

## **Information Sharing to Improve Retail Product Freshness of Perishables**

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## **Abstract**

We explore the value of information (VOI) in the context of a retailer that provides a perishable product to consumers and receives replenishment from a single supplier. We assume a periodic review model with stochastic demand, lost sales, and order quantity restrictions. The product lifetime is fixed and deterministic once received by the retailer, although the age of replenished items provided by the supplier varies stochastically over time. Since the product is perishable, any unsold inventory remaining after the lifetime elapses must be discarded (outdated). Without the supplier explicitly informing the retailer of the product age, the age remains unknown until receipt. With information sharing, the retailer is informed of the product age prior to placing an order and hence can utilize this information in its decision-making. We formulate the retailer's replenishment policies, with and without knowing the age of the product upon receipt, and measure the VOI as the marginal improvement in profit that the retailer achieves with information sharing, relative to the case when no information is shared. We establish the importance of information sharing and identify the conditions under which substantial benefits can be realized.

Keywords: value of information, inventory management, perishable inventory, product freshness, grocery industry

## 1. Introduction

The sale of perishable products makes up over 50% of the \$400 billion U.S. retail grocery industry (First Research, 2005). The importance of perishable goods is growing in terms of sales, SKUs, and the competitive importance of attracting consumers. For supermarkets, perishables are the driving force behind the industry's profitability and represent one of the last competitive advantages over the hard-charging and lower cost Wal-Mart super centers. Further, perishables have become the order winning criteria of consumers, becoming the core reason many consumers choose one supermarket over another (Heller, 2002). Despite their strategic importance, perishables subject grocery retailers to losses of up to 15 percent due to damage and spoilage. Thus, they offer a significant opportunity for improvement. These are all powerful incentives for investment in information enabling technologies for the management of perishables. Indeed many suppliers are embarking on supply chain initiatives premised on information technologies. For example,

Del Monte is focusing on making the retailer's life easier by taking on more of the work through supply partnerships... Technology has been the key to Del Monte's strategy – along with a sophisticated partnering package. Del Monte is working with retailers on accounting, packaging, merchandising, and sales – shared technology that allows broader and richer enhancement of information. (Hennessy, 2000, p. 74)

A distinguishing characteristic of perishables is that they have a finite lifetime and hence, the age of the products must be considered in their management. While our research focus is on groceries, the management of perishable inventories is an important problem confronting many other industries including blood banks, food service, pharmaceuticals, chemicals, and increasingly, biotechnology. Yet the grocery industry is particularly appropriate, given current practitioner activity and industry initiatives. In this paper, we introduce a model that extends the research on perishable inventory systems by evaluating a system where the age of the

replenished items is uncertain, the retailer orders in batches, and unmet demand results in lost sales: three highly significant aspects to the management of perishables in the grocery industry.

We measure the value-of-information (VOI) in the context of a retailer that provides a perishable product to consumers. Demand is stochastic and unsatisfied demands are lost. The retailer receives replenishments from a single supplier and there is a batch ordering constraint on the ordering decisions. The product lifetime is fixed and deterministic once received by the retailer, although the age of replenished items varies stochastically over time. These assumptions correspond to the widespread use of packaging highly perishable products with expiration dates. Without the supplier explicitly informing the retailer of the product age, the age of any replenishment remains unknown until receipt. Since the product is perishable, any unsold inventory remaining after the lifetime elapses must be discarded (outdated). With information sharing, the retailer is informed of the product age, prior to placing an order, and hence can utilize this information in its decision-making. We formulate the retailer's replenishment problem under these respective scenarios as Markov Decision Processes (MDPs). Given the complexity and computational limitations of the MDPs, we introduce and test well performing heuristic policies. We then use these heuristics to measure the VOI as the marginal improvement in profit that a retailer achieves with information sharing, relative to the case when no information is shared.

We find that the retailer benefits the most from information sharing when: 1) the variability of demand is high, 2) product lifetimes are short, and 3) the cost of the product is high. We also find that information sharing is generally more beneficial when demand is satisfied with a FIFO issuing policy than with a LIFO issuing policy. Upon further investigation, we also find that a random issuing policy (SIRO) results in measurements of the VOI that closely resemble a LIFO

issuing policy. Averaging across all parameter values, we find the average improvement from information sharing is 4.4% for FIFO, 3.6% for SIRO, and 3.4% for LIFO issuing. The benefits of information sharing, however, are not always directly shared with the supplier. Yet, the entire supply chain is either better-off or no worse-off, indicating that Pareto-improving contractual arrangements are feasible.

The rest of the paper is organized as follows. §2 reviews the literature, §3 defines the model, §4 provides and tests heuristic policies, §5 presents a numerical evaluation of the VOI for both FIFO and LIFO issuing policies along with a sensitivity analysis that isolates the main drivers of the VOI, and §6 extends the analysis to include random issuing policies, correlation in the age of the replenished product over time, and retail demand sensitivity to the product freshness. Finally, §7 measures the impact on profits an investment in information sharing has versus other common investment opportunities and §8 concludes the paper.

## **2. Literature Review**

Our research draws on two separate research streams: the literature on perishable inventory theory and the value of information. In this section, we provide a review of prominent research in each stream and position our study at the point of their intersection.

### **2.1 Perishable Inventory Theory**

Two problems addressed by the literature on fixed lifetime perishable inventory theory include determining reasonable and appropriate methods for both issuing inventory and for replenishing inventory. Since inventory may contain units of different ages, the issuing problem focuses on the order in which units of each age category are withdrawn from inventory to satisfy demand. Early work by Leiberman (1958) and Pierskalla and Roach (1972) address the conditions where issuing the oldest items first (FIFO) and youngest items first (LIFO) are

optimal. With constant product utility until outdating, as is the case with our research, FIFO issuing is optimal. Even so, we also address LIFO inventory issuing and random issuing (SIRO) since it is clear from practice that inventory issuing is not always controllable by a retailer.

Significant research has been done to derive and evaluate replenishment policies for items with a fixed lifetime. Simultaneously, yet independently, Nahmias (1975) and Fries (1975) were the first to derive and evaluate optimal policies for perishable products with lifetimes greater than two periods. In their models, the quantity of product to be outdated is expressed recursively in terms of previous outdates and demands. They formulate their respective problems as cost-minimizing dynamic programs that include both outdating and shortage costs. In both cases, the optimal ordering policy is shown to be non-stationary and dependent on the age distribution of inventory. Unlike our model, the product is assumed to be fresh on receipt (i.e. the remaining lifetime upon receipt does not vary from one replenishment to the next).

Given the multidimensional state of inventory, computation of optimal solutions using dynamic programs on long lifetime products is impractical since the state space expands exponentially with the number of possible age categories. Hence, much of the more recent work has focused on well performing heuristic policies. Nandakumar and Morton (1993) and Chui (1995) provide approximations for continuous review perishable systems. We also introduce well performing heuristics that are designed to evaluate the VOI in a periodic review system where all units do not arrive fresh at the retailer. The remaining lifetime depends on the age of stock at the supplier used to satisfy a retail order.

## **2.2 Value of Information**

While the importance of managing perishables is growing, there has also been a growing interest in the value of information sharing for supply chain management (VOI) as exemplified

by recent contributions to the academic literature by Aviv and Federgruen (1998), Cachon and Fisher (2000), Lee et al. (2000), Moïnzadeh (2002), and Aviv (2002). Most of the research has focused on the potential benefits of sharing *downstream* information on inventory stocking levels and ordering policies with upstream facilities located closer to the originating suppliers. The upstream suppliers can then incorporate this information into their decision making process to better match supply with demand. In contrast, the potential benefits with respect to the reverse flow of information (supplier to the retailer) have received scant attention in the literature.

Recently, a few articles have emerged that provide literature reviews and taxonomies that address the VOI for supply chain management. Sahin and Robinson (2002), Chen (2002) and Huang et al. (2003) are representative examples, each providing an overview of the literature and offering classification schemes. Only a few studies have addressed the value of supply information. For example, Chen and Yu (2005) consider the case where lead-time information is shared forward in the supply chain so that customers can reduce supply uncertainty. We note that both Chen (2002) and Huang et al. (2003) remark on the need for future research in this area. In this respect, we extend the literature on the VOI sharing in this important direction.

Beyond our own study, Ketzenberg and Ferguson (2005) is the only study we are aware of that addresses the value of information sharing in the context of perishable inventory. The authors address the value of information sharing in a serial supply chain consisting of a single retailer and a single supplier. Here, information is shared upstream, where the retailer shares its age-dependent inventory state, replenishment policy, and demand information with the supplier. While we also address the value of information with respect to the supply of a perishable product, in this paper we examine the reverse flow of information in which the supplier shares its inventory state with the retailer. Also, Ketzenberg and Ferguson (2005) model supply chains

where the supplier's ordering policy is highly dependent on the retailer's actions whereas we model supply chain structures where the supplier provides for a large number of retailers. Thus, the replenishment actions of a specific retailer are considered inconsequential to the choice of ordering policy of the supplier. This scenario is more appropriate for the grocery industry.

### 3. Model

The general setting is a retailer that provides a perishable product to consumers and receives replenishments from a larger supplier. We assume a periodic review inventory model, as this is the most common system used in the grocery industry. The product is perishable and has a maximum retail product shelf life of  $M$  periods, although the remaining shelf life at the time of replenishment varies between 1 and  $M$  as we later discuss. Throughout its lifetime, the utility of the product remains constant (see Ferguson and Koenigsberg (2005) for a treatment of perishable product whose provided utility degrades over time). Once the lifetime expires, the product is outdated (disposed) without any salvage value.

