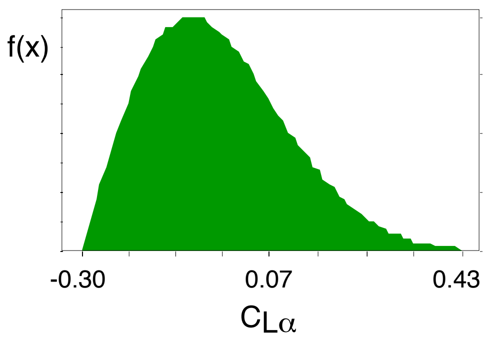

Figure 14 : Weibull Distribution for Lift Curve Slope Uncertainty

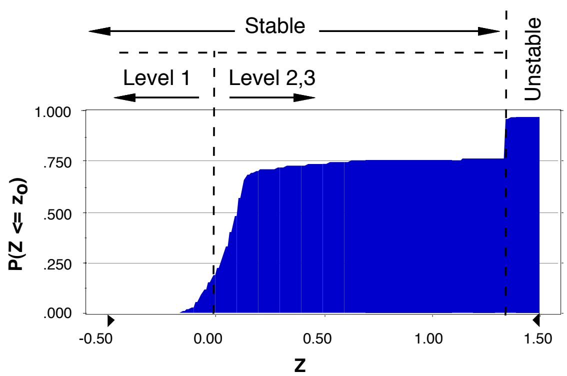

Figure 15 : CDF for Z: "Distance to Level 1" Design Space Search w/ Uncertainty Assessment

The sensitivities in this case contain a mix of design variable and uncertainty terms(Figure 16). Once again the Y1 and XWING parameters are most important in influencing the probability of achieving Level 1 HQs. However, the uncertainty in $C_{M\alpha}$ is the third greatest contributor. This indicates that an investment in reducing this uncertainty (or learning more about its structure) may significantly improve the estimate of **Z**.

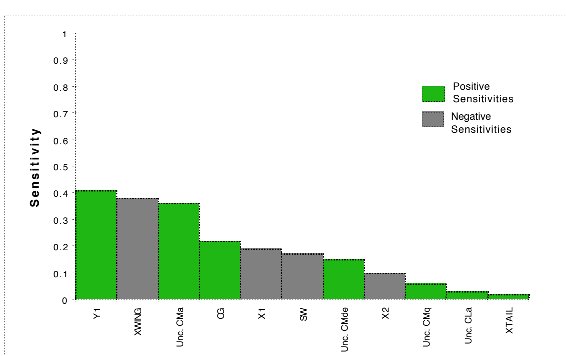

Figure 16: Sensitivities for Design Space Search with Uncertainty Assessment

Probabilistic representations are useful for evaluating the effects of uncertainty in isolation. The CDF for **Z** in Figure 17 is a result of analyzing the impact of the uncertainty of Table III on an *individual aircraft*.. The particular aircraft chosen for this example had values of 0.334 for CAP and 0.456 for ζ_{sp}, thus it is nominally Level 1. However, as the CDF shows, the probability of achieving Level 1 under fidelity uncertainty is only about 0.37. This information is extremely valuable in addressing risk in making decisions . Further, this very information is used to evaluate *system robustness*. In a robust design setting, the search is focused on finding regions of the design space with maximum probability of achieving an objective target, instead of maximizing the deterministic value of the objective. Figure 18 indicates that the uncertainty in the elevator effectiveness, in this case, has the highest contribution to the probability of **Z**.

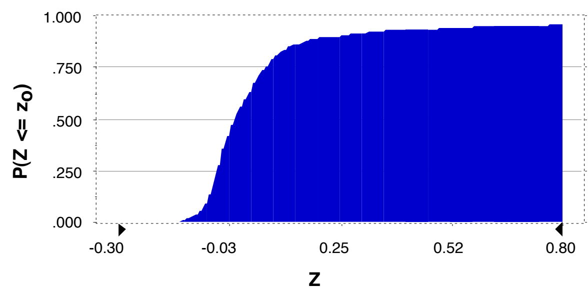

Figure 17 : CDF for Z: "Distance to Level 1" Uncertainty Assessment Only for a Nominally Level 1 Aircraft

