A LOGARITHMIC PHOTOMETER

A THESIS

Presented to

The Faculty of the Graduate Division

by

Jay Henry Schlag

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Electrical Engineering

Georgia Institute of Technology

August, 1965
be allowed without written permission.

Section 17 of the Crown Copyright Act 1958 does not

cover or copy this work, or for purposes for which a

society or other organization is permitted to copy

such copies or for the purposes of these purposes

by a person under whose direction it was written.

I, the person of whom the dissertation is

written, do not make it available for inspection.

In presenting the dissertation as a part of fulfillment of

the requirements for an advanced degree from the

Institute of Technology, I agree that the Institute of
ACKNOWLEDGMENTS

I wish to express my sincere appreciation to Dr. John W. Hooper, my thesis advisor, for his help and encouragement. Also, I want to thank Dr. F. Kenneth Hurd as a member of the reading committee for his guidance in the composition of this thesis. To Dr. Howard P. Edwards, as a member of the reading committee, I wish to express my gratitude for his interest and the interest of the Space Sciences Laboratory of the School of Aerospace Engineering. The use of the equipment loaned by Dr. Frank C. Nottingham is also sincerely appreciated.
TABLE OF CONTENTS

ACKNOWLEDGMENTS ........................................... ii
LIST OF ILLUSTRATIONS ................................. v
SUMMARY ...................................................... vii
Chapter
I. INTRODUCTION ............................................. 1
   History
   Photometer Requirements
II. SOME ASPECTS OF PHOTOMULTIPLIER TUBE THEORY ........ 8
III. INSTRUMENTATION AND EQUIPMENT ...................... 11
   Equipment Block Diagram
   Light Box
   Matrix Board
   Light Source Power Supply
   Switching Matrix
IV. PRELIMINARY EXPERIMENTS ............................ 16
   Verification of Exponential Model
   Effects of Spectral Variations on Dynode Characteristics
V. MATHEMATICAL APPROXIMATIONS ...................... 21
   The Engstrom-Fischer Circuit
   Mullivalued Response
VI. CLAMPED DEFOCUSING CIRCUIT ...................... 27
   Anode Group
   Dynode Group
   Cathode Group
VII. PHOTOMETER CIRCUIT DESIGN ....................... 41
   Design Procedure
   Experimental Circuit
   Response Considerations
<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Individual Dynode Groups</td>
<td></td>
</tr>
<tr>
<td>Linearity</td>
<td></td>
</tr>
<tr>
<td>Reliability</td>
<td></td>
</tr>
<tr>
<td>Compressed Output</td>
<td></td>
</tr>
<tr>
<td>Types of Tubes</td>
<td></td>
</tr>
<tr>
<td>Practical Considerations</td>
<td></td>
</tr>
<tr>
<td>Conclusions</td>
<td></td>
</tr>
<tr>
<td>APPENDIX I</td>
<td>50</td>
</tr>
<tr>
<td>APPENDIX II</td>
<td>53</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>55</td>
</tr>
</tbody>
</table>
# LIST OF ILLUSTRATIONS

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Feedback Control Circuit</td>
<td>3</td>
</tr>
<tr>
<td>2.</td>
<td>Series Resistor Circuit</td>
<td>5</td>
</tr>
<tr>
<td>3.</td>
<td>Response of Series Resistor Circuit</td>
<td>6</td>
</tr>
<tr>
<td>4.</td>
<td>Photomultiplier Tube Construction</td>
<td>9</td>
</tr>
<tr>
<td>5.</td>
<td>Equipment Block Diagram</td>
<td>12</td>
</tr>
<tr>
<td>6.</td>
<td>Light Box Construction</td>
<td>13</td>
</tr>
<tr>
<td>7.</td>
<td>Dynode Characteristics for RCA 931-A</td>
<td>18</td>
</tr>
<tr>
<td>8.</td>
<td>Engstrom-Fischer Circuit</td>
<td>22</td>
</tr>
<tr>
<td>9.</td>
<td>Approximate Equivalent Circuit</td>
<td>23</td>
</tr>
<tr>
<td>10.</td>
<td>Clamped Defocusing Circuit</td>
<td>28</td>
</tr>
<tr>
<td>11.</td>
<td>Separation of Clamped Defocusing Circuit</td>
<td>29</td>
</tr>
<tr>
<td>12.</td>
<td>Anode Group Detail</td>
<td>31</td>
</tr>
<tr>
<td>13.</td>
<td>Dynode Voltage Versus Anode Current</td>
<td>31</td>
</tr>
<tr>
<td>14.</td>
<td>Light Response of Anode Group</td>
<td>33</td>
</tr>
<tr>
<td>15.</td>
<td>Dynode Group Detail</td>
<td>33</td>
</tr>
<tr>
<td>16.</td>
<td>$V_8$ and $V_9$ Versus $(I_4 - I_3)$</td>
<td>33</td>
</tr>
<tr>
<td>17.</td>
<td>Circuit to Measure Gain as a Function of Dynode Voltage</td>
<td>35</td>
</tr>
<tr>
<td>18.</td>
<td>Graphical Construction</td>
<td>36</td>
</tr>
<tr>
<td>19.</td>
<td>Dynode Group Gain Versus Input Current</td>
<td>37</td>
</tr>
<tr>
<td>20.</td>
<td>Comparison of Gain Curves for Different Total Dynode Group Voltages</td>
<td>39</td>
</tr>
<tr>
<td>21.</td>
<td>Cathode Group Detail</td>
<td>39</td>
</tr>
<tr>
<td>22.</td>
<td>Test Circuit</td>
<td>42</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>23</td>
<td>Divider Resister Determination</td>
<td>44</td>
</tr>
<tr>
<td>24</td>
<td>Experimental Curve</td>
<td>46</td>
</tr>
<tr>
<td>25</td>
<td>Low Voltage Power Supply Schematic</td>
<td>51</td>
</tr>
<tr>
<td>26</td>
<td>Switching Matrix Schematic</td>
<td>52</td>
</tr>
</tbody>
</table>
SUMMARY

