AN EXPERIMENTAL INVESTIGATION OF AIR FLOW
THROUGH INSULATED TUBING AS A FUNCTION OF APPROACH
TEMPERATURE, PRESSURE RATIO, LENGTH, AND DIAMETER

A THESIS
Presented to
the Faculty of the Graduate Division
by
Richard G. Bradley, Jr.

In Partial Fulfillment
of the Requirements for the Degree
Master of Science in Aeronautical Engineering

Georgia Institute of Technology
June, 1957
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Richard G. Bradley, Jr.
AN EXPERIMENTAL INVESTIGATION OF AIR FLOW
THROUGH INSULATED TUBING AS A FUNCTION OF APPROACH
TEMPERATURE, PRESSURE RATIO, LENGTH, AND DIAMETER

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NOTATION

English

$A_2$  area of metering orifice, sq in.
$A$  area of tubing, sq in.
$C$  coefficient of discharge
$D_1$  inside diameter of pipe, in.
$D_2$  inside diameter of metering orifice, in.
$D$  inside diameter of tubing
$E$  thermal expansion factor
$g$  acceleration of gravity, 32.17 ft per sec per sec
$K_o$  flow coefficient for incompressible flow, $C/\sqrt{1 - \beta^2}$
$K$  flow coefficient for compressible flow
$P_1$  pressure upstream of the metering orifice, psia
$\Delta P$  pressure drop across the metering orifice, psia
$P_2$  pressure upstream of the tube, psia
$P_{10}$  pressure downstream of the tube, psia
$R_n$  Reynolds number, $48 \\frac{\text{w} \cdot \text{n}}{D_2 \cdot \mu_1}$
$R$  gas constant
$r$  pressure ratio, $P_{10}/P_2$
$T_1$  temperature upstream of metering orifice, $^\circ\text{R}$
$T_2$  temperature upstream of tube, $^\circ\text{R}$
$V$  velocity, ft per sec
\begin{itemize}
  \item $w$ flow rate, lbs per sec
  \item $Y$ compressibility factor
  \item Greek
    \begin{itemize}
      \item \( \beta \) diameter ratio, \( D_2/D_1 \)
      \item \( \phi \) flow factor
      \item \( \rho \) density, lbs per cubic ft
      \item \( \mu_1 \) absolute viscosity upstream of orifice, lbs per ft sec
      \item \( 1/\sqrt{1 - \beta^4} \) velocity of approach factor
      \item \( n \) \( 3.1416 \)
    \end{itemize}
  \item Other
    \begin{itemize}
      \item \( T \) thermocouple
      \item \( \circ^\circ F \) degrees Fahrenheit
    \end{itemize}
\end{itemize}
SUMMARY

This paper presents the results of an experimental investigation conducted to determine the flow rate of air through small diameter tubing of various lengths and diameters with special emphasis placed on elevated entrance temperatures. The tubing tested was representative of the type found in the plumbing of aircraft and missile pressure-sensing instrumentation systems.

The analysis was made in a manner similar to that used by the American Society of Mechanical Engineers in analyzing flow measuring devices and nozzles. The test tubing was located between sections of extra-heavy 1/4 inch pipe in which the upstream pressure and temperature were adjusted. The permanent pressure loss resulting from friction in the tube and free expansion at the exit was measured by using pipe taps located upstream and downstream of the test tube. The tubing tested ranged in diameter from approximately 0.180 to 0.555 inches and in length from 10 to 19.5 feet. The study was limited to (a) maximum head pressures of 35 psig, (b) maximum head temperatures of 500°F, (c) negligible approach velocities (small tubing to pipe diameter ratios), and (d) Reynolds numbers between 2,000 and 250,000.

It was determined that the assumption that the flow rate of air varies inversely with the square root of approach temperature was valid for small diameter tubing, at least for the tube sizes and the temperature range investigated. As a result it was possible to demon-
strate that the flow factor, $\theta$, is a pure function of pressure ratio, $r$, length, and diameter shown in plots of $\theta$ vs. $r$. A simple empirical relationship of length and diameter which would reduce the flow factor to a function of pressure ratio alone was not apparent.