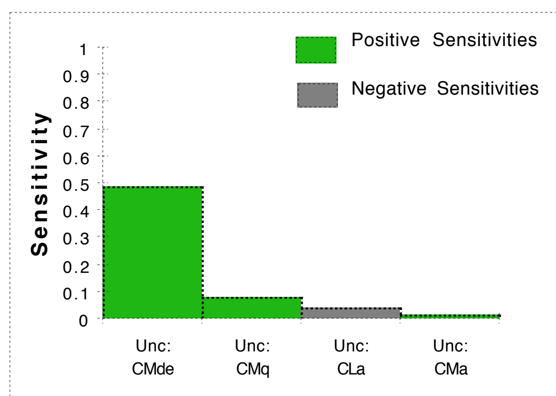

Figure 18: Sensitivities for Uncertainty Assessment

Future Work & Design Implications

The techniques presented in this paper are aimed at the ultimate goal of providing the capability for the designer to choose configuration design parameters accounting for multiple disciplines and for fidelity uncertainty, as depicted in Figure 1. Such a capability allows the search for regions of the design space which both maximize some objective and are robust to modeled uncertainty. For aircraft, this objective is likely to be multi-attribute, including weight, payload, range, closed loop stability and HQs, etc. In the specific area of stability and control, the authors are currently introducing control law design into the probabilistic setting presented in this paper. The purpose of doing so is NOT to design operational flight control laws, but instead to include closed loop design variables in the configuration designer's toolbox in his/her attempt to size, synthesize, and design robust, affordable aircraft systems.

Summary

A method has been developed and was described in this paper which allows for the viewing of longitudinal handling qualities metrics in a probabilistic fashion. The motivation for the probabilistic approach is the recognition that fidelity uncertainty exists whenever an analysis code (or a metamodel of the code) is used to predict physical phenomena. Cumulative probability distributions are employed to examine the probabilities of meeting objectives and constraints, while associated probabilistic sensitivities indicate to the designer which uncertain parameters have the largest impact on the probability.

These techniques were demonstrated by investigating a multidimensional High Speed Civil Transport design space, utilizing metamodels for four key longitudinal stability derivatives and the (approximate) short period equations of motion. The impact of fidelity uncertainty in these derivative models was evaluated by performing a series of Monte Carlo simulations. For the design space chosen, the probability of a particular aircraft in the space having Level 1 handling qualities (as defined through the control anticipation parameter) was estimated to be 0.27. When fidelity uncertainty in the derivatives is included, this probability value drops to about 0.20. The most important parameters, identified through the probabilistic sensitivities, were the wing leading edge kink location, the wing position relative to the fuselage, the uncertainty in predicting the change in pitching moment with respect to change in angle of attack. The example successfully illustrated the power and utility of the probabilistic handling qualities assessment approach.

Acknowledgments

The authors acknowledge the NASA Multidisciplinary Analysis Fellowship Program from the NASA Langley Systems Analysis Branch and the Office of Naval Research for supporting this research.

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Appendix A

Table of Regression Coefficients (Normalized)

Term	$C_{M\alpha}$	$C_{L\alpha}$	C_{Mq}
Intercept (bo)	-0.102324	2.1635319	-0.828823
X1	-0.087836	0.0346159	-0.103437
X2	-0.046123	-0.007105	-0.085522
Y1	0.1802845	-0.081305	0.1641586
SW	-0.066586	0.0077161	-0.090446
XWING	-0.163529	0.0090812	-0.174871
XTAIL	0.0014755	-0.012605	-0.009362
CG	0.097082	0.0003768	0.1079174
X1*X1	-0.001515	0.0008821	-0.008025
X2*X1	-0.001706	-0.002302	-0.012356
X2*X2	0.008036	-0.010518	0.0089753
Y1*X1	0.0144531	-0.006935	0.0379664
Y1*X2	0.0155443	-0.004854	0.0389911
Y1*Y1	-0.023482	0.0028321	-0.055525
SW*X1	-0.003292	0.0011383	-0.014404
SW*X2	-0.000801	0.0000199	-0.009116
SW*Y1	0.0072215	-0.006501	0.0325386
SW*SW	0.0003956	-0.001918	-0.008025
XWING*X1	-0.004127	0.0035383	-0.028435
XWING*X2	-0.000085	0.0024012	-0.017085
XWING*Y1	0.0129516	-0.005782	0.0698198
XWING*SW	-0.003939	0.0023728	-0.03057
XWING*XWING	-0.012976	-0.010568	-0.068025
XTAIL*X1	0.0001878	0.0001679	0.000992
XTAIL*X2	-0.000033	0.0006988	-0.000733
XTAIL*Y1	-0.000776	-0.000662	-0.005877
XTAIL*SW	0.0002385	0.0015522	-0.000561
XTAIL*XWING	0.0005972	0.0016772	0.0017204
XTAIL*XTAIL	0.0042142	0.0028321	0.0114753
CG*X1	0.0014666	0.0001324	0.0156708
CG*X2	-0.000546	0.0001883	0.0113205
CG*Y1	-0.003606	-0.000745	-0.031493
CG*SW	0.0003229	0.0006917	0.014743
CG*XWING	0.0004423	0.0008042	0.0286492
CG*XTAIL	-0.000441	-0.001767	-0.001394
CG*CG	0.00326	0.0066821	0.0019753