DECENTRALIZED DECISION-MAKING FOR REVERSE PRODUCTION SYSTEMS

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SUMMARY

Reverse production systems are often comprised of several tiers with independent members competing at each tier. This research develops and designs a decision-making process for decentralized reverse production systems where each participant in the network determines its decisions in a self-interested way.

This dissertation includes three major parts. The first part, Chapter 2, develops a prototype model for a decentralized reverse production system with two tiers, collectors and processors, focusing on the coordination of transactions of recycled items between these two tiers. The collectors determine the individual material flow allocation mechanisms, based on predictions of the range of prices from the processors, that relate the flow amount to the overall vector of acquisition prices that will be offered by the processors to all the collectors. The processors compete for the flow from the collectors and reach an equilibrium state where no entity is willing to change its decisions.

In the second part, Chapters 3 and 4, we extend the prototype model for a general multi-tiered recycling network comprised of the upstream boundary tier, several intermediate tiers, and the downstream boundary tier where each of the tiers has multiple independent entities. Recycled items flow from the top tier to the downstream tier, but acquisition prices are set from the downstream tier back to the upstream tier.

Finally the third part, Chapter 5, provides a comparison of centralized and decentralized models for reverse production systems and addresses several numerical insights of different system subsidy schemes.
CHAPTER 1 INTRODUCTION

1.1 Problem Background and Definition

A reverse production system (RPS) is a network of transportation logistics and processing functions that collect, consolidate, refurbish, and demanufacture end-of-life products. The driving forces for recycling are the recovery value of end-of-life products and the avoidance of waste disposal costs, especially in contemporary times of tight raw material markets and increasing concerns for environmental impact of disposal. In general, a RPS network is composed of several different functional entities in different tiers, connected by a transportation logistics system.

One major, and growing, recycling stream is obsolete electronic products. As we become more dependent on consumer electronics to make life more convenient, the stockpile of used, obsolete products grows. The National Safety Council\(^1\) estimates that nearly 250 million computers will become obsolete in the next five years and mobile phones will be discarded at a rate of 130 million per year by 2005. Computer monitors and cathode ray tubes (CRTs) contain an average of four\(^1\) pounds of lead and require special handling at the end of their lives. When electronics are not disposed of or recycled properly, these toxic materials can cause serious environmental problems. In addition, obsolete computers may contain significant amounts of recoverable materials including metals from wires and circuit boards, glass from monitors, and plastics from

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\(^1\) The data for this statement are available at http://www.epa.gov/ecycling/index.htm.
casings. For example, 1 metric ton of electronic scrap from personal computers contains more gold than that recovered from 17 metric ton of gold ore (U.S. Geological Survey, 2001). Due to these environmental concerns and potentially profitable incentives, several research questions emerge in the reverse production systems area.

Most of the research on RPS design and operation optimization views the system in a centralized way. The fundamental assumption of the centralized system is that one planner has the requisite information about all participating entities and has authority to determine the decision variables of entities such that the total RPS performance is optimized. Unfortunately, this assumption rarely holds.

In real-world reverse production systems, several private entities are involved in the actual situation of decision-making processes and they compete for valuable recycled items or form a coalition to achieve the optimal state of a subsystem. This type of system behavior is decentralized. Often the decision variables for each entity in a decentralized system are also influenced by other entities’ decisions. The self-interested behaviors affect and change the decision variables of the system obtained from a centralized modeling approach. The system performance of a decentralized RPS network may be overestimated using a centralized modeling approach.

Additional decision interests emerge in a decentralized system. One of the main focuses of a decentralized model is a set of internal decision variables, which are not of concern in the centralized model, since there is no net impact on the overall system.

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2 The data for this statement are available at http://pubs.usgs.gov/fs/fs060-01/fs060-01.pdf.
performance. This research is motivated by current practice in many reverse production systems where entities operate independently and act according to their own objectives and constraints. Few models or research efforts have focused on a decentralized RPS system.

There are several directions one can take in exploring these systems, and in this dissertation there are two specific contributions that are made to decentralized RPS modeling. First, it is important to understand, and propose a model for, how the different tiers of the system coordinate their activities. Second, it is also important to understand how the tiers behave with respect to other tiers and how the entities within tiers compete or coordinate their respective activities. For the first question, we propose, and then study, a relatively simple mechanism for coordination between tiers. For the second question, we propose that the entities within certain tiers are competing with each other and we are interested in finding equilibrium solutions where no entity is willing to change its decisions. It is proposed that this captures the self-interested decision-making process for the individual entity in reverse production systems. The decentralized decision-making process in this research is a step towards modeling operations of real-world recycling networks and captures elements of the individual entity in RPSs.

This research is motivated by the U.S. electronics recycling system where the system is composed of several privately-owned entities representing different functionalities such as collection, consolidation, demanufacturing, or refurbished processes. Transactions between different entities are motivated by the valuable items in the mix and based upon the correspondence between the amount of recycled items and their acquisition prices. The model proposed in this research may be generalized to other
reverse production systems with these major characteristics. The next section briefly discusses the organization of this dissertation.

1.2 Organization of the Dissertation

This dissertation develops and designs the decision-making process for a decentralized RPS network and the dissertation itself includes three major self-contained parts. The first part of “decentralized decision-making and protocol design for recycled material flows,” Chapter 2, is a prototype model for a decentralized RPS with two tiers of collectors and processors. We focus on the recycled item transactions between collectors and processors. The collectors determine the individual material flow allocation mechanism that relates the flow they provide to each processor, to the overall vector of acquisition prices, determined by the processors. The processors compete for the flow from the collectors, and a unique Nash equilibrium is reached in this competitive tier under the decentralized framework discussed in this research.

The second part includes Chapters 3 and 4. We extend the model presented in Chapter 2 for a general multi-tiered RPS network comprised of the upstream boundary tier, several intermediate tiers, and the downstream boundary tier, where each tier contains multiple independent entities and is responsible for one typical function or operation in the recycling network. Each entity in the upstream tier independently designs the material flow allocation parameters used to contract with the entities in its subsequent downstream tier sequentially from the top to the bottom of the network, before any prices are given, and each entity in the downstream tier competes and acquires the flow from its preceding upstream tier. The achieved equilibrium acquisition prices to
be offered in each tier also are obtained sequentially from the bottom to the top tier in the network. In Chapter 3, we assume the flow decision is only dependent on acquisition prices of all entities to be offered in the next tier. One of the underlying assumptions in Chapter 3 is that there is no correlation between the flow and price among different types of commodities. We further explore the multi-tiered RPS decentralized model with price correlated commodities in Chapter 4 where the flow decision is not only dependent on acquisition prices to be offered in different associated entities but also influenced by the prices of different types of commodities.

The third part, Chapter 5, compares the behavior of centralized and decentralized decision-making approaches for recycled material flow systems. In order to make a compatible comparison of the centralized and decentralized outcomes for reverse production systems, we develop a centralized model where a single planner determines the material flow through the network as well as acquired prices in the top tier. We demonstrate the comparison of the centralized and decentralized decision-making for a RPS network and draw several numerical insights from investigating different scenarios with government subsidies. Finally, we conclude the entire dissertation and provide several interesting extensions in Chapter 6.
CHAPTER 2 DECENTRALIZED DECISION-MAKING AND PROTOCOL DESIGN FOR RECYCLED MATERIAL FLOWS

Reverse logistics networks often consist of several tiers with independent members competing at each tier. This chapter develops a methodology to examine the individual entity behavior in reverse production systems where every entity considers its own objective function subject to its own constraints. We consider two tiers in the network, collectors and processors. The collectors determine individual flow functions that relate the flow they provide to each processor to the overall vector of prices that the processors determine. They do so by individually solving a robust optimization formulation where the prices from the processors are assumed to be in given ranges. The processors compete for the flow from the collectors and the Nash equilibrium is reached in this competitive tier. To demonstrate the approach, a numerical example is given for a prototypical recycling network.

2.1 Introduction

Maximizing the efficiency of recycled material flows is growing in urgency due to high demands in many raw material markets and the increasing concern for environmental impact of disposal. Supply chains are evolving from “open loop” unidirectional flows of materials, parts, and products from suppliers to end customers into more complex “closed loop” linked forward and reverse arcs (Fleischmann et al. 2000; Guide and Harrison 2003; Realf et al. 2004). Forward production systems are being
expanded to incorporate reverse production systems (RPS) that include sorting, demanufacturing and/or refurbished processes in reverse logistics systems.

Most of the research on RPS design views the system in a *centralized* way; the key assumption is that one planner has the requisite information about all the participating entities and seeks the optimal solution for the entire system (see Ammons et al. 2001; Shih 2001; Barros et al. 1998; Assavapokee et al. 2005). Wang et al. (2004) remark upon the three major drawbacks of centralized supply chain optimization models: (1) By ignoring the independence of the supply chain members, the competitive behavior between entities may lower the system efficiency and hence a centralized model may not capture the appropriate bargaining mechanisms that can mitigate the competitive behavior; (2) The cost of information processing may be expensive and the central decision maker must gather all the information from every entity; and (3) The computation of solutions to centralized optimization models can be very challenging.

An alternative structure for a RPS consists of several independent entities where individuals may have their own profit functions and may not be willing to reveal their own information to each other or the public. This type of system behavior is *decentralized*. Often the decision variables for each entity in a decentralized system are also influenced by other entities' decisions, coupling prices between members of the same tier and flows between supply chain tiers. In this chapter, we focus on the decentralized decision-making and protocol design for the RPS where entities can belong to two different tiers in the supply chain. The two tiers represent the collectors, who interact directly with the source of recycled items, and the processors who purchase the items from the collectors and convert them into more fungible commodities that are sold
to customers. We develop a mechanism for calculating the optimal (self-interested) acquisition prices and the independent optimal flow determination for recycled materials.

While forward and reverse supply chains share many similarities, there are significant differences. For forward supply chain systems, the material flow volumes are usually assumed to be functions of all prices in the final market (Nicholson 2002; Corbett and Karmarkar 2001). Once the historical data of demand and prices are available, we can predict the quantity and price relationship since retailers face a considerable number of customers in most of forward supply chain markets. However, for the RPS, the number of entities in the network is relatively small compared to a forward supply chain network. The relationship of the quantity and price in certain parts of the supply chain cannot be assumed from the historical data due to the issue of the small sample size. Instead, we present a robust approach to determine the relationship between the material flow volume and price between the collection and processing tiers of the supply chain.

The remainder of the chapter is organized as follows. In Section 2.2 we give a brief literature review. In Section 2.3 we provide the formal definition of our two-tier problem: the upstream and downstream entities and their connection. In Section 2.4 and 2.5 we develop mathematical models for determining the price and flow decisions for a decentralized RPS. In Section 2.6 we summarize the solution algorithm for the upstream and downstream models. In Section 2.7, we apply the algorithm to a numerical example to determine the equilibrium product prices and flows, and also provide a discussion of the model and results. We develop conclusions with Section 2.8 and also suggest directions for future research.
2.2 Literature Review

The past decade has seen an enormous increase in research on reverse logistics management issues. Flapper (1995, 1996), Fleishmann et al. (2000), and Guide and Harrison (2003) give systematic overviews and challenge of the logistic aspects of reuse and recycling in the closed-loop supply chains. Much of the research in RPS tends to be product, or system, specific due to the various features and complexities needed to handle the different recycling and reuse scenarios. Research on recycling and resource recovery for specific materials such as paper, plastics and sand include Pohlen and Farris (1992), Wang et al. (1995), Huttunen (1996) and Barros et al. (1998). Examples of product recovery and reuse include copy machines (Theirry et al. 1995; Theirry 1997; Krikke 1998), computers and electronics equipment (Jayaraman et al. 1997; Hong et al. 2005c), and reusable transportation containers (Kroon and Vrijens 1995). The basic underlying assumption in these papers is that the planning of reverse logistics operations is done by a central decision maker to optimize the total system performance.

There is a growing number of research papers on forward or reverse supply chains that model the independent decision-making process of each supply chain entity, specifically the interaction between pricing decisions and the material flow volume transacted in the network. Majumder and Groenevelt (2001) examine the competition behavior between an original equipment manufacturer (OEM) and the third-party local remanufacturer when the recycled products affect the demand of the original products. Guide et al. (2003) present an economic analysis for calculating the optimal acquisition prices and the optimal selling price for remanufactured products with different quality
classes in one single remanufacturing firm. Savaskan et al. (2004) model three options for collecting used products, subcontracting with retailers, outsourcing to a third-party firm, and collecting by themselves, as decentralized decision-making systems with the manufacturer being the Stackelberg leader. Ferguson and Toktay (2005) analyze a manufacturer’s recovery and entry-deterrent strategies in the face of a competitive threat from a third-party remanufacturer. The models presented in the above papers are limited in the topological network structure itself. Several researchers have presented competition models with the scope of multiple entities (Corbett and Karmarkar 2001; Nagurney and Toyasaki 2005). Corbett and Karmarkar (2001) develop a model that considers entry decisions and post-entry competition in multitier serial supply chains. Nagurney and Toyasaki (2005) use a variational inequality solution approach to solve for the equilibrium network flow and endogenous prices of recycled materials. In this chapter, instead, we consider a general RPS network structure with two tiers and multiple entities and propose an algorithm to solve for individual material flow allocation mechanism and explicit equilibrium acquisition prices within the network. We take the perspective of decentralized decision-making analysis and protocol design for the collection and processing of recycled items which may be ultimately converted into used-products or raw materials demanded by several specific markets.

2.3 A Two-tier RPS Problem: Upstream and Downstream

A RPS to reuse or recycle end-of-life products is a network of transportation logistics and processing functions that collect, refurbish, and demanufacture. In general, several entities in different tiers compose a network of collection and processing steps,
connected by a transportation logistics system. In this chapter, for simplicity, we assume a basic RPS consisting of collection and processing facilities facing sources and demand markets. Material flow allocation and product acquisition are common challenges for the reverse logistics network, where the network may be geographically complex and with a dynamic used-product market. Our experience with firms or non-profit recycling organizations in electronic scrap (e-scrap) products reveals several specific questions that go beyond the current reverse logistics models either in the strategic or operational level.

- What is the end-of-life product transaction mechanism between collectors and processors when they negotiate the price-flow contract?
- How do the collectors allocate their collected items to the processors if both of them are run by independent individuals?
- How do the processors determine their price offers if they bid for the collected items from collectors?

![Figure 1: A Two-tier RPS Network with Collection and Processing Sites](image)

We first illustrate a two-tier network problem consisting of upstream and downstream tiers for a RPS depicted in the graphical form in Figure 1. In general, the RPS is a network of several entities with functions that include collection and processing phases. The upstream tier represents the collection phase relating to sorting/consolidation
processes and the downstream tier denotes the processing phase including refurbish/demanufacture processes. Upstream entities collect end-of-life items from the residential or business sectors, and then independent downstream entities bid exchange prices for collected items from upstream entities. A successful upstream entity must carefully manage its material flow allocation of collected items, i.e., design an effective, fair and transparent material transaction mechanism between itself and downstream entities, so as to optimize its self-interest. Independent downstream entities compete for collected items from upstream entities with other members in the downstream tier. There are several value-added refurbishing/demanufacturing processes involved in the downstream entities, and items are transformed to refurbished items, subcomponents, or materials (e.g., used products, or raw materials), which are sold in several specific demand markets. An important issue for independent downstream entities is how to determine the optimal acquisition price that is used to bid for the items from upstream entities.

We focus on the transaction between upstream and downstream tiers on material flow allocation and associated price decisions. Upstream entities collect end-of-life products from residential or business sectors, which may hold positive- or negative-value recycled items. Our experience with firms in e-scrap industry shows that residential or business sources may need to pay a collection fee to collectors for discarding the obsolete e-scrap items (Hong et al. 2005c). We assume the collection amount in upstream entities is a function of the collection fee that the upstream entity charges from end-of-life product sources: the higher the fee, the lower the potential amount collected from sources. Downstream entities convert end-of-life products into several valuable raw
materials and used products as well as trash after refurbishing/demanufacturing processes. The transportation cost for the recycled item is paid by the downstream entity. The price the downstream entity pays for transportation will be taken into account by the upstream entity in flow allocation. In this research, we specifically focus on the transaction of valuable items between upstream and downstream tiers and, as a result, we assume the prices obtained in upstream entities are positive\(^3\). We also argue that the amount of raw materials resulting from the decomposition of end-of-life products and used products is relatively small compared to the quantity in the virgin raw material and brand-new product markets. This observation leads to the assumption that the selling prices of raw materials or used products in demand markets are fixed amounts, not affected by the sales quantities.

We focus on the equilibrium acquisition prices to be offered by downstream entities and the optimal material flow allocation mechanism between upstream and downstream tiers from a perspective of decentralized systems. The material flow allocation mechanism is the price-flow contract describing the correspondence between the acquisition prices offered by downstream entities and the flow amount supplied by the upstream entity to its subsequent downstream entities. The decision timeline for a two-tier problem is shown in Figure 2 where the upper arrows indicate the tasks of upstream or downstream entities, and the lower arrows state information disclosure timeline. Upstream entities first determine the price-flow contracts and communicate them to the associated downstream entities. We assume that upstream entities are unable to change

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\(^3\) This assumption is motivated from the actual situation in the U.S. electronics recycling system. The price information may be available at http://www.scrapcomputers.com/. 

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the price-flow contracts after communication. (For a discussion of what can happen when this assumption is changed, see Section 6.2.) Then, downstream entities compete with other entities within the same tier for the flow from upstream entities on the basis of the price they offer for the recycled items. The downstream tier gives a single vector of prices that apply to all the upstream tier members. The decisions of downstream entities are the acquisition prices to be offered to the upstream tier. Downstream entities simultaneously choose their respective price decisions based on the price-flow contracts given by the associated upstream entities. Finally, upstream entities determine the collection fee to acquire the corresponding amount of recycled items from the source. In the following sections, we present our modeling for independent upstream and downstream entities in the following subsequent sections followed by the summary of the algorithm.

**Figure 2:** The Decision Timeline for a Two-tier Problem
2.4 The Upstream Model: Material Flow Allocation

In this section, we present a robust optimization model for the independent upstream entity to determine the robust material flow allocation mechanism between upstream and downstream tiers. For simplicity, we refer to the material flow allocation mechanism as the *flow function*. We depict the upstream and downstream sites as nodes and the material flows as links in Figure 1. Specifically, we consider $m$ upstream sites who are involved in the collection of end-of-life products, which can then be acquired by $n$ downstream sites. A typical upstream site is denoted by $i$, and a typical downstream site by $j$. We first discuss the robust approach and scenario setting in the upstream model followed by the description of flow functions determined by the independent upstream site and the upstream model itself.

2.4.1 The Robust Approach and Scenario Setting

The goal of the upstream model for any particular site $i$, $i = 1, 2, \ldots, m$, is to design a “good” material flow allocation mechanism, or flow function. Due to the assumption of no information sharing in our decentralized model, upstream entities do not know the exact final acquisition prices to be offered by downstream entities. Each upstream entity predicts the possible range of acquisition prices as input information for determining flow functions. We assume that the transportation cost is paid by the downstream entity and recycled items coming from different upstream sites are homogeneous. As a result, the transportation cost is implicitly incorporated into the prices offered to the upstream entity. The unit reward that the upstream site receives is the acquisition price that the downstream site is willing to offer while covering its associated transportation costs. One
way to forecast lower and upper bounds of acquisition prices is based upon the information of transportation costs and market prices. A lower bound on the price is the transportation cost between upstream and downstream tiers; otherwise, the upstream sites obtain a negative price offer. We assume this is not in their interest, since the downstream site must compensate for transportation costs. Consequently, we assume the forecast acquisition prices are at least as much as the associated transportation costs in the model. One possible upper bound of the acquisition price is the highest market price of the recycled item, assuming that downstream sites are unwilling to pay more than the market price.

A particular price combination, \((P_{1o}, \ldots, P_{jw}, \ldots, P_{nw})\), of downstream entities refers to one scenario \(\omega \in \Omega\), where \(P_{jw}\) is the unit material price downstream site \(j\) willing to offer in price scenario \(\omega\). There are an infinite number of scenarios if the range of acquisition prices forecasted by the upstream entity is a continuous compact interval. This may lead to computational difficulties. In this chapter the continuous compact interval of the price range is restricted to a finite number of discrete points. A practical approach for computation is to select \(k\) points evenly in every dimension of the price range. Thus, the scenario space \(\Omega\) considered is with \(k^n\) scenarios if there are \(n\) downstream tier entities. Note that selecting \(k\) points evenly distributed over the price range for the computational analysis does not imply any assumption about the underlying probability distribution for price.

We assume each of the independent upstream entities faces price uncertainty from the downstream tier, although not necessarily the same range on each price has to be
assumed by each upstream entity. The fundamental objective of upstream entities is to construct a set of robust flow functions against the price uncertainty. In this research we use the measure of robust deviation defined by Kouvelis and Yu (1997), such that each upstream site is to minimize the maximum difference between the best it can obtain when price offers from downstream sites are realized and the robust objective value under the designed flow function. This is the well known min-max robust optimization approach (Winston 1994) and this also captures a notion of “risk” - the upstream site wants to protect itself from doing very poorly in a given price realization, which is unknown before contracting with the downstream tier.

2.4.2 The Material Flow Allocation Mechanism: Flow Functions

Intuitively the upstream entity tends to ship more flows to the downstream entity who offers the higher acquisition price. Obviously the material flow allocation mechanism from upstream sites to downstream sites is dependent on acquisition prices offered by downstream entities. Since the downstream tier entities pay the transportation costs, they compensate by subtracting the transportation cost from the prices to be offered to upstream entities.

We let $V_j^{(Tr)}$ denote the unit transportation cost from site $i$ to $j$. The unit reward that upstream site $i$ receives from downstream site $j$ is represented as the material price that the downstream entity is willing to offer while covering the associated unit transportation cost. Therefore, the unit reward of site $i$ in price scenario $\omega$ is $P_{j_\omega} - V_j^{(Tr)}$. For the material flow from upstream site $i$ to downstream site $j$ for scenario $\omega$, denoted by $x_{ij\omega}^{(Tr)}$, 

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upstream site $i$ tends to increase the flow amount on $x_{ij\omega}^{(Tr)}$ if downstream site $j$ offers a higher price. Meanwhile, upstream site $i$ may decrease the amount on $x_{ij\omega}^{(Tr)}$ to feed more flow to other arcs if other downstream sites provide more incentives in price offers. Our modeling implies any particular arc of material flow is not only a function of the price offered by its destination downstream site, but also the relative price offers of other downstream sites. The decision variables for upstream site $i$ are the coefficients of material flow determination, denoted by $\alpha_{ij}$, from upstream site $i$ to downstream site $j$ affected by downstream site $j'$ for all of downstream site pairs $j$ and $j'$. Note the decision variables of $\alpha_{ij}$ are not dependent of price scenario $\omega$; in turn, $\alpha_{ij}$ is a common set of coefficients for all of price scenarios. Using the common linear function assumption (Corbett and Karmarkar 2001; Guide et al. 2003), the material flow from upstream site $i$ to downstream site $j$ in price scenario $\omega$ is represented as

$$x_{ij\omega}^{(Tr)} = \sum_{j'=1}^{n} \alpha_{ij'} (P_{j\omega} - V_{ij'}^{(Tr)}) \quad \forall i, j.$$  

(1)

The decision variables, $\alpha_{ij}$, of upstream site $i$ are independently determined by site $i$ and designed to contract with site $i$’s downstream sites. For each particular arc $(i, j)$ associated with upstream site $i$, site $i$ designs the mechanism how downstream site $j$’s acquisition price affects the flow on arc $(i, j)$ and how other downstream sites’ acquisition prices influence the flow on arc $(i, j)$. These two effects can be described by the variable $\alpha_{ij}$, $j = j'$, and the variables, $\alpha_{ij}$, $j \neq j'$. The variable, $\alpha_{ij}$, $j = j'$, accounts for the material flow amount change on arc $(i, j)$ due to the change in the destination acquisition
price to be offered by downstream site \( j \). The variables, \( \alpha_{ij'}, j \neq j' \), explain how upstream site \( i \) relates the changes of other downstream sites’ acquisition prices to the flow change on arc \((i, j)\). The \( \alpha \)'s have units of flow/price and represent the sensitivity of flow to the prices being offered. The \( \alpha_{ij}, j \neq j' \) represent the cross-sensitivities of the flow to site \( j \) by the changing of the prices of the other members of its tier.

### 2.4.3 Potential Maximum Flow Determination

We first examine transactions between upstream sites and sources. Assume the collection amount in upstream site \( i, i = 1, 2, \ldots, m \), is characterized by a linear function \( S_{io} = a_i - b_i P_{io}^{(Co)} \), where \( a_i \) and \( b_i \) are parameters and \( a_i, b_i > 0 \). We let \( P_{io}^{(Co)} \) denote the collection fee charged by site \( i \), and \( S_{io} \) be the potential maximum flow amount collected in upstream site \( i \) corresponding to price scenario \( \omega \). To ensure that the upstream site \( i \) obtains a non-negative amount of flow, we require \( P_{io}^{(Co)} \leq a_i / b_i \) for all price scenarios. The potential profit of upstream site \( i \) in price scenario \( \omega \), \( \Pi_{io} \), is

\[
\Pi_{io} = S_{io} \left( P_{io}^{(Co)} + \max_{j=1,\ldots,n} \left\{ P_{jo} - V_{ij}^{(Tr)} \right\} \right) \\
= \left(a_i - b_i P_{io}^{(Co)} \right) \left(P_{io}^{(Co)} + \max_{j=1,\ldots,n} \left\{ P_{jo} - V_{ij}^{(Tr)} \right\} \right),
\]

where upstream site \( i \) picks the highest price offer from downstream sites as the selling price and the only unknown variable in (2) is the collection fee of \( P_{io}^{(Co)} \) that site \( i \) charged for the material from sources corresponding to price scenario \( \omega \). For notation simplicity,
we let $P_{i_o}^{(max)} = \max_{j=1,...,m} \{ P_{i_o} - r_{ij}(Tr) \}$. The potential profit function $\Pi_{i_o}$ is concave in $P_{i_o}^{(Co)}$ whenever $b_j > 0$, so (2) is maximized when the first-order condition holds, i.e., when

$$P_{i_o}^{(Co)*} = \min \left\{ \left( a_i - b_i \cdot P_{i_o}^{(max)} \right) / 2b_i, \ a_i / b_i \right\} \quad \forall i, \omega. \quad (3)$$

Thus, (3) is the optimal collection fee for upstream site $i$ in price scenario $\omega$. The potential maximum flow amount collected in upstream site $i$ corresponding to price scenario $\omega$, $S_{i_o}$, can be obtained by substituting $P_{i_o}^{(Co)*}$ into $S_{i_o} = a_i - b_i P_{i_o}^{(Co)}$.

### 2.4.4 The Robust Model for Upstream Sites

To execute the robust approach, first the optimal solution of each upstream site for each specified price scenario is found. This solution calculates the highest profit that the individual upstream site can obtain if it were to know the acquisition prices exactly. Then, we minimize the maximum deviation of objective function value between the “ideal” and the “robust” sales profit for all price scenarios. Finally, we adjust the decision variables, $\alpha'$s, to ensure those returning the best sales profit for all tight and non-tight price scenarios. We let $O_{i_o}^*$ denote the optimal objective value of upstream site $i$ for price scenario $\omega$, and $C_{ij}^{(Tr)}$ denote the shipment capacity between upstream site $i$ and downstream site $j$. We assume that each upstream site $i$ seeks to maximize the total profit associated with its collection and material allocation operations with the optimization problem given as follows for upstream site $i$ for price scenario $\omega$. 

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Maximize

\[ O_{io} \]

Subject to:

\[ O_{io} = \sum_{j=1}^{n} x_{jio}^{(Tr)} \left( P_{jio} - V_{ij}^{(Tr)} + P_{io}^{(Co)} \right) \]  \hspace{1cm} (4)

\[ x_{jio}^{(Tr)} = \sum_{j=1}^{n} \alpha_{ij} \left( P_{jio} - V_{ij}^{(Tr)} \right) \quad \forall j \]  \hspace{1cm} (5)

\[ x_{jio}^{(Tr)} \leq C_{ij}^{(Tr)} \quad \forall j \]  \hspace{1cm} (6)

\[ \sum_{j=1}^{n} x_{jio}^{(Tr)} \leq S_{io}^{*} \]  \hspace{1cm} (7)

\[ x_{jio}^{(Tr)} \geq 0 \quad \forall j \]  \hspace{1cm} (8)

\[ \alpha_{ij} > 0 \quad \forall j, j', j = j' \]  \hspace{1cm} (9)

\[ \alpha_{ij} \leq 0 \quad \forall j, j', j \neq j'. \]  \hspace{1cm} (10)

The objective function (4) is the sum of the sales profit and collection fee. Constraints (5) are the material flow function definitions for emanating arcs from upstream site \( i \). Constraints (6) and (7) provide capacity limits for each arc and for the recycled item source. Constraints (8), (9), and (10) are sign restrictions for unknown variables. Obviously, the material flow variables, \( x_{jio}^{(Tr)} \), are nonnegative, and the sign restrictions for \( \alpha \)'s require that the upstream site has more incentive to ship more flow on the arc where its destination price offer is increased, but less incentive when other downstream sites offer higher prices competing the material flow.

Next, we determine the robust flow function, or a common set of coefficients, \( \alpha \)'s, to be evaluated in every price scenario \( \omega \in \Omega \) for site \( i \). Thus, for each price scenario we subtract the robust objective function value \( (R_{io}) \) using the common set of robust coefficients from the optimal objective value \( (O_{io}^{*}) \) of realization of acquisition price.
offers. The min-max robust optimization model over all price scenarios for upstream site \( i \) can be stated as:

\[
\text{Minimize } \delta_i \\
\text{Subject to:} \\
\delta_i \geq O_{i\omega}^* - R_{i\omega} \quad \forall \omega \\
R_{i\omega} = \sum_{j=1}^{n} x_{i j \omega}^{(Tr)} (P_{j \omega} - V_{i j}^{(Tr)} + P_{i \omega}^{(Co)*}) \quad \forall \omega \\
x_{i j \omega}^{(Tr)} = \sum_{j'=1}^{n} \alpha_{ij'} \left( P_{j' \omega} - V_{i j'}^{(Tr)} \right) \quad \forall j, \omega \\
x_{ij}^{(Tr)} \leq C_{ij}^{(Tr)} \quad \forall j, \omega \\
\sum_{j=1}^{n} x_{ij}^{(Tr)} \leq S_{i\omega}^* \quad \forall \omega \\
x_{ij}^{(Tr)} \geq 0 \quad \forall j, \omega \\
\alpha_{ij'} > 0 \quad \forall j, j', j = j' \\
\alpha_{ij'} \leq 0 \quad \forall j, j', j \neq j'.
\]

The minimum maximum deviation \( \delta_i^* \) of upstream site \( i \) is realized after solving the min-max robust optimization model. The final step of the upstream model is solving the following model to optimality to ensure the decision variables, \( \alpha's \), return the best sales profit for non-effective price scenarios (non-tight price scenarios in (11)) for upstream site \( i \). The model for each upstream site \( i \) is:

\[
\text{Maximize } \sum_{\omega \in \Omega} R_{i\omega} \\
\text{Subject to:} \\
\delta_i^* \geq O_{i\omega}^* - R_{i\omega} \quad \forall \omega \\
\text{Constraints (12) - (18).}
\]

Given the robust solution values for \( \alpha \), the upstream site models determine robust flow functions for each independent upstream site. Thus, the robust flow function
describing the flow shipment from upstream site \( i \) to downstream site \( j \), denoted by \( x_{ij}^{(Tr)} \), is represented as

\[
x_{ij}^{(Tr)} = \sum_{j'=1}^{n} \alpha_{ij} \left( p_{j'} - V_{ij'}^{(Tr)} \right) \quad \forall i, j
\]

where \( p_{j'} \) is the acquisition price offered by downstream site \( j' \). Note that the price scenario \( \omega \) is not an argument in the flow function at this point, and that (19) describes the material flow relationship of the amount and acquisition price between upstream and downstream tiers. Hence, in (19), all of \( \alpha \)'s, and \( V_{ij}^{(Tr)} \)'s are known parameters and \( p \)'s, and \( x^{(Tr)} \)'s are unknown variables.

In summary, upstream sites provide downstream sites with the robust flow function to contract the material flow transactions between upstream and downstream tiers. First, the upstream site chooses several discrete price points to represent the acquisition price range forecasted by the upstream site, and calculates the potential maximum source amount that the upstream site is willing to collect. Then, each upstream site applies the min-max robust optimization approach to design the flow function between itself and downstream entities. In the next section, we present the downstream site models to solve for the equilibrium acquisition prices between the entities in these two tiers.

