DYNAMIC SELF-BALANCING OF A COMBINATION WASHER/DRYER IN HIGH-SPEED SPIN

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The Faculty of the Graduate Division
by
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Chairman

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SUMMARY

The high-speed spin portion of the operating cycle of an experimental washer/dryer machine currently being tested by Whirlpool engineers has not been implemented into a production version because of the need for an improved dynamic self-balancing technique. This new machine design utilizes a bulk redistribution method of self-balancing which would eliminate the mechanical balancing system on present production models. However, the bulk redistribution method requires a very large number of tries to attain a balanced clothes distribution.

This thesis research has utilized system analysis, computer simulation, and modern control systems theory to develop an improved self-balancing technique based on an orderly redistribution method. The entire control system design was posed as a stochastic optimization problem for which a suboptimal solution has been developed. The combination washer/dryer system was modeled mathematically by considering the component dynamic responses and using data supplied by Whirlpool engineers for a specific machine. Simulation of the model on the digital computer gave results which were consistent with runs made on the actual physical system. An orderly redistribution technique was developed, and the simulation indicated that it was a
significant improvement over the present method. In particular, whereas the present bulk redistribution method requires an average of approximately twenty-five attempts to attain an acceptable spin speed, the computer simulation of the orderly redistribution technique predicted that on the average only about seven attempts would be required.

As part of the research effort, the final testing of the orderly redistribution technique was conducted on the actual physical system at the Whirlpool Advanced Engineering Laboratories in St. Joseph, Michigan. The test machine was the one from which data had been obtained for the system model and the resulting simulation. These tests on the actual machine demonstrated that the orderly redistribution technique brought the combination washer/dryer system up to an acceptable spin speed in approximately five attempts on the average. This improvement over the predicted results may be attributed to the fact that the simulation selected an item at random to redistribute, while the actual system apparently tended to correct the location of the most poorly distributed item. Therefore, it has been shown that modern system analysis techniques could be used effectively to develop a practical design scheme for the dynamic self-balancing of the combination washer/dryer.
CHAPTER I

INTRODUCTION

Background

Motivation

The analysis and design of complex control systems where no closed-form optimal strategy exists has usually been left to the cleaverness and technical expertise of the project engineer. He must devise, implement, experimentally improve and finally produce a control system that satisfactorily meets the design requirements. His method of attack may be the expansion method of adding on controls to cover every possible shortcoming of the original system or an alternative problem re-evaluation technique that takes advantage of the concepts of system modeling, computer simulation, and optimization techniques. This research thesis has followed the second approach above to solve a control problem that was attempted but not solved by the first method.

Description of Problem

In this research the particular problem is the design of a control strategy for the dynamic self-balancing of particles in a high-speed drum. This problem originated with the need for high-speed spin control of a new generation of horizontal-axis combination washer/dryer machines that do not contain a mechanical balancing system. There is an electronic
control system for the new machines in existence today, but it has the severe limitation of the possibility of going through an extremely large number of unsuccessful attempts before attaining a spin balance. The future of the new system depends upon the solution of the control problem, and this thesis shows several new lines of approach toward that solution.

**History of the Problem**

**Present State-of-the-Art**

The Advanced Development Section of the Laundry Engineering Division of Whirlpool Corporation has had the problem of washer/dryer machine balancing under consideration for a long time, but only recently has the necessary state-of-the-art advanced to the point where the dynamic self-balancing of a machine might be realistically considered.

The current production models of horizontal-axis combination washer/dryers are used to wash and rinse clothes by tumble action at low speed, extract some of the water by centrifugal force during high-speed spin, and then thoroughly dry the clothes by hot air at a low tumble speed. The most critical phase of the machine operation is the high-speed spin-extraction process where an improperly balanced distribution of clothes can make the machine cabinet vibrate excessively and even "walk" across the floor. Current models use intermediate speed (approximately 140 rpm) stutter spins
to shed most of the water before going to a higher speed spin (400 rpm). During both the stutter-speed and high-speed spin, a mechanical system injects water into one or more of three tanks located on the circumference of the rotating drum, thereby counteracting the off-balance of the clothes distribution.

**Literature Review**

There has been extensive research into the mathematical expressions of machine dynamics [1,2] and in efforts to simulate the total machine response [3]. Most of this work has been done in: (1) predicting critical frequencies of machine operations (mechanical design), (2) predicting speed and load conditions at which the machine will "walk", and (3) studying the effects of mechanical suppression of excessive vibrations. There have been a variety of suggestions to reduce vibration [4,5] ranging from various damping schemes to development of spring suspension systems. The cause of vibration, i.e. imbalance, has also been attacked with a wide variety of mechanical systems [6,7] and even some electromechanical systems [8]. However, the question of imbalance elimination by an orderly mass redistribution scheme has not been treated in the literature.

As may be seen from the above references, most of the work has been mechanical in nature and limited to contracted or manufacturing company research. Since most of the reference material is closely guarded by each manufacturer, only access to material from the Whirlpool Corporation library, apart from published papers, has been available.
The Ideal Machine

There are several reasons for manufacturers' wishing to evolve the ultimate in combinational washer/dryers, not the least of them being profit. This ideal machine is generally thought by Whirlpool as consisting of a single drum with a drive motor directly attached and the associated control circuitry. With the exception of balancing for the high-speed spin, this machine is almost a reality at the Whirlpool Corporation. To successfully obtain an acceptable high-speed spin presently depends on the random distribution of the clothes load at the zero gravity speed, i.e. that speed at which the clothes just stop tumbling and attach themselves to the cylinder walls.

The Present Control System

The present control system which uses a tachometer and a limit switch for sensing cabinet motion, allows the motor speed to increase until one of the following three conditions exists. (1) In the first case the top speed of the motor is reached (580 rpm) and cabinet motion is below the limit switch set point. This is the condition of an almost perfectly balanced load, and the machine continues at this speed until the end of the spin cycle. (2) In the second case the motor reaches some speed above 250 rpm which is adequate for extraction, and depends on the imbalance of the load. When the speed becomes too high, the cabinet's excessive motion closes the limit switch which, in turn,
removes the voltage from the motor, allowing the drum to slow down inertially until the limit switch is opened and the spin voltage reapplied. This imbalance stutter spin is an effective way of operating this newly developed motor, and the machine continues in this manner until the end of the high-speed spin cycle. (3) In the third case the clothes are so poorly distributed that the motor speed fails to reach 250 rpm before the limit switch closes because of excessive cabinet motion. Here a dynamic brake voltage is applied to the motor, reducing its speed to zero and completely altering the clothes distribution. The motor spin voltage is then reapplied, and the new distribution formed by going through the zero gravity speed will again fall into one of the three possible situations. This third case repeats until an acceptable spin speed, i.e. above 250 rpm, is attained.

