CHARACTERIZATION AND ANALYSIS OF DAMAGE PROGRESSION IN NON-TRADITIONAL COMPOSITE LAMINATES WITH CIRCULAR HOLES

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DAMAGE PROGRESSION, FAILURE, AND ANALYSIS OF NON-TRADITIONAL COMPOSITE LAMINATES

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<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>a</td>
<td>Crack/Notch half length; WEK critical damage length parameter; Bi-linear Cohesive Zone breakpoint</td>
</tr>
<tr>
<td>a₀</td>
<td>WNAS critical damage length parameter</td>
</tr>
<tr>
<td>b</td>
<td>Bi-linear Cohesive Zone breakpoint</td>
</tr>
<tr>
<td>c</td>
<td>PWG notch sensitivity factor</td>
</tr>
<tr>
<td>d₀</td>
<td>WNPS, Karlak critical damage length parameter</td>
</tr>
<tr>
<td>CZM</td>
<td>Cohesive Zone Model</td>
</tr>
<tr>
<td>E</td>
<td>Modulus of Elasticity</td>
</tr>
<tr>
<td>FHT</td>
<td>Filled Hole Tension</td>
</tr>
<tr>
<td>G</td>
<td>Modulus of Rigidity</td>
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<td>Gₐ</td>
<td>Fracture Energy</td>
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<td>Gᵢ</td>
<td>Mode I energy release rate</td>
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<td>Hₜ</td>
<td>Composite Fracture Toughness for Mar-Lin criteria</td>
</tr>
<tr>
<td>Kᵢ</td>
<td>Mode I stress intensity factor</td>
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<td>KᵢC</td>
<td>Mode I critical stress intensity factor</td>
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<tr>
<td>k₀</td>
<td>Karlak curve fitting parameter</td>
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<tr>
<td>Kᵢ∞</td>
<td>Stress Concentration Factor for an infinite plate</td>
</tr>
<tr>
<td>L</td>
<td>Length</td>
</tr>
<tr>
<td>LEFM</td>
<td>Linear Elastic Fracture Mechanics</td>
</tr>
<tr>
<td>m</td>
<td>PWG experimental parameter relating characteristic damage length to hole radius</td>
</tr>
<tr>
<td>n</td>
<td>Exponent of singularity at crack tip for Mar-Lin</td>
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OHC  Open Hole Compression
OHT  Open Hole Tension
P    Load
PWG Pipes, Wetherhold, and Gillespie failure theory
R, r Notch/crack radius
R₀   PWG reference notch radius
S    Applied remote stress
SSB  Single shear bearing
v    Crack opening
vₖ   Critical crack opening
Vₖ   Fiber Volume Fraction
W    Width
WEK  Waddoups, Eisenmann, and Kaminski failure theory
WN   Whitney-Nuismer
WNAS Whitney-Nuismer Average Stress failure theory
WNPS Whitney-Nuismer Point Stress failure theory
x    Coordinate measured from center of notch perpendicular to direction of applied load
Xₐ   Ultimate Compressive Strength
Xₜ   Ultimate Tensile Strength
ν    Poisson’s ratio
ζ₁   R/(R+ d₀) in WNPS failure theory
ζ₂   R/(R+ a₀) in WNAS failure theory
$\sigma$  Applied Axial Stress

$\sigma_N$  Notched tensile strength of a finite width plate

$\sigma_0$  Unnotched tensile strength

$\sigma_y$  Local stress component in the y-direction
SUMMARY

Carbon fiber / epoxy laminates are increasingly being used in the primary structure of aircraft. To make effective use of these materials, it is necessary to consider the ability of a laminate to resist damage, as well as material strength and stiffness. A possible means for improving damage tolerance is the use of non-traditional composite laminates, in which the longitudinal 0° plies are replaced with ±5° or ±10° plies.

The main objectives of this collaborative Georgia Tech / Boeing research was the characterization of these non-traditional laminates, and the determination of appropriate lamina-level analytical techniques that are capable of predicting the changes caused by the use of slightly off-axis longitudinal plies. A quasi-isotropic $[45/90/-45/0/45/90/-45/\theta]_s$ and “hard” $[45/\pm \theta/45/\pm \theta/90/\pm (45)_{1/2}]_s$ lay-up, with $\theta = 0°, 5°$ or $10°$, were tested in unnotched tension, open hole tension, filled hole tension, open hole compression, and single shear bearing. These coupon level tests illustrated the effects of lay-up, notch constraint, and load type on traditional and non-traditional laminates. By utilizing die penetrant enhanced in-situ radiography to track damage progression, it was determined that use of non-traditional laminates reduced longitudinal ply cracking and delamination. This causes significant changes in stress distribution around the notch, and thus alters laminate performance. The use of non-traditional laminates also resulted in a 15%-20% improvement in bearing strength over the traditional laminates.

Several predictive techniques were implemented to evaluate their ability to predict the effect of slight changes in ply orientations. A progressive damage model was written to compare Tsai-Wu, Hashin, and Maximum Stress unnotched strength criterion. Additionally, several semi-empirical failure theories for notched strength prediction were implemented.
compared with linear and bi-linear cohesive zone models to determine applicability to non-traditional laminates.
CHAPTER 1
INTRODUCTION

Next-generation airliners promise to bring increased performance, efficiency, and durability to the commercial transportation industry. To attain these goals, airplane manufacturers will make increased use of advanced composites due to the superior characteristics these materials offer. The increased usage of composites, notably in the primary structure of the aircraft, has raised interest in the optimization of composite lay-ups. Like in metals, where tradeoffs must be made between strength, stiffness, and toughness, composites must be optimized to provide not only the high strength and stiffness they are traditionally used for, but also for robustness in damage suppression.

One possibility for improved damage suppression is the use of non-traditional composite laminates. These laminates would replace the strong, stiff $0^\circ$ plies of continuous fiber graphite / epoxy composite laminates with $\pm 5^\circ$ or $\pm 10^\circ$ plies, orientations that would be slightly less strong and stiff, but have the potential for increased damage tolerance.

This research seeks to characterize these non-traditional composite laminates, and compare their mechanical properties with that of traditional laminates. Open hole tension, filled hole tension, open hole compression, bolt bearing response, and unnotched tests have been utilized by industry for material characterization and to develop design parameters for use in the design of structures, so these coupon-level tests will be performed to evaluate these laminates. These tests can serve to screen material systems and characterize the physical and mechanical properties of a composite laminate.
Additionally, damage development will be monitored with in-situ X-ray radiography to determine if damage suppression occurs. The experimental results will be analyzed with several predictive techniques, with a focus on simpler methods that can be implemented at the design level.

This thesis is divided into six subsequent chapters. Chapter 2 will provide information on the experimental methods used to characterize composite materials, and give background into the analytical techniques used in predicting composite behavior. Chapter 3 will deal with experimental techniques and equipment used in this research. Materials, lay-ups, and specimen geometries will be described. The experimental procedures used for the open hole tension, filled hole tension, open hole compression, filled hole compression, single shear bearing, and unnotched tension tests will be given. Additionally, x-ray techniques will be described. Chapter 4 will present results from the experiments, including mechanical properties and x-ray images. Chapter 5 will compare the experimental findings with various analytical approaches. Significant findings will be summarized in chapter 6, along with conclusions that can be drawn. Chapter 7 will list recommendations for future experimental and analytical research on traditional and non-traditional laminates.
CHAPTER 2
BACKGROUND

Advanced composite materials have been used since the 1960s, and since then a great deal of experimental and analytical work has been performed. Early attempts at experimental characterization and analytical predictions for composites shared much in common with traditional metallic materials, but since then the need for different methodologies has become apparent. Testing and analysis of composites now constitutes a distinct field. The following is a review of relevant previous research, and a discussion of factors unique to composites with regard to testing and analysis.

2.1. EXPERIMENTAL BACKGROUND

Experimental testing is key to the implementation of composites in structures. Current predictive techniques are either inadequate for describing the mechanical behavior of a composite, or difficult to implement at a design level due to complexity. As such, industry relies extensively on the testing of composites at the constituent, lamina, laminate, coupon, and structural levels. This reliance necessitates robust methods of experimentation to generate accurate and consistent data. The ultimate goal of these tests is to make efficient use of time and material, while generating a sufficient amount of meaningful data to enable an understanding of the material.

Engineering structures may be subject to tensile and compressive loads, include stress concentrations, and utilize fastener and joints. Accordingly, a variety of test types are necessary to fully characterize a composite material. For example, the recommended
test matrix for notched laminate characterization given in the Composite Material
Handbook, or Mil-hdbk-17 [1], is shown in Figure 2.1. Developing the data and design
allowables necessary for creating a full structure requires even further testing.

<table>
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<th>Diameter in. (mm)</th>
<th>Width in. (mm)</th>
<th>W/D Ratio</th>
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<th>RTD Tension</th>
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<tr>
<td>(10/80/10)</td>
<td>45/45/45/45/45/45/45/45/45</td>
<td>CTD Cold Temperature Dry</td>
</tr>
<tr>
<td>(50/40/10)</td>
<td>45/45/45/45/45/45/45/45/45</td>
<td>ETW Elevated Temperature Wet</td>
</tr>
<tr>
<td>(40/20/40)</td>
<td>45/45/45/45/45/45/45/45/45</td>
<td>See Section 2.2.7</td>
</tr>
</tbody>
</table>

*See Section 2.2.7

Figure 2.1 Mil-Hdbk-17 Suggested Notch Strength Test Matrix

Open hole tension, filled hole tension, open hole compression, and bearing
response test are frequently used for comparing or screening different material systems
and lay-ups because of their overall simplicity and substantive results. They provide
basic mechanical properties, and insight into damage and failure modes a material or
particular lay-up may be susceptible to. These tests are also used to determine critical
conditions for strain limits in design. Because this program seeks to quantify the
advantages and disadvantages of slight differences in lay-up, these tests were chosen for
use in this study.
The following sections give a description of each type of test, along with a discussion of pertinent failure modes and test parameters. Composite materials can be very sensitive to test methodology, and there are ASTM, SACMA, and industry standards for determining the mechanical properties of composite laminates. Different methods of the same basic test can yield different results for properties, so a thorough understanding of the variables involved is necessary before undertaking an experimental study.

2.1.1. Notched Tension Testing

An open hole in a composite laminate significantly reduces the strength of the structure because of stress concentrations around the notch [2]. The use of a fastener can cause further changes in stress distribution and failure mechanisms, in many cases result in further reductions in strength beyond that of the open hole case [3]. The stress distribution around the hole and the resulting damage progression and failure of composite laminates is of obvious interest to engineers and designers. Numerous experimental studies have attempted to explain and quantify the response of notched composite specimens to tensile loading, and the resulting open and filled hole tensile strength of a composite laminate is often used as a material specification or design value.

2.1.1.1 Damage Mechanisms In Notched Tension

Understanding failure in composites is complicated by the variety of damage mechanisms that can occur in a laminate under loading. When a stress is applied to a composite, different types of damage can occur that may lead to failure of the laminate or simply cause local redistribution of stresses. The tensile loading of a laminate can lead to matrix cracking, fiber-matrix splitting, delamination, and fiber breakage[4]. When and
where damage occurs varies by material system, lay-up and stacking sequence, and loading.

Generally, matrix cracking is the earliest types of damage to occur. In a laminate with 90°, ±45°, and 0° plies, matrix cracking typically occurs first in the 90° plies, followed by damage in the ±45° plies. In many cases, matrix cracking does not cause significant changes in laminate properties in the fiber direction, as most of the load is carried in the fibers. However, matrix cracking can play a significant role in properties heavily influenced by the matrix, such as compression strength or transverse modulus. Additionally, delamination can result from the local strain concentrations caused by matrix cracking, and this can lead to reductions in strength and stiffness. Matrix cracking can also negatively affect areas properties such as gas and liquid permeability.

Fiber-matrix splitting or longitudinal ply cracking is cracking that occurs around the hole in the 0° plies, and is caused by shear stress concentrations at the notch resulting from the proximity of continuous load-bearing fibers to discontinuous non-load-bearing fibers. This causes notch tip blunting and local stress redistribution. This blunting reduces the stress concentration around the hole and can improve tensile strength of composite laminates. However, splitting can result in the significant reduction of properties in other loading directions.

Delaminations form at the interface between plies as the result of interlaminar stresses. These stresses transfer load between plies and in some cases result in out-of-plane stresses that attempt to peel the plies apart. These stresses are generally attributed to lay-up and laminate stacking sequence, and are often the result of stacking 90° and 0° plies together or from having 90° plies at the laminate centerline. Delamination can
occur at notches, but can also occur at the edges of a composite laminate, with the “free edge effect” [5]. When delamination occurs, the specimen may simply behave as multiple laminates. If this is the case, all plies will still carry load, and failure will be dictated by the load-bearing plies [6]. However, delaminations can result in strain concentrations, in which case failure can occur at levels lower than expected in the load bearing plies.

Advanced composites derive their superior strength and stiffness from the fiber used, and as such fiber breakage precedes most catastrophic failures. When individual fibers are stressed beyond their load carrying ability, the fiber fractures, and the load is redistributed. This redistribution may cause additional fibers to fracture, or may slow or stop damage progression.

2.1.1.2 Variables Related to Notched Tensile Testing

Unlike a traditional monolithic material, laminated composites allow the designer to choose a number of variables that will determine the performance of the material in the structure. For example, the lay-up and percentages of 0° plies, 45° plies, and 90° plies obviously affects the strength and stiffness of the laminate. Furthermore, for a given lay-up of different angle plies, how the plies are arranged – the laminate stacking sequence – can also have a significant effect on performance. When fasteners are involved factors such as bolt torques, bolt fit, and washer size all play roles in laminate strength, and all are choices that must be made or variables that should be identified. These factors combine to dictate what damage modes occur when and ultimately determine the failure of the laminate.
2.1.1.2.1 Lay-up

The lay-up is the primary factor in determining the strength and stiffness of a laminated composite. A typical laminate may contain 0° plies for longitudinal strength and stiffness, 45° plies for rotational or bending stiffness, and 90° plies for transverse strength and stiffness, and by changing the percentages of each ply, properties in each direction can be increased or decreased as needed. Damage type and progression also change with the laminate lay-up.

In general, notch effects are more severe in laminates with higher percentages of 0° plies. The stress concentration factor is known to increase as stiffness increases, and can be calculated with the equation [7]

\[
K_T^\infty = 1 + \left[ \left( \frac{E_{11}}{G_{12}} \right)^{1/2} - 2\nu_{12} + 2 \left( \frac{E_{11}}{E_{22}} \right)^{1/2} \right]^{1/2}
\]  

(2.1)

Etheridge et al [8] studied the effect of lay-up and constraint on carbon fiber / bismaleimide matrix composites. Soft, medium, and hard lay-ups were tested, with the terms “soft”, “medium”, and “hard” referring to the increasing percentage of 0° plies, and thus the relative strength and stiffness of the laminate. Additionally, a comparison was made between open hole and filled hole specimens; a more detailed discussion of the effects of a filled hole will be made in sections 2.1.1.2.3 and 2.1.1.2.4. When the soft [(±45)/90/-45/0/(±45)/45]s lay-up was loaded in tension, damage initiated in the form of cracking in the 90° degree plies and to a lesser extent in the 45° plies, as shown in Figure 2.2.
The medium \([45/90/-45/0/\])_{3s}\) quasi-isotropic lay-up experienced damage similar to the soft lay-up, namely cracking in the 90° degree plies and 45° plies. Cracking in the 45° plies is more pronounced in the medium lay-up than in the soft lay-up. Additionally, the presence of fiber-matrix splitting in the 0° degree direction is obvious. This is shown in Figure 2.3.

The hard \([45/0/\ -45/90/45/0/\ -45/0/\])_{3s}\) lay-up experienced severe splitting in the 0° plies, shown in Figure 2.4. This type of damage is typical of harder lay-ups. Hard lay-
ups have higher stress concentrations at the notch, and fiber matrix splitting redistributes and reduces stresses around the hole. However, it essentially creates two separate laminates, with very little stress occurring in the portions of the coupon above and below the hole. As such, failure stresses may be higher than expected due to elimination of the stress concentration.

Figure 2.4 Radiograph of damage in [45/0, -45/90/45/0, -45/0, -45]s, "hard" laminate [8]

2.1.1.2.2 Stacking Sequence

Laminate stacking sequence plays a significant role in determining the strength of a composite under tensile loading. As mentioned earlier, high interlaminar stresses and subsequent delamination result from stacking sequence choice. This is illustrated in Figure 2.5 [9].
For laminates in the delamination sub-group, unnotched tensile strengths were much lower than those laminates that did not experience delamination. Work has been performed (e.g. [11]) that indicates notched tensile strength demonstrates less sensitivity to stacking sequence, as failure occurs primarily due to stress concentrations around the notch. High interlaminar stresses can to some extent be controlled by measures such as avoiding 90° plies on the centerline and minimizing Poisson ratio mismatches [10].

2.1.1.2.3 Clamp-Up Force

When a torque is applied to a fastener, a stress is transferred from the nut (or washer) to the laminate. This results in what is called the clamp-up force. Clamp-up force has been shown to affect tensile strength in some, but not all, laminates. Etheridge [8] utilized open hole specimens, filled hole specimens, and filled hole specimens with a clamp-up force, and found that the increasing notch constraint caused by the bolt with a clamp-up force played a significant role in decreasing the tensile strength of hard lay-ups,
as shown in Figure 2.6. Clamp-up force caused a less dramatic effect for medium quasi-isotropic laminates, and had no effect on soft lay-ups.

Yan et al [4] found similar results, with increasing clamp-up force having a significant effect on strength reduction in hard lay-ups. Hard lay-ups are more prone to fiber-matrix splitting, which causes localized softening and stress redistribution in the laminate, delaying failure. With a clamp-up force applied, the fiber-matrix splitting was suppressed, thus leading to earlier failure. This change in damage modes is shown in Figure 2.7, and the resulting change strength shown in Figure 2.8.
Little clamp-up effect is observed in soft lay-ups, since only slight damage suppression occurred, and fiber breakage and matrix cracking were not noticeably affected by clamping force.
2.1.1.2.4 Washer Size

Like clamp-up force, washer size can also have a significant effect on filled hole tensile strength. The size of the washer dictates the area of the constraint imposed by the fastener, and the distribution of the clamp-up force. A smaller washer causes a higher amount of constraint around the notch, lessening stress redistribution and causing early failure. The effects of washer size are shown in Figure 2.9. Yan et al [4] found that the greatest strength reduction was caused by a washer diameter 2 times the hole diameter; increasing washer diameter improved tensile strength performance.

![Figure 2.9 Effect of Washer Size on T800/3900-2 Laminate [4]](image)

2.1.1.3 Discussion

As discussed above, numerous factors influence open and filled hole tensile strength. Differing lay-ups and stacking sequences, clamp-up force, and washer size all play roles in the ultimate strength of the composite. Other factors including thickness, size effects, bolt type and fit, and environmental conditioning result in additional changes to laminate strength and stiffness. ASTM D3039 and D6742 outline basic procedures for
performing open and filled hole tensile testing, but the testing techniques used depend on the information desired and the application of the material.

2.1.2. Open Hole Compression

Composites are typically weaker in compression than in tension, with compressive strengths often 30%-40% lower than the tensile strength [12]. A discontinuity or notch can further reduce the compressive strength by as much as 50%. As such, the open hole compressive strength of composites is often used as a material specification. Thus, an understanding of the compressive response of composites is key to the use of composites in structures.

2.1.2.1 Open Hole Compression Damage Modes

When a composite laminate is loaded in compression, numerous damage types can develop. Like tensile loading, compressive loading can result in delamination and fiber-matrix splitting. Additionally macroscale buckling, or buckling of the entire laminate, can lead to matrix cracking and fiber breakage in outer plies. However, this sort of damage is very dependent on the specimen geometry, indicating it is not a material property. It has been found that microbuckling is the primary failure mechanism of composites laminates loaded in compression.

Much of the early work in the field of fiber microbuckling was performed by Rosen [13]. He proposed two different types of fiber microbuckling, an extension mode with out-of-phase fiber buckling, shown in Figure 2.10, and a shear mode with in-phase fiber buckling, shown in Figure 2.11.
When compressed fibers act independently of each other, the result is extension mode fiber buckling. This is only the case in fibrous composites with a fiber volume fraction less than 5%. Since most modern composites have a fiber volume fraction much higher than that, extension mode microbuckling is rarely observed. Far more common is shear mode fiber buckling, in which fibers interact with each other and form in-phase fiber buckling. This is the case typically encountered in modern continuous fiber composites loaded in compression.

If loading continues to increase after microbuckling occurs, kink bands can form [15]. The laminate compresses until the carbon fibers fracture due to the strain caused by the microbuckling. This progression from microbuckle to kink band is shown in Figure 2.12.
It is theorized that the kink band takes this shape in order to permit both compressive and shear stress [16]. The width $W$ of the kink band in Figure 2.12 is typically on the order of 10-14 fiber diameters for carbon fiber composites, with the individual fibers rotated at an angle $\phi = 30^\circ$ from the loading direction, and the kink band growing at an incline of $\beta = 15^\circ - 30^\circ$ [17].

In a notched composite, kink bands will form at the notch tip in the $0^\circ$ plies, with fibers initially buckling into the open notch. The kink bands then grow in a manner similar to a crack, in some cases growing several millimeters before failure. As the crack grows, delamination occurs in the area around it.

2.1.2.2 Variables Related to Open Hole Compression

As in notched tension testing, there are a number of factors that contribute to open hole compressive strength. Lay-up and stacking sequence are major factors in determining the compressive performance of a laminate. Additionally, testing
composites in compression requires the use of a test fixture; the choice of fixture impacts the testing itself and the measured value of OHC strength.

2.1.2.2.1 Open Hole Compression Fixture

Difficulty arises in compressive testing due to the macrobuckling tendency of thin laminate coupons. If a specimen fails due to macrobuckling, the result obtained is size and geometry dependent, rather than an intrinsic material property. In order to ensure that a compression test yields a material strength rather a structure-dependent strength, it is necessary to constrain the specimen to prevent macrobuckling. This constraint results in laminate failure due to a microbuckling mode, which is believed to be indicative of a true material compressive strength.

There are a number of open hole compression fixtures that are used for composite laminate testing. OHC test fixtures constrain all but a small portion of the test coupon, preventing macrobuckling by using plates on either side of the laminate. The fixtures differ in implementation, coupon size, and loading type.

One of the most common test fixtures in industry originated in 1982 from Boeing Specification BSS 7260 [18]. It has since been adapted in to SACMA SRM3-88 and SRM3R-94 and, with slight modification, to ASTM D6484 [19]. The fixture is shown in Figure 2.13. This fixture uses a 12” x 1.5” coupon, usually with a .25” hole drilled in the center. The two long grips provide the constraint necessary to prevent the material from buckling, and the two short grips hold the material in place. The support plates hold the fixture together with the use of four bolts. When assembled, the fixture weighs approximately 15lbs, and measures approximately 12” x 3” x 1.5”. The fixture was originally shear loaded, meaning the fixture is gripped and load is transferred to the
specimen via shear. This requires 3” hydraulic grips that many facilities do not have, and so ASTM adapted an end loading procedure to reconcile this.