An equivalent compressible flow coefficient, $K$, was determined and is presented as a function of pressure ratio, length, and diameter.
CHAPTER I

INTRODUCTION

Modern developments in aircraft and missile design have increased velocity capabilities of such vehicles many fold. The subsequent elevated surface temperature coupled with rapid changes in surface pressures resulting from altitude changes have greatly influenced the accuracy of pressure-sensing instrumentation incorporated within the vehicle. The accuracy of such instrumentation is dependent upon the pressure lag which is present in the plumbing of the pressure-sensing system. Reported instrument inaccuracies have indicated that it is probable that increased entrance temperatures have caused a change in flow characteristics which is not negligible. This possible departure in flow properties at elevated temperature from the normal temperature flow characteristics indicates a need for high temperature investigations of the flow in typical plumbing.

An analysis of actual flow properties of air at elevated temperatures through a sharp-edged orifice and through standard plumbing fittings (tee, straight-through, and elbow) which were typical of fittings present in pressure instrumentation has been made by Ray (1) and Bennett (2). These investigations verified, for room and elevated temperature, the assumption (indicated by theory) that the flow rate through fittings is inversely related to the square root of the approach temperature. Consequently the flow factor, \( \varphi \), was shown to be
defined by an implicit expression involving only the pressure ratio, \( r \).

The objective of this research is to extend the investigation of air flow properties to small bore tubing which is representative of the type used in pressure-sensing instrumentation.
CHAPTER II
APPARATUS

All tests were conducted at the Daniel Guggenheim School of Aeronautics of the Georgia Institute of Technology. A schematic layout of the test apparatus is shown in Fig. 1. Wherever applicable ASME recommendations (3, h) were followed in the test configuration.

Air was supplied for the tests by an electrically powered, 350 cfm-capacity air compressor. After passing through a dryer the air was fed into a storage and surge tank combination, then through a pressure regulator to the supply line. The storage and surge tank combination contained more than ample storage capacity so that constant head pressure runs could be made at the selected test conditions.

The metering section and heater sections were enclosed in double extra-heavy 1/4 inch steel pipe with an inside diameter of 3.15 inches. All pipe downstream of the heater sections was extra-heavy 1/4 inch steel pipe which was found to give a negligible approach velocity for the small diameter tubing tested. The additional wall thickness for the pipe in the vicinity of the heater sections was chosen to give an adequate margin of safety at the maximum working pressures and temperatures.

Flow rates were determined with an ASME (h) standard metering orifice (see Fig. 2) having a $\beta$ ratio of 0.2, where $\beta$ is the ratio of the metering orifice diameter to the pipe inside diameter, ($D_2/D_1$).
Flange taps were employed to determine the pressure drop across the orifice which was constructed from stainless steel to conform to ASME specifications. The metering orifice was located 30 inches upstream of the heater section; resulting in negligible temperature effect on the metering section.

Drawings of a typical heater section and its components are shown in Figs. 3 and 4. Heat was supplied by twelve ferrod strip heaters rated at 1200 watts each at 220 volts. The heaters were located inside the pipe in three banks of four heaters each. Sheet metal mounting rings held the heaters in a circular configuration as shown in Fig. 4. Ignitor plug cases were used to transmit current through the heater section pipe to each bank of heaters. Copper leads were run from the plug electrodes to copper collector rings at either end of the heater banks, providing a parallel circuit for each bank. Current to each bank of heaters was controlled by a tapped auto-transformer having a continuous range from zero to one hundred per cent power. External Kaylo high temperature insulation was used to insulate the heater sections and all pipe downstream as well as the test tubing. In addition the copper pressure leads used for measuring static pressure in the tubing at various positions along the length of the tube were insulated to approximately three feet from the test tubing. A photograph of the insulated test circuit is shown in Fig. 5.

Three unshielded thermocouples constructed from 0.032 inch diameter chromel-alumel wire were used to measure temperature. A potentiometer with the balance scale calibrated in both millivolts and
$^{\circ}$F was used to record temperatures. The thermocouples were located approximately 3.5 pipe diameters upstream of the metering orifice, 3.5 pipe diameters upstream of the test specimen entrance, and 2 pipe diameters downstream of the test specimen exit. The pipe was tapped at the above locations and the thermocouple cases were screwed directly into it.