The order of events each day follows the sequence: 1) receive delivery, 2) outdate inventory, 3) observe and satisfy demand, and 4) place replenishment order. Retail demand is discrete, stochastic, and stationary over time with probability mass function (pmf)  $\phi(\cdot)$ , mean  $\mu_D$ , and coefficient of variation (cv)  $C_D$ . Define  $D$  to be a random variable denoting total demand in a period and  $d_t$ ,  $t \in \{0, 1, \dots, M\}$ , denote its realization in period  $t$ . Unsatisfied demand is lost. Let  $p$  be the unit selling price and  $w$  the unit purchase cost from the supplier. We assume that the only penalty for a lost sale is the lost margin,  $p-w$ . A holding cost  $h$  is assessed on ending inventory.



Product ordered in period  $t$  arrives in period  $t+1$ . The retailer orders from a completely reliable exogenous supplier. That is, the supplier has ample capacity so that all retail orders are fully satisfied one period later. The replenishment decision  $q_t$  is restricted to multiples of a batch quantity  $Q$  such that  $q_t = n_t Q$  in the current period, where  $n_t = 0, 1, 2, \dots$ . The batch quantity  $Q$  is given and fixed. This assumption captures certain economies of scale in transportation, handling, or packaging, although we do not model these economies explicitly (i.e., there is no fixed order cost). Such an assumption is common in practice and the literature (see Chen 1998, Cachon and Fisher 2000, Moynzadeh 2002). Although  $Q$  is exogenous in our model, we nevertheless evaluate the impact of this important parameter in our analysis.

Since the product is perishable, inventory may be composed of units with different ages. Let  $i_{x,t}$  denote inventory in period  $t$ , after demand, that expires in  $x$  periods, where  $x = 1, \dots, M$ . Let

$\mathbf{i}_t = (i_{1,t}, i_{2,t}, \dots, i_{M,t})$  represent the vector of inventory held at each age class in period  $t$  and

define  $I_t = \sum_{x=1}^M i_{x,t}$ .

We separately explore both FIFO and LIFO inventory issuing policies used to satisfy demand. While it is clear that FIFO issuing is optimal, generally retailers do not have explicit control of how demand is satisfied. Exceptions exist however, such as the load-from-the-back shelving systems often used for dairy products. When control is left to customers, they are apt to select the freshest products first.

The remaining lifetime of replenished items received in any period is an i.i.d. discrete random variable, with pmf  $\psi(\cdot)$ , mean  $\mu_A$ , and cv  $C_A$ . All replenished items received in period  $t$  have the same remaining lifetime. We do not model the supplier explicitly, but rather address the stochastic nature of the product age at the time of replenishment. These assumptions

represent the case of a large supplier that provides product to many independently controlled (in terms of ordering policies) retailers, a scenario that is common in the grocery industry today. Since the retailer makes up a fraction of the supplier's total order quantity in any given period, the supplier's inventory policy is assumed to be independent of the retailer's policy. We later relax this assumption and explore correlation in the age of replenished items over time in §6.2.

Define  $A$  to be a random variable denoting the remaining lifetime of replenished items associated with an order placed in period  $t$  and  $a_t, a_t \in \{1, 2, \dots, M\}$ , denote its realization. Further, without information sharing,  $a_t$  is unknown at the time an order is placed although the retailer does know  $\psi(\cdot)$ . Corresponding to practice, retailers do not typically know the age of replenished items until they are received, although they can estimate the age distribution from their order history. We formulate the replenishment problem as an infinite-horizon dynamic program where the objective is to find the retailer's optimal reorder policy so that its expected cost is minimized. The linkage between periods is captured through the one period transfer function of the retailer's age dependent inventory and is dependent on the current inventory level  $\mathbf{i}_t$ , any order placed in the current period  $n_t Q$ , the realization of demand in the next period  $d_{t+1}$ , and the realization of the remaining lifetime for any replenished items in the next period  $a_t$ .

For ease of exposition, let  $(z)^+ \equiv \max(z, 0)$ . Letting  $\tau(\mathbf{i}_t, d_{t+1}, n_t Q, a_t)$  denote the one period transfer function, then  $\mathbf{i}_{t+1} = \tau(\mathbf{i}_t, d_{t+1}, n_t Q, a_t)$  where

$$i_{x,t+1} = \begin{cases} \left( i_{x+1,t} - \left( d_{t+1} - \sum_{z=1}^x i_{z,t} \right)^+ \right)^+ & x \neq a_t \\ \left( i_{x+1,t} - \left( d_{t+1} - \sum_{z=1}^x i_{z,t} \right)^+ \right)^+ + n_t Q & x = a_t \end{cases} \quad \text{for FIFO inventory issuing}$$

and

$$i_{x,t+1} = \begin{cases} \left( i_{x+1,t} - \left( d_{t+1} - \sum_{z=x+2}^M i_{z,t} \right)^+ \right)^+ & x \neq a_t \\ \left( i_{x+1,t} - \left( d_{t+1} - \sum_{z=x+2}^M i_{z,t} \right)^+ \right)^+ + n_t Q & x = a_t \end{cases} \quad \text{for LIFO inventory issuing.}$$

Given the vector of ending inventory  $\mathbf{i}_t$  and an order quantity multiple of  $n_t$ , the infinite horizon cost-to-go, if future periods behave optimally, is  $f(\mathbf{i}_t)$ . The order quantity multiple that minimizes the cost-to-go is denoted by  $n_t^*(\mathbf{i}_t)$ . We represent the expected one period holding and penalty cost in period  $t$  by  $L(I_t)$  where

$$L(I_t) = h \sum_{d_t=0}^{I_t} (I_t - d_t) \phi(d_t) + (p - w) \sum_{d_t=I_t}^{\infty} (d_t - I_t) \phi(d_t). \quad (1)$$

Let the superscript *NIS* represent the no information sharing case. We can explicitly write the infinite horizon recursion as:

$$f^{NIS}(\mathbf{i}_t) = \min_{n_t \geq 0} \left\{ w i_{1,t} + L(I_t - i_{1,t} + n_t Q) + \sum_{d_{t+1}=0}^{\infty} \sum_{a_t=1}^M f(\tau(\mathbf{i}_t, n_t Q, d_{t+1}, a_t)) \psi(a_t) \phi(d_{t+1}) \right\}. \quad (2)$$

The right hand side of equation (2) computes the total expected cost that is composed of the cost of any unsold product that perishes in the next period, the one-period holding and penalty cost in the next period, and future expected cost. Note that 1) the outdated cost in the next period is independent of the replenishment decision, 2) the expectation of holding and penalty cost in the next period is predicated only on  $\phi(\cdot)$ , and 3) the expectation of future cost is predicated on both  $\phi(\cdot)$  and  $\psi(\cdot)$ . The decision space for  $n_t$  is the set of positive integer values. Since the state and decision spaces are discrete and finite and the cost is bounded, there exists an optimal policy that does not randomize (Putterman, 1994 pg 102 - 111). Let  $n_t(\mathbf{i}_t)^*$  denote the

optimal policy of order quantity multiples for period  $t$ . The resulting optimal cost-to-go is  $f^{NIS}(\mathbf{i}_t)^*$ . This formulation is similar to the approaches followed by Fries (1975) and Nandakumar and Morton (1993), where now the product lifetime is modeled as a random variable.

With information sharing, the retailer knows  $A = a_t$  prior to placing an order in period  $t$ . In this case, the state space is expanded to include this information. Let the superscript  $IS$  denote the information sharing case. The infinite horizon recursion is:

$$f^{IS}(\mathbf{i}_t, a_t) = \min_{n_t \geq 0} \left\{ w i_{1,t} + L(I_t - i_{1,t} + n_t Q) + \sum_{d_{t+1}=0}^{\infty} \sum_{a_{t+1}=1}^M f^{IS}(\tau(\mathbf{i}_t, n_t Q, d_{t+1}, a_t), a_{t+1}) \psi(a_{t+1}) \phi(d_{t+1}) \right\}$$

Let  $n_t^{IS}(\mathbf{i}_t, a_t)^*$  denote the optimal policy of order quantity multiples. The resulting optimal cost-to-go is  $f^{IS}(\mathbf{i}_t, a_t)$ . Note that while  $a_t$  is known with respect to any order placed in the current period, this information is not known for subsequent periods. Hence, the state transition probability from state  $(\mathbf{i}_t, a_t)$  to state  $(\mathbf{i}_{t+1}, a_{t+1})$  is predicated on both  $\phi(\cdot)$  and  $\psi(\cdot)$  just as it is in the no information sharing case.

Since expected profit is a more appropriate metric for the grocery industry, we interpret the VOI in terms of a change in expected profit due to information sharing by a simple conversion of our cost minimizing policies. Our switch to a profit maximization problem is simplified by the fact that we set the cost of a lost sale equal to the lost margin. Thus, the optimal ordering quantity multiples  $n(\mathbf{i}_t)^*$  and  $n^{IS}(\mathbf{i}_t, a_t)^*$  are equivalent for both the cost minimization and profit maximization problems. Letting  $\pi^{NIS}(\mathbf{i}_t)$  and  $\pi^{IS}(\mathbf{i}_t, a_t)$  represent the average expected profit

per period from the optimal policies across an infinite horizon, given a starting state of  $(\mathbf{i}_t)$  and  $(\mathbf{i}_t, a_t)$  for the respective cases, we have:

$$\pi^{NIS}(\mathbf{i}_t) = (p - w)\mu_D - f^{NIS}(\mathbf{i}_t) \text{ and } \pi^{IS}(\mathbf{i}_t, a_t) = (p - w)\mu_D - f^{IS}(\mathbf{i}_t, a_t) \quad (3)$$

## 4. Heuristic Policies

In this section, we introduce and test the performance of heuristic policies. The policies introduced in §3 are impractical to implement for many realistically sized problems given that the size of the state space expands exponentially with the age dependent vector of inventory. Hence, we provide heuristics that enable a broad evaluation on the VOI and that are more relevant to practice. In §4.1 we define our heuristic policies. In §4.2 we demonstrate through a series of tests that the heuristics perform very well over the parameter set tested. In §5 we proceed with an analysis on the VOI.

