MICROWAVE FILTERS WITH HIGH STOP-BAND PERFORMANCE AND LOW-LOSS HYBRID DEVELOPMENT

A Dissertation
Presented to
The Academic Faculty

By

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In Partial Fulfillment
Of the Requirements for the Degree
Doctor of Philosophy in Electrical and Computer Engineering

Georgia Institute of Technology
December, 2006

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MICROWAVE FILTERS WITH HIGH STOP-BAND PERFORMANCE AND LOW-LOSS
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Date Approved: November 8, 2006
ACKNOWLEDGEMENTS

I would like to express my sincerest gratitude to my advisor, Dr. Ioannis Papapolymerou, my co-advisor, Dr. Joy Laskar and my supervisor, Dr. Edward J. Wollack, who have always been providing the kindest support, encouragement, guidance and editorial advice throughout this research. Without their patience and encouragement, the accomplishment of the dissertation could not be possible. I would also like to thank all of my other committee members, Dr. John D. Cressler, Dr. Manos M. Tentzeris and Dr. Farrokh Ayazi, who have given me suggestions and helped making the dissertation more comprehensive.

I am especially grateful to my colleagues at National Aeronautic and Space Administration (NASA) Goddard Space Flight Center (GSFC) – Mathew McLinden, Kevin Horgan, David Chuss, Elmer Sharp, Jeff Piepmeier, Fernando Pellerano, Norman Phelps, George Reinhardt, Ed Means, Jared Lucey and Carey Johnson. I am also especially grateful to my colleagues at Georgia Institute of Technology – Pete Kirby, Jiahui Yuan, Minsik Ann, Jau-Horng Chen, Jong Hoon Lee, Nattapong Srirattana, Rungsun Munkong, Nantachai Kantanantha, Tiravat Assawapokee – and others whose names I may have missed. Their friendship and support contributed greatly in the completion of the dissertation.

I wish to express my gratitude to my wife, Manisa Pipattanasomporn, for her continuous support throughout these graduate academic years. I would like to give my dearest appreciation to my parents, Kalyanuwat and Kannika U-yen, who have always given me encouragement and always been there when I need.

I would like to thank Thomas Stevenson and Wen-Ting Hsieh at NASA GSFC for superconducting circuit fabrications and Stephen Horst at Georgia Electronic Design Center (GEDC) for liquid crystal polymer circuit fabrication.
Finally, I would like to thank Catherine A. Long, Terence Doiron and Samuel. H. Moseley, at NASA GSFC, for giving me an opportunity to conduct challenging research in a resourceful and supportive working environment.
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<tbody>
<tr>
<td>3-D</td>
<td>Three dimension</td>
</tr>
<tr>
<td>Al$_2$O$_3$</td>
<td>Aluminum oxide</td>
</tr>
<tr>
<td>APSI</td>
<td>Anti-parallel stepped impedance stub</td>
</tr>
<tr>
<td>CMB</td>
<td>Cosmic microwave background</td>
</tr>
<tr>
<td>CMBpol</td>
<td>Cosmic microwave background polarization</td>
</tr>
<tr>
<td>dB</td>
<td>Decibel</td>
</tr>
<tr>
<td>DSP</td>
<td>Digital signal processing</td>
</tr>
<tr>
<td>DUT</td>
<td>Device under test</td>
</tr>
<tr>
<td>E port</td>
<td>Different port</td>
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<tr>
<td>EM</td>
<td>Electromagnetic</td>
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<td>GEDC</td>
<td>Georgia electronic design center</td>
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<td>GHz</td>
<td>Gigahertz</td>
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<td>GSFC</td>
<td>Goddard space Flight center</td>
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<td>H</td>
<td>Horizontal</td>
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<tr>
<td>H port</td>
<td>Sum port</td>
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<tr>
<td>HEMT</td>
<td>High electron mobility transistor</td>
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<td>HDPE</td>
<td>High density Polyethylene</td>
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<td>Hi-Z</td>
<td>High impedance</td>
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<td>Hertz</td>
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<tr>
<td>LCP</td>
<td>Liquid crystal polymer</td>
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<td>Lo-Z</td>
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<td>MS</td>
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<td>MS-to-SL</td>
<td>Microstrip-to-slotline</td>
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<td>MUX</td>
<td>Multiplexer</td>
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<td>NASA</td>
<td>National aeronautic and space administration</td>
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<tr>
<td>Nb</td>
<td>Niobium</td>
</tr>
<tr>
<td>OMT</td>
<td>Ortho-mode transducer</td>
</tr>
<tr>
<td>SCR</td>
<td>Stepped circular ring</td>
</tr>
<tr>
<td>Si</td>
<td>Silicon</td>
</tr>
<tr>
<td>SIO</td>
<td>Stepped impedance open</td>
</tr>
<tr>
<td>SIR</td>
<td>Stepped impedance ratio</td>
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<tr>
<td>SL</td>
<td>Slotline</td>
</tr>
<tr>
<td>SOLT</td>
<td>Short-open-load-thru</td>
</tr>
<tr>
<td>T</td>
<td>Temperature (Kelvin)</td>
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<tr>
<td>T-line</td>
<td>Transmission line</td>
</tr>
<tr>
<td>TRL</td>
<td>Thru-Reflect-Line</td>
</tr>
<tr>
<td>TT</td>
<td>The temperature-temperature angular spectrum</td>
</tr>
<tr>
<td>UIR</td>
<td>Uniform impedance resonator</td>
</tr>
<tr>
<td>V</td>
<td>Vertical</td>
</tr>
<tr>
<td>WMAP</td>
<td>Wilkinson microwave anisotropic probe</td>
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## LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>%error</td>
<td>Percentage error</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Electrical length of the tapping location in of a resonator referenced from grounded-end section</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Electrical length</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Guided wavelength</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Ohm</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Permeability</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Permittivity</td>
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<tr>
<td>$\omega$</td>
<td>Angular velocity</td>
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<tr>
<td>$\lambda/4$</td>
<td>Quarter wavelength</td>
</tr>
<tr>
<td>$\phi'$</td>
<td>Electrical length of the tapping location in of a resonator referenced from opened-end section</td>
</tr>
<tr>
<td>$\Gamma_+$</td>
<td>Even-mode reflection coefficient at port 1 of a magic-T</td>
</tr>
<tr>
<td>$\Gamma_{++}$</td>
<td>Odd-mode reflection coefficient at port 1 of a magic-T</td>
</tr>
<tr>
<td>$\theta_0$</td>
<td>Electrical length of a stepped impedance resonator when the resonator has minimum length</td>
</tr>
<tr>
<td>$\lambda_0$</td>
<td>Penetration depth of a superconductor at zero Kelvin</td>
</tr>
<tr>
<td>$\omega_0$</td>
<td>The center of the operating angular velocity</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>Electrical length of the tapping location in of the SIR from the left side</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>Electrical length of the transmission line number 1</td>
</tr>
<tr>
<td>$\theta_1'$</td>
<td>Electric length of the SIR in the series Hi-Z line section</td>
</tr>
<tr>
<td>$\theta_1''$</td>
<td>Electric length of the Hi-Z anti-parallel line sections of the SIR</td>
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\( \phi_2 \)  
Electrical length of the tapping location in of the SIR from the right side

\( \theta_2 \)  
Electrical length of the transmission line number 2

\( \theta_2' \)  
Electric length of the SIR in the series Lo-Z line section

\( \theta_2'' \)  
Electric length of the Lo-Z anti-parallel line sections of the SIR

\( \lambda_L \)  
Penetration depth of a superconductor

\( \theta_p \)  
Fundamental transmission pole generated by the APSI opened-end stub

\( \varepsilon_r \)  
Relative dielectric constant

\( \theta_{sl1} \)  
Electrical length of the Hi-Z transmission line of the slotline SCR stub

\( \theta_{sl2} \)  
Electrical length of the Lo-Z transmission line of the slotline SCR stub

\( \theta_{t1} \)  
Electrical length of the Hi-Z transmission line of the SIO stub

\( \theta_{t2} \)  
Electrical length of the Lo-Z transmission line of the SIO stub

\( \theta_{z,1} \)  
The fundamental transmission zero generated by the APSI opened-end stub

\( \theta_{z,2} \)  
The second lowest transmission zero generated by the APSI opened-end stub

\( a \)  
Admittance slope

\( b \)  
Susceptance slope

\( B \)  
Susceptance

\( c \)  
Velocity of light in free space

\( c_p \)  
Coupling coefficient of a coupled line

\( f \)  
Frequency

\( f_0 \)  
Center of the operating frequency

\( f_c \)  
Cut-off frequency

\( f_{\text{high}} \)  
High frequency limit
\( f_{\text{low}} \)  
Low frequency limit

\( f_{pt,1} \)  
The lowest transmission zero frequency generated by the taped SIR

\( f_{pt,2} \)  
The second lowest transmission zero frequency generated by the taped SIR

\( f_{st1} \)  
The lowest spurious resonance frequency of the SIR

\( f_{st2} \)  
The second lowest spurious resonance frequency of the SIR

\( f_z \)  
Transmission zero frequency

\( g_i \)  
Filter coefficient section \( i \)

\( G_{n1} \)  
The spacing of the Hi-Z coupled lines of the APSI stub

\( h \)  
Plank’s constant

\( l \)  
Multipole moment

\( l_i \)  
Input current at port \( i \)

\( ISO_{\text{high}} \)  
High-frequency side stop-band isolation

\( ISO_{\text{low}} \)  
Low-frequency side stop-band isolation

\( J_{i,j+1} \)  
The impedance inverter of the filter section \( i \)

\( k \)  
Boltzmann’s constant

\( K_{i,j+1} \)  
The admittance inverter of the filter section \( j \)

\( L_{n1} \)  
The length of the Hi-Z coupled lines of the APSI stub

\( L_{n2} \)  
The length of the Lo-Z line of the APSI stub

\( L_{p1} \)  
Physical length of the grounded-end anti-parallel line of the SIR

\( L_{p2} \)  
Physical length of the opened-end anti-parallel line of the SIR

\( L_{st1} \)  
Physical length of the Hi-Z series line of the SIR

\( L_{st2} \)  
Physical length of the Lo-Z series line of the SIR

\( L_t \)  
The line length of the \( Z_t \) line

\( L_{n1} \)  
The length of the Hi-Z coupled line of the SIO stub
$L_2$  The length of the Lo-Z line of the SIO stub

$n$  Natural number

$N$  The filter order

$n_t$  Transformer turn ratio

$P_{\text{actual}}$  The detected power from the black body radiation with filtering

$P_{\text{blkbody}}$  Plank’s black body radiation per unit volume per unit frequency

$P_{\text{detect}}$  Detected power from the black body radiation for the entire frequency spectrum

$P_{\text{ideal}}$  Ideal detected power of black body radiation

$Q_{\text{ext}}$  External quality factor

$Q_{\text{si}}$  Singly-loaded quality factor

$R$  Stepped impedance ratio

$R_L$  Resonator’s load

$R_s$  Stepped impedance ratio of the SIO stub

$R_x$  The impedance ratio of the Hi-Z of the SIO stub and even-mode Hi-Z section

$S_{11}$  Reflection coefficient looking into port 1

$S_{12}$  transmission coefficient from port 2 to port 1

$S_{1E}$  transmission coefficient from port E to port 1

$S_{1H}$  transmission coefficient from port H to port 1

$S_{21}$  transmission coefficient from port 1 to port 2

$S_{22}$  Reflection coefficient looking into port 2

$S_{2E}$  transmission coefficient from port E to port 2

$S_{2H}$  transmission coefficient from port H to port 2

$S_{p1}$  The spacing between two grounded-end anti-parallel couple lines
$S_{p2}$ The spacing between two opened-end anti-parallel couple lines

$t$ Conductor thickness

$T_c$ Superconductor critical temperature

$u$ The electrical length ratio of Lo-Z line and the total electrical length of the resonator

$u_s$ The electrical length ratio of Lo-Z line and the total electrical length of the SIO stub

$V_i$ Voltage at port $i$

$w$ Percentage bandwidth

$W_{n1}$ The width of the Hi-Z coupled lines of the APSI stub

$W_{n2}$ The width of the Lo-Z line of the APSI stub

$W_{s1}$ The width of the Hi-Z line of the SIR

$W_{s2}$ The width of the Lo-Z line of the SIR

$W_t$ The width of the $Z_t$ line

$W_{t1}$ The width of the Hi-Z line of the SIO stub

$W_{t2}$ The width of the Lo-Z line of the SIO stub

$X$ Reactance

$Y_1$ Characteristic admittance of a transition line number 1

$Y_2$ Characteristic impedance of a transition line number 2

$Y_{in}$ Input admittance

$Z_{n1}$ Characteristic impedance of space

$Z_0$ Port characteristic impedance

$Z_{0,e}$ Even-mode characteristic impedance of a coupled line

$Z_{0,o}$ Odd-mode characteristic impedance of a coupled line

$Z_1$ Characteristic impedance of a transition line number 1
\( Z_{1,e} \) Even-mode characteristic impedance of the Hi-Z coupled line

\( Z_{1,o} \) Odd-mode characteristic impedance of the Hi-Z coupled line

\( Z_2 \) Characteristic impedance of a transition line number 2

\( Z_{2,e} \) Even-mode characteristic impedance of the Lo-Z coupled line

\( Z_{2,o} \) Odd-mode characteristic impedance of the Lo-Z coupled line

\( Z_3 \) Characteristic impedance of a transition line number 3

\( Z_{ij} \) Impedance value of the Z matrix in row \( i \) and column \( j \)

\( Z'_{ij} \) Impedance value of the \( Z \) matrix row \( i \) and column \( j \) of the coupled line where one end of the lines are connected together

\( Z_{\text{in}} \) Input impedance

\( Z_{\text{in}} \) Input impedance seen at the SIO stub

\( Z_t \) Characteristic impedance of a transition line used to transform slotline impedance to microstrip line impedance at port E
SUMMARY

This dissertation contains two significant investigations. One is the development of the broadband microwave bandpass filters with high out-of-band performance. The other is the development of low-loss hybrids. These researches are parts of the National Aeronautic and Space Administrator (NASA)’s mission to explore the universe.

The former is focused on the techniques used in microstrip line bandpass filter design that help achieving both low in-band insertion loss and high out-of-band attenuation level. Moreover, these filters achieve very broadband out-of-band attenuation bandwidth. These techniques are related to the improvement of stepped impedance resonators, coupling between resonators and effective methods to allocate transmission zeros to suppress filter’s out-of-band spurious responses.

The later is focused on the techniques used in planar magic-T designs such that the developed magic-T obtains high isolation between port E (difference port) and port H (sum port). Moreover, it obtains low-loss and broadband characteristics. These techniques are related to the development of the low-loss broadband microstrip-to-slotline (MS-to-SL) transition and the magic-T with a highly symmetric structure.

The theoretical analysis and experimental measurements have been performed. The experimental results of both the filter and magic-T researches show significant improvement over their prior state-of-the-art designs by number of magnitude. The designs also reduce fabrication complexity.

The dissertation consists of five chapters. Chapter one discusses the requirements of the bandpass filters and magic-Ts that are used in space applications to observe microwave cosmic background polarization. Chapter two discusses the bandpass filter literature review and its design techniques to obtain high out-of-band performance. Chapter three discusses the literature review of the microstrip-to-slotline
(MS-to-SL) transitions and magic-Ts. The design of MS-to-SL transitions using stepped circular ring and the design of broadband magic-Ts are proposed. Finally, chapter four and chapter five conclude this dissertation and provide recommendation about their applications and the future extension of the current research.
CHAPTER 1
INTRODUCTION

1.1 Cosmic Microwave Background Polarization Sensing

Electromagnetic radiation has been widely utilized in various applications, including remote sensing systems. The ability to detect a microwave signal is dependent on the sensitivity and the selectivity of the receiver. High sensitivity and selectivity can be achieved by increasing the number of sensors, suppressing out-of-band interference, minimizing the receiver insertion loss, implementing coherent detection techniques, and amplifying and filtering the received signal. In communication systems, the microwave signal typically has known patterns and high power. Therefore, detecting this signal can be simple. However, in astronomical applications, the detected signals generally have very low power. Moreover, the signals are potentially contaminated by out-of-band interference.

As part of the NASA’s mission to explore the universe, scientists measure the cosmic microwave background (CMB) polarization at various angular scales as shown in Figure 1-1. The temperature-temperature angular power spectrum (TT) has been well-measured and corresponds to the density anisotropies that provide the seeds for structure formation later in the universe’s history. The polarization is just now beginning to be explored and is believed to be present in two modes: the E-modes and the B-modes. The former represents those polarization patterns that are curl-free and can be generated from the same anisotropies that produce the TT spectrum. Conversely, the latter represents those polarization patterns that are divergence-free and can only be produced by gravitational waves created by a hypothesized period of exponential expansion early in the universe’s history referred to as “inflation”. The discovery of the B-
modes would provide solid evidence for inflation, and for this reason, they are highly anticipated. Current technologies have enabled the measurement of the (unpolarized) CMB temperature anisotropy, temperature-polarization correlation and E-mode polarization using the Wilkinson Microwave Anisotropic Probe (WMAP) (NASA 2005). At this point, WMAP and other instruments have placed and reported upper limits on the presence of CMB B-mode polarization. The minimum detectable signal is limited by number of channels, time and economic of running the instrument. To address these practical issues, the next generation of instruments will employ large arrays of incoherent detectors to achieve the required sensitivity.

![Graph showing CMB temperature, temperature-polarization correlation, E-mode and B-mode polarizations versus the multi-pole moment.](image)

Figure 1-1  The response of the CMB temperature, temperature-polarization correlation, E-mode and B-mode polarizations versus the multi-pole moment.

To detect the B-mode polarization, which has the temperature fluctuation of 0.02 μK, a direct detection system that can operate at 0.1 K or lower must be developed. At
this temperature range, the system benefits from the proper operation of passive superconducting microwave components and sensitive superconducting microwave detectors. These components will produce lower background noise than active amplifiers because the background noise is limited to the thermal noise at their operating temperature.

This dissertation focuses on the development of design techniques used to produce high-performance microwave planar bandpass filters and magic-Ts. These filters and magic-Ts are constituents of the direct detection system that will be used to search for the B-mode microwave cosmic background polarization in the frequency range from 27.5 GHz to 150 GHz. The use of direct detectors (e.g. transition-edge sensors) allows the system noise to be limited by the quantum fluctuations in the 2.725 K CMB. To increase the sensitivity of a background limited instrument, it is necessary to employ multiple detectors. Each detector shown in Figure 1-2(a) consists of a corrugated circular waveguide terminated with a quarter wavelength (\(\lambda/4\)) backshort. Microwave energy in the waveguide is collected at the orthomode transducer (OMT), a part of the planar detecting circuit, and transferred to other microstrip circuits. Microstrip circuits are suitable candidates in this application as opposed to waveguide circuits since several detector modules can be produced using fewer fabrication steps. Moreover, the modules are smaller and lighter.

Each detector system consists of several planar circuits such as the OMT, bandpass filters, magic-Ts, and thermal detectors as shown in Figure 1-2(b). The OMT is used to extract the horizontal (H) and vertical (V) components of the signals. The magic-T is used to combine the out-of-phase signals generated by the OMT. The filters are used to accept the in-band power and reject unwanted out-of-band power. Finally the signal is received at the thermal detector. The readings from several thermal detectors are multiplexed to one output line. The digital signal processing (DSP) unit is
used to extract the $Q$ and $U$ Stokes parameters from which the desired angular power spectrum can then be derived.

Figure 1-2  (a) The 3-D illustration of the CMB polarization detection system and (b) The planar circuit block diagram.

1.2 Filters’ and Magic-Ts’ General Requirements

The filters are used in the direct detection system to provide very broad attenuation bandwidth and high out-of-band suppression to reject the out-of-band infrared thermal noise. Sufficient attenuation must be provided up to seven times the
filter’s center frequency ($f_0$). This will be discussed in section 2.2. In addition, the filter must have low in-band insertion loss and must be simple to fabricate.

The magic-Ts are used in the direct detection system to combine the pairs of vertical and horizontal OMT probes. They must have low loss and very broadband response to conserve signal power. Moreover, they must have high E-H port isolation to prevent the generation of higher order modes in the wave guide structure.

To simplify the fabrication process used to produce filters and magic-Ts, it is desirable that the designs have the minimum number of via holes and metallized layers. Since the network analyzer used to measure the filter frequency response can operate up to 40 GHz, all prototypes of the planar bandpass filters were designed at 1.4 GHz, 4 GHz and 33 GHz to be able to observe the spurious frequency response of up to 40 GHz. The magic-T prototypes are designed to operate at 10 GHz and are tested from 5 GHz to 20 GHz.

1.3 Contributions

There are two areas of contribution in this dissertation.

The first is the development of bandpass filters with high out-of-band rejection. The double split-end stepped impedance resonator and its related structures are introduced for the first time. The optimal resonator coupling coefficients have been determined. The broadband bandstop filter has been studied. Its transmission poles and zeros were derived so that the bandstop filter can be integrated with the bandpass filter without degrading the in-band response. By combining all of the developed techniques, the filter simultaneously produces very high out-of-band attenuation and low in-band insertion loss. This level of out-of-band attenuation and bandwidth has not previously been reported for planar circuits.
The second is the development of the low-loss hybrids. The technique used to reduce radiation loss in a microstrip-to-slotline transition is introduced for the first time. The developed planar magic-Ts use the smallest slotline area. As a result, these magic-Ts have lower insertion loss and higher sum-to-different port isolation, at frequencies above 5 GHz, than any reported by the prior state-of-the-art broadband planar magic-Ts.