Figure 2.13 Boeing Spec. BSS 7260 OHC Fixture [18]

A newer fixture, similar to the Boeing OHC fixture, is the UCSB OHC Fixture (Figure 2.14). So named because of its origination at the University of California Santa Barbara, the UCSB Fixture sought to correct some of the common complaints with the Boeing fixture. It uses a 5”x1.5” specimen with a center drilled hole, compared to the Boeing fixture 12”x1.5” specimen. Bardis et al [20] analytically verified the loading was still sufficiently far enough from the gauge section that the smaller coupon was valid. The use of a smaller specimen also results in a physically smaller fixture, measuring 5.4” x 3” x 0.4” and weighing 1.5lb. Additionally, combined loading is utilized, with end and shear loading. Unlike the Boeing fixture, the ends of two of the four fixture parts are raised, serving as bearing surfaces to transfer load to the specimen. Furthermore these bearing surfaces facilitate proper placement of the specimen.
Another fairly common fixture in industry is the Northrop OHC fixture [21]. Two variations exist, with one utilizing a 1”x3” coupon and the other using a 1.5”x3” coupon. The fixture is assembled around the laminate, and then constraining plates are bolted around the fixture to hold it together. The assembled fixture, shown in Figure 2.15 measures approximately 3” x 3” x 2”. The fixture is end loaded, meaning load is applied directly to the specimen by the test frame through the use of end platens.

Coguill and Adams [22] experimentally evaluated several open hole compression test fixtures with a soft, medium, and hard lay-up. No clear “best” fixture emerges from
the results, shown in Figure 2.16, Figure 2.17, and Figure 2.18. Measured compressive strengths varied among the test fixtures, most likely due to differing amounts of constraint caused by the fixture. Through-thickness loads are caused by bolting the fixtures together and by test frame grips, though each fixture induces different amounts. There was also no consistent trend in scatter of data among the fixtures. The SACMA end-loaded fixture had scatter of about 3% for the soft and medium stiffness lay-ups, but 9% scatter for the hard lay-up. The Northrop fixture followed a similar trend. The ASTM shear-loaded fixture had a comparatively high scatter of approximately 8% for the soft lay-up, but only 4% for the hard lay-up.

![Figure 2.16 OHC Test Fixture Comparison (Soft Lay-up)](image-url) [22]
Figure 2.17 OHC Test Fixture Comparison (Medium Lay-up) [22]

Figure 2.18 OHC Test Fixture Comparison (Hard Lay-up) [22]
A number of factors are desirable in compression test fixtures. Ideally the OHC fixture would be easy to use. Size, weight, fixture assembly time, and fixture complexity are all factors in ease of use. Coguill and Adams report the UCSB and Northrop fixtures were easier to use than the larger Boeing fixture. Shear loading of the Boeing fixture was complicated by procuring the large grips necessary, and in actually gripping the fixture in the test frame without causing damage. The UCSB design was easy to use, as specimen placement in the fixture was repeatable.

The amount of material necessary for the OHC test should also be considered. A large number of tests may be necessary to fully characterize a new composite material, and the use of smaller coupons can reduce expense. However, as mentioned earlier, composites are notch and size sensitive, so a large enough coupon should be used to yield applicable results. The Boeing fixture, perhaps the most common in industry, uses a 12”x1.5” coupon. By contrast, the Northrop 1.5” fixture uses a 3”x1.5” coupon, or roughly a quarter of the material used by the ASTM fixture, with no apparent loss of quality.

2.1.2.2.2 Lay-Up

As is the case with tensile strength, compressive strength is lay-up dependent. Berbinau et al. [23] considered the effect of lay-up on OHC strength and, as expected, found it largely dependent on the percentage of 0° plies, as shown in Table 2.1. It is interesting to note that failure strain, however, is largely independent of lay-up. This is because the critical strain required to caused a buckle is largely a function of fiber properties and the matrix immediately surrounding the fiber, and as such the strain is not as dependant on adjacent plies. It should also be noted that
while lay-ups L2, L3, and L4 had the same percentage of 0° plies, the strengths were quite different. Of the three lay-ups L3 had the lowest strength, which is to be expected since the 90° plies in the lay-up contribute no longitudinal strength or stiffness. Adding ±45° plies increased support of the laminate and increased the strength as demonstrated by lay-up L2. Lay-up L4, consisting of 0° plies, 90° plies, and ±45° plies, had the highest strength measured of the three, perhaps due to the increased global stability offered by a mixture of ply angles.

### Table 2.1 Effect of Lay-up on Compressive Strength Properties [23]

<table>
<thead>
<tr>
<th>Lay-up</th>
<th>% 0° plies</th>
<th>$\sigma_{cr}$ (MPa)</th>
<th>$E_{cr}$ (GPa)</th>
<th>$G_{cr}$ (GPa)</th>
<th>$\epsilon_c$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L0 ([0]$_h$)</td>
<td>100</td>
<td>1485</td>
<td>160</td>
<td>6.0</td>
<td>1.1</td>
</tr>
<tr>
<td>L1 ([±45/0]$_h$)</td>
<td>67</td>
<td>(1145)</td>
<td>(115)</td>
<td>17.8</td>
<td>1.04</td>
</tr>
<tr>
<td>L2 ([±45/0]$_h$)</td>
<td>50</td>
<td>(912)</td>
<td>(92)</td>
<td>23.5</td>
<td>1.1</td>
</tr>
<tr>
<td>L3 ([±0/90]$_h$)</td>
<td>50</td>
<td>(670)</td>
<td>(78)</td>
<td>6.0</td>
<td>0.96</td>
</tr>
<tr>
<td>L4 ([±45/0/90]$_h$)</td>
<td>50</td>
<td>(838)</td>
<td>(85)</td>
<td>12.9</td>
<td>1.05</td>
</tr>
<tr>
<td>L5 ([±45/0]$_h$)</td>
<td>25</td>
<td>(916)</td>
<td>(91)</td>
<td>23.6</td>
<td>1.07</td>
</tr>
<tr>
<td>L6 ([±45]$_h$/±(45)/0(45))</td>
<td>17</td>
<td>(428)</td>
<td>(41)</td>
<td>35.4</td>
<td>1.35</td>
</tr>
</tbody>
</table>

The sensitivity of a laminate to notch size is also lay-up dependent [24]. Figure 2.19 illustrates that for softer lay-ups, the material behaves more like a brittle material. In these cases the reduction in strength is largely due to the reduction of area caused by the notch. As the lay-up becomes harder, composites demonstrate more notch sensitivity. The reduction in strength is much greater than would be caused by a reduction of area. This is in part due to the fact that a hard lay-up with an increased number of 0° plies has a higher stress concentration factor than a softer lay-up.
Additionally, thickness plays a role in determining the notched compressive strength of laminated composites [25]. Since microbuckling is influenced by fiber support and the support given by neighboring plies, the increased stability from increased thickness results in higher failure stresses as shown in Figure 2.20. The increased number of 0° plies of a thick laminate also reduces the impact of a failure of a single load-bearing 0° ply, slowing the damage progression that would more dramatic in a thinner laminate. A thick laminate will also result in three-dimensional damage propagation through the thickness, resulting in more stable damage growth.
2.1.2.3 Discussion

Like filled hole tensile strength, open hole compressive strength is influenced by many variables. Lay-up, stacking sequence, notch size, thickness, and environmental conditioning are all considerations when evaluating composite laminates. Additionally, in compressive testing the fixture utilized is a significant factor. ASTM D6484 provides information on compressive testing, but choosing a different fixture influences the methods used.

2.1.3. Bearing and Bypass Tests

In structures with mechanically fastened joints, load transfer occurs from one component to another through bolts or other fasteners. In order to understand how composite laminates will perform in such joints, bearing tests are utilized. “Bearing” refers to the load transferred from one part to another through pin, bolt, or other fastener. This results in a compressive load at the interface. Additionally, a “bypass” load is often encountered in bolted joints. For a multi-fastener joint loaded in tension, the bypass load
is the load that is not transferred through a particular fastener; in other words, it bypasses around the hole and is transferred through a different fastener.

2.1.3.1 Configuration

Depending on what information is desired out of the test, there are a number of specimen configurations that can be used. The Double Shear Single Fastener test is recommended by ASTM D5961 [26] for initial evaluation and comparison. It consists of a single coupon loaded by a lightly torqued bolt, as shown in Figure 2.21. This type of loading is not indicative of most applications and as such the results cannot be applied to single shear joints. However, this test does result in pure bearing load, which is useful for characterization of material behavior. It should be noted that a clevis must be utilized for this test, though the clevis is easily fabricated and implemented.

Figure 2.21 Double Shear Single Fastener Bearing Test Assembly [26]
The Single Fastener Single Shear configuration is widely used for initial determination of bearing properties. It features two coupons joined by a single mechanical fastener. The lap joint configuration is more typical of actual applications, though in practice multiple fasteners are more common. Single Fastener Single Shear specimens experience bearing loading, but due to the loading eccentricity the fastener is also subject to a bending moment. This can result in additional non-bearing failure modes.

![Figure 2.22 Single Shear Single Fastener Test Specimen [26]](image)

The Double Fastener Single Shear configuration shown in Figure 2.23 is similar to the Single Fastener Single Shear test, except that two fasteners are utilized instead of one. This configuration is similar to those actually used in applications. However, multiple fastener specimens experience bypass loading which can result in tensile failure instead of bearing failure; bypass loads are typically small for the configuration specified in ASTM D5961, but bypass strain should be verified to be less than 0.2% [1].
In order to avoid the bending moment experienced by bolts in the single shear configurations, it is possible to use a stabilization fixture [27]. This fixture constrains the bolt and prevents bolt rotation. An example of such a fixture is given in Figure 2.24. It is similar in design and fabrication to the Boeing open hole compression fixture.
2.1.3.2 Damage Mechanisms

Bearing tests can result in several types of failure for composite laminates. In addition to the tensile and compressive damage modes discussed earlier, and overview of the types of failures unique to bolted composite joints is given in Figure 2.25. As in filled hole tensile strength and open hole compressive strength, bearing strength is dependent on lay-up, clamp-up force, and so on, but failure strength is also very sensitive to specimen geometry. Net tension failure is usually the result of the bolt diameter being sufficiently large compared to the coupon width. In this case, the specimen fractures at or ahead of the bolt. ASTM recommends a width to hole diameter ratio greater than or equal to six. Additionally, tension failures can occur due to bypass loads exceeding the tensile strength of the material. Shear-out failure is generally avoidable in composite laminates. A ratio of the distance from the hole to the edge of the specimen of the hole, e, to the hole diameter, D, greater than or equal to three is recommended, as this provides sufficient resistance to this mode of failure. Shear-out is more likely to occur in laminates with a high percentage of 0° plies, as ±45° plies and 90° plies resist shear-out failure. In general, specimens are designed such that bearing failure occurs.
The damage progression leading to bearing failure involves several modes [28]. The compressively loaded 0° plies develop kink bands (Figure 2.26a), and 90° plies may fail in tension. Local delaminations and crushing occur in the region under the washer due to the kink bands (Figure 2.26b). Finally, structural failure occurs when the region under the washer becomes saturated with damage, large-scale delamination occurs, and joint integrity is lost.
Unlike filled hole tension, bearing strength can benefit from clamp-up force [4]. As clamp-up force is increased, bearing failure is suppressed. This causes an increase in stress concentration, and can lead to a net tension failure, which is a higher stress event.
2.2. **ANALYTICAL BACKGROUND**

When attempting to calculate the strength of a laminated composite structure, there are many factors to consider. At a micromechanical level, the interactions of the fiber, matrix, and interface determine the mechanical response and strength of individual plies. When these plies are stacked in different orientations, the stiffness and strength of the laminate differ from those on a lamina level. When loaded, plies with different orientations carry different amount of stress. Finally, notches or differing geometry can change stress fields in the laminate, causing dramatic reductions in strength. In a physical sense, micromechanical, lamina, laminate, and structural response are intertwined, as stresses caused by stress concentrations at a structural level cause damage at a micro-scale. However from a practical standpoint, simplifying assumptions must be made in order to make reasonable, efficient predictions of composite laminate behavior. Typical approaches for unnotched laminate strength and notched strength prediction, two fundamental areas of interest to engineers and designers, are discussed below.

**2.2.1. Unnotched Strength Prediction**

When composites materials are implemented in structures, it is most often in the form of multi-directional laminate. Thus, it is desirable to be able to predict the performance of a particular lay-up of a given material based on some set of material properties. Laminate failure theories are used to calculate the unnotched characteristics of multi-directional composite laminates. These theories can be incredibly detailed, with some considering the micromechanical interactions of fiber, matrix, and interface [29]. Most often, however, these theories are lamina-based and use ply-level properties.
Regardless of inputs, the theories attempt to calculate strengths of laminates subjected to uni-axial or multi-axial loads.

Significant effort has gone into generating these theories, as was highlighted in the recent World-Wide Failure Exercise [30]. The study compared 18 current theories of predicting failure in unnotched composite laminates, and evaluated each based on 14 test cases. While no single theory worked best for all situations, several performed relatively well for laminates consisting of $0^\circ$, $\pm45^\circ$, and $90^\circ$ plies, which is typical of laminates used in aerospace applications. The Maximum Stress Criteria, Tsai-Wu Criteria, Hashin Criteria, and the Hart-Smith 10% rule will be considered here based on their simplicity and performance in the exercise.

Failure of unnotched composites can be thought of as the combination of stresses necessary to cause loss of structural integrity of the laminate. Thus, for every composite system, there is a safe operating region, or envelope, of stresses inside which failure will not occur. Failure theories seek to generate these envelopes using strength and stiffness data of the composite, most typically lamina properties such as longitudinal tensile strength $X_{1T}$, longitudinal compressive strength $X_{1C}$, transverse tensile strength $X_{2T}$, transverse compressive strength $X_{2C}$, and in-plane shear strength $X_{LT}$.

2.2.1.1 Maximum Stress Criteria

Simply stated, the Maximum Stress Criterion [31] predicts failure when the stress in any of the principal directions is equal to or greater than the appropriate ultimate strength. This is one of the earliest failure criteria for multi-directional laminates, and one of the easiest to implement. Failure envelopes are generated by the equations
These conditions are applied for each lamina, and as long as the applied stress in any lamina remain in the three-dimensional space defined by the envelope, the theory does not predict failure.

A severe limitation of this approach is the lack of stress interaction; that is, a laminate under combined loading is assumed to perform exactly the same as a laminate under uniaxial loading. This is referred to as a non-interactive failure theory.

A similar approach, called the Maximum Strain Criteria, is identical in implementation except that it is based on strains, not stresses. The envelopes are generated by the equations

\[-\varepsilon_{1c} \leq \varepsilon_{11} \leq \varepsilon_{1T} \]
\[-\varepsilon_{2c} \leq \varepsilon_{22} \leq \varepsilon_{2T} \]
\[\gamma_{LT} \leq \gamma_{12} \] (2.3)

### 2.2.1.2 Tsai-Wu Criteria

The Tsai-Wu criterion [32] predicts that failure will occur when the following equation is satisfied:

\[F_1\sigma_{11} + F_2\sigma_{22} + F_{66}\tau_{12}^2 + F_{11}\sigma_{11}^2 + F_{22}\sigma_{22}^2 + 2F_{12}\sigma_{11}\sigma_{22} = 1\] (2.4)

\(F_1, F_2, \) and so on are the strength coefficient and are given by

\[F_1 = \frac{1}{X_{1T}} - \frac{1}{X_{1C}}; \quad F_2 = \frac{1}{X_{1T}} - \frac{1}{X_{1C}}; \]
\[F_{11} = \frac{1}{X_{1T}X_{1C}}; \quad F_{22} = \frac{1}{X_{2T}X_{2C}}; \quad F_{12} = -\frac{1}{2}\left(F_{11}F_{22}\right)^{1/2} \] (2.5)
These conditions are applied for each lamina and plotted on the same axes. A sample failure envelope for an E-glass MY750 [+55/-55] laminate is given in Figure 2.27.

![Sample Tsai-Wu Failure Envelope](image)

Figure 2.27 Sample Tsai-Wu Failure Envelope [33]

As long as the stresses applied to the laminate stay within the envelope defined by the overlap of the lamina failure envelopes, failure is not predicted. The Tsai-Wu Criteria is an example of an interactive criteria, because of the polynomial relating stresses in different directions.

The Tsai-Wu criteria have been widely used in industry because of its ease of use and accuracy. However, one of the disadvantages of this method is the absence of failure mode indicators. For example, implementation of the Maximum Stress Criteria will indicate whether failure was due to longitudinal compression, transverse tension, and so on. The Tsai-Wu criteria have no such feature.

Additionally, the Tsai-Wu criteria and similar interactive criteria have been criticized for their lack of physical basis. Equation 2.2 defines a smooth curve that is typically not observed in testing, as a transition from a matrix failure to a fiber failure can
be abrupt. Despite this, the Tsai-Wu remains a popular tool that is generally accepted as providing good results.

2.2.1.3 Hashin Failure Criteria

Hashin [34] proposed a failure criteria that would consider the effects of stress interaction and would include the failure mode and load direction in determining strength. It is defined by the following equations.

Fiber Mode, Tensile:

\[
\left(\frac{\sigma_{11}}{X_{1F}}\right)^2 + \left(\frac{\tau_{12}}{S_{12F}}\right)^2 = 1
\]  
(2.6)

Fiber Mode, Compressive:

\[
\left(\frac{\sigma_{11}}{X_{1C}}\right)^2 + \left(\frac{\tau_{12}}{S_{12F}}\right)^2 = 1
\]  
(2.7)

Matrix Mode, Tensile:

\[
\left(\frac{\sigma_{22}}{X_{2F}}\right)^2 + \left(\frac{\tau_{12}}{S_{12F}}\right)^2 = 1
\]  
(2.8)

Matrix Mode, Compressive:

\[
\left(\frac{\sigma_{22}}{X_{2C}}\right)^2 + \left(\frac{\tau_{12}}{S_{12F}}\right)^2 = 1
\]  
(2.9)

The Hashin criteria is a semi-interactive criteria, in that normal and shear stresses factor into the determination of each failure mode, but not all stress components are considered for each mode. The interaction of stresses can be a significant factor in lamina and laminate strength, and as such, this criterion is an improvement over the maximum stress criterion. However, it also indicates the failure mode, which may make it more desirable than the Tsai-Wu Criterion for some applications.
2.2.1.4 Ten Percent Rule

The Ten Percent Rule [35] was conceived as a design tool that could be easily implemented with simple hand calculations. Additionally, it sought to require the use of as few experimentally determined parameters as possible; it utilizes the longitudinal modulus, the longitudinal tensile strength, and the longitudinal compressive strength. All other values are chosen or calculated. The only calculations required are simple addition, subtraction, and trigonometric functions, given in [35]. A sample failure envelope is shown in Figure 2.28. The Ten Percent Rule does provide the mode of failure.

![Figure 2.28 Ten Percent Rule Failure Envelope for [+/-45/90/0]s Carbon/Epoxy Laminate [35]](image)

2.2.2. Notched Strength Prediction

The introduction of notches or stress concentrations into a composite laminate can greatly reduce the strength of a structure. As such, notched strength prediction has been
and still is an active area of research within the composites community. Given the anisotropy, variety of damage modes, and complex failure processes found in composites, strength prediction can be very difficult, and no clear consensus exists as to best way to perform analysis. Methods include the extension of linear elastic fracture mechanics typically used in metallic materials, to mechanics of materials analysis, to detailed finite element techniques that attempt to include micromechanical details and simulate individual damage modes. Each technique has advantages and disadvantages and involves differing assumptions, effort, and knowledge of material properties.

2.2.2.1 Waddoups, Eisenmann, and Kaminski Failure Theory

One of the earliest attempts at notched strength prediction of composite laminates was the Waddoups, Eisenmann, and Kaminski (WEK) failure theory [36]. The WEK method is an application of Linear Elastic Fracture Mechanics (LEFM) to composite materials. It is logical that early attempts at failure prediction in composites would be an extension of the methods used in metallic materials, and Wu [37] found this application was suitable when three conditions were met:

1. The orientation of the flaw with respect to the principal axis of symmetry must be fixed
2. The stress intensity factors defined for anisotropic cases must be consistent with the isotropic case in stress distribution and in crack displacement modes
3. The critical orientation coincides with one of the principal directions of elastic symmetry

The basis for the WEK model is the replacement of damage at the notch with an intense energy region, shown in Figure 2.29. As mentioned earlier, the evolution of damage in
composite laminates is complex and can involve multiple damage types. The WEK method circumvents the need to predict each damage type in the laminate by using this intense energy region.

Figure 2.29 WEK Fracture Model [2]

Waddoups, Eisenmann, and Kaminski applied the work of Irwin [38] relating the energy release rate $G_I$ and the stress intensity factor $K_I$ by the equation

$$ G_I = \frac{1 - \nu}{2 \cdot G} K_I^2 = \left(\frac{1 - \nu^2}{E}\right) \pi K_I^2 $$  \hspace{1cm} (2.10)

For a characteristic length “a” that is small and finite, the effect of damage zone size can be analyzed by the stress intensity factor solution developed by Bowie [39] for the problem of cracks growing from a circular hole an isotropic plate. Paris and Sih found the solution to this geometry to be [40]

$$ K_I = \sigma \sqrt{n} a f \left(\frac{a}{R}\right) $$  \hspace{1cm} (2.11)
Combining equations yields

\[ \sqrt{G_I} = \left[ \pi \sqrt{\frac{a(1-\nu^2)}{E}} \right] \sigma \cdot f\left(\frac{a}{R}\right) \]  \hspace{1cm} (2.12)

The authors then assume that the material is an ideally brittle with constant \( G_I \).

Additionally, it is assumed that the change in the characteristic length “\( a \)” is small compared to the hole radius. Thus the equation can be rearranged to show

\[ \frac{\sqrt{G_I}}{\left( \pi \sqrt{\frac{a(1-\nu^2)}{E}} \right)} = \sigma N \cdot f\left(\frac{a}{R}\right) \equiv \text{CONSTANT} \]  \hspace{1cm} (2.13)

This allows the ratio of unnotched and notched strengths to be written as

\[ \frac{\sigma_0}{\sigma_N} = f\left(\frac{a}{R}\right) \]  \hspace{1cm} (2.14)

Values of \( f\left(\frac{a}{R}\right) \) have been found by Paris and Sih [40]. Thus, for any value of “\( a \)”, the ratio of notched to unnotched strengths can be calculated for differing hole radii.

The assumptions used in applying LEFM to composites deserve attention. The assumption that flaw orientation remains fixed seems unlikely given the variety of damage mechanisms that occur. Additionally, it is not likely that the stress distribution in an anisotropic composite would be consistent with the isotropic case.

2.2.2.2 Whitney-Nuismer Failure Theory

The Whitney-Nuismer method [41] posits that failure of a notched laminate occurs when the stress at some characteristic distance away from the notch reaches the
unnotched strength of the laminate. The Whitney-Nuismer method can be implemented via either the Point Stress Criterion (WNPS) or the Average Stress Criterion (WNAS), shown schematically in Figure 2.30.

\[ \sigma_0 \leq \sigma \]

The Point Stress Criterion states that fracture occurs when the stress at the characteristic distance \( d_0 \) is equal to or greater than the unnotched strength of the laminate, given by the equation

\[
\sigma_y(x_0)_{x=\text{Red}} = \sigma_0
\]  

(2.15)

The Average Stress Criterion states that fracture occurs when the average stress over some characteristic distance \( a_0 \) is equal to or greater than the unnotched strength of the composite, and is given by the equation

Figure 2.30 2 WN Point Stress Failure Theory b.) WN Average Stress Failure Theory [2]
Whitney and Nuismer sought to address the effect of notch size in laminated composites. Timoshenko [42] originally showed the dependence of the normal stress $\sigma_y$ on hole size in an infinite, isotropic material to be

$$\frac{\sigma_y}{\sigma^\infty} = 1 + \frac{1}{2} \left( \frac{R}{x} \right)^2 + \frac{3}{2} \left( \frac{R}{x} \right)^4$$  \hspace{1cm} (2.17)$$

This results in the stress distribution shown in Figure 2.31.