Pipe taps were used to determine the head pressure and the pressure drop across the tubing test specimen. One-fourth inch copper leads were run from the pipe taps to cistern type manometers which were used to record the pressures. The rapid heat dissipation from the copper pressure leads eliminated the necessity of considering any inaccuracy resulting from heating. Pressures were recorded on 300 cm mercury manometers and the pressure drop, $\Delta P$, across the metering orifice was recorded on a 300 cm alcohol manometer. In order to improve the accuracy in reading the pressure drop across the metering orifice for low flow rate conditions an additional 16 inch alcohol vernier manometer, sensitive to 0.001 inches of alcohol, was used for low head pressure runs. This manometer was set up with a simple valve arrangement thus allowing it to be cut out of the system for high flow rate conditions.

A 0.020 inch hole was drilled in the test specimen wall at seven lengthwise stations and a standard tee fitting, bored to slip over the tube, was silver soldered into position over each hole. The third leg of each tee consisted of a female pipe thread which was connected to a $\frac{1}{4}$ inch pressure lead by means of a straight-through com-
pression fitting. Pressures were transmitted to the manometers by 1/4 inch copper tubing. Pressures along the length of test tubing were recorded with 300 cm mercury manometers and the pressure drop between each two stations was recorded with 300 cm alcohol manometers. The alcohol manometers were arranged with a needle valve at the top in order that they could be cut out of the system for high flow rates when the magnitude of the pressure drop exceeded the range of the 300 cm alcohol manometers. For these conditions the pressure drops were determined from the lengthwise pressure readings on the mercury manometers. A drawing of a typical test specimen and its manometer arrangement is shown in Fig. 6.

A 1-1/2 inch gate valve was located downstream of the test tube exit. By closing this valve and applying pressure to the test circuit a positive check for leaks could be made before each run.

The tubing tested was standard, hard-drawn, type K, copper tubing. The tubing was polished internally to reduce excess surface roughness. This was accomplished by passing a rotating tube of smaller diameter with a strip of emery cloth attached to one end through the specimen. A list of the lengths and diameters of the tubing investigated is given in Table 1. The inside diameters shown are nominal diameters since a slight variation was found over the length tested. The maximum variation was of the order of 0.005 in.

The blank flange at the end of the pipe was tapped for a 3/4 inch pipe thread. The test tubing was then connected to the pipe by means of a reducer and a straight-through compression fitting. In all
In cases the compression fitting and reducer inside diameters were equal to or larger than the test tubing inside diameter.
CHAPTER III
PROCEDURE

The following test procedure was used for all tube sizes examined. The compressor was started and allowed time to reach operating tank pressure of approximately 100 psi. The system was carefully checked for leaks by closing the gate valve at the tube exit and applying pressure with the pressure regulator. If a leak were present it would be indicated by a drop on the manometers over a short period of time. After any leaks present were corrected the gate valve was opened and the flow rate was adjusted by manually varying the head pressure, $P_1$, with the pressure regulator in increments from 0 to approximately 35 psig. The approach temperature, $T_2$, upstream of the test tube entrance was controlled by regulating the amount of current being fed into the heaters with the auto-transformers. The temperature was held constant to within plus or minus 5°F for each pressure setting. Runs were made at room temperature, 250°F, and 500°F.

When stable temperature and pressure conditions were reached the following were recorded: the values of $P_1$, $T_1$, the static pressure and temperature upstream of the metering orifice; $P_2$, $T_2$, the static pressure and temperature upstream of the test section; $P_{10}$, $T_3$, the static pressure and temperature downstream of the test section; and the pressure drop, $\Delta P$, across the metering orifice. For low mass flow conditions the pressure drop across the metering orifice was measured by
means of a 16 inch vernier alcohol manometer in order to improve the reading accuracy.