**2.5 The Downstream Model: the Equilibrium Price**

Downstream sites are involved in transactions with upstream sites and final demand markets since they wish to obtain recycled items from the upstream tier and sell the
materials or sub-components after refurbished/demanufacturing processes. Downstream sites simultaneously make decisions on their own acquisition prices subject to their constraints of processing capacities, transportation capacities, and technology restrictions. We develop an equilibrium model of competitive downstream sites to determine the Nash equilibrium price: no downstream site can improve its objective function value by a unilateral change in its price solution. In this chapter, we utilize the relaxation algorithm (see Krawczyk and Uryasev 2000; Contreras et al. 2004) or combined Karush-Kuhn-Tucker approach (see Hobbs 2001) to find the Nash equilibrium price solution.

2.5.1 The Optimization Model for the Downstream Site

We assume that recycled items coming from different upstream sites are homogeneous. Thus, the total flow shipped to downstream site \( j \), which is denoted by \( x^{(Tr)}_j \), is the sum of flows from different upstream sites to downstream site \( j \) and expressed as follows:

\[
x^{(Tr)}_j = \sum_{i=1}^{m} x^{(Tr)}_{ij} = \sum_{i=1}^{m} \sum_{j'=1}^{n} \alpha_{j'j} \left( p_{j'} - V^{(Tr)}_{j'} \right) \quad \forall j.
\]  

The independent downstream site maximizes its objective function associated with the purchasing, processing cost, and sales revenue and is subject to constraints imposed on the processing, transportation capacity, and demand restrictions. Additionally, there are constraints to ensure the conservation of flows among sites and different processes for each material. Required notation for the downstream model in addition to the upstream model notation is listed as follows.
Downstream Model Parameters:

- $V^{(Tr)}_{jc}$: Transportation cost per standard unit per distance from downstream site $j$ to customer site $c$;
- $V^{(Pr)}_{jp}$: Processing cost per standard unit for process $p$ at downstream site $j$;
- $P_{lc}$: Selling price offered per standard unit of material $l$ from customer $c$;
- $\rho_{lp}$: Proportion of material type $l$ produced by process $p$;
- $\rho_l$: Proportion of material type $l$ consumed by process $p$;
- $C^{(Pr)}_{jp}$: Maximum amount of material that process $p$ can produce at downstream site $j$;
- $C^{(Tr)}_{jc}$: Maximum amount of material that can be shipped from downstream site $j$ to customer site $c$.

Downstream Model Decision Variables:

- $p_j$: Price offered per standard unit from downstream site $j$;
- $x^{(Tr)}_j$: The aggregate flows to downstream site $j$;
- $x^{(Tr)}_{jlc}$: Amount of material shipped from downstream site $j$ to customer site $c$ of material type $l$;
- $x^{(Pr)}_{jp}$: Amount of material processed by process $p$ at downstream site $j$.

Using this notation, the optimization model for downstream site $j$ can be stated as:

Maximize

$$\sum_c \sum_l (P_{lc} - V^{(Tr)}_{jc}) x^{(Tr)}_{jlc} - p_j x^{(Tr)}_j - \sum_p V^{(Pr)}_{jp} x^{(Pr)}_{jp}$$

Subject to:

- Flow definition:
  $$x^{(Tr)}_j = \sum_{i=1}^m \sum_{j'=1}^n a_{ij'} (p_{j'} - V^{(Tr)}_{ij'})$$

- Net conservation of flow:
  $$x^{(Tr)}_j - \sum_c \sum_l x^{(Tr)}_{jlc} + \sum_p \rho_{lp} x^{(Pr)}_{jp} - \sum_p \rho_l x^{(Pr)}_{jp} = 0 \quad \forall l$$

- Processing capacity:
  $$x^{(Pr)}_{jp} \leq C^{(Pr)}_{jp} \quad \forall p$$

- Transportation capacity:
  $$\sum_l x^{(Tr)}_{jlc} \leq C^{(Tr)}_{jc} \quad \forall c$$

- Variable restrictions:
  $$x^{(Tr)}_j, x^{(Tr)}_{jlc}, x^{(Pr)}_{jp} \geq 0 \quad \text{for all } x \text{'s}$$

- $p_j \geq 0$. 

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The material flow variables for recycled items shipped to downstream site \( j \), \( x_{j}^{(Tr)} \), are substituted by the expression \( \sum_{i=1}^{m} \sum_{j'=1}^{n} a_{ij'} (p_{j'} - V_{ij'}(Tr)) \), which is a function of acquisition prices. Other types of material flow variables, \( x_{j}^{(Pr)} \), \( x_{j}^{(Pr)} \), can be expressed by \( x_{j}^{(Tr)} \) due to the flow conservation equations. The optimization model of (21) for downstream site \( j \) can be generally transformed into the model, shown in (22), expressed in acquisition price variables where \( p = (p_1, \ldots, p_n) \) are the collective price actions and where \( \phi_j \) is the payoff (or objective) function of downstream site \( j \). Let \( g_d^j \) denote the row \( d \) of constraint function and \( b_d^j \) the right-hand-side parameter of row \( d \) in downstream site \( j \)'s optimization model. It is clear that downstream site \( j \)'s optimization problem shown in (22) contains not only site \( j \)'s decision variable \( p_j \), but also incorporates other sites' price decisions.

Maximize \( \phi_j(p) \) 
Subject to: \( g_1^j(p) \leq b_1^j \)  
\vdots  
\( g_r^j(p) \leq b_r^j \)  
\( p_j \geq 0 \).  

(22)

Next, we show the convex property of downstream site models for the existence and uniqueness of the Nash equilibrium acquisition price solution.

Proposition 1 The optimization model for downstream site \( j, j = 1, \ldots, n \), has a strictly concave objective function with respect to \( p_j \) and a convex constraint set.
Proof. Trivially the set of linear constraints is a convex set with respect to price variables (Nemhauser and Wolsey 1999).

Again, the format of the material flow variable \( x_j^{(Tr)} \) is written as \[ \sum_{i=1}^{m} \sum_{j=1}^{n} \alpha_{ij} \left( p_j - V_{ij}^{(Tr)} \right) \]
where all coefficients \( \alpha \)'s are given by upstream sites and \( V_{ij}^{(Tr)} \) are transportation cost parameters. The only unknown variables are all \( p \)'s. The variable of \( x_j^{(Tr)} \) can be rewritten as:

\[ x_j^{(Tr)} = \alpha_j^1 p_1 + \cdots + \alpha_j^m p_m + C_j \]  

(23)

where \( C_j \) is a constant and \( \alpha_j^j \) is the coefficient term with \( p_j \) for downstream site \( j \)'s model. The interpretation of (23) is that the material flow shipped to downstream site \( j \) is a function of price \( p_j \) and also functions of other price variables offered by other downstream sites. Clearly, the material flow is increasing as the price offer increases, but decreasing when the competitors’ prices increase, if there exists price effect between downstream site \( j \) and other downstream sites. Thus, we have the following inequality relations

\[ \alpha_j^j > 0 \quad \text{and} \quad \alpha_j^{j'} \leq 0 \quad \forall j', j' \neq j. \]  

(24)

From (24), the sign of second derivative of the objective function \( \phi_j \) can be determined as

\[ \frac{\partial^2 \phi_j}{\partial p_j^2} < 0. \]
**Proposition 2** The impact of the change of acquisition price $p_j$, $j=1, 2, ..., n$, on the material flow $x_j^{(Tr)}$ is greater than the total impact on $x_j^{(Tr)}$ due to the price changes from the rest of downstream sites.

**Proof.** For every upstream site $i$, we have $\alpha_{ij} > 0$ for all $j, j'$ when $j$ is equal to $j'$ and $\alpha_{ij} \leq 0$ for all $j, j'$ when $j$ is not equal to $j'$. In order to ensure a positive robust objective function for each arc between the upstream and downstream tier in all of price scenarios, $\omega$, we have $\left| \alpha_{ij} \right| > \sum_{j': j' \not= j} \left| \alpha_{ij'} \right|$ for all $i$ and $j$. Therefore, $\left| \alpha_j' \right| > \sum_{j': j' \not= j} \left| \alpha_j' \right|$ for all $j$ and this completes the proof. ■

In the next section, we provide some concepts and the required notation for the illustration of the downstream site model algorithm.

### 2.5.2 Definitions and Concepts

There are $j = 1, ..., n$ downstream sites participating in competing for the material flows with the price. Each downstream site $j, j = 1, ..., n$, can adopt an individual price setting denoted by $p_j \in P_j$, where $P_j$ is the set of price actions that downstream site $j$ can choose. All downstream entities, when acting together, can take a collective action, which is a vector $p = (p_1, \cdots, p_n)$. Denote the collective price action set by $P$, and, by definition, $P \subseteq P_1 \times P_2 \times \cdots \times P_n$. Let $p = (p_1, \cdots, p_n)$ and $q = (q_1, \cdots, q_n)$ be elements of the collective price action set $P_1 \times P_2 \times \cdots \times P_n$.  

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The following notation and terminology are based upon Krawczyk and Uryasev (2000) and Contreras et al. (2004). An element \((q_j \mid p) = (p_1, \ldots, p_{j-1}, q_j, p_{j+1}, \ldots, p_n)\) of the collective price action set can be interpreted as a set of price actions where the \(j\)th downstream entity selects price offer \(q_j\) while the remaining entities are taking price \(p_j\), \(j = 1, 2, \ldots, j-1, j+1, \ldots, n\). A price action set \(p^* = (p_1^*, \ldots, p_n^*)\) is called the Nash equilibrium price if, for each downstream site \(j\),

\[
\phi_j(p^*) = \max_{(p_j, p')} \phi_j(p_j \mid p^*).
\]

Note that at the Nash equilibrium solution, no entity can improve its individual objective value by a unilateral change in its price decisions. In order to compute the Nash equilibrium, we introduce the Nikaido-Isoda function (Nikaido and Isoda 1955; Rosen 1965). This function transforms an equilibrium problem into an optimization problem (Contreras et al. 2004). The Nikaido-Isoda function \(\Psi : (P_1 \times \cdots \times P_n) \times (P_1 \times \cdots \times P_n) \to \mathbb{R}\) is defined as:

\[
\Psi(p, q) = \sum_{j=1}^{n} \left[ \phi_j(q_j \mid p) - \phi_j(p) \right].
\]

Each summand of the Nikaido-Isoda function can be viewed as the change in the objective function value when its price action changes from \(p_j\) to \(q_j\) for all sites in the downstream tier, while all other downstream sites continue to choose according to price vector \(p\). This means that one entity changes its price action while others do not. Thus, the function represents the sum of these changes in objective functions. Krawczyk and Uryasev (2000) claim that the function is non-positive for all feasible \(q\) when \(p^*\) is a Nash
equilibrium solution, since no entity can improve its objective function value at equilibrium by unilaterally alternating its solution. This observation is used to construct a termination condition for the relaxation algorithm, such that when an $\varepsilon$ is chosen, the Nash equilibrium is obtained when $\max_{q \in P} \Psi(p', q) < \varepsilon$, where $s$ is the iterative step of the relaxation algorithm.

Finally we introduce the optimum response function that returns the set of downstream entities’ price actions whereby they all try to unilaterally maximize their respective objective function values. The optimum response function (Krawczyk and Uryasev, 2000) at the price vector $p$ is expressed in (25).

$$Z(p) = \arg \max_{q \in P} \Psi(p, q) \quad p, Z(p) \in P. \quad (25)$$

Next, we illustrate the relaxation algorithm to solve for the Nash Equilibrium acquisition prices between upstream and downstream tiers.

### 2.5.3 The Relaxation Algorithm

The relaxation algorithms are used by Krawczyk and Uryasev (2000) and Contreras et al. (2004) for different applications. We apply the relaxation algorithm to iteratively search for the Nash equilibrium acquisition price solution of downstream site models. At each iteration of the algorithm, downstream sites wish to move to a price point that represents an improvement on the current price point. In other words, to obtain the final price decision, downstream sites take turns setting their price decisions, and each downstream site’s chosen price decision is a best response to the decisions its competitors chose in the last iteration. The iterations continue until no entity has incentive to change
its price decision, and thus a final price decision has been obtained. Having an initial estimate price vector, \( \mathbf{p}^0 \), the relaxation algorithm is shown as follows:

\[
\mathbf{p}^{s+1} = (1 - \beta_s)\mathbf{p}^s + \beta_s Z(\mathbf{p}^s) \quad s = 0, 1, 2, \ldots
\]

(26)

where \( 0 < \beta_s \leq 1 \). The iterative step \( s+1 \) is constructed as a weighted average of the improvement price point \( Z(\mathbf{p}^s) \) and the current price point \( \mathbf{p}^s \). The optimum response function \( Z(\mathbf{p}^s) \) returns the next best move of price solutions by solving the quadratic convex model shown in (21); in turn, each downstream site is trying to maximize its objective function by unilaterally moving its price solution given others’ price solution. The iteration at each step is constructed as a weighted average of the improvement solution \( Z(\mathbf{p}) \) and the current price solution \( \mathbf{p} \). By taking a sufficient number of iterations, the algorithm converges to the *Nash equilibrium* price \( \mathbf{p}^* \) with a specified precision.

It is also interesting to note that the concept of the algorithm itself matches the idea of a decentralized view on downstream sites. In each iteration, every entity can access all entities’ previous price actions and determines its best move in price decision based on its own interests and constraints. In other words, the problem is a calculation of the succession of price decisions, where entities choose their optimum response given the price decisions of the competitors in the previous iteration.

The following theorem states the conditions of existence and uniqueness of the Nash equilibrium solution and the convergence for the relaxation algorithm. Corollary 1 shows that our optimization downstream models satisfy these conditions. First we state
required definitions in the following (see Contreras and Krawczyk 2004). Let 
\( \Psi : P \times P \to \mathbb{R} \) be a function defined on a product \( P \times P \), where \( P \) is a convex closed 
subset of the Euclidean space \( \mathbb{R}^n \). Further, we consider that \( \Psi(p, q) \) is \emph{weakly convex-concave} if it satisfies the following inequalities.

\[
\beta \Psi(x, z) + (1 - \beta) \Psi(y, z) \geq \Psi(\beta x + (1 - \beta)y, z) + \beta(1 - \beta)r_z(x, y) \quad \forall x, y, z \in P, 0 \leq \beta \leq 1
\]

and

\[
\frac{r_z(x, y)}{||x - y||} \to 0, \quad \text{as} \quad ||x - y|| \to 0, \quad \forall z \in P,
\]

and

\[
\beta \Psi(z, x) + (1 - \beta) \Psi(z, y) \leq \Psi(z, \beta x + (1 - \beta)y) + \beta(1 - \beta)\mu_z(x, y) \quad \forall x, y, z \in P, 0 \leq \beta \leq 1
\]

and

\[
\frac{\mu_z(x, y)}{||x - y||} \to 0, \quad \text{as} \quad ||x - y|| \to 0, \quad \forall z \in P.
\]

where \( r_z(x, y) \) and \( \mu_z(x, y) \) are called the \emph{residual terms}.

In other words, a function \( \Psi \) of two vector arguments is referred to as \emph{weakly convex-concave} if it satisfies weak convexity with respect to its first argument and weak concavity with respect to its second argument. The notions of weak convexity and concavity are \emph{relaxations} of strict convexity and concavity (Berridge and Krawczyk, 1997). The residual terms, to be chosen at will, ensure that there are many concave functions that are weakly convex and many convex functions that are weakly concave.

\textbf{Theorem 1} (Uryasev and Rubinstein 1994) \emph{There exists a unique Nash equilibrium point to which the relaxation algorithm converges if:}

(1) \quad \( P \) is a convex compact subset of \( \mathbb{R}^n \),

(2) \quad the Nikaido-Isoda function \( \Psi : P \times P \to \mathbb{R} \) is a weakly convex-concave function and
\( \Psi(p, p) = 0 \) for \( p \in P \),

(3) the optimum response function \( Z(p) \) is single-valued and continuous on \( P \),

(4) the residual term \( r_z(x, y) \) is uniformly continuous on \( P \) with respect to \( z \) for all \( x, y \in P \),

(5) the residual terms satisfy
\[
 r_z(x, y) - \mu_z(y, x) \geq \lambda(\|x - y\|) \quad x, y \in P 
\]
where \( \lambda(0) = 0 \) and \( \lambda \) is a strictly increasing function,

the relaxation parameters \( \beta_s \) satisfy (a) \( \beta_s > 0 \), (b) \( \sum_{s=0}^{\infty} \beta_s = \infty \), and (c)
\[
 \beta_s \to 0 \text{ as } s \to \infty .
\]

(6)

**Corollary 1** There exists a unique Nash equilibrium price solution for downstream sites to which the relaxation algorithm converges.

**Proof.** We need to verify that our downstream models satisfy conditions (1) to (6) in Theorem 1.

**Condition (1):** it is trivially satisfied.

**Condition (2):** from (23), we have that the material flow shipped to downstream site \( j, j = 1, \ldots, n \), is expressed as \( x_j^{(P)} = \alpha_1^j p_1 + \cdots + \alpha_n^j p_n + C_j \). After algebra manipulations, the objective function of downstream site \( j \) is simply expressed in (27), where \( V_j \) is a constant parameter for downstream site \( j \).

\[
\phi_j(p) = (V_j - p_j)(\alpha_1^j p_1 + \cdots + \alpha_n^j p_n + C_j) .
\]

(27)

For any solution \( (z_j | p) = (p_1, \cdots, p_{j-1}, z_j, p_{j+1}, \cdots, p_n) \in P \) and

\[
(z_j | q) = (q_1, \cdots, q_{j-1}, z_j, q_{j+1}, \cdots, q_n) \in P ,
\]

33
\[
\beta \phi_j(z_j | p) + (1-\beta) \phi_j(z_j | q) \\
= \beta (V_j - z_j)(\alpha'_1 p_1 + \cdots + \alpha'_j p_{j-1} + \alpha'_j z_j + \alpha'_{j+1} p_{j+1} + \cdots + \alpha'_n p_n + C_j) \\
+(1-\beta)(V_j - z_j)(\alpha'_1 q_1 + \cdots + \alpha'_j q_{j-1} + \alpha'_j z_j + \alpha'_{j+1} q_{j+1} + \cdots + \alpha'_n q_n + C_j) \\
= (V_j - z_j)[\alpha'_1(\beta p_1 + (1-\beta)q_1) + \cdots + \alpha'_j(\beta p_{j-1} + (1-\beta)q_{j-1}) + \alpha'_j z_j] \\
+ \alpha'_{j+1}[\beta p_{j+1} + (1-\beta)q_{j+1}] + \cdots + \alpha'_n[\beta p_n + (1-\beta)q_n] + C_j \\
= \phi_j(z_j | \beta p + (1-\beta)q). 
\]

Thus,

\[
\beta \phi_j(z_j | p) + (1-\beta) \phi_j(z_j | q) = \phi_j(z_j | \beta p + (1-\beta)q) \
\forall j. \quad (28)
\]

Since the objective functions \( \phi_j \) for all downstream sites are concave, we have

\[
\beta \phi_j(p) + (1-\beta) \phi_j(q) \leq \phi_j(\beta p + (1-\beta)q) \\
\forall j. \quad (29)
\]

Combining (28) and (29), the following inequality is satisfied.

\[
\beta[\phi_j(z_j | p) - \phi_j(p)] + (1-\beta)[\phi_j(z_j | q) - \phi_j(q)] \geq \\
\phi_j(z_j | \beta p + (1-\beta)q) - \phi_j(\beta p + (1-\beta)q) \\
\forall j. \quad (30)
\]

Summing up all inequalities of (30) for all downstream site \( j \), it implies that

\[
\beta \sum_{j=1}^{n}[\phi_j(z_j | p) - \phi_j(p)] + (1-\beta)\sum_{j=1}^{n}[\phi_j(z_j | q) - \phi_j(q)] \geq \\
\sum_{j=1}^{n}[\phi_j(z_j | \beta p + (1-\beta)q) - \phi_j(\beta p + (1-\beta)q)]. \quad (31)
\]

The definition of the Nikaido-Isoda function is referred to (31), we have

\[
\beta \Psi(p, z) + (1-\beta) \Psi(q, z) \geq \Psi(\beta p + (1-\beta)q, z). \quad (32)
\]

From (32), the Nikaido-Isoda function \( \Psi \) is convex with respect to the first argument. Based on the same algebra manipulation, the function \( \Psi \) is also concave with respect to the second argument. That is
From (32) and (33), the Nikaido-Isoda function $\Psi$ is convex-concave that is also weakly convex-concave. Then, the optimization model for each downstream site $j$ satisfies Condition (2).

**Condition (3):** the optimum response function $Z(p)$ is single-valued and continuous on $P$ by solving the concave quadratic convex constrained model.

**Condition (4):** if the Nikaido-Isoda function $\Psi(p,q)$ is twice continuously differentiable with respect to both arguments on the set $P \times P$, the residual term is given by (Uryasev and Rubinstein 1994)

$$r_z(p,q) = \frac{1}{2} \langle A(p,p)(p-q), p-q \rangle + o(\|p-q\|^2)$$

where $\langle \cdot, \cdot \rangle$ is the notation of inner product and $A(p,p) = \Psi_{pp}(p,q)|_{q=p}$ is the Hessian of the Nikaido-Isoda function with respect to the first argument evaluated at $q = p$. Moreover, if the function $\Psi(p,q)$ is convex with respect to $p$, then $o(\|p-q\|^2) = 0$ (Uryasev 1988). The residual term of $r_z(p,q)$ is a polynomial expression that is continuous on $P$. Furthermore, $r_z(p,q)$ is uniformly continuous on $P$ since $P$ is compact (Bartle 1976).

**Condition (5):** assuming that $\Psi(p,q)$ is twice continuously differentiable, in order to prove this condition, it suffices to show that $\Psi_{pp}(p,q)|_{q=p} - \Psi_{qq}(p,q)|_{q=p}$ is positive definite (Krawczyk and Uryasev 2000), where $\Psi_{pp}(p,q)|_{q=p}$ is the Hessian of the Nikaido-Isoda function with respect to the first argument and $\Psi_{qq}(p,q)|_{q=p}$ is the Hessian...
of the Nikaido-Isoda function with respect to the second argument, both evaluated at \( q = p \). The Hessian matrices of the Nikaido-Isoda function are shown in (34) and (35) respectively.

\[
\Psi_{pp}(p,q)|_{q=p} = \begin{pmatrix}
2\alpha_1^1 & \alpha_1^1 + \alpha_1^2 & \cdots & \alpha_n^1 + \alpha_n^n \\
\alpha_1^2 + \alpha_1^1 & 2\alpha_2^2 & \cdots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
\alpha_n^1 + \alpha_n^1 & \cdots & 2\alpha_n^n 
\end{pmatrix},
\]

(34)

\[
\Psi_{qq}(p,q)|_{q=p} = \begin{pmatrix}
-2\alpha_1^1 & 0 & \cdots & 0 \\
0 & -2\alpha_2^2 & \cdots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
0 & \cdots & -2\alpha_n^n 
\end{pmatrix},
\]

(35)

As discussed in Proposition 1, we require that \( \alpha_j^j > 0 \) and \( \alpha_j^j \leq 0 \ \forall j', j' \neq j \). Since all pivots for the matrix of \( \Psi_{pp}(p,q)|_{q=p} - \Psi_{qq}(p,q)|_{q=p} \) are positive, \( \Psi_{pp}(p,q)|_{q=p} - \Psi_{qq}(p,q)|_{q=p} \) is positive definite (Strang, 1986). We can conclude that the optimization models of downstream sites satisfy Condition (5).

**Condition (6):** in order for the algorithm to converge, we may choose any sequence \( (\beta_s) \) satisfying the Condition (6) of Theorem 1.

All conditions in Theorem 1 are satisfied for the optimization model of downstream sites. It completes the proof. ■
The Relaxation Algorithm states that each downstream site is trying to maximize its objective function by unilaterally moving its price solution given others’ past price solutions. The algorithm itself essentially is an iterative scheme. In the next section, we present a simultaneous algorithm to search for the Nash equilibrium price solution.

2.5.4 The Karush-Kuhn-Tucker Approach

In this section, we demonstrate the Karush-Kuhn-Tucker (KKT) approach to find the Nash Equilibrium acquisition price. Each of downstream entities has its own optimal conditions in terms of its decision variables and others’ decision variables. A market equilibrium is defined as a set of decisions that satisfy each participant’s first-order conditions (KKT) for maximization of its profit. A solution satisfying those conditions possesses the property that no entity wants to alter its decision unilaterally and such a solution is referred to the Nash equilibrium solution (Hobbs 2001). However, we point out that KKT conditions are necessary optimal conditions for the local optimum in general, not sufficient conditions for the optimum. In order to satisfy the properties of the Nash equilibrium, we need to solve the globally sufficient KKT conditions simultaneously for the entire downstream tier instead of solving the general locally necessary KKT conditions.

The KKT approach is created by combining the KKT optimal conditions for all downstream sites with respect to their own decision variables. The combined KKT system for all downstream site \( j = 1, \ldots, n \) optimization models, referring to (22), are stated as:
\begin{align*}
g_d^j(p) + s_d^j &= b_d^j & \forall d = 1 \cdots r, j \quad (36) \\
\frac{\partial \phi_j(p)}{\partial p_j} - \sum_{d=1}^r \lambda_d^j \frac{\partial g_d^j(p)}{\partial p_j} + e_j &= 0 & \forall j \quad (37) \\
e_j p_j &= 0 & \forall j \quad (38) \\
\lambda_d^j s_d^j &= 0 & \forall d = 1 \cdots r, j \quad (39) \\
p_j &\geq 0, e_j &\geq 0 & \forall j \quad (40) \\
\lambda_d^j &\geq 0, s_d^j &\geq 0 & \forall d = 1 \cdots r, j. \quad (41)
\end{align*}

where $\lambda$'s are dual variables for constraints and both $s_d^j$ and $e_j$ are nonnegative slack variables to convert inequalities to equalities. Constraints (36) and (40) are corresponded to primal feasibility equalities, constraints (37) and (41) are dual feasibility equalities, and constraints (38) and (39) are complementary slackness conditions.

The system of combined KKT conditions is essentially a linear complementary problem (LCP) since all downstream optimization models are quadratic programming models, in which primal constraints and dual feasibility constraints are linear. Several algorithms have been developed for solving the general linear complementary problem (Lemke 1965; Bazaraa et al. 1993). The solution of the combined KKT approach is governed either by the primal feasibility equalities (the capacity constraints) or the dual feasibility equalities (the first-order conditions). If we believe that one particular downstream site reaches its capacity constraint, we may only need to take its primal feasibility equalities (36) into consideration for solution computation to avoid the more computationally challenging complementary slackness conditions (38) and (39).
**Corollary 2** There exists a unique Nash equilibrium price solution obtained by solving the combined KKT system in conditions (36) - (41).

**Proof.** Proposition 1 shows that downstream site $j$’s model is with a strictly concave objective function for $j = 1, \ldots, n$. It implies the optimum response function of $p_j$ for downstream site $j, j = 1, \ldots, n$, is unique. There are two possible combinations for solving the combined KKT system.

Case 1: None of the primal constraints are tight.

The combined KKT system is governed by all dual feasibility equalities and the resulting matrix for the linear system is stated as:

$$
\begin{pmatrix}
2\alpha^1_1 & \alpha^1_2 & \cdots & \alpha^1_n \\
\alpha^2_1 & 2\alpha^2_2 & \cdots & \alpha^2_n \\
\vdots & \ddots & \ddots & \vdots \\
\alpha^n_1 & \cdots & 2\alpha^n_n
\end{pmatrix}. 
$$

(42)

Case 2: The primal constraint for any downstream site $j$ is tight.

Row $j$ of the resulting matrix in (42) is replaced by $(\alpha^j_1 \alpha^j_2 \cdots \alpha^j_n)$. Both of the cases indicate the linear independence columns as a result of (24) and Proposition 2, which completes the proof of existence and uniqueness. ■

We note that, if all optimality conditions are known, then one can directly solve them simultaneously to find the Nash equilibrium solution. We also emphasize the key difference between the relaxation algorithm of Section 2.5.3 and the KKT approach. Conceptually, the downstream entities need to pass the information of their optimal conditions to some *invisible hand* in the system or we restrict attention to games of
complete information where each downstream site’s payoff function is common knowledge among all downstream sites (see Gibbons 1992). This implies the willingness to share information among participants with such a centralized body, in order to obtain the equilibrium solution simultaneously. Nevertheless, in the progress the relaxation algorithm, the only exogenous information for any particular downstream site is the past price solutions of other downstream sites, which follows the real-world assumption that the information of each independent participant at the current step is not revealed.

Either the iterative relaxation algorithm or the simultaneous KKT approach solves for the Nash equilibrium acquisition prices of downstream sites. We obtain the resulting material flows, \( x_{ij}^{(Tr)^*} \) for all \( i \) and \( j \), between upstream and downstream tiers by substituting the equilibrium prices \( (p_1^*, \cdots, p_n^*) \) in (19). Finally we derive the resulting material flow amount collected in upstream site \( i \), denoted by \( S_i^* \), and the resulting collection fee in site \( i \), \( P_i^{(Co)^*} \), by solving the linear equation system in (43),

\[
S_i^* = \sum_{j=1}^{n} x_{ij}^{(Tr)^*} \quad \forall i
\]

where \( S_i = a_i - b_i P_i^{(Co)} \).

In the next section, we summarize the solution algorithm consisting of upstream and downstream models and explicitly illustrate the decision-making procedures under this framework of Sections 2.4 and 2.5.
2.6 The Summary of the Solution Algorithm

The overall reverse production system is comprised of four entities: sources, collectors, processors and customers as shown in Figure 3. The material flows are represented as solid lines, information communication is denoted by the dashed line, and the steps of the solution algorithm are illustrated by the number in the rectangle box in Figure 3. The behavior of the source and the customer is assumed simple. The source supplies the collector on the basis of a fee charged by the collector, and the relationship is known by both parties. We assume that this is linear in the fee, with a decreasing flow for a higher collection fee. The source can be thought of as the aggregate behavior of many independent entities that have a given product ready for disposal that will choose, or choose not, to go to a collector based on the fee. The customers have a fixed price for a given material or sub-component, and the market is assumed to be very large such that the flow from processors to customers does not change the price. The customers communicate the price to the processors at the outset, but it is not considered private information.
The collectors do not compete among each other, but must gauge what price the processors will offer for the collected items. On the basis of the assumed price ranges of each processor, the collectors develop a set of flow functions. The collectors determine flow functions first and then communicate them to the processors. The flow functions will determine, on the basis of the actual acquisition price offered, the flow of material to each of the processors.

The processors do compete with one another for the flow from the collectors on the basis of the price they offer for the recycled item. They find their prices by iteratively picking their own price offer and posting it for all the processors to see. This is the only information each processor obtains about every other processor. The processors continue to post prices until they have no incentive to change their offer. In other words, each processor adjusts its price offer by optimizing its model given the most recently price choices of the other processors. This completes the left hand side of Figure 3. At this point the price from each processor is communicated to every collector, who then
determines the flows to the processor and the collection fee to acquire the recycled item. Then, the items are collected from the sources by the collectors, transferred to the processors, and finally the components or materials to the end customers, as depicted on the right hand side of Figure 3.

In the next section, we apply the upstream and downstream models to an example to generate the material flow allocation mechanism as well as the equilibrium acquisition price between upstream and downstream tiers. Then, we obtain the corresponding material flows and equilibrium collection fees of the collectors.

### 2.7 A Numerical Example

This example, depicted in Figure 4, illustrates the application of the above upstream and downstream models. There are three collection sites, $i = 1, 2, 3$, in the upstream tier and three processing sites, $j = 1, 2, 3$, in the downstream tier. The collection sites collect end-of-life products from sources and ship them to processing sites. The transportation costs per unit flow between collection and processing sites are given in Table 1.

![Figure 4: The Reverse Production System for the Example](image-url)
Table 1: The Unit Transportation Costs between Entities

<table>
<thead>
<tr>
<th>Unit transportation cost</th>
<th>( V_{11}^{(Tr)} )</th>
<th>( V_{12}^{(Tr)} )</th>
<th>( V_{13}^{(Tr)} )</th>
<th>( V_{21}^{(Tr)} )</th>
<th>( V_{22}^{(Tr)} )</th>
<th>( V_{23}^{(Tr)} )</th>
<th>( V_{31}^{(Tr)} )</th>
<th>( V_{32}^{(Tr)} )</th>
<th>( V_{33}^{(Tr)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>20</td>
<td>75</td>
<td>12</td>
<td>18</td>
<td>67.5</td>
<td>15</td>
<td>22</td>
<td>64.5</td>
<td></td>
</tr>
</tbody>
</table>

The final market prices for processing sites, \( j = 1, 2, 3 \), are $105, $110, and $150. The collection amount functions in collection sites, \( i = 1, 2, 3 \), are given by \( S_i = 350 - 5 P_i^{(Co)} \), \( S_2 = 320 - 4 P_2^{(Co)} \), and \( S_3 = 330 - 5 P_3^{(Co)} \). Clearly, the collection amount decreases as the collection site charges a higher collection fee per unit. In this example, we model the case that the transportation costs (or the distances) from Processing site 3 to collection sites are relatively larger than the costs of other arcs; however, the final market price for Processing site 3 provides the higher incentive to attract recycled items.