This research produces high-performance filters and magic-Ts that are not only suitable for use in radio astronomy applications, but also suitable for use in most microwave systems. Moreover, the fabrication of both filters and magic-T requires few metallized layers. The filter and magic-T designs also require no via holes, bondwires or air-bridges, which significantly reduce their fabrication complexity.
CHAPTER 2
FILTER DESIGN WITH HIGH OUT-OF-BAND PERFORMANCE

2.1 Literature Review

Microwave filter design has been a subject of interest for several decades. Several planar filter designs satisfy most of the requirements around the filter pass-band (Hong and Lancaster 1996; Liang, Shih et al. 1999; Kuo and Cheng 2004; Chang and Tam 2005). However their out-of-band performance is often limited. Since the filter is fundamentally made of sections of transmission line to imitate the ideal lumped-element filter response, the in-band response of the filter is roughly reproduced out-of-band because of the transmission line’s periodic property (Pozar 1997).

The out-of-band characteristic of the filter is also dependent on the order of the filter, pass-band bandwidth and the separation between the fundamental frequency and the lowest spurious resonance mode of the filter (Matthaei, Young et al. 1980).

To obtain the filter with wide stop-band and high stop-band attenuation, several techniques have been developed. These techniques are generally fall into three categories. One is to alter the transmission line periodicity. Second is to design filters with stepped impedance resonators. Third, the techniques use the transmission line periodicity to suppress the filter’s spurious frequency response. Finally, the filter can be embedded with dissipative elements to suppress the out-of-band response.

2.1.1 Transmission Line Periodicity Alteration Techniques

The transmission lines shape is modified to produce discontinuity in transmission line sections such that it transmits signal in the pass-band but reflects the signal out-of-band. These techniques are commonly incorporated in resonators in the bandpass filter
design such that the filter’s spurious responses are shifted away from the fundamental frequency.

The level of discontinuities ranges from small discontinuity such as in a wiggly-line filter (Lopetegi, Laso et al. 2004) and in linear tapered impedance resonators (Sagawa, Shirai et al. 1993), to large step discontinuity, which requires transmission lines with slots on a ground plane (Quendo, Rius et al. 2001; Wang and Zhu 2005).

A microstrip line section can also be patterned as in (Chang and Tam 2005), which produces a transmission zero out-of-band. The transmission pattern on a high-temperature superconductor can also reduce the physical size of the filter, as it behaves like a delay line (Lancaster, Huang et al. 1996). Moreover, using lumped elements in the planar filter design (Swanson 1989) can be considered as producing discontinuity. Since the inductor and capacitor consist of narrow and wide transmission lines, their series connection produces a step in conductor width. The semi-lumped-element technique (Kaddour, Pistono et al. 2004) can also be used. In theory, a lumped element filter produces filter response with no out-of-band spurs. However, in practical implementation, lumped elements at high frequency are considered sections of transmission line and the filters using these elements produce a spurious response.

The stop-band performance of the filter using these techniques is typically limited to 30 dB for a filter order of less than 3 and with 10 percent of bandwidth. The out-of-band suppression bandwidth is limited to four times its fundamental frequency.

2.1.2 Filter design using stepped-impedance resonators

To determine the approximate location of the spurious response of the filter, stepped-impedance resonators (SIRs) are used (Sagawa, Takahashi et al. 1989; Ishizaki and Uwano 1994; Sagawa, Makimoto et al. 1997; Wada and Awai 1999; Makimoto and Yamashita 2001; Nam, Lee et al. 2001; Kuo and Shih 2002; Lee, Park et
al. 2002; Sanada, Takehara et al. 2002; Uchida, Furukawa et al. 2002; Avrillon, Pele et al. 2003; Banciu, Ramer et al. 2003; Kuo and Shih 2003; Padhi and Karmakar 2004; Pang, Ho et al. 2004). The spurious response of the filter using this technique is directly related to the resonance frequency of the resonator. As the stepped-impedance ratio ($R$) of the SIR is reduced, the filter’s stop-band bandwidth is extended (Makimoto and Yamashita 1980; Kuo and Shih 2003; U-yen, Wollack et al. 2004). The minimum value of $R$ is set by the limited physical line width of the resonator. The spurious response of the filter using this technique can be predicted as long as the transmission line propagates in (or close to) TEM mode at high frequency. There are three types of SIR resonators: full-wave, half-wave and quarter-wave lengths.

The full-wave SIR has the highest resonance modes of the three types (Karacaoglu, Robertson et al. 1994; Sagawa, Makimoto et al. 1997). It is typically used in dual-mode filter design and has no benefit in out-of-band suppression. Moreover, it has large size.

The half-wave SIR is widely used in bandpass filter designs (Makimoto and Yamashita 1980; Sagawa, Takahashi et al. 1989; Sagawa, Makimoto et al. 1997; Wada and Awai 1999; Nam, Lee et al. 2001; Banciu, Ioachim et al. 2002; Kuo and Shih 2002; Uchida, Furukawa et al. 2002; Avrillon, Pele et al. 2003; Banciu, Ramer et al. 2003; Kuo and Shih 2003; Kuo, Hsieh et al. 2004; Padhi and Karmakar 2004), since it requires no ground termination in the resonator. It consists of both odd and even mode resonance frequencies. The maximum filter’s out-of-band suppression using this type of SIR is achieved using the coaxial-line with Saucer-loaded SIRs (Uchida, Furukawa et al. 2002). The optimal size of the half-wave SIR is determined in (Kuo and Shih 2003). This size gives the maximum separation between the fundamental frequency and its lowest spurious resonance frequency for a given value $R$, although it does not produce the minimum size SIR. The optimal-size SIR is used in the filter design and produces a filter
with a stop-band bandwidth of $8.4f_0$ and with a least 32 dB of attenuation (Kuo and Shih 2003).

Finally, the quarter-wave SIR is also used in filter designs (Ishizaki and Uwano 1994; Lee, Park et al. 2002; Sanada, Takehara et al. 2002; Pang, Ho et al. 2004). It has the smallest physical size and the fewest number of resonance frequency modes since only odd mode resonance frequencies exist. The optimal size of the quarter-wave SIR is determined in (Makimoto and Yamashita 1980). It not only gives the broadest separation of the spurious resonance frequency and its fundamental frequency, but also gives the smallest resonator size (U-yen, Wollack et al. 2004).

2.1.3 Transmission line techniques used to suppress the filter’s spurious response

Transmission zeros can be integrated into the filter in the form of quarter-wave length transmission line opened-end (Wong 1979) or stepped-impedance line opened-end (Kuo, Hsieh et al. 2004). In planar coupled-SIR filters, transmission zeros are incorporated in the end sections of the filter in the form of tapped resonator (Kuo and Shih 2003) or spur line structure (Pang, Ho et al. 2004). They are inserted in the middle section of the resonators in (Kuo, Hsieh et al. 2004). They can also be embedded in the transmission line (Chang and Tam 2005). The transmission zero can also be integrated inside the resonator through anti-parallel coupling (Matsuo, Yabuki et al. 2000), conventional parallel coupling, or the impedance transformation technique (Wada and Awai 1999).

2.1.4 The filter design embedded with dissipative elements to suppress out-of-band response.

Dissipative elements can be inserted in the filter to suppress out-of-band spurious responses. The dissipative elements can be in the form of a resistor (Lee, Ryu
et al. 2002) or a large slot on the ground plane (Kim, Kim et al. 2004). Although using a resistor can suppress spurious frequency responses by approximately 25 dB, it produces an additional 0.5 dB of insertion loss in-band. The dissipative element using a slot on a large ground plane does not contribute much loss at low frequency; however, its out-of-band radiation loss increases in-band insertion loss as the filter’s operating frequency increases. In conclusion, the dissipative techniques can be used with limited slot size on a ground plane to minimize in-band insertion loss. Resistive elements should not be used in low-loss filter designs.

### 2.2 Filter’s Out-of-band Requirement for Radio Astronomy Applications

In radio astronomy applications, the microwave filter is one of the many important components in the system, since it suppresses out-of-band noise/inference and determines the quality of the received signal. The ultimate research goal is to design low-loss filters for this application at 100 GHz and operate them at 4 Kelvin. The current research aims to develop a scaled prototype at lower frequency.

The required specifications of the filter for this application are set by the required accuracy of the instrument. The filter will be used at the front end of the instrument to detect Cosmic Microwave Background Polarization (CMBPol) radiation. This radiation is used to study the expansion of the universe from the Big Bang (Haig 1998). The energy spectrum of the CMBpol can be estimated accurately by the Plank’s black body radiation equation per unit volume per unit frequency as follows:

\[
P_{\text{blackbody}}(f, T) = \frac{8\pi hf^3}{c^3} \frac{\frac{m_f}{kT}}{e^{\frac{m_f}{kT}} - 1}.
\]  

\[\text{(2-1)}\]
where $c$ is the velocity of light in vacuum and is equal to $3 \times 10^8$ m/s. $h$ is the Plank's constant and is equal to $6.626 \times 10^{-34}$ m$^2$kg/s and $k$ is the Boltzmann's constant and is equal to $1.38 \times 10^{-23}$ Joules/K. $T$ is the temperature of the black body in Kelvin. The CMBpol's radiating temperature is 2.725 Kelvin (K). It will be measured with the presence of the sun’s far-infrared radiation at ~30K ranging from 300 to 600 GHz. The overall energy spectrum is shown in Figure 2-1. Since the sun’s infrared power is at least 30 dB higher than that of the CMBpol's, it produces strong out-of-band interference, which can significantly degrade the detecting signal quality.

The CMBpol detector, operating at the frequency ranging from 80 to 120 GHz, requires low-loss bandpass filter with wide stop-band bandwidth and with an attenuation to reject the infrared interference. Since the planar filter for this application is made of Niobium (Nb) superconductor on dielectric substrate. The minimum stop-band bandwidth of the filter is set by the cut-off frequency of the Nb superconductor ($f_c$) at ~700 GHz, where the Nb superconductor becomes normal metal and produces high signal attenuation above 700 GHz.

To investigate the error caused by the strong interference, the integral power at the detector is measured and compared with the ideal receiving power. The filter used in this investigation is assumed to have ideal transition from pass-band to stop-band and has infinite stop-band bandwidth. The low-frequency side stop-band isolation ($ISO_{low}$) and high-frequency side stop-band isolation ($ISO_{high}$) are defined relative to the in-band insertion loss, as shown in Figure 2-2.
Figure 2-1  The estimated CMBpol radiation at 2.73 K (solid line) and the black body radiation in Far-infrared frequency at 30 K (dotted line).

Figure 2-2  Idealized band-pass filter $\text{dB}|S_{21}|$ response.
The amount of detected power is the integral of radiation energy for the entire frequency spectrum as follows:

\[
P_{\text{detected}}(T, f_{\text{low}}, f_{\text{high}}) = \int_{f_{\text{low}}}^{f_{\text{high}}} P_{\text{bblkbody}}(f, T) \, df.
\]  

(2-2)

The percentage reading error can be defined as the difference between the ideal and the actual detected power as follows:

\[
\% \text{error} = \frac{P_{\text{actual}} - P_{\text{ideal}}}{P_{\text{ideal}}} \cdot 100
\]

(2-3)

where the ideal detected power is

\[
P_{\text{ideal}} = \int_{f_1}^{f_2} P_{\text{bblkbody}}(f, T = 2.73K) \, df.
\]

(2-4)

And the actual detected power is

\[
P_{\text{actual}} = \int_{f_1}^{f_2} P_{\text{bblkbody}}(f, T = 2.73K) \, df + \int_0^{f_1} P_{\text{bblkbody}}(f, T = 2.73K) \, df + \int_{f_2}^\infty P_{\text{bblkbody}}(f, T = 30K) \, df + \int_{600\text{GHz}}^{300\text{GHz}} P_{\text{ISO}}(f, T = 30K) \, df.
\]

(2-5)

The percentage error can be plotted as a function of \( ISO_{\text{low}} \) and \( ISO_{\text{high}} \) as shown in Figure 2-3. From Figure 2-3, we observed that the out-of-band interference has a strong influence on signal detection error. To achieve a percentage error of less than 1 percent, the \( ISO_{\text{high}} \) should be more than 55 dB and the \( ISO_{\text{low}} \) is more than 30 dB. This requirement set the specification of the bandpass filter used for this application.
Figure 2-3 Percentage signal detection error caused by the filter with the finite out-of-band isolation of $ISO_{high}$ and $ISO_{low}$.

2.3 Quarter-wave SIR Spurious Characteristics and Its Optimal Length

The $\lambda/4$ SIR has a superior spurious response than other types of SIR. Its size is smaller and the resonator excites fewer spurious frequency modes. The minimum size of this resonator has been determined as well as it spurious responses in (Sagawa, Makimoto et al. 1985), however no literature has been reported regarding the optimum $\lambda/4$ SIR length that provides maximum spurious-free bandwidth.

This section explains the detail of the $\lambda/4$ SIR spurious resonance mode and the optimal resonator length that maximize the filter’s spurious-free bandwidth. From a $\lambda/4$ SIR shown in Figure 2-4, the input admittance at the opened-end side can be derived as follows:
Figure 2-4 The microstrip line SIR structure.

\[
Y_{in} = j \cdot Y_2 \frac{\tan(\theta_1) \tan(\theta_2) - R}{\tan(\theta_1) + R \tan(\theta_2)}
\]  

(2-6)

Where:

- \( \theta_1 \): an electrical length of the transmission line \( Z_1 \),

- \( \theta_2 \): an electrical length of the transmission line \( Z_2 = 1/Y_2 \),

- \( Y_{in} \): input admittance from the opened end of the resonator,

- \( Z_{in} \): input impedance from the grounded end,

- \( R \): the stepped impedance ratio \( Z_2/Z_1 \).

When \( Y_{in} \) equals to 0, the resonance condition is as shown in (2-7).

\[
R = \frac{Z_2}{Z_1} = \frac{\tan(\theta_1) \tan(\theta_2)}{\tan(\theta_1) + R \tan(\theta_2)}
\]  

(2-7)

Since there is only one resonance conditions in (2-6) when the nominator of (2-6) equals 0, the number of spurious frequencies is minimized and shifted away from the fundamental frequency. The optimal length of the resonator for the most extend spurious frequency can be determined by root searching technique. By defining the ratio \( u = \theta_2/(\theta_1 + \theta_2) \) with the value of \( R \) ranging from 0 to 1 (Kuo and Shih 2003), the normalized spurious response can be determined according to the scaling of the SIR line length as shown in Figure 2-5. It is found that the spurious response of the \( \lambda/4 \) SIR is symmetric.
and the second resonance (or the first spurious) frequency is maximized when \( u = 0.5 \) (i.e. \( \theta_1 = \theta_2 = \theta_0 \)) for all \( R \) greater than 0. This is the similar condition that produces the smallest resonator length. When \( R \) equals 0 or 1, the SIR becomes a uniform quarter-wave impedance resonator and its normalized resonances are positive odd numbers.

![Graph](image)

**Figure 2-5** The lowest three spurious frequencies \( (f_{s1}, f_{s2}, \text{and } f_{s3}) \) of the \( \lambda/4 \) SIR, normalized with the center frequency \( (f_0) \), versus the ratio \( u \) when \( R = 0.2, 0.3 \text{ and } 0.5 \).

## 2.4 Parallel-coupled \( \lambda/4 \) SIR Bandpass Filter

To construct a compact \( \lambda/4 \) microstrip SIR filter, SIRs are parallel coupled as shown in Figure 2-6. There are two canonical coupled line circuits on the SIR. The opened-end and the short-end parallel coupling sections are used one after another as shown in the circuit model in Figure 2-7. In this design, the optimum \( \lambda/4 \) SIR length is chosen as discussed above. The coupler can be modeled as either the admittance inverter (\( K \)) or impedance inverter (\( J \)). To compute \( J \) and \( K \), the susceptance slope (\( b \))
and admittance slope \((a)\) of the \(\lambda/4\) SIR are required. These parameters can be calculated based on (Matthaei, Young et al. 1980). The parameters \(b\) and \(a\) can be derived as follows:

\[
b = \frac{\theta_0}{2} \frac{dB}{d\theta} \bigg|_{\theta = \theta_0} = \frac{\theta_0}{2} \frac{2Y_2^2(1 + \tan(\theta_0)^2)}{Y_1 + Y_2} = \theta_0 Y_2 \tag{2-8}
\]

\[
a = \frac{\theta_0}{2} \frac{dX}{d\theta} \bigg|_{\theta = \theta_0} = \frac{\theta_0}{2} \frac{2Z_2^2(1 + \tan(\theta_0)^2)}{Z_1 + Z_2} = \theta_0 Z_1 \tag{2-9}
\]

where susceptance \((B)\) and reactance \((X)\) are imaginary parts of \(Y_{in}\) and \(Z_{in}\) from Figure 2-4.

In the design of an \(N\)-stage band-pass filter, all SIRs have the same \(R\) \((Z_1\) and \(Z_2)\) in all stages as shown in Figure 2-6. The short-end coupled line is used at the first and the last section of the filter, since most applications require same impedance termination at both input and output (i.e. \(Z_1 = Z_0\)). Moreover, the coupling gap of the high impedance coupled line in those sections can be designed with ease without reaching
the fabrication tolerance limitation. Using (2-8) and (2-9), the impedance and the admittance inverter of coupling sections can be calculated as follow:

\[
K_{0,i} = \sqrt{\frac{w a_i Z_1}{g_0 g_i}} = Z_1 \sqrt{\frac{w \theta_0}{g_0 g_i}} \quad (2-10a)
\]

\[
J_{i,i+1} = \sqrt{\frac{w^2 b_i b_{i+1}}{g_i g_{i+1}}} = \frac{w \theta_0}{Z_2 \sqrt{g_i g_{i+1}}} \quad (2-10b)
\]

\[
K_{j,j+1} = \sqrt{\frac{w^2 a_j a_{j+1}}{g_j g_{j+1}}} = \frac{w \theta_0 Z_1}{\sqrt{g_j g_{j+1}}} \quad (2-10c)
\]

\[
K_{N,N+1} = \sqrt{\frac{w a_N Z_1}{g_N g_{N+1}}} = Z_1 \sqrt{\frac{w \theta_0}{g_N g_{N+1}}} \quad (2-10d)
\]

where \(i\) and \(j\) are odd and even numbers between 1 and \(N-1\), respectively; \(w\) is the fractional bandwidth of the center frequency. \(g_i\) and \(g_j\) are the filter coefficients of section \(i\) and \(j\), respectively. Odd and even mode impedance and admittance of coupled lines can be determined based on (Makimoto and Yamashita 1980) as follows:

\[
Z_{0,e} = Z_0 \frac{1 + JZ_0 / \sin(\theta) + (JZ_0)^2}{1 - (JZ_0)^2 \cot(\theta)^2} \quad (2-11a)
\]

\[
Z_{0,o} = Z_0 \frac{1 - JZ_0 / \sin(\theta) + (JZ_0)^2}{1 - (JZ_0)^2 \cot(\theta)^2} \quad (2-11b)
\]

for the coupled line with opened-end termination and

\[
Y_{0,e} = Y_0 \frac{1 - KY_0 / \sin(\theta) + (KY_0)^2}{1 - (KY_0)^2 \cot(\theta)^2} \quad (2-12a)
\]

\[
Y_{0,o} = Y_0 \frac{1 + KY_0 / \sin(\theta) + (KY_0)^2}{1 - (KY_0)^2 \cot(\theta)^2} \quad (2-11b)
\]

for the coupled line with grounded-end termination, where \(Y_{0,e} = 1/Z_{0,e}\) and \(Y_{0,o} = 1/Z_{0,o}\).
Two bandpass filters are fabricated to demonstrate the proposed technique. The filters shown in Figure 2-8(a) and (b) are prototypes designed for passive radiometry systems. The filters have center frequency of 1.412 GHz and have relative bandwidth of 7.5 percent. The initial designed prototypes are based on the 4th order Chebyshev response with 0.1 dB passband ripple. All design parameters are listed in Table 2-1.

Table 2-1 The design parameters of the bandpass filters

<table>
<thead>
<tr>
<th></th>
<th>Type - I</th>
<th>Type - II</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Impedance ratio</strong></td>
<td>0.2</td>
<td>0.75</td>
</tr>
<tr>
<td><strong>Z₁ (Ω)</strong></td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td><strong>Z₂ (Ω)</strong></td>
<td>10</td>
<td>37.5</td>
</tr>
<tr>
<td><strong>Resonator Length (degree)</strong></td>
<td>48.19</td>
<td>81.786</td>
</tr>
<tr>
<td><strong>Slope Parameter</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a</td>
<td>21.027</td>
<td>35.69</td>
</tr>
<tr>
<td>b</td>
<td>0.042</td>
<td>0.019</td>
</tr>
<tr>
<td><strong>Even, Odd Mode Impedance (Ω)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Z₀,1,Z₀,0₁ (Z₀,05,Z₀,05)</td>
<td>69.7, 29.75</td>
<td>65.65, 33.80</td>
</tr>
<tr>
<td>Z₀,02,Z₀,0₂ (Z₀,04,Z₀,04)</td>
<td>10.69, 9.40</td>
<td>40.23, 35.12</td>
</tr>
<tr>
<td>Z₀,03,Z₀,0₃</td>
<td>52.54, 47.46</td>
<td>52.69, 47.31</td>
</tr>
</tbody>
</table>
Figure 2-8 λ/4 SIR 4th order, 0.1 dB equal-ripple, bandpass filters operating at 1.4 GHz of center frequency (a) Type I – $R=0.2$ (b) Type II – $R=0.75$.

The filter type I has a step impedance ratio of 0.2, whereas the filter type II has a step impedance ratio of 0.75. A 0.635 mm-thick Roger’s Duriod 6010 substrate with the dielectric constant of 10.2 and the loss tangent of 0.0023 is used in both designs. The parameter $R$ is adjusted based on 50 Ohm characteristic impedance at the input/output terminal. Type-II filter has $u=0.5$ whereas the final Type-I filter has $u=0.64$. The Type-I filter is not designed with the optimal resonator length (i.e. $\theta_1=\theta_2=\theta_0$). If the optimal length is used, the coupled section with opened-end termination will not provide
sufficient coupling coefficient for the particular fabrication with minimum allowable metal spacing of 100 µm. The physical length and SIR spacing of Type-I filter is optimized to overcome the minimum metal trace spacing restriction. As a result, the first spurious frequency of the Type-I filter appears at $5.3f_0$ as opposed to $6.5f_0$ if $\nu=0.5$.