Values of pressure, $P_{16}$, for each of the seven stations on the tube being tested and the change in pressure, $AP$, between each pair of stations were also recorded to determine the pressure variation with length for each tube. For high flow rate conditions the pressure drops along the test tubing were found to be greater than could be recorded on the 300 cm alcohol manometers. These manometers were then cut out of the system by closing the needle valve and only pressure readings were recorded. It is not the purpose of this report to present the lengthwise pressure distribution in the tubing; however, a detailed analysis may be found in reference 5.

The test Reynolds numbers based on the metering orifice diameter ranged from 2,000 to 250,000. The choice of a metering orifice with a $\beta$ ratio of 0.2 permitted standard ASME methods and discharge coefficients (3) to be used to evaluate the flow rate for Reynolds numbers above 10,000. Flow rates for low Rn (below 10,000) were evaluated by using coefficients determined by Ambrosius and Spink (6).

In order to check the test procedure, runs were repeated at room temperature several days after the original runs. Agreement in data was found to be very good.

The high temperature insulation used to insulate the heated sections and the test specimen proved to be adequate. Heat loss through the tubing walls was estimated to be negligible.
CHAPTER IV

RESULTS

A development of the following basic flow equations for determining the flow rate, \( w \), through the metering orifice may be found in the Appendix. The resulting equation for flow rate is

\[
w = 1.10 \ A_2 \ K_0 \ Y \ E \ \sqrt{\frac{P_1 \ \Delta P}{T_1}} \quad (1)
\]

where

- \( A_2 \) = orifice area
- \( K_0 \) = flow coefficient \( = \frac{C}{\sqrt{1 - \beta^4}} \)
- \( Y \) = compressibility factor
- \( E \) = thermal expansion factor

Before applying the conditions of the test to equation (1) one notes that for \( \beta = 0.2 \) the velocity of approach factor, \( \sqrt{1 - \beta^4} \), is essentially unity. Therefore, \( K_0 \) is equal to the discharge coefficient, \( C \). In addition the factor \( E \) is equal to 1 for room temperature. Thus the flow equation becomes

\[
w = 1.10 \ A_2 \ C \ Y \ \sqrt{\frac{P_1 \ \Delta P}{T_1}} \quad (2)
\]

The flow rate, \( w \), was determined through the orifice and was applied to the test section.
Perry (7) made use of the relation predicted by theory that flow rate through sharp edged orifices is proportional to the orifice area and inversely proportional to the square root of the approach temperature.

That is:

\[ w = \frac{A_2}{\sqrt{T_1}} f(P_1, P_2) \]  

(3)

He defined a flow factor, \( \Omega \), such that

\[ \Omega = \frac{w \sqrt{T_1}}{P_1 A_2} = f(r) \]  

(4)

where \( r \) is the pressure ratio. Experimentally he verified the assumptions of equation (3) by showing \( \Omega \) was a function of \( r \) alone for room temperatures. This fact was further verified for temperatures from room to 500°F by Ray (1) who in addition showed that the assumption was valid for flow through standard tee, elbow, and straight-through fittings for the same temperature range. This was done experimentally by showing \( \Omega \) to be a pure function of \( r \).

A flow factor, \( \Omega \), can be defined for the flow of air through tubing in a similar manner, as

\[ \Omega = \frac{w \sqrt{T_2}}{P_2 A} \]  

(5)

where \( A \) is the area of the tube, \( T_2 \) is the approach temperature, and \( P_2 \) is the pressure upstream of the tube entrance. Values of \( \Omega \) were
computed for each test run. The resulting values for flow factor are
presented in plots of $\Omega$ vs. $r$ in Figs. 7, 8, and 9.

The assumption that the flow rate through small diameter tubing
varies inversely as the square root of the approach temperature is
seen to be valid at least to an entrance temperature of $500^\circ$F for the
range of tubing lengths and diameters examined. The maximum deviation
of test points for the high temperature runs from the room temperature
runs is of the order of 5 per cent. This deviation occurred only in
a few isolated cases for high mass flow through the 10 ft. length of
tubing and may be considered within the test accuracy.