![Figure 2.31 Normal stress distribution for circular hole in infinite isotropic plate [2]](image)

This approximation is valid for quasi-isotropic laminates with a stress concentration factor $K_T^\infty = 3$, but is inaccurate for orthotropic laminates, where $K_T^\infty \neq 3$. Konish and Whitney [43] extended Timoshenko’s work to an orthotropic plate under a uniform uniaxial stress, showing the normal stress $\sigma_y$ to be

$$\sigma_y(x,0) = \frac{\sigma^\infty}{2} \left\{ 2 + \left( \frac{R}{x} \right)^2 + 3 \left( \frac{R}{x} \right)^4 - \left( K_T^\infty - 3 \right) \left[ 5 \left( \frac{R}{x} \right)^6 - 7 \left( \frac{R}{x} \right)^8 \right] \right\} , \quad x > R$$  \hspace{1cm} (2.18)
Lekhnitskii’s solution for the stress concentration factor for an open hole in an anisotropic plate [7] can be used with the equation

$$K_T^x = 1 + \left[ \left( \frac{E_{11}}{G_{12}} \right)^{1/2} - 2\nu_{12} + 2\left( \frac{E_{11}}{E_{22}} \right)^{1/2} \right]$$  \hspace{1cm} (2.19)

For the filled hole case, Tan [3] extended Lekhnitskii’s complex variable approach to anisotropic plates with an elliptical inclusion; stress concentrations can be obtained from this method.

When this stress distribution is applied to the Point Stress Criterion, it yields

$$\frac{\sigma_N^x}{\sigma_0} = \frac{2\sqrt{2 + \xi_1^2 + 3\xi_1^4 - (K_T^x - 3)(5\xi_1^6 - 7\xi_1^8)}}{\frac{R}{R + d_o}}$$  \hspace{1cm} (2.20)

where

$$\xi_1 = \frac{R}{R + d_o}$$  \hspace{1cm} (2.21)

Similarly, applying the stress distribution to the Average Stress Criterion yields

$$\frac{\sigma_N^x}{\sigma_0} = \frac{2(1 - \xi_2^2)}{\sqrt{2 - \xi_2^2 - \xi_2^4 + (K_T^x - 3)(\xi_2^6 - \xi_2^8)}}$$  \hspace{1cm} (2.22)

where

$$\xi_2 = \frac{R}{R + a_o}$$  \hspace{1cm} (2.23)

Like the WEK model, the WN model uses two parameters – the unnotched strength of the laminate and the characteristic distance – to predict the strength of the notched laminate. The characteristic distance is believed to be a material constant, independent of lay-up and notch size. It is an experimentally determined parameter, and can be viewed as a way to “fit” a curve to the data. One advantage of the WN approaches over the WEK method is prediction of notched strength without the application of linear
elastic fracture mechanics. As discussed earlier, LEFM is of questionable applicability to composites, and the Whitney-Nuismer Point Stress and Average Stress Criterion offer a significant improvement in the study of fracture in composites.

2.2.2.3 Karlak Failure Theory

The WEK and WN Point Stress and Average Stress approaches assume that the characteristic distance is a material property, and is lay-up and notch size insensitive. Karlak [44] found that the notched strength of quasi-isotropic composite laminate was indeed dependent on stacking sequence. Karlak further observed that the characteristic length $d_0$ was not a material constant, but was related to the square root of the hole radius. Based on these observations, Karlak proposed a modified Whitney-Nuismer Point Stress Criterion. In the Karlak Fracture Model, the characteristic length is defined by

$$d_0 = k_0 R^{1/2} \quad (2.24)$$

The constant $k_0$ is a curve fitting parameter determined experimentally for a material system and particular stacking sequence. The remaining analysis can then be conducted exactly as in the Whitney-Nuismer method. The new characteristic distance is used as before in equations 2.19 and 2.20.

2.2.2.4 Pipes, Wetherhold, and Gillespie Failure Theory

The Pipes, Wetherhold, and Gillespie (PWG) failure theory [45] is a further modification of the Whitney-Nuismer method. Like the Karlak model, the PWG model does not consider the characteristic distance to be a material parameter. The PWG model
proposes a different relationship between the hole radius and characteristic distance, one given by

\[ d_0 = \left( \frac{R}{R_0} \right)^m \]  

(2.25)

This allows the WNPS failure theory to be rewritten as

\[ \frac{\sigma_{\infty}}{\sigma_0} = \frac{2}{\left( 2 + 1\lambda^2 + 3\lambda^4 - \left( K_T^e - 3 \right) \left( 5\lambda^6 - 7\lambda^8 \right) \right)} \]  

(2.26)

with

\[ \lambda = \frac{1}{1 + R^{-m-1}R_0^{-1}C^{-1}} \]  

(2.27)

This more general exponential relationship serves to better characterize the relationship between notch size and characteristic distance. \( R_0 \) is a reference notch radius that is used to non-dimensionalize the quantity in brackets, and usually taken as unity. Thus, the PWG Failure Theory is a three parameter model, with a notch-sensitivity factor \( C \) and a parameter \( m \) relating hole radius to characteristic distance.

2.2.2.5 Mar-Lin Failure Theory

As discussed earlier, the application of linear elastic fracture mechanics to composite materials is limited. For a homogeneous material, the basic LEFM equation is given by
\[ \sigma_N^\infty = K_{IC} (\pi a)^{-1/2} \]  

(2.28)

In this equation, \(K_{IC}\) is the critical mode I stress intensity factor, and \(-1/2\) is the order of the mathematical stress singularity at the crack tip. Mar and Lin [46] proposed a similar model for fracture of composites:

\[ \sigma_N^\infty = H_c (2a)^{-n} \]  

(2.29)

In this model, \(H_c\) is the composite fracture toughness, and \(n\) is the order of the singularity. The exponent \(n\) has been related to the singularity of a crack in a matrix with the tip at the fiber/matrix interface, in which case it is a function of constituent material shear moduli and Poisson ratios. This idealization is considered overly simplistic [10], but the model can still be used as a two-parameter model with both constants determined experimentally. Specimens with different sized notches are needed, but once the parameters are determined, the model has been used to successfully predict large notch performance from small notch data.

2.2.2.6 Damage Zone Model or Cohesive Zone Model

Early notched strength prediction techniques were largely based on mechanics of materials-type approaches, where a stress field was calculated based on the properties of the undamaged laminate, and fracture was determined based on some experimentally based semi-empirical method. They provide little or no insight into damage growth. While these techniques can be accurate and effective tools, they have little physical basis.

Due to the variety of damage modes and how damage evolution occurs in composite material, detailed modeling is difficult and computationally expensive. Thus, the Damage Zone Model [47] uses the fracture energy \(G_C\) to account for all energy
dissipated by the various damage mechanisms. This model was originally used by Hillerborg et al [48] for analysis of concrete. For a notched composite subjected to an external load, damage can occur in the region adjacent to the notch, as shown in Figure 2.32a.

\[ \text{Damage Zone} \quad \text{Equivalent Crack} \]

\[ \text{Cohesive Stresses} \]

Figure 2.32 a) Damage Zone at Notch and b) Equivalent Crack

This damage zone is replaced by a fictitious or equivalent crack, and analyzed via a Dugdale - Barenblatt method, with cohesive stresses acting on the crack face as shown in Figure 2.32b. As load increases, damage increases in the material, which is modeled as increased crack opening and longer crack length. Generally, the relationship between stress and displacement is linear, as shown in Figure 2.33. Other relationships can be selected based on the material. The unloaded material has no damage and as such no equivalent crack. As damage increases, the cohesive stresses decrease, with material softening occurring due to damage. With this approach, stress redistribution and stiffness degradation can be calculated with classical or finite element methods.

\[ \text{Figure 2.33 Assumed Linear Relation Between } \sigma \text{ and } v \quad [12] \]
It should be noted that Soutis et al. [12, 49, 50] have had success applying the cohesive zone model to open hole compression, replacing the kink band region and delaminated area with the equivalent crack.

More recent work by Hallett and Wisnom [51] extends the Cohesive Zone technique to model damage in each ply and between plies, rather than simply replacing all damage with a single equivalent crack. Cohesive elements are inserted between elements where damage is known or expected to grow, as shown in Figure 2.34. The damage than initiates and grows based on the cohesive zone relationships for splitting, fiber breakage, or delamination.

Figure 2.34: Schematic of Cohesive Zone Model applied to multiple damage types with Finite Element Analysis [51]

Excellent agreement between experimental and analytical predictions for both strength and damage progression was found with the above technique, for a variety of geometries,
lay-ups, thickness, and notch sizes. However, intricate finite element analysis is required, and significant computational effort is needed to obtain convergence.
CHAPTER 3

EXPERIMENTAL PROCEDURES

Following is a description of the experimental equipment, techniques and procedures used over the course of this research. The pre-preg material, lay-ups, and specimen configurations are described. This is followed by a description of the equipment and the methods used for the open hole tension, filled hole tension, open hole compression, single shear bearing, and unnotched tension tests. Finally, the use of the X-ray radiography equipment is detailed.

3.1. MATERIALS, LAY-UPS, AND SPECIMEN GEOMETRIES

The material tested was a Toray T800H /3900-2 carbon fiber /toughened epoxy resin prepreg. It is the material used in the Boeing 777 and the material that will be used in the Boeing 787. Boeing personnel fabricated the specimens used in this study. 76.2mm prepreg tape was laid up into panels by hand, and the panels were cured at 177°C according to manufacturer specifications. For the unnotched and notched specimens, the panels were then cut into 38mm x 305mm coupons using a diamond saw. Center holes were drilled using a 6.35mm center holes were drilled using a diamond-impregnated drill bit. For the bearing specimens, the panels were cut into 31.75mm x 184mm coupons with offset 6.35mm holes. Specimens were then labeled according to the following system: The first character represents the lay-up type, with quasi-isotropic being denoted by a “Q”, and the hard lay-up being denoted by an “H”. The next number indicates the longitudinal ply orientation, and is either 0, 5, or 10. The following three characters are the test type. These are UNT for unnotched tension, OHT for open hole
tension, FHT for filled hole tension, OHC for open hole compression, or SSB for single-shear bearing. The final number indicates the specimen number.

Specimen geometries are shown in Figure 3.1, Figure 3.2, and Figure 3.3.

Figure 3.1 Open Hole Specimen Configuration

Figure 3.2 Filled Hole Configuration

Figure 3.3 Single Shear Bearing Specimen Configuration
Two different types of lay-ups were considered in the current research: a quasi-isotropic \([45/90/-45/0]_s\), lay-up, and a “hard” \([45/0_2/-45/0_2/90/0_2/\pm(45)_{1/2}]_s\) lay-up. The non-traditional laminates replaced the 0° longitudinal plies with off-axis plies, such that the stacking sequences are \([45/90/-45/0/45/90/-45/\theta]_s\) for the non-traditional quasi-isotropic lay-up, and \([45/\pm0/-45/\pm0/90/\pm0/\pm(45)_{1/2}]_s\) for the non-traditional hard lay-up, where \(\theta=5^\circ\) or \(10^\circ\). It should be noted that the hard lay-up is neither balanced nor symmetric; this lay-up was chosen because it contained of a high percentage of longitudinal plies while not exceeding load-capacity of existing equipment. The presence of the \(\pm45^\circ\) plies on the centerline is the reason the laminate is non-symmetric and unbalanced, but the overall effect on the laminate is minimal. This was verified with a simple finite element analysis. An unnotched hard specimen, with properties found in [52], was modeled in ANSYS using 300 of the layered 8-node, 6 degree of freedom SHELL91 elements. Boundary conditions constrained translation and rotation in the grip region on one end of the specimen, and permitted only longitudinal translation in the other grip region. A small displacement was applied and the shear stresses in each ply considered. It was found that shear stress concentrations, shown in Figure 3.4, existed near the grips, and were approximately 50% higher than those observed in the balanced, symmetric quasi-isotropic laminates. However, the shear stresses were less than 1% of the maximum tensile stresses in the laminate and deemed sufficiently small.

Figure 3.4 ANSYS analysis of shear stresses at mid-plane of hard laminate
Fasteners were supplied by Boeing, and consisted of close tolerance Hi-Shear threaded titanium pins, frangible collars, and washers, shown in Figure 3.5. The collars are designed such that the hexagonal head shears off at a predetermined torque level of 8 N-m, ensuring consistent clamping force. Full clamp-up was used in the FHT specimens, while half clamp-up was used in the bearing specimens. These clamp-ups correspond to the worst-case scenarios described in 2.1.1.2.3.

![Image](a)

![Image](b)

Figure 3.5 Mechanical fastener used (a) pin and (b) collar

3.2. TESTING APPARATUS

The following sections describe the load frames, test accessories, and stabilization fixtures utilized in this research.

3.2.1. Hydraulic Test Frames

Initial Open Hole Tension and Filled Hole Tensile testing was performed on a 100kN SATEC servo-hydraulic test frame with hydraulic Surfalloy-coated grips. The machine, shown in Figure 3.6, was equipped with a TestStar IIIs data acquisition and control system, and the MTS Basic TestWare software package was utilized. The machine also had a 25.4mm extensometer, with a 3.81mm span.
In order to accommodate the higher loads required in both notched and unnotched testing, a 250kN MTS servo-hydraulic test frame had to be utilized. Existing grips were not capable of withstanding the loads necessary, so mechanical wedge grips rated to a load of 265kN were purchased from Curtis “Sure-Grip” Inc. The machine and grips are shown in Figure 3.7. This system was also equipped with TestStar II data acquisition and control system, and the MTS Basic TestWare software package. The same 25.4mm MTS extensometer was also calibrated for use on this machine.
3.2.2. Test Fixtures

In order to prevent out of plane deformation in the open hole compression and single shear bearing specimens, fixtures were utilized in these tests. A SACMA SRM 3R-94 open hole compression test fixture, discussed in 2.1.2.2.1, was obtained from Clark Atlanta University, and is shown in Figure 3.8. In order to conduct compression testing, the 250kN MTS frame was equipped with flat end platens to transfer load through end-loading, as discussed in 2.1.2.2.1. The compression testing setup is shown in Figure 3.9. As before, the MTS TestStar IIIs and MTS Basic Testware packages were utilized.
To constrain out-of-plane movement in the single-shear bearing specimen, a simple fixture was designed and fabricated. 76.2mm wide by 19mm thick 17-4PH stainless steel bar was purchased, and the Georgia Institute of Technology Mechanical Engineering Machine Shop performed the machining. The resulting fixture, shown in
Figure 3.10, face-supports the specimen to prevent joint rotation and ensure bearing failure.

![Figure 3.10 Single shear bearing support fixture (a) disassembled and (b) assembled](image)

The size and center notch of the SSB support fixture did not allow for mounting of the previously discussed MTS extensometer. This necessitated a different extensometer, so a 25.4mm, 2.54mm span Epsilon high-temperature extensometer (Figure 3.11) was calibrated for use with the 100kN SATEC test frame. The ceramic rods allowed the extensometer to be mounted outside the fixture to read displacement across the notch.

![Figure 3.11 High temperature extensometer used for single-shear bearing testing](image)
3.3. TEST PROCEDURES

Subsequent sections will describe the various procedures used for the different test types. Unnotched, open hole, and filled hole tensile testing were conducted using the same basic procedure. Both the open hole compression and single shear bearing procedures required different methodology and additional steps.

3.3.1. Tensile Testing Procedures

Prior to tensile testing, specimens were visually inspected for nicks, surface irregularities, warpage, and asymmetry. The width and thickness of each specimen was measured with a dial caliper at the midpoint and at both ends to ensure consistent dimensions. The diameter of the notch in the open hole specimens was also measured.

Following inspection, the specimens were inserted into the grips. Alignment was ensured by the centering guides, and confirmed with visual inspection. When hydraulic grips were used, a clamping force of 2500psi was applied. When the wedge grips were utilized, the grips were mechanically tightened. For tests requiring an extensometer, it was placed across the notch, with double-sided tape used to prevent slippage and secured with rubber bands. This was done prior to inducing any strain in the specimen. After alignment and gripping occurred, the specimen was preloaded and testing was ready to begin. The specimens were loaded at a constant displacement rate of 1.27mm/min.

3.3.2. Compression Testing Procedure

Before compression testing, specimens were inspected as explained in 3.3.1. Specimens were then placed in the OHC test fixture, and positioned such that the ends of
the specimen aligned with the edge of the fixture. The fixture was then assembled, and per Boeing standards [18] the bolts in the fixture were torqued to 1.13 N-m.

The specimen and fixture were then placed between the end platens. The hydraulic actuator was raised until the fixture was completely flush with the end platens. The fixture was centered, and alignment was verified. A slight preload was then applied, and the specimens were then loaded at a constant displacement rate of 1.27 mm/min.

### 3.3.3. Single Shear Bearing Procedures

The single-shear bearing specimens required assembly before testing. Following visual inspection, two coupons were placed bag-side to bag-side such that the holes aligned. A pin was then inserted on one side, and a washer and collar on the other. The collar was tightened to finger tight, and the specimen transferred to a vise. The vise was used to ensure alignment. With the specimen gripped lightly in the vice, the collar was tightened with a torque wrench to a torque of 4 N-m. The specimen was then visually inspected again prior to testing.

The specimen was placed in the fixture described in 3.2.2, such that 50.8 mm of material for gripping extended past either end of the fixture. The bolts in the fixture were then torqued to 0.55 N-m to ensure joint rotation did not occur, but that the specimen was relatively free to move vertically in the fixture.

Single-shear bearing tests were conducted in the 100kN SATEC test frame. The specimen was aligned, and ends of the specimen were gripped. A slight preload was applied, and loading was then applied at a constant rate of 1.27 mm/min.
3.4. X-RAY PROCEDURES

Radiographic inspection of damaged specimens was performed with a Faxitron 110kV portable X-ray unit used in conjunction with the 100kN SATEC test frame. A lead-lined plywood enclosure, fabricated by previous researchers [53] for radiation safety, was mounted on the fixture as seen in Figure 3.12. Radiation safety training was attended, and the X-ray room was certified by the Georgia Tech Office of Radiation Safety to meet applicable state and federal safety requirements.

3.4.1. Radiographic Inspection

X-rays were taken at increasing load levels to determine if damage had occurred. At the desired load increment, prior to running the x-ray, the film and film holder were positioned directly behind the center of the open hole in the specimen as shown Figure 3.13.
Because the graphite fiber and epoxy resin provide little contrast for the x-ray, zinc iodide die penetrant was used. This solution leaches into the cracks in specimen and distinguishes the damaged areas. The solution consists of

- 60 g. Zinc Iodide (ZnI2) 98% pure
- 8 mL distilled water
- 10 mL Isopropyl alcohol
- 3 mL Kodak photo flow 200

These were measured into a 125mL Erlenmeyer flask and agitated until dissolved.

Preparation of the specimen prior to x-ray depended on the type of test. No additional handling was necessary for the open hole tension tests. For the filled hole and single shear specimens, the fastener had to be removed prior to X-ray. This was accomplished by unloading the specimen and removing it from the test frame. The fastener was then removed before reinserting the specimen into the grips. The open hole
compression and single shear bearing specimens were removed from the anti-buckling fixture before x-raying.

The zinc iodide solution was then applied to the hole diameter and specimen edges using a 250 µL Hamilton luer tip syringe. Extreme care was taken to avoid surface contamination, since residual zinc iodide solution on the front or back of the specimen could obscure damage.

Depending on the type of test, an opening load was then applied. For the open hole tension tests, the opening load was equivalent to the load the specimen was exposed to. For the filled hole tension tests, an opening load of 4.5kN was applied; this low load level was applied to avoid causing any additional damage due to change in constraint from the removal of the fastener. Similarly, a 4.5kN load was applied for the open hole compression tests. No opening load was applied for the single-shear bearing specimens, as specimen geometry did not provide sufficient material to grip at the top edge of the disassembled specimen. Regardless of type of test, five minutes were allowed to permit the zinc iodide solution to seep into the specimen.

The portable x-ray unit, shown in Figure 3.14, was positioned as seen in Figure 3.15. Alignment was guaranteed by a riser arm equipped with a safety interlock, aligning marks on the floor, and visual cues on the radiation enclosure.
The parameters for the Faxitron 110 kV x-ray unit were set at voltage 33 kV, current 3 mA, and a time of 73 seconds, based on previous research [53]. After the room was cleared, the door was closed, enabling the final safety interlock. The specimen was then
irradiated to determine the damage around the notch. The image would then be processed in accordance with 3.4.2.

### 3.4.2. Film processing and Imaging

Following X-ray exposure, the Polaroid Type 55 PN sheet film was processed. The Kodak type 545i film holder was shifted from load to process, and the film removed from the holed. This action spread the developing agent on both the positive and negative images. Per manufacturer specification, thirty seconds were allowed to ensure film development. The negative and positive were then separated. The positive image was inspected to determine image quality, and if deemed acceptable, the negative image was placed in a fixing agent of 18% sodium sulfite solution made from Kodak anhydrous sodium sulfite. The negative was kept in the fixing solution for at least 30 minutes. After removal from the fixing solution, each negative was rinsed in warm tap water for 5 minutes to remove residual developing and fixing agents. Finally the negative was air dried.

The extent of damage was determined by inspecting the negative with a Leica stereographic microscope. Using the microscope, a digital image of the negative could be obtained, and any further enhancement or analysis could be performed using the ImagePro Express software.
CHAPTER 4
RESULTS

This chapter presents the results of the various tests performed on the traditional and non-traditional composite laminates. The test results are grouped first according loading type, then lay-up, laminate, and notch constraint. The findings from the unnotched tension data will be presented first, followed by open hole tension, filled hole tension, open hole compression, and single shear bearing tests. For each test type, data will be presented first in tabular form, along with an initial comparison of the traditional and non-traditional mechanical behavior. Bar graph comparisons of the strengths of the different laminates will be given. These will show the average of the three tests, and deviation bars will give the maximum and minimum results. This will be followed by a detailed description of each laminate's behavior, and the results of the radiographic inspection.

Due to the proprietary nature of the data gathered in this research, the failure load and strain-to-failure results are normalized against the traditional laminate unnotched properties provided by Boeing.