This thesis describes the development of a device which measures a large range of light intensities on a single approximately logarithmic scale. This was achieved by the use of a photomultiplier tube and a passive network. Within the framework of this study are included the evolution of the photometer circuit, an approximate mathematical model, a circuit description, a practical design procedure, and recommendations for various applications.
CHAPTER I

INTRODUCTION

The measurement of light has been an important subject for investigation for many years. During this time, light measuring devices have undergone many changes to keep abreast of technical demands, of which those created by the exploration of space are the most recent. For many space applications it is desirable to measure extremely large variations in light with a very small photometer and to achieve this large range the photometer must be non-linear. In most cases this non-linearity will be approximately logarithmic*; consequently such a device is often called a logarithmic photometer.

History

There has been considerable interest in the development of a logarithmic photometer since 1945 when the photomultiplier tube became commercially available as a photosensor. Most of the original research concerning wide range light measurement was performed by companies interested in the development of film and film processes. The light density of film has a range of about three decades and it is desirable to measure these densities on a single scale.

Feedback control can be employed to decrease the sensitivity of

*The term "Logarithmic" as it is used in this sense is discussed in Appendix II.
a photomultiplier tube as the output current increases.\textsuperscript{1,2,3,4} A simplified circuit diagram of a feedback control photometer is shown in Figure 1. The feedback will adjust the stage voltage so that the anode current is held approximately constant for variations in light intensity. If the circuit is properly designed, the output voltage will decrease logarithmically with increases in input light intensity.

Although this circuit has a logarithmic characteristic, there are certain limitations to its use. Since this characteristic is a function of the photomultiplier tube parameters alone, it cannot be changed without changing the tube. In addition, the high voltage amplifier is too large for many airborne applications.

To reduce the size, the amplifier in the feedback control circuit was replaced with a switching circuit.\textsuperscript{6} In this type of system the switch reduces the dynode voltage as the anode current reaches a predetermined limit. A relay is commonly used for this switching device because of the high supply voltage. The relay realizes a reduction in size but its switching time retards the response of the photometer. This can be very important when the light intensity changes rapidly, since a sudden burst of light could damage the tube before the relay has sufficient time to switch.

In logarithmic photometry, the efforts of recent years have been directed toward the use of passive networks in conjunction with the photomultiplier tube.\textsuperscript{5,6} This type of circuit depends on the "defocusing" action of the photomultiplier tube. (Defocusing is the decrease in sensitivity of the tube as the relative dynode voltages become unequal.)
Figure 1. Feedback Control Circuit.
A passive network developed by Engstrom and Fischer is shown in Figure 2. In this circuit $R_1$ through $R_{10}$ are used as a voltage divider to split the total supply voltage equally between the dynodes. Resistors $R_{11}$ through $R_{21}$ in series with the dynodes cause the relative dynode voltages to become unequal as the dynode currents increase. This in turn decreases the sensitivity and extends the range.

There are several disadvantages to this type of circuit, the main disadvantage being that it has a limited range of single valued response. A typical response is shown in Figure 3. Note that the anode current is single valued only over a small portion of the light intensity range. Such a photometer circuit is unsuitable as a measuring device, since the reading is ambiguous. The primary use of this circuit has been to limit the anode current of photomultiplier tubes used in on-off applications such as automobile headlight dimmers.

**Photometer Requirements**

The requirements for the photometer considered in this thesis were established by the Space Sciences Laboratory of the School of Aerospace Engineering, under the direction of Dr. Howard D. Edwards. This particular photometer would be used to study ultra-violet radiation in the upper atmosphere. The desired requirements are:

1. An intensity range of $10^3$ to $10^6$.
2. A threshold of $0.5 \times 10^{-9}$ lumens.
3. A sensitivity of $0.01$ amperes per lumen.
4. A minimum size and weight.
Figure 2. Series Resistor Circuit.
Figure 3. Response of Series Resistor Circuit.
The purpose of this research is the design of a practical circuit which fulfills these requirements and the study of the effect of circuit parameters on photometer response. The most feasible approach of the ones investigated was the utilization of the defocusing action and use of limiting to maintain a single valued response.
CHAPTER II

SOME ASPECTS OF PHOTOMULTIPLIER TUBE THEORY

The purpose of this section is the presentation of that portion of photomultiplier tube theory\textsuperscript{8,9,10,11,12} which applies most directly to the development and analysis of the circuit presented in this thesis.

