DIGITAL COMPUTATION OF
POWER SYSTEM STABILITY

A THESIS
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the Faculty of the Graduate Division

by

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Approved:

______________ Thesis Advisor

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LIST OF SYMBOLS

δ_k	Rotor angle of the K^{th} machine with respect to the synchronous rotating axes in degrees.
E_k	Equivalent voltage back of the transient reactance for the K^{th} machine in per unit.
Z_{jk}	Driving point (when $j = k$) impedance or the transfer (when $j \neq k$) impedance in per unit. Driving point impedance has positive sign and transfer impedance has negative sign in the digital program.
θ_{jk}	Phase angle of Z_{jk} in degrees.
M_k	Polar moment of inertia for the K^{th} machine in per unit.
t	Time in seconds.
Δt	Time increment in seconds.
P_{mk}	Prime mover output of the K^{th} machine in per unit.
P_{ek}	Electrical power developed by the K^{th} machine in per unit.
P_{ak}	Accelerating power for the K^{th} machine in per unit.
$\Delta \delta_n$	Change in the rotor angle for the n^{th} increment in degrees.
$\Delta \delta_{n-1}$	Change in the rotor angle for the $n-1^{th}$ increment in degrees.
K	Accelerating constant in per unit.
B-3	Number in digital program indicating the number of disturbances. Equal to one except in case of no disturbance, then zero.
B-4	Number in digital program indicating one less than the number of time intervals before occurrence of a second disturbance.
B-5	Number in digital program indicating occurrence of third disturbance. Equal to zero except in case of two disturbances, then one.
B-6	Number in digital program indicating one less than the number of time intervals between second and third disturbance.

B-7	Number in digital program indicating one less than the number of time intervals after last disturbance for which the solution is desired.
B-8	Number in digital program indicating one less than the number of machines in the stability study.

ABSTRACT

One of the ever present problems in the electric utility industry is the steady-state and transient stability analysis of power transmission systems. The steady-state analysis can be performed easily on the a-c network calculator but the transient analysis is more difficult and time-consuming. Power system engineers have long felt a need of less time-consuming methods of analysis than are now available on the a-c network calculator.

This thesis describes the application of the digital computer to the solution of transient stability studies for systems with up to seventeen machines.

For a system having N synchronous machines, the solution of the transient stability problem is basically one of solving N simultaneous, non-linear, differential equations. In the present method of solution, the network calculator is used to determine the electrical power output of each machine and these values are then assumed constant for a small finite time and the differential equations are solved for the change in the rotor angles. The new angles are then applied to the system represented on the network calculator and new electrical power outputs are obtained. This process is repeated for the number of time intervals necessary to establish the stability or nonstability of the system.

The digital computer program presented in this thesis is based on this same repetitive process except the electrical power output

CHAPTER I

INTRODUCTION

One of the ever present problems in the electric utility industry is the choice of power transmission levels so that the power system is operated with efficiency and with a high degree of reliability. In many instances, these two desirable characteristics are not compatible in that an increase in power level may seriously reduce the reliability of power continuity. One of the important limitations on reliability with high power level is the possibility of a loss of stability. Power system stability refers to the ability of the synchronous machines in the system to remain in synchronism with each other during normal conditions or following a disturbance on the system.

A system may be unstable because the synchronous machines cannot supply the loads under steady-state conditions or cannot remain in synchronism following a transient disturbance. The latter, known as transient instability, is usually the limiting factor and is the subject of this investigation.

Transient instability may be described as follows: Consider one synchronous machine in a system which is operating under steady-state conditions with its mechanical power input approximately equal to its electrical power output. When this is happening, the machine is operating at synchronous speed. At the instant a disturbance occurs, such as a fault, circuit breaker operation, or a change in load, the electrical power output will change instantaneously. Since the mechanical power input cannot change instantaneously because of prime mover characteristics, the resulting difference in the input and output power will cause the machine to accelerate or decelerate and to attain a velocity different from synchronous speed. The acceleration will continue until the input and output powers are equal. At this time, the machine may return to its original speed or it may continue to accelerate. In the latter case, the system is unstable.