Method of Attack

Mathematical Models

The elements and processes of the washer/dryer system were represented mathematically by either commonly used models or approximations developed from test results on the prototype machine. The parameters of all mathematical models were verified using specially designed test runs. The complexity of the models roughly varied inversely as the number of assumptions. The general objective was to obtain a reasonable working model.
Computer Simulation

The digital computer was used to perform mathematical analysis and to simulate the system's mathematical models. The versatility of the computer was emphasized as it was the only tool which allowed portions of this research to proceed. The various types of algorithms used were typical of those used in most simulation work, i.e. random number generators and single-step numerical integration techniques (Runge-Kutta methods).

Suboptimal Solution

The familiar technique of objective function, constraints, and optimal solution by variational calculus could not be readily applied, although analysis in view of this method led to the possible suboptimal solutions. The solutions were verified by the computer simulated model and became the partial results of this thesis.

Implementation

The computer developed control strategies were implemented directly on the prototype washer/dryer in the Whirlpool laboratories. The results of those tests are included, and the subsequent analysis comprises the main conclusions of the thesis.

Thesis Outline

The precise problem is stated at the beginning of Chapter II, and the remainder of the chapter is devoted to introducing the system models and approximations. The total
system model is then presented as the logical summary to Chapter II. The development of a control strategy is undertaken in Chapter III and the computer-aided results are presented in the summary. The implementation at the Whirlpool laboratories of the predicted control strategies and these practical results are included in Chapter IV. The total thesis conclusions are presented in Chapter V.
CHAPTER II
MODELING, REDISTRIBUTION, AND SIMULATION

Introduction

This chapter introduces and defines the mathematical models of the component subsystems, the simulation of those models, and the synthesis of the total system. Each subsystem is represented in mathematical equation form with all necessary assumptions listed and any approximations noted where the model varies from the most general case. In the implementation of the simulation from the model, assumptions that were used have been clearly noted. Remarks throughout this development attempt to relate the significance of the assumptions in terms of limitations of the more general case.

The basic washer/dryer system to be modeled is shown in Figure 1, along with an associated component description listing.

Statement of the Problem

The high-speed spin portion of the operating cycle of an experimental washer/dryer machine currently being tested by Whirlpool engineers cannot be implemented into a production version because of the possibility of the control
system requiring an extended number of cycles to attain a balanced clothes distribution. This new machine contains no mechanical balancing system and depends on the control system's use of dynamic self-balancing to produce an acceptable clothes distribution. The present Whirlpool control scheme, which is inadequate, is described in Chapter I. The problem covered by this thesis is to suggest and implement a practical non-mechanical control scheme that will solve the imbalance problem.

**Mathematical Models**

**D. C. Armature Controlled Motor and Load**

The development of the direct current motor model has been covered in many basic texts on control theory. In
particular, the reference by Dorf [9] should be consulted for further details. Note the electrical circuit form as depicted in Figure 2. Since the d.c. motor is armature controlled, the field current $i_f$ is a constant and the air gap flux $\Phi$ proportional to the field current, i.e.

$$\Phi = K_f i_f$$  \hspace{1cm} (2.1)

where $K_f$ is the proportionality constant. The torque output of the motor $T_m$ is given by the familiar linear relationship with air gap flux $\Phi$ and armature current $i_a$ as

$$T_m(t) = K_1 \Phi i_a(t)$$  \hspace{1cm} (2.2)

where $K_1$ is the constant of proportionality.

---

**Figure 2. Model of D.C. Armature Controlled Motor and Load**
Now writing the loop equation for the armature circuit, one has

\[ v_a(t) = R_a i_a(t) + L_a \frac{d i_a(t)}{dt} + v_b(t) \]  

(2.3)

where \( v_a \) is armature voltage, \( R_a \) is armature resistance, and \( L_a \) is armature inductance. The back electromotive force \( v_b \) is proportional to the motor speed \( w \), or

\[ v_b(t) = K_b \; w(t) \]  

(2.4)

where \( K_b \) is the proportionality constant.

The effective load inertia \( J \) and effective load friction \( \beta \) are both functions of load mass and distribution. For the rotating drum load, the load torque \( T_L \) can be expressed as

\[ T_L(t) = J \frac{d}{dt} w(t) + \beta \; w(t) \]  

(2.5)

Now taking the Laplace transforms of equations (2.1) through (2.5), denoted by \( L(2.1) \) through \( L(2.5) \) where the initial conditions are set to zero, one may form the following relationships. Combining \( L(2.1) \) with \( L(2.2) \) and solving for motor output torque, one obtains

\[ T_m(s) = (K_I K_f I_f) \; I_a(s) = K_T \; I_a(s) \]  

(2.7)

Combining \( L(2.3) \) with \( L(2.4) \) and solving for the motor
current yields

\[ I_a(s) = \frac{V_a(s) - K_b w(s)}{L_a s + R_a} \quad (2.7) \]

Finally solving \( L(2.5) \) for the motor speed gives

\[ w(s) = \frac{T_L(s)}{J_S + \beta} \quad (2.8) \]

Assuming that motor torque equals load torque, one can simply combine (2.6), (2.7), and (2.8) in their transfer function forms to obtain the motor and load model block diagram shown in Figure 3.

\[ \text{Figure 3. D.C. Armature Controlled Motor and Load Block Diagram} \]

**Water Removal Process**

During the high-speed spin portion of the process cycle, water is removed from the clothes in the drum by
centrifugal force. The water removal rate by high-speed spin is initially much higher than the low-speed hot air tumble drying, but the high-speed rate drops off to zero as the excess water in the clothes is removed. The ideal time to stop high-speed spin and start tumble drying is when the water extraction rates of both processes are equal (See Fig. 4).

The total fraction of the mass of water \( MW \) capable of being removed by high-speed spin alone is designated as \( \alpha \). Now let the extraction rate \( \lambda \) be considered a function of the speed at which the high-speed spin is occurring. Then considering Whirlpool test data for various types of materials, the water removal process can be assumed to be an exponential rate process and described by

\[
\frac{d_{MW}(t)}{dt} = -\lambda \alpha MW_0 e^{-\lambda t}
\]  

(2.9)

where \( MW_0 = MW(0) \).