We use \( ($60, $70, $110) \pm 20 \) as the prediction acquisition price range for processing sites, \( j = 1, 2, 3 \), in the upstream model and choose 5 evenly distributed points in each price range. The upstream model yields the following robust flow functions:

\[
\begin{align*}
X_{11}^{(Tr)} &= 2.73(p_1 - 10) - .10(p_2 - 20) - 1.36(p_3 - 75), \\
X_{12}^{(Tr)} &= -.10(p_1 - 10) + 2.73(p_2 - 20) - 1.36(p_3 - 75), \\
X_{13}^{(Tr)} &= -.35(p_1 - 10) - .35(p_2 - 20) + 3.30(p_3 - 75), \\
X_{21}^{(Tr)} &= 2.72(p_1 - 12) - 1.22(p_3 - 67.5), \\
X_{22}^{(Tr)} &= -.64(p_1 - 12) + 2.23(p_2 - 18) - .44(p_3 - 67.5), \\
X_{23}^{(Tr)} &= -.88(p_1 - 12) + 2.65(p_3 - 67.5), \\
X_{31}^{(Tr)} &= 2.78(p_1 - 15) - .73(p_2 - 22) - .31(p_3 - 64.5), \\
X_{32}^{(Tr)} &= -.08(p_1 - 15) + 2.67(p_2 - 22) - 1.06(p_3 - 64.5), \\
X_{33}^{(Tr)} &= -1.14(p_1 - 15) + 2.92(p_3 - 64.5).
\end{align*}
\]
We can apply the relaxation algorithm or the combined KKT approach to obtain the Nash Equilibrium acquisition prices determined by processing sites. The detailed steps of the relaxation algorithm are illustrated in Table 2. At iteration 7, the \( \max_{q \in P} \Psi(p^*, q) \) approaches to zero which indicates \( p_1^*, p_2^*, \) and \( p_3^* \) are the Nash Equilibrium acquisition prices for processing sites 1, 2, and 3.

**Table 2:** The Calculation of the Relaxation Algorithm for the Example

<table>
<thead>
<tr>
<th>Iteration</th>
<th>( p_1^* )</th>
<th>( p_2^* )</th>
<th>( p_3^* )</th>
<th>( Z(p_1^*) )</th>
<th>( Z(p_2^*) )</th>
<th>( Z(p_3^*) )</th>
<th>( \alpha_s )</th>
<th>( p_1^{s+1} )</th>
<th>( p_2^{s+1} )</th>
<th>( p_3^{s+1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>60.00</td>
<td>60.00</td>
<td>60.00</td>
<td>58.73</td>
<td>65.79</td>
<td>116.72</td>
<td>1.00</td>
<td>58.73</td>
<td>65.79</td>
<td>116.72</td>
</tr>
<tr>
<td>2</td>
<td>58.73</td>
<td>65.79</td>
<td>116.72</td>
<td>68.95</td>
<td>76.36</td>
<td>116.67</td>
<td>0.99</td>
<td>68.85</td>
<td>76.25</td>
<td>116.67</td>
</tr>
<tr>
<td>3</td>
<td>68.85</td>
<td>76.25</td>
<td>116.67</td>
<td>69.47</td>
<td>76.90</td>
<td>118.23</td>
<td>0.98</td>
<td>69.46</td>
<td>76.88</td>
<td>118.20</td>
</tr>
<tr>
<td>4</td>
<td>69.46</td>
<td>76.88</td>
<td>118.20</td>
<td>69.77</td>
<td>77.22</td>
<td>118.33</td>
<td>0.97</td>
<td>69.76</td>
<td>77.21</td>
<td>118.32</td>
</tr>
<tr>
<td>5</td>
<td>69.76</td>
<td>77.21</td>
<td>118.32</td>
<td>69.81</td>
<td>77.26</td>
<td>118.37</td>
<td>0.96</td>
<td>69.81</td>
<td>77.25</td>
<td>118.37</td>
</tr>
<tr>
<td>6</td>
<td>69.81</td>
<td>77.25</td>
<td>118.37</td>
<td>69.82</td>
<td>77.27</td>
<td>118.38</td>
<td>0.95</td>
<td>69.82</td>
<td>77.27</td>
<td>118.38</td>
</tr>
<tr>
<td>7</td>
<td>69.82</td>
<td>77.27</td>
<td>118.38</td>
<td>69.82</td>
<td>77.27</td>
<td>118.38</td>
<td>0.94</td>
<td>69.82</td>
<td>77.27</td>
<td>118.38</td>
</tr>
</tbody>
</table>

The combined KKT approach is also used to find the Nash Equilibrium acquisition prices. After adding slack variables \( (s_1, s_2, s_3, e_1, e_2, e_3) \) and dual variables \( (\lambda_1, \lambda_2, \lambda_3) \), the KKT optimality conditions to processing sites 1, 2, and 3 are listed in Table 3.
Table 3: The Combined KKT Optimality Conditions

<table>
<thead>
<tr>
<th>Processing Site</th>
<th>Primal feasibility</th>
<th>Dual feasibility</th>
<th>Complementary slackness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Site 1</td>
<td>8.22p_1 - .83p_2 - 2.88p_3 + 120.20 + s_1 = 400</td>
<td>-16.45p_1 + .83p_2 + 2.88p_3 + 743.35 - 8.22 \lambda_1 + e_1 = 0</td>
<td>\lambda_1 s_1 = 0, e_1 p_1 = 0</td>
</tr>
<tr>
<td>Site 2</td>
<td>-0.83p_1 + 7.64p_2 - 2.86p_3 + 56.64 + s_2 = 400</td>
<td>0.83p_1 - 15.27p_2 + 2.86p_3 + 783.23 - 7.64 \lambda_2 + e_2 = 0</td>
<td>\lambda_2 s_2 = 0, e_2 p_2 = 0</td>
</tr>
<tr>
<td>Site 3</td>
<td>-2.37p_1 - .35p_2 + 8.87p_3 - 576.13 + s_3 = 500</td>
<td>2.37p_1 + .35p_2 - 17.73p_3 + 1905.95 - 8.87 \lambda_3 + e_3 = 0</td>
<td>\lambda_3 s_3 = 0, e_3 p_3 = 0</td>
</tr>
</tbody>
</table>

The solution of \((p_1, p_2, p_3, s_1, s_2, s_3, e_1, e_2, e_3, \lambda_1, \lambda_2, \lambda_3)\) is equal to \((69.82, 77.27, 118.38, 111.4, 149.6, 218.8, 0, 0, 0, 0, 0)\) which is identical to the solution obtained from the relaxation algorithm. The corresponding material flows between collection and processing sites and equilibrium collection fees of collection sites are listed in Table 4 and Table 5 respectively.

Table 4: The Resultant Material Flows between Collection and Processing Sites

<table>
<thead>
<tr>
<th>Material flow</th>
<th>(x_{11}^{(Tr)})</th>
<th>(x_{12}^{(Tr)})</th>
<th>(x_{13}^{(Tr)})</th>
<th>(x_{21}^{(Tr)})</th>
<th>(x_{22}^{(Tr)})</th>
<th>(x_{23}^{(Tr)})</th>
<th>(x_{31}^{(Tr)})</th>
<th>(x_{32}^{(Tr)})</th>
<th>(x_{33}^{(Tr)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>98.3</td>
<td>91.1</td>
<td>101.8</td>
<td>95.2</td>
<td>72.6</td>
<td>84.1</td>
<td>95.8</td>
<td>86.2</td>
<td>94.4</td>
<td></td>
</tr>
</tbody>
</table>

Table 5: The Equilibrium Collection Fees

<table>
<thead>
<tr>
<th>Equilibrium collection fee</th>
<th>(p_1^{(Co)*})</th>
<th>(p_2^{(Co)*})</th>
<th>(p_3^{(Co)*})</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.76</td>
<td>17.02</td>
<td>10.72</td>
<td></td>
</tr>
</tbody>
</table>

The preceding example demonstrates a two-tier and single-commodity decentralized RPS problem that can be solved using the models given in Section 2.4 and 2.5. First, the upstream model for each collection site provides the material flow.
allocation mechanism used to contract with processing sites. Then, we solve for the Nash equilibrium acquisition prices between collection and processing sites either by the relaxation algorithm or the combined KKT approach. Finally, we obtain the corresponding material flows between these two tiers and the equilibrium collection fees of collection sites.

2.8 Conclusions and Extensions

This chapter presents a decentralized perspective for reverse production systems where each independent entity considers its own objective function and is subject to its own constraints. However, the objective function of each entity not only depends on its own decision variables, but also depends on decision variables, the price offers, of other entities. In this chapter, we focused on two-tier reverse production systems involving the price and material flow decisions where the material flow allocation mechanism is determined by upstream entities and the acquisition prices of material flows transacted between upstream and downstream sites are determined by downstream entities. We applied the min-max robust optimization on each of the independent upstream models to generate material flow allocation equations that are used to contract with downstream sites. One may utilize other objective criteria to seek its best flow function between the upstream and downstream tiers, such as an expected value objective, or an objective that includes risk, such as conditional-value-at-risk. A discussion on this extension can be found in Section 6.2.

Downstream entities compete for material flows from upstream tier. An iterated relaxation algorithm is used to solve for the Nash equilibrium acquisition prices between
upstream and downstream sites. Note that the algorithm itself matches the idea of a
decentralized decision-making process, where every downstream entity can access all
entities’ previous price actions and determines its next best move for its price decision.
In addition to the iterative scheme of the relaxation algorithm, a simultaneous KKT
approach is applied to find the Nash equilibrium acquisition prices. In the KKT
approach, the downstream entities need to pass their optimality conditions to some
invisible hand in the system. This requires information sharing among independent
entities, which may be an unrealistic expectation, the relaxation algorithm can avoid this
drawback. We also show the existence and uniqueness of the Nash Equilibrium under
reasonable assumptions about the underlying functions of each entity. Our numerical
example illustrates that both of the algorithms lead to the same equilibrium price solution.

One of the model limitations is that we specifically focus on valuable items. As a
result, the acquisition prices are assumed to be positive. This assumption also assures
that the quantity is valid under the flow function format proposed in this research.
Otherwise, the flow quantity may be negative when the acquisition prices are negative
due to the sign restrictions of robust variables $\alpha's$. An extension of this research is to
apply other formats of flow functions so that the model is able to handle the negative
acquisition price scenarios.

Many reverse production systems have network structures that involve more than
these two types of entities we have discussed here, and with more than one type of item
to be picked up and recycled. For example, computers, printers, monitors and other
auxiliary equipment are available from sources and may be converted into commodities
such as steel and copper through a supply chain that involves multiple processors
engaged in size reduction and smelting. The extension of our approach to these multi-tier problems with multiple item types requires further refinement of the models we have developed. Several of these refinements will be presented in subsequent chapters of this thesis.
CHAPTER 3 DECENTRALIZED DECISION-MAKING AND PROTOCOL DESIGN FOR RECYCLED MATERIAL FLOWS FOR A MULTI-TIERED NETWORK

A decentralized reverse logistics network is composed of independent entities that fulfill the functions of collectors, processors, or recyclers and who often compete with one another. We conceptualize this network in terms of tiers, where each tier represents one typical function, or operation, in the recycling network. We consider a general model of decentralized reverse production systems comprised of an upstream boundary tier, intermediate tiers, and a downstream boundary tier. Each tier is populated by multiple independent entities and each tier is connected to its upstream and downstream neighbors by material flows. The upstream tier coordinates with its downstream neighbor through a flow function whose parameters are determined by a robust optimization scheme to account for uncertainty in prices offered by the downstream tier. In this chapter, we develop a methodology to determine the equilibrium acquisition prices as well as the material flow allocation mechanisms between tiers. The individual entity behavior is to maximize its own profit functions subject to its own constraints. To demonstrate the approach, a numerical example is given for a prototypical recycling network.

3.1 Introduction

The importance of recycling end-of-life products is being recognized due to the increasing concern for the environmental impact of disposal and the economic reuse
value of materials. In the past decade, more attention has been brought to the research area of design, planning, and modeling for “closed-loop” supply chain systems (Fleischmann et al. 2000; Ammons et al. 2001; Guide and Harrison 2003). A reverse production system (RPS) is a logistics and production network that includes collection, sorting, demanufacturing, and refurbished processes for end-of-life products. Like forward supply chains, a RPS is composed of different tiers with multiple entities which conduct different operations in the reverse supply chain from end-of-life product sources to secondary product or raw material markets. Figure 5 demonstrates a typical example of material and financial flows for a RPS.

We take the electronic scrap (e-scrap) recycling system as an example to illustrate the characteristics and operations of a multi-tiered RPS. In general, an e-scrap RPS is composed of collection, consolidation, processing, and material recycling sites, which can be classified as different functional tiers. Collection sites collect e-scrap items such as central processing units (CPUs), desktop and laptop computers, monitors, televisions, cell phones, etc., from sources of end-of-life products, e.g., schools, the governments, businesses, or residential households, and then ship these recycled items to consolidation sites that may be involved in some sorting or basic dismantling processes or simply acting as intermediary brokers between collection and processing phases. Processing sites obtain materials from consolidation sites and further conduct some demanufacturing or disassembly operations. Hence, processing sites may disassemble e-scrap goods into electronic component products such as disk drives, motherboards, computer chips, metals, wires, etc. Finally, materials are transported to recycling sites that can include smelters to convert e-scrap items into raw materials or processors that further refurbish
products sold in raw material or secondary product markets such as precious metal or refurbished product demand markets. In addition to the material flow shipped from the upstream to the downstream tiers, recycled items may be sent directly to the final secondary markets. For example, processing sites may sell directly the by-products, such as hard drives, of an initial disassembly process in reuse markets, but other sub-components, e.g., motherboards and computer chips, are sent exclusively to the material recycling sites in the next tier for the further demanufacturing processing.

**Figure 5:** An E-Scrap Example of the Reverse Production System

Illustrations of the physical material and financial flows of a multi-tiered e-scrap RPS are depicted as solid and dashed arrows respectively in Figure 5. In general downstream sites acquire recycled items from upstream sites within the network. We note that collection sites, specifically in the e-scrap recycling industry, may pay or charge for collecting or processing e-scrap items collected from end-of-life product sources. For example, our experience with firms in the e-scrap recycling industry indicates that residential or business sources may need to pay a collection fee to collection sites for
discarding obsolete e-scrap products (Hong et al. 2005c). However, in some circumstances, collection sites may also need to purchase e-scrap items with high residual value.

Many researchers have studied design and modeling on reverse supply chains with multiple nodes and tiers (see Barros et al. 1998; Shih 2001; Realff et al. 2004; Fleischmann et al. 2004; Assavapokee et al. 2005). Barros et al. (1998) present a network for the recycling of sand from construction waste. Shih (2001) employs a mixed integer programming method to create an optimal collection and recycling system plan for end-of-life computers and home appliances. Realff et al. (2004) discuss the view and provide models for the general RPS problems in the carpet recycling industry. Fleischmann et al. (2004) review quantitative models for supporting reverse logistics network design. Assavapokee et al. (2005) propose an approach to solve the robust network infrastructure design under uncertainty of continuous or discrete system parameters. These previous studies, however, address the entire network scope and view the system in a centralized way. In other words, the system planner has comprehensive knowledge of the parameter values of all entities and seeks the optimal solution for the entire system.

A growing number of research papers on forward or reverse supply chains model the independent decision-making process of each supply chain entity, specifically in the interaction between pricing decisions and material flow volume transacted within the network (see Majumder and Groenevelt 2001; Guide et al. 2003; Corbett and Karmarkar 2001; Nagurney and Toyasaki 2005). Nevertheless, they are limited in the topological network structure itself or solving for endogenous prices of recycled materials instead of
explicit price decisions. Walsh and Wellman (2003) also present a decentralized model of supply chain formation which includes the process of determining the participants in the supply chain, who will exchange what with whom, and the terms of the exchanges.

In this chapter, we take the perspective of decentralized decision-making analysis and protocol design for a multi-tiered RPS where every entity considers its own objective function subject to its own constraints of collection or processing capacities and the performance of each entity is not only a function of its own decisions, but also affected by the decisions of other entities. The foundations for our analytical framework are derived from recent work in decentralized RPSs that focus on two tiers of decision-makers, where one tier is comprised of collectors, and the other tier is composed of processing sites competing amongst themselves for input. Collection sites design the material flow allocation mechanism and processing sites determine the acquisition price between them under a network equilibrium (Hong et al. 2005a). In this chapter, we generalize the approach and present a multi-tiered RPS framework for the analysis, modeling, and computation of the explicit equilibrium acquisition prices, as well as the material flow allocation mechanism, between different tiers of independent entities. We are interested specifically in pricing mechanisms and material flow determination behaviors of the various entities in the network where processing of recycled materials moves from the top tier of the entities in the network to the bottom tier entities, and material flows are driven by the financial incentives which are generated in the bottom tier of entities that reflect the secondary or raw material demand markets.

The remainder of the chapter is organized as follows. In Section 3.2 we define a multi-tiered RPS problem more formally with an upstream boundary tier, intermediate
tiers, and a downstream boundary tier. Section 3.3 provides the mathematical models for each of the independent entities in different tiers and an algorithm to solve for the equilibrium acquisition prices as well as resulting material flows between tiers. In Section 3.4 we summarize the solution algorithm for the multi-tiered RPS model. In Section 3.5, we apply our algorithm to a numerical example to illustrate both models and the approach. We conclude the chapter in Section 3.6 and also suggest directions for future research.

3.2 A Multi-tiered RPS Problem

A RPS is a network of transportation, logistics, and processing functions that collect, recycle, refurbish, and demanufacture end-of-life products. In this chapter, we represent the RPS as a multi-tiered network, depicted in Figure 6, with an upstream boundary tier, intermediate tiers, and a downstream boundary tier. We consider \( N_1 \) independent entities in the upstream boundary tier as represented by the top tier of nodes in Figure 6, \( N_2, \ldots, N_{M-1} \) entities in intermediate tiers 2,\ldots, M-1 respectively, and \( N_M \) downstream boundary tier entities associated with the bottom tier in the network. In addition, we let sources of recycled products and demand markets be two end tiers of the network which may be represented as several independent sources of end-of-life products and demand markets for secondary used products or raw materials.
Typical upstream boundary tier entities represent municipal collection sites, non-profit collection organizations, private collectors, etc. The entities in the upstream boundary tier collect recycled items from source supply, which can include, for example, residential households, businesses, schools, or the government. It is assumed there is a dynamic used-product market where the collected amount depends on the collection fee between the upstream boundary tier and the source. In other words, the upstream boundary entities may need to pay or charge an amount of money to obtain obsolete recycle items. The intermediate tiers may contain several levels of entities: for example, the tier of consolidation sites, material brokers, and processing sites who bid for collected materials from their preceding tier. They conduct some valued added processes such as sorting, or disassembly operations or simply act as an intermediate broker between tiers.
Downstream boundary tier entities associated with nodes in the bottom tier in the network can be seen as the final stage of the entire RPS where they purchase recycled items from their preceding tier and conduct further dismantling or mechanical fragmentation of items or refurbish end-of-life products for consumption purposes. Hence, downstream boundary tier entities may convert the recycled items into raw materials, refurbished products and sell them in the specific demand markets.

In general, recycled items flow from the upstream tier to the downstream tier, but acquisition prices are set by the downstream tier back to the upstream tier. Other than recycling streams within the network, we note that each entity may transport a certain fixed fraction of collected items to exogenous demand markets or other recycling streams, as depicted in dashed lines in Figure 6, since recycled items can be converted or disassembled to several different commodities which may be directly sold in an exogenous market. For simplicity, we assume that only one type of commodity is transacted between any two consecutive tiers within the network and materials must move through each tier sequentially. In other words, materials can not be directly transported across two or more tiers within the network but it is allowed for materials directly to be shipped to an exogenous market.

We assume the upstream tier specifies the material flow allocation parameters used to contract with its subsequent downstream tier and the downstream tier determines the equilibrium prices to acquire the recycled items from its preceding upstream tier. The material flow allocation mechanism is a function describing the relation of the material flow volume and the acquired prices between two consecutive tiers. We refer to the material flow allocation mechanism as the flow function. We assume that the
transportation cost for the shipment of the recycled item between any two tiers is paid by
the downstream tier entity. The price the downstream tier entity pays for transportation is
taken into account by the upstream tier entity in the flow function. In this chapter, we
specifically focus on the flow of valuable items transacted within the network; as a result,
we assume the prices obtained in entities from their subsequent tier are positive.

Decisions of the entities in intermediate tiers are the flow functions for their
subsequent downstream tier and the acquisition prices for their preceding upstream tier,
but the decisions in the upstream and downstream boundary tiers are slightly different
from intermediate tiers. We assume the collection amount of the entity in the upstream
boundary tier is a function of the collection fee that the entity charges, or pays, to sources
of end-of-life products. The entity in the upstream boundary tier determines the optimal
collection fee to obtain end-of-life products and communicates the flow function to the
subsequent tier. We also assume that the amount of raw materials resulting from the
decomposition of end-of-life products is relatively small compared to the quantity of
available virgin raw material and brand-new product markets. This observation leads to
the assumption that the selling prices of raw materials or used products in demand
markets are fixed amounts, not affected by the sales quantities. As a result, the decision
of the entity in the downstream boundary tier is only the acquisition price for its
preceding upstream tier.

The decision timeline for a $M$-tiered problem is shown in Figure 7 where the upper
arrows indicate the entity tasks and the lower arrows show the information disclosure
timeline. The flow functions are independently designed by the upstream tier sites and
communicated to the subsequent downstream tier sites. The algorithm starts at the sites
in tier 1 to determine the flow function between itself and the second tier sites given the source response functions. The sites in tier 1 communicate flow functions to the sites in intermediate tier 2. Then, each intermediate tier site also independently determines the associated flow functions and communicates them to its next tier sites. Acquisition prices are set by the downstream tier back to the upstream tier sequentially shown in Figure 7.

**Figure 7:** The Decision Timeline for a $M$-tiered Problem

Figure 8 also explicitly states the steps in the decision sequences of the algorithm. The flow functions are sequentially designed by the upstream boundary tier sites, the intermediate tier sites, and the downstream boundary tier sites. This completes the left
hand side of Figure 8. The downstream boundary tier site finds the acquisition price to be offered to its preceding upstream tier by iteratively picking its own best price offer and posting it for all sites within the same tier to see until there is no incentive to change its price offer. This is the bidding process shown in Figure 8. In a similar manner, the intermediate tier sites determine their acquisition prices to be offered to their preceding tier. Finally, the upstream boundary tier sites decide the corresponding equilibrium collection fees to acquire recycled items from sources.

\[ \text{Upstream boundary tier} \]

\[ \text{Intermediate tier} \]

\[ \text{Downstream boundary tier} \]

\[ \rightarrow \text{Material flow contract action} \]

\[ \rightarrow \rightarrow \text{Price action} \]

\[ \uparrow \text{Step in the decision sequence} \]

1. Upstream tier entities determine material flow contracts.
2. Intermediate tier entities determine material flow contracts.
3. The Nash equilibrium price is determined at the downstream boundary tier, and resulting flows from the intermediate tier are calculated.
4. The Nash equilibrium price is determined at the intermediate tier, and resulting flows from the upstream boundary tier are calculated.
5. Collection fees are determined by the entities in the upstream boundary tier.

**Figure 8:** Summary of the Decision Sequences
3.3 Models of Entities in Each Tier

In this section, we develop the multi-tiered RPS models consisting of the upstream boundary tier, intermediate tier, and downstream boundary tier models. Before we develop a general framework of the analysis for the multi-tiered RPS, we first discuss two basic concepts of the price prediction range and the material flow allocation mechanism (flow functions) between two consecutive tiers \( m-1 \) and \( m \) depicted as solid lines in Figure 9. As discussed in Section 3.2, each site in the upstream tier independently designs its flow function used to contract with the sites in its subsequent downstream tier. We make the assumption of no, or limited, information sharing in this decentralized system, so that site \( i \) in tier \( m-1 \) does not know the exact acquisition prices to be offered by the sites in tier \( m \). Upstream site \( i \) acquires estimates of scenarios for the possible acquisition prices as input information for determining the flow functions. One way to forecast the lower and upper bounds of acquisition prices is described in (Hong et al. 2005a).

**Figure 9:** The Network Structure of Tiers \( m-1 \) and \( m \)
We let $I_m = \{1, \cdots, j, \cdots, N_m \}$ denote the set of sites in tier $m$ and define $\Omega_m^i$ as the set of all specified price scenarios of tier $m$ predicted by site $i$. We also let $q_{j\omega}^{(i)}$ denote the unit material price that site $j$ in tier $m$ is willing to offer in price scenario $\omega \in \Omega_m^i$ and it is predicted by site $i$ in tier $m-1$. Considering upstream tier $m-1$ and downstream tier $m$, a particular price combination, $(q_{1\omega}^{(i)}, \cdots, q_{j\omega}^{(i)}, \cdots, q_{N_m\omega}^{(i)})$, of downstream entities, refers to one price scenario $\omega \in \Omega_m^i$. If the range of the acquisition prices forecasted by the upstream entity is a continuous compact interval, there are an infinite number of scenarios, which leads to a difficult, and currently computationally intractable, optimization problem. In this chapter the price range is restricted to a finite number of discrete points. A practical approach for computation is to select $\zeta$ points evenly in every dimension of the price range. Note that selecting $\zeta$ points evenly distributed over the price range for the computational analysis does not imply any assumption about the underlying probability distribution for price. Thus, the scenario space $\Omega_m^i$ considered is with $\zeta^{N_m}$ scenarios if there are $N_m$ downstream entities in tier $m$. We define a $N_m \times \zeta^{N_m}$ matrix, $Q^{(i)}$, to represent the considered acquisition price space predicted by site $i$ in tier $m-1$

$$Q^{(i)} = \begin{pmatrix} q_{1\omega}^{(i)} & \cdots & q_{j\omega}^{(i)} & \cdots & q_{N_m\omega}^{(i)} \\ \vdots & & \vdots & & \vdots \\ q_{j\omega}^{(i)} & q_{j\omega}^{(i)} & q_{j\omega}^{(i)} & & \vdots \\ \vdots & & \vdots & & \vdots \\ q_{N_m\omega}^{(i)} & q_{N_m\omega}^{(i)} & q_{N_m\omega}^{(i)} & & \vdots \end{pmatrix}$$

where $i \in I_{m-1}$, $j \in I_m$, and $\omega \in \Omega_m^i$.

The material flow allocation mechanism is a set of functions describing the relation of material flow volume and the acquisition prices among sites in two consecutive tiers.
Due to the assumption that materials must move through each tier sequentially, or not at all, we first examine the flow function between tiers $m-1$ and $m$, as depicted in Figure 9, and then generalize it to describe the material flow allocation mechanism between any two consecutive tiers in the network. We follow the recent work of modeling flow-price correspondence in a two-tier RPS framework (Hong et al. 2005a). Let $V_{ij}^{(Tr)}$ denote the unit transportation cost from upstream site $i \in I_{m-1}$ to downstream site $j \in I_m$ and $x_{ij_0}^{(Tr)}$ represent the material flow amount from upstream site $i$ to downstream site $j$ for price scenario $\omega \in \Omega_m$. The unit reward that site $i \in I_{m-1}$ receives from site $j \in I_m$ is represented as the material price that downstream site $j$ is willing to offer subtracting the associated unit transportation cost. Consequently, the unit reward of site $i \in I_{m-1}$ in price scenario $\omega \in \Omega_m$ is $q_{ji_0}^{(i)} - V_{ij}^{(Tr)}$. The decision variables for upstream site $i$ are the coefficients of material flow determination, denoted by $\alpha_{ij'}$, from upstream site $i$ to downstream site $j$ affected by another downstream site $j'$ for all of downstream site pairs $j$ and $j'$ where $i \in I_{m-1}$ and $j, j' \in I_m$. Note the decision variables of $\alpha_{ij'}$ are not dependent on price scenario $\omega \in \Omega_m$; in turn, $\alpha_{ij'}$ is a common set of coefficients for all of price scenarios in $\Omega_m$. Hence, for any upstream site $i$ in tier $m-1$, the material flow from site $i \in I_{m-1}$ to downstream site $j \in I_m$ in price scenario $\omega$ is represented as (Hong et al. 2005a)

$$
x_{ij_0}^{(Tr)} = \sum_{j' \in I_m} \alpha_{ij'} \left( q_{ij_0}^{(i)} - V_{ij'}^{(Tr)} \right), \quad \forall j \in I_m, \forall \omega \in \Omega_m.
$$

(44)
3.3.1 The Upstream Boundary Tier Model

First, we examine transactions of upstream boundary tier sites as depicted in Figure 10. Specifically, we refer to the upstream boundary tier as tier 1, which is involved in transactions with its subsequent tier sites, \( j = 1, \ldots, N_2 \) and associated exogenous markets. We denote a typical upstream boundary tier site by site \( i \in I_1 \). After the associated process conducted in site \( i \in I_1 \), the recycled items collected in site \( i \) are converted into two major types of commodities on the basis of a given fraction. One stream is sent to the next tier, and the other stream is sold directly in the associated exogenous market where the market price is given and assumed to be a fixed amount.

![The Network Structure of Upstream Boundary Tier Site i](image)

Intuitively the collected amount in the upstream boundary tier depends on the acquisition prices offered by its subsequent tier and the price in the exogenous market; in turn, the upstream boundary tier site has more incentive to collect recycled items due to the higher acquisition prices offered in the subsequent tier, or the higher price in the associated exogenous market and vice versa. The decisions of upstream boundary tier site \( i \) are, to determine the potential maximum amount of recycled items to be collected,
and to design its optimal material flow allocation mechanism to the sites in its subsequent tier.

We assume that the collection amount in upstream boundary tier site \( i, i \in I_1 \), is characterized by a linear function \( S_{io} = a_i + b_i p_{io} \), where \( a_i \) and \( b_i \) are parameters and \( a_i, b_i > 0 \). We let \( p_{io} \) denote the collection fee paid by site \( i \), and \( S_{io} \) be the potential maximum flow amount collected in site \( i \) corresponding to price scenario \( \omega \in \Omega_2^i \). The sign of \( p_{io} \) determines the financial direction between sources and upstream boundary tier sites. In other words, the source pays a positive collection fee to upstream boundary tier site \( i \) for discarding recycled items if \( p_{io} < 0 \) and vice versa. To ensure that upstream boundary sites obtain a non-negative amount of flow, we require \( p_{io} \geq -a_i / b_i, i \in I_1 \), for all price scenarios. We let \( P_{X_i}, i \in I_1 \), denote the price in the exogenous market for site \( i \in I_1 \) and \( \gamma_i \), where \( 0 < \gamma_i \leq 1 \), be the ratio of recycled items flowing to the next tier from site \( i \in I_1 \). Site \( i \in I_1 \) picks the highest price offered by the downstream sites in tier 2 as the selling price for the \( \gamma_i \) portion of collected items. As a result, the effective unit reward of site \( i \in I_1 \) in price scenario \( \omega \in \Omega_2^i \), denoted by \( w_{io}^{(i)} \), is

\[
\gamma_i \cdot \max_{j \in I_2} \left\{ q_{io}^{(0)} - V_j^{(Tr)} \right\} + (1 - \gamma_i) P_{X_i}.
\]

The potential profit of upstream boundary tier site \( i, i \in I_1 \), in price scenario \( \omega \in \Omega_2^i \) is
\[ \Pi_{i\omega} = S_{i\omega} \left( w_{i\omega} - p_{i\omega} \right) = \left( a_i + b_i p_{i\omega} \right) \left( w_{i\omega} - p_{i\omega} \right), \]  

(45)

where the only unknown variable in (2) is the collection fee of \( p_{i\omega} \) that site \( i \) pays for the material to sources corresponding to price scenario \( \omega \in \Omega_2^i \). The potential profit function \( \Pi_{i\omega} \) is concave in \( p_{i\omega} \) whenever \( b_i > 0 \), so (2) is maximized when the first-order condition holds, i.e., when

\[
\max \left\{ \left( b_i \cdot w_{i\omega} - a_i \right) / 2b_i, -a_i / b_i \right\} \quad \forall i \in I_i, \omega \in \Omega_2^i. 
\] 

(46)

Thus, (3) is the optimal collection fee for upstream boundary tier site \( i \) in price scenario \( \omega \in \Omega_2^i \). The potential maximum flow amount collected in site \( i \) corresponding to price scenario \( \omega \in \Omega_2^i \), \( S_{i\omega} \), can be obtained by substituting \( p_{i\omega}^* \) into \( S_{i\omega} = a_i + b_i p_{i\omega} \).