4.1. UNNOTCHED TENSION

In order to gather baseline data for modeling purposes, and to validate modulus and unnotched strength prediction techniques, unnotched tension tests were performed on each of the traditional and non-traditional lay-ups. The normalized results are given in Table 4.1, and the averaged strengths and deviation are shown in Figure 4.1. For both the quasi-isotropic and hard lay-ups, the traditional laminates were the strongest and stiffest.
The use of ±5° plies in the quasi-isotropic laminate and hard laminate caused a slight reduction in the average unnotched tensile strength. There was also a slight decrease in modulus associated with the use of ±5° plies. The ±10° plies caused an 8% reduction in average strength in the quasi-isotropic laminates, and an 11% reduction in the hard laminate. This was also accompanied by a decrease in modulus.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$S_{\text{max}}/S_{\text{traditional}}$</th>
<th>$E_{\text{exp}}$ (Gpa)</th>
<th>$E_{\text{th}}$ (Gpa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q-0-UNT-1</td>
<td>0.934</td>
<td>56.54</td>
<td>53.23</td>
</tr>
<tr>
<td>Q-0-UNT-2</td>
<td>0.926</td>
<td>54.47</td>
<td>53.23</td>
</tr>
<tr>
<td>Q-0-UNT-3</td>
<td>0.948</td>
<td>--</td>
<td>53.23</td>
</tr>
<tr>
<td>Average Q-0-UNT</td>
<td>.936</td>
<td>55.51</td>
<td>53.23</td>
</tr>
<tr>
<td>Q-5-UNT-1</td>
<td>0.870</td>
<td>54.88</td>
<td>52.54</td>
</tr>
<tr>
<td>Q-5-UNT-2</td>
<td>0.941</td>
<td>55.43</td>
<td>52.54</td>
</tr>
<tr>
<td>Q-5-UNT-3</td>
<td>0.862</td>
<td>--</td>
<td>52.54</td>
</tr>
<tr>
<td>Average Q-5-UNT</td>
<td>0.891</td>
<td>55.16</td>
<td>52.54</td>
</tr>
<tr>
<td>Q-10-UNT-1</td>
<td>0.841</td>
<td>52.88</td>
<td>50.61</td>
</tr>
<tr>
<td>Q-10-UNT-2</td>
<td>0.856</td>
<td>49.92</td>
<td>50.61</td>
</tr>
<tr>
<td>Q-10-UNT-3</td>
<td>0.902</td>
<td>--</td>
<td>50.61</td>
</tr>
<tr>
<td>Average Q-10-UNT</td>
<td>0.866</td>
<td>51.40</td>
<td>50.61</td>
</tr>
<tr>
<td>H-0-UNT-1</td>
<td>0.794</td>
<td>94.56</td>
<td>94.67</td>
</tr>
<tr>
<td>H-0-UNT-2</td>
<td>0.791</td>
<td>94.08</td>
<td>94.67</td>
</tr>
<tr>
<td>H-0-UNT-3</td>
<td>0.796</td>
<td>--</td>
<td>94.67</td>
</tr>
<tr>
<td>Average H-0-UNT</td>
<td>0.794</td>
<td>94.32</td>
<td>94.67</td>
</tr>
<tr>
<td>H-5-UNT-1</td>
<td>0.734</td>
<td>90.89</td>
<td>93.01</td>
</tr>
<tr>
<td>H-5-UNT-2</td>
<td>0.750</td>
<td>91.27</td>
<td>93.01</td>
</tr>
<tr>
<td>H-5-UNT-3</td>
<td>0.755</td>
<td>--</td>
<td>93.01</td>
</tr>
<tr>
<td>Average H-5-UNT</td>
<td>0.746</td>
<td>91.08</td>
<td>93.01</td>
</tr>
<tr>
<td>H-10-UNT-1</td>
<td>0.706</td>
<td>88.75</td>
<td>88.05</td>
</tr>
<tr>
<td>H-10-UNT-2</td>
<td>0.710</td>
<td>89.27</td>
<td>88.05</td>
</tr>
<tr>
<td>H-10-UNT-3</td>
<td>0.708</td>
<td>--</td>
<td>88.05</td>
</tr>
<tr>
<td>Average H-10-UNT</td>
<td>0.708</td>
<td>89.01</td>
<td>88.05</td>
</tr>
</tbody>
</table>
Also given Table 4.1 is the theoretical longitudinal modulus for each laminate, calculated using classical lamination theory based on the properties listed in Table 4.2 [52]. These properties. From previous discussions with Boeing, it was learned that classical lamination theory performs well with this material system, and as such, following an initial validation of CLT for each lay-up, no further experimental modulus determination was performed.
Representative images of the fractured specimens are given below. In general, failure modes were quite complex and did not yield a clear picture of where failure initiated or why it occurred. In several cases, the specimen failed in the gage length and at the grips. Some general trends emerged, however. The unnotched quasi-isotropic specimens fractured at a 45° angle, as shown in Figure 4.2. This could be indicative of a more brittle response, and would indicate failure along a plane of maximum shear stress. Conversely, the non-traditional quasi-isotropic laminates failed along a line perpendicular to the applied load, as can be seen in Figure 4.3 and Figure 4.4.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{11}$</td>
<td>142.0 GPa (20.6 msi)</td>
</tr>
<tr>
<td>$E_{22}$</td>
<td>7.79 GPa (1.13 msi)</td>
</tr>
<tr>
<td>$G_{12}$</td>
<td>3.99 GPa (0.58 msi)</td>
</tr>
<tr>
<td>$u_{12}$</td>
<td>0.34</td>
</tr>
<tr>
<td>$t_{plv}$</td>
<td>.178 mm (7 mil)</td>
</tr>
<tr>
<td>$\sigma_{1T}$</td>
<td>2.48 GPa (360 ksi)</td>
</tr>
<tr>
<td>$\sigma_{1C}$</td>
<td>1.38 GPa (200 ksi) (initial analysis)</td>
</tr>
<tr>
<td>$\sigma_{2T}$</td>
<td>33.0 MPa (4.79 ksi)</td>
</tr>
<tr>
<td>$\sigma_{2C}$</td>
<td>206.8 MPa (30 ksi)</td>
</tr>
<tr>
<td>$\tau_{12}$</td>
<td>100.0 MPa (14.5 ksi)</td>
</tr>
</tbody>
</table>
Failure in the hard laminates was characterized by delamination. Figure 4.5 shows the delamination occurring in the traditional laminate across the entire gage length. Interlaminar stresses at the free edge resulted in delamination at the 90° plies, which extended approximately 13mm into the specimen prior to failure. At failure the specimen delaminated explosively. A similar failure was observed in the non-traditional (±10°) hard laminate, as shown in Figure 4.6. Delaminations extended approximately 10mm into the specimen, again at the 90° plies. At failure, 105mm of the center portion of the specimen delaminated explosively, accompanied by fracture of the specimen at two locations in the gage length, perpendicular to the load.
The unnotched tensile strengths of the traditional quasi-isotropic and hard laminates were less than those provided by Boeing. There was a 6% difference between Boeing’s quasi-isotropic unnotched values and those found at Georgia Tech, and a 20% difference between the hard laminate strengths. This is most likely attributable to the serrated grips used for testing, as the majority of specimens failed at least partially at the grips. This is a common problem in testing of unnotched composites, and typically servo-hydraulic Surfalloy-coated grips are used to minimize grips failures. Bonded tabs were utilized in the grip region for some tests, and little difference in strength was observed.

In general, however, there was relatively little scatter in data, and several specimens did fail in the gage length. Additionally, the observed trends are consistent with what would be expected, giving credibility to the results. Despite any discrepancies, the results provide a comparison of traditional and non-traditional unnotched tensile strength.
4.2. OPEN HOLE TENSION

The following sections describe the results of the quasi-isotropic and hard laminate open hole tension tests. The tests were conducted as described in 3.3.1.

4.2.1. QUASI-ISOTROPIC OPEN HOLE TENSION

Table 4.3 summarizes the normalized results of the quasi-isotropic open hole tension tests, and the averaged results and deviations are shown in Figure 4.7. Figure 4.8 provides a comparison of the mechanical response of the traditional and non-traditional composite laminates. The traditional quasi-isotropic laminates had the highest failure loads, slightly exceeding the strength of the ±5° quasi-isotropic and ±10° quasi-isotropic non-traditional laminates. The non-traditional laminates were also less stiff than the traditional laminates, and in general had a higher strain-to-failure, as seen in Figure 4.8.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$S_{\text{max}}/S_{\text{trad}}$</th>
<th>$\varepsilon_{\text{max}}/\varepsilon_{\text{trad}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q-0-OHT-1</td>
<td>0.528</td>
<td>0.480</td>
</tr>
<tr>
<td>Q-0-OHT-2</td>
<td>0.506</td>
<td>0.496</td>
</tr>
<tr>
<td>Q-0-OHT-3</td>
<td>0.528</td>
<td>0.538</td>
</tr>
<tr>
<td>Q-0-OHT-4</td>
<td>0.533</td>
<td>NA (X-ray)</td>
</tr>
<tr>
<td>Q-0-OHT-5</td>
<td>0.533</td>
<td>NA (X-ray)</td>
</tr>
<tr>
<td><strong>Average Q-0-OHT</strong></td>
<td><strong>0.526</strong></td>
<td><strong>0.505</strong></td>
</tr>
<tr>
<td>Q-5-OHT-1</td>
<td>0.497</td>
<td>0.445</td>
</tr>
<tr>
<td>Q-5-OHT-2</td>
<td>0.528</td>
<td>0.509</td>
</tr>
<tr>
<td>Q-5-OHT-3</td>
<td>0.536</td>
<td>0.560</td>
</tr>
<tr>
<td>Q-5-OHT-4</td>
<td>0.527</td>
<td>NA (X-ray)</td>
</tr>
<tr>
<td><strong>Average Q-5-OHT</strong></td>
<td><strong>0.522</strong></td>
<td><strong>0.505</strong></td>
</tr>
<tr>
<td>Q-10-OHT-1</td>
<td>0.488</td>
<td>0.547</td>
</tr>
<tr>
<td>Q-10-OHT-2</td>
<td>0.505</td>
<td>0.501</td>
</tr>
<tr>
<td>Q-10-OHT-3</td>
<td>0.498</td>
<td>0.470</td>
</tr>
<tr>
<td>Q-10-OHT-4</td>
<td>0.479</td>
<td>NA (X-ray)</td>
</tr>
<tr>
<td><strong>Average Q-10-OHT</strong></td>
<td><strong>0.493</strong></td>
<td><strong>0.506</strong></td>
</tr>
</tbody>
</table>
The results of the die penetrant enhanced X-ray radiography are discussed next. X-ray investigation began at the anticipated onset of damage in the traditional laminate, based on the stress concentration factor. The stress concentration factor can be calculated

Figure 4.7 Average Quasi-Isotropic Open Hole Tensile Strengths

Figure 4.8 Comparison of traditional and non-traditional quasi-isotropic open hole tension response

...
as 3.00 using equation 2.18, but for a quasi-isotropic material it is the same as that found in an isotropic material with a circular hole. The specimens were X-rayed at regular intervals until failure. The traditional and non-traditional laminates were X-rayed at the same load levels to allow direct comparison of damage progression in each specimen. The load levels at each increment are given as percentages of the ultimate failure load of the specimen tested.

Prior to discussing damage observed in X-ray images, it is convenient to describe an orientation system for referencing damage location. A conventional right-hand coordinate system is chosen such that the x axis and 0° corresponds with the load direction, the y axis and 90° corresponds with the transverse direction, and the z axis is through the thickness. The location of damage can then be referenced in degrees with regards to the x axis.

![Orientation System Diagram](image)

**Figure 4.9 Definition of Orientation System**

4.2.1.1 Traditional Quasi-Isotropic Open Hole Tension Response

Figure 4.10 shows the results of the radiographic inspection of the traditional quasi-isotropic laminate loaded in open hole tension. Initial damage occurred in the form of cracking in the 90° plies, with small cracks emanating from the edge of the hole. The 90° ply cracks observed in Figure 4.10a vary in length up to 2.8mm. As loading
progressed, these 90° cracks continue to increase in density and length in the region defined from approximately 45° above and below the y axis. This can be observed in Figure 4.10 b-d. At 78% of the failure load, the 90° ply cracking had grown from the notch edge to the free edge of the specimen.

Small cracks were also observed in the 45° plies at the initial load increment. These cracks were oriented at approximately 60° and 240°. The length of these cracks at the initial increment was 1.5mm. These cracks were less dense than the 90° ply cracks, and exhibited a preferential orientation; cracking in the -45° plies was not observed until higher load levels. These cracks grew in length and density until failure. The longest of these cracks observed in Figure 4.10d was measured as 12.2mm.

At roughly half of the failure load, 0° ply cracking occurred in the form of single cracks growing from the y axis. Cracks in multiple plies are most likely superimposed over each other, but it is not possible to determine that from these images. The longitudinal cracks had a preferential orientation, with the crack at 90° growing fastest in the negative x direction, and the crack at 270° growing fastest in the positive x direction. In Figure 4.10b, the longest longitudinal ply crack was measured as 3.30mm, and eventually grew to a length of 5.6mm as seen in Figure 4.10d.

Figure 4.11 is a representative stress-strain response of a traditional quasi-isotropic specimen loaded in open hole tension. Despite substantial damage occurring in the ±45° and 90° plies prior to failure, little change is noticed in slope of the stress-strain curve in. This can be attributed to the dominance of the 0° plies.
Figure 4.10 Radiographic images of traditional quasi-isotropic open hole tension specimen at a) 36% failure load, b) 57% failure load, c) 78% failure load, and d) 93% failure load.

Figure 4.11 Typical open hole tension stress-strain response of traditional quasi-isotropic laminate.
4.2.1.2 Non-Traditional (±5°) Quasi-Isotropic Open Hole Tension Response

Figure 4.12 shows the results of the radiographic inspection of the non-traditional quasi-isotropic laminate with the ±5° longitudinal plies, loaded in open hole tension. The initial load level and subsequent intervals were the same as that of the traditional quasi-isotropic laminate.

As in the traditional laminate, initial damage occurred in the form of cracking in the 90° plies, and its location and growth are consistent with that in the traditional laminate. Small cracks can be seen in Figure 4.12a, varying in length up to 3.30mm, and as loading progressed, these cracks continue to increase in density and length. At approximately 80% of the failure load, the 90° ply cracking had grown from the notch edge to the free edge of the specimen.

Initial 45° ply cracking was observed in Figure 4.12a, oriented at 45° and 240°. The length was measured as 1.78mm. By the next load increment, 45° damage had increased, with cracks located at 45°, 135°, and 225°. Similar to the traditional laminate, a preferential orientation was observed, but by the second load increment -45° damage had initiated at 135° and 315°. ±45° damage grew in length and density, and longest of these cracks observed in Figure 4.12d was measured as 7.62mm.

At 60% of the failure load, ±5° ply cracking occurred in the form of cracks growing from the y axis. These cracks initiated at 90° and grew in the negative x direction, and at 270° and grew in the positive x direction. At each location, the cracks in the +5° and -5° plies are clearly distinguishable, as opposed to what appears to be a single crack in the traditional laminate. In Figure 4.12b, the longest longitudinal ply crack was measured as 2.03mm, and eventually grew to a length of 3.56mm. By 80% of the failure
load, longitudinal ply cracking had grown in the positive x and negative x directions at both 90° and 270°, but the initial cracks continued to be the largest.

As in the traditional laminate, little change was observed in the slope of the stress-strain curve. The slope of the curve is slightly less than that of the traditional laminate, but still extremely linear despite the damage occurring in the ±45° and 90°.

Figure 4.12 Radiographic images of non-traditional (±5°) quasi-isotropic open hole tension specimen at a) 36% failure load, b) 58% failure load, c) 79% failure load, and d) 93% failure load
4.2.1.3 Non-Traditional (±10°) Quasi-Isotropic Open Hole Tension Response

Figure 4.14 shows the results of the radiographic inspection of the non-traditional quasi-isotropic laminate with the ±10° plies, loaded in open hole tension. The initial load level and subsequent intervals were the same as that of the traditional quasi-isotropic laminate.

As in the previous two cases, the initial damage occurred in the form of cracking in the 90° plies, with small cracks emanating from the edge of the hole. The longest of the 90° ply cracks observed in Figure 4.14a is 4.06mm. As loading progressed, these 90° cracks increased in density and length in a manner similar to the previous two laminates. At approximately 60% of the failure load, the 90° ply cracking had grown from the notch edge to the free edge of the specimen.
45° cracking, oriented at 90° and 270°, can also be seen in Figure 4.14a. As before, they exhibited a preferential orientation as cracking in the -45° plies was not observed until higher load levels. The longest of these cracks at the final load increment measured 14.2mm.

By 60% of the failure load, longitudinal ply cracking occurred in the form of pairs of cracks growing from the y axis. These exhibited a preferential orientation, with the crack in the 10° ply at 90° growing fastest in the negative x direction, and the crack in the -10° ply at 270° growing fastest in the positive x direction. In Figure 4.14b, the longest longitudinal ply crack was measured as 2.0mm; this crack eventually grew to a length of 3.8mm as seen in Figure 4.14d.

As with the traditional and non-traditional quasi-isotropic laminates, there is little change in the slope of the stress-strain curve given in Figure 4.15. This indicates that the use of off-axis plies decreases the stiffness of the laminate, but has little effect on the linearity of the mechanical response.
Figure 4.14 Radiographic images of non-traditional (±10°) quasi-isotropic open hole tension specimen at a) 40% failure load, b) 64% failure load, c) 87% failure load, and d) 100% failure load.

Figure 4.15 Typical open hole tension stress-strain response of non-traditional (±10°) quasi-isotropic laminate.
4.2.2. HARD OPEN HOLE TENSION

Table 4.4 and Figure 4.16 summarize the results of the hard laminate open hole tension tests. There was a significant difference in the OHT strength of the traditional and non-traditional laminates. The laminate with the $\pm 5^\circ$ longitudinal plies experienced an approximately 10% decrease in average strength, and the average strength of the laminates with $\pm 10^\circ$ longitudinal plies was 18.5% less than that of the traditional hard laminate. Additionally, the non-traditional ($\pm 10^\circ$) laminate had a significantly lower strain-to-failure. As in the quasi-isotropic laminates, the non-traditional laminates were less stiff than the traditional laminates, as shown in Figure 4.17.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$S_{\text{max}}/S_{\text{trad}}$</th>
<th>$\varepsilon_{\text{max}}/\varepsilon_{\text{trad}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>H-0-OHT-1</td>
<td>0.474</td>
<td>0.464</td>
</tr>
<tr>
<td>H-0-OHT-2</td>
<td>0.460</td>
<td>0.433</td>
</tr>
<tr>
<td>H-0-OHT-3</td>
<td>0.486</td>
<td>0.438</td>
</tr>
<tr>
<td>H-0-OHT-4</td>
<td>0.498</td>
<td>NA (X-ray)</td>
</tr>
<tr>
<td><strong>Average H-OHT</strong></td>
<td><strong>0.480</strong></td>
<td><strong>0.445</strong></td>
</tr>
<tr>
<td>H-5-OHT-1</td>
<td>0.433</td>
<td>0.446</td>
</tr>
<tr>
<td>H-5-OHT-2</td>
<td>0.437</td>
<td>0.445</td>
</tr>
<tr>
<td>H-5-OHT-3</td>
<td>0.430</td>
<td>0.451</td>
</tr>
<tr>
<td>H-5-OHT-4</td>
<td>0.432</td>
<td>NA (X-ray)</td>
</tr>
<tr>
<td><strong>Average H-5-OHT</strong></td>
<td><strong>0.433</strong></td>
<td><strong>0.447</strong></td>
</tr>
<tr>
<td>H-10-OHT-1</td>
<td>0.394</td>
<td>0.419</td>
</tr>
<tr>
<td>H-10-OHT-2</td>
<td>0.387</td>
<td>0.404</td>
</tr>
<tr>
<td>H-10-OHT-3</td>
<td>0.386</td>
<td>0.427</td>
</tr>
<tr>
<td>H-10-OHT-4</td>
<td>0.397</td>
<td>NA (X-ray)</td>
</tr>
<tr>
<td><strong>Average H-10-OHT</strong></td>
<td><strong>0.391</strong></td>
<td><strong>0.417</strong></td>
</tr>
</tbody>
</table>
The results of the radiographic inspection of the hard open hole tension specimens are discussed in the following sections. Inspections began at the anticipated onset of damage in the traditional laminate, based on a stress concentration factor of 4.11.
Because damage was not observed at this load level, loading was increased until such a time that it was. Inspection then occurred at regular intervals, with both the traditional and non-traditional laminates being x-rayed at the same load levels. The load in each X-ray is given as a percentage relative to the failure load of the specimen tested.

4.2.2.1 Traditional Hard Open Hole Tension Response

Figure 4.18 shows the results of the inspection of the traditional hard laminate loaded in open hole tension. At 32% of the failure load, damage had occurred in the 90\(^\circ\), ±45\(^\circ\), and 0\(^\circ\) plies. The 90\(^\circ\) ply cracks observed in Figure 4.18a vary in length up to 1.78mm. As loading progressed, these 90\(^\circ\) cracks increase in density and length in the region defined from approximately 45\(^\circ\) above and below the y axis. At approximately 75% of the failure load, the 90\(^\circ\) ply cracking had grown from the notch edge to the free edge of the specimen.

Small cracks were observed in the 45\(^\circ\) and -45\(^\circ\) plies at the initial load increment. These cracks were oriented at approximately 45\(^\circ\), 135\(^\circ\), 225\(^\circ\), and 315\(^\circ\). The longest crack length at the initial increment was 1.5mm. These cracks initially grew outward from the hole, but eventually cracking in 45\(^\circ\) and -45\(^\circ\) plies intersected to form triangular regions at the notch, as seen in Figure 4.18c. Additional ±45\(^\circ\) damage grew in length up to 5.6mm. +45\(^\circ\) and -45\(^\circ\) cracks can also be seen extending from the longitudinal ply cracks in Figure 4.18b, with lengths of approximately 1.3mm.

At the initial load increment, 0\(^\circ\) ply cracking can be seen as single cracks growing from the y axis. These grew in the positive and negative x direction, eventually reaching
a length of 10.1mm measured from the axis. At high load levels, this damage became visible on the surface of the specimen, as seen in Figure 4.19.

The stress-strain curve of a typical traditional hard open hole tension test is given in Figure 4.20. The curve is linear throughout most of the loading, indicating damage in the 90° and ±45 plies had little effect on mechanical response of the specimen.

Figure 4.18 Radiographic images of traditional hard open hole tension specimen at a)32% failure load, b) 63% failure load, c)76% failure load, and d) 96% failure load
Figure 4.19  Longitudinal ply cracking observed on surface ply of traditional hard OHT laminate

Figure 4.20  Typical open hole tension stress-strain response of traditional hard laminate
4.2.2.2 Non-Traditional (±5°) Hard Open Hole Tension Response

Figure 4.21 shows the results of the radiographic inspection of the traditional hard laminate loaded in open hole tension. Initial inspection occurred at 36% of the failure load, and subsequent load levels corresponded with those in the traditional hard laminate.

Damage was observed in the 90° plies in Figure 4.21a, with small cracks emanating from the edge of the hole. The 90° ply cracks vary in length up to 4.1mm. By 73% of the failure load, the 90° ply cracking had grown from the notch to the free edge of the specimen.

Cracking in the 45° plies was also observed at the initial load increment. These cracks were oriented at approximately 45° and 225°. The length of these cracks at this load level was 2.0mm. Cracking in the -45° plies, seen in Figure 4.21c, were not observed until higher load levels. Prior to failure, the longest of the ±45° cracks eventually reached a length of 4.6mm.

5° ply cracking can be seen in Figure 4.21b. The cracks grow from a length of 1.8mm in Figure 4.21b to a length of 4.8mm in Figure 4.21d.

The open hole tension stress-strain response of a non-traditional (±5°) hard laminate is shown in Figure 4.22. The response is linear with little change in slope prior to failure.
Figure 4.21 Radiographic images of non-traditional (±5°) hard open hole tension specimen at a) 36% failure load, b) 55% failure load, c) 73% failure load, and d) 88% failure load.