The stop-band attenuation is more than 50 dB up to $4.8f_0$ for Type-I filter and up to $3.4f_0$ for Type-II filter. The spurious frequencies match well with the calculated values with few percent of error.

From Figure 2-9 and Figure 2-10, the average in-band insertion loss is 4.0 dB for $R=0.2$ and 3.0 dB for $R=0.75$. The in-band insertion loss of Type-I filter is higher than that of Type-II filter. Since Type-I filter has higher step discontinuity than in the Type-II filter, the SIRs used in the design of Type-I filter have lower quality factor than those used in the design of Type-II filter. This increases the in-band insertion loss as experimentally confirmed in (Kuo and Shih 2003) and (Stracca and Panzeri 1986).

![Figure 2-9](image_url)  
*Figure 2-9 The simulated (dash line) and measured (solid line) $S_{21}$ and $S_{11}$ (in dB) of the Type-I ($R=0.2$) filter response.*
2.5 Double Split-end Quarter-wave Stepped Impedance Resonator

The $\lambda/4$ SIR resonator shown in Figure 2-11(a) has many desirable properties for use in coupled resonator design. Its size is smaller than that of the half-wave resonator and produces fewer resonance frequencies. However, due to its small size, it is difficult to provide sufficient coupling to produce a filter response that requires wide bandwidth.

In this section, a new $\lambda/4$ SIR resonator structure is proposed. To overcome the coupling surface limitation, the grounded and opened ends of the transmission line section of the resonator can be split and folded perpendicular the structure as shown in Figure 2-11(b). The unloaded quality factor ($Q_u$) of this resonator is slightly degraded since it has a larger discontinuity and has narrow lines. The moment method simulation results show that $Q_u$ is reduced from 314 to 294 when compared with the conventional $\lambda/4$ SIR on 0.762mm-thick Roger’s Duriod 6002 substrate when both have $R=0.528$ and $Z_1=50$. 

Figure 2-10 The simulated (dash line) and measured (solid line) $S_{21}$ and $S_{11}$ (in dB) of the Type-II ($R$=0.75) filter response.
Resonance frequencies can be derived in (2-7). The condition
\[ \theta \]
resonators (as demonstrated in Section 2.7. It also gives the shortest resonator length. The fundamental
resonance condition becomes

\[ R = \tan^2 (\theta_0) \]

(2-13)

By performing a circuit analysis, it is simple to see that the proposed circuit is
identical to the conventional \( \lambda/4 \) SIR resonator. By ignoring the discontinuity effect, its
resonance frequencies can be derived in (2-7). The condition \( \theta_1 = \theta_2 = \theta_0 \) is used to
maximize the separation between the fundamental frequency and its lowest spurious
resonance frequency. It also gives the shortest resonator length. The fundamental
resonance condition becomes

\[ R = \tan^2 (\theta_0) \]

(2-13)

Using the double split-end SIRs in filter designs has several advantages over
using the conventional structures. First, it is easier to produce a strong coupling.
coefficient when two resonators are in-line coupled since more coupling area is available between two resonators. Second, the filter is more compact. Finally, this coupling topology introduces an additional transmission zero per coupling section to increase the out-of-band attenuation. As a result, for a high-order filter where many transmission zeros are present, the filter may no longer require SIRs with low $R$ value to achieve wide out-of-band attenuation, as demonstrated in the filter designs in Section 2.8.

### 2.6 Tapped Quarter-wavelength Resonator

The tapping technique is commonly used in the filter design (Lee, Park et al. 2002; Kuo and Shih 2003; Pang, Ho et al. 2004; Wang and Zhu 2005). Not only does it eliminate coupling at the end sections of the filter, it also produces extra transmission zeros and it can be used to reject spurious responses or increase the out-of-band attenuation levels (Kuo and Shih 2003). This technique was implemented in the $\lambda/4$ SIR filter in (Lee, Park et al. 2002), however no analytical solution has been reported. To determine the tapping position at the resonator for a given filter coefficient, $Q_{si}$ is required.

In this section, the $Q_{si}$ of an optimal-length $\lambda/4$ SIR is determined for the first time. The tapped $\lambda/4$ SIR shown in Figure 2-12(a) is derived based on the condition where $\theta_1=\theta_2=\theta_0$ as it greatly simplifies the $Q_{si}$ equation. The calculation is based on the lossless transmission line model. From the definition in (Kuo and Shih 2003)

\[
Q_{si} = R_L \frac{\omega_0}{2} \left. \frac{dB}{d\omega} \right|_{\omega=\omega_0}
\]  

(2-14)
SIR can be derived as follows

\[ B = \text{Im}(Y_{in}) \]

where \( Y_{in} \) is the input admittance of the resonator seen at the tapped position in Figure 2-12(a). By assuming the linear relationship between \( \omega \) and the group velocity of wave

**2.6.1 Tapping location where 0<\phi<\theta_0**

Using the transmission line technique, the susceptance at the tap point of the \( \lambda/4 \) SIR can be derived as follows

\[
B = \text{Im}(Y_{in}) = \frac{1}{Z_i} \frac{\tan(\theta_0)[\tan(\phi) + \tan(\theta_0 - \phi)] - R[1 - \tan(\theta_0 - \phi)\tan(\phi)]}{R - \tan(\theta_0)\tan(\theta_0 - \phi)\tan(\phi)}
\]  

(2-15)
traveling in the transmission medium. The $Q_{si}$ in (2-14) can be rewritten in terms of $\theta$ as follows

$$Q_{si} = R_L \frac{\theta_0}{2} \frac{dB(\theta)}{d\theta} \bigg|_{\theta=\theta_0}. \quad (2-16)$$

Since $\phi$ is also a function of frequency, it is treated as a function of $\theta$ in (2-16). By using (2-7), (2-15) and (2-16), $Q_{si}$ can be computed and simplified to as follows

$$Q_{si} = \frac{R_L \theta_0}{Z_1 \sin^2(\phi)}. \quad (2-17)$$

2.6.2 Tapping location where $\theta_0 \leq \phi \leq 2\theta_0$

In this case, the tapped location lies on the low-Z section. The $Q_{si}$ calculation can be simplified by defining a new variable

$$\phi' = 2\theta_0 - \phi. \quad (2-18)$$

The susceptance at the tap point can be derived as follows

$$B = \frac{1}{Z_2} \left[ \tan(\phi') + \frac{\tan(\theta_0) \tan(\theta_0 - \phi') - R}{\tan(\theta_0) + R \tan(\theta_0 - \phi')} \right]. \quad (2-19)$$

By using (2-7), (2-16) and (2-19), $Q_{si}$ can be computed and simplified to

$$Q_{si} = \frac{R_L \theta_0}{Z_2 \cos^2(2\theta_0 - \phi)}. \quad (2-20)$$

The $Q_{si}$ is plotted versus $\phi$ in Figure 2-13. The practical upper bound value of $Q_{si}$ (when $\phi=\theta_0=0$) is limited by the resonator’s $Q_u$. It is simple to verify the equation (2-17) and (2-20) by comparing them to the $Q_{si}$ of the $\lambda/4$ uniform resonator in (Wong 1979). The SIR becomes a uniform impedance resonator (UIR) with $R=1$ i.e. $Z_1=Z_2=Z_0$. With this condition, $\theta_0=\pi/4$, (2-17) and (2-20) are simplified to
\[ Q_{\text{si}} \bigg|_{Z_{1}=Z_{2}=Z_{0}} = \frac{R_{L} \pi}{4Z_{0} \sin^{2}(\phi)} \]  \hspace{1cm} (2-21)

which is identical to the \( Q_{\text{si}} \) equation for the UIR. Note that the \( Q_{\text{si}} \) of the SIR is continuous, however its slope is not. Its derivative has a discontinuity at the transition where \( \phi = \theta_{0} \) between \( Z_{1} \) and \( Z_{2} \) and when \( Z_{1} \) is not equal to \( Z_{2} \). The mathematical derivation for the tapped \( \lambda/2 \) SIR in (Kuo and Shih 2003) does not show continuous response at the step discontinuity which would suggest an unphysical change in the power in the system at this value.

![Graph showing \( Q_{\text{si}} \) of a \( \lambda/4 \) SIR versus variable tapping position \( \phi / \theta_{0} \) for given \( R=0.2, 0.5, 1, 2 \) and 5 and \( Z_{1}=R_{L} \).](image)

Figure 2-13 The \( Q_{\text{si}} \) of a \( \lambda/4 \) SIR versus variable tapping position \( \phi / \theta_{0} \) for a given \( R=0.2, 0.5, 1, 2 \) and 5 and \( Z_{1}=R_{L} \).
2.6.3 Transmission zero frequencies generated by the tapped SIR

A transmission zero is created at the frequency where an equivalent short appears at the tapping point. The first transmission zero frequency from section 2.6.1 and section 2.6.2 respectively can be expressed as a function of center frequency \( f_0 \) and \( \phi \) as follows.

\[
f_{pt,1} = \frac{\pi}{\phi} f_0, \quad 0 \leq \phi \leq \theta_0 \quad (2-22a)
\]

\[
f_{pt,2} = \frac{\pi}{2(2\theta_0 - \phi)} f_0, \quad \theta_0 \leq \phi \leq 2\theta_0 \quad (2-22b)
\]

The minimum value of \( f_{pt,1} \) and \( f_{pt,2} \) are limited to \( \pi f_0 / \theta_0 \) and \( \pi f_0 / 2 \theta_0 \) using tapping location in section 2.6.1 and section 2.6.2, respectively. For a given \( f_{pt,1} \) or \( f_{pt,2} \) and \( Q_{\text{si}} R_L \) can be determined. Then the filter’s port impedance \( Z_0 \) is transformed to \( R_L \) at the tapping point using a \( \lambda/4 \) impedance transformation network in Figure 2-14. The above derivations can be applied to the proposed filter. Two possible tapping configurations are shown in Figure 2-12(b) and (c).

The tapping technique can be combined with the parallel-coupled \( \lambda/4 \) SIR filter design technique (U-yen, Wollack et al. 2004) as shown in Figure 2-15. By replacing the coupling section at the ends of the filter with the tapped section, two transmission zeros are generated. Each transmission zero is used to suppress one spurious resonance frequency. The transmission zero generated by tapping from Lo-Z section is placed at 4.24\( f_0 \) (\( \phi_1 = 21.2^\circ \)), while the other zero generated by the Hi-Z section is placed at 6.1\( f_0 \) (\( \phi_2 = 29.5^\circ \)) as shown in Figure 2-15. The simulation result verifies that transmission zeros generated by tapped \( \lambda/4 \) SIR technique produce sharp attenuation at transmission zero frequencies and improve overall out-of-band attenuation around those frequencies.
Figure 2-14  The $3^{rd}$ order bandpass filter using tapped SIR technique at the filter’s end sections and two coupling topologies (a) the grounded-end anti-parallel coupling (b) the opened-end anti-parallel coupling.

Figure 2-15  The simulation results of the microstrip filter with the $3^{rd}$ order Chebyshev response, $R=0.528$ and with 10% bandwidth on 0.762 mm-thick Roger’s Duroid 6002 substrate. One uses the paralleled coupled $\lambda/4$ SIR (dash line). The other is the parallel coupled $\lambda/4$ SIR with tapped SIR technique that has transmission zeroes each of which overlaps at a peak frequency of the two lowest spurious frequencies (solid line).
2.7 Resonator Coupling Topology and Transmission Zero Generation

To introduce transmission zeros to the filter without using additional transmission line components, resonators are coupled inline as shown in Figure 2-14. This creates an anti-parallel coupling pair between a pair of resonators. There are two types of coupling in this filter design. One is the anti-parallel coupling with grounded ends (shown in Figure 2-14(a)) and the other with opened ends (shown in Figure 2-14(b)). The filter design using opened-end anti-parallel coupling was demonstrated in (Matsuo, Yabuki et al. 2000) to improve out-of-band attenuation close to in-band frequency. However, its effect in out-of-band attenuation at higher frequencies was not considered.

To study this effect, the sections, shown in Figure 2-14(a) and (b), are separated from the resonators and each section is terminated at both ends with $Z_0$ as shown in Figure 2-16(a) and (b), respectively. The effect of transmission line bends is neglected to simplify the explanation as it has negligible effect on transmission zero frequencies shift. From Figure 2-16 (a), the transmission line with $L_{s1}$ and $L_{p1}$ long are equivalent to an electrical degree $\theta_{1}'$ and $\theta_{1}''$ at $f_0$, respectively. And from Figure 2-16(b), the transmission line with $L_{s2}$ and $L_{p2}$ long are equivalent to electrical degrees $\theta_{2}'$ and $\theta_{2}''$ at $f_0$, respectively.

Consider the grounded-end (opened-end) anti-parallel coupling section (in Figure 2-16(a) and (b)), the signal traveling from port 1 to port 2 is suppressed at the frequency where $L_{p1}$ ($L_{p2}$) becomes a multiple number of a quarter-wave length long. The grounded terminals of the anti-parallel coupling section are transformed into Hi-Z or Lo-Z at the center of the structure around the split location. This blocks the signal traveling between two ports and creates a transmission zero.
These transmission zero frequencies can be expressed as follows

\[ f_{p1,n} = n \frac{\pi}{2\theta_1} f_0 \]  \hspace{1cm} (2-23a)

\[ f_{p2,n} = n \frac{\pi}{2\theta_2} f_0 \]  \hspace{1cm} (2-23b)

for the grounded-end and opened-end anti-parallel coupling section respectively, where \( n \) is a natural number.

In practical implementation, \( n \) is limited to two in (2-23a) and one in (2-23b). This is due to parasitic at the coupler ends that causes non-ideal ground/open, thus they no-longer reflect the signal effectively at frequencies much higher than \( f_0 \). Moreover, the level of attenuation at \( f_{p1} \) (or \( f_{p2} \)) become less as the coupling gap \( S_{p1} \) (or \( S_{p2} \)) become larger and vice versa. Therefore, this technique is very effective if used in the filter with relative bandwidth greater than three percent where the couplings between resonators are not weak.
Figure 2-16  The wide-band frequency responses of magnitude (dB) and phase (degree) of the $S_{21}$ of the anti-parallel coupling section on 0.762 mm-thick Rogers’ Duriod 6002 substrate when compared with the theoretical responses. The theoretical results (solid lines) use ideal opened-end and grounded termination. The simulation results (dash lines) have taken opened-end and ground via effects into account. Each section is designed to produce a transmission zero that overlaps with the SIR’s spurious resonance frequency at $4f_0$ or $6f_0$ where $f_0=1.412$ GHz. (a) Hi-Z grounded-end anti-parallel coupling (b) Lo-Z opened-end anti-parallel coupling.
2.8 Filter Construction

To simplify the filter model, we assumed that there is no interaction between the opened-end and the grounded-end coupling sections, each filter section can be constructed individually and combined to generate the desired filter response. The filter coefficients are generated from three types of sections. First, the $Q_{si}$ at the tapped sections can be calculated based on the filter’s coefficients (Kuo and Shih 2003) as follows

$$Q_{si} = Q_{ext} = \frac{g_0 g_1}{w} = \frac{g_N g_{N+1}}{w}$$  \hspace{1cm} (2-24)

where $g_i$ is the filter’s coefficient ranging from 0 to $N$.

Second, the opened-end and grounded-end anti-parallel coupling sections are modeled as impedance inverter ($J$) and admittance inverter ($K$), respectively. Based on (U-yen, Wollack et al. 2004), $J$ and $K$ can be derived as follows

$$Z_J i = \frac{K_i}{Z_1} = \frac{w \theta_0}{g_i g_{i+1}}$$  \hspace{1cm} (2-25)

where $i$ is an integer ranging from 1 to $N-1$. Grounded-end and opened-end anti-parallel coupling sections are combined in series one after another to produce $\lambda/4$ SIR structures. The even mode ($Z_{0,e}$) and odd mode ($Z_{0,o}$) impedance of the grounded-end and opened-end coupler can be determined. Based on (Matsuo, Yabuki et al. 2000), $Z_{0,e}$ and $Z_{0,o}$ of the grounded-end anti-parallel coupling section can be derived as follows

$$Z_{0,e} = 2Z_1 \left[ \frac{1 - K/Z_1 \tan(\theta_i^\prime) }{1 + K/Z_1 \cot(\theta_i^\prime)} \right]^{-1}$$  \hspace{1cm} (2-26a)
Similarly, $Z_{0,e}$ and $Z_{0,o}$ of the opened-end anti-parallel coupling section can be determined as follows

$$Z_{0,e} = 2Z_2 \frac{1 + Z_2 J \tan(\theta_2^\prime\prime)}{1 - Z_2 J \cot(\theta_2^\prime\prime)}$$  \hspace{1cm} (2-27a)

$$Z_{0,o} = 2Z_2 \frac{1 - Z_2 J \tan(\theta_2^\prime\prime)}{1 + Z_2 J \cot(\theta_2^\prime\prime)}.$$  \hspace{1cm} (2-27b)

Since there are $N+1$ transmission zeros available to suppress spurious responses, there is design flexibility in choosing the appropriate electrical length $\theta_1^\prime\prime$ and $\theta_2^\prime\prime$ of each coupling section to achieve the out-of-band suppression design goal. In this paper, we allocate all zeros to suppress the first two spurious resonance frequency modes of the SIR since they have the strongest influence on the in-band signal quality for typical communication systems. In addition, using the minimum size SIR (i.e. $\theta_1 = \theta_2 = \theta_0$), the third lowest spurious resonance frequency has the maximum extension (U-yen, Wollack et al. 2004). Using this approach, the electrical length $\theta_1^\prime\prime$ and $\theta_2^\prime\prime$ can be approximately derived at the fundamental frequency as follows

$$\theta_1^\prime\prime = \pi \frac{\theta_0}{\pi + \theta_0}.$$  \hspace{1cm} (2-28a)

$$\theta_2^\prime\prime = \frac{\pi}{2} \frac{\theta_0}{\pi - \theta_0}.$$  \hspace{1cm} (2-28b)

The exact value $\theta_1^\prime\prime$ and $\theta_2^\prime\prime$ to provide the maximum spurious response suppression are dependent on the filter bandwidth and number of available transmission zeros used to suppress a spurious resonance frequency mode.
Two microstrip filters were constructed based on the design procedures discussed in the earlier sections. The design prototypes are based on 3rd and 6th order Chebyshev filter response with 0.1 dB of in-band ripple. Their photographs are shown in Figure 2-17(a) and (b), respectively. The numbers 1-4 in Figure 2-17(a) and 1-7 in Figure 2-17(b) represent section numbers in Table 2-2. They are prototypes designed for the front-end of the passive L-band radiometer. The filters are made from 17µm-thick copper on 0.762 mm-thick Rogers’ Duroid 6002 substrate. The substrate has a dielectric constant of 2.94 and has a loss tangent of 0.0012 at 10 GHz. The center frequencies of both filters are set to 1.41 GHz. Both filters use SIRs with $R=0.528$, where $Z_1$ is set to 50 Ohm and $Z_2$ is set to 26.4 Ohm. This corresponds to $W_{s1} = 1.9$ mm and $W_{s2} = 4.7$ mm in all coupling sections. From (2-13), $\theta_b$ equals to 36º and the SIR has the lowest three normalized spurious frequencies of 4, 6 and 9 (U-yen, Wollack et al. 2004). Using (2-28a) and (2-28b) for $\theta_b = 36^\circ$, we obtain $\theta_1''=30^\circ$ and $\theta_2''=22.5^\circ$. Using these values and the filters’ $K$ and $J$ values in Table 2-2, the coupler’s odd and even mode characteristic impedances in each section are determined as in (2-26a), (2-26b), (2-27a) and (2-27b). The couplers’ physical dimensions are shown in Table 2-2.
Type-II (6th order) anti-parallel coupling section; (d) Tapped SIR in the Hi-Z section.

Table 2-2 The specifications and dimensions of the two experimental filters

<table>
<thead>
<tr>
<th>Section</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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</thead>
<tbody>
<tr>
<td>Type</td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
<td>(d)</td>
</tr>
<tr>
<td>Parameters</td>
<td>$K_{1,2} = 0.058$, $L_{s1} = 1.79\text{mm}$, $W_{p1} = 0.49\text{mm}$, $S_{p1} = 0.64\text{mm}$, Via diameter = 0.2mm, $Q_{p} = 10.315$, $\phi_1 = 21.7^\circ$</td>
<td>$J_{2,3} = 0.058$, $L_{s2} = 4.27\text{mm}$, $W_{p2} = 1.75\text{mm}$, $S_{p2} = 0.17\text{mm}$, Via diameter = 0.8mm, $Q_{p} = 10.315$, $\phi_2 = 29.0^\circ$, Via diameter = 0.8mm</td>
<td></td>
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<tr>
<th>Section</th>
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<tr>
<td>Type</td>
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<td>(a)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Parameters</td>
<td>$K_{1,2} = 0.058$, $L_{s1} = 2.68\text{mm}$, $W_{p1} = 0.51\text{mm}$, $S_{p1} = 0.86\text{mm}$, Via diameter = 0.2mm, $Q_{p} = 7.813$, $\phi_1 = 21.9^\circ$</td>
<td>$J_{2,3} = J_{4,5} = 0.041$, $L_{s2} = 4.45\text{mm}$, $W_{p2} = 1.78\text{mm}$, $S_{p2} = 0.53\text{mm}$, Via diameter = 0.2mm, $K_{3,4} = 0.039$, $L_{s3} = 2.85\text{mm}$, $W_{p3} = 0.52\text{mm}$, $S_{p3} = 1.30\text{mm}$, Via diameter = 0.2mm, $Q_{p} = 7.81$, $\phi_2 = 29.2^\circ$, Via diameter = 0.8mm</td>
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1 (a) Tapped SIR in the Lo-Z section; (b) grounded-end anti-parallel coupling section; (c) opened-end anti-parallel coupling section; (d) Tapped SIR in the Hi-Z section.
By comparing the performance of the proposed filter to the filter with no transmission zero in the band of interest (U-yen, Wollack et al. 2004) as shown in Figure 2-18, the proposed filter design reduces the maximum spurious level of the filter by at least 8 dB and improves the overall out-of-band response up to more than $6f_0$ without affecting the in-band frequency response.