The approximation is then made that the water extraction rate is directly proportional to the drum speed, or

\[
\lambda = K_2w
\]  

(2.10)

where \( K_2 \) is the constant of proportionality. This may not be exactly true for the lower speed regions, but will not seriously degrade the model because anticipated times at low speeds will be minimized. Further, the water removal process may be graphically illustrated as in Figure 4, and it may be
Figure 4. Model of Typical Water Removal Processes
seen that

\[ MW(t) - (1 - \alpha) MW_0 = \alpha MW_0 e^{-\lambda t} \quad (2.11) \]

Thus, (2.9), (2.10), and (2.11) may be combined to form the mathematical model of the process as

\[ \frac{dMW(t)}{dt} = -Kw [MW(t) - (1 - \alpha) MW_0] \quad (2.12) \]

Then the process can be represented in block diagram form as in Figure 5. The only variable not considered in this model, but which is a real part of the actual water removal process, is the type of clothing material and its ability to retain water. This variable will be considered in later sections of this chapter.

![Figure 5. Block Diagram of High-Speed Spin Water Removal Process](image)

**Clothes Distributions in the Drum**

Each item of clothing or material in the washer/dryer
drum is considered to have size and shape but its weight is assumed to be concentrated and acting at its center of gravity. For larger items the article can be considered to have been segmented into smaller connected items, each with its own center of gravity. In this manner the clothes distribution can be represented as a set of points located around and near the wall of the drum when spinning above the drum's critical speed.

A further assumption is that the distribution has only two dimensions and will be considered to be concentrated in a ring about the middle of the drum. Now it is possible to completely describe the clothes distribution by the centroid of the set of centers of gravity (See Fig. 6a).

The model of the distribution in Figure 6b has the total of the mass of clothes (MC) and the water held by the clothes (MW) located at a centroid vector length (RV), which is rotating at the speed of the drum (w). The variable RV specifies whether the clothing in the drum is perfectly balanced (RV=0), or unevenly balanced (0<RV<r), where r is the radius of the drum. Further, specifying that there are n items in the distribution each with a total mass \( m_i \) located at a distance \( l_i \) from the center and with the angular distribution \( \theta_i \) from some reference radial \( \theta_0 \), the following mathematical models may be developed. The mass of the distribution is given by
\[ MW + MC = \sum_{i=1}^{n} m_i \tag{2.13} \]

and the centroid vector in terms of the unit axial vectors \( i_x \) and \( i_y \) by

\[
\text{centroid vector} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i} i_x + \frac{\sum_{i=1}^{n} m_i y_i}{\sum_{i=1}^{n} m_i} i_y
\]

where \( x_i \) and \( y_i \) are the individual items' radial length in terms of the two-axis frame of reference, or

\[
x_i = l_i \cos \theta_i
\]
\[
y_i = l_i \sin \theta_i
\]

Thus, the vector length RV may be written as

\[
RV = \sqrt{\left( \sum_{i=1}^{n} \frac{m_i l_i \cos \theta_i}{\sum_{i=1}^{n} m_i} \right)^2 + \left( \sum_{i=1}^{n} \frac{m_i l_i \sin \theta_i}{\sum_{i=1}^{n} m_i} \right)^2} \tag{2.14}
\]

Cabinet Motion

The two-dimensional drum developed in the preceding section will be extended to the total machine model. The direct current drive motor is directly coupled to the back of the drum, and this drum and motor assembly is supported by the
Figure 6. Clothes Distribution in Drum Above Critical Speed
pylon which connects to the machine base plate. The base plate is considered stationary and the condition of machine "walking", i.e. the total cabinet shifting position about the heaviest loaded support leg, will be assumed not to occur, based on the restriction placed on total drum and pylon motion. The reed switch discussed in Chapter I measures the drum oscillation with respect to the base plate and is used to signal exceeding the preset tolerance. The cabinet depicted in Figure 7 will be used as the physical model that will be described mathematically.

The concept of a two-dimensional model will be accurate for the system description based on an extensive three-dimensional drum, motor shaft, and pylon analysis by Lo on a similar washer/dryer [5]. Since there is virtually no motor shaft in the new direct drive machine, Lo's results, that the critical speed is extremely high and unaffected by machine loading (although the unbalanced load will increase the amplitude of vibration of the motor-drum assembly), are directly applicable. Thus, for the "non-walking" case the two-dimensional model accurately describes the machine's characteristics. Using the diagram of Figure 8 and considering the motor and pylon support rigid in the horizontal axis, the equation of motion for the drum can be written as
Figure 7. Diagram of Drum Motion
\[(MG-MT) \ddot{x}(t) + MT \frac{d^2}{dt^2} (x(t) + RV \sin \omega t) = -Kx(t) - Cx(t) \tag{2.15}\]

where \(x\) is the horizontal drum motion with respect to the base plate and

\[M_T = MW + MC + MD \tag{2.16}\]

where \(MD\) is the small off-balance mass of the drum. Then (2.15) can be rearranged as

\[MG \ddot{x}(t) + Cx(t) + Kx(t) = MRV \left[\omega^2 \sin \omega t - \dot{\omega} \cos \omega t\right] \tag{2.17}\]

and directly implemented to obtain the cabinet motion.

**Redistributions**

**Orderly Methods of Redistribution**

The washer/dryer machine control system at Whirlpool makes use of a brake voltage to slow the drum down to zero speed whenever an unacceptable clothes distribution exists. This type of redistribution method can be termed bulk, in that all the clothes have left their previous locations and will seek new locations when the drum is next accelerated through its critical speed. Thus, a bulk redistribution is characterized by a totally new set of clothes item locations.

The concept of orderly redistribution is defined as the incremental change of the present distribution by some
Figure 8. Drum Motion Model
process that reorders only a small percentage of the particles in the distribution and allows enough time to measure the effects of those changes before the next change occurs. An ideal device to effect orderly redistribution might be a mechanical hand rotating at the same speed as the drum and picking one article off the wall of the drum and changing its position so as to always improve the characteristics of the distribution.

There might be many such mechanical or pseudo-mechanical methods of causing only a certain portion of the distribution to change locations, but these will be left to the inventiveness of other engineers. The objective of this thesis, dynamic self-balancing, is to investigate only methods that use the washer/dryer system as presently built and effects changes by a variation of controls.

Therefore, the range of self-balancing by particle redistribution will only exist below the drum's critical speed. As the speed of the drum is reduced to slightly below this critical speed, there exists a range of speeds where only the positions of those items closest to the center of the drum are being changed. There are so many variables acting in this dynamic situation that to attempt to account for all of them in the many combinations that they might occur would require extensive research and a great deal more complexity in the model. Some variables that would be difficult to even measure are material adhesion to surfaces,
uneven water removal, and damping effects associated with particle relocation. Thus, the actual orderly redistribution method would be extremely difficult to simulate.

The approximate model developed in the next subsection uses a number of simplifying assumptions and a single algorithm for deciding which particle is most likely to change position, where in fact an infinite number of algorithms might be found to exist during the actual orderly redistribution process.