As may be noted in Figs. 7, 8, and 9 the flow factor, $\Omega$, can
be expressed as a pure function of pressure ratio, $r$, for a given
length and diameter of tubing. In order for the results to be more
effectively utilized it would be desirable to determine some simple
empirical relation of length and diameter which combined with $\Omega$ would
define a new flow factor that could be expressed implicitly as a
function of $r$. A careful examination of the results was made to see
if such an empirical relation of length and diameter did exist. After
extensive investigation no such relation was apparent.

As has been previously noted the ratio of pipe to test tubing
diameter resulted in an approach velocity which was negligible. Con-
sequently the data presented may be reasonably applied to a system
which consists of two large chambers connected by a single length of
tubing. The pressure loss across the tube represents in addition to
the total frictional pressure loss through the tube the loss due to
free expansion at the tube exit.

The flow rate equation for the test specimen may be written as

\[ w = 1.10 A K_0 Y E \sqrt{\frac{P_2 \Delta P}{T_1}} \]

where \( \Delta P \) is the pressure drop across the tube. Rearranging and assuming the thermal expression factor, \( E \), to be unity for the temperature range considered gives

\[ K_0 Y = \frac{w}{1.10 A \sqrt{\frac{T_2}{P_2 \Delta P}}} \]  

(6)

Defining \( K_0 Y \) as the compressible flow coefficient, \( K \), and noting from equation (4) that

\[ w = \frac{\Omega P_2 A}{\sqrt{T_2}} \]  

(7)

gives

\[ K = \frac{\Omega}{1.10} \sqrt{\frac{P_2}{\Delta P}} \]  

(8)

Since \( \Delta P = (P_{10} - P_2) \), it follows that

\[ K = \frac{\Omega}{1.10 \sqrt{1 - r}} \]  

(9)

where \( r \) is the pressure ratio, \( P_{10} / P_2 \). Thus an equivalent flow
coefficient for compressible flow in small-diameter tubing is defined as a function of flow factor and pressure ratio.

The flow factor for a given length and diameter has been shown to be a pure function of pressure ratio. Therefore, the compressible flow coefficient can be expressed as a function of pressure ratio, length, and diameter. Plots of flow coefficient, $K$, versus pressure ratio, $r$, for the various tubing lengths and diameters examined are presented in Figs. 10, 11, and 12.
CHAPTER V
CONCLUSIONS

The conclusions reached are necessarily restricted by the conditions of negligible approach velocity, maximum approach pressure and temperature of 35 psig and 500°F respectively, and Reynolds numbers based on metering orifice diameter varying from 2,000 to 250,000. Within these limitations it can be concluded that:

1. For the flow of air through insulated tubing the assumption that the flow rate varies inversely as the square root of the approach temperature is valid for small diameter tubing of lengths between 10 and 19.5 feet.

2. In analyzing the flow rate of air through insulated tubing of diameter 0.180 to 0.555 inches and lengths from 10 to 19.5 feet the flow factor, $\varphi$, may be expressed as a function of pressure ratio, length, and diameter as shown in Figs. 7, 8, and 9.

3. There is no apparent simple relation of length and diameter which will, when combined with flow factor, result in an implicit function of pressure ratio.

4. A compressible flow coefficient, $K$, may be defined as a function of pressure ratio, length, and diameter as shown in Figs. 10, 11, and 12 for flow in small bore tubing of length 10 to 19.5 feet and diameter of 0.180 to 0.555 inches.
CHAPTER VI

RECOMMENDATIONS

The plumbing found in typical pressure-sensing systems is composed of lengths of small diameter tubing connected with standard plumbing fittings. It is therefore recommended that a study be conducted to determine the flow characteristics of a system consisting of tubing and standard straight-through, tee, and elbow fittings.

It is possible that surface temperatures on a high speed vehicle could exceed 500°F and that approach velocities to the pressure-sensing system may be of such a magnitude that they cannot be neglected. With these conditions in mind it is further recommended that the study of the flow of air in tubing be extended to a higher entrance temperature range and to non-negligible approach velocity conditions.
APPENDIX

DEVELOPMENT OF THE BASIC FLOW EQUATION

Following the procedure outlined by the ASME (h) it is assumed that the flow through the metering orifice is steady, incompressible, and obeys the perfect gas laws. In addition the assumption is made that there is no loss of energy from friction and no heat transfer takes place between the fluid and the surrounding walls.

