Dispersive Wave Analysis using the Chirplet Transform

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Dispersive Wave Analysis using the Chirplet Transform

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SUMMARY

Time-frequency representations (TFR) are a widely used tool to analyze signals of guided waves such as Lamb waves. As a consequence of the uncertainty principle, however, the resolution in time and frequency is limited for all existing TFR methods. Due to the multi-modal and dispersive character of Lamb waves, displacement or energy related quantities can only be allocated to individual modes when they are separated in the time-frequency plane.

The chirplet transform has been introduced as a generalization of both the wavelet and Short-time Fourier transform. It offers additional degrees of freedom to adjust time-frequency atoms which can be exploited in a model-based approach to match the group delay of individual modes.

The objective of the current thesis is to apply the algorithm proposed by Kuttig [18] to a series of candidate nondestructive evaluation problems. The accuracy and robustness of the CT based procedure is examined for each of these example problems and is benchmarked against analytical solutions (if available) and to the conventional STFT.

CHAPTER I

INTRODUCTION

Ultrasonic waves are often used in nondestructive evaluation to evaluate the integrity of structural components or to determine material properties. Chimenti [5] gives an overview of potential applications for multi-mode, dispersive guided waves such as Lamb waves.

To analyze dispersive wave signals advanced signal processing methods are required. Time-frequency representations (TFRs) can identify time-varying spectral components in these signals. Both the short-time Fourier transform (STFT) and the wavelet transform (WT) have been used effectively in the context of Lamb waves, see Niethammer [24] and Hurlebaus et al. [13, 14].

The complexity of the underlying physics of dispersive and multi-modal wave signals, however, can sometimes cause problems: the allocation of displacement amplitudes or energy related quantities to individual modes is not always accurate, especially when modes are not well separated in the time-frequency plane. Furthermore, difficulties arise in regions where a particular mode is highly dispersive. For these regions, time-frequency atoms should be adjustable.

The chirplet transform (CT) was introduced in 1991 by Mann et. al. [22] as a generalized TFR. The resulting transform space is of dimension five and includes the STFT and the WT as a subspace. Additional degrees of freedom stem from time and frequency shear operators shaping the group delay and instantaneous frequency of the time-frequency atoms by means of a chirp term. The subspaces of the CT transform space have been studied in more detail by Mann et. al. [23] and Baraniuk et.al. [2],

applications included radar signals [26] and matching pursuit algorithms [9].

Kuttig in [17] and [18] developed an adaptive algorithm based on the chirplet transform for the use in dispersive Lamb wave analysis. Based on a given dispersion relation, this knowledge is exploited to adjust the time-frequency atoms using the time shear operator. Furthermore, the algorithm determines regions for each individual mode for which the CT can be computed without interference with other modes.

The objective of the current thesis is to apply the algorithm proposed by Kuttig [18] to a series of candidate nondestructive evaluation problems. These candidate example problems include both geometric and material attenuation, leaky lamb waves, nonlinear Lamb waves and the analysis of adhesive bonded components. The accuracy and robustness of the CT based procedure is examined for each of these example problems and is benchmarked against analytical solutions (if available) and to the conventional STFT.

In the beginning, the basic concepts of wave propagation in elastic media are outlined focusing on Lamb wave propagation in plates. The interpretation of the dispersion curves in the time-frequency plane as group delay of the time signal becomes important in the implementation of the chirplet transform. Chapter 3 introduces the necessary signal processing prerequisites and describes the utilized chirplet algorithm. Applications are discussed in Chapter 4 the range of which comprises mode displacement ratios and attenuation in plates, adhesive bond properties for a two layer specimen and the determination of material nonlinearities. The thesis concludes with a summary of the results and suggestions for further research.

CHAPTER II

WAVE PROPAGATION IN ELASTIC SOLIDS

This chapter gives a brief introduction to the subject of wave propagation in elastic solids. It is intended to provide information about the phenomena observed in plate-like structures. A more detailed and broader description can be found in the literature, the interested reader is especially referred to the work of Achenbach [1], Graff [10] and Rose [25].

2.1 Linear Elasticity and Equations of Motion

First, the fundamental equations in linear elasticity should be repeated. The media studied in this research is considered to be homogeneous, isotropic and linear elastic. The constitutive equation, Hooke's law, is therefore given by

$$\sigma_{ij} = \lambda \epsilon_{kk} \delta_{ij} + 2\mu \epsilon_{ij} . \quad (2.1)$$

where ϵ_{ij} is the strain tensor and μ and λ are the Lamé constants.

The strain tensor is related to the displacement u_i by

$$\epsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) . \quad (2.2)$$

Cauchy's formula gives an expression for the traction t_i on the surface of a body

$$t_i = \sigma_{ij} n_j , \quad (2.3)$$

where σ_{ij} is the stress tensor.

The symmetry of the stress tensor,

$$\sigma_{ij} = \sigma_{ji} , \quad (2.4)$$

can be derived from the balance of angular momentum applying Reynold's transport theorem.

The balance of linear momentum for a body with volume V and surface S can be formulated as

$$\int_S \sigma_{ij} n_j dS + \int_V \rho f_i dV = \int_V \rho \ddot{u}_i dV, \quad (2.5)$$

with ρ representing the material mass density and f_i representing the body force. Applying Gauss's theorem to (2.5) leads to

$$\int_V (\sigma_{ij,j} + \rho f_i - \rho \ddot{u}_i) dV = 0. \quad (2.6)$$

As equation (2.6) has to be fulfilled for any body volume V , the stress equation of motion becomes

$$\sigma_{ij,j} + \rho f_i = \rho \ddot{u}_i. \quad (2.7)$$

Equation (2.7) is written in terms of both displacements u_i and stresses σ_{ij} . To eliminate one variable in the balance of linear momentum, plug Hooke's law into the equation of motion. Neglecting external body forces f_i , Navier's equations of motion therefore becomes

$$\mu u_{i,jj} + (\lambda + \mu) u_{j,ji} = \rho \ddot{u}_i \quad (2.8a)$$

or, in vector representation,

$$\mu \nabla^2 \mathbf{u} + (\lambda + \mu) \nabla \nabla \cdot \mathbf{u} = \rho \ddot{\mathbf{u}}. \quad (2.8b)$$

The resulting partial differential equations, (2.8), cannot be solved explicitly. Assuming a specific structure for the displacement field, however, Navier's equations of motions can be uncoupled. The Helmholtz decomposition

$$\mathbf{u} = \nabla \varphi + \nabla \times \boldsymbol{\psi}, \quad (2.9)$$

expresses the displacement $\mathbf{u}$ as the sum of the scalar potential φ and the gradient of the vector field $\boldsymbol{\psi}$. The uniqueness of this solution is guaranteed by imposing an additional constraint

$$\nabla \cdot \boldsymbol{\psi} = 0 . \quad (2.10)$$

Plugging (2.9) into the displacement equation of motion (2.8) the two uncoupled wave equations in terms of the displacement potentials φ and $\boldsymbol{\psi}$ are given by

$$\nabla^2 \varphi = \frac{1}{c_L^2} \ddot{\varphi} , \quad \nabla^2 \boldsymbol{\psi} = \frac{1}{c_T^2} \ddot{\boldsymbol{\psi}} \quad (2.11)$$

Hereby, c_L and c_T represent the wave speed of longitudinal and transverse shear waves, respectively,

$$c_L^2 = \frac{\lambda + 2\mu}{\rho} , \quad c_T^2 = \frac{\mu}{\rho} . \quad (2.12)$$

Note that there are only two independent elastic constants. The relationship between Lamé constants μ and λ and the mass density ρ on the one hand and Young's modulus E and Poisson's ratio ν on the other hand is given by

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} , \quad (2.13)$$

$$\mu = \frac{E}{2(1+\nu)} . \quad (2.14)$$

2.2 *Plane Waves*

Looking at the 2 dimensional case (Figure 2.1), the complexity of wave phenomena is reduced. Thus, the displacement field of a plane wave – a wave whose properties such as ϵ , σ , $\mathbf{u}$ are independent with respect to the plane perpendicular to its direction of propagation $\mathbf{p}$ – can be formulated as

$$\mathbf{u} = \mathbf{d}f(\mathbf{x} \cdot \mathbf{p} - ct) , \quad (2.15)$$

Figure 2.1: 2-d wave propagation for a semi-infinite medium.

where $\mathbf{d}$ represents the direction of particle movement in form of a unit vector and c stands for the wave speed. Substituting (2.15) into (2.8), the resultant expression

$$(\mu - \rho c^2)\mathbf{d} + (\lambda + \mu)(\mathbf{p} \cdot \mathbf{d})\mathbf{p} = 0 \quad (2.16)$$

can be fulfilled by two different types of waves:

Longitudinal wave: If the particle motion is parallel to the direction of propagation,

$\mathbf{d} = \pm\mathbf{p}$, equation (2.16) yields the longitudinal wave speed $c = c_L$ as defined in (2.12).

Transverse wave: Consider the case that the particle motion is perpendicular to the direction of propagation, i.e. $\mathbf{p} \cdot \mathbf{d} = 0$. Equation (2.16) is then solved for $c = c_T$ as in (2.12). Depending on whether the particle moves in an in-plane ((x_1, x_2) -plane) or out-of plane ((x_2, x_3) -plane) direction, the waves are called SV (vertically polarized transverse/shear wave) and SH (horizontally polarized) waves, respectively.

As is shown in [1], $c_L > c_T$ always holds. In a homogeneous, isotropic material, however, both longitudinal and transverse waves are non-dispersive.

2.3 Reflections of P and SV-Waves

Reflections generally occur when the medium has finite dimension in the direction of propagation so that the incident wave will ultimately hit a boundary. Depending on the type of the incident wave, different cases have to be considered as depicted in Figure 2.2. In any case, however, the possibility of mode conversion exists, i.e. the phenomenon that an incident wave produces reflected waves of a different type.