Figure 4.22 Typical open hole tension stress-strain response of non-traditional (±5°) hard composite laminate.
4.2.2.3 Non-Traditional (±10°) Hard Open Hole Tension Response

Figure 4.23 shows the results of the radiographic inspection of the non-traditional (±10°) hard laminate loaded in open hole tension. Initial inspection occurred at 40% of the failure load, and occurred at regular intervals corresponding to the load levels used for the traditional hard laminate.

Initial damage occurred in the form of cracking in the 90° plies, with small cracks emanating from the edge of the hole. The 90° ply cracks observed in Figure 4.23a vary in length up to 2.3mm. These 90° ply cracks have grown from the notch edge to the free edge of the specimen by approximately 80% of the failure load.

45° ply cracking at the initial load increment was located at approximately 60° and 225°, and measured 1.8mm. -45° ply cracking is observed at the next load increment. By failure the longest of the ±45° cracks was measured as 5.1mm.

At approximately 60% of the failure load, longitudinal ply cracking was observed. +10° and -10° cracks can be seen growing at 90° and 270°. The length of the longest longitudinal ply crack at failure was 4.3mm, seen in Figure 4.23d.

Despite substantial damage occurring in the ±45° and 90° plies prior to failure, little change is noticed in slope of the stress-strain curve in Figure 4.24. This can be attributed to the dominance of the ±10° plies.
Figure 4.23  Radiographic images of non-traditional (±10°) hard open hole tension specimen at a) 40% failure load, b) 59% failure load, c) 79% failure load, and d) 100% failure load

Figure 4.24  Typical open hole tension stress-strain response of non-traditional (±10°) hard composite laminate
4.2.2.3.1 Edge Delamination in Hard Open Hole Tension Specimens

Edge delamination was evident in the x-ray images in Figure 4.18d and Figure 4.21d, but it is difficult to characterize the exact amount present. As such, one of each type of hard OHT laminate was sectioned during post-failure inspection of the specimens. Since delamination was comparable on both halves of the broken specimen, only one half of each was analyzed. The specimen was cut into 1.25cm sections using a silicon carbide cutting disc on a water-cooled abrasive saw. Damage length was then measured at these points under a stereomicroscope. Results are shown in Figure 4.25 and Figure 4.26. A definite reduction in delamination occurred in both non-traditional laminates, and no delamination was apparent in the specimen with the ±10° plies.

Figure 4.25 Cross sectional view of delamination damage in traditional hard open hole tension specimen
4.3. FILLED HOLE TENSION

The following sections present the results of the quasi-isotropic and hard laminate filled hole tension tests. The tests were conducted as described in 3.3.1, with the fasteners torqued to full clamp-up.

4.3.1. QUASI-ISOTROPIC FILLED HOLE TENSION

Table 4.5 and Figure 4.27 summarize the results of the quasi-isotropic filled hole tension tests, and a comparison of the stress-strain responses is shown in Figure 4.28. In this case, the ±5° and the ±10° plies resulted in an improvement in average strength over the traditional laminate, but the amount fell within experimental scatter. The observed stiffnesses were similar to those observed in the open hole tension case.
Table 4.5 Quasi-Isotropic Filled Hole Tension Results

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$S_{max}/S_{trad}$</th>
<th>$\varepsilon_{max}/\varepsilon_{trad}$</th>
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<td>NA (X-ray)</td>
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<tr>
<td>S-10-FHT-3</td>
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<td>NA (X-ray)</td>
</tr>
<tr>
<td>Average S-10-FHT</td>
<td>0.471</td>
<td>0.552</td>
</tr>
</tbody>
</table>

Figure 4.27 Average Quasi-Isotropic Filled Hole Tensile Strengths
Following is a discussion of the damage observed in the quasi-isotropic filled hole tension specimens, using the die penetrant enhanced X-ray radiography. X-ray investigation began at the anticipated onset of damage in the traditional laminate, and occurred at regular intervals until failure. The initial inspection point was again based on a stress concentration factor of 3.0, but inspection at this load level and subsequent load levels, indicated that damage was not occurring until higher load levels. This was expected, as the presence of a fastener with clamp-up force reduced damage around the notch due to the constraint imposed by the fastener [4, 8]. As before, the traditional and non-traditional laminates were X-rayed at the same load levels to allow direct comparison of damage progression in each specimen. The load levels are given as percentages of the ultimate failure load of the specimen tested.
4.3.1.1 Traditional Quasi-Isotropic Filled Hole Tension Response

X-ray images of the traditional quasi-isotropic filled hole tension specimens are shown in Figure 4.29. Damage onset occurred rapidly, and included 90° cracking and ±45° cracking. A prominent +45° and -45° crack formed and overlapped, creating a triangular region at the hole edges approximately 1.4mm in width by 2.9mm in height. Fiber breakage, seen in Figure 4.29b, and notch delamination occurred shortly thereafter. Slight longitudinal ply cracking at 90° and 270° is evident prior to failure. Most damage occurs in the area under the washer, and only progresses to the rest of the specimen just prior to failure.

Figure 4.29  Radiographic images of traditional quasi-isotropic filled hole tension specimen at a) 81.3% failure load, b) 97.6% failure load, and c) 99% failure load
Figure 4.30 shows the stress-strain response of the traditional quasi-isotropic filled hole tension test. As in the open hole case, the curve is quite linear. However, prior to complete failure, a small drop in load occurs. This can be attributed to failure of the constrained material under the fastener, and was accompanied by an audible popping of the specimen. Failure of this material did not result in immediate failure of the laminate, and as such loading continued, albeit briefly, until failure of the entire specimen.

![Figure 4.30 Typical filled hole tension stress-strain response of traditional quasi-isotropic laminate](image)

4.3.1.2 Non-Traditional (±5°) Quasi-Isotropic Filled Hole Tension Response

X-ray images of the non-traditional (±5°) quasi-isotropic filled hole tension specimens are shown in Figure 4.31. As in the traditional laminate, damage was not observed until much higher load levels than seen in the open hole case. Damage onset occurred rapidly, with most damage occurring between 92% and 96% of the failure load. Prominent +45° and -45° crack formed and overlapped, similar to that observed in the
traditional laminate, and was followed by fiber breakage and notch. No longitudinal ply cracking was evident in this laminate prior to failure. As before, most damage occurs in the area under the washer, and only progresses to the rest of the specimen just prior to failure.

Figure 4.31 Radiographic images of non-traditional (±5°) quasi-isotropic filled hole tension specimen at a) 84.6% failure load, b) 92.3% failure load, and c) 96.2% failure load

Figure 4.32 shows the stress-strain response of the non-traditional (±5°) quasi-isotropic filled hole tension test. The curve is generally linear until prior to failure, when a slight reduction in load occurs. This was accompanied by audible popping, and was due to failure of the constrained material under the fastener. Failure of this material did not result in immediate failure of the laminate, and loading continued to a point higher than that at which the initial load drop occurred.
4.3.1.3 Non-Traditional (±10°) Quasi-Isotropic Filled Hole Tension Response

X-ray images of the non-traditional (±10°) quasi-isotropic filled hole tension specimens are shown in Figure 4.33. As in the previous two laminates, damage was not observed until close to failure. The observed damage was consistent with the traditional case, with severe ±45° and 90° damage occurring triangular regions at the hole edges. This was followed by delamination and fiber breakage, seen in Figure 4.33b. No longitudinal ply cracking is evident prior to failure. Initial damage occurs in the area under the washer, but can be seen progressing to the rest of the specimen in Figure 4.33b prior to failure.
Figure 4.33  Radiographic images of non-traditional (±10°) quasi-isotropic filled hole tension specimen at a) 82% failure load, and b) 98.4% failure load

Figure 4.34 shows the stress-strain response of the non-traditional (±10°) quasi-isotropic filled hole tension test. The curve is linear, but experiences a slight reduction in load prior to failure. Subsequent material response was less stiff than that of the undamaged material, but loading continued until a point higher than the initial load drop before ultimately failing.

Figure 4.34  Typical filled hole tension stress-strain response of non-traditional (±10°) quasi-isotropic laminate
4.3.2. HARD FILLED HOLE TENSION

The results of the hard filled hole tension tests are given in Table 4.6 and Figure 4.35. There was a slight reduction in average strength associated with the use of non-traditional laminates, but these fell within experimental scatter. Both non-traditional laminates also had a lower strain-to-failure than the traditional laminates. These trends are shown in Figure 4.36. There was a significant reduction in the strength of all laminates when compared to the open hole case, with a fastener resulting in a 23% decrease in strength in the traditional laminate.

Table 4.6 Hard Laminate Filled Hole Tension Results

<table>
<thead>
<tr>
<th>Specimen</th>
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<th>$\varepsilon_{\text{max}}/\varepsilon_{\text{trad}}$</th>
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<td><strong>Average H-FHT</strong></td>
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<td>H-5-FHT-3</td>
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<td>H-5-FHT-4</td>
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<td>NA (X-ray)</td>
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<tr>
<td><strong>Average H-5-FHT</strong></td>
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<td><strong>0.368</strong></td>
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<td>H-10-FHT-2</td>
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<td>H-10-FHT-3</td>
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<td>H-10-FHT-4</td>
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<tr>
<td><strong>Average H-10-FHT</strong></td>
<td><strong>0.337</strong></td>
<td><strong>0.360</strong></td>
</tr>
</tbody>
</table>
The x-ray images of the hard filled hole tension specimens are discussed below. Radiographic inspection began when damage initiation was predicted based on the stress concentration factor, but no damage was observed until higher load levels. Inspection
occurred at regular intervals until failure. Load levels for each image are given as percentages of the failure load of the x-rayed specimen.

4.3.2.1 Traditional Hard Filled Hole Tension Response

X-ray images of the traditional hard filled hole tension specimens are shown in Figure 4.37. Longitudinal ply cracking, located at 0°, 90°, and 270° and measuring 3.2mm in length, was observed at 70% of the failure load. Slight ±45° damage was also present at this load level. At the next load increment, the initial longitudinal ply cracks had increased in length, and additional longitudinal cracks had formed at the edge of the washer, measuring 3.4mm in length. The amount of ±45° damage had also increased. Additionally, very pronounced cracks oriented perpendicular to the load had also formed. Because of their severity and appearance close to the time of failure, it is believed these represent fiber breakage in the 0° plies. These cracks extend to the edge of the washer.

Figure 4.38 shows the stress-strain response of the traditional hard filled hole tension test. The curve is linear until failure.

![Figure 4.37 Radiographic images of traditional hard filled hole tension specimen at a)70% failure load, and b) 96% failure load](image-url)
4.3.2.2 Non-Traditional (±5°) Hard Filled Hole Tension Response

X-ray images of the non-traditional (±5°) hard filled hole tension specimens are shown in Figure 4.39. At 80% of the failure load, slight longitudinal ply cracking was observed at 90° and 270°, measuring 1.5mm in length. Slight 90° and ±45° damage was also observed at this increment. By 96% of the failure load, the longitudinal cracks had grown in length to 3.3mm. ±45° and 90° damage had also increased, though it was confined to the region under the washer. Very pronounced cracks oriented perpendicular to the load were seen at 90°, and were measured as 1.3mm. It is believed that these are cracks in the longitudinal plies.

Figure 4.40 shows the stress-strain response of the non-traditional (±5°) hard filled hole tension test. The curve is generally linear, but prior to complete failure a small drop in load occurs. This can be attributed to failure of the constrained material under the
fastener, and was accompanied by an audible popping of the specimen. This material did not constitute failure of the laminate, and loading continued until failure of the entire specimen.

Figure 4.39 Radiographic images of non-traditional (±5°) hard filled hole tension specimen at a) 80% failure load, and b) 96% failure load

![Radiographic images of non-traditional (±5°) hard filled hole tension specimen](image)

Figure 4.40 Typical filled hole tension stress-strain response of a non-traditional (±5°) hard laminate

![Typical filled hole tension stress-strain response](image)

4.3.2.3 Non-Traditional (±10°) Hard Filled Hole Tension Response

X-ray images of the non-traditional (±10°) hard filled hole tension specimens are shown in Figure 4.41. Initial inspection showed 1.1mm longitudinal ply cracks at 90° and 270°. Slight 90° and ±45° damage was also observed at this increment. By 95% of
the failure load, the longitudinal cracks were measured as 3.0mm in length. Small cracks oriented perpendicular to the load were observed at approximately 100° and 260°. These were measured as approximately 0.3mm. It is believed that these are cracks in the longitudinal plies.

Figure 4.42 shows the stress-strain response of the traditional hard filled hole tension test. The curve is linear until failure.

Figure 4.41 Radiographic images of non-traditional (±10°) hard filled hole tension specimen at a) 74% failure load, b) 95% failure load

Figure 4.42 Typical filled hole tension stress-strain response of non-traditional (±10°) hard laminate
4.4. OPEN HOLE COMPRESSION

The subsequent sections present the results of the quasi-isotropic and hard laminate open hole compression tests. These tests were conducted using the procedures described in 3.3.2.

4.4.1. QUASI-ISOTROPIC OPEN HOLE COMPRESSION

The results of the quasi-isotropic open hole tension tests are given below in Table 4.7 and Figure 4.43. The traditional laminates exhibited the highest strength, with the use of ±5° and ±10° plies causing slight decreases in strength. Due to the bulk of the fixture, it proved difficult to mount an extensometer across the notch to measure strain. As such, strain-to-failure results are calculated using the equation:

\[ \varepsilon = \frac{P}{EA} \]  

(4.1)

The simplification is justified by the relatively linear response of the composites, and is useful for comparison purposes.

As discussed 2.1.2.1, failure in open hole compression occurs due to fiber microbuckling. Because this buckling mechanism and its subsequent growth are by nature unstable, tracking damage progression in OHC via x-ray radiography proved very difficult. In some cases, no damage was detected prior to failure. In general, detection of a microbuckle implied failure was imminent, and further x-raying at multiple increments was often not feasible due to time and cost constraints.
Table 4.7 Results of Quasi-Isotropic Open Hole Tension Tests

<table>
<thead>
<tr>
<th>Specimen</th>
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<td><strong>Average S-0-OHC</strong></td>
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<td><strong>0.545</strong></td>
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</table>

Figure 4.43 Average Quasi-Isotropic Open Hole Compression Strengths
Following is a discussion of the results of the radiographic inspection of the quasi-isotropic open hole compression specimens. Initial inspections occurred at approximately half of the anticipated failure load, and proceeded incrementally until damage was observed. In all cases, damage not was observed until at least 80% of the failure load was reached.

4.4.1.1 Traditional Quasi-Isotropic Open Hole Compression Response

Figure 4.44 shows a open hole quasi-isotropic specimen loaded in compression, at 96% of its failure load. Longitudinal ply cracking can be seen extending in the positive x direction at 90° and extending in the negative x direction at 270°. 0° damage can also be seen at 0°, extending in the positive x direction. No other matrix damage was observed. The crack extending from the right side of the hole is a microbuckle. This was confirmed by inspection of the specimen under the stereomicroscope, which showed fibers buckling into the hole diameter (Figure 4.45). At this load level, it had reached a length of 2.4mm. Failure occurred shortly thereafter, with a sudden and complete loss of load-bearing ability.

Figure 4.44 Radiographic images of traditional quasi-isotropic open hole compression specimen at 96% failure load
4.4.1.2 Non-Traditional (±5°) Quasi-Isotropic Open Hole Compression Response

Figure 4.46 shows a non-traditional (±5°) quasi-isotropic specimen loaded in open hole compression. At the initial load, slight 45° damage can be seen located at 90°. No longitudinal ply cracking or any other matrix damage is seen at this increment. A microbuckle can be seen initiating at the left hand side of the hole. It is 1.8mm in length at this point. At 96% of its failure load, this microbuckle had grown in length to 2.8mm. Additionally, another microbuckle has formed on the left side, and one on the right side, with lengths of 3.8mm and 2.5mm, respectively. The microbuckles on the left side are accompanied by delamination of approximately 1.3mm in width. Failure occurred shortly after the second x-ray was taken.
4.4.1.3 Non-Traditional ($\pm 10^\circ$) Quasi-Isotropic Open Hole Compression Response

Figure 4.47 shows a non-traditional ($\pm 10^\circ$) quasi-isotropic laminate loaded in open hole compression, at 94% of its failure load. Slight 45° ply cracking can be seen at 90°. No other matrix damage was observed. At this load level, a microbuckle is seen extending from the left side of the hole with a length of 1.2mm. Failure occurred shortly thereafter.

4.4.2. HARD LAMINATE OPEN HOLE COMPRESSION

The results of the hard laminate open hole compression tests are given in Table 4.8 and Figure 4.48. Again, the traditional laminates were the strongest. The use of $\pm 5^\circ$
plies caused a 12% reduction in average strength, and the ±10° plies resulted in a 15% reduction in average strength.

Table 4.8 Results of Hard Laminate Open Hole Compression Tests

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<tr>
<th>Specimen</th>
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<tr>
<td>H-10-OHC-2</td>
<td>0.417</td>
<td>0.448</td>
</tr>
<tr>
<td>H-10-OHC-3</td>
<td>0.445</td>
<td>0.479</td>
</tr>
<tr>
<td>H-10-OHC-4</td>
<td>0.442</td>
<td>0.476</td>
</tr>
<tr>
<td><strong>Average H-10-OHC</strong></td>
<td><strong>0.428</strong></td>
<td><strong>0.461</strong></td>
</tr>
</tbody>
</table>

Figure 4.48 Average Hard Open Hole Compression Strengths
Following is a discussion of the results of the radiographic inspection of the hard open hole compression specimens. Initial inspections occurred at approximately half of the anticipated failure load, and proceeded incrementally until damage was observed. In all cases, no damage was observed until at least 80% of the failure load was reached.

4.4.2.1 Traditional Hard Open Hole Compression Response

Figure 4.49 shows a traditional hard laminate specimen loaded in open hole compression to 94% of its failure load. Longitudinal ply cracking can be seen extending in the positive x direction and 90° and in the negative x direction at 270°. No other matrix damage is seen at this increment. A microbuckle can be seen initiating at the left hand side of the hole, and has grown to 1.2mm in length at this point. At 99% of its failure load, the microbuckle has grown in length to 7.6mm. The microbuckle is accompanied by delamination approximately 2.2mm in width. Failure occurred shortly after the x-ray was taken, occurring with a sudden and complete loss of load-bearing ability.

![Figure 4.49 Radiographic images of traditional hard open hole compression specimen at a) 94% failure load and b) 99% failure load](image)

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4.4.2.2 Non-Traditional (±10°) Hard Open Hole Compression Response

Figure 4.50 shows a non-traditional (±10°) hard laminate loaded in open hole compression, at 96% of its failure load. No matrix damage can be seen, but microbuckles can be seen extending from the left and right side of the hole. The microbuckle on the right is 1.9mm in length, and the microbuckle on the left is 1.1mm in length. Failure occurred shortly thereafter, with a sudden and complete loss of load-bearing ability.

![Figure 4.50](image)

Figure 4.50  Radiographic images of non-traditional (±10°) quasi-isotropic open hole compression specimen at 96% failure load

4.5. SINGLE SHEAR BEARING

The following sections describe the results of the single shear bearing tests, performed as described in 3.3.3 using half clamp-up force. In addition to the maximum stress reached, an offset strength is given. While the initial response of a bolt bearing specimen is linear, eventually crushing occurs at the bearing surface, the hole elongates, and the fastener rotates, all leading to a non-linear response. As such, an offset strength often used as a design value and is calculated in addition to finding the maximum load. The offset strength is based on the slope of the linear portion of the stress-strain curve, offset by some arbitrary criterion. In this research, the offset is based on an acceptable amount of hole deformation, which is 2% of the hole diameter. The offset strength is shown graphically in Figure 4.51.
Unlike previous tests, failure of these specimens was not indicated by complete loss of load carrying ability. The tests were run until a clear maximum load had been reached, and then the test was terminated to avoid excessive crushing of the bearing surface.

4.5.1. QUASI-ISOTROPIC SINGLE SHEAR BEARING

The results of the quasi-isotropic single shear bearing tests are given in Table 4.9 and Figure 4.52. The non-traditional laminates increased both the average maximum stress and the offset strength in the quasi-isotropic laminate, although experimental scatter was significant.
Table 4.9 Results of quasi-isotropic single shear bearing tests

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$S_{\text{max}}/S_{\text{trad}}$</th>
<th>$S_{0.02}/S_{\text{trad}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-0-SSB-1</td>
<td>0.828</td>
<td>0.798</td>
</tr>
<tr>
<td>S-0-SSB-2</td>
<td>0.939</td>
<td>0.900</td>
</tr>
<tr>
<td>S-0-SSB-3</td>
<td>0.859</td>
<td>0.832</td>
</tr>
<tr>
<td>S-0-SSB-4</td>
<td>0.800</td>
<td>NA (X-ray)</td>
</tr>
<tr>
<td>Average S-0-SSB</td>
<td><strong>0.857</strong></td>
<td><strong>0.843</strong></td>
</tr>
<tr>
<td>S-5-SSB-1</td>
<td>0.862</td>
<td>0.889</td>
</tr>
<tr>
<td>S-5-SSB-2</td>
<td>0.950</td>
<td>0.823</td>
</tr>
<tr>
<td>S-5-SSB-3</td>
<td>0.903</td>
<td>0.821</td>
</tr>
<tr>
<td>S-5-SSB-4</td>
<td>0.889</td>
<td>NA (X-ray)</td>
</tr>
<tr>
<td>Average S-5-SSB</td>
<td><strong>0.901</strong></td>
<td><strong>0.844</strong></td>
</tr>
<tr>
<td>S-10-SSB-1</td>
<td>0.943</td>
<td>0.923</td>
</tr>
<tr>
<td>S-10-SSB-2</td>
<td>0.971</td>
<td>0.912</td>
</tr>
<tr>
<td>S-10-SSB-3</td>
<td>0.996</td>
<td>0.843</td>
</tr>
<tr>
<td>S-10-SSB-4</td>
<td>0.891</td>
<td>NA (X-ray)</td>
</tr>
<tr>
<td>Average S-10-SSB</td>
<td><strong>0.950</strong></td>
<td><strong>0.893</strong></td>
</tr>
</tbody>
</table>

Figure 4.52 Average Quasi-Isotropic Single Shear Bearing Strengths

Figure 4.53 shows a comparison of the traditional and non-traditional quasi-isotropic single shear bearing response. The initial portion is linear for each specimen, but becomes non-linear as bearing damage accumulates. The traditional and non-
traditional (±5°) responses are comparably stiff, and the non-traditional (±10°) response is slightly less stiff. Each laminate experiences a plateau in loading, maintaining a significant load-carrying capability after a maximum load is reached and a significant amount of bearing damage has occurred.

![Normalized Stress vs. Normalized Strain](image)

Figure 4.53 Comparison of quasi-isotropic single shear bearing response

4.5.1.1 Traditional Quasi-Isotropic Single Shear Bearing

Figure 4.54 shows the single shear bearing damage progression of the traditional quasi-isotropic laminate. In Figure 4.54a, slight damage has occurred in the 90° and ±45° plies. As loading progresses, bearing damage begins to initiate, which is indicated by the darker region at the top of the hole seen in Figure 4.54b. While an exact correlation between the quasi-static tests and the incremental x-ray tests is difficult because the repeated loading and unloading, and damage caused by fastener insertion and removal, Figure 4.54b corresponds roughly to the point of non-linearity in the stress-strain curve.
given in Figure 4.55. In the non-linear portion of response, damage consists mainly of bearing damage and hole elongation, as shown in Figure 4.54c-d. This is also accompanied by pin rotation, which in turn causes surface damage on the specimen. A cross-section of the specimen, showing bearing damage at the final load level, is given in Figure 4.56.