### 4.1 Heuristic Policies

The structure of the heuristic policies is predicated on a balance between simplicity and performance. Since a retailer can place an order each day and the lead-time is one day, our heuristics represent myopic policies where the order decision rests on whether sufficient stock exists in the current period that will carry over and minimize expected cost in the next period only. If sufficient stock exists, then the decision to order is postponed to the next day.

Let  $g^s(\mathbf{i}_t)$  denote the total estimated future outdating cost associated with inventory  $\mathbf{i}_t$  in periods  $t+s$  through  $t+M+1$ , where  $s \in \{2, 3, \dots, M+1\}$ . Hence,  $g^2(\mathbf{i}_t)$  denotes the total estimated future outdating cost of inventory  $\mathbf{i}_t$  in periods  $t+2$  through  $t+M+1$ , inclusive. Note

that we are not interested in the outdateding that occurs in period  $t+1$  since the replenishment decision in period  $t$  is independent of the outdateding cost in period  $t+1$ . Formally, we have

$$g^s(\mathbf{i}_t) = \begin{cases} wi_{1,t} + \sum_{d_{t+1}=0}^{I_t} g^{s+1}(\tau(\mathbf{i}_t, d_{t+1}, 0, a_t))\phi(d_{t+1}) & 2 \leq s \leq M \\ wi_{1,t} & s = M + 1 \end{cases}.$$

Note that zero is passed as a parameter for the order quantity for the tau function, since we are not concerned with outdateding of future orders. Now, let  $f^{HNIS}(\mathbf{i}_t)$  denote the minimum total estimated cost for the Heuristic No Information Sharing policy (HNIS), where

$$f^{HNIS}(\mathbf{i}_t) = \min_{n_t \geq 0} \left\{ L(I_t - i_{1,t} + n_t Q) + \sum_{d_{t+1}=0}^{I_t - i_{1,t} + n_t Q} \sum_{a_t=1}^M g^2(\tau(\mathbf{i}_t, d_{t+1}, n_t Q, a_t))\psi(a_t)\phi(d_{t+1}) \right\}.$$

For the Heuristic Information Sharing policy (HIS), we simply add the age of replenished items to the current state so that

$$f^{HIS}(\mathbf{i}_t, a_t) = \min_{n_t \geq 0} \left\{ L(I_t - i_{1,t} + n_t Q) + \sum_{d_{t+1}=0}^{I_t - i_{1,t} + n_t Q} g^2(\tau(\mathbf{i}_t, d_{t+1}, n_t Q, a_t))\phi(d_{t+1}) \right\}.$$

Expected profits are found by substituting  $f^{HNIS}(\mathbf{i}_t)$  with  $f^{NIS}(\mathbf{i}_t)$  and  $f^{HIS}(\mathbf{i}_t)$  with  $f^{IS}(\mathbf{i}_t)$  in (3). The advantages of the heuristics are that they are easy to implement, extremely fast computationally, and provide near optimal performance as we describe below.

## 4.2 Validation of Heuristic Performance

We test the heuristics by comparing their performance to optimality for a variety of scenarios. Consumer demand  $\phi(\cdot)$  corresponds to a truncated negative binomial distribution with a maximum value of 50 (probabilities for values exceeding 50 are redistributed proportionately within the truncated limit of the distribution). See Nahmias and Smith (1994) regarding the advantages of assuming negative binomial distributions for retail demand.

For our computational study, we set the maximum product lifetime  $M$  to 7 periods, although the age of receipts  $A$  varies according to  $\psi(\cdot)$ . We define  $\psi(\cdot)$  for each  $\mu_A$  and  $C_A$  pair in Appendix A. There is not a unique distribution for each pair, but through our discussions with practitioners, these symmetric distributions seemed most appropriate.

Each period represents a day, the selling price is \$1 and the annual holding cost is 25% of the purchase cost. We consider a set of experiments that comprise a factorial design for all combinations of the following parameter values:

$$\begin{aligned} \mu_D &\in \{3, 4\} & C_D &\in \{0.60, 0.75, 0.90\} & Q &\in \{1, 2, 4\} \\ \mu_A &\in \{2, 3, 4\} & C_A &\in \{0.2, 0.3, 0.4\} & w &\in \{0.4, 0.55, 0.70\} \end{aligned}$$

Our selection of parameter values correspond to operating characteristics of many short life-time products that include deli items, fresh cut produce, as well as packaged meats and seafood (Raper, 2003 and Pfankuch, B. 2004). Through experimentation with the policies, we found that their performance degraded when the order batch size was substantially larger relative to mean demand. Hence, we restricted our tests to conditions when the ratio was less than two. Moreover, our choice of parameter values for testing is constrained by the computational feasibility of the MDPs – our principal motivation for developing the heuristics.

We duplicate the factorial design for each issue policy: LIFO and FIFO. Hence, there are a total of 972 experiments in our test. We use value iteration to compute the results for the respective optimal and heuristic policies and then solve the accompanying state transition matrices using the method of Gaussian elimination to evaluate steady state behavior as described in Kulkarni (1995, p. 124).

We measure the performance of each heuristic policy by taking the percentage difference in expected profit, relative to the corresponding optimal policy. Overall, the results are very good.

The no information sharing heuristic achieves, on average, a total expected profit that is 0.9% less than optimal and the information sharing heuristic achieves, on average, a total expected profit that is 1.6% less than optimal. We report the performance at selected percentiles of the 972 test cases in Table 4.1.

| Percentile | No Info Heuristic | Info Sharing Heuristic |
|------------|-------------------|------------------------|
| 0.00       | 0.0%              | 0.0%                   |
| 0.05       | 0.0%              | 0.1%                   |
| 0.10       | 0.0%              | 0.2%                   |
| 0.25       | 0.1%              | 0.5%                   |
| 0.50       | 0.5%              | 1.2%                   |
| 0.75       | 1.3%              | 2.2%                   |
| 0.90       | 2.5%              | 3.6%                   |
| 0.95       | 3.3%              | 4.2%                   |
| 1.00       | 8.6%              | 8.8%                   |

Table 4.1: Heuristic Performance

As shown in Table 4.1, the worst-case performance is less than 9% from optimality for both heuristics and is less than 5% from optimality in over 95% of the test cases. We were not able to identify any patterns in the results to explain why the performance under a few sets of parameter values was worse than others (except for large order batch sizes relative to mean demand which we did not include in our test).

In a second test, we compared the VOI achieved with the heuristics to that of the optimal policies. We evaluate the VOI, measured as the % improvement in expected retailer profit, relative to the case where information is not shared. Specifically, define

$$\text{VOI} = \frac{(\pi^{IS}(\mathbf{i}_t) - \pi^{NIS}(\mathbf{i}_t, a_t))}{\pi^{NIS}(\mathbf{i}_t)}.$$

The average VOI of the heuristics across all 972 examples is 6.6%, or 0.6% less than the VOI for the optimal policies. This is not unexpected as the performance of the information sharing heuristic is, on average, 0.7% further from optimality than the performance of the no information



sharing heuristic. Hence, we would expect the VOI to be underestimated by the heuristics. Moreover, a thorough comparison of the heuristic VOI to the optimal VOI, across all parametric settings, demonstrates the same qualitative relationships. From the basis of these comparisons, we consider the heuristic policies to be well suited for our purposes and provide in the next section an extensive evaluation of the VOI.

## 5. Analysis

In this section, we report on a simulation study that evaluates the VOI using the heuristic policies defined in §4. §5.1 details the experimental design and simulation procedures, §5.2 reports our principal results and general observations, §5.3 and §5.4 respectively report our results for FIFO and LIFO issuing, §5.5 provides a sensitivity analysis, and §5.6 extends our analysis to the impact information sharing has on the supplier and the supply chain as a whole.

### 5.1 Experimental Design and Simulation Procedures

Testing via simulation allows us to choose a set of parameter values that captures the majority of cases for fresh meat, seafood, and produce based on our literature search and personnel interviews with produce managers (Pfankuch 2004, Raper 2003, Man and Jones 2000). We consider a set of experiments that comprise a factorial design for all combinations of the following parameters:

$$\begin{aligned} \mu_D &\in \{10, 15, 20\} & C_D &\in \{0.5, 0.6, 0.7\} & Q &\in \{1, 2, 4, 8\} \\ \mu_A &\in \{2, 3, 4, 5, 6\} & C_A &\in \{0.2, 0.3, 0.4\} & w &\in \{0.4, 0.55, 0.70\} \end{aligned}$$

We duplicate the factorial design for each issuing policy so that there are a total of 3,240 experiments with which to evaluate the VOI. The age distribution  $\psi(\cdot)$  that corresponds to each

$\mu_A$  and  $C_A$  pair are specified in Appendix A. The maximum product lifetime  $M$  is 11 days, the selling price is \$1 and the annual holding cost rate is fixed at 25% of the product cost.

We developed a simulation program using the PASCAL programming language. Each experiment is simulated for 1,100 days and replicated 20 times. The first 100 days of each replication are set aside as the simulation warm-up period so that statistics are calculated for 1,000 days in each replication. This 100 days period was chosen for convenience, yet larger than the number of days necessary for the system to exhibit steady-state behavior. In each replication, the random number streams across all experiments are identical in order to reduce the sampling error. The estimated standard error for the expected average daily profit using either heuristic, averages 0.5% of its mean value, and has a maximum error of 1.4%. Thus, we are over 99% confident that the true VOI in each experiment falls within 4.5% of the reported value.