For a system having N synchronous machines, the solution of the transient stability problem is basically one of solving N simultaneous, non-linear, differential equations. For many years, the a-c network calculator has been used to study the stability problem by a method of finite-difference approximations which requires a considerable amount of manual calculations. The details of the a-c network calculator solution are described later. The repetitive calculations and measurements required in the solution suggest the use of a digital computer. However, because of the complexity of modern power systems, a complete digital solution of both electrical and mechanical quantities does not appear as practical as a combination of the digital technique with the a-c network calculator.

This thesis presents two digital computer programs for the IBM 650 digital computer using the Bell General Purpose Interpretative System (1)[1]. The first program was written to solve as large a system

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[1] Numbers in parentheses refer to the Bibliography unless preceded by the word "equation". Such numbers refer to equations, unless the term "Reference" is added.

as possible with the storage available on the IBM 650 digital computer. The second program was written with the maximum number of machines in the system reduced so that the program could later be modified to remove some of the assumptions normally made in stability analyses.

CHAPTER II

ANALYSIS OF THE STABILITY PROBLEM

The equation (Reference 2) for the power output of each synchronous machine can be written as follows:

For the first and the K^{th} machines, respectively:

$$P_{e1} = \frac{E_1 E_1}{Z_{11}} \text{Cos} (\delta_1 - \delta_1 + \theta_{11}) - \frac{E_1 E_2}{Z_{12}} \text{Cos} (\delta_1 - \delta_2 + \theta_{12}) \quad (1)$$

$$- \cdots - \frac{E_1 E_k}{Z_{1k}} \text{Cos} (\delta_1 - \delta_k + \theta_{1k}) - \cdots$$

$$P_{ek} = - \frac{E_k E_1}{Z_{k1}} \text{Cos} (\delta_k - \delta_1 + \theta_{k1}) - \frac{E_k E_2}{Z_{k2}} \text{Cos} (\delta_k - \delta_2 + \theta_{k2})$$

$$- \cdots + \frac{E_k E_k}{Z_{kk}} \text{Cos} (\delta_k - \delta_k + \theta_{kk}) - \cdots$$

The power input for each machine is assumed constant over the time interval under investigation so that the following equations (Reference 3) can be written:

For the first and the K^{th} machines, respectively:

$$M_1 \frac{d^2 \delta_1}{dt^2} = P_{m1} - P_{e1} \quad (2)$$

$$M_k \frac{d^2 \delta_k}{dt^2} = P_{mk} - P_{ek}$$

The solution to this set of simultaneous non-linear differential equations is obtained by a step-by-step process. In applying this process, it is assumed that the electrical power output of each machine is constant during a small time interval, Δt, although they may have different values in different intervals. When this assumption is made, a formal solution can be obtained since the non-linear differential equations become linear differential equations. The changes in the machine rotor angles in the time interval, Δt, are obtained from the linear differential equations. The electrical power outputs at these new angles are then calculated from equation (1) and the process repeated.

It is shown in Reference (4) that the change in rotor angle and the rotor angle at the end of the n^{th} interval are given by:

$$\Delta\delta_n = \Delta\delta_{n-1} + K(P_m - P_e) \qquad (3)$$

$$\delta_n = \delta_{n-1} + \Delta\delta_{n-1} \qquad (4)$$

with $K = \frac{\Delta t^2}{M}$

In the present method of solution, the a-c network calculator is used to determine the electrical power output for each machine, as given in equation (1). These values are then assumed constant for a small finite time and equation (3) solved for the changes in the angles of the machine's rotors. The new angles are then applied to the system represented on the network calculator and new electrical power outputs are obtained. This process is repeated for the number of time intervals necessary to establish the stability or nonstability of the system. The angles as given by equation (4) are plotted as a function of time and the resulting curves, known as swing curves, are indicative of the stability of the system.

The digital computer program presented in this thesis is based on this same repetitive process of calculating electrical power outputs from equation (1) and the corresponding rotor angles from equations (3) and (4).

The voltages (E_n), the transfer and driving point impedances (Z_{nm}), their phase angles (θ_{nm}), and the initial rotor angles (δ_n) must be known in the calculation of the electrical power outputs. These values can be determined digitally but it appears that they are most easily obtained from the a-c network calculator. Hence, in the computer programs presented in this thesis, these values together with the time interval, the acceleration constants, and the number of time intervals desired constitute the input data.