Another common supply function, \( S_{i\omega} \), other than the linear assumption is the Cobb-Douglas function (see Nicholson 2002). We assume that the collection amount in site \( i \in I_i \) is given by the function \( S_{i\omega} = a_i (p_{i\omega})^b \), where \( a_i \), \( b_i \) are parameters and \( a_i > 0, 0 < b_i < 1 \). Under this framework, the collection fee, \( p_{i\omega} \), is assumed to be positive to avoid an aphysical collection amount \( S_{i\omega} \). Following a similar argument, the optimal collection fee of site \( i \in I_i \) in price scenario \( \omega \in \Omega_2^i \) for the Cobb-Douglas supply function is
\[
p_{\omega}^* = \frac{b_i}{b_i + 1} \omega^{(i)}_{\omega} \quad \forall i \in I_1, \omega \in \Omega_2^i.
\]

We define vectors \( \mathbf{p}_i^* \) and \( \mathbf{S}_i^* \) of sizes \( 1 \times \zeta_{N} \) to represent the optimal collection fees and corresponding potential maximum flows of site \( i \in I_1 \). Equivalently, \( \mathbf{p}_i^* = (p_{i\omega}^*, \ldots, p_{i\omega_{N_2}}^*) \) and \( \mathbf{S}_i^* = (S_{i1}, \ldots, S_{i\omega_{N_2}}) \).

Another decision of upstream boundary tier sites is the flow function used to contract with the sites in subsequent tier 2. We assume that the site faces significant price uncertainty from its subsequent tier. Therefore, the objective of site \( i \) is to construct a set of flow functions that recognize this price uncertainty. In this chapter we use the measure of robust deviation defined by (Kouvelis and Yu 1997), such that site \( i \in I_1 \) is to minimize the maximum difference between the best it can obtain when acquisition prices from its subsequent tier 2 are realized, and the objective value under the flow function selected by the minimax approach.

To execute the robust approach, first the optimal solution of each upstream site for each specified price scenario is found. This solution calculates the highest profit that the individual upstream site can obtain if it were to know the acquisition prices exactly. Then, we minimize the maximum deviation of the objective function value between the “ideal” and the “robust” sales profit for all price scenarios, \( \omega \in \Omega_2^i \). Finally, we adjust the decision variables, \( \alpha 's \), to ensure those returning the best sales profit for all price scenarios.
We let \( O^*_i \), \( i \in I_i \) and \( \omega \in \Omega^i \), denote the optimal objective value of site \( i \) for price scenario \( \omega \), and \( C^{(Tr)}_{ij} \) denote the shipment capacity between site \( i \in I_i \) and downstream site \( j \in I_j \). Let \( C^{(Pr)}_i \) be the processing capacity in site \( i \in I_i \). We assume that site \( i \) seeks to maximize the total profit associated with its collection and material allocation operations with the optimization problem given as follows for site \( i \in I_i \) for price scenario \( \omega \in \Omega^i \).

\[
\text{Maximize } O^*_i \quad \tag{48}
\]

\[
\text{Subject to:}
\]

\[
O^*_i = \sum_{j \in I_j} x^{(Tr)}_{ij} \left( q^{(i)}_{j} - V^{(Tr)}_{j} - p^*_i \right) \quad \tag{49}
\]

\[
x^{(Tr)}_{ij} = \sum_{j' \in I_j} \alpha_{ijj'} \left( q^{(i)}_{j} - V^{(Tr)}_{j'} \right) \quad \forall j \in I_j \quad \tag{50}
\]

\[
x^{(Tr)}_{ij} \leq C^{(Tr)}_{ij} \quad \forall j \in I_j \quad \tag{51}
\]

\[
\sum_{j \in I_j} x^{(Tr)}_{ij} \leq \gamma_i \cdot S^*_i \quad \tag{52}
\]

\[
\sum_{j \in I_j} x^{(Tr)}_{ij} \leq C^{(Pr)}_i \quad \tag{53}
\]

\[
x^{(Tr)}_{ij} \geq 0 \quad \forall j \in I_j \quad \tag{54}
\]

\[
\alpha_{ijj'} > 0 \quad \forall j, j' \in I_j, j = j' \quad \tag{55}
\]

\[
\alpha_{ijj'} \leq 0 \quad \forall j, j' \in I_j, j \neq j' \quad \tag{56}
\]

The objective function (48) is the sum of the sales profit minus acquisition cost. Constraints (50) are the material flow function definitions for arcs emanating from site \( i \in I_i \). Constraints (51), (52), and (53) provide capacity limits for each arc, the recycled item source, and the processing capability in site \( i \in I_i \) respectively. Constraints (54), (55), and (56) are sign restrictions for unknown variables. Obviously, the material flow
variables, \( x_{ij}^{(Tr)} \), are nonnegative, and the sign restrictions for \( \alpha's \) require that upstream site \( i \in I_1 \) has incentive to ship more flow on the arc where its destination price offer is increased, but less incentive when other downstream sites offer higher prices while competing for the material flow.

Next, we determine the robust flow function, or a common set of coefficients, \( \alpha's \), to be evaluated in every price scenario \( \omega \in \Omega^i_2 \) for site \( i \in I_1 \). Thus, for each price scenario we subtract the robust objective function value \( (R_{io}) \) using the common set of robust coefficients from the optimal objective value \( (O^i_{io}) \) of realization of acquisition price offers. The min-max robust optimization model over all price scenarios for site \( i \in I_1 \) can be stated as:

**Minimize** \( \delta_i \)

**Subject to:**

\[ \delta_i \geq O^i_{io} - R_{io} \quad \forall \omega \in \Omega^i_2 \]  
(57)

\[ R_{io} = \sum_{j \in I_2} x_{ij}^{(Tr)} \left( q_{jio}^{(i)} - V_{ij}^{(Tr)} - p_{io}^* \right) \quad \forall \omega \in \Omega^i_2 \]  
(58)

\[ x_{ij}^{(Tr)} = \sum_{j' \in I_2} \alpha_{ij} \left( q_{j'io}^{(i)} - V_{ij'}^{(Tr)} \right) \quad \forall j \in I_2, \omega \in \Omega^i_2 \]  
(59)

\[ x_{ij}^{(Tr)} \leq C_{ij}^{(Tr)} \quad \forall j \in I_2, \omega \in \Omega^i_2 \]  
(60)

\[ \sum_{j \in I_2} x_{ij}^{(Tr)} \leq \gamma_1 \cdot S^*_{io} \quad \forall \omega \in \Omega^i_2 \]  
(61)

\[ \sum_{j \in I_2} x_{ij}^{(Tr)} \leq C_i^{(Tr)} \quad \forall \omega \in \Omega^i_2 \]  
(62)

\[ x_{ij}^{(Tr)} \geq 0 \quad \forall \omega \in \Omega^i_2 \]  
(63)

\[ \alpha_{ij} > 0 \quad \forall j, j' \in I_2, j = j' \]  
(64)

\[ \alpha_{ij} \leq 0 \quad \forall j, j' \in I_2, j \neq j' \]  
(65)
We let $\delta^*_i$ denote the realized minimum maximum deviation of site $i \in I_1$ obtained from the earlier min-max robust optimization model. The final step of the flow function model is solving the following model with the fixed deviation, $\delta^*_i$, to optimality so that the decision variables, $\alpha'$s, return the best sales profit for scenarios that do not have active constraints, (57), for site $i \in I_1$. The model for each site $i \in I_1$ is:

\[
\text{Maximize } \sum_{\omega \in \Omega^i_2} R_{i\omega} \\
\text{Subject to:} \\
\delta^*_i \geq O^*_i - R_{i\omega} \quad \forall \omega \in \Omega^i_2 \\
\text{Constraints (58) - (65).}
\]

Given the robust solution values for $\alpha$, the upstream site models determine robust flow functions for each independent upstream site. Thus, the robust flow function describing the flow shipment from site $i \in I_1$ to its downstream site $j \in I_2$, denoted by $x^{(Tr)}_{ij}$, is represented as

\[
x^{(Tr)}_{ij} = \sum_{j \in I_2} \alpha_{ij} \left( p_{j'} - V^{(Tr)}_{ij} \right) \\
\text{(66)}
\]

where $p_{j'}$ is the acquisition price offered by downstream site $j' \in I_2$. Note that the price scenario $\omega$ is not an argument in the flow function at this point, and that (66) describes the material flow as a function of the acquisition prices between site $i \in I_1$ and site $j \in I_2$. Hence, in (66), all of $\alpha'$s, and $V^{(Tr)}$, $s$ are known parameters and $p'$s, and $x^{(Tr)}$, $s$ are unknown variables.
Each of the upstream boundary tier sites provides the sites in tier 2 with the robust flow function to contract the material transactions between these two tiers. In summary, the input parameters of the robust flow function determination are the optimal collection fee in the upstream boundary tier, the corresponding potential maximum material flow source amount in each price scenario, the price forecast of the sites in tier 2, the parameters of transportation costs and capacities between tiers 1 and 2, and the fraction of flow transported to the next tier.

For notation simplicity, we define the matrices $V_{m-1,m}^{(Tr)}$ and $C_{m-1,m}^{(Tr)}$ of sizes $N_{m-1} \times N_m$ to represent the input parameters of transportation costs, $V_{ij}^{(Tr)}$, and capacities, $C_{ij}^{(Tr)}$, respectively, between tiers $m-1$ and $m$. The set of the flow function coefficients describing the price-flow relation between site $i$ in tier $m$ and the sites in tier $m+1$ is the output solution represented by the $N_{m+1} \times N_{m+1}$ matrix of $A_i^{(O)}$ where the $j$th row in $A_i^{(O)}$ is the set of $\alpha$'s for describing the robust flow function of $x_{ij}^{(Tr)}$, $i \in I_m$ and $j \in I_{m+1}$. We let function $f$ denote the mapping from the input parameter space to the solution space of flow function coefficients.

As a result, the flow function coefficients, $\alpha$'s, describing the material flow allocation mechanism between site $i$ in tier 1 and the sites in tier 2 can be obtained by

$$A_i^{(O)} = f(P_i^*, S_i^*, Q^{(p)}, V_{1,2}^{(Tr)}(i,\cdot), C_{1,2}^{(Tr)}(i,\cdot), C_i^{(Pr)}, \gamma_i)$$

where $i \in I_1$, $V_{1,2}^{(Tr)}(i,\cdot)$, and $C_{1,2}^{(Tr)}(i,\cdot)$ are the $i$th rows of the matrices.
3.3.2 The Intermediate Tier Model

The intermediate tier site, denoted by \( j \) in Figure 11, which is involved in transactions with its preceding tier sites in site set \( I_{m-1} \), its subsequent tier sites in site set \( I_{m+1} \), and the associated exogenous market. As discussed in the upstream boundary tier model, a fraction, \( \gamma_j \), of recycled items is continuously transported to the next tier and the rest is directly sold in the associated exogenous market.

![Diagram of Intermediate Site](image)

**Figure 11:** The Network Structure of Intermediate Site \( j \)

Two major decisions of intermediate tier site \( j \in I_m \) are to determine the potential maximum flow amount from sites in tier \( m-1 \) to itself and to design the flow functions between site \( j \in I_m \) and the sites in tier \( m+1 \). As in the above discussion, site \( j \in I_m \) predicts the range of acquisition prices offered by the sites in tier \( m+1 \). We let \( \Omega_{m+1}^j \) denote the set of all specified price scenarios of tier \( m+1 \) predicted by site \( j \in I_m \). We assume that recycled items coming from the different sites in tier \( m-1 \) are homogeneous.
Thus, the total flows shipped to site $j \in I_m$ for price scenario $\omega \in \Omega_{m+1}^j$, which is denoted by $x_{jio}^{(Tr)}$, is the sum of flows from the different sites in tier $m-1$ and expressed as follows:

$$x_{jio}^{(Tr)} = \sum_{i \in I_{m-1}} x_{ijio}^{(Tr)} = \sum_{i \in I_{m-1}} \sum_{j' \in I_m} \alpha_{ij'} \left( p_{j'\omega} - V_{ij'}^{(Tr)} \right) \quad \forall j \in I_m, \omega \in \Omega_{m+1}^j. \quad (67)$$

In (67), all coefficients $\alpha_i's$ are given by the preceding sites in tier $m-1$ and $V_{ij}^{(Tr)}$ are transportation cost parameters where $i \in I_{m-1}$ and $j \in I_m$. The only unknown variables are all $p_i's$. The variable of $x_{jio}^{(Tr)}$ can be rewritten as:

$$x_{jio}^{(Tr)} = \alpha_j^i p_{i\omega} + \cdots + \alpha_j^{N_a} p_{N_a\omega} + C_j = \sum_{j' \in I_m} \alpha_j^{j'} p_{j'\omega} + C_j \quad (68)$$

where $C_j$ is a constant and $\alpha_j^{j'}$ is the coefficient term with $p_{j'\omega}$ in the expression of the aggregate flow $x_{jio}^{(Tr)}$. Combining (64) and (65) we have the following inequality relations for site $j \in I_m$.

$$\alpha_j^j > 0 \text{ and } \alpha_j^{j'} \leq 0 \quad \forall j' \in I_m, j' \neq j. \quad (69)$$

The interpretation of (24) is that the material flows shipped to site $j \in I_m$ is increasing as the price offer increases but decreasing when the competitors' prices increase, if there exist price effects between site $j \in I_m$ and other sites in tier $m$.

The material amount flowing into site $j \in I_m$ depends on the acquisition prices for associated price scenario $\omega \in \Omega_{m+1}^j$ and the price in the exogenous market, $PX_j$, $j \in I_m$. 

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For notation simplicity, we let \( w_{\omega}^{(j)} \) denote the effective unit reward of site \( j \in I_m \) in price scenario \( \omega \in \Omega_{m+1} \), which is equal to \( \gamma_j \cdot \max_{k \in I_m} \left( q_{k \omega}^{(j)} - V_{j k}^{(\gamma_j)} \right) + (1 - \gamma_j) P X_j \). The potential profit of site \( j \in I_m \) is

\[
\Pi_{j \omega} = x_{j \omega} \left( w_{\omega}^{(j)} - p_{j \omega} \right) \\
= w_{\omega}^{(j)} \sum_{j' \in I_m} \alpha_{j'}^j p_{j' \omega} + w_{\omega}^{(j)} C_j - p_{j \omega} \sum_{j' \in I_m} \alpha_{j'}^j p_{j' \omega} - C_j p_{j \omega}.
\]

The profit function \( \Pi_{j \omega} \) is concave in \( p_{j \omega} \) since \( \alpha_j^j > 0 \), so (70) is maximized when the first-order condition holds, i.e., when

\[
p_{j \omega}^* = \left( w_{\omega}^{(j)} \alpha_j^j - \sum_{j' \in I_m} \alpha_{j'}^j p_{j' \omega} - C_j \right) / 2 \alpha_j^j \\
\forall j \in I_m, \omega \in \Omega_{m+1}.
\]

Thus, (71) is the optimum response function of acquisition price of site \( j \in I_m \) in price scenario \( \omega \in \Omega_{m+1} \). For any price scenario \( \omega \in \Omega_{m+1} \), site \( j \in I_m \) generates the optimum response functions for all sites in tier \( m \) on the basis of its belief on the acquisition price forecast. Then, combining these response functions and solving them yield the optimal acquisition prices, \( p_{j \omega}^* \), for site \( j \in I_m \) in price scenario \( \omega \in \Omega_{m+1} \); that is

\[
\left( p_{1 \omega}^*, \ldots, p_{j \omega}^*, \ldots, p_{N \omega}^* \right). \]

The potential maximum material amount flowing into site \( j \in I_m \) in price scenario \( \omega \in \Omega_{m+1} \), \( S_{j \omega}^* \), can be obtained by substituting \( \left( p_{1 \omega}^*, \ldots, p_{j \omega}^*, \ldots, p_{N \omega}^* \right) \) into

\[
S_{j \omega}^* = \sum_{j' \in I_m} \alpha_j^j p_{j' \omega}^* + C_j.
\]
The set of the flow function coefficients describing the price-flow relation from the sites in tier \( m-1 \) to site \( j \in I_m \) is the input parameter for \( j \in I_m \) to obtain the potential maximum flow amount. We define a matrix \( A_j^0 \) of size \( N_{m-1} \times N_m \) matrix to represent the flow function coefficients for site \( j \in I_m \) where the \( i \)th row in \( A_j^0 \) is the set of \( \alpha \)'s for describing the robust flow function of \( x_{ij}^{(Tr)} \), \( i \in I_{m-1} \) and \( j \in I_m \). We let function \( h \) denote the mapping from the input parameter space to the solution space of \( S^{\star}_{j\omega} \) for all \( \omega \in \Omega_{m+1} \). Consequently, the potential maximum material amount flowing into site \( j \in I_m \) can be obtained by

\[
S_j^\star = h(A_j^0, Q_j^{(j)}, V_{m-1,m}^{(Tr)}(\cdot, j), V_{m,m+1}^{(Tr)}(\cdot, j), \gamma_j) \text{ associated with } p_j^\star = (p_j^{\star_1}, \ldots, p_j^{\star_{N_{m+1}}})
\]

where \( j \in I_m \), and \( V_{m-1,m}^{(Tr)}(\cdot, j) \) is \( j \)th column of the matrix.

Next, we determine the robust flow functions between site \( j \in I_m \) and the sites in tier \( m+1 \), which can be obtained by

\[
A_j^0 = f(P_j^\star, S_j^\star, Q_j^{(j)}, V_{m,m+1}^{(Tr)}(j, \cdot), V_{m,m+1}^{(Tr)}(j, \cdot), C_{m,m+1}^{(Pr)}(j, \cdot), C_{m,m+1}^{(Pr)}(j, \cdot))
\]

where \( j \in I_m \), \( V_{m,m+1}^{(Tr)}(j, \cdot) \), and \( C_{m,m+1}^{(Pr)}(j, \cdot) \) are the \( j \)th rows of the matrices.

**3.3.3 The Downstream Boundary Tier Model**

The downstream boundary tier site, denoted by \( k \) in Figure 12, which is involved in transactions with sites in tier \( M-1 \) and final demand markets where tier \( M \) is the last bottom tier in the network. The downstream boundary tier site converts recycled items into different materials such as raw materials or refurbished products and makes the
decision on its own acquisition price subject to its constraints of processing capacities, transportation capacities, and technology restrictions.

![Diagram of network structure](image)

**Figure 12:** The Network Structure of Downstream Boundary Tier Site $k$

We develop an optimization model for competitive downstream boundary tier sites and utilize the relaxation algorithm, or the combined Karush-Kuhn-Tucker (KKT) approach, to determine the Nash equilibrium price where no downstream boundary tier site can improve its objective function value by a unilateral change in its price solution (Hong et al. 2005a).

The total flows shipped to site $k \in I_M$, denoted by $x_k^{(Tr)}$, is the sum of flows from the different sites in tier $M-1$ to site $k \in I_M$ and expressed as follows:

$$x_k^{(Tr)} = \alpha_{k} \sum_{j \in I_{M-1}} \sum_{j' \in I_M} \alpha_{jk'} (p_k - \gamma_{k'}^{(Tr)}) \quad \forall k \in I_M. \quad (72)$$

We let $p_k$ denote the acquisition price offered by site $k \in I_M$ to its preceding upstream tier. We assume that the selling prices of raw materials or used products in demand markets are fixed amounts, not prediction of price ranges leading to a single material flow variable $x_k^{(Tr)}$. Downstream boundary tier site $k \in I_M$ maximizes its
objective function associated with the purchasing, processing cost, and sales revenue and is subject to constraints imposed on the processing, transportation capacity, and demand restrictions. Additionally, there are constraints to ensure the conservation of flows among sites and different processes for each material. The details of parameters, variables and model for downstream boundary tier site $k \in I_M$ are listed as follows from (Hong et al. 2005a).

**Downstream Boundary Tier Site Model Parameters:**

$V_{kc}^{(Tr)}$ Transportation cost per standard unit per distance from downstream boundary tier site $k$ to customer site $c \in C$;

$V_{kp}^{(Pr)}$ Processing cost per standard unit for process $p$ at downstream site boundary tier $k$;

$P_{lc}$ Selling price offered per standard unit of material $l \in L$ from customer $c$;

$\rho_{lp}$ Proportion of material type $l$ produced by process $p$;

$\rho_{lp}$ Proportion of material type $l$ consumed by process $p$;

$C_{kp}^{(Pr)}$ Maximum amount of material that process $p$ can process at downstream boundary tier site $k$;

$C_{kc}^{(Tr)}$ Maximum amount of material that can be shipped from downstream boundary tier site $k$ to customer site $c$.

**Downstream Model Decision Variables:**

$P_k$ Price offered per standard unit from downstream boundary tier site $k$;

$x_k^{(Tr)}$ The aggregate flows to downstream boundary tier site $k$;

$x_{kc}^{(Tr)}$ Amount of material shipped from downstream boundary tier site $k$ to customer site $c$ of material type $l$;

$x_{kp}^{(Pr)}$ Amount of material processed by process $p$ at downstream boundary tier site $k$. 


Maximize
\[
\sum_{c \in C} \sum_{l \in L} (P_{lc} - V_{hc}^{(Tr)}) x_{klc}^{(Tr)} - p_k x_k^{(Tr)} - \sum_{p \in Pr} V_{kp}^{(Pr)} x_{kp}^{(Pr)}
\]
Subject to:
\[
x_k^{(Tr)} = \sum_{j \in I_{M-1}} \sum_{k' \in I_M} \alpha_{jkk'} \left( p_{k'} - V_{jkk'}^{(Tr)} \right)
\]
\[
x_k^{(Tr)} - \sum_{c \in C} \sum_{l \in L} x_{klc}^{(Tr)} + \sum_{p \in Pr} \rho_{kp}^{(Pr)} x_{kp}^{(Pr)} - \sum_{p \in Pr} \rho_{kp}^{(Pr)} x_{kp}^{(Pr)} = 0 \quad \forall l \in L
\]
\[
x_{kp}^{(Pr)} \leq C_{kp}^{(Pr)} \quad \forall p \in Pr
\]
\[
\sum_{l \in L} x_{klc}^{(Tr)} \leq C_{kc}^{(Tr)} \quad \forall c \in C
\]
\[
x_k^{(Tr)}, x_{klc}^{(Tr)}, x_{kp}^{(Pr)} \geq 0 \quad \text{for all } x's
\]
\[
p_k \geq 0.
\]

The material flow variables for recycled items shipped to downstream boundary tier site \( k \), \( x_k^{(Tr)} \), are substituted by the expression \( \sum_{j \in I_{M-1}} \sum_{k' \in I_M} \alpha_{jkk'} \left( p_{k'} - V_{jkk'}^{(Tr)} \right) \), which is a function of acquisition prices. Other types of material flow variables, \( x_{klc}^{(Tr)}, x_{kp}^{(Pr)} \), can be expressed by \( x_k^{(Tr)} \) due to the flow conservation equations. The optimization model of (73) for downstream boundary tier site \( k \in I_M \) can be generally transformed into a quadratic programming model expressed in acquisition price variables \( p = (p_1, \cdots, p_k, \cdots, p_{N_M}) \).

There are \( N_M \) quadratic programming models for the sites in tier \( M \). We utilize the relaxation algorithm or the combined KKT approach to solve for the Nash equilibrium price solution \( (p_1^*, \cdots, p_k^*, \cdots, p_{N_M}^*) \), \( k \in I_M \). Hong et al. (2005a) provide the detailed
procedure of the algorithm and address the key conceptual difference between these two methods: information sharing among competing participants. The relaxation algorithm only requires competitors’ past price decisions, but the combined KKT approach needs information divulgence of all participants’ optimality conditions.

3.3.4 Synthesis of Computation for Equilibrium Acquisition Prices and Associated Material Flows

This section addresses the algorithm to solve for the equilibrium acquisition prices offered in each tier and the resulting material flows between tiers in a backward direction from the bottom to the top tier within the network. First, each site in the downstream boundary tier determines the Nash equilibrium price solution to be offered in tier $M$ to the sites in tier $M-1$. The associated material flows between the last two tiers $M$ and $M-1$ can be obtained by substituting the equilibrium prices $\left(p_1^*, \cdots, p_k^*, \cdots, p_{N_M}^*\right)$, $k \in I_M$, into the flow functions given by the sites in tier $M-1$, or formally,

$$X_{jk}^{(Tr)} = \sum_{k \in I_M} \alpha_{jkk} \left(p_k^* - V_{jk}^{(Tr)}\right) \quad \forall j \in I_{M-1}, k \in I_M. \quad (74)$$

Next, we solve for the price decisions of the sites in tier $M-1$ (the second last tier in the network). The revenue per unit collected in site $j \in I_{M-1}$, denoted by $PR_j$, is the weighted average of the unit revenue of the transactions between site $j$ and the sites in tier $M$, and the price in the associated exogenous market for site $j \in I_{M-1}$. The unit revenue $PR_j$ can be stated as
\[
PR_j = \gamma_j \cdot \frac{\sum_{k \in I_M} x_{jk}^{(Tr^*)} \left( p_k^* - v_{jk}^{(Tr)} \right)}{\sum_{k \in I_M} x_{jk}^{(Tr)^*}} + \left(1 - \gamma_j\right) P_j \quad \forall j \in I_{M-1}.
\] (75)

In this chapter, we focus on the single period equilibrium problem. This results in the assumption of the flow conservation rule for each site within the network; in other words, each site is not allowed to have imbalance in input and output flow after transactions. In addition, there are also other constraints from the transportation and process capacities in site \( j \in I_{M-1} \). We assume that each site in tier \( M-1 \) seeks to maximize the profit associated with its management of recycled items with the optimization problem given as follows for site \( j \in I_{M-1} \).

Maximize

\[
\sum_{i \in I_{M-2}} (PR_i - p_j)x_{ij}^{(Tr)}
\] (76)

Subject to:

\[
x_{ij}^{(Tr)} = \sum_{j \in I_{M-1}} \alpha_{ij} \left( p_j^* - v_{ij}^{(Tr)} \right) \quad \forall i \in I_{M-2} \text{ Flow definition (77)}
\]

\[
\gamma_j \sum_{i \in I_{M-2}} x_{ij}^{(Tr)} = \sum_{k \in I_M} x_{jk}^{(Tr)^*} \quad \forall j \in I_{M-1} \text{ Flow conservation (78)}
\]

\[
x_{ij}^{(Tr)} \leq C_{ij}^{(Tr)} \quad \forall i \in I_{M-2} \text{ Transportation capacity (79)}
\]

\[
\sum_{i \in I_{M-2}} x_{ij}^{(Tr)} \leq C_j^{(Pr)} \text{ Processing capacity (79)}
\]

\[
x_{ij}^{(Tr)} \geq 0 \quad \forall i \in I_{M-2} \text{ Variable restrictions (80)}
\]

\[
p_j \geq 0.
\]

The material flows between the sites in tier \( M-2 \) and site \( j \in I_{M-1} \), \( x_{ij}^{(Tr)} \), are functions of the acquisition prices to be offered in tier \( M-1 \), as shown in the flow
definition equation (77). The optimization model of site \( j \in I_{M-1} \) can be transformed into a quadratic programming model expressed in acquisition price variables \( \mathbf{p} = (p_1, \cdots, p_j, \cdots, p_{N_{M-1}}) \).

There are \( N_{M-1} \) quadratic programming models for the sites in tier \( M-1 \). We utilize the relaxation algorithm or the combined KKT approach to solve for the equilibrium acquisition prices \( \left(p_1^*, \cdots, p_j^*, \cdots, p_{N_{M-1}}^*\right), j \in I_{M-1} \). However, instead of solving the quadratic programming models iteratively or the combined optimality conditions simultaneously, the price solutions to be offered in tier \( M-1 \) are determined by the flow conservation equations of the sites in tier \( M-1 \) since there are \( N_{M-1} \) unknown price variables in the \( N_{M-1} \) equations of flow conservation. This is a consequence of the uniqueness and existence property of the linear system in tier \( M-1 \)’s problem (see Hong et al, 2005a). Therefore, we observe that, in equilibrium, the shipments of materials that site \( j \in I_{M-1} \) transports to the sites in its subsequent tier \( M \) must be equal to the shipments from the sites in its preceding tier \( M-2 \). In general, this argument also applies on shipments between other tiers. As a result, for any particular site \( j \in I_m \) of the intermediate tiers, we have the following flow conservation equations.

\[
\gamma_j \sum_{i \in I_{M-1}} x_{ij}^{(Tr)^*} = \sum_{k \in I_{M-1}} x_{jk}^{(Tr)^*} \quad \forall j \in I_m.
\]  

(81)

Then, we can obtain the equilibrium acquisition prices \( \left(p_1^*, \cdots, p_j^*, \cdots, p_{N_m}^*\right) \) of all sites in tier \( m \) by simultaneously solving the linear equation system (81). However, this procedure implies willingness to share information of associated flow functions and
collected amounts among competing participants. Alternatively, the Jacobi method (see Kress 1998), an iterative algorithm can be used to solve the linear equations (81) without sharing associated information. Another way of writing (81) is: \( \sum_{j \neq j^*} \alpha_j^j p_j = F_j \) for all \( j \in I_m \) where \( \alpha_j^j \) is the coefficient term with variable \( p_j \), and \( F_j \) is a parameter term, which is \( \sum_{k \in I_{m-1}} x_{jk}^{(Tr)} / \gamma_j \). Having an initial estimate price \( p_j^{(0)} \), \( j^* = 1, ..., N_m \), the iterative Jacobi method can be stated as

\[
P_j^{(s+1)} = \frac{F_j}{\alpha_j^j} - \sum_{j \neq j^*} \frac{\alpha_j^j}{\alpha_j^j} p_j^{(s)} \quad \forall j \in I_m, s = 0, 1, 2, ...
\]

**Corollary 1**  
The Jacobi method converges for each site \( j \in I_m \) to the unique solution of (81).

**Proof.** We let \( x_j^{(Tr)} \) denote the aggregate material flow transported to the sites in tier \( m \) and represent (81) as the matrix form \( \alpha p = F \). The impact of the change of acquisition price \( p_j \), \( j \in I_m \), on \( x_j^{(Tr)} \) is greater than the total impact on \( x_j^{(Tr)} \) due to the price changes from the rest of sites \( 1, ..., j-1, j+1, ..., N_m \) in tier \( m \) (Hong et al. 2005a). This statement is equivalent to the inequality \( |\alpha_j^j| > \sum_{j \neq j^*} |\alpha_j^j| \) for all \( j \in I_m \) which indicates matrix \( \alpha \) is a *diagonally dominant matrix*. This completes the proof (Kress 1998; Strang 1986).

The successive iterations of the Jacobi method imply that each site in tier \( m \) finds its price by iteratively picking its own price and posting it for all of the sites in tier \( m \) to see.
This is the only information that each site in tier $m$ needs to access from all other sites in the same tier. Hence, equilibrium acquisition prices and resulting material flows for sites in intermediate tiers can be derived in a backward direction from tier $M-1$ to tier 2.

Finally, we focus on the decision variables of the sites in the upstream boundary tier, represented as tier 1 in the model. We assume that each site in tier 1 seeks to maximize the profit associated with its collection of recycled items with the optimization problem given as follows for site $i \in I_1$.

Maximize

$$S_i(PR_i - p_i)$$  

Subject to:

$$S_i = a_i + b_i p_i^*$$  

Source flow response  

$$(83)$$

$$\gamma_i S_i = \sum_{j \in J_2} x_{ij}^{(tr)}$$  

Flow conservation  

$$(84)$$

$$S_i \leq C_i^{(pr)}$$  

Collection capacity  

$$(85)$$

$$S_i \geq 0.$$  

The optimization model of site $i \in I_1$ is a single-variable quadratic programming problem since we assume the collection amount in site $i \in I_1$ only depends on the collection fee paid by site $i \in I_1$ to sources. The equilibrium collection fee $p_i^*, i \in I_1$, can be obtained by simply solving the flow conservation equation in (85) and, then, the

* The result for the Cobb-Douglas source response function, $a_i (p_i)^{\gamma_i}$, can be derived by a similar way.
resulting material flow, \( S'_i \), is also obtained from the source flow response function in (84).

In the next section, we summarize the solution algorithm consisting of the upstream boundary, intermediate, and downstream boundary tier models and explicitly illustrate the decision-making procedures under the framework of Section 3.3.

### 3.4 The Summary of the Solution Algorithm

The overall generic reverse production system is comprised of five categories of entities: sources, upstream boundary tier sites, intermediate tier sites, downstream boundary tier sites, and customers as shown in Figure 13. The steps of the solution algorithm are illustrated by the numbers in the rectangle boxes of Figure 13. The input information and output solution for each step are represented as solid and dashed lines respectively. The behavior of the source and the customer is assumed simple. The source supplies the upstream boundary tier site on the basis of a fee paid by the site in the upstream boundary tier. We assume that the relationship is known by both parties and is the function of a fee, with an increasing flow for higher collection fees. The customers in demand markets have a fixed price for a given material or sub-component and the market is assumed to be very large, such that the flow from downstream boundary tier sites to customers does not change the price. The customers communicate the price to the downstream boundary tier sites at the outset, but it is not considered private information.

The algorithm starts at site \( i \in I_1 \) to determine the flow function between itself and the sites in tier 2 given the source supply function, the prediction range of acquisition
prices, and the price in the associated exogenous market. Then, the sites in the upstream boundary tier communicate flow functions to the sites in the next subsequent tier. Intermediate tier site $j \in I_m$ determines the associated flow functions describing the price-flow relations between site $j \in I_m$ and the sites in tier $m+1$ on the basis of the flow functions between tier $m-1$ and $m$, price prediction ranges of sites to be offered in tier $m+1$, and the price in the associated exogenous market.