**The Orderly Redistribution Model**

The two-dimensional clothes distribution model developed in an early section of this chapter will be used here. Examining the action of one article in the drum, one can see that as the speed of the drum is reduced to the critical speed for the single item on the wall, all the forces acting on that item are first balanced at top dead-center of the drum. Thus, if an item breaks loose at a speed close to the critical speed, it will leave a position at the top of the drum and fall in a ballistic trajectory until it again reaches the wall of the drum. At the completion of this relocation event, the particle will have changed its location in the drum by some angle (delta θ).

The orderly redistribution model considers the test item's center of gravity to be located near the wall of the drum, but it is assumed that the item is physically resting on top of another article. When the drum is slowed to the
critical speed of the bottom article (closest to the wall), it will be below the critical speed of the article closer to the center. Therefore, the test item will break away from its position as it passes through the top of the drum and fall to its new position.

The model assumes no drag on the test item, that its horizontal velocity is the same as that of a particle on the wall of the drum, and only gravity acts on the item during its free fall to its new position. The graphical study in Figure 9 shows the trajectories for the test article and the associated reference position change at three drum rotation speeds.

During the test particle's time of transit from the reference position to its new location, the initial reference changes position also, and this angular change depends on the particle flight time and drum rotation speed. The time in trajectory is longer for lower initial velocity particles and therefore the reference change is greater for slower drum speeds. The three test points and a linear approximation are developed in Figure 10. The approximation for this angular change, which is valid only in the speed range close to and below the critical speed, is given by

$$\delta \theta = -2.5 (9.66w) + 135 \quad \text{(2.18)}$$

where \(w\) is in radians per second and \(\delta \theta\) is in degrees.
Figure 9. Graphical Model of Orderly Redistribution

Figure 10. Particle Angular Relocation by the Orderly Redistribution Process

\[ \Delta \theta = -2.5(9.66w) + 135 \]
The model will be completed by the development of the algorithm which selects the item in the clothes distribution that will change position by delta \( \theta \) degrees. When there are three or more items whose positions in the drum are within a short segment of the wall arc, then it can be assumed that one of those items is resting on top of the other two and will be redistributed at a speed for which the bottom two articles are held firmly against the wall. This model selects the middle item of the grouping of three items that are within a six inch arc of the drum wall. This item so determined will then have its axial position \( \theta_1 \) changed by the angle delta \( \theta \), which depends upon the speed of rotation of the drum.

**Simulation**

**System Physical Parameters**

The following general constants were taken from the system as built in the Whirlpool laboratories and were used in computations to be described in the following sections:

- Total machine weight (empty) = 275 lbf
- Drum weight (empty) = 36 lbf
- Radius of drum (inside) = 1.08 ft.
- Reed switch set point = 0.030 inches

**The Motor and Load Model**

The performance curves of the d.c. armature controlled motor used in this system are given in Figure 11. Another source of simulation data is a Whirlpool project report [10]
which used this same motor in an upright washer simulation on the analog computer. The motor constants were confirmed by an analog computer simulation and comparison with the known motor curves. The constants are

\[
\begin{align*}
R_a &= 2.35 \text{ ohms} \\
L_a &= 0.110 \text{ henries} \\
K_t &= 1.5 \text{ ft-lbf/ampere} \\
J_a &= 0.16 \text{ ft-lbf-sec}^2 \\
B_a &= 0.1 \text{ (full load) ft-lbf-sec} \\
K_b &= 1.48 \text{ volt seconds}
\end{align*}
\]

The effective inertia and friction of the total system are known to vary with the amount of loading and the off-balance of the loading in the drum. To determine these values a set of tests were run on the actual machine in the Whirlpool laboratories and were compared with the data from a digital computer program which used the constants above and varied only the inertia and friction. Both the test results and the simulation results were plotted together and appear as Figures 12 and 13.

The equation which approximates the inertia of a thin-walled cylinder is

\[
J \approx \frac{m}{12} [3r^2 + h^2]
\]

where \( r \) is the radius, \( m \) is the mass, and \( h \) is the height. Knowing that the inertia varies directly with the mass, a linear scheme can be used to represent the change in inertia
Figure 11. Direct Drive Motor Curves

Armature Current

Motor Voltage $v_a$ (in volts)

- $v_a = 10$
- $v_a = 30$
- $v_a = 50$
- $v_a = 70$

Motor Torque (ft-lbf)
from no-load to full-load or

\[ J = J_{NL} + \frac{MW + MC}{36.0} (RV) (\text{delta} J) \]  

(2.19)

where delta J is the inertia change between no-load and full-load. From the data of Figure 12, it was determined that \( J_{NL} \) equals approximately 2.5 lb-ft-sec\(^2\) and using the above relationship, the entire range of possible loadings gives a compatible range of inertias.

The effective friction was determined by using the above relationship for inertia and varying the parameter \( \beta \). From the comparison of test data and simulation results in Figure 13, it can be seen that the full load friction is 0.1 lb-sec-ft. Assuming that the same loading and distribution function affects the friction as well as inertia, then the effective system friction is

\[ \beta = \beta_{NL} + \frac{MW+MC}{36.0} (RV) (\text{delta} \beta) \]  

(2.20)

where \( \beta_{NL} \) is 0.005 lb-ft-sec, and delta \( \beta \) is the friction change between no-load and full-load.

The Water Removal Model

From extensive tests in the Whirlpool laboratories on the water retention of various types of materials during the high-speed spin process, (the data for towels is included in Figure 14) it can be seen that the process removes about 12 percent of the initial water load in a period of about six
Figure 12. Comparison of System and Simulation to Determine Effective Inertia
Figure 13. Comparison of System and Simulation for Determination of Effective Friction
minutes. The model developed earlier will accurately approximate this spin process if the constants of (2.12) are determined as

$$\frac{4 \text{ time constants}}{\lambda} = 6 \text{ minutes}$$

and from (2.10) and the test data

$$K_2 = \frac{\lambda}{w} = \frac{(.66 \text{ min}^{-1}) (9.66 \text{ rpm/rad/sec})}{(400 \text{ rpm}) (60 \text{ sec/min})} = 0.000268$$

Finally the constant $\alpha$ can be seen to be 0.12.