It is frequently desirable to study a system in which the driving point and transfer impedances of the system are changed while the system is in the transient state. Such changes are required to represent circuit breaker operations, changes in loads, or other subsequent disturbances after the first disturbance. The programs have provisions for two such changes which may occur any time after the initiation of the calculations.

CHAPTER III

PROGRAM

There have been two general programs developed in this thesis to solve the stability problem with provisions for up to two changes in system impedances which may occur at any time after the initiation of the calculations. The two programs were considered desirable so that modifications could be made later to the smaller program. The first program, given in Table 1, permits calculation of the swing curves for systems having up to seventeen synchronous machines. The second program, given in Table 2, permits calculation of the swing curves for systems with any number of machines up to ten. The programs also differ in the addresses of the input data, output data, and calculation locations.

The digital computer programs written here make provisions for three different sets of impedance data. The first set of impedances, with their phase angles, is a part of the initial input data. These are the transfer and driving point impedances of the system as it exists immediately after the first disturbance. Later changes in the system resulting from circuit breaker operations or other action of the system's equipment may result in different sets of impedances. Two such sets may be read into the computer after a time interval corresponding to T_2 and T_3 seconds.

The flow diagrams for both programs are shown in Figures 1 - 5.

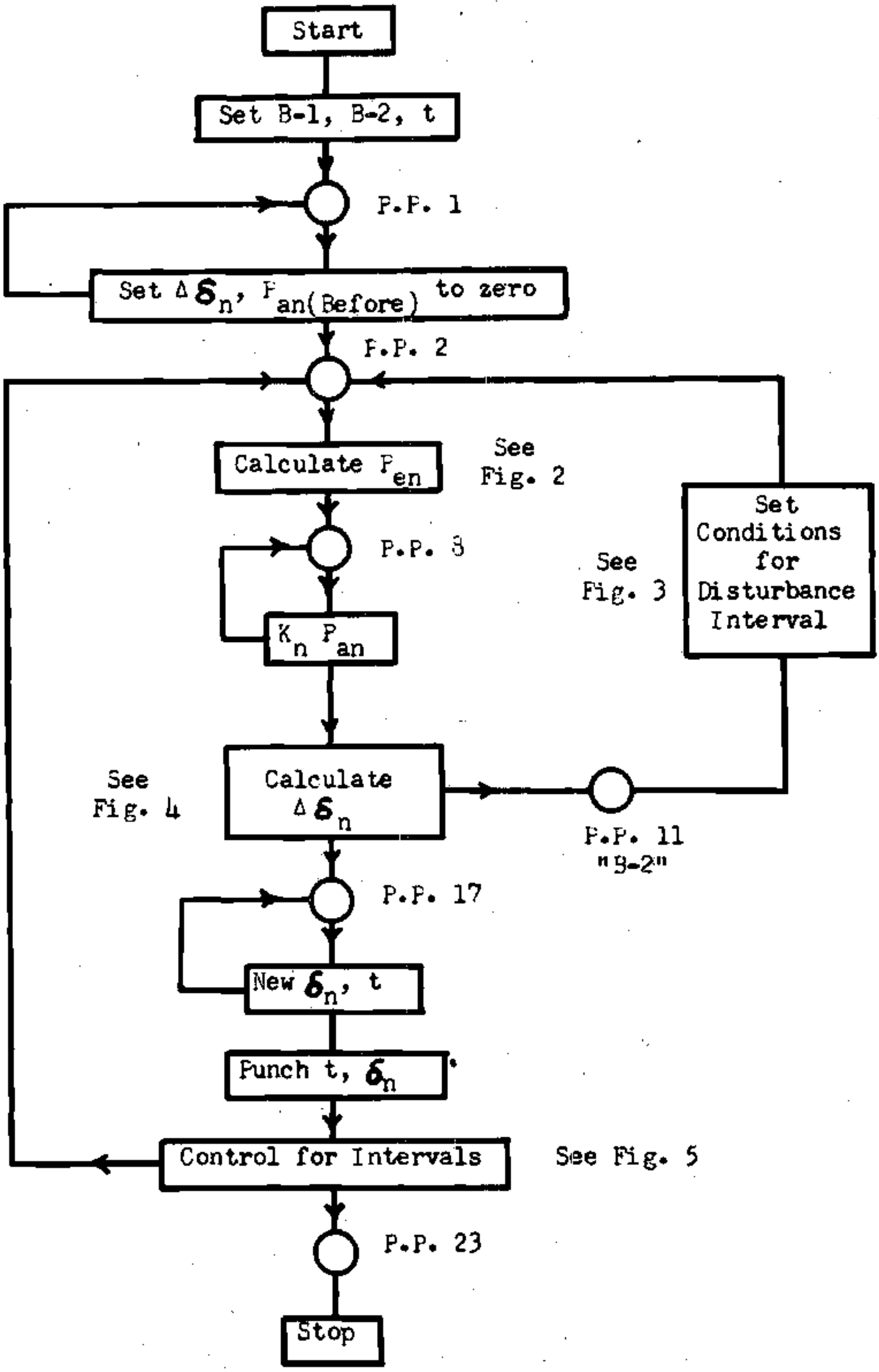

Fig. 1 General Flow Diagram for Digital Program

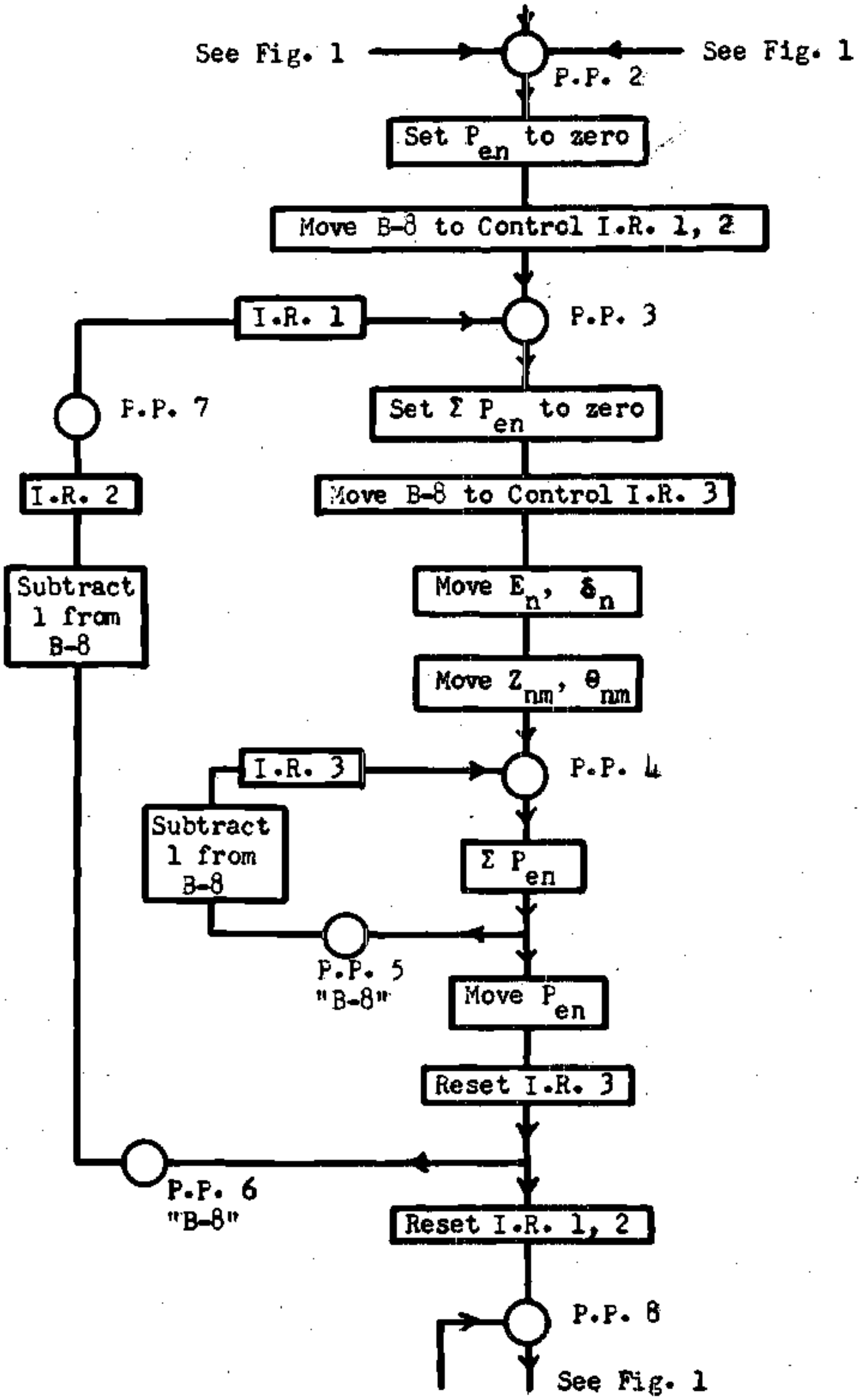