Figure 2.2: Reflection interfaces. (a) Reflection of a P-wave. (b) Reflection of an SV-wave.

Studying the interface for time-harmonic excitation, the displacement field for an incident plane wave is given by

$$\mathbf{u}^{(n)} = A_n \mathbf{d}^{(n)} \exp \left[i k_n (x_1 p_1^{(n)} + x_2 p_2^{(n)} - c_n t) \right], \quad (2.17)$$

where n denotes the respective wave ($0 \hat{=}$ incident wave, $1 \hat{=}$ reflected P-wave, $2 \hat{=}$ reflected SV-wave), $k_n = \frac{\omega}{c_n}$ is the wavenumber and c_n is the speed of the respective wave. The boundary condition for a stress-free interface at $x_2 = 0$ yields

$$\sigma_{21} = 0, \quad \sigma_{22} = 0 \quad \text{and} \quad \sigma_{23} = 0. \quad (2.18)$$

Normally, both an incident SV- and a P-wave can cause both reflected P- and SV-waves each. θ_0 as the incident angle and the angles of the reflected waves, θ_1 and θ_2 , are related by Snell's law,

$$k_0 \sin \theta_0 = k_1 \sin \theta_1 = k_2 \sin \theta_2 . \quad (2.19)$$

For the amplitudes A_n , ratios relative to the amplitude of the incident wave can be determined as

$$\begin{aligned} \frac{A_1}{A_0} &= \frac{\sin 2\theta_0 \sin 2\theta_2 - \kappa^2 \cos 2\theta_2}{\sin 2\theta_0 \sin 2\theta_2 + \kappa^2 \cos 2\theta_2} \\ \frac{A_2}{A_0} &= \frac{2\kappa \sin 2\theta_0 \sin 2\theta_2}{\sin 2\theta_0 \sin 2\theta_2 + \kappa^2 \cos 2\theta_2} \end{aligned} \quad (2.20)$$

for an incident P-wave and as

$$\begin{aligned} \frac{A_1}{A_0} &= -\frac{\kappa \sin 4\theta_0}{\sin 2\theta_0 \sin 2\theta_1 + \kappa^2 \cos 2\theta_0} \\ \frac{A_2}{A_0} &= \frac{\sin 2\theta_0 \sin 2\theta_1 - \kappa^2 \cos 2\theta_0}{\sin 2\theta_0 \sin 2\theta_1 + \kappa^2 \cos 2\theta_0} \end{aligned} \quad (2.21)$$

for an incident SV-wave, where $\kappa = \frac{c_L}{c_T}$. Investigation of (2.20) and (2.21) also reveals cases for which no mode conversion occurs, i. e. for $\theta_0 = 0$.

2.4 Guided Waves

Let's consider a body with a cross-section of finite dimension as in Figure 2.3. A wave traveling in such a waveguide will be reflected multiple times from the boundaries. With mode conversion occurring at both interfaces, the sum of incident and reflected time-harmonic waves forms a standing wave across the thickness of the waveguide.

To solve for the displacement field in an elastic waveguide, potential functions are usually chosen. According to the coordinate system in Figure 2.3, the waves are propagating in the x_1 -direction leading to the decomposition

$$\varphi = \Phi(x_2) \exp [i(kx_1 - \omega t)] , \quad \psi_3 = \Psi(x_2) \exp [i(kx_1 - \omega t)] . \quad (2.22)$$

Figure 2.3: Waveguide.

Following the derivation in Achenbach [1] for stress-free boundaries at $x_2 = \pm h$ and plane strain motion in the (x_1, x_2) -plane, the Rayleigh-Lamb frequency equations for a plate-like structure are given by

$$\frac{\tan(qh)}{\tan(ph)} = -\frac{4k^2pq}{(q^2 - k^2)^2} \quad (2.23a)$$

$$\frac{\tan(qh)}{\tan(ph)} = -\frac{(q^2 - k^2)^2}{4k^2pq}, \quad (2.23b)$$

where

$$p^2 = \frac{\omega^2}{c_L^2} - k^2, \quad q^2 = \frac{\omega^2}{c_T^2} - k^2 \quad (2.24)$$

and $2h$ is the plate thickness. Equations (2.23a) and (2.23b) characterize the symmetric and antisymmetric Lamb modes, respectively. Thereby, symmetry is defined with respect to the direction of propagation, the x_1 -axis. Each Lamb mode describes the amplitude distribution over the plate thickness oscillating with an angular frequency ω . The corresponding phase velocity $c = \frac{\omega}{k}$ is given by the solution of the Rayleigh-Lamb spectrum for a specific frequency. Note, however, that solutions for these equations (2.23a) and (2.23b) can only be found numerically. Furthermore, the dispersive nature of Lamb waves implies that the propagation velocity of a Lamb mode depends upon its oscillation frequency. In the following, symmetric modes should be labeled $s_0, s_1, \dots$ in ascending order as to lowest angular frequency ω , whereas antisymmetric modes are called $a_0, a_1, \dots$.

Having solved the Rayleigh-Lamb equations numerically, differentiation of $f = \frac{\omega}{2\pi}$

Figure 2.4: Theoretical solution of Raleigh-Lamb equation (2.23) in slowness-frequency domain. s_i denote symmetric, a_i denote antisymmetric Lamb modes.

with respect to k for each individual mode gives group velocity c_g ,

$$c_g(f) = 2\pi \frac{\partial f}{\partial k} . \quad (2.25)$$

The physical meaning of c_g is the velocity of the energy transport, as opposed to the phase velocity c_p which quantifies the velocity of energy packets with equal phase. Only for dispersive waves, however, are group and phase velocities different. Often, the inverse of the group velocity, the energy slowness sl_e , is used to represent the dispersion relation,

$$sl_e(f) = \frac{1}{c_g(f)} . \quad (2.26)$$

In Figure 2.4, the dispersion curves for a 1mm thick aluminum plate are plotted in the slowness-frequency domain.

To compare the theoretical solution with experimental or simulation results, slowness as a function of frequency has to be expressed in terms of the propagation time t ,

$$sl_e(f) = \frac{t(f)}{d} , \quad (2.27)$$

Figure 2.5: Point source and receiver set-up for wave propagation in plates.

where d represents the distance between source and receiver as depicted in Figure 2.5. The frequency components $f(t)$ that are present in a receiver signal at a particular instant of time – called instantaneous frequency – can be determined applying signal processing techniques. Its inverse $t(f)$ indicates the arrival time of a specific frequency component at the receiver point and is therefore referred to as the group delay of the signal. The mathematical definition of both instantaneous frequency and group delay can be found in Cohen [6].

The influence of the excitation source is also of great importance. Even if the same type of excitation is used – in this work point or point-like sources – the amplitude of the displacement field depends upon the amplitude of the excitation force. Comparing data produced with different excitation forces, the results have to be normalized, e.g. to a reference mode, to eliminate the source effect.

2.5 Attenuation

Attenuation is a characteristic property of lossy materials. It quantifies the decay of a component, in this case out-of-plane displacement, over propagation distance in dimensions of neper/length. However, both material and geometric effects contribute to attenuation. Therefore, these two distinct factors of influence have to be considered separately.

Figure 2.6: Aluminum plate coupled to a water half-space.

2.5.1 Geometric spreading

Consider a single plate as in Figure 2.5 with infinite extension and point-like source and receiver. The amplitude of the out-of plane displacement diminishes for a wave traveling over the distance d . For two different source-receiver distances $d_1 > d_2$, the corresponding time signals obey the relation

$$\frac{u_1(x, t)}{u_2(x, t)} = \sqrt{\frac{d_2}{d_1}}, \quad (2.28)$$

which describes cylindrical geometric spreading for a surface or Lamb wave.

2.5.2 Attenuation in Absorbing Materials

Now consider the example of a plate coupled to a water half-space as depicted in Figure 2.6. Lamb waves traveling in the plate will experience attenuation due to energy leakage into the water half-space. Therefore, they are often called leaky Lamb waves.

The out-of displacement field is described by

$$u_i(x, t) = \frac{A_i}{\sqrt{d_i}} e^{-\alpha_i d_i} e^{i(kx - \omega t)}, \quad (2.29)$$

with k and ω solving the Rayleigh-Lamb equations. For two given propagation distances d_1 and d_2 , solving for the attenuation coefficient α yields

$$\alpha(x, t) = \frac{1}{d_2 - d_1} \ln \left(\frac{|u_1(x, t)| \sqrt{d_1}}{|u_2(x, t)| \sqrt{d_2}} \right). \quad (2.30)$$

Thereby, α depends not only on the individual mode, but also on time/frequency and propagation distance.

CHAPTER III

SIGNAL PROCESSING

Analysis of dispersive wave signals demands advanced signal processing methods. While classical Fourier transform maps the signal into the frequency domain, time-frequency representations like the chirplet transform analyze the frequency content of a signal with respect to its occurrence in time.

In the following, a brief overview of some of the existing methods is given finally focusing on the description of the chirplet transform which is used in all the applications to be studied.

3.1 Fourier Transform

The Fourier transform is the basis for some of the time-frequency representations described in the next sections. A time signal is decomposed into its different frequency components by calculating the Fourier integral

$$X(\omega) = \int_{-\infty}^{+\infty} x(t)e^{-i\omega t} dt . \quad (3.1)$$

Expanding the same signal in terms of sinusoidal components gives the inverse transform

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(\omega)e^{i\omega t} d\omega . \quad (3.2)$$

The above relations are sometimes formulated differently as to the normalizing factor of $\frac{1}{2\pi}$, but for this work (3.1) and (3.2) will be used.