![Figure 13: Summary of the Solution Algorithm](image-url)
The sites in the downstream boundary tier determine the equilibrium acquisition price on the basis of the flow functions given by the sites in the preceding tier and the final known market price. This completes the left hand side of Figure 13. Then, the resulting flow into the downstream boundary tier site can be obtained by substituting the equilibrium price into the flow function.

We compute the equilibrium acquisition prices in a backward direction from tier $M-1$ to tier 2. The revenue per unit collected in intermediate tier sites is a known variable after taking the weighted average of the equilibrium prices and the exogenous market price on the basis of the resulting flows. Instead of solving the optimization problem to obtain the equilibrium acquisition prices of the intermediate tiers, we state that equilibrium prices are governed by flow conservation equations. Therefore, due to the flow conservation equilibrium, the resulting flows and equilibrium prices of the rest of tiers can be computed in a backward direction as depicted in the right hand side of Figure 13. In computing the equilibrium prices in each tier, competing sites find their equilibrium prices by iteratively picking their own price offer on the basis of other sites’ past posted prices. Finally, we solve for the collection fee and the resulting material flow amount collected in the upstream boundary tier site. In the next section, we apply the multi-tiered RPS model to an example to determine the equilibrium acquisition prices and the corresponding resulting material flows within the network.

3.5 A Numerical Example

In this section, we provide a numerical example to illustrate the use of the above multi-tiered RPS model to analyze the behavior of a recycling network, whose structure
is depicted in Figure 15. We assume there are five collection sites, $i = 1, \ldots, 5$, in tier 1, three consolidation sites, $j = 1, 2, 3$, in tier 2, and four processing sites, $k = 1, \ldots, 4$, in tier 3. The transportation costs per unit flow between any associated two sites are given in Table 12. For simplicity, we do not consider the exogenous market for each site in this numerical example. In other words, all of the recycled items collected in collection sites are transported to the consolidation sites in tier 2 and the items shipped to the consolidation sites are fully transported to the processing sites in tier 3.

Figure 14: The Reverse Production System for the Example
Table 6: The Unit Transportation Costs between Sites

<table>
<thead>
<tr>
<th></th>
<th>$j \in I_2$</th>
<th>$k \in I_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i \in I_1$</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>10.0</td>
<td>15.0</td>
</tr>
<tr>
<td>2</td>
<td>10.0</td>
<td>13.0</td>
</tr>
<tr>
<td>3</td>
<td>13.0</td>
<td>10.0</td>
</tr>
<tr>
<td>4</td>
<td>15.0</td>
<td>13.0</td>
</tr>
<tr>
<td>5</td>
<td>17.0</td>
<td>14.0</td>
</tr>
</tbody>
</table>

The source response functions describing the collection amounts of obsolete products in collection sites, $i = 1, \ldots, 5$, are given by $S_i = 400 + 5p_i$, $S_2 = 420 + 6p_2$, $S_3 = 440 + 6p_3$, $S_4 = 430 + 6p_4$, and $S_5 = 410 + 5p_5$ where $p_i$ is the collection fee per unit of the obsolete product paid by collection site $i$ and $S_i$ is the collection amount in site $i$. After a series of operations through collection and consolidation sites in the recycling network, processing sites in tier 3 in this example obtain recycled items and may further conduct some refurbished processes so that obsolete products are converted into refurbished products sold in the secondary demand markets. The prices of refurbished products in these secondary markets for processing sites, $k = 1, \ldots, 4$, are $155, 145, 147, \text{ and } 150$ respectively.

We assume that all collection sites predict the ranges of acquisition prices offered by consolidation sites, $j = 1, 2, 3$, in tier 2 as ($65, 60, 65$) ± 20. In other words, collection sites, $i = 1, \ldots, 5$, in tier 1 predict the prices offered by consolidation sites to acquire obsolete products collected in sites in tier 1 are within the ranges of [$45, 85$], [$40, 80$] and [$45, 85$] for consolidation sites, $j = 1, 2, 3$, in tier 2 respectively. We choose 4 evenly distributed points in each price range so we consider 64 price scenarios.
for collection site \( i \in I_1 \) and compute corresponding potential maximum flow amount \( S_{i\omega}^* \) and collection fee \( p_{i\omega}^* \) for all \( i \in I_1 \) and \( \omega \in \Omega_2 \). Given \( S_{i\omega}^* \) and \( p_{i\omega}^* \), then, the upstream boundary tier model yields the following robust flow function between tiers 1 and 2 or equivalently

\[
A_i^{(0)} = f(P_i^*, S_i^*, Q^{(i)}, V_{12}^{(Tr)}(i, \cdot), C_{12}^{(Tr)}(i, \cdot), C_i^{(Tr)}, \gamma_i).
\]

\( x_{ij}^{(Tr)}: i \in I_1, j \in I_2; \)

\[
\begin{align*}
 x_{11}^{(Tr)} &= 2.77 \ (p_1-10) -0.56 \ (p_2-15) -0.90 \ (p_3-18), \\
 x_{12}^{(Tr)} &= 3.38 \ (p_2-15) -1.26 \ (p_3-18), \\
 x_{13}^{(Tr)} &= -1.47 \ (p_2-15) +3.53 \ (p_3-18), \\
 x_{21}^{(Tr)} &= 3.14 \ (p_1-10) -0.47 \ (p_2-13) -0.77 \ (p_3-16), \\
 x_{22}^{(Tr)} &= -0.61 \ (p_1-10) +3.63 \ (p_2-13) -0.75 \ (p_3-16), \\
 x_{23}^{(Tr)} &= -1.51 \ (p_2-13) +3.49 \ (p_3-16), \\
 x_{31}^{(Tr)} &= 3.34 \ (p_1-13) -0.13 \ (p_2-10) -1.19 \ (p_3-14), \\
 x_{32}^{(Tr)} &= -1.22 \ (p_1-13) +3.48 \ (p_2-10) -0.14 \ (p_3-14), \\
 x_{33}^{(Tr)} &= -1.42 \ (p_2-10) +3.42 \ (p_3-14), \\
 x_{41}^{(Tr)} &= 3.45 \ (p_1-15) -1.55 \ (p_2-13), \\
 x_{42}^{(Tr)} &= -0.66 \ (p_1-15) +3.64 \ (p_2-13) -0.70 \ (p_3-11), \\
 x_{43}^{(Tr)} &= -0.63 \ (p_1-15) -0.54 \ (p_2-13) +3.15 \ (p_3-11), \\
 x_{51}^{(Tr)} &= 3.51 \ (p_1-17) -1.49 \ (p_2-14), \\
 x_{52}^{(Tr)} &= -1.36 \ (p_1-17) +3.57 \ (p_2-14), \\
 x_{53}^{(Tr)} &= -0.72 \ (p_1-17) -0.79 \ (p_2-14) +2.80 \ (p_3-9).
\end{align*}
\]
distributed points in each price range. As a consequence, we consider 256 price scenarios \((4^4 = 256)\) in \(\Omega^j_3\) for consolidation site \(j \in I_2\) and compute the corresponding potential maximum flow amount \(S^*_{jo}\) and acquisition price \(p^*_{jo}\) for all \(j \in I_2\) and \(\omega \in \Omega^j_3\). Given \(S^*_{jo}\) and \(p^*_{jo}\), then, the intermediate tier model yields the following robust flow functions between tiers 2 and 3 or equivalently

\[
A^{(0)}_j = f(P^*_j, S^*_j, Q^{(j)}, V^{(Tr)}_{2,3}(j, \cdot), C^{(Tr)}_{2,3}(j, \cdot), C^{(Tr)}_{j}(\cdot), \gamma^j).
\]

\[
x_{jk}^{(Tr)}: \ j \in I_2, \ k \in I_1;
\]

\[
x_{11}^{(Tr)} = 2.42 (p_1 - 8) -0.30 (p_2 - 8) -0.78 (p_3 - 10) -0.20 (p_4 - 12),
\]

\[
x_{12}^{(Tr)} = 2.13 (p_2 - 8) -0.50 (p_3 - 10) -0.16 (p_4 - 12),
\]

\[
x_{13}^{(Tr)} = -0.42 (p_2 - 8) +2.61 (p_3 - 10) -0.82 (p_4 - 12),
\]

\[
x_{14}^{(Tr)} = -0.35 (p_1 - 8) -0.80 (p_2 - 8) +2.51 (p_4 - 12),
\]

\[
x_{21}^{(Tr)} = 1.67 (p_1 - 10),
\]

\[
x_{22}^{(Tr)} = -0.49 (p_1 - 10) +2.45 (p_2 - 8) -0.62 (p_4 - 11),
\]

\[
x_{23}^{(Tr)} = -0.36 (p_1 - 10) -0.41 (p_2 - 8) +2.81 (p_3 - 7) -0.86 (p_4 - 11),
\]

\[
x_{24}^{(Tr)} = -0.04 (p_1 - 10) +1.74 (p_4 - 11),
\]

\[
x_{31}^{(Tr)} = 2.39 (p_1 - 12) -0.48 (p_2 - 10) -0.62 (p_3 - 8),
\]

\[
x_{32}^{(Tr)} = -0.30 (p_1 - 12) +2.55 (p_2 - 10) -0.30 (p_3 - 8) -0.58 (p_4 - 7),
\]

\[
x_{33}^{(Tr)} = -0.91 (p_2 - 10) +2.73 (p_3 - 8) -0.57 (p_4 - 7),
\]

\[
x_{34}^{(Tr)} = -0.82 (p_1 - 12) -0.06 (p_3 - 8) +2.22 (p_4 - 7).
\]

In (88), all of \(p^*\)'s refer to the acquisition prices offered by the processing sites in tier 3. We utilize the relaxation algorithm, or the combined Karush-Kuhn-Tucker approach, to determine the Nash equilibrium prices in the downstream boundary tier model and yield the Nash equilibrium prices for processing sites 1,…, 4 of tier 3 in Table 7. The computation at this step completes the left hand side algorithm of Figure 13.
Table 7: The Equilibrium Prices in Tier 3

<table>
<thead>
<tr>
<th>( p_k^* )</th>
<th>( p_1^* )</th>
<th>( p_2^* )</th>
<th>( p_3^* )</th>
<th>( p_4^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k \in I_3 )</td>
<td>98.81</td>
<td>94.96</td>
<td>100.32</td>
<td>94.08</td>
</tr>
</tbody>
</table>

Then, the resulting flows into processing sites 1,…, 4 in tier 3 can be obtained by substituting the equilibrium prices into the flow functions of (88). We list the detail resulting material flow between tiers 2 and 3 in Table 8.

Table 8: The Resulting Material Flows between Tiers 2 and 3

<table>
<thead>
<tr>
<th>( x_{jk}^{(Tr)} )</th>
<th>( x_{11}^{(Tr)} )</th>
<th>( x_{12}^{(Tr)} )</th>
<th>( x_{13}^{(Tr)} )</th>
<th>( x_{14}^{(Tr)} )</th>
<th>( x_{21}^{(Tr)} )</th>
<th>( x_{22}^{(Tr)} )</th>
<th>( x_{23}^{(Tr)} )</th>
<th>( x_{24}^{(Tr)} )</th>
<th>( x_{31}^{(Tr)} )</th>
<th>( x_{32}^{(Tr)} )</th>
<th>( x_{33}^{(Tr)} )</th>
<th>( x_{34}^{(Tr)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( j \in I_2 ), ( k \in I_3 )</td>
<td>106.6</td>
<td>126.4</td>
<td>131.6</td>
<td>103.9</td>
<td>148.0</td>
<td>118.3</td>
<td>123.8</td>
<td>140.7</td>
<td>109.8</td>
<td>112.1</td>
<td>125.4</td>
<td>116.6</td>
</tr>
</tbody>
</table>

Due to the flow conservation equilibrium, the resulting flows and equilibrium prices as well as the equilibrium collection fees are computed in a backward direction as depicted in the right hand side of Figure 13. The detail solutions are listed in Table 9, Table 10, and Table 11 respectively.

Table 9: The Equilibrium Prices in Tier 2

<table>
<thead>
<tr>
<th>( p_j^* ), ( j \in I_2 )</th>
<th>( p_1^* )</th>
<th>( p_2^* )</th>
<th>( p_3^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( j \in I_2 )</td>
<td>62.75</td>
<td>61.20</td>
<td>62.86</td>
</tr>
</tbody>
</table>

Table 10: The Resulting Material Flows between Tiers 1 and 2

| \( x_{ij}^{(Tr)} \) | \( x_{11}^{(Tr)} \) | \( x_{12}^{(Tr)} \) | \( x_{13}^{(Tr)} \) | \( x_{14}^{(Tr)} \) | \( x_{21}^{(Tr)} \) | \( x_{22}^{(Tr)} \) | \( x_{23}^{(Tr)} \) | \( x_{24}^{(Tr)} \) | \( x_{31}^{(Tr)} \) | \( x_{32}^{(Tr)} \) | \( x_{33}^{(Tr)} \) | \( x_{41}^{(Tr)} \) | \( x_{42}^{(Tr)} \) | \( x_{43}^{(Tr)} \) | \( x_{51}^{(Tr)} \) | \( x_{52}^{(Tr)} \) | \( x_{53}^{(Tr)} \) |
|----------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \( i \in I_1 \), \( j \in I_2 \) | 79.7 | 99.7 | 90.7 | 106.4 | 107.3 | 90.7 | 101.7 | 110.3 | 94.3 | 90.4 | 107.3 | 107.6 | 90.3 | 106.0 | 80.6 |
Table 11: The Equilibrium Collection Fees in Tier 1

<table>
<thead>
<tr>
<th>$p_i^*$</th>
<th>$p_1^*$</th>
<th>$p_2^*$</th>
<th>$p_3^*$</th>
<th>$p_4^*$</th>
<th>$p_5^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i \in I_1$</td>
<td>-25.98</td>
<td>-19.26</td>
<td>-22.28</td>
<td>-20.78</td>
<td>-26.60</td>
</tr>
</tbody>
</table>

* The negative equilibrium collection fee indicates that the collection site pays a minus price for the collected items. In other words, the collection site charges the source a positive fee for collecting items.

The preceding example demonstrates a three-tier and single-commodity decentralized RPS problem that can be solved using the models given in Sections 3.3 and 3.4. Each independent site provides its subsequent tier with the material flow allocation mechanism. Then, we solve for the Nash equilibrium acquisition prices in the downstream boundary tier. Finally, we obtain the corresponding material flows and equilibrium acquisition prices in intermediate tiers and the upstream boundary tier by substituting the price into the flow functions.

We note the solution of equilibrium acquisition prices may not return a value located within the prediction price range in some extreme cases. If the equilibrium price is above the range, it may lead to infeasible flows due to the flow capacity of sites. To avoid this, we assume that the associated entities within each tier conservatively predict the range of acquisition prices so that the equilibrium acquisition prices are within corresponding price ranges. Otherwise, they may cause a significant loss due to the potential penalty of unsatisfactory supply to associated parties. We leave the penalty design or setup of policy analysis for future work and summarize this chapter in the next section.
3.6 Summary and Extensions

This chapter presents a model of a decentralized multi-tiered reverse production system where each independent site considers its own objective function and is subject to its own constraints. We consider a general model of decentralized reverse production systems with an upstream boundary tier, intermediate tiers, and a downstream boundary tier where each tier has multiple independent sites.

The main focus of the chapter is to propose mechanisms to coordinate the multi-tiered system under the assumption that no, or limited, information is shared between the entities in a given tier and between tiers. The source supplies the upstream boundary tier site on the basis of a fee paid by the site in the upstream boundary tier. We assume that the relationship is known by both parties, and a function of a fee, with an increasing flow for higher collection fees. In addition, the prices of raw materials or used-products in the final demand market are assumed to be fixed amounts due to the large demand market such that the flow from downstream boundary tier sites do not change the price. In each site, recycled items are mainly separated into two streams: one is sent to the next tier, and the other one is sold directly in the associated exogenous market. To coordinate between upstream and downstream tier pairs, each site in the upstream tier independently designs the material flow allocation mechanism used to contract with the sites in its subsequent downstream tier. This is presumed to occur sequentially from the top to the bottom of the reverse production system.

For the competition between entities in a given tier, we propose a mechanism to find the equilibrium acquisition prices sequentially from the downstream tier to the
upstream tier. The downstream boundary tier site determines the equilibrium acquisition price based on the final known market price of raw materials or refurbished products and the flow function given by the sites in the preceding tier. Then, we obtain the resulting material flows and equilibrium prices between tiers by backward substituting computations from the bottom to the top tier in the network. Finally, we solve for the collection fee and resulting material flow amount collected in the upstream boundary tier site.

The model can be further extended in several directions. Instead of assuming a fixed final market price, we may further explore the system with a dynamic final market where the price depends on the selling quantity sold. In addition, many reverse production systems have network structures that are more complicated such that material can flow across two or more tiers. Moreover, due to the value of recycled items or sub-components, the acquisition prices within tiers are not necessarily positive, a situation that cannot be handled by our current model. Another extension of the research in this chapter is to investigate multi-commodity networks where the price behavior of the different types of recycled items is influenced by the correlations between them. In this research, we focus on a single period of transaction problem where capacity expansion and new entrants are not allowed, but research could extend these ideas to multiple periods where capacity expansion and/or entrant entities could be considered (Corbett and Karmarkar 2001). The investigation on the equilibrium of the entry game in recycling systems is also an interesting extension that could be explored further.
CHAPTER 4 DECENTRALIZED DECISION-MAKING AND PROTOCOL DESIGN FOR RECYCLED MATERIAL FLOWS FOR A MULTI-TIERED NETWORK WITH PRICE CORRELATED COMMODITIES

Chapter 3 presents a decentralized model for a multi-tiered RPS where we assume that each type of commodities is independent and there is no price correlation among different types of commodities. In this chapter, we investigate the price behavior of the recycled item when there exists price correlation between different types of recycled items. The material flow amount between two consecutive upstream and downstream sites may not only be influenced by competitors’ prices but also affected by the acquisition price of the other type of recycled items. Section 4.1 discusses the concept of price correlated commodities. We extend the model presented in Chapter 3 to represent a multi-tiered RPS network with price correlated commodities in Section 4.2, and provide an e-scrap example to illustrate the modified model in Section 4.3. Finally, we discuss computational complexity of the model and present a screening algorithm to reduce the price scenario space in Section 4.4.

4.1 Concept of Price Correlated Commodities

In Chapter 3, we examine how changes in its own acquisition price and competitors’ acquisition prices of a particular type of the recycled item affect the flow amount of one particular commodity. However, it should be clear that a change in the price of one commodity may also affect the quantities of other types of commodities.
Parkin and Bade (2002) summarize the price-quantity relations across different commodities. A well-known example in the real-world is that the sales amount of automobiles, especially in sports utility vehicles (SUVs), might be expected to decline when the price of gasoline rises. The formal terminology for such relation of automobiles and gasoline is called *complement-in-consumption* where if the price of one commodity rises, the quantity consumed of the other falls. Another common type of relationship between different commodities is *substitute-in-consumption*, where if the price of one commodity increases, more of the other commodity will be demanded (Nicholson 2002). Typical examples of substitute-in-consumption commodities include wheat and rice, or petroleum and natural gas (used for heating or electricity). In addition to the complement and substitute relations in the demand market, there are also similar relations among different commodities in the supply market: *substitute-in-production* and *complement-in-production*. In substitute-in-production, a good can be produced in place of another good relative to the capacity constraint, e.g., a truck and a SUV in an automobile factory. The supply of a good increases if the price of one of its substitutes in production falls and vice versa. On the other hand, a good is produced along with another good in complement-in-production. For example, straw is a complement in production of wheat. The supply of a good increases if the price of one of its complements in production rises and vice versa (Parkin and Bade 2002).

In an e-scrap recycling industry, there are similar situations of cross price-flow relations for different commodities. For example, considering a two-tiered recycling network with collectors and processors, the flow of obsolete desktop computers supplied to processors may be decreased if the acquisition price of obsolete laptop computers is
increased since the collector may sacrifice the collection capacity for desktop computers due to a high acquisition price of laptop computers (substitute-in-production). On the other hand, the flow of motherboards supplied to processors may be increased if the acquisition price of disk drives is increased since the commodity of motherboards is a by-product of disk drives (complement-in-production). These two examples explain the situations that there exist price-flow correlations among different commodities other than competition effects of different entities in the network. In this research we specifically focus on a substitute-in-production case in the RPS network where all commodities are utilizing the common resource of collection, disassembly, or demanufacturing capacities in associated entities.

4.2 Modification of the Decentralized Model for a Multi-tiered Network

The decentralized model for a multi-tiered network with price correlated commodities follows the previous work presented in Chapter 3. The major modification is on the material flow allocation mechanism format discussed in Section 2.4.2. We refer to the material flow allocation mechanism as flow functions which describe the material flow relationship of the amount and acquisition price between sites in two consecutive tiers. First we review the material flow function format for the flow without price correlation among different types of commodities. We let \( x^{(Tr)}_{ij} \) denote the material flow amount from upstream site \( i \in I_{m-1} \) to downstream site \( j \in I_m \) where \( I_m \) is the set of the sites in tier \( m \). Since different commodities are seen to be independent, it is not necessary
to add in the commodity argument in the flow function of (89). We also let $p_j$ denote the acquisition price to be offered by site $j \in I_m$, and $V_{ij}^{(Tr)}$ be the unit transportation cost shipped from site $i \in I_{m-1}$ to site $j \in I_m$. The robust coefficients, $\alpha_{ij}^{uv}$, are decision variables for upstream site $i \in I_{m-1}$. The function form of (89) represents the material flow amount which is only a function of its own acquisition price and other competitors’ acquisition prices.

$$x_{ij}^{(Tr)} = \sum_{j \in \Lambda_m} \alpha_{ij}^{uv} (p_j - \bar{V}_{ij}^{(Tr)}) \quad \forall i \in I_{m-1}, j \in I_m.$$  

(89)

We generalize the flow function form of (89) to describe the flow amount relationship for price correlated commodities. We let $\Lambda_m$ denote the set of associated commodities handled in tier $m$. In other words, commodities in set $\Lambda_m$ are shipped from tier $m-1$ to tier $m$ and the shipment flow amount of the commodity in set $\Lambda_m$ is correlated with the acquisition price of other commodities in set $\Lambda_m$ in addition to the competition effect from other competitors. Considering the transaction of commodity $u \in \Lambda_m$ between site $i \in I_{m-1}$ and site $j \in I_m$, we let $x_{iju}^{(Tr)}$ denote the material flow amount of commodity $u$ from site $i$ to $j$, and $p_{jv}$ be the acquisition price of commodity $v \in \Lambda_m$ to be offered by site $j' \in I_m$. The modification of flow functions for price correlated commodities is shown in (90) where the decision variables, $\alpha_{ij}^{uv}$, are the coefficients of material flow determination for commodity $u$ from site $i$ to $j$ affected by site $j' \in I_m$ and commodity $v \in \Lambda_m$. An intuitive explanation of (90) is that the material flow amount for
one particular commodity is not only influenced by all acquisition prices to be offered by the next tier but also affected by other commodities’ acquisition prices.

\[
\chi_{ij}^{(Tr)} = \sum_{v \in \Lambda_m} \sum_{j \neq j'} \alpha_{ij}^{uv} \left( p_{ij} - V_{ij}^{(Tr)} \right) \quad \forall i \in I_{m-1}, \ j \in I_m, \ u \in \Lambda_m. \quad (90)
\]

In (90), we explicitly require that \( \alpha_{ij}^{uv} \) is strictly positive if \( j = j' \) and \( u \) equals to \( v \); \( \alpha_{ij}^{uv} \) is nonpositive otherwise. The economic interpretation of this sign restriction can be stated as follows. In the case of the identical commodities \( (u = v) \), intuitively site \( i \) has more incentive to ship more flow on the arc where a higher price of that particular commodity is offered by site \( j \). This results in (91) and (92).

\[
\alpha_{ij}^{uv} > 0 \quad \forall i \in I_{m-1}, \ j \in I_m, \ i = j; \ u, v \in \Lambda_m, \ u = v, \quad (91)
\]

\[
\alpha_{ij}^{uv} \leq 0 \quad \forall i \in I_{m-1}, \ j \in I_m, \ i \neq j; \ u, v \in \Lambda_m, \ u = v. \quad (92)
\]

In the other case of the different commodities \( (u \neq v) \), the supply of commodity \( u \) increases if the price of commodity \( v \) falls since commodities \( u \) and \( v \) are substitute-in-production, and vice versa. As a result, we have

\[
\alpha_{ij}^{uv} \leq 0 \quad \forall i \in I_{m-1}, \ j \in I_m; \ u, v \in \Lambda_m, \ u \neq v. \quad (93)
\]

We follow the same computation algorithm developed in Chapter 3 to solve for the equilibrium acquisition price and resulting material flow for each commodity in different tiers. In the next section, we utilize the generalized flow function format and the algorithm presented in Chapter 3 to solve an example of the multi-tiered network with price correlated commodities.
4.3 A Numerical Example

In this section, we provide a numerical example to represent an e-scrap recycling network, depicted in Figure 15, and it illustrates the application of the above multi-tiered RPS model with price correlated commodities. We consider a three-tier RPS with collection, consolidation, and processing sites of e-scrap, specifically in obsolete laptop and desktop computers, recycling network where each site is operated by the individual party. In other words, every site in the network independently determines its own associated decision variables. We follow the same network structure presented in Section 3.5 and assume there are five collection sites, \( i = 1, \ldots, 5 \), in tier 1, three consolidation sites, \( j = 1, 2, 3 \), in tier 2, and four processing sites, \( k = 1, \ldots, 4 \), in tier 3. The transportation costs per unit flow between any two associated sites are given in Table 12.

![Diagram of Reverse Production System](image)

**Figure 15:** The Reverse Production System for the Example
We consider two types of the price correlated commodities in this example: obsolete desktop and laptop computers. Collection sites collect obsolete desktop and laptop computers from the source. For simplicity purposes, we assume collection (tier 1) and consolidation sites (tier 2) are simply acting as intermediary brokers within the network. As a result, the commodity sets for tiers 1 and 2, \( \Lambda_1 \) and \( \Lambda_2 \), are simply equal to \( \{D, L\} \) where desktop and laptop computers are denoted by \( D \) and \( L \) respectively.

We let \( p_{(Co)}^{(u)} \) denote the collection fee of commodity \( u \) paid by site \( i \) in tier 1 and let \( S_{(u)} \) denote the source response function of commodity \( u \) collected in site \( i \) in tier 1. The source response functions describing the collection amounts of obsolete desktop and laptop computers in collection sites in tier 1 are given by

\[
\begin{align*}
S_{1D} &= 400 + 5p_{(Co)}^{(D)} - 1p_{(Co)}^{(L)} , \\
S_{2D} &= 420 + 6p_{(Co)}^{(D)} - 1p_{(Co)}^{(L)} , \\
S_{3D} &= 440 + 6p_{(Co)}^{(D)} - 1p_{(Co)}^{(L)} , \\
S_{4D} &= 430 + 6p_{(Co)}^{(D)} - 1p_{(Co)}^{(L)} , \\
S_{5D} &= 410 + 5p_{(Co)}^{(D)} - 1p_{(Co)}^{(L)} ,
\end{align*}
\[
\begin{align*}
S_{1L} &= 420 - 2p_{(Co)}^{(D)} + 5.5p_{(Co)}^{(L)} , \\
S_{2L} &= 440 - 2p_{(Co)}^{(D)} + 6.5p_{(Co)}^{(L)} , \\
S_{3L} &= 460 - 2p_{(Co)}^{(D)} + 6.5p_{(Co)}^{(L)} , \\
S_{4L} &= 450 - 2p_{(Co)}^{(D)} + 6.5p_{(Co)}^{(L)} , \\
S_{5L} &= 430 - 2p_{(Co)}^{(D)} + 5.5p_{(Co)}^{(L)} .
\end{align*}
\]
Processing sites in tier 3 in this example obtain desktop or laptop computers from consolidation sites in tier 2 and may further conduct some refurbished processes so that obsolete recycled items are converted into refurbished desktop or laptop computers sold in the secondary demand markets. Table 13 lists the prices of refurbished desktop and laptop computers in these secondary markets for processing sites. For illustration purposes, the data we provide in this example may not currently represent the actual prices or source amount responses for the desktop and laptop computer recycling streams.

**Table 13: The Prices of Refurbished Desktop and Laptop Computers**

<table>
<thead>
<tr>
<th>Demand Markets</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Desktop</td>
<td>$155</td>
<td>$145</td>
<td>$147</td>
<td>$150</td>
</tr>
<tr>
<td>Laptop</td>
<td>$160</td>
<td>$150</td>
<td>$152</td>
<td>$155</td>
</tr>
</tbody>
</table>

We follow the same computation algorithm developed in Chapter 3 to solve for the equilibrium acquisition prices and resulting material flow for each commodity in different tiers. We let $x_{ju}^{(Tr)}$ and $x_{ku}^{(Tr)}$ denote the aggregate material flow to site $j \in I_2$ and $k \in I_3$, where $x_{ju}^{(Tr)} = \sum_{i \in I_1} x_{jui}^{(Tr)}$ and $x_{ku}^{(Tr)} = \sum_{j \in I_2} x_{jki}^{(Tr)}$, for commodity $u \in \{D, L\}$. The material flow functions between tiers are listed as follows.

\[
\begin{align*}
  x_{j}^{(Tr)} & : j \in I_2, \ u \in \{D, L\}; \\
  x_{1D}^{(Tr)} & = 14.61 \ p_{1D} - 3.97 \ p_{2D} - 2.86 \ p_{3D} - 97.16, \\
  x_{2D}^{(Tr)} & = -2.03 \ p_{1D} + 12.35 \ p_{2D} - 2.78 \ p_{3D} - 85.17, \\
  x_{3D}^{(Tr)} & = -3.78 \ p_{1D} - 2.70 \ p_{2D} + 14.19 \ p_{3D} - 101.18, \\
  x_{1L}^{(Tr)} & = -3.49 \ p_{1D} - 0.58 \ p_{2D} - 0.07 \ p_{3D} + 12.57 \ p_{1L} - 1.76 \ p_{2L} - 85.97, \\
  x_{2L}^{(Tr)} & = -0.48 \ p_{1D} - 1.06 \ p_{2D} - 0.18 \ p_{3D} - 1.53 \ p_{1L} + 8.61 \ p_{2L} - 0.58 \ p_{3L} - 56.71, \text{ and} \\
  x_{3L}^{(Tr)} & = -0.40 \ p_{1D} - 0.47 \ p_{2D} - 3.93 \ p_{3D} - 1.16 \ p_{2L} + 11.58 \ p_{3L} - 74.91.
\end{align*}
\]
$x^{(Tr)}_{ku}, \quad k \in I_3, \; u \in \{D,L\};$

$x^{(Tr)}_{1D} = 8.84 \ p_{1D} - 2.73 \ p_{2D} - 1.83 \ p_{3D} - 1.46 \ p_{4D} - 0.07 \ p_{1L} - 0.02 \ p_{2L} - 31.15,$

$x^{(Tr)}_{2D} = -0.72 \ p_{1D} + 7.50 \ p_{2D} - 2.13 \ p_{3D} - 2.03 \ p_{4D} - 22.40,$

$x^{(Tr)}_{3D} = -2.51 \ p_{1D} - 0.44 \ p_{2D} + 7.54 \ p_{3D} - 1.43 \ p_{4D} - 0.14 \ p_{1L} - 0.06 \ p_{2L} - 0.06 \ p_{3L} - 0.07 \ p_{4L} - 17.86,$

$x^{(Tr)}_{4D} = -2.34 \ p_{1D} - 1.10 \ p_{2D} - 1.92 \ p_{3D} + 8.03 \ p_{4D} - 28.72,$

$x^{(Tr)}_{1L} = -0.40 \ p_{1D} - 0.33 \ p_{2D} - 0.32 \ p_{3D} - 0.36 \ p_{4D} + 7.34 \ p_{1L} - 1.89 \ p_{2L} - 1.25 \ p_{3L} - 0.66 \ p_{4L} - 24.57,$

$x^{(Tr)}_{2L} = -0.28 \ p_{1D} - 0.18 \ p_{2D} - 0.16 \ p_{3D} - 0.21 \ p_{4D} - 0.23 \ p_{1L} + 6.85 \ p_{2L} - 1.95 \ p_{3L} - 1.61 \ p_{4L} - 16.82,$

$x^{(Tr)}_{3L} = -0.31 \ p_{1D} - 0.20 \ p_{2D} - 0.14 \ p_{3D} - 0.23 \ p_{4D} - 1.42 \ p_{1L} - 0.68 \ p_{2L} + 6.77 \ p_{3L} - 1.43 \ p_{4L} - 13.92,$

$x^{(Tr)}_{4L} = -0.24 \ p_{1D} - 0.18 \ p_{2D} - 0.16 \ p_{3D} - 0.25 \ p_{4D} - 1.59 \ p_{1L} - 1.01 \ p_{2L} - 1.26 \ p_{3L} + 6.86 \ p_{4L} - 25.94.$

We examine the cross-price effect of a change in one commodity’s price on the other commodity’s flow amount and the own-price effect of a change in one commodity’s prices on its flow amount. An interesting finding in the flow functions shown in this example indicates that most of the cross-price effects have less impact on the aggregate flow compared to the own-price effects in this example. The other details of the price and flow solutions are listed in Table 14, Table 15, and Table 16. It is clear that the acquisition prices of laptop computers are relatively higher than the prices of desktop computers as a result of the higher final market prices of the laptop computers in this example.
Table 14: The Equilibrium Prices and Collection Fees

<table>
<thead>
<tr>
<th>$p_{iu}^{(Co)^*}$: $i \in I_1$, $u \in {D, L}$</th>
<th>$p_{iu}^*$: $j \in I_2$, $u \in {D, L}$</th>
<th>$p_{iu}^*$: $k \in I_3$, $u \in {D, L}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{1D}^{(Co)^*} = -31.29$</td>
<td>$p_{1D}^* = 59.32$</td>
<td>$p_{1D}^* = 117.95$</td>
</tr>
<tr>
<td>$p_{2D}^{(Co)^*} = -34.33$</td>
<td>$p_{2D}^* = 64.91$</td>
<td>$p_{2D}^* = 110.95$</td>
</tr>
<tr>
<td>$p_{3D}^{(Co)^*} = -43.98$</td>
<td>$p_{3D}^* = 61.85$</td>
<td>$p_{3D}^* = 111.06$</td>
</tr>
<tr>
<td>$p_{4D}^{(Co)^*} = -33.16$</td>
<td></td>
<td>$p_{4D}^* = 114.85$</td>
</tr>
<tr>
<td>$p_{5D}^{(Co)^*} = -31.78$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p_{1L}^{(Co)^*} = -45.74$</td>
<td>$p_{1L}^* = 66.22$</td>
<td>$p_{1L}^* = 122.79$</td>
</tr>
<tr>
<td>$p_{2L}^{(Co)^*} = -36.67$</td>
<td>$p_{2L}^* = 68.17$</td>
<td>$p_{2L}^* = 115.72$</td>
</tr>
<tr>
<td>$p_{3L}^{(Co)^*} = -34.51$</td>
<td>$p_{3L}^* = 68.15$</td>
<td>$p_{3L}^* = 115.76$</td>
</tr>
<tr>
<td>$p_{4L}^{(Co)^*} = -40.79$</td>
<td></td>
<td>$p_{4L}^* = 119.64$</td>
</tr>
<tr>
<td>$p_{5L}^{(Co)^*} = -31.78$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* The negative equilibrium collection fee indicates that the collection site pays a minus price for the collected items. In other words, the collection site charges the source a positive fee for collecting items.