The photomultiplier tube is constructed of a photoemissive cathode, nine to fifteen dynodes, and an anode, as shown in Figure 4. Light enters the tube and strikes the photoemissive cathode. Electrons in the cathode are given sufficient energy to escape the cathode surface by the incoming photons.

The electrons that escape the cathode are accelerated to the first dynode through a positive potential. Unlike the photon, which can transfer its energy to only one electron, the incident or primary electron can excite several dynode electrons to a sufficient energy to escape the dynode.

The electrons which leave the first dynode are accelerated to the second dynode and excite a greater number of electrons. The process continues through all the dynodes until the electrons are collected at the anode.

As the electrons pass through the tube, each dynode multiplies the number of electrons, thus making the tube gain the product of all the dynode gains, as long as no electrons are lost. This "focused condition" exists if the differential dynode voltages are equal. If
Figure 4. Photomultiplier Tube Construction.
the differential dynode voltages are unequal, the electrostatic fields between the dynodes becomes distorted and some of the electrons miss the succeeding dynode. This is the defocused condition.

The gain of the dynode is a function of the impact energy of the primary electrons and, therefore, a function of the acceleration voltage. The gain versus voltage relationship can be approximated by the empirical expression:

\[ G = (V/V_c)^B \]  

(1)

Where \( V \) is the dynode voltage and \( V_c \) and \( B \) are constants of the dynode. Verification of this relationship is treated in Chapter IV. When the differential dynode voltages become unequal, Equation (1) does not accurately describe the gain relationship because defocusing occurs.

For the purpose of this thesis the reduction in gain caused by changes in impact energy will be called energy attenuation, and reductions in gain caused by defocusing will be called defocusing attenuation. The sum of these reductions will be called the nonsymmetric attenuation.
CHAPTER III

INSTRUMENTATION AND EQUIPMENT

This chapter presents a discussion of the non-standard items of equipment used in the instrumentation of this research. A discussion of the equipment block diagram will indicate the interconnections between the items of equipment employed.

**Equipment Block Diagram**

The block diagram of the equipment used is shown in Figure 5. The source power supply delivers 0-5 volts d.c. to the light source located in the light tunnel. The light shines down the tunnel to the photomultiplier tube located in the detector housing. The photomultiplier tube is connected to the matrix board on which the passive network under study is built. The high voltage power supply delivers 0-2500 volts d.c. to the matrix board for the dynode voltages. A five microampere meter with a variable shunt are connected to the matrix board to read the dynode and anode currents.

**Light Box**

The light box, illustrated in Figure 6, consists of a detector housing and a light tunnel. The detector housing contains the photomultiplier tube with its associated mounting assembly. At the entrance to the housing is an adjustable light aperture. The light tunnel contains a light source which can be moved forward or backward along a rail.
Figure 5. Equipment Block Diagram.
Figure 6. Light Box Construction.
The three volt incandescent lamp used as a light source produces a usable intensity range of approximately 5000, and an additional range of 23 can be achieved by moving the light source. The total range of $10^5$ is one decade less than the upper limit desired for the photometer, but proved sufficient to verify the photometer design procedure. A linear photometer circuit was used to calibrate the light tunnel so that the nonlinearity of the source and the reflection of the tunnel walls would be included in the calibration.

**Matrix Board**

The matrix board was used to temporarily connect a large number of different circuits. Its base is a piece of plexiglass with legs. Connections are made from the bottom of the board, and network components are connected by means of screw terminals to the top. The simplicity of the board provided both ease of connection and dependability in operation. The open construction insured low leakage currents between adjacent terminals.

**Light Source Power Supply**

The light source power supply* was a 0-5 volt regulated power supply employing a full wave silicon rectifier and a transistor regulator. Source temperature compensation was possible by means of a thermistor in the voltage feedback circuit.

**Switching Matrix**

The switching matrix* was used in place of the matrix board

---

*A schematic diagram is shown in Appendix I.
when the ammeter was to be switched between dynodes during circuit operation. The switching was performed by relays activated by a set of push buttons. A holding circuit was used to keep the selected relay energized until a release button was depressed.
The preliminary experiments in this research were a study of those characteristics of the photometer tube which seemed important in the design of photometers. The first experiment was a verification of a mathematical model and a determination of its limits of validity. The second was an investigation of the effects of spectral variations on dynode characteristics.