**The Cabinet Motion Model**

An extensive analysis of system vibration for the two-dimensional model was reported in a design analysis made by USS Machinery and Allied Industries [3]. For lower speeds (range of normal machine operation) the horizontal machine displacement can be approximated as

$$x \approx \frac{MT}{K} (RV) w^2$$

(2.20)

Also at the machine's maximum self-oscillation, the magnitude of the displacement can be expressed as

$$x \approx \frac{MT}{C} \sqrt{\frac{K}{MG}}$$

(2.21)

A set of test runs were made on the machine, measuring cabinet horizontal motion with a one pound off-balance weight on the wall of the drum, and Figure 15 shows these results. Since the speed of 250 rpm is the crossover between
Figure 14. Water Removal by High-Speed Spin

Percent Water Retention (@ 400 rpm)

Time (minutes)

8 lbf Towel Load

4 lbf Towel Load
good and undesired clothes distributions, the values of $K$ and $C$ were calculated at these points using (2.20) and (2.21).

$$K = \frac{1.0 \text{ lbf}}{32.2 \text{ lbf} \frac{\text{ft}}{\text{lbm}} \frac{(1.08 \text{ ft})(12 \text{ in.})}{\text{ft}} \left[\frac{250 \text{ rpm} \cdot 2\pi}{60 \text{ sec/min}}\right]^2}{0.010 \text{ inches}}$$

$$K = 27,600 \text{ lbf/ft}$$

and where $M_g = 276 \text{ lbf}$, and $x_{max} = 0.3 \text{ inches}$

$$C = \frac{1.0 \text{ lbf}}{32.2 \text{ lbf} \frac{\text{ft}}{\text{lbm}} \frac{(1.08 \text{ ft})(12 \text{ in.})}{\text{ft}} \sqrt{27,600 \frac{\text{lbf}}{\text{ft}}}}{0.3 \text{ inches}} \sqrt{\frac{276.0 \text{ lbf}}{32.2 \frac{\text{lbf}}{\text{lbm}}}}$$

$$C = 229 \frac{\text{lbf-sec}}{\text{ft}}$$

The maximum cabinet displacement $x_{max}$ was not obtained by experiment but from results in a project report [3]. The accuracy of these values of spring constant $K$ and damping coefficient $C$ seem reasonable but will be verified and adjusted, if necessary, in the total model simulation section.

**The Control System Model**

The detailed schematics of the present control system have been available and are accurately described in Chapter I. The block diagram simulation of the controls in Figure 16 was completed by development of the logic functions of system
Figure 15. Washer/Dryer Cabinet Motion with a Fixed One Pound Off-Balance Load
operation and the testing of the actual system in the Whirlpool laboratories to determine the time delays within the controller.

The Karnaugh maps and logic functions are easily developed, where the only assumption is that brake voltage is applied until the washer/dryer slows to less than 5 rpm, which simplifies the computer implementation with no loss of generality.

Tests at the Whirlpool laboratories have given the following values of time delays

- Tachometer circuit: 0.014 seconds
- Reed switch circuit: 0.009 seconds

The time delays include all delays from the time a tolerance is exceeded until the required control action has just started.
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The Total System

Synthesis of the System

The total model was completed by the combination of the previously developed sub-processes and the total block diagram is shown in Figure 17.

To implement this system on a digital computer, it is necessary to postulate the conditions of action of the bulk redistribution process. The assumption to be used here and later verified is that each particle after the bulk redistribution process has as much chance of being at one location in the drum as being at any other. This can be stated as the particle location \( \theta_i \) having a uniform probability density function between 0 and \( 2\pi \) radians. This process can be simulated by a uniform random number generator which gives the new set of particle locations after a bulk redistribution. This idea and the assumptions used to develop the simplified centroid vector length \((\text{RV})\) calculations will be covered in more detail in Chapter III.

The system states \( Y[1] \) through \( Y[5] \) are defined in Table 1. The model can now be simulated by the following five first-order nonlinear differential equations where the system variables, \( J, \beta, MW, \) and \( MT \) are defined by previously developed linear relationships and other constants are as modeled.
Table 1. Definitions of System States

<table>
<thead>
<tr>
<th>State</th>
<th>Description</th>
<th>Model variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y[1]$</td>
<td>motor armature current</td>
<td>$i_a$</td>
</tr>
<tr>
<td>$Y[2]$</td>
<td>drum speed</td>
<td>$w$</td>
</tr>
<tr>
<td>$Y[3]$</td>
<td>mass of water in clothes</td>
<td>$MW$</td>
</tr>
<tr>
<td>$Y[4]$</td>
<td>magnitude of drum oscillation</td>
<td>$z$</td>
</tr>
<tr>
<td>$Y[5]$</td>
<td>velocity of drum oscillation</td>
<td>$\dot{z}$</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\frac{d}{dt} Y[1] &= \frac{-R_a}{L_a} Y[1] - \frac{K_b}{L_a} Y[2] + \frac{V_a}{L_a} \\
\frac{d}{dt} Y[3] &= -K_2 Y[2] (Y[3]-(1-\alpha) MW_0) \\
\frac{d}{dt} Y[4] &= Y[5] \quad \text{(2.22)} \\
\frac{d}{dt} Y[5] &= \frac{-1}{MG} (C Y[5] + K Y[4]) \\
&\quad -\text{MT RV } [(Y[2])]^2 \sin(Y[2]t) - \\
&\quad \frac{d}{dt} Y[2] \cos(Y[2]t)) \]


Figure 17. Total Washer/Dryer Computer Simulation
The actual computer program used on the Burrough B5500 digital computer was written in the ALGOL language. The numerical integration technique used was a single-step fourth-order Runge-Kutta method with a fixed step size.

Verification of the Model

The total simulated system was operated with loads similar to those for which test data was available from the Whirlpool laboratories. An improvement necessary to show better agreement between the actual system and the model was in the machine spring constant K. The model calculations used only the drum weight, while it is known that motor shaft and coupling mechanisms should have also been considered in MT. Therefore, the value of $K = 30,000 \text{ lbf/ft}$ was determined iteratively to give the correct response. This new K is only 8.7 percent higher than predicted and changes the damping coefficient $C$ to $245 \frac{\text{lbf-sec}}{\text{ft}}$.

Test runs were conducted at the Whirlpool laboratories where this present bulk redistribution control system was subjected to various amounts of concentrated off-balance weights to develop the data given in Figure 18. In these tests the brake voltage was disconnected and the system allowed to stabilize at its maximum speed for its off-balance load condition. From this test it can be seen that the 250 rpm crossover point or critical off-balance loading is 3.00 ft-lbf. This idea can be extended using the two-dimensional
Figure 18. Off-Balance Loading of Washer/Dryer Without Brake Voltage Activated
clothes distribution model such that whenever

\[(MT)g(RV) \leq 3.00, \tag{2.23}\]

where \( g = 32.2 \text{ lbf/lbm} \), the load and distribution will reach a speed greater than 250 rpm and be classified as successful. The total system simulation was given the same conditions as the above described laboratory test and a comparison of the results is plotted in Figure 19.

