3D Metrology

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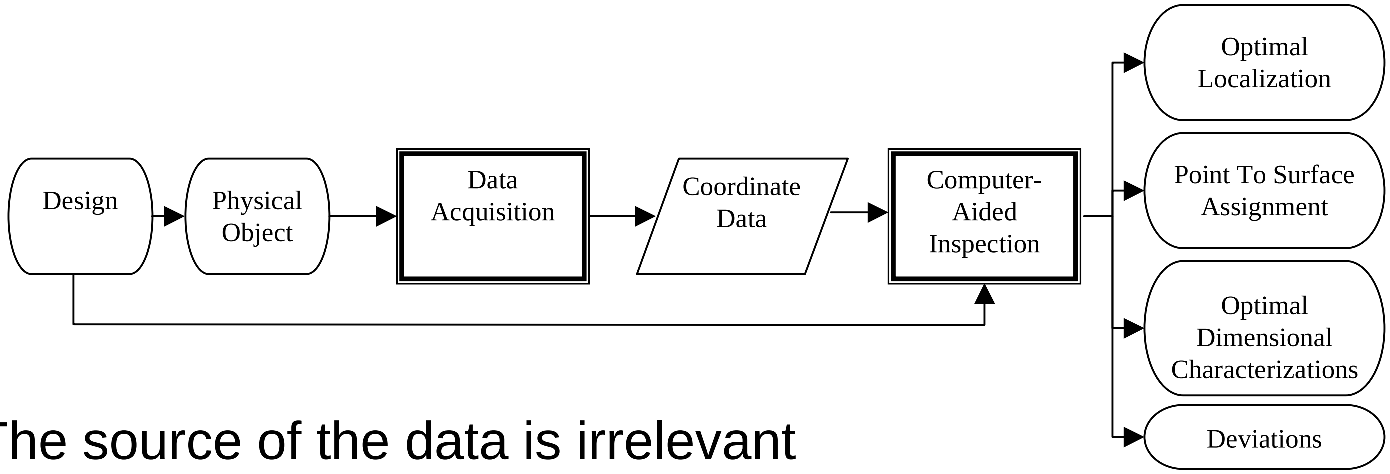
Three Dimensional Metrology

- Evolution of 1 and 2 dimensional metrology techniques
- Goal: Make inferences on three dimensional characteristics of manufactured artifacts
- Separates the measurement task into two parts
  - Data acquisition (Hardware)
  - Data analysis (Software)

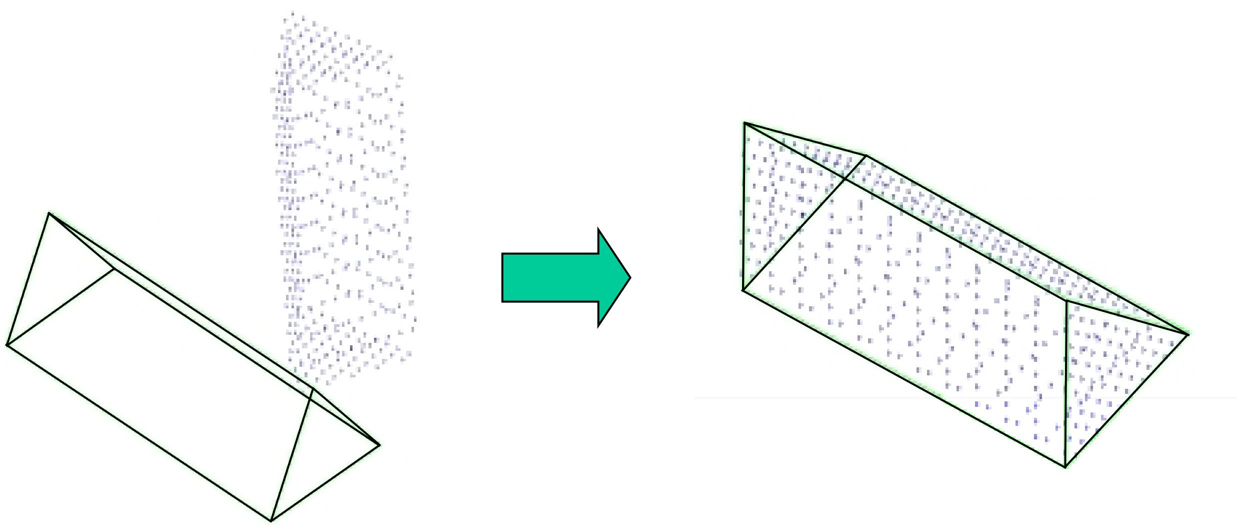
3D Metrology



- The source of the data is irrelevant
- Localization transformation facilitates dimensional characterization
- Need the right combination of mechanical engineering, computer science and manufacturing knowledge

Data Localization

- Foundation of 3D data analysis
- Must be performed before any other inferences can be made



Elements of Localization

- Nominal geometry
- Measurement data
- Measure of separation between model and data
- Minimization procedure