![Radiographic images of traditional quasi-isotropic single shear bearing specimen at different loads.](image)

Figure 4.54 Radiographic images of traditional quasi-isotropic single shear bearing specimen at a) 55% maximum load, b) 69% maximum load, c) 83% maximum load, and d) 96% maximum load.
Figure 4.55  Typical single shear bearing stress-strain response of traditional quasi-isotropic specimen

![Normalized Stress vs Normalized Strain](image)

Figure 4.56  Bearing damage in quasi-isotropic single shear bearing specimen

4.5.1.2  Non-Traditional (±5°) Quasi-Isotropic Single Shear Bearing

Figure 4.57 shows the single shear bearing damage progression for the non-traditional (±5°) quasi-isotropic laminate. In Figure 4.57a, ±45° damage can be seen at
the top of the hole, but little other damage is evident. The initiation of bearing damage, in addition to more pronounced $\pm 45^\circ$ and $90^\circ$ damage, can be seen in Figure 4.57b. This coincides with the transition to non-linear behavior shown in Figure 4.58. Subsequent damage is predominantly bearing damage and hole elongation, as shown in Figure 4.57c-d. This is also accompanied by pin rotation.

![Figure 4.57 Radiographic images of non-traditional ($\pm 5^\circ$) quasi-isotropic single shear bearing specimen at a) 48% maximum load, b) 60% maximum load, c) 72% maximum load, and d) 84% maximum load](image-url)
4.5.1.3 Non-Traditional (±10°) Quasi-Isotropic Single Shear Bearing

Figure 4.59 shows the single shear bearing damage progression of the non-traditional (±10°) quasi-isotropic laminate. In Figure 4.59a, slight ±45° damage has occurred at 135° and 315°. Significant bearing damage is not observed until Figure 4.59c, which would correlate with point of non-linearity in Figure 4.60. Additional loading results primarily in an increase in the amount of bearing damage, though more ±45° and 90° damage is also observed.
Figure 4.59 Radiographic images of traditional quasi-isotropic single shear bearing specimen at a) 46% maximum load, b) 58% maximum load, c) 69% maximum load, and d) 81% maximum load.

Figure 4.60 Typical single shear bearing response of non-traditional (±10°) quasi-isotropic laminate.
4.5.2. HARD LAMINATE SINGLE SHEAR BEARING

The results of the single shear bearing tests with the hard laminates are given below in Table 4.10 and Figure 4.61. The non-traditional laminates increased both the maximum stress and the offset strength in these tests. The non-traditional (±5°) SSB specimens had an average strength 15% higher than the traditional laminate, and the non-traditional (±10°) SSB specimens had an average strength 17% higher, although again scatter was significant. It should be noted that in the non-traditional laminates, the specimens that were x-rayed failed at significantly lower strengths than the quasi-static specimens. This is most likely a result of the repeated loading and loading and fastener removal causing additional damage, resulting in a lower strength. If these values were not included in the averages, the difference between traditional and non-traditional laminates would be even greater.

### Table 4.10 Results of hard laminate single shear bearing tests

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$S_{\text{max}}/S_{\text{trad}}$</th>
<th>$S_{\text{0.02D}}/S_{\text{trad}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>H-0-SSB-1</td>
<td>0.387</td>
<td>0.343</td>
</tr>
<tr>
<td>H-0-SSB-2</td>
<td>0.366</td>
<td>0.333</td>
</tr>
<tr>
<td>H-0-SSB-3</td>
<td>0.408</td>
<td>0.327</td>
</tr>
<tr>
<td>H-0-SSB-4</td>
<td>0.368 NA (X-ray)</td>
<td></td>
</tr>
<tr>
<td><strong>Average H-0-SSB</strong></td>
<td><strong>0.382</strong></td>
<td><strong>0.334</strong></td>
</tr>
<tr>
<td>H-5-SSB-1</td>
<td>0.465</td>
<td>0.395</td>
</tr>
<tr>
<td>H-5-SSB-2</td>
<td>0.444</td>
<td>0.402</td>
</tr>
<tr>
<td>H-5-SSB-3</td>
<td>0.446</td>
<td>0.405</td>
</tr>
<tr>
<td>H-5-SSB-4</td>
<td>0.396 NA (X-ray)</td>
<td></td>
</tr>
<tr>
<td><strong>Average H-5-SSB</strong></td>
<td><strong>0.438</strong></td>
<td><strong>0.401</strong></td>
</tr>
<tr>
<td>H-10-SSB-1</td>
<td>0.453</td>
<td>0.411</td>
</tr>
<tr>
<td>H-10-SSB-2</td>
<td>0.469</td>
<td>0.405</td>
</tr>
<tr>
<td>H-10-SSB-3</td>
<td>0.461</td>
<td>0.417</td>
</tr>
<tr>
<td>H-10-SSB-4</td>
<td>0.407 NA (X-ray)</td>
<td></td>
</tr>
<tr>
<td><strong>Average H-10-SSB</strong></td>
<td><strong>0.447</strong></td>
<td><strong>0.411</strong></td>
</tr>
</tbody>
</table>
Figure 4.61 Average Hard Single Shear Bearing Strength

Figure 4.62 compares the single shear bearing responses of the traditional and non-traditional hard laminates. The initial portion of each stress-strain curve is linear, but as damage accumulates, the response becomes non-linear. The laminates all appear to have a similar stiffness, but the traditional laminate becomes non-linear first. Testing was continued until a noticeable drop in load occurred; the specimen still maintained some load carrying ability after this point, but testing was terminated to avoid causing damage that would mask useful information about failure modes.
4.5.2.1 Traditional Hard Single Shear Bearing

Figure 4.63 shows the single shear bearing damage progression of the traditional hard laminate. In Figure 4.63a, very slight $\pm 45^\circ$ was observed. Figure 4.63b shows evidence of bearing failure. Additionally, longitudinal ply cracking is seen extending from the top of the hole, and measures approximately 6.0mm. Figure 4.63c shows the specimen just prior to reaching the maximum load. Severe bearing damage is evident, and has resulted in delamination from approximately $0^\circ$ to $90^\circ$. The longitudinal splitting has reached the edge of the specimen at this load.
Figure 4.63 Radiographic images of traditional quasi-isotropic single shear bearing specimen at a) 58% maximum load, b) 75% maximum load, and c) 92% maximum load.

Figure 4.64 shows a typical stress-strain curve for the hard traditional laminate in single shear bearing. Following the initial linear portion, the response becomes non-linear as bearing damage and fastener rotation occurs. The tests were generally terminated following a noticeable decrease in load, but if joint displacement continued beyond this point, the 0° plies began to shear out, as shown in Figure 4.65.
4.5.2.2 Non-Traditional ($\pm 5^\circ$) Single Shear Bearing

Figure 4.66 shows the single shear bearing damage progression of the non-traditional ($\pm 5^\circ$) hard laminate. In Figure 4.66a, slight damage has occurred in the $90^\circ$
and ±45° plies at the bearing surface. As loading progresses, bearing damage becomes more prominent in Figure 4.66b. Longitudinal ply cracking at 90° and 270° can be seen at this load increment, and measures approximately 3.2mm. Additionally, longitudinal cracking can be seen at the top of the hole. By 95% of the maximum load, severe bearing damage has occurred, as indicated in Figure 4.66c. The longitudinal ply cracking at the top of the hole has grown to approximately 11.5mm, but has not reached the edge of the specimen.

Figure 4.66  Radiographic images of traditional quasi-isotropic single shear bearing specimen at a) 59% maximum load, b) 77% maximum load, and c) 95% maximum load
Figure 4.67 shows a typical stress-strain curve for the hard traditional laminate in single shear bearing. Following the initial linear portion, the response becomes non-linear as bearing damage and fastener rotation occurs.

4.5.2.3 Non-Traditional (±10°) Hard Single Shear Bearing

Figure 4.68 shows the single shear bearing progression of the non-traditional hard (±10°) laminate. In Figure 4.68a, slight damage has occurred in the ±45° plies at the top of the hole. As loading progresses, bearing damage occurs as seen in Figure 4.68b. Slight longitudinal ply cracking can also be seen at the top of the hole in Figure 4.68b, and grows to a length of 9.5mm at 93% of the failure load, as seen in Figure 4.68c.
Figure 4.68 Radiographic images of traditional quasi-isotropic single shear bearing specimen at a) 58% maximum load, b) 75% maximum load, and c) 93% maximum load.
Figure 4.69  Typical single shear bearing stress-strain response of non-traditional (±10°) hard laminate
4.6. COMPARISON OF TEST PARAMETER EFFECTS

There are several test parameters to consider when evaluating the results of the unnotched tension, open hole tension, filled hole tension, open hole compression, and single shear bearing trials. The two different lay-ups cause considerable changes in response among the different test types, and within each lay-up, the use of $0^\circ$, $\pm 5^\circ$, or $\pm 10^\circ$ longitudinal plies causes further changes in response. Additionally, loading and notch constraint caused significant changes in damage progression and mechanical properties. Following is a discussion of each of these variables.

4.6.1. Effect of lay-up

The difference between lay-ups caused changes in multiple areas of interest. As the percentage of longitudinal plies increased, the strength and stiffness of the laminate increased for both tension and compression. Additionally, as discussed in 2.1.1.2.1, a higher percentage of $0^\circ$ plies results in a higher stress concentration factor. This was evident in all of the notched coupon tests, as the quasi-isotropic specimens failed at higher percentages of the unnotched strength compared to the hard laminates, illustrating the notch effect.

The effect of fasteners was also more pronounced in the hard lay-up, as shown in Figure 4.70. While fasteners caused a reduction in strength for each laminate tested, the reduction was greatest in the hard lay-ups.
The bolt bearing strengths of the hard lay-up were less than those of the quasi-isotropic laminate. This resulted from a reduction in the number of 90° and ±45° plies, which provide much of the bolt bearing resistance. The response of the quasi-isotropic laminate was also linear longer, and had a longer plateau of load carrying ability.

Radiographs of the specimens illustrated the effect of lay-up on damage progression. In notched tension, the primary difference in damage between the quasi-isotropic laminates and the hard laminates was the amount of longitudinal ply crackling, illustrated in Figure 4.71. The quasi-isotropic specimens exhibited little longitudinal fiber-matrix splitting at the notch, which in turn meant that little difference was observed between the traditional and non-traditional laminates. The hard lay-ups had pronounced splitting at the notch, which resulted in large contrast between traditional and non-
traditional laminates. Edge delamination was also more pronounced in the hard laminates, although this could also be the result of the stacking sequence.

![Comparison of traditional quasi-isotropic and hard laminate splitting damage](image)

**Figure 4.71** Comparison of traditional quasi-isotropic and hard laminate splitting damage

### 4.6.2. Effect of non-traditional laminates

The use of slightly off-axis longitudinal plies had several effects. For unnotched tension, the non-traditional laminates had a lower strength and stiffness. The same was true for notched tension and compression. Representative notched tensile stress-strain curves of the six different specimen types are shown in Figure 4.72 and Figure 4.73. In general, the filled hole tension performance of the non-traditional laminates was better than the open hole when compared to the traditional laminates. The non-traditional quasi-isotropic filled hole tension specimens exhibited a slight increase in strength over the traditional laminates, and the non-traditional hard filled hole tension specimens had comparable strengths but significantly less damage than the traditional specimens.
Effect of Longitudinal Ply Orientation and Notch Constraint on Laminate Behavior

Figure 4.72 Comparison of stress-strain curves of quasi-isotropic laminates under open and filled hole tension

Figure 4.73 Comparison of stress-strain curves for hard laminates under open and filled hole tension
The non-traditional laminates caused an increase in bearing strength in both the quasi-isotropic and hard laminates. This is most likely the result of slightly off-axis plies providing more bearing resistance than 0° plies.

X-ray radiography showed the use of non-traditional laminates reduced the severity of longitudinal fiber-matrix splitting in each case where splitting was observed in the traditional laminate. In notched laminates under uniaxial load, this splitting causes a reduction in the stress concentration factor, resulting in a higher ultimate strength. When splitting was severe as was the case in the hard laminate seen in Figure 4.74, the suppression of splitting resulted in a significant reduction in strength.

Figure 4.74 Comparison of splitting damage in hard a) traditional, b) non-traditional (±5°), and c) non-traditional (±10°) laminates
4.6.3. Effect of notch constraint

The effect of notch constraint on each laminate was shown in Figure 4.70. For each case, the filled hole tensile strength was lower than the open hole tension case. Because a fastener with clamp-up constrains the area around the notch, the softening and stress redistribution that happens in an open hole does not occur. As such, damage appears later but failure occurs earlier. The difference in damage prior to failure can be seen in Figure 4.75. The difference between OHT and FHT strength is most severe in the hard traditional laminate. The high OHT strength of this laminate results from splitting at the edges of the notch, which leads to the elimination of the notch effect. When this splitting is suppressed by the fastener, the strength is greatly reduced. Since the non-traditional laminates suppress splitting in the OHT case, the effect of the fastener in the FHT case is less pronounced.

Figure 4.75 Comparison of damage in tradition quasi-isotropic laminate loaded in a) open hole tension and b) filled hole tension
The following discussion of predictive techniques is divided into three sections. First, the implementation of laminate failure theories is discussed, and the results are compared with the unnotched tensile strength tests. Second, the semi-empirical failure theories are discussed, and compared against each other. Because these theories have a “fitting” parameter, each can produce a result that perfectly matches the experimental data; thus, it is most meaningful to compare the fitting parameters used. Lastly, the fracture mechanics approaches are compared with experimental data.

As with the experimental results, any proprietary data has been normalized against unnotched properties.

5.1. IMPLEMENTATION OF UNNOTCHED STRENGTH PREDICTION

Based on Tsai’s contribution to the World Wide Failure Exercise[30, 33], discussed in 2.2.1.2, the “unnotched.m” MATLAB program was written to predict unnotched strength. The program, given in Appendix A, implements the Maximum Stress, Hashin, and Tsai-Wu failure criterion discussed in Chapter 2.

The flow of the progressive damage methodology discussed in [33] is shown in Figure 5.1. Based on ply properties found in [52] and [54], the stresses and strains in each lamina are calculated using Classical Lamination Theory. A failure criterion is then applied on a ply-by-ply for an assumed load. If no plies fail, a higher load is applied. If a ply is found to fail, a selective degradation of material properties takes place, as given in Table 5.1. The initial failure is assumed to occur in matrix-dominated directions, and
thus the modulus of the matrix in the damaged ply is reduced. The failure criterion is then reapplied, and if failure occurs again, the fiber properties are greatly reduced, indicating a loss of load-bearing capability of the ply. Load is then increased incrementally until all plies fail.

![Figure 5.1 Flow Chart of the Progressive Failure Modeling Used [33]](image)

Table 5.1 Lamina Degradation factors used in progressive damage methodology

<table>
<thead>
<tr>
<th>Intact</th>
<th>Matrix degradation</th>
<th>Fiber degradation</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>Baseline</td>
<td>Modified</td>
</tr>
<tr>
<td>$E_1^s$</td>
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<td>210</td>
</tr>
<tr>
<td>$E_m^s$</td>
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</tr>
<tr>
<td>$E_y$</td>
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<td>126.00</td>
</tr>
<tr>
<td>$E_t$</td>
<td>11.00</td>
<td>1.930</td>
</tr>
<tr>
<td>$v_y$</td>
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<td>0.042</td>
</tr>
<tr>
<td>$E_{tt}$</td>
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<td>1.070</td>
</tr>
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<td>$Y$</td>
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<td>200</td>
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<td>79</td>
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<tr>
<td>$P_{xx}$</td>
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<td>-0.08</td>
</tr>
</tbody>
</table>

Table 5.2 contains a comparison of the predictions and the experimentally determined strengths.
Table 5.2  Unnotched Strength Predictions

<table>
<thead>
<tr>
<th>Lay-up</th>
<th>Experimental $\sigma_{un}/\sigma_{un0}$</th>
<th>Max Stress $\sigma_{un}/\sigma_{un0}$</th>
<th>Hashin $\sigma_{un}/\sigma_{un0}$</th>
<th>Tsai-Wu $\sigma_{un}/\sigma_{un0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>% Error</td>
<td>% Error</td>
<td>% Error</td>
<td>% Error</td>
</tr>
<tr>
<td>Quasi (0°)</td>
<td>0.938 1.020 8.7</td>
<td>0.952 1.5</td>
<td>0.953 1.6</td>
<td></td>
</tr>
<tr>
<td>Quasi (±5°)</td>
<td>0.891 1.017 14.1</td>
<td>0.932 4.6</td>
<td>0.938 5.3</td>
<td></td>
</tr>
<tr>
<td>Quasi (±10°)</td>
<td>0.866 1.009 16.5</td>
<td>0.877 1.27</td>
<td>0.895 3.3</td>
<td></td>
</tr>
<tr>
<td>Hard (0°)</td>
<td>0.794 0.958 20.7</td>
<td>0.936 17.9</td>
<td>0.904 13.9</td>
<td></td>
</tr>
<tr>
<td>Hard (±5°)</td>
<td>0.746 0.951 27.5</td>
<td>0.911 22.1</td>
<td>0.855 14.6</td>
<td></td>
</tr>
<tr>
<td>Hard (±10°)</td>
<td>0.708 0.931 31.5</td>
<td>0.837 18.2</td>
<td>0.723 2.12</td>
<td></td>
</tr>
</tbody>
</table>

Because of the inconsistencies in unnotched tension results as noted in section 4.1, it is difficult to determine a “best” criterion from the above comparisons. However, a qualitative analysis of the results is still possible.

The Maximum Stress Criterion, described in 2.2.1.1, performs best when used to predict traditional laminate behavior, and encounters increasing error as the angle of the longitudinal plies diverge from 0°. This may result from the non-interactive nature of the criterion. Fiber failure is determined based on the following equation:

$$\frac{\sigma_{11}}{S_{11}} = 1 \quad (5.1)$$

As such, the Maximum Stress Criterion only considers stress in the fiber direction in determining fiber failure. However, a shear stress component would exist in the longitudinal plies due to the off-axis fiber orientation. This component is also present mathematically due to the stress transformation, but is not used in the criterion. This could result in the increase in error observed in the transition to non-traditional laminate.
The Hashin Criterion performs equally well predicting traditional and non-traditional laminate behavior. Unlike the Maximum Stress Criterion, the Hashin Criterion is semi-interactive and considers shear stress component in the determination of fiber failure, as shown in the following equation.

\[
\left( \frac{\sigma_{11}}{X_{11}} \right)^2 + \left( \frac{\tau_{12}}{S_{12}} \right)^2 = 1 \quad (5.2)
\]

Mathematically, the only difference between equations 5.1 and 5.2 is the inclusion of the shear stress term. However, equation 5.2 performs much better in determining fiber failure. This implies that even though the shear stress component would be relatively small, it plays a role in failure strength.

The Tsai-Wu Criterion also performs equally well for traditional and non-traditional laminates. This too implies the shear stress interaction plays a role in fiber failure. Additionally, the Tsai-Wu criterion does a much better job of predicting the hard laminate behavior than the other two criterion. This could stem from the fully interactive nature of the criterion, as seen in equation .

\[
F_1\sigma_{11} + F_2\sigma_{22} + F_6\tau_{12}^2 + F_1\sigma_{11}^2 + F_2\sigma_{22}^2 + 2F_1\sigma_{11}\sigma_{22} = 1 \quad (5.3)
\]

As the lay-up becomes more orthotropic, transverse stresses may become a significant factor in fiber failure, and the ability of the Tsai-Wu criterion to consider these stresses could account for the improved accuracy.

In general, each of the three criterion performed best in the quasi-isotropic case. This may result solely because of better experimental results; higher experimentally observed strengths would certainly improve the percent error. However, as noted in section 4.1, the hard laminates exhibited substantial delamination damage while the
quasi-isotropic laminates did not. This delamination would result in considerable strain concentrations and a lower strength than that predicted when assuming all plies have constant displacement.

Because of inconsistencies between Boeing and Georgia Tech data, and because Boeing data did not exist for the non-traditional laminates, it was decided to use calculated unnotched strength values for all following notched strength predictions. Since the Tsai-Wu criterion compared most favorably to the experimental data, and has performed well in numerous other studies, it was selected to generate unnotched strength value. The failure criterion predicted the general trends of differences between traditional and non-traditional laminates, and provided a compromise between Boeing and GT determined values.

5.2. SEMI-EMPIRICAL FAILURE THEORY IMPLEMENTATION

As discussed in Chapter 2, semi-empirical composite failure theories have been studied extensively, have been shown to give reasonable predictions, and are relatively easy to implement [2]. For these reasons, several are considered here for the prediction of Open Hole Tensile strength.

In general, semi-empirical failure criterion require an unnotched strength, a characteristic length, and possibly a notch parameter that have experimentally determined for one lay-up. The relationship can than be used to predict the notched strength of a laminate with different lay-up, notch size, or both. Since both notched and unnotched strengths of the laminates are known in this study, a comparison of the characteristic lengths is given. One can compare the characteristic length from one lay-up with another to determine how close of a prediction could be obtained using a given failure theory.
Table 5.3 calculates the Whitney-Nuismer Point Stress, Whitney-Nuismer Average Stress, and Karlak characteristic distance lengths based on the equations given in 2.2.2.

Table 5.3  Comparison of Characteristic Distance Parameters based on OHT results

<table>
<thead>
<tr>
<th>Lay-up</th>
<th>$\sigma_{un}/\sigma_{un,trad}$</th>
<th>$\sigma/\sigma_{un}$</th>
<th>$K_t$</th>
<th>$d_0$</th>
<th>$a_0$</th>
<th>$k_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quasi-Isotropic (0°)</td>
<td>0.953</td>
<td>0.526</td>
<td>3.00</td>
<td>0.0381</td>
<td>.0968</td>
<td>.1077</td>
</tr>
<tr>
<td>Quasi-Isotropic (±5°)</td>
<td>0.938</td>
<td>0.522</td>
<td>2.98</td>
<td>0.0391</td>
<td>.0996</td>
<td>.1106</td>
</tr>
<tr>
<td>Quasi-Isotropic (±10°)</td>
<td>0.895</td>
<td>0.493</td>
<td>2.92</td>
<td>0.0383</td>
<td>.0957</td>
<td>.1083</td>
</tr>
<tr>
<td>Hard (0°)</td>
<td>0.904</td>
<td>0.480</td>
<td>4.11</td>
<td>0.0303</td>
<td>.0899</td>
<td>.0857</td>
</tr>
<tr>
<td>Hard (±5°)</td>
<td>0.855</td>
<td>0.433</td>
<td>4.03</td>
<td>0.0272</td>
<td>.0771</td>
<td>.0770</td>
</tr>
<tr>
<td>Hard (±10°)</td>
<td>0.723</td>
<td>0.391</td>
<td>3.84</td>
<td>0.0213</td>
<td>.0549</td>
<td>.0601</td>
</tr>
</tbody>
</table>

The wide range of values in Table 5.3 illustrates the difficulty in choosing a single characteristic distance value for notched strength prediction, and the danger in assuming the characteristic distance value is a material property. As an example, the characteristic distance parameters obtained from the traditional quasi-isotropic laminate are applied to the other lay-ups in Table 5.4, using the Whitney-Nuismer Point Stress and Average Stress Criterion. Because the Karlak Criterion is essentially identical to the WNPS criterion unless multiple notch sizes are involved, it is omitted here.