**Verification of the Exponential Model**

Several mathematical models have been proposed for the voltage-current characteristics of the photomultiplier tube. One of the simplest relates the dynode current gain to the dynode voltage by the exponential formula:

\[ G = \left( \frac{V}{V_c} \right)^B \] (2)

In this formula \( G \) is the current gain of the dynode, \( V \) is the dynode voltage, \( V_c \) is a voltage constant of the dynode, and \( B \) is an exponential constant of the dynode. If the photomultiplier tube has \( N \) identical dynodes then the total equivalent current gain of the tube is given by:

\[ G_T = \left( \frac{V}{V_c} \right)^{NB} \] (3)

The anode, or output, current can then be expressed as:
\[ I_a = FS \left( \frac{V}{V_c} \right)^{NB} \]  

where \( S \) is the cathode sensitivity in amperes per lumen and \( F \) is the light intensity in lumens.

Taking the logarithm of both sides of (10) one obtains

\[ \log I_a = \log SF + NB \log \left( \frac{V}{V_c} \right) \]  

and

\[ \log I_a = \log SF + NB \log V - NB \log V_c \]  

If the input light intensity is held constant and the differential of both sides is taken, one obtains

\[ d(\log I_a) = NB \, d(\log V) \]  

Solving for \( NB \) yields

\[ NB = \frac{d(\log I_a)}{d(\log V)} \]  

Hence, if \( \log I_a \) is plotted versus \( \log V \) then the slope of the curve should equal \( NB \).

Verification of this model can be made by plotting experimental values of \( \log I_a \) versus \( \log V \), with constant \( F \), to determine whether the slope of the curve is a constant. This technique allows \( NB \) to be determined without knowing \( F \), \( S \), or \( V_c \).

The anode current of RCA 931-A photomultiplier tube was measured as the voltage per dynode was varied from 35 to 125 volts. The experimental points of \( \log I_a \) versus \( \log V \) is shown in Figure 7, with a
Figure 7. Dynode Characteristics for RCA 931-A.
constant slope drawn through the points. The fact that the experimental points fall so close to a straight line indicates that this model is quite accurate over this range of operation. In this case the experimental value of NB was 6.93 or B = .770. This procedure was repeated with a CBS CL-1002 photomultiplier tube obtaining a straight line with a slope of 8.92.

The voltage-current characteristics of the seventh, eighth, and ninth dynodes of the RCA 931-A were found experimentally by measuring log I₇, log I₈, and log I₉ as a function of log V as above. The following values were determined by plotting the measured points and calculating the slopes and intercepts.

1. seventh dynode - B₇ = .820, V₇ = 12.0 v.
2. eighth dynode - B₈ = .873, V₈ = 19.0 v.
3. ninth dynode - B₉ = .708, V₉ = 19.8 v.

B for identical dynodes should be .770. It can be seen from these values that all dynodes do not have identical characteristics. It should be noted that in each experiment all the dynodes were varied together. It is important to show whether the exponential model can be used when defocusing occurs. To determine this, the relative gain of the RCA 931-A was measured as a function of the eighth dynode voltage. This was compared to the relative gain as computed by the exponential model. The actual gain was as much as 40% lower than the calculated gain. This would indicate that considerable error should be expected when using the exponential model in defocusing circuits.
Effects of Spectral Variations on Dynode Characteristics

It is well known that the sensitivity of a photomultiplier tube changes with different wavelengths of incident light. It was the purpose of this experiment to show that the dynode characteristics are independent of the light wavelength.

In order to determine whether the input wavelength effects the dynode characteristics, the tube characteristics were measured with different filters over the light source. Measurements were made with a red filter, with a blue filter, and with the source filament at a lower temperature. The bulb temperature was reduced by reducing the source voltage. The source was moved closer to the detector to maintain the same intensity.

The slope, NB, of the measured points was the same in each case as the original NB of Figure 7. Hence it can be concluded that the changes in light wavelength had no appreciable effect on the dynode characteristics. This means that the input light wavelength and the dynode characteristics are independent functions over the range of wavelengths experimentally employed. It follows, that a control network dependent on the dynode characteristic would be independent of the wavelength.
CHAPTER V

MATHEMATICAL APPROXIMATIONS

It was shown in the previous chapter that the exponential model could be used to approximate the response of a photomultiplier tube under certain conditions. It is instructive then, to study a simple network mathematically, using this model. It should be noted that this is not a valid design approach since the model is a poor approximation when defocusing occurs. This study will show, however, some of the characteristics of photomultiplier tubes and passive networks.

The Engstrom-Fischer Circuit

Engstrom and Fischer have described a circuit which utilizes a passive network in connection with a photomultiplier tube. The circuit was designed to protect photomultiplier tubes from overload conditions. In this circuit resistors are placed in series with the dynodes as shown in Figure 8. As the current through the dynodes increase, a voltage across the series resistor is developed which defocuses the tube and reduces its gain. This circuit is of importance because a resistor network is the simplest type of control network. Therefore, a study of this circuit will be instructive in determining the actual network to be used in the photometer circuit.