## 5.2 General Observations

In general, we find that the sharing of supply information enables a retailer to purchase fresher product and consequently, information can be valuable. In Table 5.1, we separately report the VOI at given percentiles of the set of 1,620 experiments evaluated for each issuing policy. For example, the 0.50 percentile denotes the median values for VOI. We also report additional performance measures of interest that include the absolute difference in expected profit, percentage change in the average remaining product lifetime of replenished items, level of outdating, and service (fill-rate), where all % change measures are relative to the no information sharing case. Note that the values for *each* performance measure are ranked according to the percentile (from lowest to highest) and not according to the VOI.

| Percentile | % Change in |       |       |       |                      |       |           |        |         |       |
|------------|-------------|-------|-------|-------|----------------------|-------|-----------|--------|---------|-------|
|            | Δ Profit    |       | VOI   |       | Lifetime of Receipts |       | Outdating |        | Service |       |
|            | FIFO        | LIFO  | FIFO  | LIFO  | FIFO                 | LIFO  | FIFO      | LIFO   | FIFO    | LIFO  |
| 0.00       | 0.00        | -0.04 | 0.1%  | -1.2% | 0.4%                 | -3.4% | -90.1%    | -56.5% | -4.0%   | -6.0% |
| 0.25       | 0.09        | 0.05  | 1.3%  | 1.0%  | 4.3%                 | 2.8%  | -66.8%    | -16.7% | 0.2%    | 0.2%  |
| 0.50       | 0.15        | 0.10  | 2.9%  | 2.2%  | 8.1%                 | 6.7%  | -51.4%    | -6.3%  | 0.7%    | 0.9%  |
| 0.75       | 0.28        | 0.19  | 5.6%  | 4.5%  | 12.8%                | 11.8% | -35.6%    | -1.7%  | 1.7%    | 2.4%  |
| 1.00       | 0.81        | 0.59  | 39.5% | 37.2% | 25.0%                | 25.5% | 0.1%      | 0.1%   | 38.1%   | 40.6% |

Table 5.1: Performance metrics at percentiles of the 1,620 experiments for each issue policy

For FIFO issuing, the range of the VOI is between 0.1% and 39.5%, with a mean of 4.4% and a median of 2.9%. For LIFO issuing, the range of the VOI is between -1.2% and 37.2%, with a mean of 3.4% and a median of 2.2%. In a few experiments, expected profit from the information sharing heuristic was less than the no information sharing heuristic, which we attribute to the use of a heuristic policy.

Although it is clear from Tables 5.1 that the VOI can at times be large, the range that is reported also reveals that any realization of value is sensitive to model parameterization. Next, we discuss the drivers of value for each issuing policy and follow with a sensitivity analysis to understand the conditions in which information sharing is most beneficial.

### 5.3 FIFO Results

With FIFO issuing, the retailer has explicit control of its inventory so that product outdating is minimized and it is profitable to maintain (based on the average over all parameter values explored) a 91% service fill rate without information sharing. By using the supplier's age of inventory in its replenishment decision, the retailer will increase the expected lifetime of replenished items by ordering more in periods when the supplier has fresher product and less in periods when the supplier has older product. On average, the expected improvement in replenishment lifetime increases from 4.0 days to 4.3 days (8%). In turn, the level of outdating that arises from product expiration decreases from an average of 0.68 units per period to 0.41

(40% improvement). The key is that in any replenishment period, an improvement in the freshness of replenished items decreases the likelihood of product outdating in future periods.

The improvement in product freshness is not necessarily shared with consumers. Although, on average, consumers realize a 1.5% improvement in the remaining lifetime at the point of sale, it ranges from -10% to 9% as shown in Table 5.1. We find that the change in product freshness to consumers is largely a function of a change in retailer service. When the service level increases, the average inventory levels also increase so that product freshness decreases at the point of sale. On average, the retailer observes a slight improvement in the expected service fill rate (1%) since the expected cost of over–stocking, relative to the opportunity cost of a lost sale, is reduced with a fresher product. Yet this is not always the case as shown in Table 5.1 where we observe that the service fill-rate actually decreases in approximately 10% of the experiments. We find that information sharing enables a systematic tradeoff between a decrease in the cost of outdating and an increase in profit contribution. On average, we observe that 72% of the improvement in average expected profit arises from a reduction in outdating and 28% arises from higher service. Figure 5.1 shows how these two components (outdating cost and service) are influenced by information sharing. Specifically, we break out the proportion in increased expected profit due to each component at fixed intervals of the reported increase in expected profit across experiments (values on the x-axis multiplied by 100 for readability).

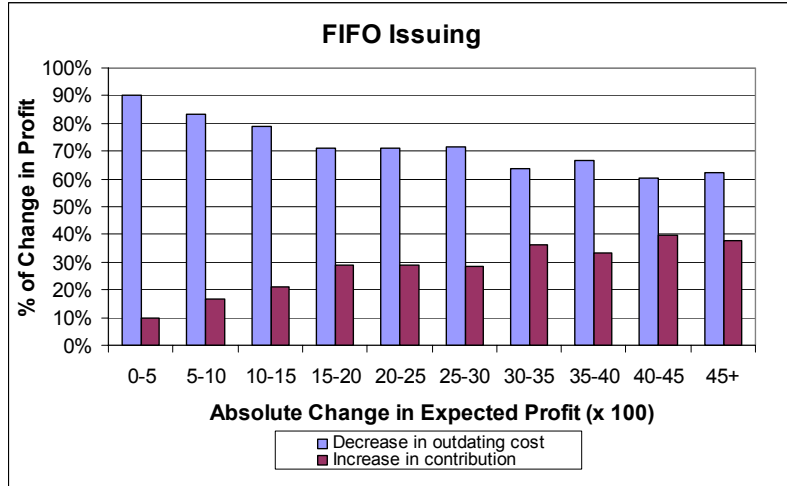


Figure 5.1: The components of the VOI for FIFO issuing

As Figure 5.2 illustrates, a reduction in the cost of outdating is largely responsible for the VOI that we observe. However, when information is most valuable, the retailer is able to substantively increase service while simultaneously reduce outdating. We elaborate later with a sensitivity analysis, but first position our results with those that arise for LIFO issuing.

## 5.4 LIFO Results

The results indicate the VOI is generally greater with FIFO issuing than LIFO issuing, although in 15% of the experiments the VOI with LIFO issuing is greater. These instances correspond to scenarios where the VOI is smallest. Consider that when the VOI with LIFO issuing exceeds FIFO issuing, the VOI is, on average, 2.3% with a maximum of 13.6%. When the converse is true, the VOI is, on average, 5.1%, with a maximum of 39.5%.

With LIFO issuing, the retailer has inherently less control of product outdating so that the cost of holding inventory is greater than we observe with FIFO issuing. Consequently, the retailer maintains a lower service level on average (86% fill rate) without information sharing. Just as with FIFO issuing, when information is shared, it results in an improvement in the freshness of replenishment. Yet here, any improvement will not necessarily result in a decrease

in product outdating because consumers buy the freshest product first. Moreover, any new replenishment may increase the likelihood of outdating product that is already in stock. Hence, with LIFO issuing the retailer is more constrained in its ability to reduce product outdating. Across experiments, information sharing enables a reduction in outdating from an average 1.4 units per day to an average of 1.2 units (14% improvement) which is considerably less than that observed with FIFO issuing (40% improvement).

As we did in Figure 5.1 for FIFO issuing, Figure 5.2 breaks out the proportion of the VOI attributed to a reduction in outdating and an increase in contribution. Comparing Figure 5.1 with Figure 5.2 shows that across all levels of the VOI, increasing expected profit through higher service plays a greater role with LIFO issuing than with FIFO issuing. Across all experiments, 53% of the increase in expected profit is attributed to an increase in service fill-rate as compared to only 28% with FIFO issuing. Consider that with FIFO issuing, service levels are already relatively high without information so that there is less opportunity to increase them.

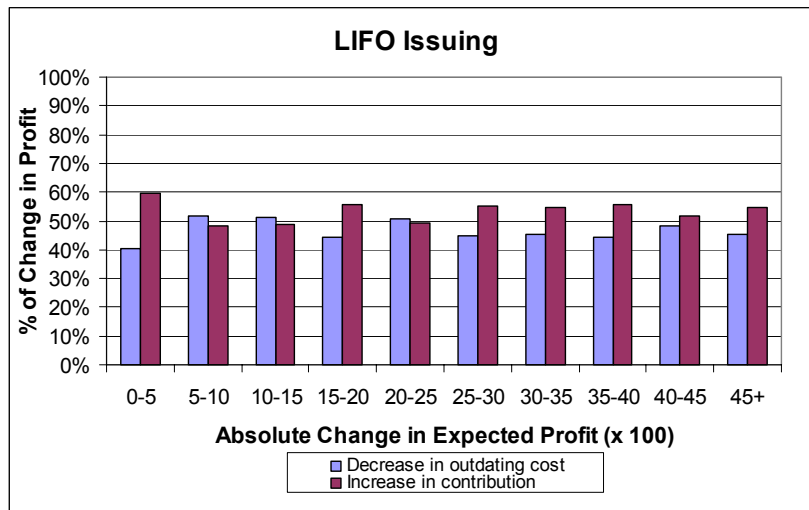


Figure 5.2: The components of the VOI for LIFO issuing

In the next section, we proceed to elaborate on our findings through a sensitivity analysis that explores the conditions in which information is most valuable.