Least Squares Minimization Problem

- Goal: Minimize the measure of model-data separation.
- Mechanism to minimize the separation is a rigid body transformation

$$\begin{bmatrix} P^*_{i,x} \\ P^*_{i,y} \\ P^*_{i,z} \end{bmatrix} = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\phi & 0 & \sin\phi \\ 0 & 1 & 0 \\ -\sin\phi & 0 & \cos\phi \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} P_{i,x} \\ P_{i,y} \\ P_{i,z} \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

- Find the vector of rigid body transformation parameters, $<\theta, \phi, \psi, t_x, t_y, t_z>$, that minimize the separation.

$$\min_{\langle \phi, \phi\psi, t_x, t_y, t_z \rangle} \left\{ \sum_{i \in D} \min_{j \in G} \left[e^2_{i,j} \right] \right\}$$

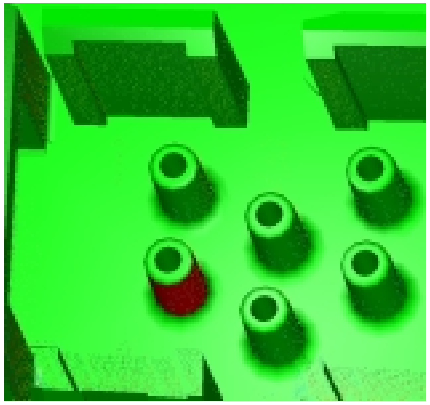
Geometric Parameter Fitting

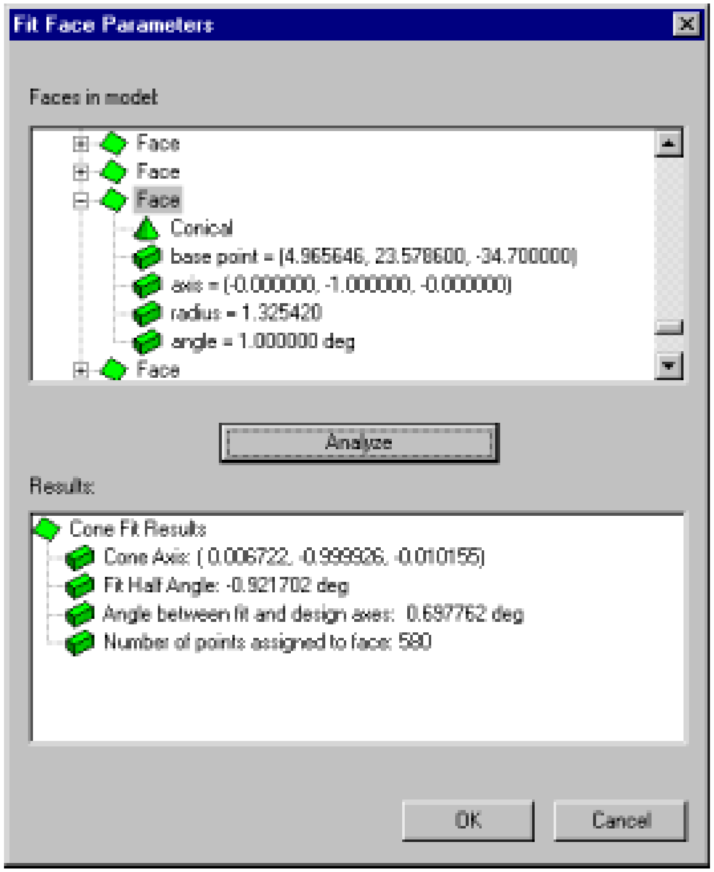
- Parameters of analytics (plane, sphere, tori, cylinder, cone)
- Metrologists are usually more interested in these shapes
- Requires localized data and point to surface assignment
- Also a minimization problem
- Geometric parameters are included as variable in addition to transformation
- Torus minimization statement

$$\min_{(\theta,\phi,\varphi,t_x,t_y,t_z,r_1,r_2)}\left[\sum_{p_i^*\in P^*}\left(\left\|G(r_1,r_2)-p_i^*\left(\theta,\phi,\varphi,t_x,t_y,t_z\right)\right\|\right)^2\right]$$

Geometric Parameter Fitting

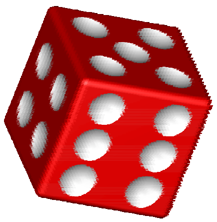
- For Example





Avenues for Speed Improvement

- First order information used extensively
  - Finite difference derivatives common
  - Analytic partials are much faster
- Point To Surface Assignment
  - Volume sweep method
- Assignment Trust Region
  - Reduces redundant reassignment
- Preliminary Results



Part	New	Imageware	# Analytics	# Points
Coffee Cup	1:09	44:45	14	86587
Dice	0:38	13:05	47	10492

Summary

- New algorithms provide capability to analyze large sets of data provided my modern measurement equipment
- 3D analysis provides more information about manufactured artifacts than traditional 1 & 2D methods
- Methods developed are applied to surfaces with closed form point to surface deviation equations, but can be applied to spline surfaces with additional computational cost