Table 5.4  Predictions of OHT strength based on traditional quasi-isotropic characteristic distance

<table>
<thead>
<tr>
<th>Lay-up</th>
<th>Experimental $\sigma/\sigma_{un}$</th>
<th>WNPS $\sigma/\sigma_{un}$</th>
<th>WNAS $\sigma/\sigma_{un}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quasi-Isotropic (0°)</td>
<td>0.526</td>
<td>0.526</td>
<td>0%</td>
</tr>
<tr>
<td>Quasi-Isotropic (±5°)</td>
<td>0.522</td>
<td>0.517</td>
<td>1.0%</td>
</tr>
<tr>
<td>Quasi-Isotropic (±10°)</td>
<td>0.493</td>
<td>0.492</td>
<td>0.2%</td>
</tr>
<tr>
<td>Hard (0°)</td>
<td>0.480</td>
<td>0.528</td>
<td>10.0%</td>
</tr>
<tr>
<td>Hard (±5°)</td>
<td>0.433</td>
<td>0.498</td>
<td>15.0%</td>
</tr>
<tr>
<td>Hard (±10°)</td>
<td>0.391</td>
<td>0.417</td>
<td>6.7%</td>
</tr>
</tbody>
</table>
Table 5.4 illustrates that the semi-empirical failure criterion considered here are capable of predicting only small changes in strength due to the use of non-traditional laminates. In the quasi-isotropic laminates, the reduction in notched strength was largely due to the reduction in unnotched strength. Because the notched strength varies proportionately to the unnotched strength in the WNPS and WNAS criterion, these reductions were accurately predicted. However, the reductions in strength observed in open hole tension in the non-traditional hard laminates resulted from the change in stress state at the notch. The WNPS and WNAS criterion only utilize the stress concentration factor to determine the stress distribution in the laminate, and because the SCF does not vary much from traditional to non-traditional laminates, no large change is predicted. Thus, it is apparent that the semi-empirical theories considered here are not sensitive enough to consider the non-traditional laminates.

5.3. IMPLEMENTATION OF COHESIVE ZONE MODEL

Based on the work of Soutis [12], the a Cohesive Zone Model was implemented in the program BL_CZM.m. Following is a description of the algorithm used in the program given in Appendix B. All damage is replaced by a line crack, growing perpendicular to the loading direction. The crack is discretized, and grows in increments; at crack initiation, there is one increment. The crack begins to grow when the stress at the notch equals the unnotched strength of the laminate. Stress over each increment is assumed constant, but can vary from increment to increment. As the crack grows, a linear load-displacement relationship is assumed to govern the cohesive stress acting on the crack surfaces. This relationship is related to the fracture toughness $G_c$, as shown in Figure 5.2.
For a given load, the total crack opening displacement can be written as the sum of the displacements due to remote loading and local loading:

\[ v_i = v_i^e + v_i^p = f_i S + \alpha_{ij} \sigma_j \]  

(5.4)

which in turn can be written as

\[ v_i = (\beta_{ji} f_i + \alpha_{ij}) \sigma_j \]  

(5.5)

Based on the linear stress-displacement relationship, \( v_i \) can be written as

\[ v_i = v_c \left( I_i - \frac{\sigma_i}{\sigma_{un}} \right) \]  

(5.6)

By combining equations 5.5 and 5.6 the stress at each increment, \( \sigma_i \), can be related to \( v_c \) with the following equation.

\[ v_c = \left( \frac{v_c}{\sigma_{un}} \delta_{ij} + \beta_{ji} f_i + \alpha_{ij} \right) \sigma_j \]  

(5.7)

Equation 5.7 is a series of \( j \) equations with \( j \) unknowns, and can be solved the local stress \( \sigma_j \).

The assumption that stress remains finite everywhere then allows the stress intensity factors, derived by Newman [55], are then applied to relate local and remote stresses. Thus, for an assumed crack length, the remote load \( S \) can be found as a function of fracture toughness, modulus, unnotched strength, crack length, and geometry.
Two improvements are made to the Cohesive Zone Model in the present work. In [12], Soutis uses the longitudinal modulus of the composite, but does not account for the orthotropy of the material. Following the recommendations in [56], an effective longitudinal stiffness is used in the present work to incorporate material anisotropy. This incorporates longitudinal, transverse, and shear modulus, in addition to Poisson’s ratio. Additionally, the load displacement curve is modified, as discussed in the following section.

5.3.1. Modification of load-displacement relationship

A linear load-displacement relationship has been shown to work well in some cases, but difficulty in scaling has resulted in attempts to improve accuracy [57]. One such method is a bilinear load-displacement relationship.

![Figure 5.3 Bi-Linear Cohesive Zone Model](image)

The values of $G_c$, “a”, and “b” can be determined experimentally by testing panels with 2.54cm and 10.16cm notches and performing a best-fit analysis. A further description of this procedure can be found in [57].

Thus, equation 5.6 becomes
\[ v_i = \frac{b v_c}{1-a} \left( 1 - \frac{\sigma_i}{\sigma_{un}} \right), \sigma_i > a \sigma_{un} \]

\[ v_i = v_c \left( 1 - \frac{1-b \sigma_i}{a \sigma_{un}} \right), \sigma_i < a \sigma_{un} \] (5.8)

This is then utilized with equation 5.7 as before, along with a criteria for determining which portion of the curve the stress state is at. The rest of the procedure remains identical.

### 5.3.2. Comparison with Open Hole Tension Results

The results of the CZM model predictions are compared to experimental results in Table 5.5.

<table>
<thead>
<tr>
<th>Lay-up</th>
<th>( \sigma_{un}/\sigma_{un,trad} )</th>
<th>Experimental ( \sigma/\sigma_{un,trad} )</th>
<th>Linear CZM ( \sigma/\sigma_{un,trad} )</th>
<th>Bi-Linear CZM ( \sigma/\sigma_{un,trad} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quasi-Isotropic (0°)</td>
<td>0.953</td>
<td>0.526</td>
<td>0.495</td>
<td>5.6%</td>
</tr>
<tr>
<td>Quasi-Isotropic (±5°)</td>
<td>0.938</td>
<td>0.522</td>
<td>0.491</td>
<td>5.9%</td>
</tr>
<tr>
<td>Quasi-Isotropic (±10°)</td>
<td>0.895</td>
<td>0.493</td>
<td>0.478</td>
<td>3.0%</td>
</tr>
<tr>
<td>Hard (0°)</td>
<td>0.904</td>
<td>0.480</td>
<td>0.442</td>
<td>7.9%</td>
</tr>
<tr>
<td>Hard (±5°)</td>
<td>0.855</td>
<td>0.433</td>
<td>0.434</td>
<td>0.2%</td>
</tr>
<tr>
<td>Hard (±10°)</td>
<td>0.723</td>
<td>0.391</td>
<td>0.408</td>
<td>4.4%</td>
</tr>
</tbody>
</table>

Both cohesive zone models give reasonable predictions for notched strength, based only on experimentally determined parameters. However, as shown in Table 5.5, the models do not fully capture the strength reductions observed in non-traditional laminates. In the hard laminates, there was an 18.5% reduction in strength, but the cohesive zone model...
only predicts an 8% reduction. Changes in unnotched strength and stiffness are again the primary factors in notched strength reduction. The cohesive zone model does not, at this point, predict the variation of the stress state at the notch between the traditional and non-traditional laminate.

The bi-linear cohesive zone does, however, have the ability to change the stress state at the notch through adjustment of the breakpoint described in 5.3.1. By altering points “a” and “b”, the stress in the damaged zone near the notch can be increased or decreased. For the purpose of this research, “a” is fixed as 0.5, and “b” allowed to vary. The relationship between “a”, “b”, $G_c$, and $v_c$ is given by

$$v_c = \frac{1}{(a+b)} \left( \frac{2G_c}{\sigma_{un}} \right) \quad (5.9)$$

Based on the radiographic inspection of the hard laminates, it is known that the longitudinal ply cracking reduces stresses around the notch through elimination of the stress concentration factor. Thus, the non-traditional laminates, which experienced less longitudinal ply cracking, should have a higher stress in the damaged region. This can be accounted for by adjusting the breakpoint upwards for the traditional laminates. By increasing “b” 25% for the quasi-isotropic laminates, the results in Table 5.6 can be obtained. The breakpoint of the hard laminate was adjusted downward for the hard non-traditional laminate to provide experimentally observed 18.5% drop in strength.
### Table 5.6 Results of modified bi-linear CZM predictions for OHT strength

<table>
<thead>
<tr>
<th>Lay-up</th>
<th>$\sigma_{\text{rel}}/\sigma_{\text{un,trad}}$</th>
<th>Experimental $\sigma/\sigma_{\text{un,trad}}$</th>
<th>Linear CZM $\sigma/\sigma_{\text{un,trad}}$</th>
<th>Bi-Linear CZM $\sigma/\sigma_{\text{un,trad}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quasi-Isotropic (0°)</td>
<td>0.953</td>
<td>0.526</td>
<td>0.504</td>
<td>5.6% 0.522 0.8%</td>
</tr>
<tr>
<td>Quasi-Isotropic (±5°)</td>
<td>0.938</td>
<td>0.522</td>
<td>0.502</td>
<td>5.9% 0.515 1.3%</td>
</tr>
<tr>
<td>Quasi-Isotropic (±10°)</td>
<td>0.895</td>
<td>0.493</td>
<td>0.497</td>
<td>3.0% 0.500 1.4%</td>
</tr>
<tr>
<td>Hard (0°)</td>
<td>0.904</td>
<td>0.480</td>
<td>0.458</td>
<td>7.9% 0.442 7.9%</td>
</tr>
<tr>
<td>Hard (±5°)</td>
<td>0.855</td>
<td>0.433</td>
<td>0.457</td>
<td>0.2% 0.411 5.1%</td>
</tr>
<tr>
<td>Hard (±10°)</td>
<td>0.723</td>
<td>0.331</td>
<td>0.457</td>
<td>4.4% 0.377 13.9%</td>
</tr>
</tbody>
</table>

#### 5.3.3. Comparison with Open Hole Compression results

The Soutis-Fleck Cohesive zone model was originally derived for use in predicting notched compressive strength. Because the damage observed in compression, microbuckling, is essentially a single crack extending from the hole in a line perpendicular to the load, the Stress Intensity Factor solutions used above seem a good fit to this case.

Based on fracture toughness values provided by Boeing, and predicted unnotched strengths, the linear and bilinear Cohesive Zone Models were used to predict notched compressive behavior for the traditional and non-traditional laminates. The results of these predictions are given in Table 5.7.
Table 5.7  Cohesive Zone Model predictions of Open Hole Compressive strength

<table>
<thead>
<tr>
<th>Lay-up</th>
<th>$\frac{\sigma_{\text{un}}}{\sigma_{\text{un_trad}}}$</th>
<th>Experimental $\frac{\sigma}{\sigma_{\text{un_trad}}}$</th>
<th>Linear CZM $\frac{\sigma}{\sigma_{\text{un_trad}}}$</th>
<th>Bi-Linear CZM $\frac{\sigma}{\sigma_{\text{un_trad}}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quasi-Isotropic (0°)</td>
<td>0.948</td>
<td>0.548</td>
<td>0.493</td>
<td>9.9%</td>
</tr>
<tr>
<td>Quasi-Isotropic (±5°)</td>
<td>0.941</td>
<td>0.527</td>
<td>0.491</td>
<td>6.8%</td>
</tr>
<tr>
<td>Quasi-Isotropic (±10°)</td>
<td>0.920</td>
<td>0.519</td>
<td>0.482</td>
<td>7.1%</td>
</tr>
<tr>
<td>Hard (0°)</td>
<td>1.000</td>
<td>0.505</td>
<td>0.458</td>
<td>9.3%</td>
</tr>
<tr>
<td>Hard (±5°)</td>
<td>0.978</td>
<td>0.447</td>
<td>0.453</td>
<td>1.3%</td>
</tr>
<tr>
<td>Hard (±10°)</td>
<td>0.917</td>
<td>0.428</td>
<td>0.444</td>
<td>3.7%</td>
</tr>
</tbody>
</table>

The results in Table 5.7 are generally conservative and for the most part within 10% of the experimentally determined values. The bilinear stress-displacement curve performs worse than the linear case in OHC, but the breakpoints used were the same as those for the tensile case. Since the stress states in the damaged region of the two load cases are obviously different, this is another opportunity to change the location of these points. In this case, the quasi-isotropic laminates appear capable of sustaining more damage, and the hard laminates less. Thus, the breakpoint was increased 33% for the quasi-isotropic laminates, and reduced by 33% for the hard laminates. In both cases, accuracy was significantly improved, as seen in Table 5.8. The point of this is not to make the breakpoint location a fitting parameter, although it can be used that way. The goal is better approximate the physical reality of the stress state in the damaged composite.
Table 5.8 Results of modified bi-linear CZM predictions for OHC strength

<table>
<thead>
<tr>
<th>Lay-up</th>
<th>$\sigma_{ul}/\sigma_{ul,trad}$</th>
<th>Experimental $\sigma/\sigma_{ul,trad}$</th>
<th>Linear CZM $\sigma/\sigma_{ul,trad}$</th>
<th>Bi-Linear CZM $\sigma/\sigma_{ul,trad}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quasi-Isotropic (0°)</td>
<td>0.948</td>
<td>0.548</td>
<td>0.493</td>
<td>0.552</td>
</tr>
<tr>
<td>Quasi-Isotropic (±5°)</td>
<td>0.941</td>
<td>0.527</td>
<td>0.491</td>
<td>0.548</td>
</tr>
<tr>
<td>Quasi-Isotropic (±10°)</td>
<td>0.920</td>
<td>0.519</td>
<td>0.482</td>
<td>0.538</td>
</tr>
<tr>
<td>Hard (0°)</td>
<td>1.000</td>
<td>0.505</td>
<td>0.458</td>
<td>0.509</td>
</tr>
<tr>
<td>Hard (±5°)</td>
<td>0.978</td>
<td>0.447</td>
<td>0.453</td>
<td>0.467</td>
</tr>
<tr>
<td>Hard (±10°)</td>
<td>0.917</td>
<td>0.428</td>
<td>0.444</td>
<td>0.444</td>
</tr>
</tbody>
</table>

Because the CZM calculates applied stress as a function of crack length, it is possible to compare the measured microbuckle lengths from 4.4.1 and 4.4.2 with the model. As shown in Figure 5.4, the cohesive zone model provides fairly reasonable predictions of damage length in addition to accurate predictions of strength.

Figure 5.4 Predicted damage length from bi-linear cohesive zone model compared with experimental data
The above OHC and OHT strength predictions demonstrate the effectiveness of cohesive zone models. Despite a number of simplifications, it can be used to predict open hole tension and compression strengths using only stiffness, strength, and fracture toughness, and without fitting parameters. It should also be noted that some of the error results from the lack of unnotched compressive strength values for the non-traditional laminates. For example, using Boeing values for unnotched compressive strength for the traditional quasi-isotropic laminate, the quasi-isotropic OHC strength prediction becomes 0.504, an error of 8%. However, the advantages of the CZM model are many. It can be implemented at a simple level as done here, with easily performed calculations or even closed-form equations. At a more complex level, laminate-based or lamina-based finite element analysis can be performed on the geometries described above. If one so desires, cohesive zone relations can be used to model 0°, ±45°, 90°, and delamination damage as done in [51]. The adaptability of the cohesive zone model in combination with its reliance on experimentally determined parameters makes it a excellent tool for composite strength prediction.
CHAPTER 6
CONCLUSIONS

An experimental research program was designed to characterize non-traditional CFRP composite laminates. Mechanical properties and damage development in traditional and non-traditional lay-ups were compared and contrasted. Unnotched tension, open hole tension, filled hole tension, open hole compression, and single-shear bearing tests were conducted on multi-directional laminates with $0^\circ$, $\pm5^\circ$, and $\pm10^\circ$ longitudinal plies. Laminate strengths and stiffnesses were experimentally determined. Damage progression was monitored via in-situ radiograph to investigate the effect of longitudinal ply orientation on the initiation and growth of damage. Several unnotched and notched strength prediction techniques were evaluated to determine the sensitivity of analysis techniques to slightly off-axis ply orientations.

*The unnotched properties of the non-traditional laminates were found to decrease somewhat as off-axis plies were substituted for $0^\circ$ longitudinal plies.* Slight decreases in strength and modulus occurred in the quasi-isotropic laminates when using $\pm5^\circ$ and $\pm10^\circ$ longitudinal plies. Failures were quite complex, often occurring in multiple locations. A general trend of failing on a $45^\circ$ plane was observed in the traditional laminates, while the non-traditional laminates failed perpendicular to the load. Using $\pm5^\circ$ or $\pm10^\circ$ plies in a hard laminate resulted in modulus decreasing and average unnotched tensile strength decreasing. These unnotched tension failures were also very complex, in some cases occurring at multiple points and at the grips. The traditional hard laminate and the non-traditional ($\pm10^\circ$) laminate both experienced explosive delamination from grip to grip, a
feature that was not observed in the non-traditional hard laminate with the $\pm 5^\circ$ plies. Results of the unnotched tension tests are somewhat inconclusive, due to failures occurring in the grip region, and discrepancies between this experimental study and Boeing data.

In general, strength and stiffness were also found to decrease in notched non-traditional laminates. The unnotched strengths of the non-traditional laminates were lower, contributing in part to this reduction. More influential, though, was the suppression of longitudinal matrix cracking through the use of off-axis plies. Because this splitting results in the reduction of the stress concentration factor, suppressing it reduced the notched strength. In a quasi-isotropic open hole tension test, when the $0^\circ$ longitudinal plies were replaced with $\pm 5^\circ$ or $\pm 10^\circ$ plies, a slight reduction in strength of occurred. In-situ X-ray radiography of the quasi-isotropic open hole tension specimens revealed that longitudinal ply cracking in the non-traditional laminates was approximately 50% less than that observed in the traditional laminate. In the hard laminate under open hole tension, where splitting was more pronounced, a significant reduction of 10% and 19% was found in average strength in laminates with the $\pm 5^\circ$ and $\pm 10^\circ$ plies. Again, in-situ radiography revealed suppression of longitudinal ply cracking. In the traditional laminate, this splitting was so severe as to essentially eliminate the notch effect. However, the suppression of splitting in the non-traditional laminates caused the stress concentration to remain, resulting in a more severe stress state at the notch. Of additional interest was the appearance of delamination at the free edge of the hard open hole tension laminates. Severe delamination was observed in the traditional laminate, and this delamination appeared to be reduced in the non-traditional laminate.
with the $\pm 5^\circ$ plies, and eliminated in the non-traditional laminate with $\pm 10^\circ$ plies. This is most likely due to reductions in interlaminar shear and Poisson effect.

In filled hole tension, there was not a considerable difference between the traditional and non-traditional laminates. Slight improvements in strength were observed in the laminate with the $\pm 5^\circ$ plies and in the laminate with the $\pm 10^\circ$ plies. While an improvement was not observed in the hard non-traditional filled hole tensile strength, the difference between traditional and non-traditional strength was less than that of the open hole tension case; only slight reductions were found using $\pm 5^\circ$ and $\pm 10^\circ$ longitudinal plies. In-situ radiography of the filled hole tension specimens indicated damage initiation in all cases was delayed until higher load levels than observed in open hole tension tests, and that damage was mostly confined to the area under the fastener. This results from the fastener eliminating much of the notch effect and suppressing damage.

Under open hole compression, failure strengths were reduced by using non-traditional laminates. A slight reduction in average strength was found in the quasi-isotropic laminates with $\pm 5^\circ$ and $\pm 10^\circ$ plies. A more severe reduction was encountered in the hard laminates, as using $\pm 5^\circ$ and $\pm 10^\circ$ longitudinal plies resulted in 11% and 18% reductions in strength. Radiographic inspection revealed that the slightly off-axis plies did not appear to have an effect on the critical damage mode, microbuckling, and as such did not provide any advantage in this case.

Single-shear bearing performance was improved by using non-traditional laminates. This is most likely due to the added bearing resistance of the slightly off-axis plies when compared to a $0^\circ$ longitudinal ply. The quasi-isotropic non-traditional laminate with the $\pm 5^\circ$ resulted in a 5% increase in average maximum load, and the non-
traditional laminate with ±10° plies provided an 11% improvement, although scatter was significant. The non-traditional hard laminate with ±5° plies resulted in a 23% percent increase in strength over the traditional hard laminate, and the non-traditional laminate with ±10° longitudinal plies resulted in a 25% increase in strength.

A variety of analytical techniques were evaluated to determine their ability to predict non-traditional laminate response. Classical Lamination Theory was capable of accurately predicting traditional and non-traditional laminate longitudinal modulus, and of predicting the trends of non-traditional laminates. While unnotched tension experimental results were inconclusive, they did allow for qualitative comparison of unnotched laminate failure criterion. The non-interactive Maximum Stress Criterion did not do a good job of predicting non-traditional laminate strength, perhaps because it did not consider the shear stress component induced by the slightly off-axis longitudinal ply. The semi-interactive Hashin Criterion and the fully interactive Tsai-Wu Criterion both did as well in predicting non-traditional laminate strengths as traditional laminate strengths. The Hashin Criterion performed better in predicting quasi-isotropic results than hard laminate results. This could be because it does not consider transverse stresses in determining fiber failure; these stresses would be more of a factor in the hard laminate. The Tsai-Wu Criterion most closely predicted the observed experimental results for both the quasi-isotropic and hard laminates. This may be due to the fully interactive nature of the criteria, which takes into account longitudinal, transverse, and shear stresses in each lamina.

A comparison of notched strength prediction techniques demonstrated the difficulty in accurately predicting notched composite residual strength. Because the
semi-empirical methods evaluated do not consider the physical mechanisms occurring in the laminate, they proved incapable of predicting the difference between traditional and non-traditional laminates. A simple cohesive zone model was implemented, and showed promising results. The linear load-displacement provided reasonably accurate results for traditional and non-traditional laminates using only experimentally determined values. The implementation of a bi-linear cohesive zone model further improved accuracy, although certain parameters were chosen based on intuitive reasoning rather than experimental data.

This experimental investigation showed that by replacing 0° plies with slightly off-axis plies, non-traditional laminates can be utilized to suppress certain damage types and improve bearing strength. This new class of laminate shows promise for use in multi-directional composite laminate applications where damage tolerance is required. The radiographic inspection of both traditional and non-traditional laminates yielded insight into the physical mechanisms at work in notched composite laminates and highlighted both the need for refinement of existing analytical techniques, and techniques that show great potential for modeling of composites.
CHAPTER 7
RECOMMENDATIONS

There are several aspects of non-traditional composite laminates that could warrant further investigation. The initial characterization of non-traditional composite laminates provided a basic understanding of non-traditional laminate behavior, but further work is needed to determine in what applications these lay-ups would provide the most benefit. Additional testing of the same configurations and laminates are needed to add statistical significance to the preliminary observations made in this work, and to ensure the observed changes in strength are due to laminate response and not experimental scatter.