The overall performance of the present washer/dryer control system and the computer simulation are compared in Figure 20 for the same load condition of eight towels. The data compares the number of times bulk redistribution was required to produce a successful distribution and the top speed attained by that distribution. It must be realized that both the machine performance and the simulation are random processes and should be compared with respect to the form of the data and not as direct reproductions. The comparison shows the validity of the simulation concerning the bulk redistribution process, i.e. each particle has a uniform probability density function between 0 and \( 2\pi \) radians under the action of the bulk process.
Figure 19. Comparison of Simulation and Actual Machine for Off-Balance Without Brake Voltage
Figure 20. Verification of the Overall System Model
CHAPTER III

SYSTEM ANALYSIS AND IMPROVEMENT

Introduction

This chapter analyzes in detail the bulk redistribution process and develops a new insight into the expected value of the centroid vector length (RV). The optimal feedback problem for the washer/dryer system is posed and some suboptimal approaches to its solution are developed. The computer-aided results summarizing this chapter, which are based on the system model developed in Chapter II and the control strategy of this chapter, predict the action of implementation at the Whirlpool laboratories.

Analysis of the Bulk Redistribution Process

Development

The two-dimensional model of the clothes distributions developed in Chapter II will be the starting point. To this general model the following ideas will be added to develop a simpler model which will allow analysis.

A washer/dryer load of clothes made up of identical type items, not individually as large as bed sheets but something about the size of bath towels, can be thought of in terms of the model development of single, unconnected items with
size and shape, but with their weight acting at the items' center of gravity. This case will be followed closely in further development, but first consider two possible variations of the clothes load, one with mixed-size items, the largest being a bath towel, and the other load consisting of a few large items the size of bed sheets. No attempt will be made to accurately model either of these latter loads but assumptions will be given that make the future analysis also applicable to these cases. The mixed load can be thought of as being dominated by a few of the heaviest items. By giving each item the same average weight and placing more restrictive constraints on their distribution, one can attain a satisfactory distribution regardless of the positions of the unconsidered items. The other load condition of few large items can be considered by restricting the angular distribution of the connected equally-weighted centers of gravity associated with each large item.

The bulk redistribution process will use the uniform probability density concept described in Chapter II. Again, this concept assumes that after bulk redistribution each item has an equal probability of being in any location in the distribution, i.e. that each \( \theta_i \), the angular position of the \( i \)-th item, is uniformly distributed between 0 and \( 2\pi \) radians.

Now one may apply the two-dimensional model of the clothes distribution to the clothes load of \( n \) identical items.
It is assumed that all items initially have the same water retention and that the centers of gravity are located on the inside radius of the drum. The centroid vector length for the distribution (2.13) becomes

\[ RV = \frac{1}{n m_i} \sqrt{\left[ m_i r \Sigma_{i=1}^n \cos \theta_i \right]^2 + \left[ m_i r \Sigma_{i=1}^n \sin \theta_i \right]^2} \]  

(3.1)

which can be further reduced to

\[ RV = \frac{N}{n} \sqrt{\left[ \Sigma_{i=1}^n \cos \theta_i \right]^2 + \left[ \Sigma_{i=1}^n \sin \theta_i \right]^2} \]  

(3.2)

Knowing the probability distribution of \( \theta_i \), the analysis can proceed by using a mathematical method where only a limited number of items are considered. Furthermore, the analysis may be applied to any number of items by using a random number generator on the digital computer.

**Mathematical Analysis**

The centroid vector length (RV) can be described as a function of \( n \) independent random variables \( \theta_1, \theta_2, \ldots, \theta_n \), and is itself a random variable. If one can describe the frequency distribution of RV, then the probability of obtaining a better clothes distribution on the next cycle of the bulk redistribution process can be predicted.

To describe this frequency distribution, the mean and standard deviation will be calculated for a number of items of clothing. For the trivial case of one item of clothing
(n = 1) in the drum at angular position $\theta_1$ the centroid vector is

$$ RV = r \sqrt{\cos^2 \theta_1 + \sin^2 \theta_1} = r $$  \hfill (3.3)

and it is obvious that the mean $E[RV] = r$ and the standard deviation $\sigma_{RV} = 0$. The case of two items (n = 2), where the particle positions are $\theta_1$ and $\theta_2$, becomes a little more difficult when calculating the random variable $RV$ from (3.2) as

$$ RV = \frac{\xi_1^2 + 2 \cos (\theta_1 - \theta_2)}{2} $$  \hfill (3.4)

The new random variable $W = |\theta_1 - \theta_2|$ is introduced, which modifies (3.4) to

$$ RV = r \sqrt{\frac{1 + \cos W}{2}} = r \cos \frac{W}{2} $$  \hfill (3.5)

The frequency distribution of $W$ can be shown to be uniform between 0 and $\pi$. Thus, the expected value of $RV$ is

$$ E[RV] = \int_0^\pi (\frac{1}{\pi}) r \cos \frac{W}{2} dW = \frac{2r}{n} $$  \hfill (3.6)

The standard deviation of $RV$ when $n = 2$ follows from

$$ E[(RV)^2] = \int_0^\pi (\frac{1}{\pi}) r^2 \cos^2 \frac{W}{2} dW = \frac{r^2}{2} $$  \hfill (3.7)

and

$$ \sigma_{RV}^2 = E[(RV)^2] - (E[RV])^2 $$  \hfill (3.8)
Thus,
\[
\sigma_{rv}^2 = \frac{r^2}{2} - \frac{4r^2}{\pi^2} = 0.095r^2
\]
or
\[
\sigma_{rv} = 0.308r \quad (3.9)
\]

As the number of particles continues to increase, obtaining the closed form distribution characteristics by mathematical techniques becomes exceedingly more difficult and thus the approximation method using the digital computer was employed. However, before the computer analysis is presented in the next section, there is an approximation that will help in understanding how the standard deviation varies for increasing numbers of particles.

The \( E[(RV)^2] \) can be calculated for \( n = 3, 4, \ldots \), and the general relationship
\[
E[(RV)^2] = \frac{r^2}{n} \quad (3.10)
\]
shown to be true by considering (3.8) and writing
\[
\sigma_{rv} \approx r \sqrt{\frac{1}{n} - (E[RV])^2} \quad (3.11)
\]
This will be shown to hold in the next section.

**Computer Analysis**

The random number generator used to simulate the set of independent, uniformly distributed particle locations was implemented from work done by Brown and Rowland [11]. The
program was implemented on the Burroughs B5500 computer using the ALGOL 60 programming language.

The mean and standard deviation of the frequency distributions of RV, developed from 1000 runs at each number of items from 1 to 15, were calculated using

\[ \text{MEAN} = \frac{1}{\text{RUNS}} \sum_{I=1}^{40} \text{RV}[I] \text{MP} \]  \hspace{1cm} (3.12)

(where the histogram used forty intervals of 0.125 each as RV varied from 0 to 1), and the

\[ \text{STD Deviation} = \sqrt{\frac{1}{\text{RUNS}} \sum_{I=1}^{40} (\text{RV}[I])^2 \text{MP}} - (\text{MEAN})^2 \]  \hspace{1cm} (3.13)

where RV[I] is the number of values of RV within that interval, I\text{MP} = midpoint of the interval and RUNS = total runs conducted. Since the radius of the actual drum is 1.08 feet, the use of a normalized radius allowed the analysis to proceed on the basis of 0 ≤ RV ≤ 1.0.