Consider the stations 1 and 2 in the element shown in Fig. 13.
The incompressible Bernoulli equation gives

\[ P_1 + \frac{\rho V_1^2}{2 g} = P_2 + \frac{\rho V_2^2}{2 g} \]  \hspace{1cm} (1)

where \( P_1, V_1, P_2, V_2 \) are pressure and velocity at stations 1 and 2 respectively, \( \rho \) is the density, and \( g \) is the acceleration due to gravity.

Rearranging gives

\[ (V_2^2 - V_1^2) \frac{\rho}{2 g} = P_1 - P_2 = \Delta P \]  \hspace{1cm} (2)

From continuity considerations the rate of flow, \( w \), is given by

\[ w = \rho A_1 V_1 = \rho A_2 V_2 \]  \hspace{1cm} (3)

where \( A_1 \) and \( A_2 \) are the cross sectional areas at stations 1 and 2 respectively.

Analyzing the circular cross section gives

\[ w = \rho V_1 \frac{\pi D_1^2}{4} = \rho V_2 \frac{\pi D_2^2}{4} \]  \hspace{1cm} (4)

or

\[ V_1 = V_2 \left( \frac{D_2}{D_1} \right)^2 = V_2 \beta^2 \]  \hspace{1cm} (5)

where \( \beta \) is the ratio of orifice diameter to pipe diameter, \( \frac{D_2}{D_1} \).
Substituting (5) into (2) and simplifying results in

\[ V_2 = \frac{1}{\sqrt{1 - \beta^4}} \sqrt{\frac{2 \cdot g \cdot \Delta P}{\rho}} \]  

(6)

It follows from the equation of state \((P_1 = \rho R T_1)\) and equation (3) that

\[ w = \frac{A_2}{\sqrt{1 - \beta^4}} \sqrt{\frac{2 \cdot g \cdot P_1 \cdot \Delta P}{R \cdot T_1}} \]  

(7)

Since the actual flow varies from the above theoretical relations the ASME (4) suggest three empirical corrections \((K_0, Y, E)\) to the above equation. The incompressible flow coefficient

\[ K_0 = \frac{C}{\sqrt{1 - \beta^4}} \]

combines the coefficient of discharge, \(C\), with the velocity of approach factor, \(\sqrt{1 - \beta^4}\). The compressibility factor, \(Y\), is a function of the \(\beta\) ratio and the pressure ratio, \(r\), and \(E\) is a coefficient which corrects for the thermal expansion of the element.

Now for air the flow rate, \(w\), is given by

\[ w = 1.10 \cdot A_2 \cdot K_0 \cdot Y \cdot E \cdot \sqrt{\frac{P_1 \cdot \Delta P}{T_1}} \]  

(8)
Table 1

Tubing Tested

<table>
<thead>
<tr>
<th>Length (ft)</th>
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<tbody>
<tr>
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TEST APPARATUS

FIG. 1
METERING ORIFICE

FIG. 2
2 HOLES TAPPED FOR HEATER LEADS

SECTIONS AA & BB

HEATER SECTION

FIG. 3
HEATER COMPONENTS

FIG. 4
INSULATED TEST CIRCUIT

FIG. 5
TYPICAL TEST TUBE & MANOMETER ARRANGEMENT

FIG. 6
FLOW FACTOR vs PRESSURE RATIO
LENGTH = 19.5 FEET
FIG. 7
FLOW FACTOR vs PRESSURE RATIO
LENGTH = 100 FEET
FIG. 9
FLOW COEFFICIENT vs PRESSURE RATIO
LENGTH = 195 FEET
FIG. 10
FLOW COEFFICIENT vs PRESSURE RATIO
LENGTH = 15.0 FEET
FIG. 11
FLOW COEFFICIENT vs PRESSURE RATIO
LENGTH = 10.0 FEET
FIG. 12
REFERENCES