## 5.5 Sensitivity Analysis

In Table 5.2, we report the average VOI across all 3,240 experiments for each fixed parameter value and separately for each issuing policy. In addition, we report the associated average change in the cost of outdating and the average change in profit contribution. These results indicate that the VOI is largely a function of the level of uncertainty the retailer experiences and the sensitivity of its costs to uncertainty.

As expected, the VOI increases as the expected lifetime of replenished items  $\mu_A$  decreases. Improvements in product freshness reduce the potential for outdating, allowing the retailer to carry more inventory for the same amount (or less) of product outdating. To see this, consider the extreme case of a non-perishable product. Here, there is no outdating and information sharing has no effect on retailer behavior because product freshness is no longer material to the problem. Consequently, the VOI is zero.

| Parameter         | Value | FIFO Issuing |                       |                    | LIFO Issuing |                       |                    |
|-------------------|-------|--------------|-----------------------|--------------------|--------------|-----------------------|--------------------|
|                   |       | VOI          | $\Delta$ Contribution | $\Delta$ Outdating | VOI          | $\Delta$ Contribution | $\Delta$ Outdating |
| Mean Demand       | 10    | 4.4%         | \$0.04                | \$0.09             | 3.4%         | \$0.05                | \$0.04             |
|                   | 15    | 4.4%         | \$0.06                | \$0.14             | 3.4%         | \$0.07                | \$0.07             |
|                   | 20    | 4.3%         | \$0.08                | \$0.19             | 3.4%         | \$0.10                | \$0.09             |
| Demand CV         | 0.5   | 2.9%         | \$0.04                | \$0.11             | 2.5%         | \$0.02                | \$0.10             |
|                   | 0.7   | 4.3%         | \$0.06                | \$0.14             | 3.2%         | \$0.08                | \$0.06             |
|                   | 0.9   | 5.9%         | \$0.09                | \$0.16             | 4.6%         | \$0.13                | \$0.05             |
| Expected Lifetime | 2     | 7.3%         | \$0.05                | \$0.23             | 6.0%         | \$0.04                | \$0.16             |
|                   | 3     | 5.5%         | \$0.07                | \$0.18             | 4.1%         | \$0.08                | \$0.09             |
|                   | 4     | 4.1%         | \$0.08                | \$0.13             | 2.8%         | \$0.09                | \$0.04             |
|                   | 5     | 3.2%         | \$0.07                | \$0.10             | 2.4%         | \$0.08                | \$0.03             |
|                   | 6     | 1.8%         | \$0.05                | \$0.06             | 1.7%         | \$0.07                | \$0.01             |
| Lifetime CV       | 0.2   | 1.7%         | \$0.01                | \$0.07             | 1.2%         | \$0.02                | \$0.03             |
|                   | 0.3   | 4.0%         | \$0.05                | \$0.14             | 2.9%         | \$0.07                | \$0.05             |
|                   | 0.4   | 7.4%         | \$0.12                | \$0.21             | 6.2%         | \$0.14                | \$0.11             |
| Product Cost      | 0.40  | 2.1%         | \$0.03                | \$0.13             | 1.6%         | \$0.05                | \$0.06             |
|                   | 0.55  | 3.8%         | \$0.06                | \$0.15             | 2.9%         | \$0.07                | \$0.07             |
|                   | 0.70  | 7.2%         | \$0.10                | \$0.14             | 5.7%         | \$0.10                | \$0.07             |
| Batch Size        | 1     | 4.3%         | \$0.06                | \$0.14             | 3.4%         | \$0.07                | \$0.07             |
|                   | 2     | 4.3%         | \$0.06                | \$0.14             | 3.4%         | \$0.07                | \$0.07             |
|                   | 4     | 4.3%         | \$0.06                | \$0.14             | 3.4%         | \$0.07                | \$0.07             |
|                   | 8     | 4.6%         | \$0.06                | \$0.14             | 3.5%         | \$0.08                | \$0.06             |

Table 5.2: Sensitivity of the VOI to parameters

Two factors that affect the retailer's ability to efficiently match supply with demand are the coefficients of variations in demand  $C_D$  and in the lifetime of replenished items  $C_A$ . As shown in Table 5.2, the VOI increases with respect to both parameters. While the fact that the VOI increases with an increase in uncertainty of demand has been well studied, we observe the same, if not stronger, relationship between supply uncertainty and the VOI. That is, the more uncertainty there is with regard to the age of replenished items, the higher the VOI. Again, an extreme example is sufficient to demonstrate. Consider the case where  $C_A = 0$ . Here, there is no variability over time with respect to freshness of replenishment and hence the VOI is zero.

While uncertainty is main driver of the VOI, the magnitude of this effect depends on the sensitivity to mismatches in supply and demand. A clear example is that of the product cost. When the product cost is high (contribution margins low), the cost of holding inventory relative



to a lost sale is larger since the cost of unit outdating is larger. Thus, service levels are lower even with FIFO issuing. Consequently, information sharing that enables a reduction in the cost of outdating will also be accompanied by an increase in service. Consider that with FIFO issuing, information sharing enables an average 0.4% increase in service when the product cost is \$0.40 but a 2.7% increase when the product cost is \$0.70. These improvements are comparable with LIFO issuing. We can also see from Table 5.2 that an increase in contribution has a larger role in the total profit improvement due to information sharing when the product cost is high. Thus, the VOI is largest when the retailer is able to substantially improve its service.

Given that one of the drivers of value resides with the retailer's ability to match supply with demand, it may seem surprising that the VOI demonstrates no sensitivity with respect to the order batch size. However, to draw such a conclusion may be partially misleading since we have restricted our analysis to evaluating scenarios where the order batch size does not significantly exceed the mean demand rate (because heuristic performance degrades). Hence, when demand rates are low compared to the order batch size, the VOI *may* be more sensitive to the order batch size. Certainly, large batch sizes make it more difficult for a retailer to effectively match supply and demand. Given our prior analysis, one would expect the VOI to be more valuable and we do have some limited experience with the optimal policies to support this assertion. However, a large order batch size itself will constrain a firm's ability to take advantage of information. Since it remains unclear which effect dominates and under what conditions, analysis of the VOI with large order batch sizes is an important avenue for future research.

## **5.6 Impact on the Supplier**

Our analysis would not be complete without studying the impact of information sharing on the supplier. While the supplier is exogenous to the model, we can nevertheless measure the

impact that information sharing has on its performance by considering the net change in expected retail orders and the net change in product outdating at the supplier. Across experiments, we observe a range of between  $-13.5\%$  and  $+41.6\%$  and a mean of  $0.0\%$  in the change in expected size of retail orders per period. The size of the change depends largely on the relative improvements in retailer outdating and retailer service. Improvements in retailer outdating translate to a decrease in orders to the supplier while improvements in service translate to an increase in orders to the supplier. In 43% of the experiments, the average expected order size to the supplier increases.

As for outdating, we take a conservative approach to measuring the impact on the supplier and assume that whenever product at the supplier has a remaining lifetime of one day and the retailer does not place an order, it expires. We assume the quantity that expires is equal to the average order size placed by the retailer. This assumption is conservative as it assumes that no other retailers buy the soon-to-expire stock. Across experiments, we observe a change in supplier outdating due to information sharing that ranges from 0.0 units per day to 1.2 units per day, with a mean of 0.09 units. Hence, a reduction in retailer outdating (with a mean of 0.20 units) translates to an increase in supplier outdating, but the impact is much less on the supplier.

Given both the change in retailer orders and supplier outdating that arises from information sharing, the supplier is worse off on average. In Table 5.3, we report the impact that information sharing has on the supply chain by reporting, at given percentiles across the 3,240 experiments, the change in orders to the supplier, the change in supplier outdating, and the change in supplier revenue. We also report the impact on both the supplier's expected profit and the combined expected profit for both the retailer and the supplier. We do so by evaluating two cases: one in which the supplier's product margin is 10% and another in which it is 50%. This provides a

relative comparison between cases when the supplier is a distributor (low margin) and a manufacturer (high margin).

| Percentile | Supplier       |                  |                    |                              |                              | Supply Chain                 |                              |
|------------|----------------|------------------|--------------------|------------------------------|------------------------------|------------------------------|------------------------------|
|            | $\Delta$ Order | $\Delta$ Revenue | $\Delta$ Outdating | $\Delta$ Profit (10% Margin) | $\Delta$ Profit (50% Margin) | $\Delta$ Profit (10% Margin) | $\Delta$ Profit (50% Margin) |
| 0.00       | -1.04          | -0.53            | 0.00               | -0.30                        | -0.33                        | -0.02                        | -0.08                        |
| 0.05       | -0.57          | -0.28            | 0.00               | -0.18                        | -0.19                        | 0.01                         | 0.00                         |
| 0.10       | -0.38          | -0.18            | 0.01               | -0.13                        | -0.13                        | 0.02                         | 0.01                         |
| 0.25       | -0.16          | -0.08            | 0.02               | -0.06                        | -0.06                        | 0.05                         | 0.04                         |
| 0.50       | -0.02          | -0.01            | 0.05               | -0.02                        | -0.02                        | 0.09                         | 0.10                         |
| 0.75       | 0.12           | 0.07             | 0.13               | -0.01                        | 0.02                         | 0.17                         | 0.21                         |
| 0.90       | 0.36           | 0.23             | 0.25               | 0.00                         | 0.07                         | 0.26                         | 0.35                         |
| 0.95       | 0.55           | 0.37             | 0.34               | 0.00                         | 0.13                         | 0.33                         | 0.47                         |
| 1.00       | 3.08           | 2.15             | 1.20               | 0.02                         | 0.79                         | 0.64                         | 1.46                         |

Table 5.3: Impact of information sharing on the supply chain

As shown in Table 5.3, while the supplier is harmed in a preponderance of the cases, the supply chain as a whole generally improves or is no worse-off. While nearly 5% of the cases show a negative change to expected supply chain profit, we attribute these negative values to our use of a heuristic policy. Consider that in the worst case reported, supply chain profit decreases from \$7.27 to \$7.19 (-1.3%) with information sharing. Overall the change in supply chain profit, expressed as a percentage relative to the no information case, ranges from -0.4% to 19.2% with a 10% supplier margin and from -1.3% to 32.9% with a 50% supplier margin.