The frequency distributions for 5, 10, and 15 items are plotted in Figure 21 and the plots showing all means and standard deviations from 1 to 15 items are included in Figure 22. A comparison of mathematical and computer developed results appears in Table 2.
Table 2. Analytical and Computer Calculations of Certain Means and Standard Deviations

<table>
<thead>
<tr>
<th>No. of items</th>
<th>Mean Value</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>Analytical</td>
<td>Computer</td>
</tr>
<tr>
<td>1</td>
<td>1.000</td>
<td>0.987</td>
</tr>
<tr>
<td>2</td>
<td>0.636</td>
<td>0.618</td>
</tr>
<tr>
<td>5</td>
<td>---</td>
<td>0.398</td>
</tr>
<tr>
<td>10</td>
<td>---</td>
<td>0.274</td>
</tr>
<tr>
<td>15</td>
<td>---</td>
<td>0.228</td>
</tr>
</tbody>
</table>

In calculating the mathematical standard deviation, (3.11) was used with the computer mean for 5, 10, and 15 items.

Development of the Optimization Problem

The high-speed spin water removal process described and modeled in Chapter II can be thought of as the basis of the problem. The optimum system would allow the maximum water removal in the minimum time subject to the various physical constraints of the washer/dryer system. The total system is formulated as shown in Figure 23, where the plant consists of the system (2.22) and the controller is the implementation of the optimum control strategy.
Figure 21. Frequency Distribution of RV for $n = 5, 10, \text{ and } 15$ Items
Figure 22. Mean and Standard Deviation of RV Distributions
Initial Conditions
\((n, MC, MW_0, Y[0])\)

Figure 23. The Total Washer/Dryer System Model

To determine the optimum control, the performance index

\[ J(u) = \int_0^{t_f} dt \]  

(3.14)

is suggested, where the system is driven from the initial state \(Y[3] = MW_0\) to the final state \(Y[3] = (1 - \alpha) MW_0\) in the minimum amount of time \(t_f\). During operation the system is subjected to the constraints

\[-40 \text{ volts} \leq u \leq +90 \text{ volts} \]

\[ Y[4] \leq 0.030 \text{ inches} \]  

(3.15)

where the bulk and orderly redistribution processes are effective when \(Y[2]\) is less than the drum's critical speed.

This optimum nonlinear stochastic control problem has not been solved theoretically to date [12], and it is not the objective of this thesis to solve this formal problem. In
addition, the complexities of the redistribution process do not lend themselves to a closed form analysis. In the next section, it is shown what approximations and additional constraints can be used to form a suboptimal control strategy.

**Suboptimal Solution**

*Formulation of the Problem*

If all the system states could be measured during the high-speed spin cycle, in particular \( Y[3] \) (weight of water retained in the clothes), then perhaps a different approach might be used to obtain a suboptimal solution. Since there is no direct way to measure \( Y[3] \), it is necessary to formulate the control problem based on the other states that can be measured.

From the Whirlpool laboratory tests, it is known that a process with a minimum spin speed of 250 rpm will give an adequate water removal rate and, if continued for a fixed length of time, will always give an adequate amount of water removal. This constitutes an additional constraint on the washer/dryer system as

\[
Y[2] \mid t_f \geq 250 \text{ rpm} \tag{3.16}
\]

Now the optimization problem is only concerned with attaining a high enough system spin-speed in the minimum amount of time.

Using the concept developed in Chapter II and expressed by (2.23), the successful final state can be stated as total
mass in the distribution (MT) multiplied by the centroid vector length (RV). Thus, an alternate performance index can be expressed as

\[ J(u) = (MC + Y[3]) \cdot RV + \int_0^{t_1} dt \]  

(3.17)

where \( t_1 \) is the time at which the drum speed first exceeds 250 rpm, subject to constraints (3.15) and (3.16) and the system equation (2.22).

**Suboptimal Analysis**

The performance index (3.17) can now be minimized and described in terms of the known characteristics of the bulk and orderly redistribution processes. Taking the derivative of (3.17) with respect to time and equating to zero, the minimum time \( t_1 \) satisfies

\[ RV \frac{d}{dt} Y[3] + (MC + Y[3]) \frac{d}{dt} (RV) + t_1 = 0 \]  

(3.18)

and, in the practical situation, will be minimum when the first two terms are minimized.

The first term of (3.18) indicates the manner in which to proceed if the distribution is very close to being successful. In such a case the approach is to continue the high-speed spin at the limit until the water removal reduces (2.23) to the critical value. There is a limit to this approach and it can be applied to only a very few of the unacceptable distributions. The second term states the other possible way of attaining success, i.e. changing the particle locations.
within the distribution. When considered with the bulk redistribution process analyzed at the beginning of this chapter, it leads to the proper method of redistribution to be employed. When the centroid vector (RV) is larger than the mean value of its frequency distribution considering the number of items involved, then bulk redistribution is most likely to produce a better distribution. However, when the RV is less than the mean, then the orderly redistribution process is the quickest way to proceed. This line of reasoning could be expanded using the standard deviation of the frequency distribution but will not be considered here.

**Implementation**

The above suboptimal strategy will be implemented in the following manner. When the reed switch first opens, if

(a) drum speed ≥ 250 rpm, the present Whirlpool control scheme described in Chapter I will settle out at a successful speed,

(b) 200 rpm ≤ drum speed < 250 rpm, the drum will be slowed by braking to a speed slightly less than critical (about 49 rpm) where the orderly redistribution will produce a successful distribution,

(c) drum speed < 200 rpm, the brake voltage will be applied and the bulk redistribution process used to start the system over again.

The crossover speed of 200 rpm was based on the mean value for the n = 8 distribution, shown in Figure 22, and the
corresponding speed from Figure 18. The remaining problem is knowing when in the orderly redistribution process to reapply the high-speed spin voltage. This practical problem will be covered in Chapter IV, but for the computer implementation it is possible to use the criteria of (2.23).

**Computer-Aided Results**

The control scheme of the preceding section was implemented on the simulation model developed in Chapter II and runs were made for various loading conditions.

For the eight-pound towel load discussed in Chapter II, the predicted washer/dryer response is given in Figure 24. Here the average time spent in orderly redistribution was 20 seconds, and the longest case was 75 seconds. This is a significant improvement over the bulk redistribution process discussed in Chapter II.