To study the operation of this circuit, the approximate equivalent circuit, shown in Figure 9, was developed based on the exponential model. In this equivalent circuit, the current generators represent the dynode
Figure 8. Engstrom-Fischer Circuit.
Figure 9. Approximate Equivalent Circuit.
current gain and the fixed voltage generators represent the voltage across the divider resistors. The main disadvantage of this circuit is that the anode current as a function of light intensity is multivalued as was discussed in Chapter I and shown in Figure 3. It is important to find the reason for the multivalued response and what circuit additions can be made to keep the response single valued.

**Multivalued Response**

The mathematical basis for the multivalued response can be shown by finding what factors affect the dynode currents. Referring to the equivalent circuit, $I_2$ can be expressed as:

$$I_2 = I_1 \left( \frac{V_1}{V_c} \right)^B$$  \hspace{1cm} (8)

The first dynode voltage is

$$V_1 = V_s + R_2(I_2 - I_1) - R_1 I_1$$ \hspace{1cm} (9)

For the special case of $B$ equal to one, the expression for $V_1$ can be substituted into Equation (8) to obtain:

$$I_2 = \frac{I_1}{V_c} \left[ V_s + R_2(I_2 - I_1) - R_1 I_1 \right]$$ \hspace{1cm} (10)

Solving this expression for $I_2$ results in the following expression:

$$I_2 = \frac{V_s I_1 - (R_2 + R_1)I_1^2}{V_c - R_2 I_1}$$ \hspace{1cm} (11)

The point at which the response becomes multivalued can be determined by setting the derivative of the $I_2$ expression equal to zero and thereby finding the point at which $I_2$ is a maximum. Differentiating
Equation (11) with respect to $I_1$ yields:

$$\frac{dI_2}{dI_1} = \frac{[V_cI_1 - (R_1 + R_2)I_1^2][R_2] + [V_c - R_2I_1][V_s - 2(R_1 - R_2)I_1]}{(V_c - R_2I_1)^2}$$  \hspace{1cm} (12)$$

Setting (12) equal to zero and combining terms yields:

$$R_2(R_1 + R_2)I_1^2 - 2V_c(R_1 + R_2)I_1 + V_cV_s = 0$$  \hspace{1cm} (13)$$

Solving for $I_1$,

$$I_1 = \frac{V_c}{R_2} \pm \sqrt{\frac{V_c}{R_2} - \frac{V_cV_s}{R_2(R_1 + R_2)}}$$  \hspace{1cm} (14)$$

$I_2$ has a maximum and is thus multivalued when there is a real solution for (13). Therefore, when the square root is positive the response is multivalued, and when the square root is negative, the response is single valued. This means that the response becomes single values as:

$$V_c/R_2 = V_s/(R_1 + R_2)$$  \hspace{1cm} (15)$$

Solving for $V_c$ in (15) and substituting into Equation (11) yields the following expression for $I_2$:

$$I_2 = (R_1 + R_2)I_1/R_2$$  \hspace{1cm} (16)$$

Note that this indicates that at the point at which the response becomes single valued, the gain becomes constant and the response no longer logarithmic.

One other fact can be seen by investigation of Figure 9; that is, the response of a particular dynode is a function of all preceding dynodes. $I_3$ is a function of $V_2$ and $I_2$, but $V_2$ is also a function of $I_1$. 
Therefore, $I_3$ is a function of $I_2$ and $I_1$. This can be extended to the $n$th dynode so that $I_n$ is a function of $I_1$ through $I_{n-1}$.

To summarize, the results of this chapter show that in the Engstrom-Fischer circuit there is a mathematical justification for the multivalued dynode response, and that the response of a dynode cannot be made logarithmic and still single valued for all values of $I_1$. It was also shown that the response of a particular dynode is a function of all preceding dynodes.
CHAPTER VI

CLAMPED DEFOCUSING CIRCUIT

Study of the Engstrom-Fischer circuit reveals two basic disadvantages. First, the dynode response tends to be multivalued, and second, the response of each dynode is a function of all preceding dynodes. The clamped defocusing circuit, as shown in Figure 10 has been designed to eliminate these.

By using diode clamping circuits to limit the dynode voltage the multivalued response is eliminated. Interaction between dynodes is reduced by using series resistors only in alternate dynodes so that interaction is limited to dynode pairs.

Since pairs of dynodes have little interaction, their response is independent; therefore, the circuit can be broken into separate groups as illustrated in Figure 11. The indicated independence of these groups has been verified experimentally by comparing the characteristics of dynode group No. 2 for various dynode voltages in adjoining dynode groups. It was found that variations in the adjoining dynode groups had little effect upon dynode group No. 2. As these groups have little interaction, they can be studied separately.