In general, $X(\omega)$ is a complex-valued function – also called the spectrum – and can therefore be written as

$$X(\omega) = A(\omega)e^{i\phi(\omega)} \quad (3.3)$$

with $A(\omega)$ as spectral amplitude and $\phi(\omega)$ as spectral phase. This leads to the definition of the energy density spectrum as the squared absolute value of the spectrum $|X(\omega)|$. The total energy of the signal then becomes

$$C = \int_{-\infty}^{+\infty} X(\omega)e^{i\omega t} d\omega. \quad (3.4)$$

Consider now the signals $x_1(t)$, $x_2(t)$, $x_3(t)$ and their respective Fourier Transform $X_1(\omega)$, $X_2(\omega)$, $X_3(\omega)$. Then the following properties hold [21]:

Table 3.1: Properties of the Fourier transform.

property	time domain $x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(\omega)e^{i\omega t} d\omega$	frequency domain $X(\omega) = \int_{-\infty}^{+\infty} x(t)e^{-i\omega t} dt$
Time shift	$x_2(t) = x_1(t - t_0)$	$X_2(\omega) = e^{-it_0\omega} X_1(\omega)$
Frequency shift	$x_2(t) = e^{i\omega_0 t} x_1(t)$	$X_2(\omega) = X_1(\omega - \omega_0)$
Convolution $(\star)$ in time	$x_3(t) = x_1(t) \star x_2(t) = \int_{-\infty}^{+\infty} x_1(\tau)x_2(t - \tau)d\tau$	$X_3(\omega) = X_1(\omega)X_2(\omega)$
Convolution $(\star)$ in frequency	$x_3(t) = x_1(t)x_2(t)$	$X_3(\omega) = X_1(\omega) \star X_2(\omega) = \int_{-\infty}^{+\infty} X_1(\nu)X_2(\omega - \nu)d\nu$
Scaling	$x_2(t) = x_1(\frac{t}{s})$	$X_2(\omega) = s X_1(s\omega)$
Parseval's formula	$\int_{-\infty}^{+\infty} x_1(t)x_2^*(t)dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X_1(\omega)X_2^*(\omega)d\omega$	

3.2 Time-Frequency Representations

As stated before, the frequency spectrum does not contain information about the time-dependent behavior of a signal. That explains the need for time-frequency representation, especially in the context of dispersive wave analysis. The idea is to correlate a signal with time-frequency atoms. These atoms extend to a concentrated

region of the time-frequency plane to determine the frequency content of the original signal averaged over that area.

3.2.1 The Uncertainty Principle

The resolution of time-frequency representations, however, is limited by the uncertainty principle. Let's consider a short duration signal which is constructed by multiplying the signal with an appropriate window function $h(t)$,

$$y(t_0) = x(t_0)h(t - t_0) . \quad (3.5)$$

The uncertainty principle then states that the product of the standard deviations in time and frequency for the signal in (3.5) cannot be made arbitrarily small. Instead, the following relation holds

$$\sigma_t \sigma_\omega \leq 1/2 . \quad (3.6)$$

The uncertainty principle was first derived by Werner Heisenberg in 1927 a brief description of which can be found in [6]. For a signal of the form (3.5), high resolution in the time domain corresponds to low resolution in the frequency domain and vice versa. It is also shown in [6] that for the normalized Gaussian window,

$$h(t) = \frac{1}{\sqrt[4]{\pi s^2}} e^{-\frac{1}{2} \left(\frac{t-t_0}{s} \right)^2} \quad (3.7a)$$

$$H(\omega) = \sqrt[4]{\pi} \sqrt{2s} e^{-\frac{(s\omega)^2}{2} - i\omega t_0} \quad (3.7b)$$

Equation (3.6) is equally fulfilled given the standard deviations $\sigma_t = \frac{s}{\sqrt{2}}$ and $\sigma_\omega = \frac{1}{\sqrt{2}s}$. In this sense, the Gaussian window represents an optimum window and will therefore be used in the remainder of this work.

3.2.2 Short-Time Fourier Transform

The short-time Fourier transform (STFT) is a commonly used time-frequency representation. The general formulation in time domain is given by

$$C^{\text{stft}}(t_0, \omega_0) = \int_{-\infty}^{+\infty} x(t) g_{t_0, \omega_0}^*(t) dt \quad (3.8)$$

with the time-frequency atom

$$g_{t_0, \omega_0}(t) = h(t - t_0)e^{i\omega_0 t} \quad (3.9)$$

using an appropriate window function $h(t)$. With the properties of Table 3.1, the corresponding frequency domain expression becomes

$$C^{\text{stft}}(t_0, \omega_0) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(\omega) G_{t_0, \omega_0}^*(\omega) d\omega \quad (3.10)$$

$$G_{t_0, \omega_0}(\omega) = H(\omega - \omega_0) e^{-i(\omega - \omega_0)t_0}. \quad (3.11)$$

According to (3.4), the energy density spectrum of the STFT is given by

$$P^{\text{stft}}(t_0, \omega_0) = |C^{\text{stft}}(t_0, \omega_0)|^2 = \left| \int_{-\infty}^{+\infty} x(t) h(t - t_0) e^{-i\omega_0 t} dt \right|^2 \quad (3.12)$$

which is also called the *spectrogram*. To summarize, the spectrogram averages the frequency content of a specific time interval of the original signal using a window shifted in time and frequency.

3.2.3 Wavelet Transform

In contrast to the STFT, the Wavelet transform (WT) is capable of analyzing signal structures of different sizes. To achieve this, a basis function, the so-called mother wavelet $\psi(t)$, is translated and dilated to obtain a family of time-frequency atoms with varying time support,

$$\psi_{t_0, s}(t) = \frac{1}{\sqrt{s}} \psi\left(\frac{t - t_0}{s}\right). \quad (3.13)$$

The application of these operation does not affect the properties of the mother wavelet which has zero mean and is normalized such that $\|\psi\|_2 = 1$. The WT can then be written as the inner product of the original signal with a member of the family of wavelets,

$$C^{\text{wt}}(t_0, s) = \int_{-\infty}^{+\infty} x(t) \psi_{t_0, s}^*(t) dt = \int_{-\infty}^{+\infty} x(t) \frac{1}{\sqrt{s}} \psi^*\left(\frac{t - t_0}{s}\right) dt. \quad (3.14)$$

Figure 3.1: Tiling of the time-frequency plane by time-frequency representations. Each ellipse represents one time-frequency atom. (a) WT (Section 3.2.3). (b) STFT (Section 3.2.2).

The energy density function of the WT is called *scalogram* which becomes

$$E^{\text{wt}}(t_0, s) = \left| \int_{-\infty}^{+\infty} x(t) \frac{1}{\sqrt{s}} \psi^* \left(\frac{t - t_0}{s} \right) dt \right|^2. \quad (3.15)$$

Thus, the WT depends on the variables time and scale which leads to a tiling of the respective plane as depicted in Figure 3.1 as opposed to the tiling of the time-frequency plane imposed by the STFT. However, the analogy to the STFT can be seen in the analysis equation when choosing a frequency modulated real and symmetric window as mother wavelet [21], $\psi(t) = h(t)e^{i\omega_0 t}$. Equation (3.14) then becomes

$$C^{\text{wt}}(t_0, s) = \int_{-\infty}^{+\infty} x(t) \frac{1}{\sqrt{s}} h\left(\frac{t - t_0}{s}\right) e^{-i\omega_0 \left(\frac{t - t_0}{s}\right)} dt \quad (3.16)$$

compared to

$$C^{\text{stft}}(t_0, \omega_0) = \int_{-\infty}^{\infty} x(t) h(t - t_0) e^{-i\omega_0 t} dt \quad (3.17)$$

for the STFT.

3.2.4 Chirplet Transform

The chirplet transform (CT) generalizes the concepts of the previously described time-frequency representations. The terminus was coined by Mann et. al. [22] in 1991 to characterize the kernel function of the transform as a “portion of a chirp”.

3.2.4.1 Definition

The CT can be defined analogous to the WT as the inner product of the original signal $x(t)$ with a time-frequency atom of the class of chirplets. Originating from a basis function $g(t)$, the CT offers five operations to adjust the atom in a desired fashion.

Time Shift: The time shift operation

$$\begin{aligned}\mathbf{T}_{t_0}h(t) &= h(t - t_0) \\ \mathbf{T}_{t_0}H(\omega) &= e^{-i\omega t_0}H(\omega)\end{aligned}\tag{3.18}$$

corresponds to a shift of the time axis

$$\begin{aligned}\bar{t} &= t - t_0 \\ \bar{\omega} &= \omega .\end{aligned}\tag{3.19}$$

Frequency Shift: Similarly, the frequency shift operation

$$\begin{aligned}\mathbf{F}_{\omega_0}h(t) &= e^{i\omega_0 t}h(t) \\ \mathbf{F}_{\omega_0}H(\omega) &= H(\omega - \omega_0)\end{aligned}\tag{3.20}$$

corresponds to a shift of the frequency axis

$$\begin{aligned}\bar{t} &= t \\ \bar{\omega} &= \omega - \omega_0 .\end{aligned}\tag{3.21}$$

Scale: The scaling operation

$$\begin{aligned}\mathbf{S}_s h(t) &= \frac{1}{\sqrt{s}} h\left(\frac{t}{s}\right) \\ \mathbf{S}_s H(\omega) &= \sqrt{s} H(s\omega)\end{aligned}\tag{3.22}$$

dilates the window function $h(t)$

$$\begin{aligned}\bar{t} &= \frac{t}{s} \\ \bar{\omega} &= s\omega.\end{aligned}\tag{3.23}$$