Because the non-traditional single shear single fastener bearing specimens demonstrated an increase in strength, this would be a logical starting point. To further characterize this material, multiple fastener specimens could be tested to give insight into the bearing / bypass relationship for these material, and simulate more closely the joints used in actual aircraft.

The suppression of delamination is an interesting aspect of these laminates. Further experimental work is needed to gain a better understanding of the onset and progression of this phenomena in both the traditional and non-traditional laminates. Analysis of the interlaminar stresses, using relatively simple techniques such as those described in [58-60], could be used to predict location and severity of this damage.

The non-traditional laminates may offer an improvement in fatigue loading. Because delamination can be a significant factor in fatigue of composites, this could be
an area of future research. Likewise, the effect of the suppression of splitting and the increased bearing strengths may improve fatigue performance in notched or bolted joint specimens.

This research would seem to indicate that changes in notched strength in non-traditional laminates could be accounted for in either the apparent fracture toughness of the laminate or in the strain-softening response of the material. These can, to at least some extent, be determined experimentally through large notch testing of panels.

The use of cohesive zone modeling has, in this work and several others, shown considerable promise. It can be implemented at a relatively simple level as in this research, or through increasingly complicated finite element techniques to provide insight into damage progression and different failure modes.

This work has highlighted the lack of robust failure theories, in that a truly robust technique would be capable of predicting the changes in strength due to slight changes in lay-up. This is not to say that existing theories are incapable of predicting notched composite strengths. However, by necessity most models over-simplify the physical mechanisms of damage that occur in a composite, and, as demonstrated in this work, very slight changes in lay-up invalidate these simplifications. Damage and strength of composites are determined at a micromechanical level, and most notched composite theories neglect these interactions. A further understanding of both micromechanical damage evolution, and an evaluation of how to model damage at this level, is necessary.

A substantial amount of testing has been done in the areas of notched compression and tension. At present, however, relatively little multi-axial testing of notched composites has been published. Since few applications are truly uni-axial, this
field should be explored further. This would allow objective evaluation of existing failure criterion and offer potential for development of new models. In combination with the X-ray radiography techniques discussed in this work, further insight into the behavior of composite materials could be achieved.
APPENDIX A

This program calculates the unnotched strength of a composite laminate based on the progressive damage methodology of Tsai and Wu. Tsai-Wu, Hashin, and Maximum Stress Criterion are considered here.

NOTE: The separate criteria are not meant to be implemented at one time. Two of the three criteria must be commented out in order to run.

clear all; clc;

% Prompts user for lamina material properties
n       = input('Enter number of plys: ');
ply_E11 = input('Enter lamina longitudinal stiffness modulus E11 (MPa or Ksi) : ');
ply_E22 = input('Enter lamina transverse stiffness modulus E22 (MPa or Ksi): ');
ply_G12 = input('Enter lamina shear modulus G12 (MPa or Ksi): ');
ply_v12 = input('Enter lamina Poisson ratio v12: ');
ply_v21 = ply_E22*ply_v12/ply_E11;
a11 = input('Enter lamina longitudinal CTE alpha11: '); 
a22 = input('Enter lamina transverse CTE alpha22 : ');
deltaT = input('Enter cure temperature deltaT or 0 (C or F): ');
ply_t   = input('Enter ply thickness (mm or in): ');

orient = input('Enter fiber orientation angles, in degrees, as a vector [0 45 90 ... ] ');

for k=1:n;

% This line converts orientation angle theta to radians
theta(k)=3.14159*theta(k)/180;

% Individual ply properties are defined.
E11(k) = ply_E11;
E22(k) = ply_E22;
G12(k) = ply_G12;
v12(k) = ply_v12;
v21(k) = ply_v21;
t(k) = ply_t;
\[ h(k) = (k-n/2)\times t(k); \]

\[ X1t(k) = \text{ply X1t}; \]
\[ X1c(k) = \text{ply X1c}; \]
\[ X2t(k) = \text{ply X2t}; \]
\[ X2c(k) = \text{ply X2c}; \]
\[ S12f(k)= \text{ply S12f}; \]

\[ F_{xy}(k)=-.5; \]

\% These variables track damage in each ply
\[ \text{matrix(k)}=0; \]
\[ \text{fiber(k)} =0; \]
\[ \text{ply\_fail(k)}=0; \]

end

failed\_plies = 0;
\[ N = [0;0;0]; \]
\[ q=0; \]
while failed\_plies < n
\[ q=q+1; \]

\% Loading increment in longitudinal, transverse, and shear directions.
\[ N = N+[10;0;0]; \]

\% This for loop calculates the transformed stiffness coefficients for each of the n plies
\[ \text{for}\ L=1:n \]
\[ \% This calculates the reduced stiffness coefficients \( Q \)
\[ Q11(L) = E11(L)/(1-v12(L)*v21(L)); \]
\[ Q12(L) = (v12(L)*E22(L))/(1-v12(L)*v21(L)); \]
\[ Q22(L) = E22(L)/(1-v12(L)*v21(L)); \]
\[ Q66(L) = G_{12}(L); \]
\[ \]
\[ Q11bar(L) = Q11(L)\times \cos(\theta(L))^4 + 2\times (Q12(L)+2\times Q66(L))\times \sin(\theta(L))^2\times \cos(\theta(L))^2 + Q22(L)\times \sin(\theta(L))^4; \]
\[ Q12bar(L) = (Q11(L) + Q22(L) - 4\times Q66(L))\times \cos(\theta(L))^2 + 2\times \sin(\theta(L))^2 + Q12(L)\times (\sin(\theta(L))^4 + \cos(\theta(L))^4); \]
\[ Q16bar(L) = (Q11(L) - Q12(L) - 2\times Q66(L))\times \sin(\theta(L))^3 + Q12(L)\times \sin(\theta(L))^3; \]
\[ Q22bar(L) = Q11(L)\times \sin(\theta(L))^4 + 2\times (Q12(L)+2\times Q66(L))\times \sin(\theta(L))^2\times \cos(\theta(L))^2 + Q22(L)\times \cos(\theta(L))^4; \]
\[ Q26bar(L) = (Q11(L) - Q12(L) - 2\times Q66(L))\times \sin(\theta(L))^3 + Q12(L)\times \sin(\theta(L))^3 + (Q12(L) - Q22(L) + 2\times Q66(L))\times \sin(\theta(L))^3 \times \cos(\theta(L)) + Q22(L)\times Q66(L)\times \sin(\theta(L))^3; \]
\[
Q_{66}^{\text{bar}}(L) = (Q_{11}(L) + Q_{22}(L) - 2*Q_{12}(L) - 2*Q_{66}(L))^2 \cos(\theta(L))^2 \sin(\theta(L))^2 + Q_{66}(L)^2 \sin(\theta(L))^4 + \cos(\theta(L))^4);
\]

end

% This for loop sums the various Qijbar to calculate A, B, and D matrices for p=1:n;

if p==1
    A_{11} = Q_{11}^{\text{bar}}(p)(h(p) - (-n*t(p)/2));
    A_{12} = Q_{12}^{\text{bar}}(p)(h(p) - (-n*t(p)/2));
    A_{16} = Q_{16}^{\text{bar}}(p)(h(p) - (-n*t(p)/2));
    A_{22} = Q_{22}^{\text{bar}}(p)(h(p) - (-n*t(p)/2));
    A_{26} = Q_{26}^{\text{bar}}(p)(h(p) - (-n*t(p)/2));
    A_{66} = Q_{66}^{\text{bar}}(p)(h(p) - (-n*t(p)/2));
    B_{11} = Q_{11}^{\text{bar}}(p)(h(p)^2 - (-n*t(p)/2)^2);
    B_{12} = Q_{12}^{\text{bar}}(p)(h(p)^2 - (-n*t(p)/2)^2);
    B_{16} = Q_{16}^{\text{bar}}(p)(h(p)^2 - (-n*t(p)/2)^2);
    B_{22} = Q_{22}^{\text{bar}}(p)(h(p)^2 - (-n*t(p)/2)^2);
    B_{26} = Q_{26}^{\text{bar}}(p)(h(p)^2 - (-n*t(p)/2)^2);
    B_{66} = Q_{66}^{\text{bar}}(p)(h(p)^2 - (-n*t(p)/2)^2);
    D_{11} = Q_{11}^{\text{bar}}(p)(h(p)^3 - (-n*t(p)/2)^3);
    D_{12} = Q_{12}^{\text{bar}}(p)(h(p)^3 - (-n*t(p)/2)^3);
    D_{16} = Q_{16}^{\text{bar}}(p)(h(p)^3 - (-n*t(p)/2)^3);
    D_{22} = Q_{22}^{\text{bar}}(p)(h(p)^3 - (-n*t(p)/2)^3);
    D_{26} = Q_{26}^{\text{bar}}(p)(h(p)^3 - (-n*t(p)/2)^3);
    D_{66} = Q_{66}^{\text{bar}}(p)(h(p)^3 - (-n*t(p)/2)^3);
else
    A_{11} = Q_{11}^{\text{bar}}(p)(h(p) - h(p-1)) + A_{11};
    A_{12} = Q_{12}^{\text{bar}}(p)(h(p) - h(p-1)) + A_{12};
    A_{16} = Q_{16}^{\text{bar}}(p)(h(p) - h(p-1)) + A_{16};
    A_{22} = Q_{22}^{\text{bar}}(p)(h(p) - h(p-1)) + A_{22};
    A_{26} = Q_{26}^{\text{bar}}(p)(h(p) - h(p-1)) + A_{26};
    A_{66} = Q_{66}^{\text{bar}}(p)(h(p) - h(p-1)) + A_{66};
    B_{11} = [Q_{11}^{\text{bar}}(p)(h(p)^2 - h(p-1)^2)] + B_{11};
    B_{12} = [Q_{12}^{\text{bar}}(p)(h(p)^2 - h(p-1)^2)] + B_{12};
    B_{16} = [Q_{16}^{\text{bar}}(p)(h(p)^2 - h(p-1)^2)] + B_{16};
    B_{22} = [Q_{22}^{\text{bar}}(p)(h(p)^2 - h(p-1)^2)] + B_{22};
    B_{26} = [Q_{26}^{\text{bar}}(p)(h(p)^2 - h(p-1)^2)] + B_{26};
    B_{66} = [Q_{66}^{\text{bar}}(p)(h(p)^2 - h(p-1)^2)] + B_{66};
    D_{11} = [Q_{11}^{\text{bar}}(p)(h(p)^3 - h(p-1)^3)] + D_{11};
\]
D12=[Q12\bar{p}*(h(p)^3-h(p-1)^3)]+D12;
D16=[Q16\bar{p}*(h(p)^3-h(p-1)^3)]+D16;
D22=[Q22\bar{p}*(h(p)^3-h(p-1)^3)]+D22;
D26=[Q26\bar{p}*(h(p)^3-h(p-1)^3)]+D26;
D66=[Q66\bar{p}*(h(p)^3-h(p-1)^3)]+D66;
end
end

% Multiplying by factor completes the calculations for A, B, and D.

A11=A11;
A12=A12;
A16=A16;
A22=A22;
A26=A26;
A66=A66;
B11=1/2*B11;
B12=1/2*B12;
B16=1/2*B16;
B22=1/2*B22;
B26=1/2*B26;
B66=1/2*B66;

D11=1/3*D11;
D12=1/3*D12;
D16=1/3*D16;
D22=1/3*D22;
D26=1/3*D26;
D66=1/3*D66;

A=[A11, A12, A16; A12, A22, A26; A16, A26, A66];
%disp('10^6 N/m or *10^6 lb/in');

B=[B11, B12, B16; B12, B22, B26; B16, B26, B66];
%disp('10^3 N or 10^3 lb');

D=[D11, D12, D16; D12, D22, D26; D16, D26, D66];
%disp('Nm or lb*in');
a=A^1;

%Calculation of strain in laminate
\[ e_{\bar{L}} = a^*N; \]
for i=1:n;

%Calculation of stress in laminate
s11bar = Q11bar(i)*ebarL(1)+Q12bar(i)*ebarL(2)+Q16bar(i)*ebarL(3);
s22bar = Q12bar(i)*ebarL(1)+Q22bar(i)*ebarL(2)+Q26bar(i)*ebarL(3);
t12bar = Q16bar(i)*ebarL(1)+Q26bar(i)*ebarL(2)+Q66bar(i)*ebarL(3);

s = [cos(theta(i))^2, sin(theta(i))^2, 2*cos(theta(i))*sin(theta(i));
    sin(theta(i))^2,cos(theta(i))^2,-2*cos(theta(i))*sin(theta(i));
    -sin(theta(i))*cos(theta(i)),cos(theta(i))*sin(theta(i)), cos(theta(i))^2-
    sin(theta(i))^2]*[s11bar; s22bar; t12bar];

%Calculation of stress in each ply
s11(i) = s(1);
s22(i) = s(2);
t12(i) = s(3);

%Failure Criterion defined and applied in each ply.

%TSAI-WU
F1 = (1/X1t(i)-1/X1c(i));
F11 = 1/(X1t(i)*X1c(i));
F2 = (1/X2t(i)-1/X2c(i));
F22 = 1/(X2t(i)*X2c(i));
F66 = 1/(S12f(i)^2);
F12 = Fxy(i)*((F11*F22)^.5;%.5/(X1t(i)*X1c(i));%
F(i) = F1*s11(i)+F2*s22(i)+F66*t12(i)^2+F11*s11(i)^2+F22*s22(i)^2+2*F12*s11(i)*s22(i);

%MAX-STRESS
Ffmax(i) = s11(i)/X1t(i);
Ffmin(i) = -s11(i)/X1c(i);
Fmmax(i) = s22(i)/X2t(i);
Fmmin(i) = -s22(i)/X2c(i);

%HASHIN
Fft(i) = (s11(i)/X1t(i))^2+(t12(i)/S12f(i))^2;
Ffc(i) = (s11(i)/X1c(i))^2+(t12(i)/S12f(i))^2;
Fmt(i) = (s22(i)/X2t(i))^2+(t12(i)/S12f(i))^2;
Fmc(i) = (s22(i)/X2c(i))^2+(t12(i)/S12f(i))^2;

end
% Ply degradation of Tsai-Wu criteria
for i=1:n
    if F(i)>=1
        ply_fail(i)=1;
        if matrix(i)==0
            E22(i)=E22(i)*.175;
            v12(i)=v12(i)*.15;
            v21(i)=E22(i)*v12(i)/E11(i);
            G12(i)=G12(i)*.162;
            X1c(i)=.85*X1c(i);
            Fxy(i)=Fxy(i)*.15;
            matrix(i)=1;
        else
            E11(i)=E11(i)*.01;
            E22(i)=E22(i)*.01;
            v12(i)=v12(i)*.01;
            G12(i)=G12(i)*.01;
            X1c(i)=.66*X1c(i);
            Fxy(i)=.01*Fxy(i);
            fiber(i)=1;
            ply_fail(i)=1;
        end
    end
end

% Ply degradation for %Max-Stress criteria
for i=1:n;
    if Ffmax(i) >= 1;
        E11(i)=E11(i)*.01;
        E22(i)=E22(i)*.01;
        v12(i)=v12(i)*.01;
        G12(i)=G12(i)*.01;
        X1c(i)=.66*X1c(i);
        fiber(i)=1;
        ply_fail(i)=1;
    end
    if Fmmax(i) >= 1;
        E22(i)=E22(i)*.175;
        v12(i)=v12(i)*.15;
        v21(i)=E22(i)*v12(i)/E11(i);
        G12(i)=G12(i)*.162;
    end
X1c(i) = 0.85 * X1c(i);
matrix(i) = 1;
ply_fail(i) = 1;
end

if Ffmin(i) >= 1;
    E11(i) = E11(i) * 0.01;
    E22(i) = E22(i) * 0.01;
    v12(i) = v12(i) * 0.01;
    G12(i) = G12(i) * 0.01;
    X1c(i) = 0.66 * X1c(i);
    fiber(i) = 1;
    ply_fail(i) = 1;
end

if Fmmin(i) >= 1;
    E22(i) = E22(i) * 0.175;
    v12(i) = v12(i) * 0.15;
    v21(i) = E22(i) * v12(i) / E11(i);
    G12(i) = G12(i) * 0.162;
    X1c(i) = 0.85 * X1c(i);
    matrix(i) = 1;
    ply_fail(i) = 1;
end
end

%Ply degradation for Hashin Criteria
for i = 1:n;
    if Fft(i) >= 1;
        E11(i) = E11(i) * 0.01;
        E22(i) = E22(i) * 0.01;
        v12(i) = v12(i) * 0.01;
        G12(i) = G12(i) * 0.01;
        X1c(i) = 0.66 * X1c(i);
        fiber(i) = 1;
        ply_fail(i) = 1;
    end
end
if Fmt(i) >= 1;

    E22(i)=E22(i)*.175;
    v12(i)=v12(i)*.15;
    v21(i)=E22(i)*v12(i)/E11(i);
    G12(i)=G12(i)*.162;
    X1c(i)=.85*X1c(i);

    matrix(i)=1;
    ply_fail(i)=1;
end

end

ex(q) = ebarL(1);
Exx(q) =(A11*A22-A12^2)./(t(1)*n*A22);
failed_plies=sum(ply_fail);
end
APPENDIX B

% This program determines the strength of a composite laminate with a circular notch, using a bi-linear cohesive zone model.
clear all; clc;
Gc= % Fracture toughness value entered here
R = % Hole radius entered here
W = % Specimen width entered here
E = % Longitudinal modulus entered here
sigma_un = % Unnotched Strength entered here
A= % Bi-linear breakpoint A entered here
B= % Bi-linear breakpoint B entered here
imax = 40; % Number of elements crack is discretized into

vc =1/(A+B)*2*Gc/sigma_un; % Calculation of critical crack displacement

lf=.22; % Final crack length

sigmat = zeros(imax,imax); % Initialize stress matrix to 0
vmat = zeros(imax,imax); % Initialize crack displacement matrix to 0
vsig = zeros(imax,imax); % Initialize crack opening due to local stress to 0
vrem = zeros(imax,imax); % Initialize crack opening due to remote stress to 0

l=lf/imax:lf/imax:lf;
d=l+R;

% Finite Width Correction Factor 1
F2 = (sec(pi*R/(2*W))*sec(pi*d./(2*W))).^(1/2);
% Finite Width Correction Factor 2

A2 = 0.221*(R./d).^2+0.046*(R./d).^4;

for i=1:40;
    for j=1:i;
        b(j) = (R+l(i))-j/i*l(i);
        c(j) = (R+l(i))-(j-1)/i*l(i);
        x(j) = (R+l(i))-(j-.5)/i*l(i);
    end
end
\[ G_1 = (1+A_1(i)/(1-R/d(i))+3*A_2(i)/(2*(1-R/d(i))^2))*\sin(b(j)/d(i))+(A_1(i)/(1-R/d(i))+(4-(b(j)/d(i)))*A_2(i)/(2*(1-R/d(i))^2))*(1-(b(j)/d(i))^2)^{1/2}; \]
\[ G_2 = (1+A_1(i)/(1-R/d(i))+3*A_2(i)/(2*(1-R/d(i))^2))*\sin(c(j)/d(i))+(A_1(i)/(1-R/d(i))+(4-(c(j)/d(i)))*A_2(i)/(2*(1-R/d(i))^2))*(1-(c(j)/d(i))^2)^{1/2}; \]
\[ B_1(j) = \sin(\pi*b(j)/(2*W))/\sin(\pi*d(i)/(2*W)); \]
\[ B_2(j) = \sin(\pi*c(j)/(2*W))/\sin(\pi*d(i)/(2*W)); \]
\[ G(j) = G_2-G_1; \]
\[ F_3(j) = G(j)/{\sin(\pi*c(j)/d(i))}-\sin(b(j)/d(i))}; \]

**Finite Width Correction Factor 3**
\[ F_4(j) = ((\sin(\pi*B_2(j))-\sin(B_1(j)))/{\sin(\pi*c(j)/d(i))}-\sin(b(j)/d(i))})*(\sec(\pi*d(i)/(2*W)))^{1/2}; \]

**Finite Width Correction Factor 4**
\[ \beta(j) = -(2/\pi)*((\sin(\pi*c(j)/d(i))-\sin(b(j)/d(i)))*(F_3(j)*F_4(j))/(F_1(i)*F_2(i)); \]
\[ f(j) = (2/E)*(d(i)^2-x(j)^2)^{1/2}*F_1(i)*F_2(i); \]

**end**

**for m=1:i;**
**for n=1:i;**
\[ \text{posA} = \text{real}((2/(\pi*E))*((+c(n)-x(m))*\cosh((d(i)^2-c(n)*x(m))/(d(i)*abs(+x(m)-c(n))))-\cosh((d(i)^2-b(n)*x(m))/(d(i)*abs(+x(m)-b(n))))) + \cosh((d(i)^2-x(m)^2)^{1/2})*\sin(+c(n)/d(i))-\sin(+b(n)/d(i)))\}; \]
\[ \text{minA} = \text{real}((2/(\pi*E))*((+c(n)+x(m))*\cosh((d(i)^2+c(n)*x(m))/(d(i)*abs(-x(m)-c(n)))) - \cosh((d(i)^2+b(n)*x(m))/(d(i)*abs(-x(m)-b(n))))) + \cosh((d(i)^2-x(m)^2)^{1/2})*\sin(+c(n)/d(i))-\sin(+b(n)/d(i)))\}; \]
\[ \text{alpha}(m,n) = \text{posA}+\text{minA}; \]
**end**
**end**

**for m=1:i**
**for n=1:i;**
\[ \text{matrix1}(m,n) = (1-A)/B*(B/(1-A)*vc/sigma_un-alpha(m,n)-f(m)*beta(n) \}; \]
\[ \text{matrix2}(m,n) = ((1-B)/A*vc/sigma_un-alpha(m,n)-f(m)*beta(n) \}; \]
**else**
\[ \text{matrix1}(m,n) = -(1-A)/B*(alpha(m,n)+f(m)*beta(n) \}; \]
\[ \text{matrix2}(m,n) = -(alpha(m,n)+f(m)*beta(n) \}; \]
**end**
**end**

\[ \sigma_1 = vc*\text{sum(matrix1}^{1/2}); \]
\[ \text{Calculation of cohesive stress according to S-v relationship 1} \]
\[ \sigma_2 = vc*\text{sum(matrix2}^{1/2}); \]
\[ \text{Calculation of cohesive stress according to S-v relationship 2} \]
% This loop prevents displacement beyond vc
for m=1:i
    if sign(sigma1(m))~=sign(sigma_un)
        sigma1(m)=0;
    end
    if sign(sigma2(m))~=sign(sigma_un)
        sigma2(m)=0;
    end
end

caseB=0;

% This loop evaluates the stress to determine if it is in region 1 or 2 of the bi-linear load-displacement relationship.
for m=1:i
    if sigma1(m)< sigma2(m)
        caseB=1;
    end
    if caseB==1
        sigma(m) = sigma1(m);
    else
        sigma(m) = sigma2(m);
    end
end

S(i)=beta*sigma';  % Remote load is calculated
vrem(i,1:i) = S(i)*f;  % Crack opening is determined.
vsig(i,1:i) = alpha(1:i,1:i)*sigma(1:i);
vmat(i,1:i) = vrem(i,1:i)+vsig(i,1:i);
sigmat(i,1:i)=sort(sigma(1:i));  % Cohesive stresses at current increment are stored for analysis.
end
REFERENCES