The diodes must have extremely low leakage current since the dynodes represent a very high impedance. For experimental work 1N596 diodes were used. They have a leakage current of .025 µA at 200 volt.
Figure 10. Clamped Defocusing Circuit.
Figure 11. Separation of Clamped Defocusing Circuit.
A few overall circuit considerations should be discussed before a detailed description of the separate groups is undertaken. The resistors $R_1$ through $R_{20}$ should be selected so that the voltage across each of these resistors is essentially constant throughout the operating range. This means that the current through these resistors must be much larger than the maximum dynode current. One simple approach is to let the current through $R_1$ to $R_9$ be equal to the current through $R_{11}$ to $R_{20}$ and equal to ten times the maximum dynode current. If the current through each chain of divider resistors is the same, then $R_{11}$ plus $R_{12}$ must equal $R_1$, $R_{13}$ plus $R_{14}$ must equal $R_2$ plus $R_3$, and so on for the remaining dividers. For most common photomultiplier tubes the voltages across resistors $R_2$ through $R_{10}$ should be the same, so that $R_2$ through $R_9$ should be equal to each other and equal to twice $R_{10}$.

**Anode Group**

Figure 12 shows the anode group in detail with normal current and voltage values. From the discussion above, $I_1'$ and $I_2'$ are equal and $V_1$ is equal to $V_{11}$ plus $V_{12}$. When the anode current is zero, the diode is back biased, and the anode voltage $V_a$ is equal to $V_1$. As $I_a$ increases, $V_a$ is reduced by the voltage drop across $R_{21}$. When $V_a$ falls to the value of $V_{12}$, the diode becomes forward biased, and $V_a$ remains equal to $V_{12}$ for further increases in $I_a$. Figure 13 shows $V_a$ as a function of $I_a$. The value of $I_a$ at which $V_a$ is clamped is:

$$I_a = \frac{V_1 - V_{12}}{R_{21}}$$

(17)

The gain of the photomultiplier tube is independent of the
Figure 12. Anode Group Detail

Figure 13. Dynode Voltage Versus Anode Current.
voltage between the anode and the last dynode above approximately 40 volts. Below this value, the gain of the tube starts to decrease because the anode collection efficiency decreases. Figure 14 shows a typical current response of the anode group for increasing light intensities. When the intensity is low, the gain is approximately linear; but as the intensity increases, the gain decreases until the anode voltage is clamped and the gain remains constant. The net result is logarithmic response over a limited portion of the anode response.

**Dynode Group**

Figure 15 shows the dynode group in detail with normal current and voltage values. The divider currents are equal, and large compared to the maximum dynode current. The voltage \( V_{22} \) is:

\[
V_{22} = R_{22}(I_4 - I_3) \tag{18}
\]

Since the gain of the dynode is maintained above unity, \( I_4 \) minus \( I_3 \) is positive, yielding the voltage polarity as shown in Figure 15. As the intensity increases from zero, \( V_6 \) starts at \( V_3 \) and increases by \( V_{22} \), while dynode nine voltage starts at \( V_2 \) and decreases by \( V_{22} \). This operation continues until \( V_6 \) is clamped at \( V_{14} \) and \( V_9 \) is clamped at \( V_{13} \). Figure 16 shows the action of the dynode voltages as the dynode current increases. It should be noted that:

\[
V_{13} + V_{14} = V_2 + V_3 = V_8 + V_9 = \text{Constant} \tag{19}
\]

The gain of the dynode group as a function of \( V_8 \) and \( V_9 \) cannot
Figure 14. Light Response of Anode Group.

Figure 15. Dynode Group Detail

Figure 16. $V_8$ and $V_9$ Versus $(I_4-I_3)$. 
be accurately determined by the exponential model because defocusing occurs. However, a study of the dynode group can be realized by graphical analysis. Although this technique is rather involved for a design procedure, it gives insight into the circuit characteristics.

The two experimental curves required for this procedure are obtained using the circuit shown in Figure 17. The first curve represents the relative dynode group gain \( \frac{I_a}{I_n} \) as a function of \( V_8 \) and \( V_9 \) with no series resistor in dynode eight. The gain characteristic shown in the top half of Figure 18 is independent of \( I_n \). The second curve represents the eighth dynode curve as a function of \( V_8 \) and \( V_9 \). It is linear with respect to \( I_n \); therefore, a family of curves can be constructed from one experimental curve, as shown in the bottom half of Figure 18.

When a resistor is added in series with the eighth dynode, and the potentiometer is set at the midpoint, then \( V_8 \) and \( V_9 \) are:

\[
V_8 = 100 + R I_8 \\
V_9 = 100 - R I_9
\]

(20)

this can be represented by letting a load line with a slope \( \frac{1}{R} \) intersect the dynode current curves. The points of intersection are the equilibrium points of this dynode group. By projecting them to the relative gain curves, a plot of dynode group gain versus input current is obtained for a particular series resistor as shown in Figure 19.

The response of the dynode group will be single valued as long as \( \frac{dI_a}{dI_n} \) is positive. Since \( I_a = G I_n \) and:
Figure 17. Circuit to Measure Gain as a Function of Dynode Voltage.
Figure 18. Graphical Construction.
Figure 19. Dynode Group Gain Versus Input Current
\[
\frac{dI_a}{dI_n} = \frac{dG}{dI_n} \cdot I_n + G \tag{21}
\]

it is obvious that multivalued response occurs when the negative slope of the \( G \) versus \( I_n \) curve is greater than \( \frac{G}{I_n} \).