Table 15: The Resulting Flows of Desktop Computers

<table>
<thead>
<tr>
<th>$x_{iDj}^{(Tr)^*}$</th>
<th>$j \in I_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>$i \in I_1$</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$x_{jDk}^{(Tr)^*}$</th>
<th>$k \in I_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>$j \in I_2$</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>3</td>
</tr>
</tbody>
</table>

Table 16: The Resulting Flows of Laptop Computers

<table>
<thead>
<tr>
<th>$x_{iLj}^{(Tr)^*}$</th>
<th>$j \in I_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>$i \in I_1$</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$x_{jLk}^{(Tr)^*}$</th>
<th>$k \in I_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>$j \in I_2$</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>3</td>
</tr>
</tbody>
</table>

The preceding example demonstrates a three-tier decentralized RPS problem with two price correlated commodities. The major modification is on the material flow.
allocation mechanism format discussed in Chapters 2 and 3. The next section presents the analysis of computational complexity and suggests an alternative to improve the computational efficiency.

### 4.4 Discussion of Computational Complexity

This section provides the analysis for computational complexity. Each independent site forecasts the acquisition price offered by the sites in its next tier. As discussed in Section 2.3, there are an infinite number of price scenarios if the prediction range is a continuous compact interval. This may lead to computational difficulties. In this research, the continuous compact interval of the price range is restricted to a finite number of discrete points. A practical approach for the computation of this study is to select $\zeta$ points evenly in every dimension of the price range. Thus, the price scenario space $\Omega'_m$ considered by site $i \in I_{m-1}$ is with $\zeta^{N_m}$ scenarios if there are $N_m$ sites in tier $m$. We let $|\Lambda_m|$ denote the number of price correlated commodities acquired in tier $m$. The price scenario space is with $\zeta^{|\Lambda_m|}$ scenarios for one single independent site. The model may also lead to computation difficulties due to the exponential growth in the number of price scenarios.

We suggest one way to reduce the number of the price scenarios considered in each tier. The main reason we explore the price scenario space is price uncertainty of acquisition prices for different sites in one tier. Due to the assumption of no information sharing in the decentralized problem, each entity does not know the exact final acquisition prices to be offered by the sites in its next tier when the entity determines the
material flow allocation mechanism between itself and sites in the next tier. However, the entity can reduce the candidate list of entities to whom the entity will supply recycled items. For example, the entity is unwilling to supply recycled items to a downstream tier site who always offers a lower acquisition price compared to other downstream tier sites in every circumstance after taking account of transportation cost effect. We take the transaction between site \( i \in I_{m-1} \) and sites in tier \( m \), depicted in Figure 16, as an example to illustrate this reduction rule.

![Diagram](image)

**Figure 16:** The Acquisition Price Range Forecasted by Site \( i \)

We let \( q_{ju}^{(i)} \) and \( \bar{q}_{ju}^{(i)} \) denote the lower and upper bounds of the acquisition price of commodity \( u \in \Lambda_m \) offered by site \( j \in I_m \) forecasted by site \( i \in I_{m-1} \). The effective lower and upper bounds of the acquisition price are \( q_{ju}^{(i)} - V_{q_j}^{(Tr)} \) and \( \bar{q}_{ju}^{(i)} - V_{\bar{q}_j}^{(Tr)} \) respectively.

Site \( i \) in tier \( m-1 \) screens all possible sites in tier \( m \) and excludes the sites whose effective upper bound is always lower than effective lower bounds of all other sites. We let \( B_i \) denote the final set of candidate sites for upstream site \( i \in I_{m-1} \). The screening algorithm for site \( i \in I_{m-1} \) is stated as follows.
Set $B' := I_m$

For all $j \in I_m$

If $q^{(i)}_{ju} - V^{(Tr)}_{ij} < q^{(j)}_{ju} - V^{(Tr)}_{ij}$ for all $j' \in I_m$ and $u \in \Lambda_m$

Then $B := B' \setminus \{j\}$

This screening algorithm can be accomplished in $O(|\Lambda_m| \cdot N_m^2)$ time. By doing the screening algorithm in advance, it may effectively reduce the number of price scenarios which is a significant factor affecting computation time of models developed in Section 4.2. The next section summarizes and concludes this chapter of a multi-tiered RPS network with price correlated commodities.

### 4.5 Conclusions and Extensions

This chapter presents a modified model for a multi-tiered RPS with price correlated commodities where recycled material flow allocation mechanism is not only influenced by competitors’ prices but also affected by the acquisition price of the other type of recycled items. The major modification to the models of the earlier chapters is on the material flow allocation mechanism function. The function is modified to include additional coefficients that represent the impact of other commodities on the material flow. We follow the same computational algorithm developed in Chapter 3 to solve for the equilibrium acquisition prices and the resulting flows for different commodities in different tiers. Finally we suggest a screening algorithm used to reduce the price scenario space to improve computational efficiency for a multi-tiered RPS network with price correlated commodities.
One extension of this chapter is to explore price-correlated commodities where one good is a complement in production of another good in a recycling network. For example, the commodity of motherboards is a complement in production of disk drives when the recycler conducts disassembly processes of computers. An excess flow of some sub-components may arise in this particular case and this extension requires further refinement of the models we have developed.
CHAPTER 5 CENTRALIZED VS. DECENTRALIZED DECISION-MAKING FOR RECYCLED MATERIAL FLOWS

This chapter examines centralized and decentralized decision-making models for reverse logistics networks. We compare the application of a centralized model for planning reverse production systems, where a single planner is acquainted with all of system information and has authority to determine decision variables of the entire system, to a decentralized approach. In the decentralized approach the entities coordinate between tiers of the system using a parameterized flow function, and compete within tiers based on reaching a price equilibrium. We numerically and mathematically investigate the impact on decision variables and the system net profit due to changes in the final market prices or alternate schemes of recycling subsidies.

5.1 Introduction

Reverse production systems (RPS) include collection, sorting, demanufacturing, and refurbished processes networked by reverse logistics operations. For many products, the infrastructure for RPSs is still in its early stages of organization, and understanding the advantages and disadvantages of different approaches to structuring it is important. In this chapter we examine two extremes of organizational approaches, centralized and decentralized, in order to gain insight into their respective behaviors. In a centralized decision-making process, a single planner or organization is acquainted with all system information including transportation capacities, processing capabilities, or associated sales prices of recycled items. The planner has authority to determine decision variables
of the system; for example, how recycled materials are flowing through the RPS network or how much the system can spend to acquire recycled items. One example of the decision maker in the centralized setup is a local government which owns the municipal collection and processing sites in a recycling network. The government may be acting as a central planner to determine the RPS network behavior. In the past decade, many researchers have discussed reverse logistics system planning for end-of-life products in a centralized framework (e.g., Fleischmann et al. 2000; Ammons et al. 2001; Guide and Harrison 2003). The major tasks of RPS planning consider collection, sortation, consolidation, disassembly, demanufacturing processes within system limitations of the RPS network. Several studies have applied mathematical programming methods to find an optimal system plan and design for reverse supply chain systems (see Barros et al. 1998; Shih 2001; Realff et al. 2004; Fleischmann et al. 2004; Assavapokee et al. 2005). Most of these studies propose mathematical programming models that solve the problem as a reverse network flow problem to obtain the optimal infrastructure design as well as associated material flow allocations or other decision variables within the network.

In a decentralized decision-making reverse supply chain system, a RPS consists of several independent entities operated by different private parties, who are unwilling to reveal their own confidential information for processing capacities or cost structures to others or the public. In addition, the decision variables for each entity are often influenced by other entities’ decisions. There is also a growing number of research papers on forward or reverse supply chains that model the independent decision-making process of each entity in supply chain, specifically the interaction between pricing decisions and material flow volume transacted in the network (see Majumder and
There are also several studies on the comparison of the centralized and decentralized setups for the forward supply chain, especially for inventory problems (Chang and Harrington 2000; Jorgensen and Kort 2002; Chen and Chen 2005). However, little research has addressed the comparison of centralized and decentralized problems for reverse production systems or reverse logistics networks. In this chapter, we examine the individual entity and system behaviors for centralized versus decentralized RPS setups, and present several numerical implications and insights drawn from the centralized and decentralized models. The remainder of the chapter is organized as follows. In Section 5.2 we provide a formal definition of a multi-tiered RPS problem with an upstream boundary tier, several intermediate tiers, and a downstream boundary tier for centralized and decentralized RPS models. Section 5.3 provides mathematical models for the centralized problem setting and overviews the decentralized models developed in our recent work (Hong et al. 2005b). In Section 5.4 we provide a numerical illustration of the relative impacts of centralized versus decentralized approaches. In Section 5.5 we numerically and mathematically investigate the impact on the decision variables and net profits for different subsidizing scenarios or the increased final market prices in both the centralized and decentralized problem settings. We conclude this chapter in Section 5.6 and also suggest directions for future research.
5.2 A Multi-tiered RPS Problem

A RPS is a network of transportation logistics and processing functions that collect, recycle, refurbish, and demanufacture end-of-life products. In this chapter, we model the RPS as a multi-tiered network, depicted in Figure 6, which consists of an upstream boundary tier, several intermediate tiers, and a downstream boundary tier. We consider \( N_1 \) independent entities in the upstream boundary tier as represented by the top tier of nodes in Figure 6, \( N_2, \ldots, N_{M-1} \) entities in intermediate tiers 2, \ldots, \( M-1 \) respectively, and \( N_M \) downstream boundary tier entities associated with the bottom tier in the network. In addition, we let sources of recycled products and demand markets be two end tiers of the network which may be represented as several independent and possibly geographically distinct sources of end-of-life products and demand markets for secondary used products or raw materials.
Sources of recycle materials

\[ S_i = a_i + b_i p_i^{(Co)} \]
\[ S_i = a_i + b_i p_i^{(Co)} \]
\[ S_{N_i} = a_{N_i} + b_{N_i} p_{N_i}^{(Co)} \]

Figure 17: A General Multi-tiered RPS Network Structure

Typical upstream boundary tier entities can be represented as municipal collection sites, non-profit collection organizations, private collectors, etc. The entities in the upstream boundary tier collect recycled materials from the source supply, which can include, for example, residential households, businesses, schools, or the government, with a dynamic used-product market where the amount collected is dependent on the transaction collection fee between the upstream boundary tier and the source. The intermediate tiers may contain several levels of entities: for example, the tier of consolidation sites, material brokers, processing sites who bid for collected materials
from their preceding tier and conduct some valued added processes such as sorting, or disassembling operations or simply act as an intermediary between tiers. Downstream boundary entities associated with nodes in the bottom tier in the network can be seen as the final stage of the entire RPS, where they purchase recycled items from their preceding tier and conduct further dismantling/mechanical fragmentation of items or refurbish end-of-life products for consumption purposes. Hence, downstream boundary entities may convert the recycled items into raw materials, refurbished products and sell them to the specific demand markets. In general, recycled items flow from the upstream tier to the downstream tier of entities but financial incentives are driven from the downstream tier back to the upstream tier of entities. For simplicity, we assume that materials must move through each tier sequentially. In other words, materials can not be transported directly across two or more tiers within the network.

We let $I_m = \{1, \ldots, j, \ldots, N_m \}$ denote the set of sites in tier $m$. The entities in the upstream boundary tier collect recycled items from the source supply and the source supplies the upstream boundary tier site on the basis of a fee paid by the upstream boundary tier site. We let $S_i$ denote the collection amount in upstream boundary tier site $i \in I_1$ and $p_i^{(Co)}$ be the collection fee per unit of the recycled item paid by site $i \in I_1$. The collection amount in upstream boundary tier site $i \in I_1$ is characterized by a linear function $S_i = a_i + b_i p_i^{(Co)}$, where $a_i$ and $b_i$ are parameters and $a_i, b_i > 0$. Note that the collection fee, $p_i^{(Co)}$, of site $i \in I_1$ is without sign restriction; in other words, the upstream boundary tier site may pay or charge for collecting recycled items if $p_i^{(Co)}$ is positive or negative respectively. We also argue that the amount of raw materials resulting from the
decomposition of end-of-life products and used products is relatively small compared to the quantity in the virgin raw material and brand-new product markets. This observation leads to the assumption the selling prices of raw materials or used products in final demand markets are fixed amounts, not affected by the sales quantities. We let $P^{(Su)}_k$ denote the selling price obtained in downstream boundary tier site $k \in I_M$. In other words, two main exogenous information streams to the system are the response functions of the collected recycled item amount in the upstream boundary tier, represented by $S_i = a_i + b_i p^{(Co)}_i$ for site $i \in I$, and the selling price obtained in the downstream boundary tier, denoted by $P^{(Su)}_k$ for site $k \in I_M$. Here, $a_i$, $b_i$, and $P^{(Su)}_k$ are known parameters, but $S_i$ and $p^{(Co)}_i$ are unknown variables to the system.

We first consider a setup in which management is centralized. A single decision maker (e.g., the state or local government) has the requisite information about all the participating entities and seeks the optimal solution for the entire system. The underlying assumption of the centralized problem setting is that the decision maker has authority to manage associated operations or processes of all entities within the network. In a centralized setup, the decision maker determines the optimal level of the collection amount from the source, and the most efficient way of material flow allocation through the network so that the system profit is maximized. In addition, there are some internal transaction variables among entities in the network such as internal transaction prices; however, these are not relevant in the centralized setting.

The principle of the decentralized setup is that the network system is composed of several independent entities operated individually. Each independent entity has its own
profit function subject to its own processing or transportation constraints, and may not be willing to reveal its own information to other entities or the public. Often the decision variables for each entity in a decentralized system are also influenced by other entities’ decisions. The foundations of the decentralized RPS models are derived from our recent work in the multi-tiered RPS network (Hong et al. 2005b). Using this decentralized RPS network framework, we obtain the equilibrium collection fee paid by the upstream boundary tier site and the resulting material flow allocation within the network. In this chapter, we examine the comparison of behaviors for a centralized versus decentralized RPS approaches and investigate implications and insights for public policy determination.

5.3 The Centralized and Decentralized Models

In this section, first, we develop a centralized model for a RPS consisting of an upstream boundary tier, intermediate tiers, and a downstream boundary tier, followed by an overview of the decentralized RPS model. The centralized model finds the optimal collection fees and the material flow allocation so that the system profit function is maximized subject to the individual entity and system constraints. The decentralized model solves for the equilibrium collection fee and resulting material flow while each entity determines its own associated decision variables of acquisition prices and flow allocation mechanism.
5.3.1 The Centralized Quadratic Programming Model

In the centralized RPS model, a single planner has the requisite information for all the participating entities and seeks the optimal solution for the entire system. We let $x_{ij}^{(Tr)}$ denote the material flow from site $i \in I_m$ to site $j \in I_{m+1}$ and $x_k^{(Tr)}$ denote the aggregate flow shipped from the sites in tier $M-1$ to downstream boundary tier site $k \in I_M$. The decision variables for the system are the optimal material flow allocation, $x_{ij}^{(Tr)}$ and $x_k^{(Tr)}$, within the system and the collection fee, $p_{ij}^{(Co)}$, in the upstream boundary tier. We also define the following system parameters known by the decision maker as

$V_{ij}^{(Tr)}$ Transportation cost per standard unit from site $i$ to site $j$;

$C_{ij}^{(Tr)}$ Maximum amount of material that can be shipped from site $i$ to site $j$; and

$P_{ij}^{(Pr)}$ Maximum amount of material can be processed in site $i$.

Using this notation, the centralized RPS optimization model for the entire system can be stated as:

Maximize

$$\sum_{k \in I_M} x_{ij}^{(Tr)} P_k^{(Sa)} - \sum_{i \in I_1} p_{ij}^{(Co)} (a_i + b_i p_{ij}^{(Co)}) - \sum_{m=1}^{M-1} \sum_{i \in I_m} \sum_{j \in I_{m+1}} V_{ij}^{(Tr)} x_{ij}^{(Tr)}$$ (94)

Subject to

$$\sum_{j \in I_i} x_{ij}^{(Tr)} = a_i + b_i p_{ij}^{(Co)} \quad \forall i \in I_1$$ (95)

$$\sum_{i \in I_{m+1}} x_{ij}^{(Tr)} = \sum_{k \in I_{m+1}} x_{jk}^{(Tr)} \quad \forall j \in I_m, \forall m = 2 \ldots M - 1$$ (96)

$$\sum_{j \in I_{m+1}} x_{jk}^{(Tr)} = x_k^{(Tr)} \quad \forall k \in I_M$$ (97)

$$x_{ij}^{(Tr)} \leq C_{ij}^{(Tr)} \quad \forall i \in I_m, j \in I_{m+1}, \forall m = 1 \ldots M - 1$$ (98)

$$\sum_{j \in I_{m+1}} x_{ij}^{(Tr)} \leq C_{ij}^{(Pr)} \quad \forall i \in I_m, \forall m = 1 \ldots M - 1$$ (99)

$$\sum_{j \in I_{m+1}} x_{jk}^{(Tr)} \leq C_{jk}^{(Pr)} \quad \forall k \in I_M$$ (100)
\[
\begin{align*}
\chi_{ij}^{(Tr)} &\geq 0 \quad \forall i \in I_n, j \in I_{n+1}, \forall m = 1 \cdots M - 1 \\
\chi_{k}^{(Tr)} &\geq 0 \quad \forall k \in I_M.
\end{align*}
\]

The objective function (94) maximizes the system net profit which is the sum of the sales profit from the destination demand markets, collection fees incurred between the upstream boundary tier and sources, and transportation costs of all shipments through the system. Constraints (95), (96), and (97) are the flow conservation among sites within the network. Constraints (98), (99), and (100) are the transportation and processing capacity limitations respectively. We also intuitively require all material flow variables, \(x's\), to be nonnegative in the centralized model in constraints (101) and (102). The centralized model has a concave quadratic objective function and a convex constraint set since we require \(b_i\) to be positive and the model itself is subject to a linear constraint set. Several algorithms can be used to solve quadratic programming problems (see Bazaraa and Shetty 1993). Constraint (95) specifies the recycled item amount from sources to the system and the volume is increasing as the upstream boundary tier site increases the collection fee. Obviously, because the total amount collected is a linear function of the unit collection fee, the corresponding unit fee must be large when a large amount is collected. Consequently, recycled items flowing into the system are limited to either the system capacity itself or the optimal acquisition amount determined by the concave quadratic net profit objective function.

5.3.2 The Decentralized Model

In the decentralized decision-making framework, each entity within the RPS concentrates on optimizing its own profit subject to its own transportation and processing
capacity constraints. The decentralized RPS model is developed in detail in (Hong et al. 2005b). The upstream entities in one tier provide information in the form of a function that connects the downstream price information to the flow they will provide. Each upstream entity acts individually to determine the function used to contract with each member of the next tier. The flow function is determined using a robust optimization formulation that captures the idea that the upstream entity does not have exact price information from the downstream entities, and wants to minimize the worst outcome it can have.

The downstream tier sites are assumed to reveal their bids for the items from the preceding tier until they have no incentive to change them. This allows a Nash equilibrium to be reached within the tier. An algorithm for finding this equilibrium is presented in (Hong et al. 2005a, b). The algorithm respects the structure of the system by only having the previous bids of each entity available for inspection when the next bid is being determined by each independent entity. Under this framework, entities in the system reach the equilibrium of the acquisition prices as well as the resulting material flow allocation through the network. The decentralized model contains this set of internal equilibrium acquisition prices, which are not present in the centralized problem setting. In the following sections, we investigate numerical results of the comparison between the centralized and decentralized settings and draw several insights of examining different fee subsidizing schemes in a three-tiered RPS.
5.4 The Numerical Study

An example, depicted in Figure 15, demonstrates the mathematical behavior of the centralized and decentralized models and provides several insights to compare these two models. We follow the numerical example presented in (Hong et al. 2005b) to compare the system and individual behavior of the RPS network in the centralized and decentralized problem setting. We consider a three-tier RPS with collection, consolidation, and processing sites. There are five collection sites, \( i = 1, \ldots, 5 \), in tier 1, three consolidation sites, \( j = 1, 2, 3 \), in tier 2, and four processing sites, \( k = 1, \ldots, 4 \), in tier 3. The transportation costs per unit flow between any two associated sites are given in Table 12.

![Diagram of the Reverse Production System for the Example](image)

**Figure 18:** The Reverse Production System for the Example
Table 17: The Unit Transportation Costs between Sites

<table>
<thead>
<tr>
<th>Unit Transportation Cost</th>
<th>$j \in I_2$</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i \in I_1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>10.0</td>
<td>15.0</td>
<td>18.0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>10.0</td>
<td>13.0</td>
<td>16.0</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>13.0</td>
<td>10.0</td>
<td>14.0</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>15.0</td>
<td>13.0</td>
<td>11.0</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>17.0</td>
<td>14.0</td>
<td>9.0</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Unit Transportation Cost</th>
<th>$k \in I_3$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$j \in I_2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>8.0</td>
<td>8.0</td>
<td>10.0</td>
<td>12.0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>10.0</td>
<td>8.0</td>
<td>7.0</td>
<td>11.0</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>12.0</td>
<td>10.0</td>
<td>8.0</td>
<td>7.0</td>
<td></td>
</tr>
</tbody>
</table>

The final market prices for processing sites, $k = 1, \ldots, 4$, are $155, 145, 147, \text{ and } 150$ respectively. The collection amount functions in collection sites, $i = 1, \ldots, 5$, are given by $S_1 = 400 + 5p_1^{(Co)}$, $S_2 = 420 + 6p_2^{(Co)}$, $S_3 = 440 + 6p_3^{(Co)}$, $S_4 = 430 + 6p_4^{(Co)}$, and $S_5 = 410 + 5p_5^{(Co)}$. We consider two cases of capacitated and uncapacitated settings for the arc transportation and site processing capacities. In the capacitated case, we limit the arc transportation capacity to 200 units, the collection site capacity to 600 units, the consolidation site capacity to 800 units, and the processing site capacity to 800 units.

The centralized model solution is derived from solving the quadratic programming problem presented in Section 5.3 and the decentralized model solution is obtained using solution methodology described in our previous work (see Hong et al. 2005b). We examine the decision variables of the optimal collection fees paid by collection sites, $i = 1, \ldots, 5$, in tier 1 and the material flow allocations within the network as well as the net profit values for the centralized and decentralized problems. Table 18, Table 19, and Table 20 summarize the numerical solutions of the net profits, collection fees, and material flow allocations for centralized and decentralized problems in capacitated and uncapacitated cases.
Table 18: Numerical Results of the Centralized and Decentralized Problems

<table>
<thead>
<tr>
<th>Capacitated case</th>
<th>Centralized model</th>
<th>Decentralized model*</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Net profit</strong></td>
<td>280,096</td>
<td>152,160</td>
</tr>
<tr>
<td><strong>Collection fee</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( p_1^{(Co)^*} )</td>
<td>7.36</td>
<td>-25.98</td>
</tr>
<tr>
<td>( p_2^{(Co)^*} )</td>
<td>13.36</td>
<td>-19.26</td>
</tr>
<tr>
<td>( p_3^{(Co)^*} )</td>
<td>12.69</td>
<td>-22.28</td>
</tr>
<tr>
<td>( p_4^{(Co)^*} )</td>
<td>12.52</td>
<td>-20.78</td>
</tr>
<tr>
<td>( p_5^{(Co)^*} )</td>
<td>6.36</td>
<td>-26.60</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Uncapacitated case</th>
<th>Centralized model</th>
<th>Decentralized model*</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Net profit</strong></td>
<td>308,779</td>
<td>201,380</td>
</tr>
<tr>
<td><strong>Collection fee</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( p_1^{(Co)^*} )</td>
<td>28.50</td>
<td>-15.08</td>
</tr>
<tr>
<td>( p_2^{(Co)^*} )</td>
<td>33.50</td>
<td>-9.12</td>
</tr>
<tr>
<td>( p_3^{(Co)^*} )</td>
<td>30.83</td>
<td>-12.20</td>
</tr>
<tr>
<td>( p_4^{(Co)^*} )</td>
<td>30.17</td>
<td>-10.60</td>
</tr>
<tr>
<td>( p_5^{(Co)^*} )</td>
<td>26.00</td>
<td>-15.85</td>
</tr>
</tbody>
</table>

* The negative equilibrium collection fee indicates that the collection site pays a minus price for the collected items. In other words, the collection site charges the source a positive fee for collecting items.

Table 19: The Material Flow Allocation of Capacitated Case

<table>
<thead>
<tr>
<th>( x_{ij}^{(Tr)} ): ( i \in I_1, j \in I_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_{11}^{(Tr)^*} )</td>
</tr>
<tr>
<td>-------------------------------</td>
</tr>
<tr>
<td><strong>Centralized</strong></td>
</tr>
<tr>
<td><strong>Decentralized</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( x_{jk}^{(Tr)} ): ( j \in I_2, k \in I_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_{11}^{(Tr)^*} )</td>
</tr>
<tr>
<td>-------------------------------</td>
</tr>
<tr>
<td><strong>Centralized</strong></td>
</tr>
<tr>
<td><strong>Decentralized</strong></td>
</tr>
</tbody>
</table>
Table 20: The Material Flow Allocation Uncapacitated Case

\[ x_{ij}^{(Tr)}: \; i \in I_1, \; j \in I_2 \]

<table>
<thead>
<tr>
<th>Flows</th>
<th>( x_{11}^{(Tr)} )</th>
<th>( x_{12}^{(Tr)} )</th>
<th>( x_{13}^{(Tr)} )</th>
<th>( x_{21}^{(Tr)} )</th>
<th>( x_{22}^{(Tr)} )</th>
<th>( x_{23}^{(Tr)} )</th>
<th>( x_{31}^{(Tr)} )</th>
<th>( x_{32}^{(Tr)} )</th>
<th>( x_{33}^{(Tr)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centralized</td>
<td>542.5</td>
<td>0</td>
<td>0</td>
<td>621.0</td>
<td>0</td>
<td>0</td>
<td>625.0</td>
<td>0</td>
<td>191.2</td>
</tr>
<tr>
<td>Decentralized</td>
<td>110.5</td>
<td>99.0</td>
<td>115.1</td>
<td>121.7</td>
<td>114.4</td>
<td>129.1</td>
<td>140.1</td>
<td>103.1</td>
<td>123.6</td>
</tr>
</tbody>
</table>

| Flows | \( x_{jk}^{(Tr)} \): \; j \in I_2, \; k \in I_3 |
|-------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Centralized | 1354.7 | 0 | 0 | 0 | 810.3 | 0 | 0 | 0 | 386.5 | 0 | 0 | 0 | 388.1 |
| Decentralized | 212.5 | 152.0 | 120.7 | 145.9 | 148.6 | 122.3 | 120.7 | 119.4 | 159.2 | 125.7 | 140.0 | 186.9 |

There are several insights that can be drawn from the numerical results. As expected, both of capacitated and uncapacitated cases show that the net profit of the centralized model outperforms the net profit of the decentralized model. The net profit of the centralized model serves an upper bound on the net profit of the decentralized model for both the capacitated and uncapacitated cases. For the centralized model, the net profit of the capacitated case is bounded by arc capacities of transportation, but the net profit of the uncapacitated case is constrained by the first-order condition of its quadratic concave objective function.

Table 18 indicates that the collection sites in the centralized problem pay a positive collection fee to sources to acquire recycled items, but sources pay the collection fee to collection sites for discarding end-of-life products in the decentralized problem. This implies that the centralized approach acquires more recycled items compared to the decentralized problem. Moreover, the net profit ratios of the decentralized to centralized problem settings are 54.4% and 65.2% in the capacitated and uncapacitated cases.
respectively. In other words, especially in the capacitated case, one may overestimate the system profit and/or the volume of recycled items processed by the system if it is assumed that the decisions are made centrally in a system of independent entities.

Our results also capture the notion of *double marginalization* of the vertical supply chain where two independent firms, upstream and downstream, may end up with lower profits in the decentralized setting (Durham 2000). The decentralized model (Hong et al. 2005b) also considers *price uncertainty* since the price information is not revealed between two independent entities or to the public. Price uncertainty in decentralized problems is the other factor which leads to the difference of net profit values in centralized and decentralized settings and price uncertainty essentially plays a more critical factor in the capacitated case since the price-flow dependence is sensitive to the flow capacity, which is limited in the capacitated case. As a result, we make the observation that the net profit value difference between the centralized and decentralized problems in the capacitated case is larger than the net profit value difference in the uncapacitated case.

The collection fees paid by the upstream boundary tier sites are unknown variables in the above discussion in the centralized problem. As a next step, we examine the most efficient material flow allocation under the centralized model given the equilibrium collection fees from the decentralized model. Here we are interested in the optimal material flow allocation within the network, from a centralized perspective, given the same source amount as that in the decentralized problem. This investigation provides us, given the same amount of source supply, the comparison between the best case the system can achieve in a centralized setting and all independent entities can obtain in the
decentralized problem setting. The decision variables of the collection fees in the centralized model are substituted by the equilibrium collection fees derived by the decentralized model. Under this setting, the total amounts of recycled items are identical in the centralized and decentralized problems, and the centralized model is essentially a linear programming model. The material flow allocations under this framework are listed in Table 21. The net profits of the centralized model given the equilibrium collection fees in tier 1 under the capacitated and uncapacitated cases are 224,850 and 258,543 respectively. Both the capacitated and uncapacitated cases show that the optimal net profit of the centralized model given the equilibrium collection fees fall between the optimal net profits of the centralized and decentralized models discussed earlier (see Table 18). The optimal net profit difference between the decentralized model and the centralized model given the equilibrium collection fees can be interpreted as the system gain due to the efficiency of material flow allocation in the centralized problem setting.