Time Shear: Convolution with a chirp signal in time domain

$$\begin{aligned}\mathbf{P}_p h(t) &= (\imath p)^{-\frac{1}{2}} \exp\left[\imath \frac{1}{2p} t^2\right] \star h(t) \\ \mathbf{P}_p H(\omega) &= \exp\left[\imath \frac{p}{2} \omega^2\right] H(\omega)\end{aligned}\tag{3.24}$$

has the effect of rotating the time axis by an angle of $\alpha = \arctan p$. Looking at the frequency domain product $\exp\left[\imath \frac{p}{2} \omega^2\right] H(\omega)$, the time shear operation influences its group delay $grd(\omega)$ which becomes

$$grd(\omega) = -p\omega\tag{3.25}$$

Consequently, the time-frequency plane is tiled as expressed by the coordinate transform

$$\begin{aligned}\bar{t} &= t - p\omega \\ \bar{\omega} &= \omega.\end{aligned}\tag{3.26}$$

Instead of a linear or first order chirp, higher order terms can be included yielding

$$\begin{aligned}\mathbf{P}_p H(\omega) &= \exp\left[\imath \left(\frac{p_1}{2} \omega^2 + \frac{p_2}{3} \omega^3 + \dots\right)\right] H(\omega) \\ \bar{t} &= t - p_1 \omega - p_2 \omega^2 - \dots\end{aligned}\tag{3.27}$$

Frequency Shear: Analogous to (3.24), frequency shear is defined as

$$\begin{aligned}\mathbf{Q}_q h(t) &= h(t) \exp \left[i \frac{q}{2} t^2 \right] \\ \mathbf{Q}_q H(\omega) &= (iq)^{-\frac{1}{2}} \exp \left[i \frac{2}{q} \omega^2 \right] \star H(\omega)\end{aligned}\tag{3.28}$$

The instantaneous frequency $\omega_{inst}(t)$ for the signal $\mathbf{Q}_q h(t)$ is given by

$$\omega_{inst}(t) = -qt \tag{3.29}$$

which is equivalent to the coordinate transform

$$\bar{t} = t \tag{3.30}$$

$$\bar{\omega} = \omega - qt .$$

Again, a higher order chirp can be used as well:

$$\begin{aligned}\mathbf{Q}_\mathbf{Q} h(t) &= \exp \left[i \left(\frac{q_1}{2} t^2 + \frac{q_2}{3} t^3 + \dots \right) \right] h(t) \\ \bar{\omega} &= \omega - q_1 t - q_2 t^2 - \dots\end{aligned}\tag{3.31}$$

Thus, the transformed signal can be written in the form

$$C^{ct}(t_0, \omega_0, s, q, p) = \int_{-\infty}^{\infty} x(t) g_{t_0, \omega_0, s, q, p}^*(t) dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) G_{t_0, \omega_0, s, q, p}^*(\omega) d\omega \tag{3.32}$$

with the time-frequency atom

$$g_{t_0, \omega_0, s, q, p}(t) = \mathbf{T}_{t_0} \mathbf{F}_{\omega_0} \mathbf{S}_s \mathbf{Q}_q \mathbf{P}_p h(t) \tag{3.33a}$$

$$G_{t_0, \omega_0, s, q, p}(\omega) = \mathbf{T}_{t_0} \mathbf{F}_{\omega_0} \mathbf{S}_s \mathbf{Q}_q \mathbf{P}_p H(\omega) \tag{3.33b}$$

in time (3.33a) and frequency (3.33b), respectively.

The resulting transform space is of dimension 5 and comprises as a subspace both the WT-space and the STFT-space. It should also be noted that the operators $\mathbf{T}_{t_0}$, $\mathbf{F}_{\omega_0}$, $\mathbf{S}_s$, $\mathbf{Q}_q$ and $\mathbf{P}_p$ are not commutative.

The energy of the transformed signal can be calculated as

$$P^{ct}(t_0, \omega_0, s, q, p) = |C^{ct}(t_0, \omega_0, s, q, p)|^2 . \tag{3.34}$$

Figure 3.2: Visualization of operators for the chirplet transform.

3.2.4.2 Chirplet transform for dispersive wave analysis

Based on the assumption that the material characteristics (E, ν, ρ) are known, an algorithm exploiting the benefits of the CT was developed by Kuttig [17]. Having at hand the theoretical dispersion relation, e.g. from solving the Rayleigh-Lamb equations (2.23a) and (2.23b), the time-frequency atoms can be adjusted for every single mode individually. Plotted in the time-frequency plane, the dispersion relation $t(f)$ represents the group delay for each individual mode. Applying the time shear operator of Equation (3.24), the resulting basis function of the CT can locally approximate the group delay of the signal component to be investigated, i.e. an individual mode in a region of average around the center (f_0, t_0) as illustrated in Figure 3.3. Conversely, the frequency shear operator can be used to approximate the instantaneous frequency of any particular mode in the frequency-time plane. For ease of computation, time shear is performed in the frequency domain and frequency shear in the time domain

Figure 3.3: CT basis functions adjusted to the s_0 -mode using 5th-order time shear.

to avoid convolution.

Figure 3.4 in turn shows the time-frequency atoms as used by the STFT. Irrespective of the dispersion behavior of individual modes, energy averaging could be compromised not only by misallocation at intersection points but also by the phase contribution varying with the dispersion pattern.

The *region of average* is thereby understood as the extension of the basis function of the CT in the time-frequency plane. In this research, a Gaussian window as given by (3.7) is used whose isopleths are elliptical. 98.1 % of its power are concentrated in an ellipse with a $2s$ semi-axis in time direction and a $\frac{1}{2s}$ semi-axis in frequency direction, while 99.9 % are within an ellipse of semi-axes $3s$ and $\frac{1}{3s}$, respectively. Difficulties occur, however, when individual modes are not widely separated from each other or even intersect in the time-frequency plane. The region of averaging could consequently include signal components that are part of a different mode.

Figure 3.4: Time-frequency atoms as used in the STFT plotted for the s_0 -mode.

The adaptive algorithm therefore first determines frequency ranges where modes are well separated based on a predefined criterion – the extension of the basis function represented by ellipses with semi-axes between $(2s, \frac{1}{2s})$ and $(3s, \frac{1}{3s})$. At potential points of interference, the scale operator is used to stretch the window function in one variable and compress in the other. Only if scaling does not resolve the problem of intersection, the computation of the CT at that particular point in time-frequency plane is abandoned. Figure 3.5 indicates the frequency ranges for which the respective CT is computed. These ranges increase with increasing propagation distance.

To summarize, the variable parameters to be specified when using the adaptive algorithm are

- the default scale s ,
- the mode intersection criterion in terms of the standard deviation of the window

Figure 3.5: Frequency ranges for which CT can be computed (x) for an aluminum plate using the $(2s, \frac{1}{2s})$ - criterion.

function,

- the order of linear approximation for the time shear operator.

The triple $(s, \sigma, O(p))$ has to be chosen according to the dispersion relation of the material under consideration.

Finally, the problem should be addressed of how the quantities computed with the CT are related to physical variables. Equation (3.34) suggests that the energy of the original signal at a specific point (t_0, ω_0) is given by the squared absolute value of the CT. $P^{\text{ct}}(t_0, \omega_0, s, q, p) = |C^{\text{ct}}(t_0, \omega_0, s, q, p)|^2$ is proportional to the energy of an individual mode, but the constant of proportionality is mode and frequency dependent. Therefore, normalization to a reference mode cannot result in relative mode energy. Instead, the absolute value of the CT $|C^{\text{ct}}(t_0, \omega_0, s, q, p)|$ is extracted as a quantity

Figure 3.6: Amplitude plot of the CT for the s_0 -mode of an aluminum plate.

proportional to the absolute modal displacement and normalized to a reference mode to obtain a relative displacement ratio. Figure 3.6 shows the amplitude plot of the CT computed for the s_0 -mode of a 1 mm aluminum plate.

3.2.4.3 Theoretical Comparison

Having introduced the CT for the purpose of dispersive wave analysis the question is risen up what the potential advantages could be in comparison to STFT and WT. Generally speaking, the benefit of adjusting the time-frequency atom at every point in the (t, f) -plane should result in a more consistent transformation of the original signal. While it seems to be difficult to exactly quantify this effect in general, it can be demonstrated for a special case.

Let's consider the case of a signal $x(t)$, $X(\omega)$ with a certain group delay $grd(\omega)$ which

can be approximated by its Taylor-series

$$grd(\omega) = t_0 + \underbrace{\frac{dt}{d\omega}}_{2\alpha} \bigg|_{\omega_0} (\omega - \omega_0) + \frac{1}{2!} \frac{d^2t}{d\omega^2} \bigg|_{\omega_0} (\omega - \omega_0)^2 + \dots \quad (3.35)$$

In the frequency domain, the complex-valued FT $X(\omega)$ can be written as

$$X(\omega) = A(\omega)e^{i\phi(\omega)}, \quad (3.36)$$

with $\phi(\omega) = \gamma - \int grd(\omega)d\omega$ and the constant γ . Let's choose a Gaussian window function $H(w)$ as in (3.7b) since it has the best resolution in both time and frequency. As a further assumption, the group delay $grd(\omega)$ of the signal should be approximated with the first two terms of (3.35) in the frequency intervall covered by the time-frequency atom. Then, ϕ becomes

$$\phi(\omega) = \gamma - \omega(t_0 - 2\alpha\omega_0) - \alpha\omega^2. \quad (3.37)$$