The slope of this curve decreases as the slope of the gain versus dynode voltage curve decreases. It was found experimentally that the slope of the latter curve can be decreased by decreasing the total voltage across the dynode group as shown in Figure 20. However, this reduction also tends to make the dynode response less logarithmic.

The response of the dynode group can be adjusted by varying the total voltage and the series resistor. The resistor will adjust the point at which the logarithmic response begins and the voltage will adjust the shape of the response. When these parameters are adjusted for the best logarithmic response with the greatest usable range, the clamping circuit is adjusted to hold the dynode voltage constant at the point at which the response tends to become multivalued.

**Cathode Group**

The cathode group is shown in detail with normal current and voltage values in Figure 21. The voltage across \( R_{10} \) is constant because \( I_3 \) is large compared to the maximum dynode current. The dynode voltage \( V_{D1} \) is equal to \( V_{10} \) minus \( V_{26} \). \( V_{26} \) is \( R_{26}I_1 \) and equals zero when \( I_1 \) equals zero, and increases as \( I_1 \) increases. As the input increases, the dynode gain decreases, and the response is logarithmic. With no clamping circuit, the response is multivalued, setting an absolute maximum on the output current and protecting the tube against overload.
Figure 20. Comparison of Gain Curves for Different Total Dynode Group Voltages.

Figure 21. Cathode Group Detail.
This completes the description of the different circuit groups and their current responses. The total response of the circuit will be the product of all the group responses. The overall design of a practical unit will be discussed in Chapter VII.
CHAPTER VII

PHOTOMETER CIRCUIT DESIGN

Because of the extreme nonlinearity of photomultiplier circuits with unequal dynode voltages, a strict mathematical design procedure would not be feasible. It was shown that a graphical procedure could be used with experimental curves to predict the response of the clamped defocusing circuit; but even this is time consuming. It is desirable, therefore, to have a simple procedure for designing practical circuits of this type. The design procedure presented in the following sections is an experimental one, where the circuit is connected and its parameters adjusted in a specified manner until acceptable values are obtained.

Design Procedure

The photomultiplier tube is connected in the circuit shown in Figure 22. It is necessary first to establish a desired power supply voltage and anode current range. For the purposes of this example suppose that it is desirable to use a power supply of approximately one thousand volts, and an anode current range from five microamperes to one hundred microamperes. Both $I_1'$ and $I_2'$ should be about ten times the maximum anode current or, in this case, one milliampere. Figure 23 shows the division of 940 volts across the divider resistors with one milliampere through each divider chain. The 40 volts in the anode circuit is chosen because the anode collection efficiency does not decrease until the anode voltage falls below this value. The clamping
Figure 22  Test Circuit.
point of the anode is set at one fifth of the anode voltage, or eight volts. Further reduction in anode voltage might increase the interaction with the first dynode group. The resistors are calculated by Ohms Law and are listed in the figure.

Potentiometer $P_5$ is selected so that, with the minimum desirable anode current, about one half of the anode voltage will be dropped across the full potentiometer. In this case:

$$P_5 = \frac{\frac{1}{2}(\text{anode voltage})}{\text{min. anode current}} = \frac{\frac{1}{2}(40)}{5 \times 10^{-6}} = 4 \text{ megohms} \quad (22)$$

$P_6$ should be 2 to 3 times $P_5$, $P_7$ equal 2 to 3 times $P_6$, and so on to $P_9$. This completes the selection of the test circuit parts.

To start the design procedure, the wiper arm of potentiometers $P_1$ through $P_4$ are turned to the more positive end, and $P_5$ through $P_9$ are turned to zero ohms. This disables all the clamping circuits and removes the series dynode resistance, resulting in maximum sensitivity. Using a calibrated light source, increase the light intensity until the anode current reaches the minimum value in the desired range. Increase the series resistance, $P_5$, in the anode circuit until the anode current starts to decrease. Next increase the light intensity until the anode becomes clamped, which is marked by a sudden increase in anode current. Then increase $P_6$ until the anode current starts to decrease. Increase the light intensity until the anode current becomes multivalued, then adjust the clamping potentiometer $P_1$ until the response is clamped just before it becomes multivalued. This completes the adjustment of $P_6$ and $P_1$ which determine the response of the first dynode group; in like manner $P_7$ and $P_2$ are adjusted for dynode group two; $P_8$ and $P_3$ for group
Figure 23. Divider Resistor Determination.
three; $P_9$ and $P_4$ for group four. The logarithmic response of each dynode group should start simultaneous with the clamping of the group above.

This completes the actual design of the photometer circuit. After the potentiometers are adjusted, the values can be measured and the potentiometers replaced with fixed value resistors in the construction of the finished circuit, which can then be calibrated using the calibrated light source.