Since the supplier is generally worse-off, some form of contract beyond the normal price-only contract is needed to induce the supplier to participate in the information sharing. Gerchak and Wang (2004) and Cachon and Lariviere (2005) discuss revenue sharing contracts, where the product is sold to the retailer at the supplier's cost and the retailer shares a pre-determined percentage of the revenue with the supplier. The percentage of revenue shared is generally set such that the contract is Pareto improving. In our scenario, retailers are typically much smaller than suppliers, thus retailers may be more risk adverse. Gan et al. (2005) discuss how

coordination contracts may be modified to account for this reality. Given the common practice of sharing point-of-sale data, the cost of implementing and monitoring such a contract should not be prohibitive.

## **6. Extended Analysis**

In this section, we explore some practical extensions to our base model assumptions. Specifically, we study the impact of a random issuing policy, correlation in the age of replenished items over time, and demand sensitivity to product freshness.

### **6.1 Random Issuing**

While we have modeled consumer behavior with respect to LIFO and FIFO issuing policies, some product displays allow consumers to be random in their selection of products with respect to their freshness. Hence, an interesting and practical line of inquiry is to examine the VOI with a service-in-random-order (SIRO) issuing policy. To do so, we assume that the probability associated with a unit of demand being satisfied with a unit of product in a given age category is equivalent to the proportion of total inventory represented by the given age category. For example, if 20% of the units in inventory have a remaining lifetime of three days, then a particular unit of demand has a probability of 0.20 of being satisfied with a unit of inventory with a remaining lifetime of three days.

To explore the VOI in the context of SIRO issuing, we duplicate the full set of experiments conducted for FIFO and LIFO issuing as defined in §5. All parameter settings and simulation methods are identical to that described in §5 (as are subsequent studies in §6.2 and 6.3).

The results of our experiments were somewhat surprising, particularly when compared to the VOI for the other issuing policies. We found that the average VOI for the SIRO policy across experiments is 3.6%, considerably closer to the 3.4% average for the LIFO policy than the 4.4%

for the FIFO policy. Looking beyond the averages, a more comprehensive analysis of the VOI across individual scenarios provides the same picture: the VOI for the SIRO policy is closer to the LIFO policy than the FIFO policy.

Duplicating the sensitivity analysis presented for the other issuing policies in §5.4, in Table 6.1, we report sensitivity of the VOI with SIRO issuing to the parameters. Here, we find the same relationships between the VOI and parameters for all issuing policies, but notice that not only are the averages reported at each parameter value closer to those reported for LIFO issuing, one is even less ( $C_D = 0.5$ ) and several averages are identical.

| Parameter         | Value | SIRO |                       |                    | FIFO | LIFO |
|-------------------|-------|------|-----------------------|--------------------|------|------|
|                   |       | VOI  | $\Delta$ Contribution | $\Delta$ Outdating | VOI  | VOI  |
| Mean Demand       | 10    | 3.7% | \$0.05                | \$0.06             | 4.4% | 3.4% |
|                   | 15    | 3.6% | \$0.07                | \$0.08             | 4.4% | 3.4% |
|                   | 20    | 3.5% | \$0.10                | \$0.11             | 4.3% | 3.4% |
| Demand CV         | 0.5   | 2.1% | \$0.03                | \$0.07             | 2.9% | 2.5% |
|                   | 0.7   | 3.6% | \$0.07                | \$0.09             | 4.3% | 3.2% |
|                   | 0.9   | 5.1% | \$0.11                | \$0.09             | 5.9% | 4.6% |
| Expected Lifetime | 2     | 6.0% | \$0.06                | \$0.16             | 7.3% | 6.0% |
|                   | 3     | 4.4% | \$0.08                | \$0.11             | 5.5% | 4.1% |
|                   | 4     | 3.3% | \$0.08                | \$0.07             | 4.1% | 2.8% |
|                   | 5     | 2.8% | \$0.07                | \$0.06             | 3.2% | 2.4% |
|                   | 6     | 1.7% | \$0.06                | \$0.03             | 1.8% | 1.7% |
| Lifetime CV       | 0.2   | 1.2% | \$0.02                | \$0.04             | 1.7% | 1.2% |
|                   | 0.3   | 3.2% | \$0.06                | \$0.08             | 4.0% | 2.9% |
|                   | 0.4   | 6.4% | \$0.13                | \$0.14             | 7.4% | 6.2% |
| Product Cost      | 0.4   | 1.7% | \$0.05                | \$0.08             | 2.1% | 1.6% |
|                   | 0.55  | 3.2% | \$0.07                | \$0.09             | 3.8% | 2.9% |
|                   | 0.7   | 5.9% | \$0.10                | \$0.08             | 7.2% | 5.7% |
| Batch Size        | 1     | 3.6% | \$0.06                | \$0.10             | 4.3% | 3.4% |
|                   | 2     | 3.6% | \$0.07                | \$0.08             | 4.3% | 3.4% |
|                   | 4     | 3.5% | \$0.07                | \$0.08             | 4.3% | 3.4% |
|                   | 8     | 3.7% | \$0.08                | \$0.08             | 4.6% | 3.5% |

Table 6.1: Sensitivity analysis

Without information sharing, we find that the retailer is able to achieve a level of expected profit for SIRO that is generally midway between the FIFO and LIFO issuing policies. On average, expected profit for SIRO is \$5.53 compared to \$5.21 for LIFO and \$5.88 for FIFO.

However, the average absolute improvement in expected profit due to information sharing for SIRO is much closer to LIFO. On average, we find the VOI for SIRO is \$0.153, while for FIFO it is \$0.199 and for LIFO it is \$0.137. These results indicate that, as with LIFO, the retailer is more constrained in its ability to take advantage of fresher product since doing so may increase outdating of product held in inventory. In fact, when we examine components of the change in profit that arise from information sharing, we observe that they behave similar to LIFO issuing as we demonstrate in Figure 6.1 (which mirrors the analysis presented in Figure 5.2 for LIFO).

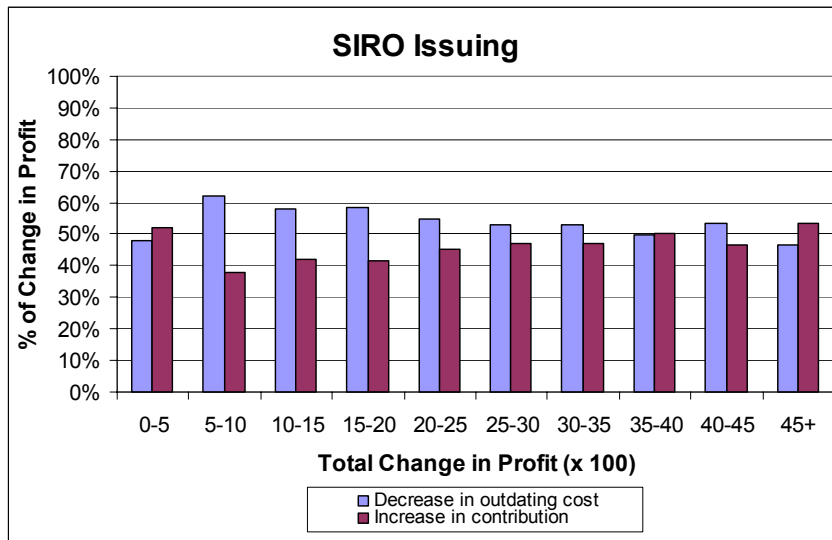


Figure 6.1: The components of the VOI for SIRO issuing

While SIRO generally demonstrates a greater absolute improvement in expected profit with information sharing, the fact that the retailer is better off with SIRO issuing (even without information sharing) can result in lower reported VOI. In Table 6.2, we provide analysis on the cases in which 1)  $LIFO \geq SIRO$ , 2)  $LIFO \geq FIFO$ , and 3)  $SIRO \geq FIFO$ . We also report the conditions (parameter values) that correspond to the scenarios, as indicated by the row headers.

| VOI              | Scenarios | % Scenarios | Mean Demand | Demand CV | Product Lifetime | Order Batch Size | Order Age CV |
|------------------|-----------|-------------|-------------|-----------|------------------|------------------|--------------|
| LIFO $\geq$ SIRO | 543       | 33.5%       | Higher      | Higher    |                  |                  |              |
| LIFO $\geq$ FIFO | 239       | 14.8%       |             |           | Longer           | Lower            | Lower        |
| SIRO $\geq$ FIFO | 146       | 9.0%        | Lower       | Lower     | Longer           | Lower            |              |

Table 6.2: Analysis of cases in which the VOI for one issuing policy is greater than another