Let's now calculate the resulting energy distribution for the STFT, WT and CT. The energy density for the signal $X(\omega)$ is given by

$$\begin{aligned} P &= \left| \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[A(\omega) e^{-i(-\gamma + \omega(t_0 - 2\alpha\omega_0) + \alpha\omega^2)} \sqrt[4]{\pi} \sqrt{2s} e^{\frac{s^2}{2}(\omega - \omega_0)^2} e^{i\omega t_0} \right] d\omega \right|^2 \quad (3.38) \\ &= \left| \frac{e^{i\gamma} s}{\sqrt{2}\pi^{\frac{3}{4}}} \int_{-\infty}^{\infty} \left[A(\omega) e^{\left(\frac{2\alpha^2\omega_0^2 + i\alpha\omega_0^2 s^2}{s^2 - 2i\alpha} \right)} e^{\left(\omega \sqrt{s^2/2 - i\alpha} - \frac{\omega_0(2i\alpha - s^2)}{2\sqrt{s^2/2 - i\alpha}} \right)^2} \right] d\omega \right|^2 \\ &= \frac{s}{2\pi^{\frac{3}{2}}} \left| e^{i\alpha\omega_0^2} \right|^2 \left| \int_{-\infty}^{\infty} \left(A(\omega) e^{\left(\omega \sqrt{s^2/2 - i\alpha} - \frac{\omega_0(2i\alpha - s^2)}{2\sqrt{s^2/2 - i\alpha}} \right)^2} \right) d\omega \right|^2. \end{aligned}$$

Assuming $A(\omega)$ to be quasistationary in the frequency region covered by the time-frequency atom, the integral can be evaluated as

$$P = \frac{1}{2\pi^{\frac{3}{2}}} A^2(\omega) \left| \frac{\sqrt{\pi}}{\sqrt{s^2/2 - i\alpha}} \right|^2 = \frac{1}{\sqrt{\pi}(s^4 + 4\alpha^2)} A^2(\omega) \quad (3.39)$$

Equation (3.39) holds for both the STFT and the WT. However, the scale s is fixed for the whole frequency range when using the STFT, while s could vary in the case of the WT.

The time-frequency atom used in the CT can match the phase of the original signal $X(\omega)$ because of the additional time shear operator. The energy density becomes

$$\begin{aligned} P^{\text{ct}} &= \left| \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[A(\omega) e^{-i(\omega_0 + \omega t_0 + \alpha \omega^2)} \sqrt[4]{\pi} \sqrt{2s} e^{\frac{s^2}{2}(\omega - \omega_0)^2} e^{i\omega t_0} e^{i\alpha \omega^2} \right] d\omega \right|^2 \\ &= \left| \frac{e^{i\omega_0 s}}{\sqrt{2\pi}^{\frac{3}{4}}} \int_{-\infty}^{\infty} \left[A(\omega) e^{\frac{s^2}{2}(\omega - \omega_0)^2} \right] d\omega \right|^2, \end{aligned} \quad (3.40)$$

which can be evaluated assuming quasi stationarity for the Amplitude $A(\omega)$ as

$$P^{\text{ct}} = \frac{1}{2\pi^{\frac{3}{2}}} A^2(\omega) \left| \frac{\sqrt{2\pi}}{s} \right|^2 = \frac{1}{\sqrt{\pi}s} A^2(\omega). \quad (3.41)$$

Comparison with (3.41) and (3.39) reveals that for the given assumptions – Gaussian window function, linear approximation for $grad(\omega)$, quasistationarity of $A(\omega)$ – the correction factors to obtain the desired energy distribution over frequency, $A^2(\omega)$, becomes $s\sqrt{\pi}$ for the CT and $\sqrt{\pi(s^4 + 4\alpha^2)}$ for the STFT and WT. The main difference, however, is that the correction factor for the CT – under the assumptions made – does not depend on the dispersion relation since this has been included implicitly.

CHAPTER IV

RESULTS

In the following, the adaptive algorithm based on the chirplet transform will be applied to plate-like structures made from different aluminum alloys. The frequency region of interest lies in the interval up to 10 MHz. With increasing frequency, however, individual modes are less separated from each other which makes it difficult to analyze these regions.

4.1 Single Aluminum Plate

The CT has been applied by Kuttig [17] to analyze synthetic signals of a single aluminum plate in order to obtain normalized displacement ratios. The Lamb waves generated in the plate are dispersive and multi-modal in nature. Therefore, the single aluminum plate should serve as a benchmark to test the proposed adaptive chirplet algorithm and compare the results to a classical technique, the STFT. Apart from the displacement ratios, geometric attenuation due to the propagation pattern should also be determined.

The parameters for the adaptive CT algorithm are chosen to $(5.3\mu s, 2.2s, 5)$ for the scale parameter, the intersection criterion and the order of the time shear operator.

4.1.1 Signals Examined

Lamb waves traveling in a plate-like structure are derived analytically based on normal-mode expansion which has been proposed by Weaver and Pao [27] and is used by Luangvilai et. al. [20]. First, the freely vibrating plate is considered. The corresponding partial differential equations for the displacements can be integrated

Figure 4.1: Synthetic signal describing the out-of plane displacement of surface particles for an aluminum plate and a source-receiver distance $d = 90\text{mm}$.

numerically for each mode and frequency separately. Applying point-like sources normal to the surface results in an expression for the out-of plane displacement field. Superposing the displacements of all modes gives the desired time signals, an example of which is shown in Figure 4.1. Spurious frequency components present before the actual arrival of the wave front are due to numerical noise.

Experimental measurements for a single aluminum plate were conducted by Benz [3]. The Lamb wave signals were generated with a non-contact, point-like pulse laser with broadband excitation capability. A dual probe laser interferometer served as a point-like, non-contact detection system for high fidelity measurements. Figure 4.2 shows a typical time-domain signal recording the out-of plane particle velocity.

4.1.2 Displacement Ratio

Taking the amplitude of the CT at the location of dispersion curves in the time-frequency plane extracts a displacement-related quantity. However, the excitation

Figure 4.2: Experimental signal describing the out-of plane displacement of surface particles for an aluminum plate and a source-receiver distance $d = 90\text{mm}$.

source affects the displacement of individual modes. Normalization to a particular mode eliminates the source effect and makes results comparable.

First, the synthetic signal of the out-of plane displacement for a source-receiver distance $d = 90\text{mm}$ is considered.

Figure 4.3 shows the displacement ratios for the first two symmetric and anti-symmetric modes normalized to the a_0 -mode for which CT values can be computed over a large frequency interval. For the ratios a_1/a_0 and s_0/a_0 , the CT results are very close to the theoretical solution, while the amplitude ratio extracted from the STFT deviates from the theoretical solution, especially when individual modes are highly dispersive such as the s_0 -mode for frequencies between $2 - 3\text{ MHz}$. The time-frequency atoms used for the STFT don't match the dispersion behavior of the mode under consideration. Therefore, drastic changes in the group delay of a single mode might lead to inconsistent values using the STFT, whereas the CT can keep its level of accuracy. The results shown in Figure 4.4 support this interpretation: low frequencies

Figure 4.3: Out-of plane displacement ratios for modes a_1 , s_0 , s_1 and s_2 normalized to mode a_0 for the synthetic signal.

Figure 4.4: Out-of plane displacement ratios for modes a_0 , a_1 , s_1 and s_2 normalized to mode s_0 for the synthetic signal.

of a_0/s_0 and the range from 2-3 MHz for a_1/s_0 are resolved better with the CT than the STFT. When individual modes have almost constant group delay in the (t, f) -plane, however, both transforms produce similar amplitude ratios.

In order to quantify the level of accuracy, a simple metric is introduced for a function $x(t)$ by

$$x(t) \mapsto p(x(t)) = \frac{\sqrt{\langle x(t), x(t) \rangle}}{L}, \quad (4.1)$$

where $L = \int_{-\infty}^{\infty} dt$ and $\langle \cdot, \cdot \rangle$ is defined as the inner product for functions [15], $\langle x(t), y(t) \rangle = \int_{-\infty}^{\infty} x^*(t)y(t)dt$. Taking the energy densities of the CT and STFT, P^{ct} and P^{stft} , defined in (3.34), and (3.12) as x^2 , the mean absolute deviations from the exact solution normalized to the value of the exact solution become

$$\begin{aligned} \sigma^{\text{ct}} &= p \left(\frac{\sqrt{P^{\text{ct}}} - \sqrt{P^{\text{theoretical}}}}{\sqrt{P^{\text{theoretical}}}} \right) \\ \sigma^{\text{stft}} &= p \left(\frac{\sqrt{P^{\text{stft}}} - \sqrt{P^{\text{theoretical}}}}{\sqrt{P^{\text{theoretical}}}} \right), \end{aligned} \quad (4.2)$$

where $P^{\text{theoretical}}$ stands for the theoretically expected energy density.

Table 4.1 compares the performance of CT and STFT in terms of the introduced metric. Overall, the CT deviates about half as much as the STFT from the theoretical solution.

Table 4.1: Average deviation from theoretical mode displacement ratio in % for the synthetic signal.

	$\frac{a_1}{a_0}$	$\frac{s_0}{a_0}$	$\frac{s_1}{a_0}$	$\frac{s_2}{a_0}$	$\frac{a_0}{s_0}$	$\frac{a_1}{s_0}$	$\frac{s_1}{s_0}$	$\frac{s_2}{s_0}$
CT (in %)	9.49	67.89	8.46	12.00	10.54	4.00	12.19	5.60
STFT (in %)	18.60	686.36	14.65	19.48	51.33	66.28	10.24	9.69

The same quantities are computed for the experimental signals collected by Benz [3]. Figures 4.5 and 4.6 show the displacement ratios normalized to the a_0 - and s_0 -mode, respectively.

Figure 4.5: Out-of plane displacement ratios for modes a_1 , s_0 , s_1 and s_2 normalized to mode a_0 for the experimental signal.

Figure 4.6: Out-of plane displacement ratios for modes a_0 , a_1 , s_1 and s_2 normalized to mode s_0 for the experimental signal.