Figure 2-18  Comparison between the frequency response of $\text{dB}|S_{21}|$ of the 3rd order filter design using parallel coupled technique that has no transmission zero (dash line) and that of the proposed filter design (solid line) that has 4 transmission zeros. Both filters are 3rd order filters with $\omega=0.1$. 
Figure 2-19  The measured (solid lines) and simulated (dash lines) frequency response of $|S_{21}|$ and $|S_{11}|$ of the Type-I filter with 2 transmission zeros placed around the lowest spurious resonance frequency (at 5.65 GHz) and 2 transmission zeros placed around the second lowest spurious resonance frequency (at 8.47 GHz).

The measurement results shown in Figure 2-19 and Figure 2-20 agree well with the moment method simulation using Ansoft Designer. The connectors are de-embedded from the measurement. From Table 2-2, the tapping locations in the SIRs at both ends of the filters are placed differently to optimally minimize the peak of the two lowest spurious frequencies. For the 3rd order filter, the lowest spurious mode was suppressed to below 41.7 dB, while the second spurious mode was suppressed to below 27.7 dB. In Type-I filter, it has a minimum in-band insertion loss of 0.6 dB. Although the two lowest spurious resonance frequency modes of the Type-I filter are suppressed by the same number of transmission zeros, they have different level of suppression due to several factors.

First, the lowest spurious resonance frequency is additionally suppressed by the transmission zero generated by the grounded-end anti-parallel coupling section as it is located close to the first spur as shown in Figure 2-16(a). Second, at high frequency, the
transmission zeros generated by coupling sections are not as effective as those at low frequency due to high parasitic at grounded/opened end as discussed in section 2.7. Moreover, the Lo-Z coupling section provides strong coupling between two input ports at frequencies above the lowest spur mode than it does in-band as shown in Figure 2-16 (b). This causes difficulty in suppressing the second lowest spurious mode.

![Figure 2-20](image)

Figure 2-20  The measured (solid lines) and simulated (dash lines) frequency response of $d|B_{S_{21}}|$ and $d|B_{S_{11}}|$ of Type-II filter with 3 transmission zeros placed around the lowest spurious resonance frequency (at 5.65 GHz) and 4 transmission zeros placed around the second lowest frequency (at 8.47 GHz).

For the 6th order filter, ten transmission zeros are generated below the third lowest spurious resonance frequency. Two zeros are caused by two tapped SIR sections. Six zeroes are cause by three grounded-end anti-parallel coupling sections. The final two zeros are caused by the opened-end anti-parallel coupling sections.

Three transmission zeros are used to suppress the lowest spur while four zeros are used to suppress the second lowest spur. The last three non-controlled zeros from
the grounded-end anti-parallel coupling are at frequency lower than the lowest spurious resonance frequency. As a result, the over-all out-of-band suppression is at least 37.8 dB up to 8.5 times the fundamental frequency. Moreover, the filter has a low in-band insertion loss of 1.9 dB.

2.9 Filter Design Using Half-wavelength Stepped-impedance Resonator with Even-mode Spurious Resonance Suppressor

Consider the conventional \(\lambda/2\) SIR, as shown in Figure 2-21(a), it consists of three lines. The line in the middle section has the characteristic impedance of \(Z_1\). The others have the characteristic impedance of \(Z_2\). At \(f_0\), the opened-end of the resonator is transformed to a virtual ground at the center of the resonator. Then as shown in Figure 2-21(b), both \(Z_1\) and \(Z_2\) lines are split and folded in perpendicular to its structure to produce a more compact structure. The split \(Z_1\) and \(Z_2\) sections have electrical lengths of \(\theta_1''\) and \(\theta_2''\), respectively. As shown in Figure 2-21(c), the SIR’s internal coupling is formed by inserting SIO stubs around the center. The SIO stub is constructed from two lines connected in series, each of which has characteristic impedances of \(Z_{s1}\) and \(Z_{s2}\) and electrical lengths of \(\theta_{1t}\) and \(\theta_{2t}\), respectively. These electrical lengths are tuned such that the SIO stub provides a virtual ground at \(f_0\). When connected to the parallel line with the characteristic impedance of \(2Z_1\) in (2-29), it forms a grounded-end anti-parallel coupler where \(2Z_{1,e}\) and \(2Z_{1,o}\) are its even- and odd-mode characteristic impedance.

\[
2Z_1 = \sqrt{2Z_{1,e} \cdot 2Z_{1,o}} \quad (2-29)
\]
Using the SIO stub allows the lowest even-mode resonance of the SIR to shift away from $f_0$ as discussed in section 2.9.1. Since the proposed SIR is symmetric in both x and y axis, it can be modeled using the quarter of the circuit (see the dark gray area in Figure 2-21(c)) in even and odd modes, as shown in Figure 2-21(d) and Figure 2-21(e), respectively. The fundamental resonance condition of this SIR is as in (2-7). The minimum length of the $\lambda/2$ SIR is used in this design to reduce the overall filter size (i.e.
\( \theta_1 = \theta_2 = \theta_0 \). The split-end sections on the left and the right side of the SIR have the characteristic impedance of

\[
2Z_z = \sqrt{2Z_{z,e} \cdot 2Z_{z,o}}.
\]  
(2-30)

Where \( 2Z_{z,e} \) and \( 2Z_{z,o} \) are even- and odd-mode impedance of the opened-end line \( 2Z_z \). They are used for coupling between SIRs to form a filter response. The admittance inverter for this coupler, which is used for filter designs, is derived in (2-25).

2.9.1 The Resonator’s Spurious Suppression Capability from SIO Stubs

In this filter design is focused on suppressing the lowest spurious resonance frequency as this filter design will be incorporated with broadband bandstop filter in section 2.12. The filter’s out-of-band suppression capability depends on two factors. First, it depends on the \( R \) value as it defines the separation between \( f_0 \) and its lowest resonance mode. Second, it depends on the SIO stub which has the input impedance as follows:

\[
Z_{s_{in}} = -j2Z_s \left( \frac{R_s - \tan(\theta_t) \tan(\theta_{t2})}{\tan(\theta_{t2}) + R_s \tan(\theta_t)} \right)
\]  
(2-31)

where \( R_s = Z_{s2}/Z_{s1} \). At \( f_0 \), it behaves as a virtual ground at A-A’ in Figure 2-21(d), thus \( Z_{s_{in}} = 0 \). The effect of the variable \( R_x = Z_{s1}/Z_{1,e} \), \( R_s \) and \( u_x = \theta_{s2}/(\theta_{t1} + \theta_{s2}) \) on filter responses as described below.

2.9.2 The Effect of the \( R_x \) Variable

The \( R_x \) value in SIRs controls the bandwidth of the filter, as well as its out-of-band suppression capability, given \( R = R_s \).

First, the bandwidth of the filter relies on \( R_x \) to be close to zero at \( f_0 \) in order for each \( \lambda/2 \) SIR in the filter to behave as two coupled quarter-wavelength (\( \lambda/4 \)) SIRs. When the SIO stub is combined with the main resonator at A-A’ in Figure 2-21(d), a
transmission zero frequency \((f_z)\) is also generated on the low frequency side of \(f_0\) as shown in Figure 2-22 when \(R_x=1, 0.2\) and 0. \(f_z\) is formed when the opened end of \(2Z_{s2}\) line is transformed to a virtual ground at the connection between \(2Z_{1,e}\) and \(2Z_1\) lines. \(f_z\) approaches zero as \(R_x\) decreases to zero.

Second, the attenuation at the lowest spurious frequency \((f_{s1})\) of the filter also relies on the value \(R_x\). As \(R_x\) is close to zero, the main resonator behaves close to a \(\lambda/4\) SIR and its lowest even-mode resonance frequency \((f_{s1})\) is suppressed.

2.9.3 The Effect of the \(R_s\) and \(u_s\) Variable

The \(R_s\) and \(u_s\) values can be adjusted such that \(f_{s1}\) of the SIR is shifted away from \(f_0\). The maximum separation between \(f_0\) and \(f_{s1}\) is obtained when \(u_s=2/3\) as well as using small \(R_s\) value. This effect is demonstrated in Figure 2-22 where \(R_s=1\) and \(u_s=1\) and where \(R_s=0.3\) and \(u_s=2/3\).

![Figure 2-22](image)

Figure 2-22  Frequency responses of the \(\text{dB}|S_{21}|\) of the 4\(^{th}\) order filters using the proposed SIRs with \(R=0.528\). The nominal design (bold solid line) has \(R=R_s=0.528, R_x=1, \theta_0=\theta_1=\theta_2=36^\circ\). Other responses are obtained by only adjusting either \(R_x\) (where \(R=R_x=0.528\) and \(\theta_1=\theta_2=36^\circ\)) or \(R_s\) and \(u_s\) (where \(R_x=1\) and \(R=0.528\)) from the nominal design.
2.9.4 Filter Design and Implementation

A 4th order bandpass filter can be constructed as shown in Figure 2-23. The filter’s coefficients are based on an equal-ripple filter prototype. The prototype microstrip filter has a center frequency at 1.41 GHz and has 10% bandwidth. It will be used in the NASA’s Aquarius satellite to reject out-of-band spurious response up to 6 GHz while providing rejection from the on-board radar instrument at 1.265 GHz. A 0.635 mm-thick Roger’s Duroid 6010 substrate is used in the design. Overall, the filter has a dimension of 44 mm by 35mm. Its detailed dimension is provided in Table 2-3.

Figure 2-23 The photograph of the 4th order bandpass filter on 0.635 mm-thick Roger’s Duroid 6010 substrate.

The internal spacing \( (G_s) \) and the inter-stage spacing \( (G_o) \) shown in Figure 2-24 are adjusted to provide the proper filter coupling coefficient. Two high impedance \( \lambda/4 \) lines with the line width of \( W_{r1} \) and \( W_{r2} \) are tapped from the left and right SIR respectively. \( R, R_s \) and \( R_x \) are set to 0.528, 0.3 and 1, respectively, thus \( \theta_0=36^\circ, \theta_1=20^\circ \) and \( \theta_{d2}=40^\circ \). From the given parameters, \( f_{s1} \) of the SIR, the lowest even-mode and odd-mode frequencies can be determined to be at \( 3f_0 \) and \( 4f_0 \), respectively. The line lengths of \( L_{o1}, L_{o3} \) and \( L_{o5} \) are adjusted such that the coupling sections generate transmission zeros at \( 3f_0, 3.9f_0 \) and \( 4f_0 \), respectively, to optimally reject the spurious response of the
filter. The line length $L_{s2}$ is adjusted in co-ordination with the SIO stubs’ width $W_{L1}$ and $W_{L2}$ such that a transmission zero is generated close to 1.265 GHz. The filter is designed and simulated using Ansoft Designer. The theoretical result in Figure 2-25 agrees with the method-of-moments simulation. Using the propose SIRs alone in filter design can suppress $f_{s1}$ by more than 20 dB. The filter provides at least 49 dB of attenuation at 1.265 GHz and has the minimum in-band insertion loss of 1.75 dB, as shown in Figure 2-26. The deviation of the $f_z$ value from 1.265 GHz is caused by asymmetric parasitic couplings from $L_{o1}$ and $L_{o3}$ to $A-A'$ and from $L_{o3}$ and $L_{o5}$ to $B-B'$ as shown in Figure 2-24, whereas the parasitic coupling between $A-A'$ and $B-B'$ has a negligible effect on $f_z$. The proposed filter produces a broadband attenuation of at least 39.7 dB up to 3.9$f_0$. The suppression around 4.24$f_0$ was as not optimal as in (Kuo and Shih 2003) due to transmission zeroes slight misplacement.

Figure 2-24  The physical layout with dimensions of the 4th order filter on 0.635 mm-thick Roger’s Duroid 6010 substrate, $Z_0=50$ Ohm.
Table 2-3  The Filter’s detail dimension in millimeter

<table>
<thead>
<tr>
<th></th>
<th>Main resonators</th>
<th>SIO stubs &amp; spacing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$W_{o1} = 1.63, W_{o2} = 4.17, W_{s1} = 0.47, W_{s2} = 1.91,\newline W_1 = 0.11, W_2 = 0.1, L_{o1} = 5.36, L_{o2} = 1.24,\newline L_{o3} = 3.78, L_{o4} = 2.67, L_{o5} = 3.16, L_{o6} = 3.33,\newline L_{s1} = 0.39, L_{s2} = L_{s3} = 6.89$</td>
<td>$W_{l1} = 1.71, W_{l2} = 7.93,\newline L_{ST1} = 3.89, L_{ST2} = 7.17 G_s = 0.83,\newline G_o = 2.63$</td>
</tr>
</tbody>
</table>

Figure 2-25  The simulated frequency responses of the $|S_{21}|$ of the filter in Figure 2-23 with a transmission zero placed around $f_{s1}$ (solid line) and without transmission zero at $f_{s1}$ (dashed line). The dotted line is the theoretical filter response using a transmission line model, including a transmission zero at $f_{s1}$. 
Figure 2-26  The measured and simulated $|S_{11}|$ and $|S_{21}|$ in dB versus the frequency of the 4th order bandpass filter in Figure 2-23.

2.10 Anti-parallel Stepped-Impedance Opened-end Stub

The structure, called the anti-parallel stepped-impedance (APSI) opened-end stub, has been implemented in addition to the bandpass filter to reject spurious responses. The preliminary analysis is performed by (Hsieh and Wang 2005), however their analysis did not cover the structure’s transmission pole locations. Moreover, their analysis on the structure’s transmission zero locations is not complete. In this section, more complete analysis of the APSI opened-end stub is performed such that it can be sufficiently used as a component in a bandpass filter design to improve the bandpass filter’s out-of-band performance.

The APSI opened-end stub consists of an anti-parallel coupling pair connected to a low-impedance transmission line ($Z_2$) open end, as shown in Figure 2-27(a). $Z_{1,e}$ and $Z_{1,o}$ are the even and odd mode characteristic impedances of the coupler. The coupler
has a coupling coefficient of \( c \) and is normalized to the characteristic impedance of \( Z_1 \). \( \theta \) is an electrical length of both the \( Z_1 \) and \( Z_2 \) section.

![Diagram of APSI stub and equivalent circuit]

Figure 2-27  (a) The physical layout of the APSI stub; and (b) its equivalent circuit.

2.10.1 APSI Opened-end Stub Circuit Modeling and Its Frequency Response

The APSI opened-end stub generates the maximum number of transmission poles and zeros when its electrical length of the parallel coupling section equals to that of the low-impedance stub. This condition also simplifies several derivations related to this structure significantly. The APSI opened-end stub transmission poles and zeros are shown in Figure 2-28.

Consider the \( Z \) matrix equation of a parallel coupled line (Pozar 1997):

\[
\begin{bmatrix}
V_1 \\
V_2 \\
V_3 \\
V_4 \\
\end{bmatrix} =
\begin{bmatrix}
Z_{11} & Z_{12} & Z_{13} & Z_{14} \\
Z_{21} & Z_{22} & Z_{23} & Z_{24} \\
Z_{31} & Z_{32} & Z_{33} & Z_{34} \\
Z_{41} & Z_{42} & Z_{43} & Z_{44} \\
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
I_3 \\
I_4 \\
\end{bmatrix}
\tag{2-32}
\]

Where

\[
Z_{11} = Z_{22} = Z_{33} = Z_{44} = \frac{-1}{2} \left( Z_{1,e} + Z_{1,o} \right) \cot(\theta)
\tag{2-33a}
\]
\[
Z_{12} = Z_{21} = Z_{34} = Z_{43} = \frac{-i}{2} (Z_{1,e} - Z_{1,o}) \cot(\theta) \quad (2-33b)
\]

\[
Z_{13} = Z_{31} = Z_{24} = Z_{42} = \frac{-i}{2} (Z_{1,e} - Z_{1,o}) \csc(\theta) \quad (2-33c)
\]

\[
Z_{14} = Z_{41} = Z_{23} = Z_{32} = \frac{-i}{2} (Z_{1,e} + Z_{1,o}) \csc(\theta) \quad (2-33d)
\]

\(Z_{1,e}\) and \(Z_{1,o}\) are the even- and odd mode characteristic impedances of the coupler, respectively. \(V_i\) and \(I_i\) are voltage and input current at port \(i\), respectively. \(\theta\) is the electrical length of both the coupler and the open stub \(Z_2\), as shown in Figure 2-27(b). \(\theta\) is computed at the center of this structure.

By applying the conditions \(V_3 = V_4\) and \(V_3 = -(I_3 + I_4) \frac{Z_2}{j \tan(\theta)}\) in (2-32), The \(Z\) matrix can be simplified to a two-port matrix

\[
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix} =
\begin{bmatrix}
Z_{11} & Z_{12} \\
Z_{21} & Z_{22}
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix}
\]

\(2-34\)

where

\[
Z_{11}^* = Z_{22}^* = j \frac{\sin^2(\theta) [Z_{1,e}^2 + Z_{1,o}(Z_{1,e} + 2Z_2) - 2Z_{1,e}Z_2 \cos^2(\theta)]}{2\sin(\theta)\cos(\theta)(Z_{1,e} + 2Z_2)} \quad (2-35a)
\]

\[
Z_{12}^* = Z_{21}^* = j \frac{\sin^2(\theta) [Z_{1,e}^2 - Z_{1,o}(Z_{1,e} + 2Z_2) - 2Z_{1,e}Z_2 \cos^2(\theta)]}{2\sin(\theta)\cos(\theta)(Z_{1,e} + 2Z_2)} \quad (2-35b)
\]

2.10.2 Transmission Zeros Generated by the APSI Opened-end Stub

Transmission zeros are generated around \(f_0\) and can derived as follows. Using the \(Z\) matrix transformation to the S-parameter matrix (Pozar 1997), the transmission zeroes of a structure are determined using the condition

\[
S_{21} = \frac{2Z_{12}^* Z_{10}}{\Delta Z} = 0 \quad (2-36)
\]
where

$$\Delta Z = (Z'_{11} + Z_0)^2 - Z_{12}^{-2}.$$  \hspace{1cm} (2-37)

Using (2-35b), (2-36) and (2-37), the electrical length, where transmission zeros are generated, can be determined as follows:

$$\theta_{c,1} = n\pi + \tan^{-1}\frac{2Z_{1,e}Z_2}{Z_{1,e}^2 - Z_{1,o}(Z_{1,e} + 2Z_2)}$$  \hspace{1cm} (2-38a)

$$\theta_{c,2} = n\pi - \tan^{-1}\frac{2Z_{1,e}Z_2}{Z_{1,e}^2 - Z_{1,o}(Z_{1,e} + 2Z_2)}$$  \hspace{1cm} (2-38b)

$$\theta_{c,3} = (n+1)\frac{\pi}{2}$$  \hspace{1cm} (2-39)

Where \(n\) is a positive integer including zero. Consider the fundamental mode where \(n=0\), from (2-38a), we observe that \(\theta_{c,1}\) cannot be greater than \(\pi/2\). By comparing this structure to the conventional quart-wave length open, we observe that the total length of this structure is smaller than the conventional opened-end stub when \(\theta_{c,1}\) is less than \(\pi/4\). From (2-38a) and \(\theta_{c,1}<\pi/4\) gives

$$\frac{2Z_2}{Z_{1,e}} < \frac{Z_{1,e} - Z_{1,o}}{Z_{1,e} + Z_{1,o}} = c_p$$  \hspace{1cm} (2-40)

where \(c_p\) is the coupling coefficient of the coupler.

### 2.10.3 Transmission Poles Generated by the APSI Opened-end Stub

The structure can produce transmission poles at the desired location. Consider the \(S_{11}\) parameter of the structure using the two-port network conversion table 4.3 in (Pozar 1997).

$$S'_{11} = \frac{(Z'_{11} + Z_0)(Z_{11} - Z_0) - Z_{12}^{-2}}{\Delta Z}$$  \hspace{1cm} (2-41)

By substituting (2-35a) and (2-35b) into (2-41) with \(S_{11}=0\) gives.
\[-Z_{l,e}^2 Z_{l,o} \tan(\theta)^2 - Z_{l,e} Z_0^2 + 2 Z_{l,e} Z_{l,o} Z_0 - 2 Z_0^2 Z_2 = 0 \] \hspace{1cm} (2-42)

Solving (2-42) for \( \theta \) gives the transmission pole location at

\[ \theta_{p,1} = n \pi + \tan^{-1} \left( \frac{2Z_2}{Z_{l,e} Z_{l,o} - Z_0^2} - Z_{l,e} Z_0^2 \right) \] \hspace{1cm} (2-43a)

\[ \theta_{p,2} = n \pi - \tan^{-1} \left( \frac{2Z_2}{Z_{l,e} Z_{l,o} - Z_0^2} - Z_{l,e} Z_0^2 \right) \] \hspace{1cm} (2-43b)

\[ \theta_{p,3} = n \pi \] \hspace{1cm} (2-44)

We can observe from (2-43a) that \( \theta_{p,1} \) is valid only when coupler impedance value is greater than \( Z_0 \), thus

\[ \sqrt{Z_{l,e} Z_{l,o}} > Z_0 \sqrt{\frac{Z_{l,e}}{2Z_2}} + 1 \] \hspace{1cm} (2-45)

\( \theta_{p,1} \) when \( n = 0 \), can be used to set the required pass-band bandwidth. In addition, \( \theta_{p,3} \) always presents in the response and it is independent of the value of the coupler and the stub impedance.