The computer simulation also predicted the best time to reapply the high-speed spin voltage during the orderly redistribution process. The typical process, as recorded in Figure 25, showed that if one waited until the system reached a maximum speed, the distribution was not good enough to support a successful spin-up. The best time to reapply the spin voltage is at the point of maximum increasing rate of the motor speed.
Figure 24. Computer Predicted Response of Improved Control Scheme with Eight Pound Towel Load
Figure 25. Variables During Orderly Redistribution Process
CHAPTER IV

IMPLEMENTATION AND DISCUSSION OF RESULTS

Introduction

The objective of this chapter is to outline the implementation procedures and problems that were encountered during final laboratory testing. The test results using orderly redistribution will be presented along with comparisons with the computer predicted and the bulk redistribution process results. Finally, the discussion will focus on computer prediction of actual system operation.

The Whirlpool engineering laboratory staff provided great assistance in this final testing and, through open information exchange and extensive past experience, agreed on the uniqueness and effectiveness of the solution. This actual implementation was extremely important both to test the validity of the control strategy and to point out problem areas where more work will be needed.

Implementation

General Conditions

The final testing was conducted in the Whirlpool Advanced Engineering Laboratories at St. Joseph, Michigan, on 30 and 31 July 1970. The test washer/dryer was the same
as used on previous experiments with the exception of the control circuit which was modified to implement the new control strategy and the use of a voltmeter to constantly monitor the magnitude of motor voltage.

The control scheme used was that developed in Chapter III with 200 rpm as the crossover speed between orderly and bulk redistribution. This speed was measured from the motor shaft by using a hand-held tachometer, which was read directly.

The orderly redistribution motor speed control consisted of the standard tumble speed circuit with an additional potentiometer allowing slightly higher motor voltages. The tumble speed motor control has down-speed control only and was operative to restore the drum to the desired speed when below the set speed. Thus, the motor voltage becomes a series of pulses, the magnitude of which is proportional to the effective load off-balance weight and the pulse frequency that of the drum speed. This circuit was particularly useful for two reasons. First the information relative to off-balance load could be obtained, and secondly, the critical nature of the orderly redistribution method could be observed. For the orderly redistribution method, speeds much below the set speed could not be permitted without degenerating into a semi-bulk method where large groups of clothes items would break away from the drum wall.

In determining the proper time to reapply the spin voltage the concept developed at the end of Chapter III and
illustrated in Figure 25 was combined with the motor voltage as described above. When the magnitude of the motor voltage passed through a local maximum value and started to decrease rapidly, that point was selected to reapply spin voltage.

**Results**

It was determined that a specific speed range of orderly redistribution exists, and for the washer/dryer system described this range was 56 to 58 rpm. In this speed range only single items were changing position within the drum, and the drum speed was a function of the quality of the distribution. If the distribution was relatively poor then the motor voltage pulses were large and the speed stayed between 56 to 57 rpm, while if the distribution was good the speed stayed higher, approximately 57 to 58 rpm, and the voltage pulses were much smaller.

A typical time plot of the motor voltage, Figure 26, shows the effects of orderly redistribution. The plot is oscillatory, although not in the single frequency predicted by the simulation, but the same low frequency characteristics can be detected and effectively used. The points marked as A and B in Figure 26 indicate the best times to reapply spin voltage, and this criteria proved extremely successful in the test runs.

An additional experimental note is that it was possible to detect the motor loading by listening to the motor.
However, attempts at postulating the quality of the load based on this method were only fairly successful because of the detection method.

![Motor Voltage vs Time](image)

**Figure 26. Motor Voltage Versus Time During Orderly Redistribution**

The final results were the effect on system performance of the new control strategy. These results, for the eight-pound towel load, are shown in Figure 27.

**Discussion of Results**

The improvement by control strategy between the bulk redistribution data, Figure 20a, and the computer predicted results, Figure 24, indicated the order of magnitude improvement expected. The comparison between the actual implementation, Figure 27, and the computer prediction points out that the actual results were somewhat better than predicted. The computer simulation was limited to one algorithm in selecting the item to be redistributed, while in fact the
Figure 27. Orderly Redistribution Implementation Results with Eight Pound Towel Load
worst distributed item's position was being changed in the implementation.

The longest time required during the orderly redistribution to attain a successful distribution was about 65 seconds, and there were 17 out of the 50 test runs that attained a speed above 250 rpm without requiring orderly redistribution. This test information compares well with the simulation predictions and is important in confirming the minimum time control strategy.

The performance of the actual implementation and the simulation predictions are in good agreement, and the overall results are, in fact, better than expected. It has been demonstrated conclusively that this control strategy is a distinct improvement over using the bulk redistribution process alone.
CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

General Conclusion

The method of system analysis using computer simulation and modern control theory was shown to be extremely helpful in developing fundamental research data concerning washer/dryer high-speed spin operation and in predicting system performance in the complex environment of orderly redistribution.

Specific Conclusions

The two-dimensional mathematical model and computer simulation of the total washer/dryer system was shown to yield an accurate method for predicting actual machine performance. The model was adequate for systems analysis methods to develop significant results that improved the machine performance.

The high-speed spin portion of the operating cycle of a washer/dryer machine with bulk redistribution alone was determined to be a random process, the expected performance of which could be determined from a straight-forward application of probability theory with the aid of a digital computer.
The bulk redistribution process of clothes items that were small enough to be described by a single center of gravity is a process in which the angular location of each item is a random variable. Each item has an equal probability of being at any location in the distribution, and therefore the angular location may be described by a uniform probability density function.

It was determined that it is not possible to dynamically self-balance the washer/dryer system in a minimum time with only the bulk redistribution process. However, the suboptimal solution using the orderly redistribution process significantly improved the machine performance.

**Recommendations**

This thesis represents fundamental research and basic analysis of the washer/dryer system and should be used for further research. One should proceed to model the clothes distribution by using variable weighted items, then randomly select the item which fills the randomly selected angular location. This more accurate model should be compared with the uniform weighted model used in this thesis. Additionally, the use of location restraints for large segmented items should be explored.

Another area of interest is the stochastic optimization problem posed by this thesis. When solutions of the theoretical problem become available, they should be applied to this
problem and attempts made to implement the results on a variable speed motor controller.

The prediction analysis developed by the simulation of the orderly redistribution, as well as the discussion of those results, should lead to the development of a more sophisticated distribution quality detector. This device could sense the variation of the distribution during orderly redistribution and predict the correct time to reapply spin-up voltage.

Finally, attempts should be made to more accurately model the orderly redistribution process by predicting particle motion throughout the entire sub-critical speed range. Analysis with this tool may allow another set of suboptimal solutions which further reduce the maximum number of tries to attain dynamic self-balance.
BIBLIOGRAPHY