## 6.2 Correlation in the Age of Replenished Items

In our base model, we assume that the age distribution of replenished items is stationary over time. In many supply chains of perishable produce; there are hundreds of retailers served by a single supplier so that it is reasonable to assume that the ordering policy of the retailer does not significantly affect the state of the inventory carried at the supplier. There are cases however, where this is not true. Thus, we examine the robustness of our model and findings with respect to the correlation in the age of replenished items. First, we require additional notation. Let  $\rho$  denote the one period correlation in the age of replenished items and let  $\psi_t(\cdot)$  denote the distribution for the lifetime of replenished items for an order placed in the current period. If no replenishment arrives at the beginning of period  $t$ , then

$$\psi_t(x) = \begin{cases} \rho\psi_{t-1}(x+1) + (1-\rho)\psi(x) & 0 < x < M \\ \rho\psi_{t-1}(1) + (1-\rho)\psi(x) & x = M \end{cases}$$

otherwise

$$\psi_t(x) = \begin{cases} (1-\rho)\psi(x) & x \neq a_{t-1} - 1 \\ \rho + (1-\rho)\psi(a_{t-1}) & x = a_{t-1} - 1, a_{t-1} > 1. \\ \rho + (1-\rho)\psi(M) & x = a_{t-1} - 1, a_{t-1} = 1 \end{cases}$$

To explore the VOI in the context of correlation, we use a subset of the experiments we explored in §5 duplicated for values of  $\rho$  where  $\rho \in \{0.0, 0.1, \dots, 1.0\}$ . Since we expect that increasingly higher values of  $\rho$  will decrease the VOI, we restrict the range of other parameter

values to those we know from prior results for which either the VOI is not measurably sensitive or otherwise correspond to conditions of high VOI. In this way, the influence of  $\rho$  is more readily transparent. Hence, we fix the parameters  $\mu_D = 10, C_D = 0.4, w = 0.7, Q = 1$  and vary the parameters  $\mu_A \in \{2, 3, 4, 5, 6\}$  and  $C_A \in \{0.2, 0.3, 0.4\}$ , along with  $\rho$  for the FIFO and LIFO issuing policies in a full factorial design. In total, there are 180 experiments with which to explore the impact of correlation on the VOI.

The results were somewhat of a surprise. While we expect that the VOI would decrease with respect to the correlation, we find a concave relationship. That is, the VOI is greatest at intermediate values of  $\rho$ . Clearly, when  $\rho = 1.0$ , there is no VOI since the age of replenished items is known exactly without information sharing. As for, the concave relationship between  $\rho$  and the VOI, we find that for intermediate values, the cv in the age of replenished items actually increases. Since the VOI is proportional to the cv, we observe the concave relationship. To illustrate our findings, in Figure 5.1 we report the average VOI and average realized  $C_A$  across experiments, at each value of  $\rho$ .

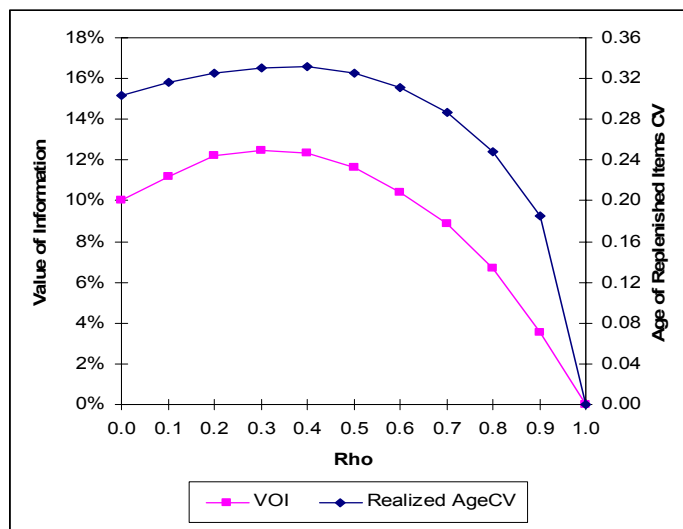


Figure 6.2: The VOI and Age CV as a function of  $\rho$



Another surprising finding that is also readily transparent from Figure 6.2 is that the VOI can remain quite substantial for high values of  $\rho$  and, upwards to  $\rho = 0.6$  the VOI can be higher than the VOI at  $\rho = 0$ . Clearly, we have tilted the balance towards high VOI in our experiments given our choice of parameter values. Even so, we find the same relationship holds over the entire range of parameter values.

### 6.3 Demand Sensitivity to Product Freshness

In our base model, we assume that demand is i.i.d. over time. However, based on the observed behavior of consumers selecting the freshest products first and that perishables, along with their product freshness, have become order winning criteria for food purveyors, it seems to some extent that demand is sensitive to product freshness. That is, we expect a store selling fresher product experiences a higher level of demand than a store selling older product. Hence, we test the robustness of our model and findings with respect to demand sensitivity. To do so, we adopt a simple linear model of demand sensitivity where mean demand  $\mu_{D,t}$  in day  $t$  is a function of 1) a maximum rate of demand  $\mu_D$ , 2) the average lifetime of inventory available for sale at the retailer  $\lambda_t$  relative to maximum lifetime  $M$ , and 3) a constant  $\alpha$  that conceptually represents demand sensitivity to product freshness, where  $0 \leq \alpha \leq 1$ . Here,

$$\lambda_t = \frac{\sum_{x=1}^M xi_{x,t}}{I_t} \quad \text{and} \quad \mu_{D,t} = \mu_D - \mu_D \alpha \left(1 - \frac{\lambda_t}{M}\right).$$

We assume that the  $C_D$  in each period  $t$  is independent of the mean demand rate so that for any  $t$ , total demand  $D$  is a random variable with mean  $\mu_{D,t}$  and cv  $C_D$ . Note that if  $\alpha = 0$ , then  $\mu_{D,t} = \mu_D$  for all  $t$ , corresponding to the case where demand is insensitive to product freshness.

For our experiments, we choose a subset of the experiments defined in §5. Here, we fix the parameters  $\mu_D = 10$  and  $Q = 1$  since our prior results showed no sensitivity to these parameters and choose a factorial design based on the following parameters:

$$C_D \in \{0.5, 0.7, 0.9\} \quad C_A \in \{0.2, 0.3, 0.4\} \quad w \in \{0.4, 0.55, 0.7\}$$

$$\mu_A \in \{3, 4, 5, 6\} \quad \alpha \in \{0.0, 0.25, 0.5\} .$$

The full set of experiments are duplicated for both the FIFO and LIFO issuing policies so that there are a total of 972 experiments with which to explore the impact of  $\alpha$  on the VOI.

Our results show that when demand is sensitive to product freshness, the VOI can be quite substantial and that the VOI increases at an increasing rate with respect to  $\alpha$ . In Table 6.3, we report our summary results for each issuing policy that identifies the average VOI and percentage change in demand, outdating, service, and lifetime of replenished items across experiments.

| % Change in     | FIFO           |                 |                 | LIFO           |                 |                 |
|-----------------|----------------|-----------------|-----------------|----------------|-----------------|-----------------|
|                 | $\alpha = 0.0$ | $\alpha = 0.25$ | $\alpha = 0.50$ | $\alpha = 0.0$ | $\alpha = 0.25$ | $\alpha = 0.50$ |
| VOI             | 3.5%           | 4.8%            | 7.2%            | 2.8%           | 4.0%            | 6.4%            |
| Demand          | 0.0%           | 0.7%            | 2.0%            | 0.0%           | 0.1%            | 0.9%            |
| Outdating       | -56.1%         | -61.6%          | -66.1%          | -9.1%          | -11.6%          | -12.8%          |
| Service         | 1.2%           | 1.3%            | 1.6%            | 1.6%           | 2.1%            | 3.1%            |
| Age of Receipts | 7.6%           | 9.6%            | 11.0%           | 6.6%           | 8.2%            | 10.6%           |

Table 6.3: Summary Results of the VOI with respect to  $\alpha$

With demand sensitivity, information sharing that provides a fresher product provides the capability of increasing the mean demand rate, in addition to reducing product outdating and reducing the service level. As  $\alpha$  increases, increasing the demand rate plays an increasingly greater role in the net profit improvement due to information sharing. These summary results are also representative of the sensitivity of the VOI to parameters and matches quite closely to the results provided to our sensitivity analysis in §5.5. For comparison, we illustrate the sensitivity of the VOI to each parameter and level of  $\alpha$  for both issuing policies in Figure 6.3. The height

of each bar corresponds to the average VOI across experiments for the parameter and value specified on the x-axis and there are three bars for each, corresponding to each value of  $\alpha$ .

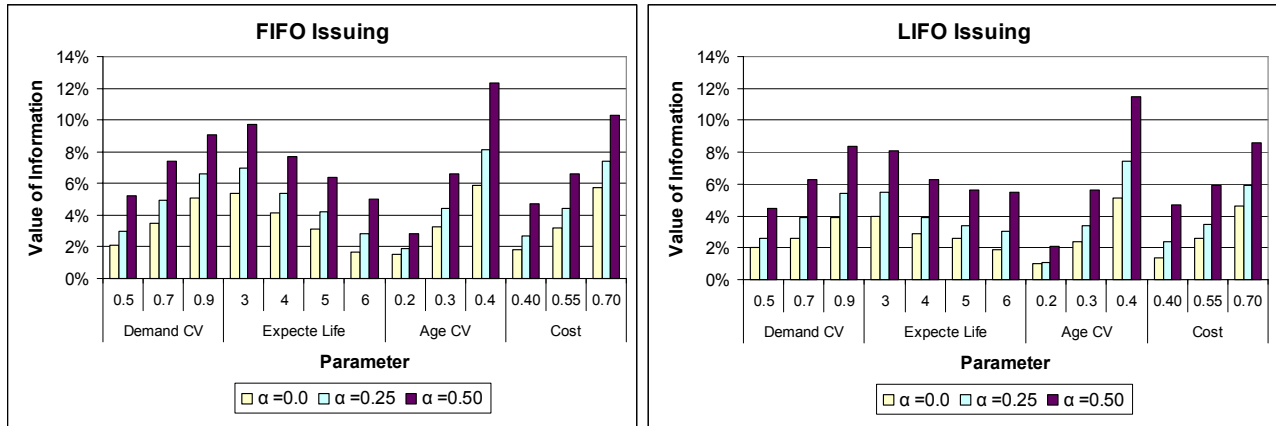


Figure 6.3: Sensitivity of the VOI to the parameters at each level of  $\alpha$ .