The frequency response of this structure can be compared with a conventional stub with the same total electrical length, as shown in Figure 2-29. The APSI opened-end stub provides significantly higher suppression than the conventional opened-end stub since three transmission zeroes are concentrated in the same frequency range. Moreover, low return loss can be obtained in-band with no impedance matching required.

The signal suppression capability of the APSI stub depends on the separation between the electrical lengths \( \theta_z \) and \( \pi \theta_z \). The suppression level becomes higher as \( \theta_z \) and \( \pi \theta_z \) approach \( \pi/2 \). They can be controlled by independently adjusting the values \( c_p \), \( Z_1 \) and \( Z_2 \).
Figure 2-28 The dB|S_{21}| response of the APSI stub and its associated transmission zeros and poles in the fundamental mode where \( n=0, Z_0=50 \text{ Ohm}, Z_1=100 \text{ Ohm}, \ c_p=0.3 \) and \( Z_2=50 \text{ Ohm} \).

Figure 2-29 The frequency response of the conventional opened-end stub and the APSI stub. Both have the same total electrical length.
2.10.4 High-frequency Blocking Filter Implementation

A band-stop filter can be constructed based on APSI opened-end stubs. Several sections of the ASPI opened-end stubs can be combined in series, as shown in Figure 2-30. This filter consists of four APSI opened-end stubs with variable $Z_2$ and $\theta$ such that the combined response provides the minimum out-of-band rejection of 50 dB.

Figure 2-30 The high-frequency blocking filter constructed using four sections of the APSI opened-end stub on 30 µm-thick silicon substrate.
The filter is designed based on the Niobium superconductor line on a 30 µm-thick silicon substrate. The EM simulation shows that the multiple circuit models of the APSI opened-end stubs can roughly predict the response of the overall bandstop filter as shown in Figure 2-31. The deviations from the circuit model are due to the parasitic caused by step discontinuities in the structure. The isolation of the filter is limited by the physical separation between the two ports of an APSI opened-end stub. Moreover, spurious responses can be observed when APSI opened-end stubs are serially connected although the response level is more than 30 dB below the pass-band. These are caused by the resonance frequencies between several pairs of APSI stubs. Since the filter is designed to provide high-frequency blocking from 3 GHz to 30 GHz, it was not optimized for uniform in-band response.

![The frequency response of the high-frequency blocking filter using the method of moments simulation (solid line) and the ideal transmission line model (dashed line).](image)

Figure 2-31   The frequency response of the high-frequency blocking filter using the method of moments simulation (solid line) and the ideal transmission line model (dashed line).
2.11 Superconductor Modeling for Use in EM Simulators

The EM simulator is a very important tool used in the microwave circuit designs. With proper setup, the simulator gives accurate solutions that agree well with the measurement results. Since almost all simulators are designed for the circuits using normal metal, some modifications must be applied to the circuit using superconductor. In this section, proper superconducting transmission line model is introduced in a method of moment circuit simulator to compensate for kinetic inductance terms in superconductor. The effect of kinetic inductance in the bandpass filter response is studied.

Superconductor was discovered in 1911 by H. Kamerlingh Onnes. Its loss less property is being used in modern microwave instruments to reduce loss in their systems. In superconductivity, electrons are paired and travel under the influence of electric field with no loss (Lancaster 1977). The classical superconductor model is the “two fluid model”, consisting of normal and complex conductivity terms. The complex conductivity term influences superconducting devices that operate in microwave or sub-mm frequencies, since It causes the superconducting transmission line to be more inductive than the normal transmission line. The inductive term in superconductor is known as kinetic inductance. It is dependent of the London penetration depth ($\lambda_L$), the conductor thickness ($t$), width-to-height ratio of the microstrip line and the substrate’s dielectric constant (Yassin and Withington 1995).
In this dissertation, Niobium (Nb) superconductor on the 1.5 µm thick Al₂O₃ substrate is used in the filer designs. \( \lambda_L \) is approximately 90nm at 0 K and it is comparable to \( t \) of 0.1µm. The critical temperature \( (T_c) \) is approximately 9.3 K. The characteristic impedance of the superconducting microstrip line was derived using the analytical solutions (Yassin and Withington 1995) provided in the appendix. It is compared with the characteristic impedance and its phase constant of the microstrip line using loss less metal as shown in Figure 2-32 and Figure 2-33, respectively. The percentage variation in characteristic impedance is shown in Figure 2-34.

![Characteristics Impedance Graph](image)

**Figure 2-32** The characteristic impedance of the microstrip line using Niobium superconductor (solid line) and loss-less metal (dashed line) on 1.5 µm thick Al₂O₃ substrate. The line width varies from 1 to 100 µm. For \( \lambda_L=90 \text{ nm} \), \( t=0.1 \text{ µm} \) and temperature = 4.2 K.
Figure 2-33  Phase constant versus frequency of the Nb superconducting line with line width of 6 µm.

Figure 2-34  The percentage variation of the microstrip line’s characteristic impedance using Nb superconductor and that using loss less conductor.
From Figure 2-34, we observed that the superconducting microstrip characteristic impedance is approximately 5% higher than that of the lossless microstrip line. Moreover, the phase constant of the superconductor microstrip line is higher than that of the lossless case due to kinetic inductance effect (Lancaster 1977) as shown in Figure 2-33. In filter responses, this effect causes a shift in its center frequency as well as a reduction in in-band return loss. For the EM simulator to predict the filter response accurately, a parameter to compensate for the kinetic inductance is required. The compensation using surface impedance technique (Kerr 1999) is used since this technique provides the parameter that can be combined with the available commercial EM simulation software such as HFSS and Designer from Ansoft Corporation.

In the superconducting fabrication process at NASA GSFC, \( t \) is not much greater than \( \lambda_L \). The field incident wave on conductor close to the substrate is more than that on the other side of the conductor. In this case, the surface impedance equation, where the incident wave is excited on one side of the conductor plane, is used as follows

\[
Z_s = j \alpha \mu L \left( \frac{ie^{\frac{t}{\lambda}}}{Z_\eta} + \frac{Z_\eta - j \alpha \mu L}{Z_\eta + j \alpha \mu L} e^{\frac{t}{\lambda}} \right)
\]

where \( Z_\eta \) is the characteristic impedance of space (377 ohm in vacuum). In this fabrication process,

\[
Z_\eta = \sqrt{\frac{\mu}{\varepsilon}} = \frac{377}{10} = 119.2
\]

\( \lambda_L \) is also a function of the temperature as follows
\[ \lambda_z(T) = \frac{\lambda_0}{\sqrt{1 - \left(\frac{T}{T_c}\right)^2}} \]  \tag{2-48}

where \( \lambda_0 \) is the penetration depth at 0 Kelvin. \( T \) is the operating temperature.

Using (2-47), (2-48) and (2-49), the surface impedance is computed as a function of frequency. Since the surface impedance only has the imaginary part, it is called surface reactance as shown in Figure 2-35.

![Figure 2-35](image)

Figure 2-35  The frequency response of the surface reactance of the microstrip line versus frequency.

From Figure 2-35, the compensation due to the kinetic inductance can be neglected at frequency below 1 GHz. However at frequency above microwave, the compensation is required and can be included in the simulator as sheet impedance. At 4 GHz and 33 GHz, sheet impedance are \( j2.3 \times 10^{-3} \) and \( 0.019 \) Ohm, respectively.
The EM simulations with and without sheet impedance compensation are compared with the result obtained from (Yassin and Withington 1995). Ansoft Designer, which is a method of moment simulator, is used in this evaluation. The results in Figure 2-36 show a very good agreement with the analytical solutions. However, the characteristic impedance shown in Figure 2-37 agrees within 6% from the predicted value for the line width ranges from 1 µm to 30 µm.

![Figure 2-36](image)

Figure 2-36 The comparison between the microstrip line phase delay obtained from the EM simulation (dashed lines) and that derived from equations in (Yassin and Withington 1995) (solid lines). The Nb line is 100 µm long with $\lambda_0=90$ nm, $t=0.1$ µm, $T=4.2$ K. The Al$_2$O$_3$ dielectric thickness = 1.5 µm and $\varepsilon_r=10$. 
The characteristic impedance in Ohm of the Nb superconducting microstrip line obtained by the EM simulation and that from the analytical solution.

The kinetic compensation was included in the 33 GHz bandpass filter design that will be discussed in section 2.12. The EM simulation results in Figure 2-38 show that the pass-band filter response without compensation is significantly altered from the original design with the compensation.

In the pass-band response, the bandwidth of the filter without the compensation has a broader bandwidth as the transmission line becomes effectively longer due to smaller phase velocity value; given both filters have the same physical dimensions. Therefore the electrical length of the couple sections in the filter becomes longer and produces stronger coupling in the passband. Moreover, the in-band return loss increases in the filter model without kinetic inductance compensation due to improper coupling filter coefficient generated in-band.
Figure 2-38 The pass-band frequency response of the 33 GHz bandpass filter in Figure 2-47(b) with and without the kinetic inductance compensation in the microstrip lines.

In the out-of-band response, changes in the kinetic inductance result in the variation in the effective electrical length of the filter. This causes shifts in non-optimum locations of the transmission zeros to suppress the out-of-band spurious responses. Since the bandpass filter used in the final design has integrated bandstop filters that are used to suppress spurious response over wide frequency range, non-optimal location of the transmission zeros have little effect in suppressing the out-of-band response as shown in Figure 2-39.
In conclusion, the compensation of kinetic inductance using sheet impedance model can accurately predict the propagation constant of the superconducting line. However, the method of moment simulation shows a small variation of characteristic impedance from the analytical solution. The kinetic inductance compensation becomes more important as the filter’s center frequency increases. The pass-band response has strong effect in this compensation while the out-of-band has negligible effect for the bandpass filter design with proper out-of-band spur suppression techniques such as the one discussed in section 2.12.
2.12 The Bandstop Filter and Bandpass Filter Integration

In this dissertation, the ultimate goal is to construct the bandpass filter with very high out-of-band performance. Moreover, the filter design must be compatible with the provided superconductor fabrication process at NASA Goddard Space Flight Center (GSFC) such that it is cost effective and qualified for use in the NASA missions. The out-of-band specifications, discussed in section 1.2, require that the filter provides at least 50 dB of attenuation up to 700 GHz for the filter with $f_0$ of 100 GHz. Moreover, the lowest possible number of SIRs should be used in a filter design to minimize insertion loss and to produce a compact filter layout. In this section, a circuit model is developed to design filters with $f_0$ of 4 GHz and 33 GHz. The 4 GHz filter is designed to demonstrate the out-of-band suppression capability while the 33 GHz filter is one of the designs to be used in the CMBpol detector system.

In the narrow band filter where the bandwidth is less than 15%, spurious responses produced by the filter can be suppressed using transmission zeroes generated by the SIRs as described in section 2.7. However, in the broadband filter where the bandwidth is greater than 15%, these spurs are more difficult to suppress as the isolation between $f_0$ and $f_{s1}$ decreases in proportion to the filter bandwidth as shown in Figure 2-40.

From Figure 2-40, the isolation between $f_0$ and $f_{s1}$ is reduced by 14 dB with the percentage pass-band bandwidth increases from 0.1 to 0.2. Therefore transmission zeros generated by the SIRs must also be used to increase the isolation between $f_0$ and $f_{s1}$ to an acceptable level. Since, the notch responses created by the transmission zeros from SIRs are narrow band, more than one transmission zeros are required to suppression the spurious response and enhance isolation, simultaneously. Using the $N^{th}$ order DSOE filter that has $N+1$ transmission zeros can not sufficiently suppress $f_{s1}$.
Figure 2-40  The frequency response of $|S_{21}|$ of the circuit-modeled $3^{\text{th}}$-order coupled-$\lambda/4$ SIRs filter with $R=0.528$ in Figure 2-15. They are designed for three different percentage bandwidths ($w$).

and $f_{s2}$ or increase the overall isolation, when the value $N$ is small. Therefore additional zeros are required out-of-band and these zeros can be obtained by inserting bandstop filters. The bandstop filters consists of APSI opened-end stub combined in series as discussed in section 2.10.4.

The integration of the bandstop filter with the bandpass filter has several benefits. First, the out-of-band response of the bandpass filter design can be relaxed, since the bandstop filter suppresses the out-of-band spurious response more effectively than the notch responses produced internally by the SIRs in the bandpass filter. Therefore, these internal notches are no longer needed to be exactly overlapped with the spurious response at $f_{s1}$ or at $f_{s2}$. As a result, the broadband bandpass filter can be designed using fewer resonators and some transmission zeros from the bandpass filter can be used to suppress interference close to in band. Second, the bandstop filter allows the
attenuation bandwidth to be extended beyond $f_{s2}$ without affecting the filter’s in-band response. The bandstop filter can be designed to generate a transmission pole around $f_0$ and three transmission zeros at the frequency beyond $f_0$. Therefore it can be connected directly in series with the bandpass filter. Additional loss produced by the bandstop filter is less than the loss in the filter with higher order.

2.12.1 The Bandpass Filter Design Using Integrated Broadband Bandstop Filter

To design the filter that meets both in-band and out-of-band requirements, the filter has the dependent in-band and out-of-band specifications.

For the In-band specification, the bandpass filter must be designed such that $f_{s1}$ is furthest away from $f_0$ such that it overlaps with the lowest notch frequency produced by broadband bandstop filter. $f_0$ is also required to be overlapped with the transmission pole of the bandstop filter such that the bandpass filter’s pass band response is not effected. This can be demonstrated in Figure 2-41.

For the out-of-band specification, the minimum number of stages of bandstop filters is used to minimize the in-band insertion loss. The number of stages is also dependent on the suppression bandwidth and the level of attenuation.

2.12.2 Bandpass Filter Design

The prototype Chebyshev 4th order filter with 20% bandwidth was designed using the $\lambda/2$ SIR with $R$ of 0.528 where $\theta_1=\theta_2=\theta_0$. This SIR generates spurious resonance frequencies at $f_{s1}=2.5f_0$, $f_{s2}=4f_0$, $f_{s3}=5f_0$ and $f_{s4}=6f_0$. The filter with 0.1 dB pass-band ripple has the coefficient as follows: $g_0=1$, $g_1=1.1088$, $g_2=1.3061$, $g_3=1.7703$, $g_4=0.8180$, $g_5=1.3554$. 
The $\lambda/2$ SIRs are used as opposed to the $\lambda/4$ SIRs to avoid via holes connected from traces to ground, although they generates additional spurious resonance frequencies. Using via-less design increases the superconductor fabrication yield and reduces number of fabrication’s photo lithography steps. The filter is designed based on the procedure discussed in section 2.8. However, the odd and even-mode characteristic impedances of the grounded-end anti-parallel coupling section are computed using the following equations

$$Z_{1,e} = Z_o \left[ \frac{1 - K/Z_i \tan(\theta_i^{(e)})}{1 + K/Z_i \cot(\theta_i^{(e)})} \right]^{-1}$$  \hspace{1cm} (2-49)

$$Z_{1,o} = Z_o \left[ \frac{1 + K/Z_i \tan(\theta_i^{(o)})}{1 - K/Z_i \cot(\theta_i^{(o)})} \right]^{-1}$$  \hspace{1cm} (2-50)
where $K$ is computed in (2-10c). And the odd and even-mode characteristic impedances of the opened-end anti-parallel coupling section are computed using the following equations

$$Z_{2,e} = \frac{Z_2}{1-Z_2 J \tan(\theta'_2)} \left(1 + \frac{Z_2 J \tan(\theta''_2)}{1-Z_2 J \cot(\theta''_2)}\right)$$  \hspace{1cm} (2-51)

$$Z_{2,o} = \frac{Z_2}{1+Z_2 J \tan(\theta''_2)} \left(1 - \frac{Z_2 J \tan(\theta''_2)}{1-Z_2 J \cot(\theta''_2)}\right)$$  \hspace{1cm} (2-52)

where $J$ is computed in (2-10b). The equivalent circuit and its parameter values are shown in Figure 2-42 and in Table 2-4, respectively. $\phi'$ is a function of $Q_{sh}$, $Z_2$, $\theta_b$ and $R$ as in (2-18). $\phi$ is adjusted such that $Z_i$ is equal to the port input impedance $Z_0$. Therefore the quarter-wavelength line is no long needed to transform impedance from the input port to the tapped location. This technique also reduces the size of the filter.

![Figure 2-42](image-url) The circuit model of the 4th order coupled-SIR filter.
Table 2-4 The design parameters at $f_0$ of the 4th order coupled-SIR band pass filter with $R=0.528$. The ports’ input impedance are 20 Ohm.

<table>
<thead>
<tr>
<th>Section</th>
<th>Computed filter parameter</th>
<th>Circuit parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tapped</td>
<td>$Q_{si}=5.544$</td>
<td>$Z_1=14.36 , \Omega$, $Z_2=7.58 , \Omega$, $\phi=35.4^\circ$, $Z_\pi=20 , \Omega$, $\theta=36^\circ$</td>
</tr>
<tr>
<td>Grounded-end anti-parallel coupling</td>
<td>$Z_2^*J=0.083$</td>
<td>$Z_{1,e}=24.54 , \Omega$, $Z_{1,o}=9.64$, $\theta_1'=6^\circ$, $\theta_1''=30^\circ$</td>
</tr>
<tr>
<td>Opened-end anti-parallel coupling</td>
<td>$K/Z_1=0.104$</td>
<td>$Z_{2,e}=10.26 , \Omega$, $Z_{2,o}=5.02 , \Omega$, $\theta_2'=13.5$, $\theta_2''=22.5$</td>
</tr>
</tbody>
</table>

The 4 GHz and the 33 GHz bandpass filters are designed based on the parameters above. Their physical layouts on 1.5µm-thick Al$_2$O$_3$ substrate are shown in Figure 2-43.

![Figure 2-43](image)

In this filter design, resonators are required to overlap to produce sufficient coupling for the proper pass-band response. Resonators are placed in the middle and top layer in alternative sequence. The top and the middle metal layers are separated by 0.25 µm-thick Al$_2$O$_3$. The stepped impedance stub in the 4 GHz filter is split into two
sections to minimize the overlapping area between two metal layers. The 4 GHz and 33 GHz filters’ frequency responses are shown in Figure 2-44(a) and (b), respectively.

Figure 2-44 The frequency response of (a) the 4 GHz and (b) the 33 GHz bandpass filters using the SIRs with internal coupling.
The transmission zeros of both 4 GHz and 33 GHz bandpass filters are placed at $4f_0$ and $6f_0$, to suppress their odd-mode spurious resonances. A transmission zero is placed at $2.5f_0$ to suppress the lowest even-mode spurious resonance frequency. Other higher-order even-mode resonance frequencies are partially suppressed by Lo-Z SIO stubs. The location of these transmission zeros are not at optimum at the spurious frequencies as they will finally be suppressed by the broadband bandstop filter.

The sharp attenuation that appeared close to the high frequency side of the pass-band of the 4 GHz filter is generated by the ground-end coupling section that combines with the non-overlap SIO stubs. The location of this transmission zero varies as a function of the parasitic capacitance at the SIO stubs' overlapping area.

### 2.12.3 Broadband Bandstop Filter Design

The broadband bandstop filter is designed to suppress the filter’s spurious response beyond $f_{s1}$. The effective suppression range is dependent of the difference between $Z_{n1}$ and $Z_{n2}$ and the coupling coefficient $c_n$ derived in section 2.10.

Two bandstop filters are designed and combined in series with the bandpass filter. One filter is designed such that its transmission zeros are used to suppress the bandpass filter's $f_{s1}$, $f_{s2}$ and $f_{s3}$. The other is used to suppress spurious response beyond $f_{s3}$. The parameters of the bandstop filters are provided in Table 2-5.

<table>
<thead>
<tr>
<th>Table 2-5</th>
<th>The physical parameters of the bandstop filters in Figure 2-45 that are integrated with the bandpass filters.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bandstop filter type-I</td>
<td>Bandstop filter type-II</td>
</tr>
<tr>
<td>Physical parameters for the 4 GHz filter</td>
<td>$W_{n2}=1.75 , \mu m$, $L_{n2}=1239 , \mu m$, $G_{n1}=1 , \mu m$, $W_{n2}=40 , \mu m$, $L_{n2}=622.5 , \mu m$</td>
</tr>
<tr>
<td>Physical parameters for the 33 GHz filter</td>
<td>$W_{n1}=1.75 , \mu m$, $L_{n1}=198 , \mu m$, $G_{n1}=1 , \mu m$, $W_{n2}=25 , \mu m$, $L_{n2}=112 , \mu m$</td>
</tr>
</tbody>
</table>
For the 33 GHz filter, transmission poles of the bandstop filters type-I and type-II are located at 33 GHz and 75 GHz respectively. The EM simulation in Figure 2-46(a) of the bandstop filters shows that the combined response can provide a suppression level of 38 dB from 95 GHz to 365 GHz. The bandstop filter type-I for the 4 GHz filter and the bandstop filter type-II for the 33 GHz were modified to provide sufficient low in-band loss and broad attenuation band. Their electrical lengths in the $L_{n1}$ and $L_{n2}$ do not equal as they are optimized to provide low return loss in the pass-band while providing very broadband out-of-band attenuation simultaneously. Therefore, these transmission zeros are not as clearly presented in the attenuation band as those demonstrated in section 2.10.
Figure 2-46 The frequency response of (a) the bandstop filters type-I and type-II used in the 33 GHz bandpass filter; (b) the bandstop filter type-I used in the 4 GHz bandpass filter.
The broadband bandstop filters are placed at each end of the bandpass filter as shown in Figure 2-47(a) and (b). The frequency responses of the 4 GHz and 33 GHz filters show an improvement in the out-of-band response from $2f_0$ to $8f_0$, when compared with the conventional bandpass filter without the integrated bandstop filter as shown in Figure 2-48 and Figure 2-49, respectively. However, the broadband bandstop filter slightly reduces the return loss of the bandpass filter as shown in Figure 2-50. The 4 GHz filter has higher overall isolation than that of the 33 GHz due to a larger physical size and has less parasitic from its operation at lower frequencies.