In this design process the anode starts defocusing at the minimum current. Just as the anode circuit starts to clamp, the first dynode group starts to defocus. As it clamps, the second dynode group starts to defocus. This sequence continues through all the dynode groups.

**Experimental Circuit**

Figure 24 shows the experimental response of a circuit designed according to the above outlined procedure. It can be seen that the curve has an overall logarithmic response, although not linearly logarithmic. The anode current varied between 1 and 100 microamperes as the normalized light intensity varied between 1 and 70,000. This device has 700 times the range of a linear photometer.

**Response Considerations**

Even though the stated design procedure produces a usable device, it may not be ideal for all applications. It may be possible to achieve other response characteristics by varying some of the circuit parameters from the design procedure values on a trial and error basis.

**Individual Dynode Groups**

The shape of an individual dynode group response can be altered
Some of the 45 Experimental Points Taken

Figure 24. Experimental Curve.
by changing the total divider voltage across that group. In general, increasing the voltage tends to make the group response more logarithmic and more multivalued.

**Linearity**

The point at which a dynode group response becomes multivalued is usually the most nonlinear part of the response. Therefore, additional linearity can often be gained by adjusting the dynode group clamping points to eliminate the more nonlinear portions of the response. This procedure will tend to reduce the usable tube range. Piecewise linearity can be achieved with a commercially available diode function generator placed before the recording device employed.

**Reliability**

In the design procedure, the dynode group clamping point was set just before the response became multivalued. For this reason slight variations in the circuit could cause the response to be multivalued. Therefore, the reliability can be improved in most cases by setting the clamping points slightly before the multivalued points. This also tends to reduce the photometer range.

**Compressed Output**

If the output range must be very small the response can often be compressed. This is achieved by starting the dynode group defocusing before the preceding group has clamped.

**Types of Tubes**

A particular response that cannot be achieved with a particular photomultiplier tube might be achieved using a different tube. No attempt was made in this research to categorize the different photo-
multiplier tubes. Four tubes were examined and their characteristics were similar. Two of these tubes, RCA 931-A and CBS CL-1002, were studied in more detail. It was found that the CBS CL-1002 was more logarithmic but at the same time more multivalued than the RCA 931-A.

Practical Considerations

It is important to consider some practical factors which might dictate the type of response necessary for a particular application. Tube aging could be an important factor in a design that must be used for long periods of time. Here circuit reliability would be important. In permanent applications, it might be desirable to change the photomultiplier tube without readjusting the circuit. Since there are considerable differences between tubes of the same type, circuit reliability is here again important. Calibration of the photometer circuit must be performed frequently because the characteristics of the photomultiplier tube are subject to change. In important airborne application, the photometer should be calibrated before each flight. Environmental changes such as temperature or magnetic field variations, must be considered in each photomultiplier tube application.

Conclusions

No attempt was made to exhaust all possible types of tubes, combinations of voltages, ranges of operation, and so on. The designer of a practical circuit must decide which characteristics are most important for a certain application, then employ several combinations until an acceptable circuit is achieved. This research shows that a wide range photometer can be built with the photomultiplier tube and a
passive network. It also demonstrates how various factors tend to affect the circuit behavior.
APPENDIX I

SCHEMATIC DIAGRAMS

Presented in this appendix are the schematic diagrams of the Low Voltage Power Supply and the Switching Matrix which were discussed in Chapter II.

Low Voltage Power Supply

The schematic diagram of the Low Voltage Power Supply is shown in Figure 25. The circuit consists of a full wave silicon rectifier and a transistor regulator. Source temperature compensation is possible by means of a thermistor in the voltage feedback circuit.

Switching Matrix

The schematic diagram of the Switching Matrix is shown in Figure 26. The circuit consists of ten relays which are used to place a current meter in series with any dynode. There is a set-reset holding circuit to hold the selected dynode relay.
Figure 25. Low Voltage Power Supply.
Figure 26. Switching Matrix
APPENDIX II

LOGARITHMIC

It is important to qualify the term "logarithmic" as it is usually applied to photometer response. In this case "logarithmic" refers to the general class of monotonically increasing functions in which the range of the function is smaller than the domain of definition. This can be thought of as a logarithmic function in which the base of the logarithm is a function of the log variable. The case in which the base is a constant is distinguished as the linear logarithmic function.

As an example, consider the output of the feedback control photometer circuit. The output voltage can be expressed as

\[ V = \left( \frac{K_2}{F} \right)^{1/B} \]  

This shows that the range of the function (variations in \( V \)) will be smaller than the domain of definition of the function (variations in \( F \)). Since \( V \) is monotonically decreasing, it can be expressed as the reciprocal of a logarithm.

\[ V = \frac{1}{\log_w F} \]  

where \( W \) can be expressed as a function of \( F \) as

\[ W = F \left( \frac{K_2}{F} \right)^{1/B} \]
Therefore, the output voltage decreases logarithmically with increases in input light intensity.