## 7. Other Investments

Our exploration of the VOI also enables a comparison to the value of other investments that may be available to the firm to improve the operations of selling perishable products. For example, many firms have opportunities for investments that could influence the issuing policy or that could increase product life times. Since all investments compete for a limited budget of available funds, it is important to compare the return on the capital investment needed for obtaining the age of the product versus these other investment opportunities. Thus, in §7.1 we explore the value of switching the issuing policy used to satisfy demand and in §7.2 we explore the value of increasing the product lifetime.

### 7.1 Switching the Issuing Policy

Practitioners are well aware of the value in controlling inventory issuing with a FIFO issuing policy. Extensive investments are made into specialized equipment including rear-loading shelving systems and gravity wells, in addition to extensive training and labor expenditures to

ensure that perishables are continuously rotated. Here, we provide a comparison on the value of influencing (switching) the issuing policy to that of sharing information.

For our comparisons, we use the set of experiments defined in §5, as well as our extended results with the SIRO policy presented in §6.1. Table 7.1 is representative of the overall results where we show the average percent change in expected profit that arises from switching the issuing policy from 1) LIFO to FIFO, 2) LIFO to SIRO, and 3) SIRO to FIFO. For convenience, we also report the average VOI for each issuing policy.

|                          | FIFO   | SIRO   | LIFO   | LIFO to FIFO<br>% Change | LIFO to SIRO<br>% Change | SIRO to FIFO<br>% Change |
|--------------------------|--------|--------|--------|--------------------------|--------------------------|--------------------------|
| Without information      | \$5.88 | \$5.53 | \$5.21 | 12.7%                    | 6.1%                     | 6.2%                     |
| With information sharing | \$6.13 | \$5.73 | \$5.39 | 13.7%                    | 6.3%                     | 7.0%                     |
| VOI                      | 4.4%   | 3.6%   | 3.4%   | -                        | -                        | -                        |

Table 7.1: The value of switching the issue policy versus the VOI

We find the value of switching the issue policy to FIFO or SIRO is more valuable than the VOI both on average and for a vast majority of the cases we evaluated. Only in approximately 5% of the cases do we find that the VOI is greater than switching from LIFO to FIFO (17% for LIFO to SIRO and 15% for SIRO to FIFO) and these instances, not surprisingly, correspond to where the VOI is greatest – low expected product lifetime, high variability in the age of replenished items, high product cost, and high demand variability. This result indicates that retailers who have not implemented FIFO issuing may be better off trying to do so first, before making investments in information sharing.

## 7.2 Increasing the Product Lifetime

Practitioners often can also invest in equipment that increases product lifetime. Examples include specialized cold storage equipment, chemical treatments, use of preservatives, food

irradiation, and even specialized lighting. Here, we provide a comparison on the estimated benefits of these investments for increasing the product lifetime to that of sharing information.

In Table 7.2 we report the percentage change in expected profit by increasing the product lifetime. The first row of the table denotes the base product lifetime and each subsequent row denotes the expected average improvement in profit by increasing the product lifetime to the number of days indicated by the row header. For comparison, the final row reports the corresponding average VOI for the lifetime indicated in the column header. For example, increasing the product lifetime from 2 days to 3 days increases expected profit by an average of 16.5%. This compares to an increase of 6.7% in average expected profit due to information sharing when the lifetime is two days.

| Expected Life | 2     | 3     | 4    | 5    |
|---------------|-------|-------|------|------|
| 3             | 16.5% |       |      |      |
| 4             | 25.4% | 7.6%  |      |      |
| 5             | 30.6% | 12.1% | 4.2% |      |
| 6             | 34.3% | 15.3% | 7.1% | 2.8% |
| VOI           | 6.7%  | 4.8%  | 3.5% | 2.8% |

Table 7.2: Value of Increasing Product Lifetime

A review of Table 7.2 shows that on average, investments that improve the product lifetime for all lifetimes we evaluate provide a greater benefit than that of information sharing. A further comparison of all individual experiments reveals that in only a few cases is the VOI greater than the value of increasing the product lifetime - even by one day. In 50 (7.7%) of the relevant scenarios, the value of information is greater when increasing the lifetime from 5 days to 6 days. This figure drops to 31 scenarios (4.8%) when increasing the lifetime from 4 days to 5 days. For shorter product lifetimes, there are only 3 scenarios. Collectively, all of these scenarios for which the VOI is greater than increasing the product lifetime correspond to the conditions

demonstrating the highest VOI ( $C_A = 0.4$ ,  $w = 0.7$ , and FIFO issuing) and the average benefit of information sharing relative to increasing the product lifetime is 1.1%.

## 8. Conclusion

In this paper, we study the benefits of information sharing to a retailer that sells a perishable product with a fixed lifetime and is constrained to order in fixed lot sizes. We first describe exact policies for determining the optimal batch size multiple at the retailer for each time period and inventory state. Since the product is perishable, the need to track the age of inventory makes this policy intractable for reasonable sets of parameter values. Thus, we propose heuristic policies for the retailer under both no information sharing and information sharing of the age of the inventory arriving upon replenishment. The heuristic solutions are compared against the exact results and shown to perform well. The heuristics are then used to measure the value of information under a wide range of parameter value settings. We find that the retailer benefits the most from information sharing when: 1) the variability of either demand or the remaining lifetime of replenished items is high, 2) product lifetimes are short, and 3) the cost of the product is high. We also find that information sharing is generally more beneficial when demand is satisfied with a FIFO issuing policy than with a LIFO issuing policy. Upon further investigation, we also find that a random issuing policy (SIRO) results in measurements of the VOI that closely resemble a LIFO issuing policy. In fact, we observe that it is generally more profitable to switch from LIFO (or SIRO) to FIFO issuing (if possible) than from sharing information.

Averaging across all parameter values, we find the average improvement from information sharing is 4.4% for FIFO, 3.6% for SIRO, and 3.4% for LIFO issuing. The benefits of information sharing, however, are not always shared with the supplier. Although the supplier is exogenous to our model, we observe that information sharing may result in a net decrease in

retailer replenishment orders due to a reduction in the amount of retailer outdating and an increase in outdating at the supplier's facility. The benefits of information sharing to the whole supply chain are almost always positive however, indicating the possibility for Pareto improvement through some form of coordination contract. We also study the effect on the VOI when the age of replenished items is correlated over time and when retail demand is sensitive to the product freshness. We conclude with a comparison of the payoffs when investments are available that change the choice of issuing or increase the lifetime of the product.

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## Appendix A

Distributions of  $\psi(\cdot)$  used in the design of experiments:

| $C_A$ | $\mu_A$ | $P\{A = a\}$ |      |      |      |      |      |      |      |      |      |      |  |
|-------|---------|--------------|------|------|------|------|------|------|------|------|------|------|--|
|       |         | 1            | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 10   | 11   |  |
| 0.2   | 2       | 0.08         | 0.84 | 0.08 |      |      |      |      |      |      |      |      |  |
|       | 3       | 0.03         | 0.06 | 0.82 | 0.06 | 0.03 |      |      |      |      |      |      |  |
|       | 4       | 0.02         | 0.03 | 0.05 | 0.81 | 0.05 | 0.03 | 0.02 |      |      |      |      |  |
|       | 5       | 0.01         | 0.02 | 0.03 | 0.04 | 0.80 | 0.04 | 0.03 | 0.02 | 0.01 |      |      |  |
|       | 6       | 0.01         | 0.01 | 0.02 | 0.03 | 0.03 | 0.79 | 0.03 | 0.03 | 0.02 | 0.01 | 0.01 |  |
| 0.3   | 2       | 0.18         | 0.64 | 0.18 |      |      |      |      |      |      |      |      |  |
|       | 3       | 0.07         | 0.14 | 0.60 | 0.14 | 0.07 |      |      |      |      |      |      |  |
|       | 4       | 0.04         | 0.07 | 0.11 | 0.57 | 0.11 | 0.07 | 0.04 |      |      |      |      |  |
|       | 5       | 0.02         | 0.05 | 0.07 | 0.09 | 0.55 | 0.09 | 0.07 | 0.05 | 0.02 |      |      |  |
|       | 6       | 0.02         | 0.03 | 0.05 | 0.06 | 0.08 | 0.54 | 0.08 | 0.06 | 0.05 | 0.03 | 0.02 |  |
| 0.4   | 2       | 0.32         | 0.36 | 0.32 |      |      |      |      |      |      |      |      |  |
|       | 3       | 0.12         | 0.24 | 0.28 | 0.24 | 0.12 |      |      |      |      |      |      |  |
|       | 4       | 0.06         | 0.13 | 0.19 | 0.23 | 0.19 | 0.13 | 0.06 |      |      |      |      |  |
|       | 5       | 0.04         | 0.08 | 0.12 | 0.16 | 0.20 | 0.16 | 0.12 | 0.08 | 0.04 |      |      |  |
|       | 6       | 0.03         | 0.06 | 0.08 | 0.11 | 0.14 | 0.18 | 0.14 | 0.11 | 0.08 | 0.06 | 0.03 |  |

Table A.1: Values for  $\psi(\cdot)$