Figure 2-47 The photographs of the (a) 4 GHz and (b) 33 GHz bandpass filter with integrated bandstop filters.
Figure 2-48 The broad-band frequency response of the 4 GHz bandpass filter with and without the broadband bandstop filter.

Figure 2-49 The broad-band frequency response of the 33 GHz bandpass filter with and without the broadband bandstop filter.
Figure 2-50  The pass-band frequency response of (a) the 4 GHz and (b) the 33 GHz bandpass filters with and without the broadband bandstop filters.

2.12.4 Superconducting filter fabrications

The filter is fabricated at NASA GSFC in Greenbelt, MD. The measurement chip sample was designed to allow on-chip thru-reflect-line (TRL) calibration at 4.3 K. Since the filters have 20 Ohm input impedance while the probe impedance is 50 Ohm, an impedance transformer is designed to provide low return loss from the probe to the filter input ports as shown in Figure 2-51(a). The photograph of the fabricated sample is shown in Figure 2-51(b).

Due to several variations in fabrication processes, all parameters obtained are different from the designed values as shown in Table 2-6.
Figure 2-51  (a) The layout of the standard calibration lines, 4 GHz and 33 GHz bandpass filters; (b) The photograph of the layout fabricated at NASA GSFC.

Table 2-6  The superconductor fabrication parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Designed values</th>
<th>Measured values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al₂O₃ substrate thickness</td>
<td>1.5 µm</td>
<td>1.76 µm</td>
</tr>
<tr>
<td>Al₂O₃ thickness between the top and middle layers</td>
<td>73 nm</td>
<td>280 nm</td>
</tr>
<tr>
<td>Middle metal thickness</td>
<td>0.1 µm</td>
<td>0.095 µm</td>
</tr>
<tr>
<td>Top metal thickness</td>
<td>0.1 µm</td>
<td>0.11 µm</td>
</tr>
<tr>
<td>Ground thickness</td>
<td>0.5 µm</td>
<td>0.1 µm</td>
</tr>
</tbody>
</table>
2.12.5 Superconducting filter measurement

The 4 GHz and 33 GHz filters were measured using the cryogenic probe station model TTP6 from Lake Shore Cryotronics, Inc. The probe station is provided by Georgia Electronic Design Center in Atlanta, Georgia. The vacuum chamber inside the probe station is shown in Figure 2-52.

![Image of vacuum chamber inside the probe station model TTP6.](image)

**Figure 2-52** The original setup of vacuum chamber inside the probe station model TTP6.

This probe station allows the temperature of the device under test (DUT) to be close to 4.3 K. However, in the current configuration, the temperature at the probe tip is at above 10 K which increases the temperature of the DUT around the contacting area. This causes the Nb superconductor to become a normal conductor that has high resistance. Therefore, the probe station was modified such that the probe tip can achieve lower temperature than the $T_c$ (9.3 K) of the Nb. This allows the Nb line to become superconductor.
To be able to measure superconducting filter responses, multiple copper straps are tied to the probe body to reduce the temperature of the probe. In addition, 72 mil-thick high density polyethylene is used as a separation between the probe body and the probe holder, which is made of copper. This setup reduces the high temperature heat flow from the probe holder to the probe body and thus cooling the probe more quickly. Additional copper straps are used to dissipate the heat from the coaxial cables (connected to the probe) to the chuck, where its temperature is at 4.3 K. Moreover, intermediate radiation heat shield is installed around the chuck to isolate the warm area from the cold area at the chuck. Belleville washers are used between all screws and mounting to provide reliable contact between two surfaces where thermal contraction occurs as temperature reduces. Three silicon diode sensors were mounted at the sample holder, probe body and the coaxial cable to monitor their temperatures. The photographs of the setup of the chamber and the probe station used for the filter measurement are shown in Figure 2-53 and Figure 2-54, respectively. The cross-sectional view of the setup is shown in Figure 2-55. From Figure 2-55, the DUT and the standard calibration substrate are mounted at the center of the substrate holder using thermal grease.

Using this setup, the temperature at the sample holder is as low as that at the chuck. Moreover, the probe body temperature is reduced from >10 K to 6.3 K when the chuck temperature is reduced to 4.3 K, as shown in Figure 2-56. The standard calibration substrate (white sample at the center of the photo) is placed above the DUT to perform short-open-load-thru (SOLT) calibration up to the probe tip at 300 K and served as a contact substrate at 4.3 K.
Figure 2-53  The probe station chamber setup for the superconducting measurement at 4.3 K.

Figure 2-54  The probe station setup to measure superconducting filters at GEDC.
Due to the vibration of the vacuum pump line of the probe station, stress on the stainless steel cable generated by copper straps and the unsymmetrical contraction of the probe arms among x, y and z axes, the probes were not able to obtain stable contacts to the substrate. Moreover, thin oxide layer is formed on the Nb line, which makes it difficult for the probe ground pins (with weak contacting strength) to make DC connections to the Nb ground. As a result, the measurement has so much noise that the measurement using TRL calibration at 4.3 K is not an acceptable quality. Therefore, the measurements are performed using the SOLT calibration at 300K as discussed earlier.
The measurement results of the 4 GHz and 33 GHz filters are shown in Figure 2-57 and Figure 2-58, respectively. The measurement frequency ranges from 1 GHz to 48 GHz. The high-end of the frequency is limited by the network analyzer and the coaxial cables used in the probe station.

The measurement results include the response of the impedance transformer that is used to transform 50 Ohm impedance at the probe tip to 20 Ohm impedance at the inputs of the filter. Due to calibration instability, the impedance transformer cannot be removed from the calibration. As a result, the passband response of the 4 GHz bandpass filter was suppressed due to high return loss at the low frequency side of the band. $f_0$ of the bandpass filter shifts slightly higher to 4.1 GHz. The bandwidth and the insertion loss of the filter are higher than those in the model due to the substrate’s high loss tangent. Moreover, the Nb lines have finite loss as the temperature of the probe is not much lower than the Nb’s $T_c$. The average out-of-band attenuation level is
approximately at 45 dB which is limited by the background noise of the measurement and the lossy ground plane around the probe landing area.

![Graph showing frequency response](image)

Figure 2-57 The measured response of the superconducting 4 GHz bandpass filter that includes the 50 Ohm to 20 Ohm impedance transformers. This measurement uses the SOLT calibration at 300 K.

The measured 4 GHz filter response has lower stopband bandwidth and has a strong spurious response at 32 GHz. This is caused by the incorrect coupling coefficient in the stopband filter as the Al₂O₃ thickness changed from the desired value. Therefore, the stopband filter’s transmission zero $f_{m}$ was no longer effective. High noise levels and measurement ripples are observed in the measurements using the SOLT calibration at 300 K. These noises and ripples are due to unstable measurement as discussed earlier.

The measurement of the 33 GHz filter response in Figure 2-58 shows several ripples similar to those in the 4 GHz filter. Due to the high frequency limit of 48 GHz of the network analyzer, the out-of-band response of the 33 GHz filter cannot be measured.
The in-band insertion loss is caused by measurement errors, poor contact on Nb lines and loss in Nb lines as they do not become superconducting completely due to high temperature at the probe contact.

Figure 2-58  The measured response of the superconducting 33 GHz bandpass filter that includes the 50 Ohm to 20 Ohm impedance transformers. This measurement uses the SOLT calibration at 300 K.

2.12.6 The Filter’s Performance in Detector Systems

To test the filter’s isolation in the presence of interference in the actual environment, the simulated 33 GHz filter’s response is rescaled to operate at the center frequency of 100 GHz. Microwave energy from the sky, as shown in Figure 2-59, is applied to the filter with the presence of 30 K far-infrared black body radiation from 300 to 600 GHz. The integral power from the filter is measured.
The simulation results in Figure 2-59 show that the integral power caused by this interference is significantly suppressed such that it produces less than 0.1% increment from the desired measurement level. This exceeds the specification of the CMBpol measurement program at NASA. However, the current measurement of the superconducting filters shows low out-of-band isolation due to high background noise in the calibration. To meet the specification, the filter requires an improved fabrication process and measurement capability, in which will be discussed in Chapter 5.
CHAPTER 3

BROADBAND AND LOW-LOSS MAGIC-T DEVELOPMENT

3.1 Literature Review

A microwave hybrid is an important component in a microwave system and has been a subject of interest for several decades. In a radiometer system, the microwave hybrid is used as one of the front-end components to extract the stroke parameters (Skou, Laursen et al. 1999) in microwave signals. This dissertation focuses on the design of the passive broadband 180° hybrid on the thin-film substrate that has low in-band loss, broadband response and high sum-to-difference port isolation. The operating frequency is in the Ka-Band (From 26 GHz to 40 GHz) or higher. The origin and history of the broadband hybrid are addressed. Then, several approaches are studied.

The 180° hybrid can be implemented using passive and active components. In this proposal, the 180° hybrid is referred to as “hybrid” hereafter. Although active hybrids (Tokumitsu, Hara et al. 1989) are significantly smaller than passive hybrids, they are not used in passive radiometer applications. Since the active hybrids are made from transistors, they have limited operating power. The hybrids’ transistor noise is also added to the output signal. Moreover, the output responses are not as linear as those in passive hybrids.

The literature review begins with the simplest passive hybrid design is called the retrace hybrid. (Reed and Wheeler 1956). Most research focuses on reducing the size of rat-race hybrids (Hirota, Minakawa et al. 1990; Kamitsuna 1992; Eccleston and Ong 2003; Chen and Tzuang 2004; Ghali and Moselhy 2004; Hettak, Morin et al. 2004; Ng, Chongcheawchamnan et al. 2004; Okabe, Caloz et al. 2004; Sung, Ahn et al. 2004; Yun
Among the research are studies related to reducing the physical length of the T-line while maintaining the same electrical length (Eccleston and Ong 2003; Chen and Tzuang 2004; Sung, Ahn et al. 2004; Yun 2004). The T-line with the defected ground structure (DGS) is used in (Sung, Ahn et al. 2004; Yun 2004). The DGS in (Sung, Ahn et al. 2004) also produces signal suppression out-of-band of interest. The hybrid size can be reduced by 32 to 46 percent using artificial T-lines (Eccleston and Ong 2003), which consist of shunt and series T-line stubs connected in series. Lumped capacitors can also be incorporated in a T-line (Hirota, Minakawa et al. 1990; Kamitsuna 1992; Hettak, Morin et al. 2004; Ng, Chongcheawchamnan et al. 2004). Using this technique, a $\sqrt{2} Z_0 \lambda/4$ line can be replaced by the $2Z_0$ T-line with $\lambda/8$ long and lumped capacitors at both ends of the T-line (Kamitsuna 1992). The meandering lines can also be used to reduce the overall size (Chen and Tzuang 2004; Ghali and Moselhy 2004). The techniques described above do not give broadband response. Their bandwidth is equivalent to the conventional retrace hybrid. Because of the discontinuities added to the hybrid, such as multiple-bends, narrow T-line and DGS, these hybrids produce higher in-band insertion loss than that of the conventional hybrid.

To improve the bandwidth and in-band return loss of the retrace hybrid, stepped-impedance transformers are used (Kim and Naito 1982; Mgombelo 1989; Ahn and Wolff 2001). The values of the stepped-impedance transmission line are determined using optimization techniques. Moreover, extra shunt T-line sections can be introduced to improve the phase balance and increase passband bandwidth (Piernas, Hayashi et al. 2000).

Most research, which focuses on producing broadband response hybrids, incorporates broadband 180° phase shifters in the hybrids. Broadband phase shifters can be produced using left-handed T-lines (Okabe, Caloz et al. 2004). However, using
today’s technology these T-lines are in the form of distributed lumped elements. Therefore, the left-handed T-line has limited use at the millimeter or sub-millimeter frequency. Other techniques are related to the transition between TEM line and slotline. Moreover, the 180° phase shifters can be in the form of coupled lines (Rehnmark 1977) and balun structure (Jones 1960; Laughlin 1976; Ang and Leong 2002).

Broadband 180° phase shifters using co-planar waveguides (CPW) to slotlines (SL) transition are widely used, since both CPW and SL are on the same layer, which are simple to fabricate. The hybrids using this technique provide broadband response and are more compact than the conventional retrace hybrid (Hirota, Tarusawa et al. 1987; Ho, Fan et al. 1994; Murgulescu, Moisan et al. 1994; Fan, Ho et al. 1995; Fan, Kanamaluru et al. 1995; Fan, Heimer et al. 1997; Chang, Yang et al. 1999; Wang 1999). Moreover, the length of each T-line section of these hybrids can be reduced to less than λ/4 long (Murgulescu, Moisan et al. 1994; Fan, Ho et al. 1995). The magnitude and phase imbalances of these hybrids are minimal and have the 3 dB bandwidth of more than one octave. Despite their good performance, these hybrids are expensive to fabricate since they require bondwires or airbridges to connect between two ground planes of the CPW stripes. The airbridges/bondwires need to be placed at the locations that have discontinuity and where long CPW lines are used. This placement reduces the production yield of the hybrid. Moreover, at sub-millimeter wave frequencies, resulting parasitics due to bondwires/airbridges becomes so significant that they introduce phase and amplitude imbalances in the upper operating frequency of the hybrid.

To eliminate the use of bondwires/airbridges in a hybrid design, MS-to-SL transition technique is implemented (Ronde; Aikawa and Ogawa 1980; Ogawa, Hirota et al. 1985; Hiraoka, Tokumitsu et al. 1989; Kim and Park 2002). The designs using this technique produce a broadband response and can also be very compact (Hiraoka, Tokumitsu et al. 1989). However, three of the ports are SL and require transition to the
MS line layer. Moreover, the sum and difference ports of the magic-T can be located on the opposite side of the input ports (Aikawa and Ogawa 1980). This technique requires that most T-lines in the structure be slot lines; therefore, the hybrid using this technique has high slotline radiation. De Ronde proposed a very compact magic-T structure (Ronde). However, no analytical design is determined. Since a large circular slot is directly placed under the MS line in the structure, it produces high radiation loss if operated at high frequency. Other types of hybrids are asymmetric coupled-transmission lines (Carpenter; Gruszczynski; Kraker 1964; Nakajima and Tanabe 1996). Using this technique requires two broadside coupled lines. Therefore, the hybrid requires two metallization layers in addition to a ground plane layer. Although broadband response is achieved, this hybrid produces high in-band phase and magnitude ripple.

3.2 Microstrip-to-slotline Transitions Using Slotline Stepped Circular Ring

Microstrip-to-slotline (MS-to-SL) transitions (Knorr 1974) find use in a variety of microwave applications. Many techniques have been developed to extend the transition bandwidth using different types of terminations such as parallel sections of $\lambda/4$ stubs (Akhavan and Mirshekar-Syahkal 1996) or lumped element terminations, or using the impedance transformation techniques at the termination (Zhang, Wang et al. 2004). Although transitions using these techniques have broadband characteristics, their insertion loss typically increases gradually as frequency increases. This is partly due to slotline radiation loss and the change in slotline impedance with frequency.

For the MS-to-SL transition (see Figure 3-6(a)) to effectively transfer power, the microstrip requires a grounded-end termination, whereas the slotline requires an opened-end termination at the transition. The effective lengths of the line used to
produce a grounded-end/opened-end termination determine the transition bandwidth. In the lower transition frequency limit, the perimeter of the slotline “open” is small compared to the guided wavelength, the termination will be effectively grounded. In the upper transition frequency limit, as the perimeter approaches the scale of the wavelength, the slotline termination acts as an unintended resonant ring-slot antenna. Similar considerations apply to the complementary microstrip structure’s limitations; however, as a practical matter, due to microstrip lines greater field confinement, radiation losses are a lesser concern.

In order to reduce radiation loss in the MS-to-SL transition, this dissertation propose the use of slotline stepped impedance circular ring (SCR) termination. The circuit model and radiation loss characteristics are studied. Three MS-to-SL transitions are constructed using three SCR terminations that have different physical dimensions. Two-port measurements are performed in the band of interest from 2 to 26 GHz. Finally, the performance of the transition using the SCR termination is compared with the transitions using slotline circular rings, circular pads and radial pads.

3.2.1 Slotline Stepped Circular Ring Termination

To suppress radiation, the electrical length of the slotline structure must be small relative to the guided wave length over the operating frequency band (Stutzman and Thiele 1998). From this perspective, the slotline SCR shown in Figure 3-1(a) has several advantages over \( \lambda/4 \)-long slotline terminations such as radial or circular stubs. The slotline SCR is smaller in size, provides broadband response and reduces radiation loss
Figure 3-1  (a) The proposed slotline SCR (b) electric fields in the slotline SCR (arrow line). (c) the equivalent transmission-line circuit model of the slotline SCR. Grey areas represent ground plane.

The slotline SCR consists of three slotline sections with the characteristic admittances of \( Y_0, Y_1 \) and \( Y_2 \). Their physical lengths are \( l_0, l_1 \) and \( l_2 \) and their electrical lengths are \( \phi, \theta_1 \) and \( \theta_2 \), respectively. By symmetry, the circular structure forces the electric field (E-field) to cancel at center, creating low-loss virtual ground as shown in Figure 3-1(b) over the operating band. Its equivalent circuit is shown in Figure 3-1(c). When \( \phi=0 \), the slotline section \( Y_0 \) is discarded and the input admittance of the termination can be expressed as:

\[
Y_{in} = -jY_2 \frac{R - \tan(\theta_1)\tan(\theta_2)}{2\tan(\theta_2) + R\tan(\theta_2)}
\]  

(3-1)

where \( R \) is the stepped admittance ratio \( Y_1/Y_2 \). In practice, the slotline admittance values (\( Y_1 \) and \( Y_2 \)) contain much smaller imaginary parts relative to their real parts. Thus, their imaginary parts are negligible in the circuit analysis. At the center of the operating frequency \( f_0 \), i.e., \( Y_{in}=0 \) in (1), we obtain the relationship:

\[
R = \tan(\theta_1)\tan(\theta_2)
\]

(3-2)
for a lossless structure. In practical implementation, the slotline SCR is designed at the frequency above $f_0$ and transformed to $f_0$ using $Y_0$ and a non-zero $\phi$ value. When the non-zero $\phi$ value is used, we achieve a smaller SCR area with slightly narrower operating bandwidth. The circuit model and the electro-magnetic (EM) simulation of the slotline SCR are in agreement over a broad frequency range as shown in Figure 3-2. The deviations from the model are mainly due to the SCR’s admittance stepped discontinuities and the slotline’s characteristic impedance that changes with frequency.

![Figure 3-2](image)

Figure 3-2 The input admittance of the SCR on the 0.102 mm-thick Rogers liquid crystal polymer (LCP) substrate using the EM simulation with the lossless transmission line model ($Y_1=6.8\cdot10^{-3}-j1.25\cdot10^{-4}$ Siemens, $Y_0=Y_2=1.1\cdot10^{-2}-j1.23\cdot10^{-4}$ Siemens, $\theta_1=26.0^\circ$ and $\theta_2=29.7^\circ$ at 10 GHz). $\phi$ values are at 10 GHz.

### 3.2.2 Reducing Radiation Loss with the Slotline SCR

To study radiation loss in a slotline SCR, the following loss factors are defined as $L_{1-\text{port}}=1-|S_{11}|^2$ and $L_{2-\text{port}}=1-|S_{11}|^2|S_{21}|^2$. These equations are for a one-port slotline termination and a back-to-back MS-to-SL transition, respectively, where $S_{11}$ and $S_{21}$ are
the reflection and transmission coefficients of the slotline structure, respectively. The back-to-back MS-to-SL transition's ports can be interchanged (i.e. $S_{11}=S_{22}$ and $S_{21}=S_{12}$).

If a lossless conductor and a loss-less substrate are used, $L_{1\text{-port}}$ and $L_{2\text{-port}}$ represent radiation losses. The EM simulations of the $L_{1\text{-port}}$ of a slotline termination using Ansoft Designer in Figure 3-3 show that the maximum $L_{1\text{-port}}$ value occurs at $f_{r1}$ when $Y_{in}$ is equivalent to the magnitude of the slotline’s port admittance $Y_s$. This also occurs around the frequency where $Y_{in}$ reaches the maximum value at $f_{r2}$.

![Figure 3-3](image)

Figure 3-3  Simulated $L_{1\text{-port}}$ of the slotline SCR in Figure 3-2 connected to a slotline with the characteristic impedance of $Y_s=0.01$, 0.02 and 0.05 Siemens.
Moreover, radiation loss is reduced as the port admittance increases. Therefore, to increase the low-radiation loss bandwidth, a high $Y_s$ value is used and the separation between $f_{r1}$ and $f_{r2}$ must be maximized. This can be achieved by reducing the effective total length of the slotline SCR while maintaining low $Y_{in}$ around the operating frequency. From (3-2), a low $R$ value is used, such that $\theta_1$ and $\theta_2$ are minimized. Moreover, the condition $\theta_1=2\theta_2$ is used to extend $f_{r2}$ away from $f_0$ (Kuo and Shih 2003). For the layout simplicity, this condition is approximated by $l_1=2l_2$. The simulation results in Figure 3-4 show that the radiation loss is reduced when the slotline length ratio of $l_1/l_2$ changes from 1 to 2 (in Type-I and Type-II terminations) and when $R$ is decreased from 0.645 to 0.556 (in Type-II and Type-III terminations).