**Table 21:** The Material Flow Allocations in the Centralized Model Given Collection Fees

| Flows | $x_{ij}(Tr)^*$ | $x_{i1}(Tr)^*$ | $x_{i2}(Tr)^*$ | $x_{i3}(Tr)^*$ | $x_{i4}(Tr)^*$ | $x_{i5}(Tr)^*$ | $x_{i6}(Tr)^*$ | $x_{i7}(Tr)^*$ | $x_{i8}(Tr)^*$ | $x_{i9}(Tr)^*$ | $x_{i10}(Tr)^*$ | $x_{i11}(Tr)^*$ | $x_{i12}(Tr)^*$ | $x_{i13}(Tr)^*$ | $x_{i14}(Tr)^*$ | $x_{i15}(Tr)^*$ | $x_{i16}(Tr)^*$ | $x_{i17}(Tr)^*$ | $x_{i18}(Tr)^*$ | $x_{i19}(Tr)^*$ | $x_{i20}(Tr)^*$ | $x_{i21}(Tr)^*$ | $x_{i22}(Tr)^*$ | $x_{i23}(Tr)^*$ | $x_{i24}(Tr)^*$ | $x_{i25}(Tr)^*$ | $x_{i26}(Tr)^*$ | $x_{i27}(Tr)^*$ | $x_{i28}(Tr)^*$ | $x_{i29}(Tr)^*$ | $x_{i30}(Tr)^*$ | $x_{i31}(Tr)^*$ | $x_{i32}(Tr)^*$ | $x_{i33}(Tr)^*$ | $x_{i34}(Tr)^*$ | $x_{i35}(Tr)^*$ | $x_{i36}(Tr)^*$ | $x_{i37}(Tr)^*$ | $x_{i38}(Tr)^*$ | $x_{i39}(Tr)^*$ | $x_{i40}(Tr)^*$ |
|-------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| Cap   | 200.0          | 70.1           | 200.0          | 104.4          | 0              | 0              | 200.0          | 106.3          | 0              | 105.3          | 200.0          | 0              | 0              | 77.0           | 200.0          |
| Uncap | 324.6          | 0              | 365.3          | 0              | 0              | 366.8          | 0              | 366.4          | 0              | 330.7          |
| Flows | $x_{jk}(Tr)^*$ | $x_{j1}(Tr)^*$ | $x_{j2}(Tr)^*$ | $x_{j3}(Tr)^*$ | $x_{j4}(Tr)^*$ | $x_{j5}(Tr)^*$ | $x_{j6}(Tr)^*$ | $x_{j7}(Tr)^*$ | $x_{j8}(Tr)^*$ | $x_{j9}(Tr)^*$ | $x_{j10}(Tr)^*$ | $x_{j11}(Tr)^*$ | $x_{j12}(Tr)^*$ | $x_{j13}(Tr)^*$ | $x_{j14}(Tr)^*$ | $x_{j15}(Tr)^*$ | $x_{j16}(Tr)^*$ | $x_{j17}(Tr)^*$ | $x_{j18}(Tr)^*$ | $x_{j19}(Tr)^*$ | $x_{j20}(Tr)^*$ | $x_{j21}(Tr)^*$ | $x_{j22}(Tr)^*$ | $x_{j23}(Tr)^*$ | $x_{j24}(Tr)^*$ | $x_{j25}(Tr)^*$ | $x_{j26}(Tr)^*$ | $x_{j27}(Tr)^*$ | $x_{j28}(Tr)^*$ | $x_{j29}(Tr)^*$ | $x_{j30}(Tr)^*$ | $x_{j31}(Tr)^*$ | $x_{j32}(Tr)^*$ | $x_{j33}(Tr)^*$ | $x_{j34}(Tr)^*$ | $x_{j35}(Tr)^*$ | $x_{j36}(Tr)^*$ | $x_{j37}(Tr)^*$ | $x_{j38}(Tr)^*$ | $x_{j39}(Tr)^*$ | $x_{j40}(Tr)^*$ |
| Cap   | 200.0          | 0              | 200.0          | 200.0          | 0              | 200.0          | 156.8          | 200.0          | 0              | 106.3          | 200.0          |
| Uncap | 689.9          | 0              | 0              | 366.8          | 0              | 0              | 697.2          | 0              | 0              |

Cap: capacitated case
Uncap: uncapacitated case
5.5 Insights from Centralized and Decentralized Problems

This section develops insights from the comparison of different recycling scenarios in centralized and decentralized problems for the special circumstances when entities within the RPS are subsidized by the government or some organizations, or when the final market prices are increased. First, we discuss the situation when entities within the RPS are subsidized. For example, a recent California act\textsuperscript{4} establishes an Advance Recycling Fee (ARF) of $6-$10 on all electronic products containing hazardous materials, which would be used to fund an electronics recycling system (Gable and Shireman 2001). Maine also has mandated a similar $6 fee on televisions from 2005 to 2012\textsuperscript{5}. These legislative developments for the recycling of electronic devices are certain to have an impact on the individual behavior of each entity in the RPS for these regions.

We follow our base numerical example presented in Section 5.4 and investigate impacts on decision variables due to different subsidizing scenarios as well as the increases in final market prices. Table 22 summarizes the different scenarios we examine in this section. In scenario (I), we assume that the government subsidizes all entities in the RPS throughout on the equal basis of amount processed; in other words, each entity in the system is subsidized by $5 per unit of materials. As a consequence, each unit of materials obtains a total subsidy of $15 in this three-tier example since we assume that materials must move through each tier sequentially. Scenarios (II, III, and IV) examine the different subsidizing scenarios where the government only subsidizes the first, second, or third tier by a total subsidy of $15 per unit of materials. For comparison, we also

\textsuperscript{4} The statement for this act is available at http://www.ciwmb.ca.gov/electronics/act2003/.

investigate a scenario, (V), where the final market prices are increased by $15 per unit of materials.

Table 22: Scenario Descriptions of Numerical Examples

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td>The original numerical example presented in Section 5.4.</td>
</tr>
<tr>
<td>(I)</td>
<td>Each entity in the system is subsidized by $5 per unit of materials.</td>
</tr>
<tr>
<td>(II)</td>
<td>Each entity in the first tier is subsidized by $15 per unit of materials.</td>
</tr>
<tr>
<td>(III)</td>
<td>Each entity in the second tier is subsidized by $15 per unit of materials.</td>
</tr>
<tr>
<td>(IV)</td>
<td>Each entity in the third tier is subsidized by $15 per unit of materials.</td>
</tr>
<tr>
<td>(V)</td>
<td>The final market prices are increased by $15 per unit of materials.</td>
</tr>
</tbody>
</table>

In the decentralized setting, we assume that materials are transported from the upstream tier to its subsequent downstream tier and acquired by the downstream tier. Under this framework, a key issue is the set of internal equilibrium acquisition prices which are not the decision variables in the centralized problem. One can refer to (Hong et al. 2005b) for the details on the methodology to compute equilibrium acquisition prices within the network. We let \( p_j^* \) denote the equilibrium acquisition price to be offered by site \( j \) and \( S_i^* \) be the amount of source flow in upstream boundary tier site \( i \). The details of equilibrium prices and source flows in the decentralized setting for the capacitated case are listed in Table 23 and Table 24.
Table 23: The Equilibrium Prices in the Decentralized Scenarios for the Capacitated Case

<table>
<thead>
<tr>
<th>Equilibrium Price</th>
<th>Original</th>
<th>(I) $5 all</th>
<th>(II) $15 tier 1</th>
<th>(III) $15 tier 2</th>
<th>(IV) $15 tier 3</th>
<th>(V) final market $15</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_1^{(Co)} )</td>
<td>-25.98</td>
<td>-23.70</td>
<td>-26.04</td>
<td>-25.88</td>
<td>-20.13</td>
<td>-20.13</td>
</tr>
<tr>
<td>( p_4^{(Co)} )</td>
<td>-20.78</td>
<td>-18.49</td>
<td>-20.29</td>
<td>-20.71</td>
<td>-15.31</td>
<td>-15.31</td>
</tr>
</tbody>
</table>

Tier 1

| \( p_1^* \)     | 62.75    | 63.20      | 57.42          | 63.17           | 68.13          | 68.13               |
| \( p_2^* \)     | 61.20    | 61.91      | 58.25          | 60.42           | 66.44          | 66.44               |
| \( p_3^* \)     | 62.86    | 63.29      | 59.11          | 63.13           | 68.17          | 68.17               |

Tier 2

| \( p_1^* \)     | 98.81    | 94.96      | 94.59          | 84.16           | 108.10         | 108.10              |
| \( p_2^* \)     | 94.96    | 89.02      | 89.36          | 82.97           | 104.40         | 104.40              |
| \( p_3^* \)     | 100.32   | 98.21      | 98.25          | 90.42           | 110.27         | 110.27              |

Tier 3

| \( p_4^* \)     | 94.08    | 92.39      | 90.63          | 80.58           | 103.08         | 103.08              |

Table 24: The Details of Source Flows in the Decentralized Scenarios for the Capacitated Case

<table>
<thead>
<tr>
<th>Source Flow</th>
<th>Original</th>
<th>(I) $5 all</th>
<th>(II) $15 tier 1</th>
<th>(III) $15 tier 2</th>
<th>(IV) $15 tier 3</th>
<th>(V) final market $15</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_1^* )</td>
<td>270.1</td>
<td>281.5</td>
<td>269.8</td>
<td>270.6</td>
<td>299.4</td>
<td>299.4</td>
</tr>
<tr>
<td>( S_2^* )</td>
<td>304.4</td>
<td>318.1</td>
<td>307.5</td>
<td>304.8</td>
<td>337.1</td>
<td>337.1</td>
</tr>
<tr>
<td>( S_3^* )</td>
<td>306.3</td>
<td>320.3</td>
<td>310.9</td>
<td>306.3</td>
<td>338.9</td>
<td>338.9</td>
</tr>
<tr>
<td>( S_4^* )</td>
<td>305.3</td>
<td>319.1</td>
<td>308.3</td>
<td>305.7</td>
<td>338.1</td>
<td>338.1</td>
</tr>
<tr>
<td>( S_5^* )</td>
<td>277.0</td>
<td>288.5</td>
<td>279.0</td>
<td>277.3</td>
<td>306.3</td>
<td>306.3</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>1463.1</strong></td>
<td><strong>1527.5</strong></td>
<td><strong>1475.5</strong></td>
<td><strong>1464.7</strong></td>
<td><strong>1619.8</strong></td>
<td><strong>1619.8</strong></td>
</tr>
</tbody>
</table>

It is clear that scenarios (IV) and (V) have the same impact on the results of source flows and equilibrium acquisition prices. In other words, there is no difference between these two perturbation scenarios where the third tier (the bottom tier) is subsidized by $15 per unit of material and the final market prices are increased by $15 per unit of material respectively. The other interesting finding is that the acquired prices in tier 3 \( (p_k^*, k \in I_3) \)
in scenario (III) show a relatively lower level compared to other scenarios. An explanation is that the entities in tier 3 are not pushing to acquire materials from tier 2 with high acquisition prices since the entities in tier 2 gain the *exogenous* financial sources which are the subsidies from the government. We also investigate impacts on the results of different scenarios we discuss earlier in the decentralized problem setup for the uncapacitated case, which are shown in Table 25 and Table 26 respectively.

**Table 25:** The Equilibrium Prices in the Decentralized Scenarios for the Uncapacitated Case

<table>
<thead>
<tr>
<th>Equilibrium Price</th>
<th>Original</th>
<th>(I) $5 all</th>
<th>(II) $15 tier 1</th>
<th>(III) $15 tier 2</th>
<th>(IV) $15 tier 3</th>
<th>(V) final market $15</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_1^{(Co)*} )</td>
<td>-15.08</td>
<td>-5.60</td>
<td>-8.84</td>
<td>-3.12</td>
<td>-8.10</td>
<td>-8.10</td>
</tr>
<tr>
<td>( p_2^{(Co)*} )</td>
<td>-9.12</td>
<td>-0.18</td>
<td>-2.87</td>
<td>1.94</td>
<td>-2.67</td>
<td>-2.67</td>
</tr>
<tr>
<td>( p_3^{(Co)*} )</td>
<td>-12.20</td>
<td>-3.17</td>
<td>-5.92</td>
<td>-0.93</td>
<td>-5.70</td>
<td>-5.70</td>
</tr>
<tr>
<td>( p_4^{(Co)*} )</td>
<td>-10.60</td>
<td>-1.50</td>
<td>-4.33</td>
<td>0.81</td>
<td>-4.01</td>
<td>-4.01</td>
</tr>
<tr>
<td>( p_5^{(Co)*} )</td>
<td>-15.85</td>
<td>-6.36</td>
<td>-9.60</td>
<td>-3.78</td>
<td>-8.87</td>
<td>-8.87</td>
</tr>
<tr>
<td>( p_1^* )</td>
<td>73.08</td>
<td>79.72</td>
<td>73.07</td>
<td>84.08</td>
<td>79.58</td>
<td>79.58</td>
</tr>
<tr>
<td>( p_2^* )</td>
<td>69.80</td>
<td>76.23</td>
<td>69.79</td>
<td>80.66</td>
<td>76.00</td>
<td>76.00</td>
</tr>
<tr>
<td>( p_3^* )</td>
<td>72.64</td>
<td>79.23</td>
<td>72.63</td>
<td>83.76</td>
<td>79.06</td>
<td>79.06</td>
</tr>
<tr>
<td>( p_1^* )</td>
<td>117.92</td>
<td>121.68</td>
<td>117.90</td>
<td>117.87</td>
<td>129.22</td>
<td>129.22</td>
</tr>
<tr>
<td>( p_2^* )</td>
<td>111.04</td>
<td>114.73</td>
<td>111.04</td>
<td>110.94</td>
<td>122.13</td>
<td>122.13</td>
</tr>
<tr>
<td>( p_3^* )</td>
<td>110.64</td>
<td>114.29</td>
<td>110.65</td>
<td>110.62</td>
<td>121.71</td>
<td>121.71</td>
</tr>
<tr>
<td>( p_4^* )</td>
<td>114.74</td>
<td>118.49</td>
<td>114.74</td>
<td>114.89</td>
<td>125.99</td>
<td>125.99</td>
</tr>
</tbody>
</table>
Table 26: The Source Flows in the Decentralized Scenarios for the Uncapacitated Case

<table>
<thead>
<tr>
<th>Source Flow</th>
<th>Original</th>
<th>(I) $5 all</th>
<th>(II) $15 tier 1</th>
<th>(III) $15 tier 2</th>
<th>(IV) $15 tier 3</th>
<th>(V) final market $15</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1^*$</td>
<td>324.6</td>
<td>372.0</td>
<td>355.8</td>
<td>384.4</td>
<td>359.5</td>
<td>359.5</td>
</tr>
<tr>
<td>$S_2^*$</td>
<td>365.3</td>
<td>418.9</td>
<td>402.8</td>
<td>431.6</td>
<td>404.0</td>
<td>404.0</td>
</tr>
<tr>
<td>$S_3^*$</td>
<td>366.8</td>
<td>421.0</td>
<td>404.5</td>
<td>434.4</td>
<td>405.8</td>
<td>405.8</td>
</tr>
<tr>
<td>$S_4^*$</td>
<td>366.4</td>
<td>421.0</td>
<td>404.0</td>
<td>434.8</td>
<td>405.9</td>
<td>405.9</td>
</tr>
<tr>
<td>$S_5^*$</td>
<td>330.7</td>
<td>378.2</td>
<td>362.0</td>
<td>391.1</td>
<td>365.6</td>
<td>365.6</td>
</tr>
<tr>
<td>Total</td>
<td>1753.8</td>
<td>2011.1</td>
<td>1929.1</td>
<td>2076.4</td>
<td>1940.9</td>
<td>1940.9</td>
</tr>
</tbody>
</table>

The uncapacitated case indicates several interesting findings in a clearer way when we avoid potential capacity complications. Again, we obtain the identical results for scenarios (IV) and (V) which means that a $15 increased final market price can be seen as a subsidy of $15 to the entities in tier 3 (the bottom tier). We focus on the comparison of different subsidizing scenarios ((I), (II), (III), and (IV)). Subsidizing scenario (IV) shows relatively higher acquired prices to be offered in tier 3 ($p_k^*, k \in I_3$) compared to other scenarios. A similar outcome appears for the acquisition prices offered by tier 2 in scenario (III) and the prices offered by tier 1 in scenario (II). The observation indicates that subsidizing one particular tier $m$ increases the willingness of entities in tier $m$ to offer a higher acquired price to obtain recycled items from its preceding tier $m-1$ due to the competition within tier $m$.

One of the reasons that the government subsidizes the individual entity in the RPS is to increase the total amount of recycled items flowing into the recycling network. Table 26 compares the impact on the total source amount in the collection sites of tier 1 due to the different subsidizing scenarios. It is obvious that all subsidizing scenarios ((I), (II), (III), and (IV)) increase the total amount of recycled items flowing into the entire
system compared to the original scenario, and scenario (III), in which tier 2 is subsidized, shows the highest amount of recycled items collected. From this numerical finding, we conjecture that subsidizing directly, instead of spreading the subsidizing fund throughout the whole system, to the entities in the middle tier may be the most efficient way to achieve the highest amount of recycled items flowing into the system.

Corbett and Karmarkar (2001) develop a model that considers competition, particularly the relation among prices, quantities, and profits, in multi-tiered serial forward supply chains. We further construct a general $M$-tiered RPS serial chain, depicted in Figure 19, to explore the impact on the total flow amount of different subsidizing scenarios in a RPS. Recycled items are collected in site 1 in tier 1 and are flowing sequentially to downstream sites in the chain. The collection amount in site 1, $S$, is characterized by a linear function $S = a + b p$, where $a, b > 0$ and $p$ is the collection fee paid by site 1 to acquire recycled items from the exogenous source. Each downstream site offers a price to acquire the material from its preceding upstream site and finally site $M$ in the end of the chain sells the item to the final market in a given price, $P^{(Sc)}$. We let $p_m$ denote the acquisition price to be offered by site $m$ where $p_m$ is unknown to site $m-1$ for $m = 2, \ldots, M$. Upstream site $m-1$ forecasts a price range, denoted by $[\underline{q}_m, \bar{q}_m]$, of $p_m$ to construct the robust price-flow relation used to contract with its subsequent downstream site $m$. We define $\Omega_m$ as the set of all specified price scenarios of site $m$ predicted by site $m-1$; in turn, the considered price scenario space, $\Omega_m$, is a mapping from the prediction price range of $[\underline{q}_m, \bar{q}_m]$. Let $q_{m\omega}$ denote a particular price
scenario $\omega \in \Omega_m$. In other words, $q_m$'s for all price scenarios $\omega \in \Omega_m$ are within the forecast range $[q_m, \bar{q}_m]$.

\[ S = a + b p \]

**Figure 19:** A General $M$-tiered RPS Serial Chain

The flow response to the acquisition price between upstream site $m$ and downstream site $m+1$ is assumed to be a simple linear function, $x_{m,m+1} = \alpha_m p_{m+1}$, where $\alpha_m$ is the decision variable of site $m$ for describing how site $m$ responds to the acquired price, $p_{m+1}$, offered by site $m+1$ when site $m$ designs the price-flow function. For simplicity, we neglect the transportation costs and relax the capacity constraints in this serial chain. We assume that site $m$ in the serial chain is subsidized by a certain amount of the exogenous financial source denoted by $t_m$ per unit of materials.

First, we consider site 1’s transaction with the source and site 2. The profit function of site 1 in price scenario $\omega \in \Omega_2$ is

\[ \Pi_{1\omega} = (q_{2\omega} - p_{1\omega} + t_1)(a + bp_{1\omega}) \quad \forall \omega \in \Omega_2, \quad (103) \]

where $p_{1\omega}$ is the response collection fee to be offered by site 1 to the source corresponding to acquisition price $q_{2\omega}$, $\omega \in \Omega_2$. The profit function $\Pi_{1\omega}$ is concave in
$p_{1\omega}$ whenever $b > 0$, so (103) is maximized when the first-order condition holds, i.e.,

when

$$p_{1\omega}^* = \frac{q_{2\omega} \cdot b - a + t_1 b}{2b} \quad \forall \omega \in \Omega_2. \quad (104)$$

Thus, (104) is the optimal collection fee paid by site 1 in price scenario $\omega \in \Omega_2$. The maximum flow amount collected in site 1 corresponding to price scenario $\omega \in \Omega_2$, $S_{\omega}^*$, can be obtained by substituting $p_{1\omega}^*$ into $S_{\omega}^* = a + b p_{1\omega}^*$.

$$S_{\omega}^* = a \frac{q_{2\omega} \cdot b - a + t_1 b}{2} \quad \forall \omega \in \Omega_2. \quad (105)$$

Site 1 is to design a robust price-flow relation, $x_{12} = \alpha_1 p_2$, used to contract with site 2; in other words, site 1 is to determine the robust coefficient, $\alpha_1$, such that the maximum loss due to uncertainty of acquisition price $p_2$ is minimized. The details of the robust optimization model for determining $\alpha_1$ are presented in (Hong et al. 2005b). We let $x_{12\omega}$ denote the material flow between sites 1 and 2 corresponding to price scenario $\omega \in \Omega_2$. Since $x_{12\omega} = \alpha_1 q_{2\omega}$ for all $\omega \in \Omega_2$, the material flows supplied to site 2 from site 1 for all price scenarios $\omega \in \Omega_2$ are restricted by the corresponding source amounts, $S_{\omega}^*$.

Therefore, we have

$$\alpha_1 q_{2\omega} \leq S_{\omega}^* \quad \forall \omega \in \Omega_2. \quad (106)$$
Suppose that $\omega_2 \in \Omega_2$ is the constrained price scenario in $\Omega_2$; that is
\[ \alpha_1 q_{2\omega_2} = S^*_n, \omega_2 \in \Omega_2. \]
Then by dividing (105) with $q_{2\omega_2}$, we obtain
\[ \alpha_i = \frac{q_{2\omega_2} \cdot b \cdot a + t_i \cdot b}{2q_{2\omega_2}}. \quad (107) \]

Site 1’s flow function used to contract with site 2 is $x_{12} = \alpha_1 p_2$ where $\alpha_1$ is shown in (107). Next, we consider site $m$’s transaction with its preceding site $m-1$ and subsequent site $m+1$. The flow site $m$ can supply is equal to the flow obtained from site $m-1$. Therefore, the profit function of site $m$ in price scenario $\omega \in \Omega_{m+1}$ is
\[ \Pi_{m,\omega} = \left(q_{m+1,\omega} - p_{m,\omega} + t_m\right) x_{m-1,m,\omega} \quad \forall \omega \in \Omega_{m+1} \quad (108) \]
where $x_{m-1,m,\omega} = \alpha_{m-1} \cdot p_{m,\omega}$. By similar reasoning as site 1, the optimal acquisition price to be offered by site $m$ in price scenario $\omega \in \Omega_{m+1}$ is
\[ p_{m,\omega}^* = \frac{q_{m+1,\omega} + t_m}{2} \quad \forall \omega \in \Omega_{m+1}. \quad (109) \]

The corresponding maximum amount flowing into site $m$ in price scenario $\omega \in \Omega_{m+1}$ is
\[ x_{m-1,m,\omega}^* = \frac{\alpha_{m-1} (q_{m+1,\omega} + t_m)}{2} \quad \forall \omega \in \Omega_{m+1}. \quad (110) \]

Again, we require the material flow supplied to site $m+1$ from site $m$ is constrained by the amount obtained from site $m-1$ to site $m$ for all price scenarios $\omega \in \Omega_{m+1}$.
\[ x_{m,m+1,\omega} \leq x_{m-1,m,\omega}^* \quad \forall \omega \in \Omega_{m+1} \quad (111) \]
where \( x_{m,m+1,\omega} = \alpha_m q_{m+1,\omega} \). Suppose that \( \omega_{m+1} \in \Omega_{m+1} \) is the constrained price scenario in \( \Omega_{m+1} \); that is \( \alpha_m q_{m+1,\omega_{m+1}} = x_{m-1,m,\omega_{m+1}} \), \( \omega_{m+1} \in \Omega_{m+1} \). Hence, the robust coefficient \( \alpha_m \) can be obtained by dividing (110) with \( q_{m+1,\omega_{m+1}} \). Also, the robust coefficients \( \alpha_m, m = 2, \ldots, M-1, \) can be generalized in (112). We note that (112) is a recursive function where \( \alpha_m \) is a function of \( \alpha_{m-1} \).

\[
\alpha_m = \alpha_{m-1} \frac{q_{m+1,\omega_{m+1}} + t_m}{2q_{m+1,\omega_{m+1}}} \quad \forall m = 2, \ldots, M-1. \tag{112}
\]

Finally, considering site \( M \), the selling price, \( P^{(Sc)}_M \), in the final market is a fixed given amount. The profit function of site \( M \) can be stated as (113) without a price scenario argument.

\[
\Pi_M = \left( P^{(Sc)}_M - P_M + t_M \right) x_{M-1,M} \tag{113}
\]

where \( x_{M-1,M} = \alpha_{M-1} p_M \). The optimal acquisition price \( p^*_M \) to be offered by site \( M \) is

\[
p^*_M = \frac{P^{(Sc)}_M + t_M}{2}. \tag{114}
\]

The resulting equilibrium material flow amount can be obtained by substituting the equilibrium acquisition price to be offered by site \( M, p^*_M \), into \( x_{M-1,M} = \alpha_{M-1} P_M \).

\[
x_{M-1,M}^* = \alpha_{M-1} \cdot \frac{P^{(Sc)}_M + t_M}{2}. \tag{115}
\]

By substituting the recursive function \( \alpha_{M-1} \) into (115), we obtain the resulting equilibrium material flow \( x_{M-1,M}^* \) expressed in (116). Due to the flow conservation assumption, the collection amount \( x_1 = \cdots = x_{M-1,M} \) in a \( M \)-tiered serial chain. Finally, we have
By examining (116), we observe several insights on the collection amount for a $M$-tiered serial RPS chain.

**Corollary 1** Among all price forecast ranges of sites $2, \ldots, M$, the collection amount is only influenced by the forecast, predicted by site 1, of the acquisition price to be offered in site 2 under the no-subsidizing scenario.

**Proof.** The parameters $t_1 = \cdots = t_m = 0$ under the no-subsidizing scenario. The material flow $x^*_{M-1,M}$ can be rewritten as

$$x^*_{M-1,M} = \frac{P^{(Sc)} + t_M}{2} \frac{q_{M,o_M} + t_{M-1}}{2q_{M,o_M}} \cdots \frac{q_{3,o_2} + t_2}{2q_{3,o_2}} \frac{q_{2,o_2} \cdot b + a + t_1 b}{2q_{2,o_2}}.$$

(116)

**Corollary 2** Except for the first tier, the increment of the collection amount due to a subsidy to an upstream tier is larger than the collection increment due to a subsidy to anyone of its downstream tiers for a $M$-tiered serial chain.

**Proof.** We assume that price forecast ranges for all tiers do not overlap; in turn,

$$q_{2,o_2} < q_{3,o_2} < \cdots < q_{M,o_M}.$$

(117)

We let $x^{(m)*}_{M-1,M}$ denote the material flow amount under the subsidizing scenario that only site $m$ is subsidized by an amount $t$. Here $t_i = 0 \; \forall i \neq m$ and $t_i = t$ when $i = m$. As a result of (117), the material flow amount $x^{(m)*}_{M-1,M}$ is larger than $x^{(m+1)*}_{M-1,M}$ for $m = 2, \ldots, M-1$.

Similarly, we conclude that the flow increment due to a subsidy to an upstream site is larger than the increment due to a subsidy to a downstream site. ■
Corollary 2 states that, except for the first tier, a subsidy to an upstream tier is more effective than a subsidy to anyone of its downstream tiers in terms of the total collection amount in a multi-tiered serial chain. By examining (116), the impact on the collection amount increment due to a subsidy to the first tier is affected by the parameters $a$ and $b$. In other words, if $a$ and $b$ ($a, b > 0$) are small enough, the increment due to a subsidy to the first tier may achieve the highest collection amount compared to other subsidizing scenarios where any of tiers $2, \ldots, M$ is subsidized by an identical unit amount. To verify the above claim, we modify the source supply function $S_i$, $i = 1, \ldots, 5$, in the previous three-tiered example. The detail modifications of the source supply functions are shown in Table 27 where $a_i = 0$ and $b_i = 1$ for $i = 1, \ldots, 5$.

**Table 27: The Details of the Source Supply Functions**

<table>
<thead>
<tr>
<th></th>
<th>$S_1$</th>
<th>$S_2$</th>
<th>$S_3$</th>
<th>$S_4$</th>
<th>$S_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Previous</td>
<td>$400 + 5p_i^{(Co)}$</td>
<td>$420 + 6p_2^{(Co)}$</td>
<td>$440 + 6p_3^{(Co)}$</td>
<td>$430 + 6p_4^{(Co)}$</td>
<td>$410 + 5p_5^{(Co)}$</td>
</tr>
<tr>
<td>New</td>
<td>$p_1^{(Co)}$</td>
<td>$p_2^{(Co)}$</td>
<td>$p_3^{(Co)}$</td>
<td>$p_4^{(Co)}$</td>
<td>$p_5^{(Co)}$</td>
</tr>
</tbody>
</table>

The numerical results (Table 28) using the new supply function setting indicate that subsidizing the first tier sites returns the highest total collection amount for this source supply setting, which implies the base flow, $a_i$, is assumed to be a low level and the collection amount rate is also low, $b_i$, in terms of the collection fee paid by the first tier sites.
This observation indicates that the best location of the subsidy is a function of the specific parameters that govern the flow and price behavior. From a policy perspective, this makes the most cost-effective subsidy scheme harder to design and implement.

The above discussions are based upon the framework of the decentralized problem setting. In this section, we also investigate the impact on the total amount of recycled items collected and the system net profit in the centralized problem setting due to the different subsidizing scenarios. The detail results of equilibrium collection fees paid by the collection sites in tier 1 and the source amounts of recycled items for the capacitated and uncapacitated cases are listed in Table 29 and Table 30.