The performance of both methods drops significantly as Table 4.2 indicates. For all the ratios considered, the average deviation from the theoretical solution is much higher than for the synthetic signal. Issues like the choice of dimensions for the specimen – in this case a plate of dimensions 610mm x 305mm x 0.99mm is used – or measurement environment conditions might explain the degradation in measurement result which will be persistent throughout all the experimental data analyzed in this research.

In particular, the results for the CT seem to be much more affected since only for the a_0 - and s_0 -mode, the CT remains advantageous over the STFT. However, the results for a_0/s_0 and s_0/a_0 are obtained for broader frequency ranges and are therefore more significant. The model-based approach for the CT algorithm is inherently more sensitive to measurement errors or varying material properties of the specimens tested.

The inexact estimation of displacement-related quantities for individual modes could also propagate and adversely influence the results for geometric attenuation, for example, which will be discussed in the next section.

Table 4.2: Average deviation from theoretical mode displacement ratio for the experimental signal.

	$\frac{a_1}{a_0}$	$\frac{s_0}{a_0}$	$\frac{s_1}{a_0}$	$\frac{s_2}{a_0}$	$\frac{a_0}{s_0}$	$\frac{a_1}{s_0}$	$\frac{s_1}{s_0}$	$\frac{s_2}{s_0}$
CT (in %)	173.73	46.96	343.88	591.13	35.75	272.29	484.40	936.09
STFT (in %)	115.36	83.06	223.07	448.48	52.99	289.76	385.70	758.46

4.1.3 Geometric Attenuation

Luangvilai et. al. [19] reported that the amplitude decay due the propagation pattern cannot be recalculated exactly using the spectrogram. Looking at Figure 4.7, their judgment is affirmed. Even for the synthetic signal calculated with source-receiver distances from 4 cm to 9 cm, the STFT results differ from the theoretically expected $\frac{1}{\sqrt{d}}$ -spreading if only those regions are considered where modes are well-separated.

Figure 4.7: Geometric attenuation for the a_0 -mode comparing different propagation distances of the synthetic signal.

yellow dotted/dashed ($\triangle$) line: 40 mm / 90 mm theoretical/measured
green dotted/dashed ($\diamond$) line: 50 mm / 90 mm theoretical/measured
black dotted/dashed (+) line: 60 mm / 90 mm theoretical/measured
blue dotted/dashed (∇) line: 70 mm / 90 mm theoretical/measured
red dotted/dashed (x) line: 80 mm / 90 mm theoretical/measured

On the contrary, the CT results are very accurate in these frequency intervals, the respective figures are shown in Appendix A. Especially the low-frequency regions (0–2 MHz) of the a_0 - and s_0 -mode match the theoretical solution almost exactly, which also holds for the 4–5 MHz band of mode a_1 (c. f. Table 4.3).

Table 4.3: Average deviation in % from theoretical geometric attenuation for the synthetic signal.

distance	mode a_0		mode a_1		mode s_0		mode s_1	
	CT	STFT	CT	STFT	CT	STFT	CT	STFT
80/90 mm	0.31	3.64	1.65	2.54	1.71	9.08	5.61	19.96
70/90 mm	0.43	6.40	2.80	3.62	1.99	11.78	8.39	24.37
60/90 mm	0.49	8.48	4.65	5.20	6.23	24.37	13.19	24.09
50/90 mm	0.24	10.12	1.79	7.80	6.56	28.91	38.22	32.26
40/90 mm	0.28	14.04	3.14	7.28	6.33	28.26	132.4	40.03

Repeating this analysis for the experimental signals, the greater consistency in the CT data becomes visible. Although the displacement-related quantities extracted from the CT for a single measurement didn’t match the theoretical solution well, the comparison for different source-receiver distances from 5 cm to 15 cm reveals a more accurate recalculation of geometric spreading than for the STFT. As an exemplary figure, 4.8 for the a_0 -mode also demonstrates the difficulties coping with experimental data: measurement noise leads to less smooth, oscillating curves. The respective figures for the a_1 -mode, s_0 -mode and s_1 -mode are attached in Appendix A.

Only for the s_1 -mode, the CT results are worse than the corresponding STFT values the reason being only small frequency regions were evaluated. Thus, single erratic values can influence the mean deviation. Besides, the laser interferometer detection device captures the out-of plane surface displacement, while the displacement field for the symmetric modes has large in-plane components.

Despite the difficulties of generating valuable experimental data, it is clear that all in all, the CT resolves geometric attenuation better than the STFT also for measured signals, see Table 4.4.

Figure 4.8: Geometric attenuation for the a_0 -mode comparing different propagation distances of the synthetic signal.

green dotted/dashed (x) line: 50 mm / 90 mm theoretical/measured
black dotted/dashed (∇) line: 60 mm / 90 mm theoretical/measured
blue dotted/dashed (+) line: 70 mm / 90 mm theoretical/measured
red dotted/dashed ($\diamond$) line: 80 mm / 90 mm theoretical/measured

Table 4.4: Average deviation in % from theoretical geometric attenuation for the experimental signal.

distance	mode a_0		mode a_1		mode s_0		mode s_1	
	CT	STFT	CT	STFT	CT	STFT	CT	STFT
120/150 mm	1.91	4.16	15.03	22.78	17.01	20.91	32.92	18.7
90/150 mm	7.26	14.95	15.83	22.08	9.63	20.63	24.63	16.38
60/150 mm	2.50	16.48	9.71	26.91	13.56	32.68	67.88	14.46
50/150 mm	3.44	19.21	3.12	14.56	10.78	31.37	76.49	5.29

4.2 *Aluminum Plate Coupled to a Water Half-space*

Apart from geometric attenuation, amplitude decay can also be caused by energy leakage. For an aluminum plate bounded by a water-half-space, the attenuation coefficient α should be determined in the following. Luangvilai et. al. [19] used a laser measurement system to generate leaky Lamb waves in the specimen for source-receiver distances between 3.8 cm and 4.4 cm. They also simulated the absorbing plate structure numerically by means of normal-mode expansion.

Figures 4.9 –4.16 show the results for the synthetic signal for which again the first two symmetric and anti-symmetric modes are analyzed. Apparently, both techniques manage to determine α very exactly in the low frequency interval (0.5–1.5 MHz) for the a_0 - and s_0 -mode and around 3.8 MHz for the s_1 -mode. The average absolute deviation from the analytically derived solution is below $5 \frac{N_p}{m}$, see Table 4.5. For the a_1 -mode, only a small band below 5 MHz is computable with the parameters (5.3 μ s, 2.5s, 5) for the CT algorithm, but accuracy is low for both methods in this range.

Note that the values represent the average unnormalized deviation from the theoretical solution in $[\frac{N_p}{m}]$,

$$\begin{aligned}\sigma^{\text{ct}} &= p(\sqrt{P^{\text{ct}}} - \sqrt{P^{\text{theoretical}}}) \\ \sigma^{\text{stft}} &= p(\sqrt{P^{\text{stft}}} - \sqrt{P^{\text{theoretical}}}),\end{aligned}\tag{4.3}$$

while the values in brackets stand for the normalized deviations in % as defined

in (4.2). The very high percentages for the symmetric modes are due to their low attenuation in the frequency band under consideration, so that even small absolute deviations cause large relative deviations.

Figure 4.9: Attenuation curve of the a_0 -mode for the synthetic signal comparing STFT and CT with the theoretical solution.

Figure 4.10: Attenuation curve of the a_1 -mode for the synthetic signal comparing STFT and CT with the theoretical solution.

Figure 4.11: Attenuation curve of the s_0 -mode for the synthetic signal comparing STFT and CT with the theoretical solution.

Figure 4.12: Attenuation curve of the s_1 -mode for the synthetic signal comparing STFT and CT with the theoretical solution.

Table 4.5: Average deviation of attenuation dispersion for the synthetic signal in $\frac{N_p}{m}(\%)$.

mode	Average deviation for STFT			Average deviation for CT		
	$\frac{d_1}{d_2} = \frac{38\text{mm}}{44\text{mm}}$	$\frac{d_1}{d_2} = \frac{40\text{mm}}{44\text{mm}}$	$\frac{d_1}{d_2} = \frac{42\text{mm}}{44\text{mm}}$	$\frac{d_1}{d_2} = \frac{38\text{mm}}{44\text{mm}}$	$\frac{d_1}{d_2} = \frac{40\text{mm}}{44\text{mm}}$	$\frac{d_1}{d_2} = \frac{42\text{mm}}{44\text{mm}}$
a_0	6.4(7.4)	5.2(6.0)	3.5(3.9)	5.9(7.0)	2.1(2.3)	1.8(1.9)
a_1	267.8(200.3)	105.8(79.1)	100.2(75.1)	120.0(90.4)	82.5(61.9)	98.0(73.4)
s_0	63.4(271.7)	26.7(274.8)	13.9(148.2)	41.8(140.0)	5.0(145.3)	8.2(97.4)
s_1	2.5(558.2)	1.5(305.4)	1.5(268.5)	2.4(591.5)	0.5(65.7)	0.9(125.8)

Experimental detection of attenuation requires a special set-up. Large differences in source-receiver distances should reduce the influence of measurement noise when calculating attenuation curves. On the other hand, leaky Lamb waves are exposed to geometric attenuation in addition to energy leakage which decreases signal amplitudes for large propagation distances even more and therefore affects the signal-to-noise ratio.

Luangvilai et. al. [19] generated signals for source-receiver distances from 38 mm to 44 mm. Although attenuation curves could be determined successfully for the synthetic signals for the very same distances, the results for the experimental signals suggest that under real conditions, these very close distances don't allow for an exact calculation of α . Negative attenuation for the s_0 - and s_1 -mode at low frequencies indicates low signal-to-noise ratios for the respective measurement signals. For completeness the average absolute (relative) deviations from the analytical solution are given in Table 4.6. As in Table 4.5, the high percentages in the bracket expressions result from the normalization to the analytical solution, which for frequencies less than 1.5 MHz for the s_0 -mode and 4.5 MHz for the s_1 -mode is below $20 \frac{N_p}{m}$.