![Figure 3-4 Simulated input admittance and $L_{1,\text{port}}$ of the slotline SCRs.](image)

- **Type-I**: $R=0.645$, $W_1=219 \ \mu\text{m}$ and $l_1=l_2=2.03 \ \text{mm}$.
- **Type-II**: $R=0.645$, $W_1=219 \ \mu\text{m}$ and $l_1=2l_2=1.37 \ \text{mm}$.
- **Type-III**: $R=0.556$, $W_1=500 \ \mu\text{m}$ and $l_1=2l_2=1.04 \ \text{mm}$.

For all SCR types, $l_0=0.45\text{mm}$, $W_0=W_2=102\mu\text{m}$ and $Y_s=0.02$ Siemens.
The radiation loss of the slotline SCRs can also be visualized from the perception of E-field confinement around the termination. Using Ansoft HFSS software, the simulated E-field in Figure 3-5 shows that the slotline circular ring provides the least confinement while the SCR Type-III has the highest confinement. Therefore, the slotline SCR Type-III provides the minimum radiation loss among the three.

![E-field magnitude at 10 GHz and 19 GHz of the slotline terminations](image)

Figure 3-5 The simulated E-Field magnitude at \( f_0 = 10 \text{ GHz} \) and at 19 GHz of the slotline terminations (a) circular ring, (b) SCR Type-I (c) SCR Type-II and (d) SCR Type-III. Slot areas are shown in white.

### 3.2.3 Hardware Implementation of Slotline SCRs in MS-to-SL Transitions

MS-to-SL transitions are fabricated on a 102\( \mu \text{m} \)-thick LCP substrate. Their center frequency is at 10 GHz. The footprints of the transition with three different slotline SCR terminations are shown in Figure 3-6. The photograph of the test structure is shown in Figure 3-7(a) and (b).
Figure 3-6  The layout of back-to-back MS-to-SL transitions using the slotline SCR terminations (a) Type-I, (b) Type-II (c) and Type-III. \( W_1=W_0=100 \) µm and \( L_s=1.78 \) mm on all types above. Type-I, Type-II and Type-III have the same microstrip line dimensions.

Figure 3-7  The photograph of the (a) top view and (b) bottom view of the seven MS-to-SL transitions and calibration lines on 0.102 mm-thick Roger’s LCP substrate. The sample’s overall dimension is 86 mm × 70 mm.

The LCP substrate has the relative dielectric constant of 2.9. The microstrip line sections in these transitions are terminated with stepped impedance stubs that have dimensions: \( W_{m1}=0.34 \) mm, \( W_{m2}=0.85 \) mm and \( L_{m1}=1.25 \) mm. At input ports, the microstrip line with the characteristic impedance of 50 Ohm is transformed to the \( Y_s \)
value of 0.011 Siemens using a $\lambda/4$-long line ($L_{mg}=4.86$ mm). This $Y_s$ value is set by the minimum slotline width allowed in our fabrication process. Transitions are connected to SMA connectors and the Thru-Reflect-Line (TRL) calibration is used. The measured minimum frequency is limited by the TRL calibration standards while the measured maximum frequency is limited by the SMA connector’s operating frequency. For the measurements, cavity resonances between the test fixture and instrument ground were damped by placing the transitions 5mm above a 0.76 mm-thick ECCOSORB GDS sheet (ECCOSORB 2006).

The MS-to-SL transition loss is calculated based on $L_{2\text{-port}}$ which includes the loss from conductor and dielectric. By comparing the total loss of the MS-to-SL transition using the circular pad, radial pad or circular ring terminations, the experimental results in Figure 3-8 show that the transition using the slotline SCR produces the lowest in-band insertion loss. At 12 GHz, the insertion loss of the transition using Type-III SCRs is 0.57 dB compared with 0.82 dB of the transition using radial stubs. Strong radiation can be observed in the MS-to-SL transition using circular rings at 21 GHz as the terminations become effective slotline antennas as shown in the $L_{2\text{-port}}$ plot in Figure 3-8.

Among the three SCR designs, the Type-III transition in Figure 3-9 has the least pass-band radiation loss and is in the acceptable agreement with the one-port simulation results. The $f_r$ and $f_o$ of the Type-III slotline SCR termination are approximately at 3 GHz and 25 GHz, respectively.
Figure 3-8  Measured frequency responses of (a) $\text{dB}|S_{21}|$ and (b) the $L_{2,\text{port}}$ of the MS-to-SL transitions with the slotline circular pad, circular ring, 50° radial pad, or Type-I SCR terminations.
3.3 A Low-loss Planar Magic-T using Microstrip-to-slotline Transition

A magic-T is one type of four-port microwave junction. In the ideal case it is lossless and has a port sum ($H$) and port difference ($E$) which allow incident signals to be divided or combined with a well defined relative phase. The structure approximating these ideal properties has been widely used as a circuit element in correlation receivers, frequency discriminators, balanced mixers, four-port circulators, microwave impedance bridges, reflectometers, etc (Montgomery, Dicke et al. 1948).

To achieve the magic-T with desirable properties over a broad-band, both phase and amplitude from two input ports must be identical as they are combined in-phase and out-of-phase at port $H$ and port $E$, respectively. Choosing the proper magic-T’s electromagnetic topology with the correct symmetry is important and helps achieve the desirable isolation and bandwidth.
The symmetry in the in-phase combining section can be achieved using microstrip line (MS) (Ang and Leong 2002) or co-planar wave guide (CPW) (Fan, Ho et al. 1995), whereas the symmetry in the out-of-phase combining section can be achieved using slottine (SL) (Kim and Park 2002), (Aikawa and Ogawa 1980). MS-to-SL or CPW-to-SL transitions are necessary to connect between the in-phase and out-of-phase combiners and to bring all ports to the same metal layer (Knorr 1974).

Several techniques were developed to effectively implement MS-to-SL or CPW-to-SL transitions in magic-Ts. These techniques provide magic-Ts with broadband response. Although the theoretical E-H port isolation of these magic-Ts is infinite, the practical isolation level of these designs is typically limited to 35dB by the following physical factors.

Firstly, it is limited by the magic-T layout asymmetry such as in (Fan, Ho et al. 1995) and errors from fabrication process as they produce unequal parasitic couplings between port 1 and port 2. Secondly, it is limited by the transmission line dispersion as the transmission line impedances in magic-T change with frequency (Gupta, Gang et al. 1996). This dispersion occurs more noticeable in a wide slotline and in a wide CPW than in a microstrip line. Moreover, the magic-T using the CPW-to-SL transition also has an additional isolation limit due to bondwires or airbridges parasitic and their slight misplacement on the transmission line discontinuities in the structure. Finally, the isolation is limited by electric-field (E-field) confinement and loss on the ground plane. The magic-Ts using the transmission line mode conversion with a large slot area on ground plane (Kim and Park 2002), (Aikawa and Ogawa 1980) have a limited E-H port isolation as the high level of E-field coupling occurs directly between port E and port H.
3.3.1 Circuit Configuration

The proposed magic-T is shown in Figure 3-10. The in-phase combiner in Figure 3-10(a) consists of two quarter-wavelength (\(\lambda/4\)) microstrip lines, with the characteristic impedance of \(Z_1\), combined at the \(H\) port. The out-of-phase combiner in Figure 3-10(b) consists of two (\(\lambda/4\)) microstrip lines with the characteristic impedance of \(Z_2\) combined with the half-wavelength (\(\lambda/2\)) line with the characteristic impedance of \(Z_3\). The out-of-phase combined signal can be obtained from the slotline with the characteristic impedance of \(Z_{sl}\) at the center of the structure below the \(Z_3\) line. Finally, the slotline section is transformed to the microstrip line output at port \(E\).

![Diagram of the proposed broadband magic-T](image)

Figure 3-10 The proposed broadband magic-T consisting of (a) the in-phase combiner and (b) the out-of-phase combiner using microstrip-to-slotline transition.
The proposed magic-T has several advantages over conventional magic-Ts as follows. It requires only one short section of the MS-to-SL transition to achieve a broadband 180 degree phase shift and an out-of-phase power combiner. Secondly, the structure has a small total slotline area, thus minimizing radiation loss and parasitic coupling to microstrip lines. The magic-T layout is also symmetric along the y-axis up to port $E$ at $Z_{sl}$. As a result, the parasitic coupling from slotline sections to microstrip line sections at port 1 and port 2 are equal. Thus, the E-H port isolation of the magic-T exhibits broad-band characteristics. Moreover, it does not require via holes, bondwires or airbridges which increase fabrication complexity and allow broadband operation in mm-wave frequency. The magic-T is analyzed in odd and even modes up to the slotline $Z_{sl}$ section as shown in Figure 3-11(a) and (b) respectively.

![Figure 3-11](image)

Figure 3-11 The (a) odd-mode and (b) even-mode electric field and the current flow in the proposed magic-T and in the microstrip and slotline junction at A-B.
In the odd mode, the signals from port 1 and port 2 are out-of-phase. This creates a microstrip virtual ground plane along the y-axis of the magic-T. The slotline SCR termination connected to the slotline $Z_{sl}$ also allows the MS-to-SL mode conversion to occurs as demonstrated by electric-field and current directions around the A-B cross section as shown in Figure 3-11(a).

In the even mode, the signals from port 1 and port 2 are in phase, thus creating a microstrip virtual open along the y-axis of the magic-T as shown in Figure 3-11(b). Electric-fields in the slotline at the A-B cross section are canceled thus creating a slotline virtual short that prevents the signal flow to or from port $E$.

### 3.3.2 Magic-T Port Impedance Matching

The microstrip line section of the magic-T can be modeled as a half circuit for the odd and even modes due to its symmetry along the y axis. By ignoring step impedance discontinuities, these circuits are shown in Figure 3-11(a) and 3-10(b), respectively.

This model is valid at the center frequency ($f_0$) and it approximates the hybrid's response around $f_0$. The magnitude of the isolation between port 1 and port 2 and the magnitude of input return loss at port 1 and port 2 are computed as follows

\[
\text{Isolation} = -20\log \left( \frac{\Gamma_- - \Gamma_+}{2} \right) \quad (3-3)
\]

\[
\text{Returnloss} = -20\log \left( \frac{\Gamma_+ + \Gamma_-}{2} \right) \quad (3-4)
\]

where $\Gamma_-$ and $\Gamma_+$ are odd-mode and even-mode reflection coefficient at port 1 in Figure 3-12(a) and (b) respectively.
In the odd mode circuit shown in Figure 3-12(a), the port H becomes a virtual ground. Using a λ/4 transformation through $Z_1$ line, the virtual ground becomes an open at port 1. To match $Z_0$ at port 1 with $Z_{sl}/2$ at port E, port 1 impedance is transformed to the slot line impedance of $n_t^2 Z_{sl}/2$ using a $Z_2$ line, $Z_3$ line and the transformer ratio $n_t$. In the single mode limit, $n_t$ is dependent of the substrate thickness, the transmission line characteristic impedance and the MS-to-SL physical alignment (Kim and Park 1997). The slotline SCR is connected to the $Z_{sl}$ section to create a virtual open termination and enables the MS-to-SL mode conversion. The slotline SCR characteristic is described in section B. The relationship of $Z_0$, $Z_2$, $Z_3$ and $Z_{sl}$ can be determined at $f_0$ as follows:

$$Z_0 = n_t^2 Z_{sl} \left(\frac{Z_2}{Z_3}\right)^2$$  \hspace{1cm} (3-5)

It is desirable that $n_t^2 Z_{sl}/2$ equals to $Z_0$ to eliminate the discontinuity of microstrip lines (i.e. $Z_0 = Z_{sl} = Z_0$). However, for some fabrication processes, the value $Z_{sl}$ is limited to
the allowable minimum slot width and the substrate thickness. To minimize radiation loss of the transition, $Z_{sl}$ is set to the minimum value.

In the even mode circuit shown in Figure 3-12(b), port $E$ becomes a virtual open and it is $\lambda/2$ transformed to an open at port 2. Moreover, port 2 impedance is transformed to $2Z_0$ using $Z_1$ line with the characteristic impedance of $\sqrt{2Z_0}$. In this mode, port $H$ has narrow band frequency responses since it requires long transmission line length with stepped impedance discontinuity to achieve perfect impedance transformation from port 1 and port 2. The frequency response of the magic-T is shown in Figure 3-13.

![Figure 3-13](image)

**Figure 3-13** The magic-T frequency responses based on the circuit model in Figure 3-12(a), case 1 in Table 3-1.
To increase the magic-T bandwidth at port $H$, the values $Z_1$, $Z_2$ and $Z_3$ can be numerically optimized as shown in case 2 in Table 3-1, for the magic-T on the 105 µm-thick Roger’s liquid crystal polymer (LCP) substrate. The value of 94.1 Ohm is used for $Z_{sl}$. The frequency responses of the magic-T using the optimized parameters show an improved bandwidth while producing small in-band ripples at the transmissions 1-$H$ and 2-$H$ as shown in Figure 3-14.

Table 3-1  Magic-T’s parameters used in Figure 3-12, the impedance unit is in Ohm

<table>
<thead>
<tr>
<th>Magic-T section</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Microstrip line</td>
<td>$Z_0=50$, $Z_1=70.7$, $Z_2=50$, $Z_3=49$</td>
<td>$Z_0=50$, $Z_1=58.3$, $Z_2=53.5$, $Z_3=51.5$</td>
</tr>
<tr>
<td>Slotline</td>
<td>$Z_{sl}=94.1$, $n=1$, $Z_{sl0}=94.6$, $Z_{sl1}=94.6$, $Z_{sl2}=348.4$, $\theta=17.3^\circ$, $\theta_0=8.6^\circ$, $\theta_1=12.7^\circ$, $\theta_2=24.9^\circ$</td>
<td></td>
</tr>
</tbody>
</table>

Figure 3-14  The magic-T frequency responses based on the circuit model in Figure 3-12(b), case 2 in Table 3-1.
3.3.3 Microstrip-to-Slotline transition using Stepped Impedance Circular Ring

The SCR termination is used in the magic-T as opposed to other types of slotline terminations since it is compact and generates low in-band insertion loss. Its input admittance can be modeled using transmission lines. Using the slotline parameter values in Table 3-1, the model shows good agreement with the EM simulation as shown in Figure 3-15.

Figure 3-15  The frequency response of the input admittance of the slotline SCR on the 0.102 mm-thick Roger’s LCP substrate using the slotline circuit model and the EM simulation. Its physical dimensions are shown in Table 3-2.

The complete magic-T circuit model that includes the MS-to-SL transition at port $E$ is shown in Figure 3-16. The microstrip line stepped impedance stub with the characteristic impedance of $Z_{t1}=40$ Ohm and $Z_{t2}=20$ Ohm and $\theta_{t1}=23.3^\circ$ and $\theta_{t2}=46.6^\circ$ is used to produce a virtual short at the MS-to-SL transition at Port $E$. The magic-T’s physical dimensions shown in Figure 3-17(a) are computed based on the parameters in Table 3-1 and they are shown in Table 3-2.
Figure 3-16  The full circuit model of the proposed magic-T.

Figure 3-17  The layout and dimensions of the proposed magic-T on the Roger’s LCP substrate.

Table 3-2  The magic-T’s physical dimensions in millimeters.

<table>
<thead>
<tr>
<th>Microstrip line sections</th>
<th>Slotline sections</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_0=0.24$, $W_1=0.19$, $W_2=0.22$, $W_t=0.14$,</td>
<td>$W_{sl}=0.10$, $W_{sl1}=0.10$,</td>
</tr>
<tr>
<td>$W_{sl}=0.34$, $W_{sl2}=0.84$, $L_1=4.76$, $L_2=4.73$,</td>
<td>$L_{sl}=0.38$, $L_{sl1}=1.04$, $L_{sl2}=4.17$</td>
</tr>
<tr>
<td>$L_3=4.73$, $L_{t1}=4.81$, $L_{t1}=1.25$, $L_{t2}=1.85$</td>
<td></td>
</tr>
</tbody>
</table>
Two slotline half circuit sections are used in the magic-T model to preserve the structure symmetry. The circuit model shows a very good agreement with the EM-simulation as shown in Figure 3-18. The E-H isolation is infinite in the circuit model. The 1-E insertion loss has higher loss than that in the 1-H due to additional radiation loss from the MS-to-SL transition at port E.

3.3.4 The Effect of Layout Asymmetry in Magic-T’s E-H Port Isolation

The isolation of the proposed magic-T is dependent on the phase and impedance mismatch between port 1 and port 2 and the parasitic couplings between ports E and H.

The phase and impedance mismatch is caused by the fabrication misalignment between microstrip and slotline, as well as asymmetric parasitic coupling from slotline to micro-strip line in port 1 and port 2 around the out-of-phase combiner. Moreover, the parasitic coupling is produced from the magic-T’s slotline section to microstrip line sections at port H.

Although small fabrication misalignment can not be avoided, the E-H port isolation can be improved by increasing the physical distance from port E to port H. From Figure 3-19, the isolation is increased by more than 26 dB in the low frequency side of $f_0$ as the slotline length $L_{sl}$ increases from 50 to 200 µm. However, small increase in isolation is observed at the frequency above $f_0$, since the parasitic coupling from port E and H has more significant effect than the asymmetric parasitic coupling to port 1 and port 2.
Figure 3-18  The frequency response of the magic-T using transmission model (dashed line) and using EM simulation (solid line).
Figure 3-19  The simulated port E-H isolation of the magic-T with variable slotline length ($L_{sl}$).

### 3.3.5 Hardware Implementation of the Proposed Broadband Magic-T

The magic-T is designed on a 0.25mm-thick Duriod 6010 substrate. The design is at 10 GHz and is based on the Duroid 6010 substrate as opposed to the LCP substrate mentioned earlier since the fabrication using the LCP substrate is not available. The circuit parameters and physical parameters of this magic-T are shown in Table 3-3 and Table 3-4, respectively. The minimum slotline width used in the design is 0.1mm which is equivalent to the $Z_{sl}$ of 72.8 Ohm. The $\lambda/4$-long line with an impedance value of $Z_l$ to used to transform $Z_{sl}$ to $Z_0$ at port E. The photograph of the magic-T is shown in Figure 3-20. The magic-T is calibrated using TRL method to de-embed parasitic at the connectors and lines connected to the magic-T. The calibration reference plane is shown in Figure 3-20(a). The TRL calibration standard on Duriod 6010 substrate used in this measurement is shown in Figure 3-21.
Table 3-3  The circuit parameters used in the magic-T design on 0.254 mm-thick Duroid 6010 substrate

<table>
<thead>
<tr>
<th>Microstrip line section</th>
<th>Slotline section</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Z_0 = 50 ), ( Z_1 = 57.52 ), ( Z_2 = 58.92 ), ( Z_3 = 47.693 ), ( Z_I = 57.26 )</td>
<td>( Z_s = 72.8 , \Omega ), ( Z_{sl1} = 72.8 , \Omega ), ( Z_{sl2} = 72.8 , \Omega ), ( \theta_{sl1} = 6.2^\circ ), ( Z_{sl2} )</td>
</tr>
<tr>
<td>( Z_s = 72.8 , \Omega ), ( Z_{sl1} = 72.8 , \Omega ), ( Z_{sl2} = 72.8 , \Omega ), ( \theta_{sl1} = 6.2^\circ ), ( Z_{sl2} )</td>
<td>( Z_{sl1} = 163.4 ), ( \theta_{sl1} = 34.95^\circ )</td>
</tr>
</tbody>
</table>

Table 3-4  The physical parameters in millimeter of the magic-T on 0.254 mm-thick Duroid 6010 substrate

<table>
<thead>
<tr>
<th>Microstrip line sections</th>
<th>Slotline sections</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W_0 = 0.238 ), ( W_1 = 0.175 ), ( W_2 = 0.165 ), ( W_I = 0.16 ), ( L_1 = 2.92 ), ( L_2 = 2.90 ), ( L_3 = 2.87 ), ( L_I = 2.79 ), ( L_{m1} = 0.68 ), ( W_{m1} = 0.37 ), ( L_{m2} = 1.30 ), ( W_{m2} = 1.05 )</td>
<td>( L_s = 1.02 ), ( W_s = 0.10 ), ( L_{sl} = 0.58 ), ( W_{sl0} = 0.10 ), ( L_{sl1} = 0.23 ), ( W_{sl1} = 0.1 ), ( L_{sl2} = 0.91 ), ( W_{sl2} = 0.71 )</td>
</tr>
</tbody>
</table>

Figure 3-20  The photographs show (a) the top and (b) the bottom view of the proposed magic-T.
The magic-T provides an average in-band insertion loss of 0.3 dB and 0.7 dB in the in-phase and the out-of-phase power combining sections, respectively as shown in Figure 3-22. The in-band frequency ranges from 6.6 GHz to 13.8 GHz, which is equivalent to 72% bandwidth. The out-of-phase power combining section has higher insertion loss than the in-phase combining section due to additional loss caused by slotline radiation and microstrip line loss. Moreover, the magic-T has both low amplitude and phase imbalance of less than 0.5 dB and 2 degree as shown in Figure 3-23 and Figure 3-24, respectively. The return loss is greater than 10 dB in the operating frequency bandwidth as shown in Figure 3-25 and Figure 3-26. The amplitude and phase balance of the magic-T are computed as follows

\[
\text{Out-of-phase amplitude balance} = |S_{1E}| - |S_{2E}| \quad (3-8a)
\]

\[
\text{In-phase amplitude balance} = |S_{1H}| - |S_{2H}| \quad (3-8b)
\]

\[
\text{Out-of-phase amplitude balance} = \angle S_{1E} - \angle S_{2E} \quad (3-9a)
\]

\[
\text{In-phase amplitude balance} = \angle S_{1H} - \angle S_{2H} \quad (3-9b)
\]
where $S_{1E}$ and $S_{2E}$ are forward transmission from port E to port 1 and from port E to port 2, respectively. $S_{1H}$ and $S_{2H}$ are forward transmission from port H to port 1 and from port E to port 2, respectively. The port 1-2 isolation is in good agreement with the simulated results in Figure 3-27, however the minimum port E-H isolation is 32 dB, which is 20 dB lower than predicted by the simulation due to infinite ground conductivity and area.