### Table 28: The Source Flows in the Decentralized Scenarios for the Uncapacitated Case

<table>
<thead>
<tr>
<th>Source Flow</th>
<th>Original</th>
<th>(I) $5 all</th>
<th>(II) $15 tier 1</th>
<th>(III) $15 tier 2</th>
<th>(IV) $15 tier 3</th>
<th>(V) final market $15</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1^*$</td>
<td>31.4</td>
<td>37.2</td>
<td>37.7</td>
<td>37.2</td>
<td>34.8</td>
<td>34.8</td>
</tr>
<tr>
<td>$S_2^*$</td>
<td>31.5</td>
<td>37.3</td>
<td>37.9</td>
<td>37.3</td>
<td>34.8</td>
<td>34.8</td>
</tr>
<tr>
<td>$S_3^*$</td>
<td>29.9</td>
<td>35.6</td>
<td>36.2</td>
<td>35.5</td>
<td>33.2</td>
<td>33.2</td>
</tr>
<tr>
<td>$S_4^*$</td>
<td>31.0</td>
<td>36.7</td>
<td>37.3</td>
<td>36.7</td>
<td>34.2</td>
<td>34.2</td>
</tr>
<tr>
<td>$S_5^*$</td>
<td>31.8</td>
<td>37.6</td>
<td>38.1</td>
<td>37.7</td>
<td>35.1</td>
<td>35.1</td>
</tr>
<tr>
<td>Total</td>
<td>155.6</td>
<td>184.4</td>
<td>187.2</td>
<td>184.4</td>
<td>172.2</td>
<td>172.2</td>
</tr>
</tbody>
</table>
Table 29: The Details of Equilibrium Collection Fees and Source Flows in the Centralized Scenarios for the Capacitated Case

<table>
<thead>
<tr>
<th>Equilibrium Price</th>
<th>Original</th>
<th>(I) $5 all</th>
<th>(II) $15 tier 1</th>
<th>(III) $15 tier 2</th>
<th>(IV) $15 tier 3</th>
<th>(V) final market $15</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_1^{(Co)^*}$</td>
<td>7.36</td>
<td>7.36</td>
<td>7.36</td>
<td>7.36</td>
<td>7.36</td>
<td>7.36</td>
</tr>
<tr>
<td>$p_3^{(Co)^*}$</td>
<td>12.69</td>
<td>12.69</td>
<td>12.69</td>
<td>12.69</td>
<td>12.69</td>
<td>12.69</td>
</tr>
<tr>
<td>$p_4^{(Co)^*}$</td>
<td>12.52</td>
<td>12.52</td>
<td>12.52</td>
<td>12.52</td>
<td>12.52</td>
<td>12.52</td>
</tr>
<tr>
<td>$p_5^{(Co)^*}$</td>
<td>6.36</td>
<td>6.36</td>
<td>6.36</td>
<td>6.36</td>
<td>6.36</td>
<td>6.36</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Source Flow</th>
<th>Original</th>
<th>(I) $5 all</th>
<th>(II) $15 tier 1</th>
<th>(III) $15 tier 2</th>
<th>(IV) $15 tier 3</th>
<th>(V) final market $15</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1^*$</td>
<td>436.8</td>
<td>436.8</td>
<td>436.8</td>
<td>436.8</td>
<td>436.8</td>
<td>436.8</td>
</tr>
<tr>
<td>$S_2^*$</td>
<td>500.1</td>
<td>500.1</td>
<td>500.1</td>
<td>500.1</td>
<td>500.1</td>
<td>500.1</td>
</tr>
<tr>
<td>$S_3^*$</td>
<td>516.1</td>
<td>516.1</td>
<td>516.1</td>
<td>516.1</td>
<td>516.1</td>
<td>516.1</td>
</tr>
<tr>
<td>$S_4^*$</td>
<td>505.1</td>
<td>505.1</td>
<td>505.1</td>
<td>505.1</td>
<td>505.1</td>
<td>505.1</td>
</tr>
<tr>
<td>$S_5^*$</td>
<td>441.8</td>
<td>441.8</td>
<td>441.8</td>
<td>441.8</td>
<td>441.8</td>
<td>441.8</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>2400.0</td>
<td>2400.0</td>
<td>2400.0</td>
<td>2400.0</td>
<td>2400.0</td>
<td>2400.0</td>
</tr>
</tbody>
</table>

Table 30: The Details of Equilibrium Collection Fees and Source Flows in the Centralized Scenarios for the Uncapacitated Case

<table>
<thead>
<tr>
<th>Equilibrium Price</th>
<th>Original</th>
<th>(I) $5 all</th>
<th>(II) $15 tier 1</th>
<th>(III) $15 tier 2</th>
<th>(IV) $15 tier 3</th>
<th>(V) final market $15</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_1^{(Co)^*}$</td>
<td>28.50</td>
<td>36.00</td>
<td>36.00</td>
<td>36.00</td>
<td>36.00</td>
<td>36.00</td>
</tr>
<tr>
<td>$p_2^{(Co)^*}$</td>
<td>33.50</td>
<td>41.00</td>
<td>41.00</td>
<td>41.00</td>
<td>41.00</td>
<td>41.00</td>
</tr>
<tr>
<td>$p_3^{(Co)^*}$</td>
<td>30.83</td>
<td>38.33</td>
<td>38.33</td>
<td>38.33</td>
<td>38.33</td>
<td>38.33</td>
</tr>
<tr>
<td>$p_4^{(Co)^*}$</td>
<td>30.17</td>
<td>37.67</td>
<td>37.67</td>
<td>37.67</td>
<td>37.67</td>
<td>37.67</td>
</tr>
<tr>
<td>$p_5^{(Co)^*}$</td>
<td>26.00</td>
<td>33.50</td>
<td>33.50</td>
<td>33.50</td>
<td>33.50</td>
<td>33.50</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Source Flow</th>
<th>Original</th>
<th>(I) $5 all</th>
<th>(II) $15 tier 1</th>
<th>(III) $15 tier 2</th>
<th>(IV) $15 tier 3</th>
<th>(V) final market $15</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1^*$</td>
<td>542.5</td>
<td>580.0</td>
<td>580.0</td>
<td>580.0</td>
<td>580.0</td>
<td>580.0</td>
</tr>
<tr>
<td>$S_2^*$</td>
<td>621.0</td>
<td>666.0</td>
<td>666.0</td>
<td>666.0</td>
<td>666.0</td>
<td>666.0</td>
</tr>
<tr>
<td>$S_3^*$</td>
<td>625.0</td>
<td>670.0</td>
<td>670.0</td>
<td>670.0</td>
<td>670.0</td>
<td>670.0</td>
</tr>
<tr>
<td>$S_4^*$</td>
<td>611.0</td>
<td>656.0</td>
<td>656.0</td>
<td>656.0</td>
<td>656.0</td>
<td>656.0</td>
</tr>
<tr>
<td>$S_5^*$</td>
<td>540.0</td>
<td>577.5</td>
<td>577.5</td>
<td>577.5</td>
<td>577.5</td>
<td>577.5</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>2939.5</td>
<td>3149.5</td>
<td>3149.5</td>
<td>3149.5</td>
<td>3149.5</td>
<td>3149.5</td>
</tr>
</tbody>
</table>
In the capacitated case, the centralized setting does not result in any difference in collection fees or source flow amounts obtained from all scenarios since the solution of the original problem is constrained by the capacity of the system. The amount of recycled items collected is not changed even if the system is subsidized or the final market prices are increased. This also suggests that the government should take the system capacity into account while it legislates to subsidize the recycling network. Otherwise, subsidizing may lead to no impact on the total collection amount in a short term due to the capacity restriction.

The results for collection fees and source amounts of recycled items collected in the centralized system for the uncapacitated case are summarized in Table 30. Obviously, the collection fees paid by collection sites in tier 1 and source amounts are increased due to the exogenous subsidizing fund or the increased final market prices. Finally, we note that, in the centralized problem setting, there is no difference among any subsidizing scenarios either in the capacitated or uncapacitated cases since the results of scenarios (I) – (V) are the same in both cases.

In summary, these numerical results illustrate the variety of issues that can be examined using the centralized and decentralized models. One can evaluate, numerically, the effect of numerous changes to the data of a specific example, in terms of changes in the final market prices, transportation costs, or capacity limitations. We also investigate the impact on the individual entity or the system behavior due to different subsidizing scenarios. We demonstrate that the location of the subsidy may not be obvious, and the structure of the system is insufficient to determine the best location.
5.6 Summary and Extensions

There are considerable differences in the results of net profits and material flow allocations derived from the centralized and decentralized RPS models. This chapter demonstrates the comparison of the individual and the system behavior between the centralized and decentralized decision-making for a RPS network. We develop a centralized framework for the recycling network system where a single decision maker is acquainted with all system information including transportation capacities, processing capabilities, or associated sales prices of recycled materials, and the planner also has authority to determine system decision variables, which are the collection fees paid by the upstream boundary tier sites to acquire recycled items from sources and the material flow allocation throughout the entire network. The centralized RPS model presented in this chapter can be used to generate the compatible numerical results to compare the equilibrium solution obtained from the decentralized multi-tiered RPS model. As expected, the centralized solution is superior to the decentralized solution in terms of the net profit, especially in the capacitated case. It implies that one may overestimate the system profit if the decision maker utilizes the centralized approach to model a decentralized RPS network. The difference of the results between the centralized and decentralized solutions is mainly attributed to price uncertainty and double marginalization. Unfortunately, our model structure does not permit the separate examination of the roles of price uncertainty and double marginalization in the decentralized model.
We also investigate the individual and the system behavior of centralized and decentralized problem settings under several different scenarios where the government subsidizes the associated entities in a RPS or the final market prices are increased. Several insights are drawn from these numerical results. The specific numerical example suggests that placing the subsidy at the middle tier of the three tiered system leads to the highest recycling flows. However, arguments in a simpler multi-tiered, but one single entity at each tier, the model in the uncapacitated case indicates that the subsidy can be the most effective at the top tier based upon the source supply characteristics between the first tier entity and the source. Moreover, we also show that, except for the first tier, a subsidy to an upstream tier is more effective than a subsidy to anyone of its downstream tiers in terms of the total collection amount in a multi-tiered serial chain. Therefore, we conclude that the specific entity responses, and not only the structure, have to be considered for the subsidy and no simple formula is apparent for a multi-tiered network.

A key extension of this work is to incorporate additional types of related recycled items or conduct a more complicated network such that the materials may or may not move through all tiers sequentially. Another extension of the research in this chapter is to examine the individual or the system behavior of the semi-centralized or semi-decentralized network which may contain several independent recycling organizations or firms and several municipal collection sites or recyclers. Finally it is also interesting to try to separate the effects of price uncertainty and double marginalization and examine the individual effect on the results. Also, an interesting analysis can be performed on the subsidy effect in the capacitated case.
CHAPTER 6 CONCLUSIONS AND EXTENSIONS

6.1 Summary and Conclusions

Most of the previous research on RPS design and operation construct a centralized model of the system. It is assumed that one planner has the requisite information about all the participating entities and has the authority to determine the decision variables of participating entities such that the system performance is optimized. Unfortunately, this assumption rarely holds in real-world recycling systems, which are, in general, composed of several private entities. The system performance obtained from a centralized model leads to an optimistic view that may not represent the actual way the system operates.

This research presented in this thesis captures properties of self-interested entities in a RPS, and provides a decentralized framework for the decision-making process and protocol design for a RPS constituted by several privately-owned entities in different tiers within the network. In particular, the question of how the entities coordinate between tiers is of central concern, and the role of uncertainty in price on the coordination is significant. We present an algorithm to solve for an equilibrium of price and flows where, when the coordination function between tiers is fixed, no entity is willing to change its solution within the decentralized RPS network.

This dissertation includes three major parts. In the first part, Chapter 2, we develop an initial prototype model for a RPS with two tiers of upstream entities (collectors) and downstream entities (processors) where downstream entities acquire recycled items from upstream entities. We focus on the price and material flow decisions where the material
flow allocation mechanism is determined by upstream entities and the acquisition prices of material flows transacted between upstream and downstream entities are determined by downstream entities. We apply a min-max robust optimization model approach for each of the independent upstream entities to generate material flow allocation mechanism equations used to contract with downstream sites. Downstream entities compete for material flows from upstream entities. The iterated relaxation algorithm or the combined Karush-Kuhn-Tucker (KKT) approach is used to solve for the Nash equilibrium acquisition prices between upstream and downstream entities. We address the fundamental difference between these two algorithms even though both of them reach the identical equilibrium price solution vector. The relaxation algorithm itself matches the idea of a decentralized decision-making process where every downstream entity can access all entities’ previous price actions and determines its next best move for its price decision, but the downstream entity is not acquainted with other entities’ next price decisions. Alternatively, in the combined KKT approach, the downstream entities need to pass their optimality conditions to some invisible hand in the system, but this requires information sharing among independent entities.

In the second part, Chapters 3 and 4, we further explore a decentralized multi-tiered RPS where recycled items flow from the top tier to the downstream tier, but acquisition prices are set by the downstream tier back to the upstream tier. The upstream tier specifies the material flow allocation parameters used to contract with its subsequent downstream tier and the downstream tier determines the equilibrium prices to acquire the recycled items from its preceding upstream tier. In Chapter 4, by modifying the flow function format, we extend the model presented in Chapter 3 to the multi-tiered RPS
network with price correlated commodities where the material flow amount is also influenced by the acquisition price of the other type of commodities.

The third part, Chapter 5, studies the behavior of centralized and decentralized decision-making setups for reverse production systems. We develop a compatible centralized RPS model where system decision variables are the material flow amount within the entire network as well as the optimal collection fee required to acquire exogenous recycled items. We demonstrate the comparison of the centralized and decentralized decision-making for a RPS network and numerically show the initial hypothesis that the centralized system performance serves as an upper bound on the system net profit of the decentralized model. This comparison also indicates a possible bias on decisions and the system performance due to the application of a centralized model on a decentralized system.

The thesis also investigates outcomes of the centralized and decentralized models under several different scenarios when the government subsidizes the associated entities in different schemes for a RPS, or the final market prices are increased. Several insights are drawn from these numerical results. The specific numerical example suggests that placing the subsidy at the middle tier of the three tiered system leads to the highest recycling flows. We mathematically show that, except for the first tier, a subsidy to an upstream tier is more effective than a subsidy to anyone of its downstream tiers in terms of the total collection amount in a multi-tiered serial chain; however, the subsidy can be the most effective at the top tier based upon the source supply characteristics between the first tier entity and the source. Therefore, we conclude that the specific entity responses, and not only the structure, have to be considered for the subsidy and no simple formula is
apparent for a multi-tiered network. This insight may provide a valuable suggestion for policy making when governments consider subsidies for the recycling network in order to increase the amount of recycled items handled by the system.

Overall, this research presents a framework to view a recycling network in a decentralized manner, which provides a more plausible solution to predict the behaviors of individuals in reverse production systems. In addition, most decentralized modeling research solves for the equilibrium solution by combining the equilibrium conditions of all associated participants. This approach may violate the lack of willingness to share information in a decentralized system, even though the approach embeds a self-interested perspective for all participants. This thesis, unlike other decentralized setting literature, explicitly provides a decentralized coordination mechanism, which reflects a more realistic view of real decision-making for an independent participant in a system.

6.2 Extensions and Future Research Directions

Results from this dissertation raise new questions and several potential directions of future research. The first extension in terms of the whole framework is to investigate a semi-centralized (semi-decentralized) system, which may include several independent recycling organizations or firms and several coalitions composed of municipal entities.

Other extensions concern model formulation. Price uncertainty plays an important role in the flow function determination. We assume that associated upstream entities design the material flow allocation mechanism by using a robust optimization approach,
but there are other criteria that may be used by different upstream entities. Different modeling approaches on the flow function determination will lead to different results.

In this research, upstream entities independently determine the material flow allocation mechanism (price-flow contracts) and communicate them to their downstream entities. We assume that upstream entities are unable to change the price-flow contracts after communication. In other words, upstream entities decide the contracts before any information is revealed. An interesting extension is to relax this assumption. The following two-tiered example, shown in Figure 20, illustrates this potential extension.

\[ S_1 = 350 - 5p_1^{(Co)} \]
\[ S_2 = 350 - 5p_2^{(Co)} \]

**Figure 20**: A Two-tiered Example

The transportation costs per unit are shown on the arcs and the source response functions are listed in Figure 20. The final market prices for downstream sites 1 and 2 are $110 and $110. The upstream sites yield the following material flow allocation mechanism using the model proposed in this research:

---

\(^6\) Special thanks to Dr. Julie Swann for pointing out this extension.
\[ x_{11}^{(Tr)} = 3.21(p_1 - 10) - .87(p_2 - 12), \]
\[ x_{12}^{(Tr)} = -.38(p_1 - 10) + 3.11(p_2 - 12), \]
\[ x_{21}^{(Tr)} = 3.11(p_1 - 12) - .38(p_2 - 10), \text{ and} \]
\[ x_{22}^{(Tr)} = -.87(p_1 - 12) + 3.21(p_2 - 10). \]

We relax the assumption that the upstream site is unable to modify the material flow allocation mechanism after the acquisition prices are revealed, and intentionally perturb the robust flow functions associated with upstream site 1 \((x_{11}^{(Tr)} \text{ and } x_{12}^{(Tr)})\) by simultaneously changing the \(\alpha\) values of coefficients in \(x_{11}^{(Tr)}\) and \(x_{12}^{(Tr)}\) (3.21, -.87, -.38, and 3.11 respectively) while upstream site 2’s flow functions remain the same. The values used to change the upstream site 1’s \(\alpha\) values in different perturbations are -.1, +.1, +.2, and +.3. Figure 21 shows the effect on profits of upstream sites 1 and 2 respectively due to the perturbation of upstream site 1’s flow functions. Upstream site 1 may be better off (perturbation points of .1 and .2 increments) by the unilateral change in its flow function determination after the price information is revealed when upstream site 2’s flow function remains the same. An extension is to allow all upstream entities to modify the price-flow contracts, i.e., the value of \(\alpha's\), after the price information is revealed until there are no incentives to change the contracts for the upstream entities. Also, there is a one-to-one correspondence between the material flow amounts and the \(\alpha's\). We may further explore the algorithm to determine the flow function directly without using \(\alpha's\) arguments.
As discussed in Chapter 3, we follow the conservation rule of flow balance. In other words, entities do not collect recycled items in excess of the amount they need to supply to the next tier. A future research opportunity is modifying the model to allow entities being capable to collect recycled items more than the amount needed if it is profitable to do so. However, issues of the storage cost and multiple planning periods emerge in this extension. This opens up the whole question of multi-period games between the entities, which is not explored in this research.

The current model allows recycled items moving through different tiers sequentially or directly flowing through another recycling stream in a predefined ratio. A future research topic is to model the situation where it is allowed for the material to be transported across two or more tiers within the network. Other interesting future
extensions may include the modification of flow function formats or source response functions and a more efficient approach to explore the possible forecast price scenarios.

This research has been motivated by the current U.S. electronics recycling system where a system is composed of several independent privately-owned firms and organizations. An extension of this research would be to generalize the model to other reverse production system sectors: automobile, office furniture, tires, or toner cartridges. In order to carry out this generalization, the features of the specific recycling industry will need to be extracted and incorporated into the model. For example, the automobile and office furniture remanufacturing industries would have different concerns based on the technological complexity of the product and the maturity of the existing infrastructure for the resale of cars and furniture where the decision to reprocess the item is not fixed but may be dependent on its condition. The model proposed in this research therefore may require further refinement to account for the different characteristics in specific reverse production systems.

Chapter 5 provides a comparison of the results between a centralized and our decentralized modeling approaches. The effect of price uncertainty is confounded with the effect of double marginalization in the decentralized modeling approach. In future research, we may try to separate these effects and examine the impact on the result caused by a single effect. We also discuss the effect of subsidy on the total collection amount in Chapter 5. An interesting extension regarding the subsidy issue is to explore the relation between the subsidy amount and total collection amount and further to find
the “social value” of the optimum subsidy level for the society as a whole\textsuperscript{7}. This raises the important question of the optimum subsidy across many sectors of recycling as opposed to the optimal distribution of the subsidy across the supply chain.

An issue we may further investigate is the benchmark of monopoly and competitive systems in these markets\textsuperscript{8}. We conduct an initial analysis shown in Figure 22 where the left hand side is the competitive case (two upstream sites and two downstream sites), and the right hand side is a monopoly downstream tier case (two upstream sites and one monopoly downstream site). In this particular numerical comparison, the profit of the downstream site in the competitive case ($12,287) is lower than that in the monopoly case ($14,203), and the downstream site in the monopoly case has less incentive to offer a higher acquisition price to its upstream sites. A more formal discussion of competitive and monopoly systems requires further investigation.

\[
S_1 = 350 - 5p_1^{(Co)} \quad S_2 = 350 - 5p_1^{(Co)} \quad S_1 = 350 - 5p_2^{(Co)} \quad S_2 = 350 - 5p_2^{(Co)}
\]

**Figure 22**: The Competitive and Monopoly Cases

\textsuperscript{7} This is a very complex and challenging social engineering problem. Special thanks to Dr. Paul Griffin for raising this question.

\textsuperscript{8} Special thanks to Dr. Vivek Ghosal for pointing out this extension.
Finally, an issue or a question emerges in this research: how closely does a self-interested participant behave in the manner we present in this research? This issue has not been explored empirically by us, or in the literature. It is clear that many collectors follow the model of not knowing the prices they will get for their raw input, and have to negotiate the prices with the tier they supply. The proposal in this thesis is to make their negotiation take the form of the flow functions presented to each member of the next tier. The form of this negotiation is something that could be set by an “invisible hand,” such as the government, particularly if it could be shown that the results have a higher social benefit than by leaving them free, or if the government is providing the service at one particular tier.
## APPENDIX: NOTATION SUMMARY

### Table 31: The Summary of Notations in Chapter 2

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m )</td>
<td>The number of sites in the upstream tier.</td>
</tr>
<tr>
<td>( n )</td>
<td>The number of sites in the downstream tier.</td>
</tr>
<tr>
<td>( \omega )</td>
<td>Price scenario index.</td>
</tr>
<tr>
<td>( P_{j}^{\omega} )</td>
<td>The unit material price downstream site ( j ) willing to offer in price scenario ( \omega ).</td>
</tr>
<tr>
<td>( v_{ij}^{(Tr)} )</td>
<td>The unit transportation cost from site ( i ) to ( j ).</td>
</tr>
<tr>
<td>( x_{ij}^{(Tr)} )</td>
<td>The material flow from upstream site ( i ) to downstream site ( j ) for scenario ( \omega ).</td>
</tr>
<tr>
<td>( \alpha_{ij} )</td>
<td>The robust variable describing the flow from upstream site ( i ) to downstream site ( j ) affected by downstream site ( j' ).</td>
</tr>
<tr>
<td>( S_{i}^{\omega} )</td>
<td>The potential maximum flow amount collected in upstream site ( i ) corresponding to price scenario ( \omega ).</td>
</tr>
<tr>
<td>( P_{i}^{(Co)} )</td>
<td>The collection fee charged by site ( i ) corresponding to price scenario ( \omega ).</td>
</tr>
<tr>
<td>( \Pi_{i}^{\omega} )</td>
<td>The potential profit of upstream site ( i ) in price scenario ( \omega ).</td>
</tr>
<tr>
<td>( O_{i}^{\omega} )</td>
<td>The optimal objective value of upstream site ( i ) for price scenario ( \omega ).</td>
</tr>
<tr>
<td>( C_{ij}^{(Tr)} )</td>
<td>The shipment capacity between upstream site ( i ) and downstream site ( j ).</td>
</tr>
<tr>
<td>( R_{i}^{\omega} )</td>
<td>The robust objective function value of site ( i ) in price scenario ( \omega ) using the common set of robust coefficients.</td>
</tr>
<tr>
<td>( v_{jp}^{(Pr)} )</td>
<td>Processing cost per standard unit for process ( p ) at downstream site ( j ).</td>
</tr>
<tr>
<td>( P_{lc} )</td>
<td>Selling price offered per standard unit of material ( l ) from customer ( c ).</td>
</tr>
<tr>
<td>( \rho_{lp}^{1} )</td>
<td>Proportion of material type ( l ) produced by process ( p ).</td>
</tr>
<tr>
<td>( \rho_{lp} )</td>
<td>Proportion of material type ( l ) consumed by process ( p ).</td>
</tr>
<tr>
<td>( C_{jp}^{(Pr)} )</td>
<td>Maximum amount of material that process ( p ) can produce at downstream site ( j ).</td>
</tr>
<tr>
<td>( C_{jc}^{(Tr)} )</td>
<td>Maximum amount of material that can be shipped from downstream site ( j ) to customer site ( c ).</td>
</tr>
<tr>
<td>( x_{jc}^{(Tr)} )</td>
<td>Amount of material shipped from downstream site ( j ) to customer site ( c ) of material type ( l ).</td>
</tr>
<tr>
<td>( x_{jp}^{(Pr)} )</td>
<td>Amount of material processed by process ( p ) at downstream site ( j ).</td>
</tr>
<tr>
<td>( \phi_{j} )</td>
<td>The payoff (or objective) function of downstream site ( j ).</td>
</tr>
<tr>
<td>( \alpha_{j} )</td>
<td>The coefficient term with ( p_{j} ) in the aggregate flow function of downstream site ( j ).</td>
</tr>
<tr>
<td>( C_{j} )</td>
<td>The constant in the aggregate flow function of downstream site ( j ).</td>
</tr>
<tr>
<td>( s )</td>
<td>The iterative step of the relaxation algorithm.</td>
</tr>
<tr>
<td>( Z(p) )</td>
<td>The optimum response function to price ( p ).</td>
</tr>
</tbody>
</table>
$\beta_s$ : The weighted factor at iteration $s$.

**Table 32: The Summary of Notations in Chapter 3**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>The tier index.</td>
</tr>
<tr>
<td>$N_m$</td>
<td>The number of sites in tier $m$.</td>
</tr>
<tr>
<td>$I_m$</td>
<td>The set of sites in tier $m$.</td>
</tr>
<tr>
<td>$\Omega_m^i$</td>
<td>The set of all specified price scenarios of tier $m$ predicted by site $i$.</td>
</tr>
<tr>
<td>$q_{j\omega}^{(i)}$</td>
<td>The unit material price that site $j$ is willing to offer in price scenario $\omega$ predicted by site $i$.</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>The number of selected discrete price points.</td>
</tr>
<tr>
<td>$Q^{(i)}$</td>
<td>The matrix representing the considered acquisition price space predicted by site $i$.</td>
</tr>
<tr>
<td>$V_{ij}^{(Tr)}$</td>
<td>The unit transportation cost from upstream site $i$ to downstream site $j$ .</td>
</tr>
<tr>
<td>$\chi_{ij\omega}^{(Tr)}$</td>
<td>The material flow amount from upstream site $i$ to downstream site $j$ for price scenario $\omega$ .</td>
</tr>
<tr>
<td>$\alpha_{ij\omega}^{*}$</td>
<td>The robust variable describing the flow from upstream site $i$ to downstream site $j$ affected by downstream site $j^*$.</td>
</tr>
<tr>
<td>$p_{i\omega}$</td>
<td>The collection fee paid by site $i$ corresponding price scenario $\omega$ .</td>
</tr>
<tr>
<td>$S_{i\omega}$</td>
<td>The potential maximum flow amount collected in site $i$ corresponding to price scenario $\omega$ .</td>
</tr>
<tr>
<td>$p_i^{\omega}$</td>
<td>The matrix representing the optimal collection fees of site $i$.</td>
</tr>
<tr>
<td>$S_i^*$</td>
<td>The matrix representing corresponding potential maximum flows of site $i$</td>
</tr>
<tr>
<td>$O_{i\omega}^{*}$</td>
<td>The optimal objective value of site $i$ for price scenario $\omega$ .</td>
</tr>
<tr>
<td>$C_{ij}^{(Tr)}$</td>
<td>The shipment capacity between site $i$ and downstream site $j$ .</td>
</tr>
<tr>
<td>$C_{ij}^{(Pr)}$</td>
<td>The processing capacity in site $i \in I_1$ .</td>
</tr>
<tr>
<td>$R_{i\omega}^{*}$</td>
<td>The robust objective function value of site $i$ in price scenario $\omega$ using the common set of robust coefficients.</td>
</tr>
<tr>
<td>$\delta_i^{\omega}$</td>
<td>The realized minimum maximum deviation of site $i$ obtained from the realized min-max robust optimization model.</td>
</tr>
<tr>
<td>$p_j$</td>
<td>The acquisition price offered by downstream site $j$ .</td>
</tr>
<tr>
<td>$V_{m-1,m}^{(Tr)}$</td>
<td>The matrix representing the input parameters of transportation costs, $V_{ij}^{(Tr)}$, where site $i$ is in tier $m-1$ and site $j$ is in tier $m$.</td>
</tr>
</tbody>
</table>
The matrix representing the input parameters of capacities, $C_{ij}^{(Tr)}$, where site $i$ is in tier $m-1$ and site $j$ is in tier $m$.

The matrix representing the flow function coefficients emanating from site $i$.

The matrix representing the flow function coefficients flowing to site $j$.

The mapping from the input parameter space to the solution space of flow function coefficients.

The mapping from the input parameter space to the solution space of potential maximum flows.

The total flows shipped to site $j$ for price scenario $\omega$.

The coefficient term with $p_j$ in the aggregate flow function of downstream site $j$.

The constant in the aggregate flow function of downstream site $j$.

The total flows shipped to site $k$.

Transportation cost per standard unit per distance from downstream boundary tier site $k$ to customer site $c \in C$.

Processing cost per standard unit for process $p$ at downstream site boundary tier $k$.

Selling price offered per standard unit of material $l \in L$ from customer $c$.

Proportion of material type $l$ produced by process $p$.

Proportion of material type $l$ consumed by process $p$.

Maximum amount of material that process $p$ can process at downstream boundary tier site $k$.

Maximum amount of material that can be shipped from downstream boundary tier site $k$ to customer site $c$.

Amount of material shipped from downstream boundary tier site $k$ to customer site $c$ of material type $l$.

Amount of material processed by process $p$ at downstream boundary tier site $k$.

The weighted average of the unit revenue of the transactions between site $j$ and the sites in the next tier.

### Table 33: The Summary of Notations in Chapter 4

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>The tier index.</td>
</tr>
<tr>
<td>$x_{ij}^{(Tr)}$</td>
<td>The material flow amount from upstream site $i$ to downstream site $j$.</td>
</tr>
<tr>
<td>$I_m$</td>
<td>The set of the sites in tier $m$.</td>
</tr>
<tr>
<td>$p_j$</td>
<td>The acquisition price to be offered by site $j$.</td>
</tr>
<tr>
<td>$V_{ij}^{(Tr)}$</td>
<td>The unit transportation cost shipped from site $i$ to site $j$.</td>
</tr>
</tbody>
</table>
\[ \alpha_{ij}^{*} : \text{The robust variable describing the flow from upstream site } i \text{ to downstream site } j \text{ affected by downstream site } j'. \]
\[ \Lambda_m : \text{The set of associated commodities handled in tier } m. \]
\[ \alpha_{ij}^{(Tr)} : \text{The material flow amount of commodity } u \text{ from site } i \text{ to } j. \]
\[ p_{jv} : \text{The acquisition price of commodity } v \text{ to be offered by site } j. \]
\[ \alpha_{ij}^{uv} : \text{The coefficients of material flow determination for commodity } u \text{ from site } i \text{ to } j \text{ affected by site } j' \text{ and commodity } v. \]
\[ q_{ju}^{(i)} : \text{The lower bound of the acquisition price of commodity } u \text{ offered by site } j \text{ forecasted by site } i. \]
\[ q_{ju}^{-}^{(i)} : \text{The upper bound of the acquisition price of commodity } u \text{ offered by site } j \text{ forecasted by site } i. \]
\[ B_i : \text{The final set of candidate sites for upstream site } i. \]

**Table 34:** The Summary of Notations in Chapter 5

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m )</td>
<td>The tier index.</td>
</tr>
<tr>
<td>( I_m )</td>
<td>The set of the sites in tier ( m ).</td>
</tr>
<tr>
<td>( S_i )</td>
<td>The collection amount in upstream boundary tier site ( i ).</td>
</tr>
<tr>
<td>( p_i^{(Co)} )</td>
<td>The collection fee per unit of the recycled item paid by site ( i ).</td>
</tr>
<tr>
<td>( p_i^{(Sc)} )</td>
<td>The selling price obtained in downstream boundary tier site ( k ).</td>
</tr>
<tr>
<td>( \alpha_{ij}^{(Tr)} )</td>
<td>The material flow from site ( i ) to site ( j ).</td>
</tr>
<tr>
<td>( \alpha_{ij}^{(Tr)} )</td>
<td>The aggregate flow shipped to downstream boundary tier site ( k ).</td>
</tr>
<tr>
<td>( V_{ij}^{(Tr)} )</td>
<td>Transportation cost per standard unit from site ( i ) to site ( j ).</td>
</tr>
<tr>
<td>( C_{ij}^{(Tr)} )</td>
<td>Maximum amount of material that can be shipped from site ( i ) to site ( j ).</td>
</tr>
<tr>
<td>( C_i^{(Pr)} )</td>
<td>Maximum amount of material that can be processed in site ( i ).</td>
</tr>
<tr>
<td>( p_j^* )</td>
<td>The equilibrium acquisition price to be offered by site ( j ).</td>
</tr>
<tr>
<td>( S )</td>
<td>The collection amount in site 1.</td>
</tr>
<tr>
<td>( p_i )</td>
<td>The collection fee paid by site 1 to acquire recycled items from the exogenous source.</td>
</tr>
<tr>
<td>( p_i^{(Sc)} )</td>
<td>The final market price.</td>
</tr>
<tr>
<td>( p_m )</td>
<td>The acquisition price to be offered by site ( m ).</td>
</tr>
<tr>
<td>( q_m )</td>
<td>The lower bound on the acquisition price to be offered by site ( m ).</td>
</tr>
<tr>
<td>( q_m )</td>
<td>The upper bound on the acquisition price to be offered by site ( m ).</td>
</tr>
<tr>
<td>( \Omega_m )</td>
<td>The set of all specified price scenarios of site ( m ).</td>
</tr>
<tr>
<td>( q_{\omega m} )</td>
<td>A particular price scenario in ( \Omega_m ).</td>
</tr>
<tr>
<td>Variable</td>
<td>Description</td>
</tr>
<tr>
<td>---------------</td>
<td>-----------------------------------------------------------------------------</td>
</tr>
<tr>
<td>( \alpha_m )</td>
<td>The decision variable of site ( m ) for describing how site ( m ) responds to the acquired price to be offered in the next tier.</td>
</tr>
<tr>
<td>( x_{m,m+1} )</td>
<td>The material flow amount from site ( m ) to site ( m+1 ).</td>
</tr>
<tr>
<td>( t_m )</td>
<td>The subsidy amount to tier ( m ).</td>
</tr>
<tr>
<td>( \Pi_{m,\omega} )</td>
<td>The profit function of site ( m ) corresponding price scenario ( \omega ).</td>
</tr>
<tr>
<td>( p_{1,\omega} )</td>
<td>The response collection fee to be offered by site 1 to the source corresponding price scenario ( \omega ).</td>
</tr>
<tr>
<td>( x_{m,m+1,\omega} )</td>
<td>The material flow amount from site ( m ) to site ( m+1 ) corresponding price scenario ( \omega ).</td>
</tr>
</tbody>
</table>
REFERENCES


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I-Hsuan Ethan Hong was born in Taichung, Taiwan on February 12, 1975. He received his B.S. in bioenvironmental systems engineering (formerly agricultural engineering) from National Taiwan University, Taipei, Taiwan in 1997. After graduation of the undergraduate program, he decided to study in the area of industrial engineering and obtained the M.S. degree in industrial engineering from National Taiwan University in 1999. From 1999 to 2001, he served as a second-lieutenant in the army for vehicle dispatching and ground transportation planning work. In August 2001, he joined Georgia Institute of Technology continuing his Ph.D. program in industrial and systems engineering. He received the M.S. degree in industrial engineering from Georgia Institute of Technology, Atlanta, Georgia, USA in May, 2003. His primary area of interest is the modeling and analysis of practical logistics problems and competition models, which led to a minor in Economics as a part of his Ph.D. requirements. In December 2005, he received his doctoral degree in industrial and systems engineering from Georgia Institute of Technology, Atlanta, Georgia, USA. His career plans are to seek an industrial job in the Asia-pacific region, especially in the electronics industry, and he may plan to go back to the academic area after a couple of years in the industry. Apart from his academic interests, he likes to play tennis and basketball, and enjoys swimming and skiing.