The dissatisfying results for either method hint at repeating experiments for larger source-receiver distances. Given the very accurate results of both the CT and the STFT for the synthetic signal, the potential to determine attenuation dispersion exists. Limitations of the measurement procedure seem to inhibit the analysis more than the idiosyncrasies of a specific signal processing technique.

Figure 4.13: Attenuation curve of the a_0 -mode for the experimental signal comparing STFT and CT with the theoretical solution.

Figure 4.14: Attenuation curve of the a_0 -mode for the experimental signal comparing STFT and CT with the theoretical solution.

Figure 4.15: Attenuation curve of the s_0 -mode for the experimental signal comparing STFT and CT with the theoretical solution.

Figure 4.16: Attenuation curve of the s_1 -mode for the experimental signal comparing STFT and CT with the theoretical solution.

Table 4.6: Average deviation of attenuation dispersion for the experimental signal in $\frac{N_p}{m}(\%)$.

mode	Average deviation for STFT			Average deviation for CT		
	$\frac{d_1}{d_2} = \frac{38\text{cm}}{44\text{cm}}$	$\frac{d_1}{d_2} = \frac{40\text{cm}}{44\text{cm}}$	$\frac{d_1}{d_2} = \frac{42\text{cm}}{44\text{cm}}$	$\frac{d_1}{d_2} = \frac{38\text{cm}}{44\text{cm}}$	$\frac{d_1}{d_2} = \frac{40\text{cm}}{44\text{cm}}$	$\frac{d_1}{d_2} = \frac{42\text{cm}}{44\text{cm}}$
a_0	38.9(38.9)	27.3(26.3)	43.4(42.4)	73.2(78.8)	31.3(33.3)	34.0(35.6)
a_1	266.2(207.9)	130.8(102.0)	55.9(44.4)	396.0(310.7)	239.7(187.3)	145.7(115.3)
s_0	151.4(784.9)	74.1(336.0)	90.5(547.4)	139.7(2646.0)	75.0(2012.4)	95.6(837.4)
s_1	18.7(529.0)	7.5(332.2)	7.9(265.6)	27.1(800.2)	11.1(377.4)	3.0(96.5)

4.3 2-layer Specimen

Heller [11] conducted experiments with 2-layer specimens to characterize adhesive bond properties using Lamb waves. He coupled an Adhesive Transfer Tape manufactured by the 3M company bonded to an aluminum plate as depicted in Figure 4.17.

Figure 4.17: 2-layer specimen as used in the experiments.

A point-like non-contact laser detection system as described in Section 4.1.1 captured the out-of plane surface velocity of the 2-layer specimen which was excited by a ablation laser source (point-like, non-contact). To determine the influence of heating on the material properties of the bond, the whole specimen was examined in two different conditions:

- condition 1 – un-aged, i.e. adhesive bond applied following manufacturer's guidelines

- condition 2 – aged, i.e. specimen heated for 4 h at 285°C exceeding the long-term temperature tolerance of 149° C specified by the manufacturer

Using the 2d-FFT, Heller found that specimens in condition 2 (aged) were exposed to energy leakage through the adhesive which had been irreversibly damaged by excessive heating. The respective contour plots of the frequency-wavenumber spectrum are shown in Figure 4.18 and 4.19 for two identical specimens, *Tape A* and *Tape B*. Especially for higher frequencies some of the modes present in the un-aged state vanish, see for example the a_1 -mode for Tape A in the interval 2–4 MHz and for Tape B between 4 and 5.5 MHz.

Figure 4.18: 2d-FFT of 2-layer specimen A in different conditions.

The whole data set of 50 equally distributed measurement points has to be analyzed for each specimen with the 2d-FFT routine to get the above contour plots. Using STFT and CT, however, only two measurements with different source-receiver distances are needed to analyze attenuation in the 2-layer specimen.

The results obtained by STFT and CT are comparable and are in agreement with [11]: the two different states of aging show distinct attenuation behavior in relevant

Figure 4.19: 2d-FFT of 2-layer specimen B in different conditions.

frequency regions. Figure 4.20 shows attenuation of the a_1 -mode of Tape A in the regions from 2–3 MHz and around 5 MHz for which the CT could be computed using the parameters $(3.3\mu s, 2.0s, 5)$ for the adaptive algorithm. Similarly, attenuation at about 5 MHz (CT) or even between 5–7 MHz (STFT) is higher for the un-aged specimen B. The same fact can be observed for the a_0 - and s_0 -mode of Tape B, see Figure 4.21 and 4.23 in the 0 to 2 MHz band. Changes from un-aged to aged condition in the low frequency regime were not as clearly visible using 2d-FFT analysis.

Figure 4.20: Attenuation curves for the a_1 -mode of Specimen A as extracted from the CT (top) and STFT (bottom).

The significant changes that become visible using the various time-frequency representations indicate that the heated bond was indeed irreversibly damaged. CT and STFT results showed higher attenuation for the aged specimens. Altogether, excessive heating seems to stiffen the adhesive layer.

FEM data for 2-layer specimens with both stiff and weak adhesive layers was also analyzed the results of which are presented in Appendix B. They affirm the assumption that the heating induced damage results in a stiffening of the adhesive bond.

Due to the lack of an exact theoretical solution, though, it is not possible to quantitatively evaluate the performance of either the CT or the STFT. From a qualitative standpoint, however, both transforms produced comparable results. The conclusions drawn as to characterizing adhesive bond properties result from an evaluation of attenuation behavior of aged (stiff) as opposed to un-aged (weak) specimens. In this respect, both methods allowed for a detailed analysis which could hopefully be extended in future research aiming for a quantitative description.

Figure 4.21: Attenuation curves for the a_0 -mode of specimen B as extracted from CT (top) and STFT (bottom).

Figure 4.22: Attenuation curves for the a_1 -mode of specimen B as extracted from CT (top) and STFT (bottom).

Figure 4.23: Attenuation curves for the s_0 -mode of specimen B as extracted from CT (top) and STFT (bottom).

Figure 4.24: Ultrasonic transducer set-up.

4.4 *Detection of material nonlinearities*

Only the linear characteristics of Lamb-wave propagation have been used in the applications presented in this work. However, the effect of higher harmonics generated by Lamb wave propagation might prove to become important in NDE. In particular, Deng [7] and Deng et. al. [8] analyzed cumulative second-harmonic generation in elastic plates theoretically and experimentally.

Bermes [4] conducted experiments with an ultrasonic transducer and a heterodyne laser interferometer as depicted in 4.24 to observe second harmonic generation in an aluminum plate. His data will be processed in the following comparing CT and STFT.

The nonlinear effects of Lamb-wave propagation are due to the bulk nonlinearity of the material the specimen consists of, in this case an aluminum plate. Interaction between the partial bulk waves in the plate generates second harmonics. Briefly summarizing the results of Deng's theoretical analysis [7], the ultimately generated second harmonic has to be symmetrical. To be able to generate second harmonics, the phase velocity c_p of the first harmonic at excitation frequency and the second harmonic at twice the fundamental frequency have to be equal. In order to fulfill these existence conditions, Bermes in his experiments excited the 1.6 mm thick aluminum plate at a frequency f_1 of either 2.205 MHz or 2.15 MHz to observe second harmonic generation at $f_2=4.41$ MHz and $f_2=4.3$ MHz, respectively. For these frequencies, the s_1 - and s_2 -mode are traveling with a similar phase velocity of about $6500 \frac{\text{m}}{\text{s}}$, see Figure 4.26.

Figure 4.25: Out-of plane velocity of plate specimen for a propagation distance $d = 25\text{cm}$ excited by an ultrasonic transducer.

Figure 4.26: Dispersion relation for a 1.6 mm thick aluminum plate with first and second harmonic excitation frequencies.

The nonlinearity parameter $\tilde{\beta}$ is given by the ratio of the amplitudes of the first and second harmonic:

$$\tilde{\beta} = \frac{A_2}{A_1^2}, \quad (4.4)$$

where A_1 denotes the amplitude of the first harmonic and A_2 the amplitude of the second harmonics. $\tilde{\beta}$ is only a relative parameter, however, in contrast to an absolute nonlinearity parameter $\beta = c \frac{A_2}{A_1^2}$ where $c = \frac{2c_L^2}{f^2 \pi^2 d}$ is a constant depending on wave speed, propagation distance and frequency. Such an absolute parameter β can only be derived for longitudinal waves so far. Nevertheless, the amplitude ratio $\frac{A_2}{A_1^2}$ should be a measure for material nonlinearity. Due to the inverse proportionality to propagation distance, the amplitude ratio $\frac{A_2}{A_1^2}$ should increase linearly with propagation distance.

Figure 4.27 shows how the amplitudes A_1 and A_2 are extracted from the CT. Both for the fundamental frequency and the frequency of the expected second harmonic the corresponding spectrum obtained from the TFR is examined. The peak value at the intersection with the s_1 -mode is extracted as the amplitude A_1 . Correspondingly, the maximum value at frequency f_2 next the intersection with mode s_2 is taken as the amplitude of the second harmonic A_2 . The theoretical dispersion curves don't exactly match these spectral maxima neither in the CT images nor in the spectrograms. This is due to the influence of the wedge transducer which is used as an excitation source. Nonetheless, distinctive peaks at excitation frequency and the frequency of the expected second harmonic can be identified.