Figure 3-22 The magnitude of the in-phase and out-of-phase power dividing in dB of the magic-T. The referenced power dividing magnitude is 3 dB.
Figure 3-23 The measured frequency responses of the in-phase and out-of-phase phase balance of the magic-T.

Figure 3-24 The measured frequency responses of the amplitude balance of the in-phase and out-of-phase power diving sections of the magic-T.
Figure 3-25  The frequency response of the return loss of port E and port H of the magic-T.

Figure 3-26  The frequency response of the return loss of port 1 and port 2 of the magic-T.
3.4 A Compact Magic-T Design Using MS-to-SL Transitions

The magic-T design discussed in section 3.3 provides broadband response, low in-band insertion loss and high isolation simultaneously. However, the total microstrip line length required to construct the magic-T is more than $3/2\lambda$ long. To produce a more compact structure, the magic-T can be redesigned to reduce line length as shown in Figure 3-28.
From Figure 3-28, the magic-T consists of five $\lambda/4$ microstrip lines with the characteristic impedances of $Z_1$, $Z_2$ and $Z_i$. It also consists of a slotline length $L_s$ with the slotline characteristic impedance of $Z_s$. All ports are terminated with the microstrip lines with the characteristic impedance of $Z_0$. The slotline section is terminated with the slotline SCR termination at both ends to provide broadband and low-loss MS-to-SL transition and to allow out-of-phase combining to occur. $Z_i$ is used to transform slotline $Z_s$ to the microstrip line $Z_0$ at port E.

The design of the magic-T shown in Figure 3-28 uses similar concept to that discussed in section 3.3. The signals from port 1 and port 2 are combined in phase at the sum port and combined out-of-phase at the MS-to-SL transition along $A-B$ as shown in Figure 3-29(a) and (b), respectively.
3.4.1 Compact Magic-T’s Operation

In the odd mode, the signals from port 1 and port 2 are out-of-phase. This creates a microstrip virtual ground plane along the y-axis of the magic-T. The slotline SCR termination connected to the slotline $Z_{sl}$ also allows the MS-to-SL mode conversion to occur as demonstrated by electric-field and current directions around the A-B cross section as shown in Figure 3-29(a).

In the even mode, the signals from port 1 and port 2 are in-phase, thus creating a microstrip virtual open along the y-axis of the magic-T as shown in Figure 3-29(b). Electric-fields in the slotline at the A-B cross section are canceled thus creating a slotline virtual short that prevents the signal flow to or from port $E$.

3.4.2 Magic-T Port Impedance Matching

In order to match the impedance of all four ports of the magic-T. The magic-T is analyzed in odd-mode and even-mode circuits up to $Z_s$ as shown in Figure 3-30.
In the odd-mode, $\lambda/4$-line $Z_1$ is used to transform the input characteristic impedance $Z_0$ at port 1 to the desired value of $Z_0/2$. The value of $Z_1$ can be derived as follows:

$$Z_1 = \sqrt{n_i^2 \frac{Z_s}{2} \cdot Z_0} \quad (3-10)$$

where $n_i$ is the MS-to-SL transformer ratio. The $\lambda/4$-line $Z_2$ is used to transform the grounded-end to a virtual open at $Z_s$. It is desirable that $Z_2$ value is high relative to $Z_s$ to create a broadband virtual open. The practical value of $Z_2$ is set by the matching in the even-mode analysis.

In the even-mode, the input impedance $Z_0$ at port 1 is transformed to the in-phase port impedance of $2Z_0$. Since the line $Z_1$ is used to transform impedance $Z_0$ to $Z_0/2$ in odd-mode, the line $Z_2$ must be used to transform the impedance $Z_s$ to $2Z_0$. Therefore, $Z_2$ can be computed as follows:
\[ Z_2 = \sqrt[2]{2Z_0 \cdot n_t^2 \frac{Z_i}{2}} = \sqrt{Z_i}. \]  

(3-11)

The isolation and the return loss of port 1 and port 2 are derived in terms of \( \Gamma^{++} \) and \( \Gamma^{+-} \) as in (3-3) and (3-4), respectively.

The magic-T is designed on a 10-mil thick Duroid 6010 substrate with the dielectric constant of 10.2. The slotline is 0.1mm wide, which is the minimum width allowable in this fabrication process. This corresponding \( Z_s \) of 72.8 Ohm and all four ports impedances are 50 Ohm. From (3-10) and (3-11) and \( n_t = 1 \), we obtain \( Z_1 \) and \( Z_2 \) of 42.7 Ohm and 60.33 Ohm, respectively. Using this circuit model, the frequency response of the magic-T can be determined up to port E as shown in Figure 3-31.

![Image of frequency response graph](image-url)

**Figure 3-31** The frequency response of the magic-T using odd and even-mode half-circuit model.
The response shows that this magic-T provides broadband out-of-phase combining response than the in-phase combining response. The in-phase combining bandwidth is limited by two impedance transformation section in $Z_1$ and $Z_2$ used to transform $Z_0$ at port 1 to $2Z_0$ at port H in even mode. Moreover, the $Z_2$ value needs to satisfy the odd-mode match condition.

The slotline SCRs are used in this magic-T as terminations for the MS-to-SL transition. The slotline SCR in this design is slightly more compact than that used earlier by increasing the ratio $l_{s1}/l_{s2}$ from two to four. As a result, its loss is slightly reduced compared with the previously proposed magic-T as shown in Figure 3-32.

![Figure 3-32](image)

**Figure 3-32** The frequency response of the L1-port and the magnitude of the input impedance $|Z_{in}|$ of slotline SCR stubs with $l_{s1}/l_{s2}=2$ (solid line) and with $l_{s1}/l_{s2}=4$ (dashed line). Both of which have the same $W_{s0}$, $L_{s0}$, $W_{s1}$ and $W_{s2}$ values provided in Table 3-6

This slotline SCR can be modeled using transmission lines as shown in section 3.2. Its equivalent circuit parameters and its physical parameters are provided in Table
3-5 and Table 3-6, respectively. The circuit model shows a good agreement with that obtained from the method of moment simulation as shown in Figure 3-33.

<table>
<thead>
<tr>
<th>Table 3-5</th>
<th>The compact magic-T circuit design parameters at 10 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Microstrip line section</strong></td>
<td><strong>Slotline section</strong></td>
</tr>
<tr>
<td>$Z_1=42.7 , \Omega$, $Z_2=60.33 , \Omega$, $Z_n=40 , \Omega$</td>
<td>$Z_s=72.8 , \Omega$, $Z_{s0}=72.8 , \Omega$, $Z_{s2}=72.8 , \Omega$,</td>
</tr>
<tr>
<td>$\theta_1=23.3^\circ$, $\theta_2=46.6^\circ$, $Z_2=20 , \Omega$</td>
<td>$\theta_{s0}=13.57^\circ$, $\theta_{s2}=6.2^\circ$, $Z_{s1}=163.4 , \Omega$,</td>
</tr>
<tr>
<td></td>
<td>$\theta_{s1}=34.95^\circ$, $\theta_s=113.3^\circ$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 3-6</th>
<th>The physical parameters of the compact magic-T in millimeters.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Microstrip line section</strong></td>
<td><strong>Slotline section</strong></td>
</tr>
<tr>
<td>$L_1=2.62$, $W_1=0.26$, $L_2=1.83$, $W_2=0.14$, $L_3=2.80$, $W_3=0.16$, $L_4=0.68$, $W_4=0.37$, $L_5=1.30$, $W_5=1.05$</td>
<td>$L_s=1.92$, $W_s=0.10$, $L_{s0}=0.58$, $W_{s0}=0.10$, $L_{s1}=0.23$, $W_{s1}=0.10$, $L_{s2}=0.91$, $W_{s2}=0.71$</td>
</tr>
</tbody>
</table>

Figure 3-33 The input impedance of the slotline SCR in the compact magic-T using the parameters provided in Table 3-5.
3.4.3 Hardware Implementation and Experimental Results

The design is based on the circuit model of the magic-T and is shown in Figure 3-34. The magic-T is fabricated on a 0.25mm-thick Duroid 6010 substrate. The physical layout and dimensions are shown in Figure 3-35(a) and Table 3-6, respectively.

![Figure 3-34 The equivalent circuit model of the compact magic-T.](image)

The microstrip line stepped impedance stub is used to provide a virtual ground at the MS-to-SL transition at port E. Since the lines $L_1$ and $L_2$ are close to the slotline and the slotline SCR has less electric-field to the ground plane and it becomes more inductive. Their electrical lengths are slightly shorter than the conventional microstrip line with the same length. Therefore $L_1$ and $L_2$ are adjusted to be slightly longer than a $\lambda/4$ line to compensate for this effect.

The photograph of the top and the bottom side of the compact magic-T is shown in Figure 3-36(b) and (c), respectively. Each port is connected to a 2.4mm end-launch connector. These connections are de-embedded using the TRL calibration. The measurement is performed from 5 GHz to 20 GHz. The measurement results shown in Figure 3-36, Figure 3-37, Figure 3-38 and Figure 3-39 are in a good agreement with the simulation.
Figure 3-35  (a) The physical layout, the photograph of (b) the top and (c) the bottom view of the compact magic-T on 0.254 mm-thick Duroid 6010 substrate.

The phase imbalance of the in-phase and out-of-phase combining section is less than 1° and 1.6°, respectively as shown in Figure 3-40. The amplitude imbalance of the magic-T is less than 0.3 dB as shown in Figure 3-41. The magic-T provides the minimum isolation of 31 dB in the pass band as shown in Figure 3-39. Its isolation is comparable to that of the magic-T in section 3.3 since the minimum isolation is limited by the accuracy of the measurement and the non-ideal finite ground plane. The slotline SCR is close to the microstrip line section such that it produces parasitic that allows some signal from port E to transmit directly to port H without goes through the MS-to-SL transitions.
Figure 3-36  The frequency response of the in-phase and the out-of-phase power dividing of the compact magic-T.

Figure 3-37  The frequency response of the return loss at port E and port E of the compact magic-T.
Figure 3-38  The frequency response of the return loss at port 1 and port 2 of the compact magic-T.

Figure 3-39  The frequency response of measured (solid line) and simulated (dashed line) of port 1-2 and port E-H isolation of the compact magic-T.
Figure 3-40 The frequency response of the in-phase and out-of-phase phase balances in degree of the compact magic-T.

Figure 3-41 The frequency response of the in-phase and out-of-phase amplitude balances in dB of the compact magic-T.
The performance of the both broadband magic-T and compact magic-T can be compared in the prior state-of-the-art planar magic-T designs as shown in Table 3-7. Both broadband magic-T and compact magic-T have the lowest in-band insertion loss and the highest E-H port isolation in the operating band compared with prior designs. Moreover, the proposed magic-Ts have the lowest smallest phase imbalance in the operating bandwidth with return loss of more than 12 dB.

<table>
<thead>
<tr>
<th>Magic-T designs</th>
<th>Center frequency (GHz)</th>
<th>Insertion loss $S_{1E}, S_{2E}$ (dB)</th>
<th>Phase balance $S_{1E}, S_{2E}$</th>
<th>Isolation Port 1-2</th>
<th>Isolation E-H</th>
</tr>
</thead>
<tbody>
<tr>
<td>Broadband Magic-T</td>
<td>10</td>
<td>0.5</td>
<td>$&lt; \pm 1^\circ$</td>
<td>$&gt;15$</td>
<td>$&gt;30$</td>
</tr>
<tr>
<td>Compact Magic-T</td>
<td>10</td>
<td>0.3</td>
<td>$&lt; \pm 1.6^\circ$</td>
<td>$&gt;20$</td>
<td>$&gt;30$</td>
</tr>
<tr>
<td>(Hiraoka, Tokumitsu et al. 1989)</td>
<td>10</td>
<td>0.9</td>
<td>n/a</td>
<td>$&gt;14$</td>
<td>$&gt;27$</td>
</tr>
<tr>
<td>(Ho, Fan et al. 1994)</td>
<td>3</td>
<td>1.2</td>
<td>$&lt; \pm 1.5^\circ$</td>
<td>$&gt;12$</td>
<td>$&gt;35$</td>
</tr>
<tr>
<td>(Wang 1999)</td>
<td>2</td>
<td>0.8</td>
<td>$&lt; \pm 5^\circ$</td>
<td>$&gt;20$</td>
<td>$&gt;20$</td>
</tr>
<tr>
<td>(Fan, Heimer et al. 1997)</td>
<td>3</td>
<td>0.9</td>
<td>$&lt; \pm 1.5^\circ$</td>
<td>$&gt;16$</td>
<td>$&gt;28$</td>
</tr>
<tr>
<td>(Aikawa and Ogawa 1980)</td>
<td>6</td>
<td>0.9</td>
<td>$&lt; \pm 2.1^\circ$</td>
<td>$&gt;12$</td>
<td>$&gt;30$</td>
</tr>
<tr>
<td>(Kim and Park 2002)</td>
<td>2</td>
<td>1</td>
<td>$&lt; \pm 2^\circ$</td>
<td>$&gt;23$</td>
<td>$&gt;30$</td>
</tr>
</tbody>
</table>
CHAPTER 4
CONCLUSIONS

The dissertation presents the development of bandpass filters with high out-of-band performance and loss-low magic-Ts. The conclusion consists of two sections: the bandpass filter design and the magic-T design as follows.

In the bandpass filter research, developed are the techniques to produce the compact bandpass filter with very high out-of-band attenuation and bandwidth with minimal loss in the operating band.

New microstrip filter design techniques have been introduced. These techniques are implemented in the filters with coupled resonators. They simplify and provide an analytical guidance for the filter design with high out-of-band suppression. The double split-end structure relaxes the coupling requirement between filter stages while providing additional transmission zeros out-of-band. The $Q_{si}$ of the minimum-size SIR is analytically derived for the first time. The proposed techniques allow at least $N+1$ transmission zeros to exist in an $N^{th}$ order filter design below the third lowest spurious resonance frequency. Using these techniques, the filter can simultaneously produces low in-band loss and wide stop-band bandwidth.

The SIR with the built-in internal coupler has been developed. It reduces the number of $\lambda/2$ SIRs required by the filter design. This SIR increases the filter's lowest spurious resonance frequency to a higher frequency than the maximum limit of the conventional $\lambda/2$ SIR. Only one metal patterned layer is required to construct the filter using these SIRs.

The broadband bandstop filter has been introduced and integrated with the SIR bandpass filter for the first time. Transmission poles and zeros of the bandstop filter
have been derived and properly allocated to suppress attenuation out-of-band and minimize bandstop filter effect on the filter passband response. The response of this filter shows a significant improvement in the out-of-band response.

In the magic-T research, several techniques are developed such that the magic-T provides low loss and broadband responses. To design the magic-Ts, the minimum size microstrip-to-slotline transition that has high layout symmetry between port 1 and port 2 is used. The magic-T using these techniques provides high E-H port isolation and has low in-band insertion loss and has broadband responses. Moreover, it is simple to fabricate since it requires only two metallized layers and requires no via holes.

Slotline stepped circular rings have been introduced and used in microstrip-to-slotline transitions. The transitions using this technique provide low in-band insertion loss and broadband response by eliminating gradual radiation loss close to in-band. Moreover, the structure is more compact than the conventional slotline radial pad or circular pad. The magic-T using the slotline SCR has an improved in-band response over other prior known state-of-the-art planar magic-Ts.
CHAPTER 5
RECOMMENDATIONS

This dissertation introduces techniques used in filter and magic-T designs that improve their performance over the existing state-of-the-art designs. The filter designed using the techniques described in this dissertation is recommended for use in the applications that require low in-band loss and very high out-of-band attenuation such as in radio-astronomy. It can also be used in military systems where strong out-of-band interference is presence. On the other hand, the magic-T designed using the techniques described in this dissertation is recommended for use in the frequency multiplier systems where the input and output signals must be isolated from local oscillators. In addition, it can be used in microwave polarimeters.

As recommended future works, there are two possible ways to improve the bandpass filters. Firstly, in the design perspective, the elliptic function can be used in the filter’s passband response as opposed to conventional Butterworth or Chebychev functions. Moreover, the bandpass filters can be constructed from a broadband bandstop filter combined with a high-pass filter. The proper transmission pole locations of both bandstop and high-pass filter must be determined such that a proper bandpass filter response is achieved. The second improvement can be in the measurement and fabrication since high noise level was observed when measured at 4.3K. The cryogenic measurement can be improved by reducing the probe temperature and the vibration from the vacuum pump line. To provide a reliable contact between the probe and the substrate, thin layer of gold deposition can be added on the top of the Nb layer where the measurement probes land on the sample. This protects the Nb line from oxidation and guarantees a direct connection contact.
The recommended future development of the magic-T includes the improvement in input isolation. In addition, possible is extending the techniques described in this dissertation to be used in the quadrature hybrid to provide high input and output isolation.
APPENDIX A: SUPERCONDUCTING MICROSTRIP LINE MODELING

The close form solution of the superconducting microstrip line characteristic impedance and phase constant derived by G. Yassin and S. Withington are as follows.

By defining

\[ p = 2b x^2 - 1 + 2b \sqrt{b x^2 - 1} \]  \hspace{1cm} (A1) \\
\[ b = 1 + t/h \] \hspace{1cm} (A2)

Where \( t \) is the thickness of the film and \( h \) is the thickness of the dielectric.

\( ra \) is given by

\[
\ln(ra) = -1 - \frac{\pi w}{2h} - \frac{p + 1}{p^{1/2}} \tanh^{-1}\left(p^{-1/2}\right) - \ln\left(\frac{p-1}{4p}\right) 
\]  \hspace{1cm} (A3)

Where \( w \) is the microstrip line width. \( rb \) is given by

\[ rb = rbo \] \hspace{1cm} (A4)

For \( w/h \geq 5 \) and

\[
rb = rbo - \left[(rbo - 1)(rbo - p)\right]^{1/2} + (p + 1)\tanh^{-1}\left(\frac{rbo - p}{rbo - 1}\right)^{1/2} \\
- 2 p^{1/2} \tanh^{-1}\left(\frac{rbo - p}{p(rbo - 1)}\right)^{1/2} + \frac{\pi w}{2h} p^{1/2} 
\]  \hspace{1cm} (A5)

Otherwise, where

\[
rb = \eta + \frac{p + 1}{2} \ln \Delta
\] \hspace{1cm} (A6)

\[
\eta = p^{1/2} \left\{ \frac{\pi w}{2h} + \frac{p + 1}{2 p^{1/2}} \left[ 1 + \ln\left(\frac{4}{p - 1}\right)\right] - 2 \tanh^{-1} p^{-1/2} \right\}.
\] \hspace{1cm} (A7)

\( \Delta \) equals to whichever is the largest of \( \eta \) and \( p \).

The penetration factor (\( \chi \)) is defined as follows
\[ \mathcal{X} = \begin{cases} \frac{Is_1 + Is_2 + Ig_1 + Ig_2 + \pi}{2\ln(rb/ra)} & \text{w/h < 2} \\ \frac{Is_1 + Is_2 + Ig_1 + Ig_2 + \pi}{2\ln[2rb/ra]} & \text{otherwise.} \end{cases} \] (A8)

Where, for the bottom surface of the strip, we get

\[ Is_1 = \ln\left(\frac{2p - (p+1)ra + 2(pRa)^{1/2}}{ra(p-1)}\right) \] (A9)

\[ Ra = (1-ra)(p-ra) \] (A10)

For the top surface of the strip we get

\[ Is_2 = -\ln\left(\frac{-2p + (p+1)rb - 2(pRb)^{1/2}}{rb(p-1)}\right) \] (A11)

\[ Rb = (rb-1)(rb - p) \] (A12)

And for the ground plane we have

\[ Ig_1 = -\ln\left(\frac{2p + (p+1)rb + 2(pRb')^{1/2}}{rb(p-1)}\right) \] (A13)

\[ Rb' = (rb+1)(rb + p) \] (A14)

\[ Ig_2 = \ln\left(\frac{2p + (p+1)ra + 2(pRa')^{1/2}}{ra(p-1)}\right) \] (A15)

\[ Ra' = (ra + 1)(ra + p) \] (A16)

The fringing factor is defined as follows

\[ K_f = \frac{h}{w} \frac{2}{\pi} \ln\left(\frac{2rb}{ra}\right). \] (A17)

The geometrical factor \( g_1 \) is defined as follows

\[ g_1 = \frac{h}{wK_f}. \] (A18)

The propagation constant \( \beta \) and characteristic impedance \( \eta \) can be found as follows
\[
\left( \frac{\beta}{\eta} \right) = \left( \frac{\beta_m}{\eta_m} \right) \left( 1 + 2 \chi \frac{\lambda}{h} \right)^{1/2}.
\]  \hspace{1cm} (A19)

Where

\[
\beta_m = k_0 \sqrt{\varepsilon_{\text{reff}}}
\]  \hspace{1cm} (A20)

\[
\eta_m = \frac{n_0 \varepsilon_{\text{reff}}^{1/2}}{\varepsilon_{\text{reff}}^{1/2}}.
\]  \hspace{1cm} (A21)

The effective dielectric constant of the microstrip line is defined as follows

\[
\varepsilon_{\text{reff}} = \varepsilon_r + 1 + \varepsilon_r - 1 - \frac{1}{2} \frac{1}{\sqrt{1 + 12 \frac{h}{w}}}
\]  \hspace{1cm} (A22)

Close form solution of the characteristic impedance of the loss-less microstrip line as in (Pozar 1997) are as follows

\[
Z_0 = \begin{cases} 
\frac{60}{\sqrt{\varepsilon_{\text{reff}}}} \ln \left( \frac{8h}{w} + \frac{w}{4h} \right) & \text{for } w/h < 2 \\
\frac{120 \pi}{\sqrt{\varepsilon_{\text{reff}}}} & \text{for } w/h > 2.
\end{cases}
\]  \hspace{1cm} (A23)
REFERENCES


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