Figure 4.28 shows the results of the first set of data which was taken from an Al6061 plate. The amplitude ratio $\frac{A_2}{A_1^2}$ is growing with increasing propagation distance. Linear regression gives a good fit for the amplitudes extracted from the CT, while the amplitudes extracted from the STFT produce a larger residual. For a distance of 10 cm, the frequency peaks at f_2 coincides with a larger peak at the intersection with the s_3 -mode and is therefore discarded. For both methods, consistent results have been achieved for the first set of three independent measurements with an Al6061 plate,

Figure 4.27: Extraction of amplitudes A_1 (top row) and A_2 (bottom row) from the CT. The dotted black lines represent the frequency of the first and second harmonic, respectively, and the extracted values are marked with * both in the CT image (left) and amplitude plots of the spectra (right).

the CT results spreading less than the results obtained from the STFT, however.

For the second set of data, two different types of aluminum, Al1100 and Al6061, were examined. CT and STFT differ only marginally with respect to the linear best fit. The amplitude ratio for the last propagation distance of 50 cm suffered from measurement attenuation and was therefore excluded.

Figures 4.29 shows the amplitude ratios for both aluminum alloys in comparison. Despite the differences in slope, taking the quotient of the nonlinearity parameters for both alloys, $\frac{\beta_{Al1100}}{\beta_{Al6061}}$, allows for a comparison between CT and STFT. Table 4.7 indicates very similar results of about 2.5 for the above ratio for both methods which is in good agreement with a value obtained by Yost et. al. [28]. Yost et. al. used longitudinal waves in their experiments so that the absolute nonlinearity parameter β could be determined. Even though this is not possible for Lamb waves the consistency of the results also supports the assumption that the amplitude ratio $\frac{A_2}{A_1^2}$ is a measure

Figure 4.28: $\frac{A_2^2}{A_1}$ for all measurements of the first data set with an Al6061-plate.

Figure 4.29: $\frac{A_{22}^2}{A_1}$ for both alloys Al1100 and Al6061 of the second set of data in comparison.

for material nonlinearity.

Table 4.7: Comparison of normalized $\tilde{\beta}$ -ratio for Aluminum 1100 and Aluminum 6061.

	STFT	CT	YostCantrell
$\frac{\beta_{Al1100}}{\beta_{Al6061}}$	2.63	2.60	2.12

All in all, the differences between CT and STFT in this nonlinear wave propagation application were only marginal. Both methods seem to detect cumulative second harmonics generated with Lamb waves in an aluminum plate as the final results come close to values determined by other researchers.

CHAPTER V

CONCLUSION

This research continues the work done by Kuttig [17] by applying the chirplet transform to various problems in NDE and structural health monitoring. Kuttig demonstrated that with the CT, displacement- and energy-related quantities could be determined from synthetic signals. His results for numerically simulated Lamb waves in an aluminum plate have been reproduced in this work.

The main goal of the current research is to further evaluate the potential of the CT in order to quantify its robustness and accuracy in a variety of practical applications. To that end, numerically and experimentally generated signals of Lamb waves traveling in plate-like structures are examined. Whenever possible, numerical simulation data is taken as a reference to represent the analytical solution. For these synthetic signals, CT results achieve a high level of accuracy. Normalized mode displacement ratios, geometric spreading and attenuation of leaky Lamb waves could be determined for specific frequency regions. Deviations from the theoretically expected solutions were marginal thus improving results obtained from the STFT.

The results for experimentally generated signals are ambiguous: on the one hand, it was possible to recalculate geometric attenuation more exactly than with the STFT-method in regions where modes are well separated. Attenuation calculations for a 2-layer specimen with an adhesive in different aging conditions are in accordance with the STFT and 2d-FFT analysis enabling characterizations of the adhesive bond properties. Furthermore, the relative nonlinearity parameter $\tilde{\beta}$ could be successfully determined using the adaptive algorithm based on the CT.

On the other hand, attenuation dispersion for an aluminum plate bounded by a water

half-space could not be completely determined for the experimental data available. The CT did not perform any better than the STFT, nor did either of the two methods approximate the theoretically expected attenuation accurately.

To summarize, the CT has shown potential in dispersive wave analysis. Taking the dispersion behavior into account, the CT produces consistent results especially for highly dispersive modes. Conversely, the differences between CT and STFT vanish in regions where individual modes have almost a constant group delay. Knowledge of the material properties has to be provided in order to run the adaptive algorithm. Coping with experimental data, difficulties can arise due to the complexity of the set-up itself. Further research should concentrate on improving experimental procedures, e.g. increasing source-receiver distances, to obtain more valuable results. Aiming at relaxing the constraint of a priori knowledge of the dispersion relation could be another field of future developments. Concepts used in image processing or matching pursuits can be used to extract the dispersion relation from the time-frequency representation of a wave signal. Hong et. al. [12] already proposed such an iterative algorithm for a single-mode signal. Finally, computational efficiency will be crucial for any real-world application which opens up another field of research to further improve the proposed CT algorithm for its use in dispersive wave analysis.

APPENDIX A

GEOMETRIC SPREADING OF LAMB WAVES IN AN ALUMINUM PLATE

Geometric spreading of Lamb waves in an aluminum plate obtained was analyzed using the CT and STFT and compared to the theoretical solution. Figures A.1 – A.3 depict the calculated $\frac{1}{\sqrt{d}}$ -spreading effect for the a_1 -, s_0 - and s_1 -mode, respectively, using the synthetic signal.

Similar Calculations were also performed for the experimentally generated signals. Figures A.4, A.5 and A.6 show the results for the a_1 -, s_0 - and s_1 -mode.

Figure A.1: Geometric attenuation for the a_1 -mode comparing different propagation distances of the synthetic signal.

yellow dotted/dashed ($\triangle$) line: 40 mm / 90 mm theoretical/measured
green dotted/dashed ($\diamond$) line: 50 mm / 90 mm theoretical/measured
black dotted/dashed (+) line: 60 mm / 90 mm theoretical/measured
blue dotted/dashed (∇) line: 70 mm / 90 mm theoretical/measured
red dotted/dashed (x) line: 80 mm / 90 mm theoretical/measured

Figure A.2: Geometric attenuation for the s_0 -mode comparing different propagation distances of the synthetic signal.

yellow dotted/dashed ($\triangle$) line: 40 mm / 90 mm theoretical/measured
green dotted/dashed ($\diamond$) line: 50 mm / 90 mm theoretical/measured
black dotted/dashed (+) line: 60 mm / 90 mm theoretical/measured
blue dotted/dashed (∇) line: 70 mm / 90 mm theoretical/measured
red dotted/dashed (x) line: 80 mm / 90 mm theoretical/measured

Figure A.3: Geometric attenuation for the s_1 -mode comparing different propagation distances of the synthetic signal.

yellow dotted/dashed ($\triangle$) line: 40 mm / 90 mm theoretical/measured
green dotted/dashed ($\diamond$) line: 50 mm / 90 mm theoretical/measured
black dotted/dashed (+) line: 60 mm / 90 mm theoretical/measured
blue dotted/dashed (∇) line: 70 mm / 90 mm theoretical/measured
red dotted/dashed (x) line: 80 mm / 90 mm theoretical/measured

Figure A.4: Geometric attenuation for the a_1 -mode comparing different propagation distances of the synthetic signal.

green dotted/dashed (x) line: 50 mm / 90 mm theoretical/measured
 black dotted/dashed (∇) line: 60 mm / 90 mm theoretical/measured
 blue dotted/dashed (+) line: 70 mm / 90 mm theoretical/measured
 red dotted/dashed ($\diamond$) line: 80 mm / 90 mm theoretical/measured

Figure A.5: Geometric attenuation for the s_0 -mode comparing different propagation distances of the synthetic signal.

green dotted/dashed (x) line: 50 mm / 90 mm theoretical/measured
black dotted/dashed (∇) line: 60 mm / 90 mm theoretical/measured
blue dotted/dashed (+) line: 70 mm / 90 mm theoretical/measured
red dotted/dashed ($\diamond$) line: 80 mm / 90 mm theoretical/measured

Figure A.6: Geometric attenuation for the s_1 -mode comparing different propagation distances of the synthetic signal.

green dotted/dashed (x) line: 50 mm / 90 mm theoretical/measured
black dotted/dashed (∇) line: 60 mm / 90 mm theoretical/measured
blue dotted/dashed (+) line: 70 mm / 90 mm theoretical/measured
red dotted/dashed ($\diamond$) line: 80 mm / 90 mm theoretical/measured

APPENDIX B

FEM RESULTS FOR THE 2-LAYER SPECIMENS

To additionally illustrate the experimental results for 2-layer specimens, numerical data from an FEM simulation was examined. Koreck [16] modeled a two layer specimen of an aluminum plate and an adhesive bond in Abaqus. Both a stiff bond with a Young's modulus of $E = 1\text{GPa}$ and a weak bond with $E = 5\text{MPa}$ are studied. Assuming plane strain for the plate structure, the out-of plane velocity was simulated at the same source-receiver distances as in Heller's experiments [11].

Figures B.2, B.1 and B.3 for the a_0 -, a_1 - and s_0 -mode depict the results obtained from the FEM data. Again, high attenuation is visible for the stiff layer for low frequencies (0–2 MHz) of the a_0 - and s_0 -mode and for frequencies around 5 MHz for the a_1 -mode.

The analysis of the FEM data produces results that are very similar to what has been determined for the experimental signals. Thus, the stiffening assumption for the bond exposed to excessive heating seems to be justified comparing both the experimental and numerical results.

Figure B.1: Attenuation curves for the a_1 -mode from FEM simulation as extracted from CT (top) and STFT (bottom).

Figure B.2: Attenuation curves for the a_0 -mode from FEM simulation as extracted from CT (top) and STFT (bottom).

Figure B.3: Attenuation curves for the s_0 -mode from FEM simulation as extracted from CT (top) and STFT (bottom).

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