Fig. 2 Flow Diagram for Calculating Electrical Output Power

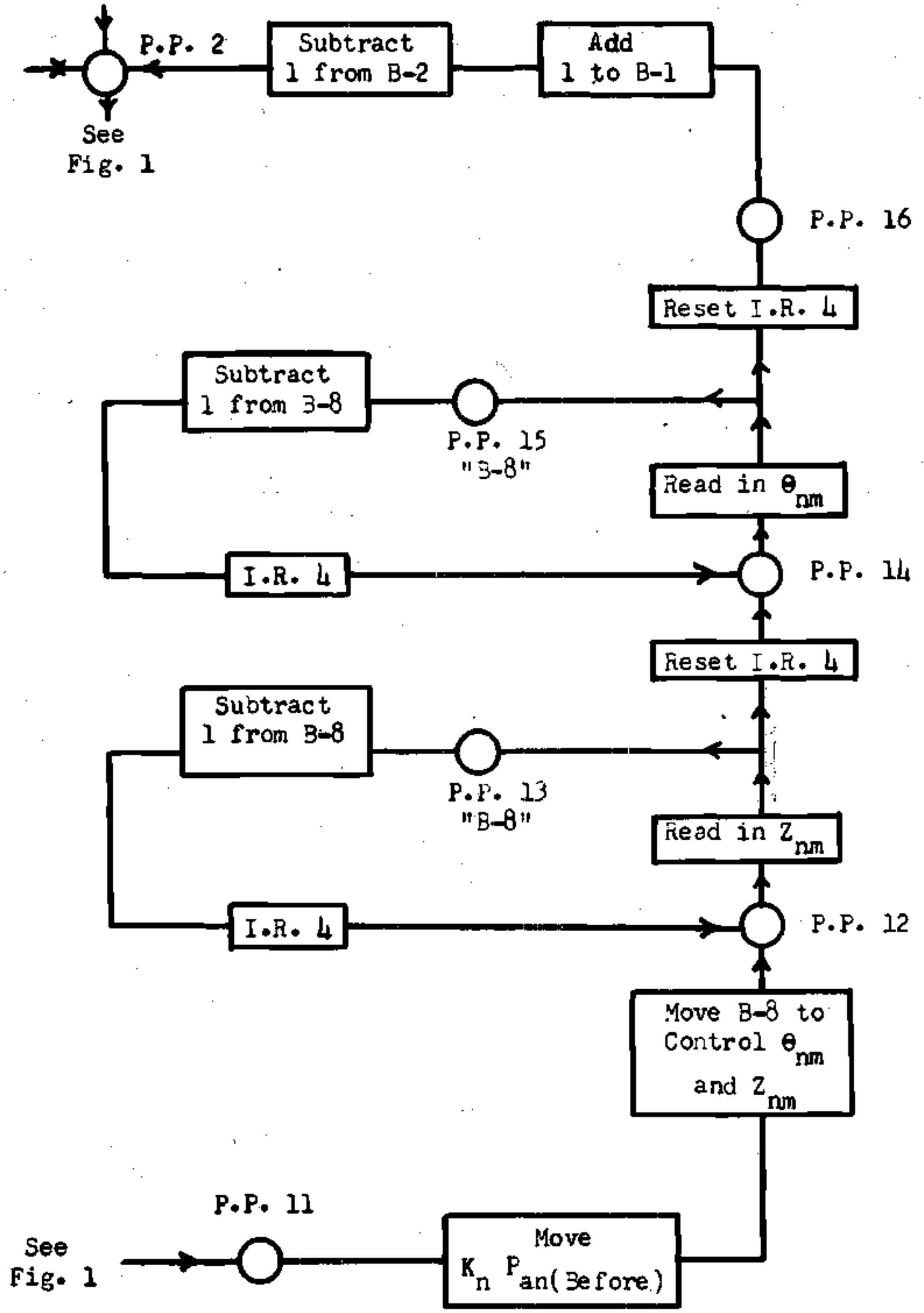

Fig. 3 Flow Diagram for Setting Conditions During Disturbance Interval

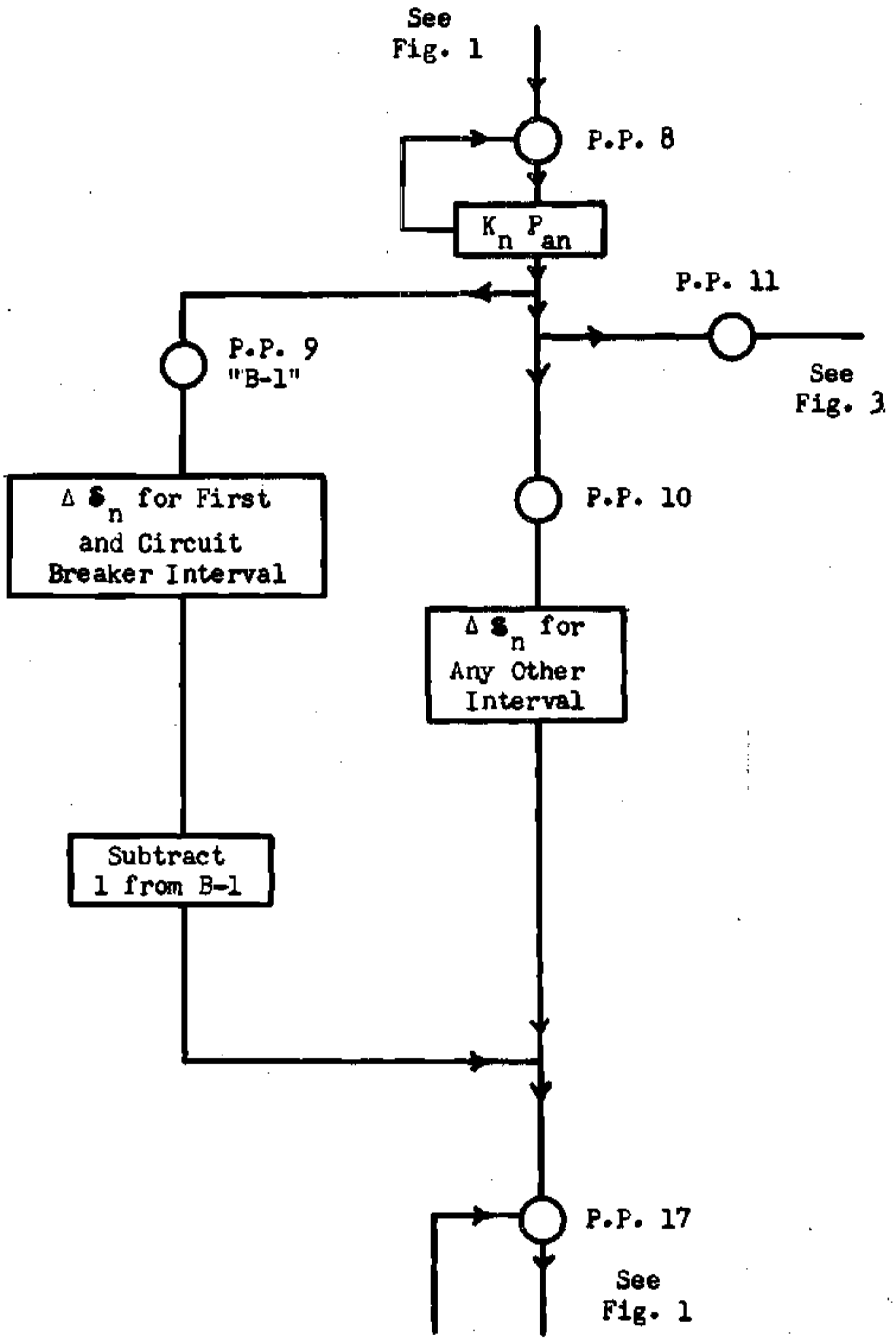

Fig. 4 Flow Diagram for Calculating Change in Rotor Angle

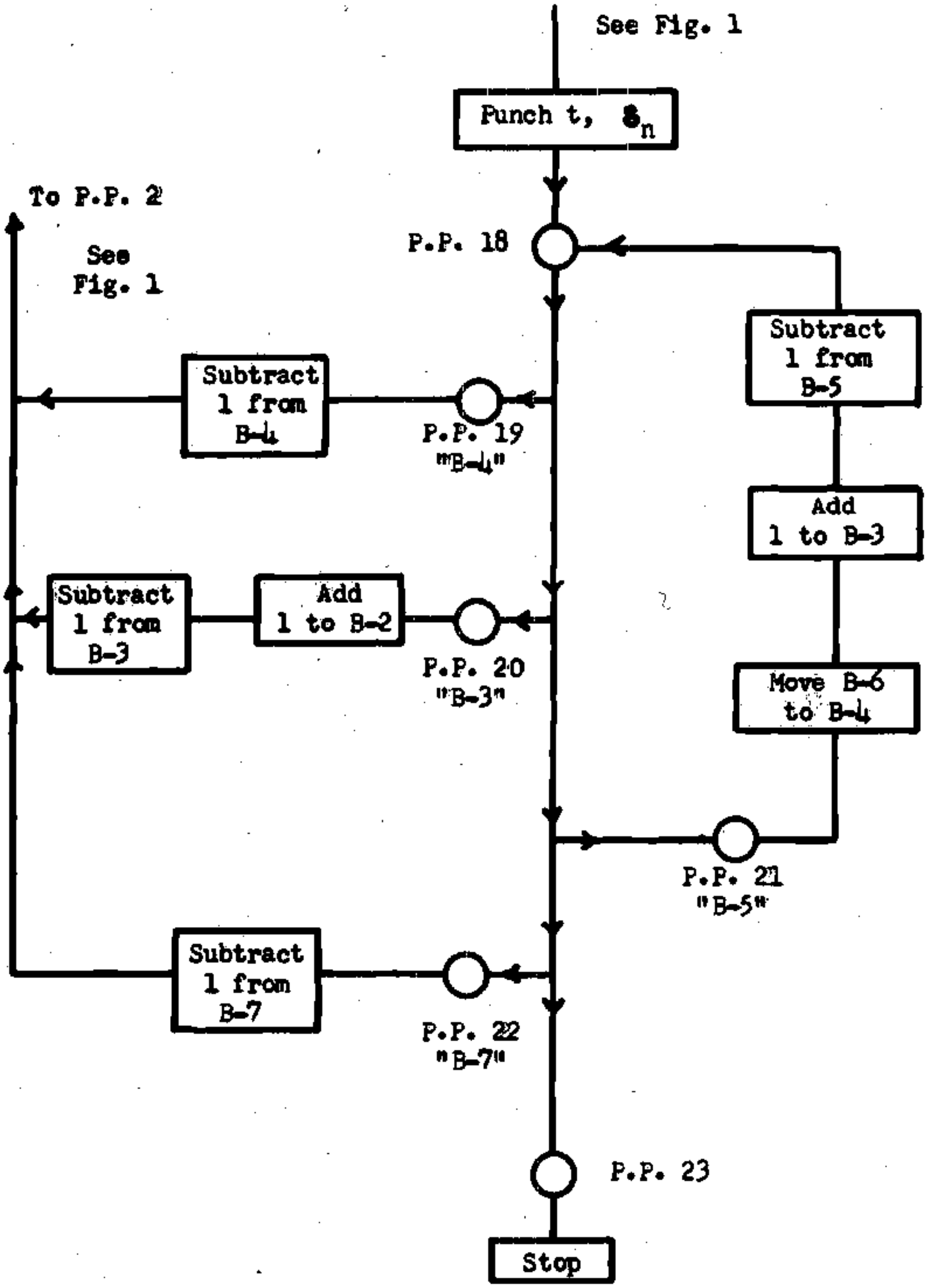

Fig. 5 Flow Diagram for Controlling Intervals

The deck arrangement is shown in Figure 6. Tables 3 - 8 contain the breakdown of the computer memory locations.

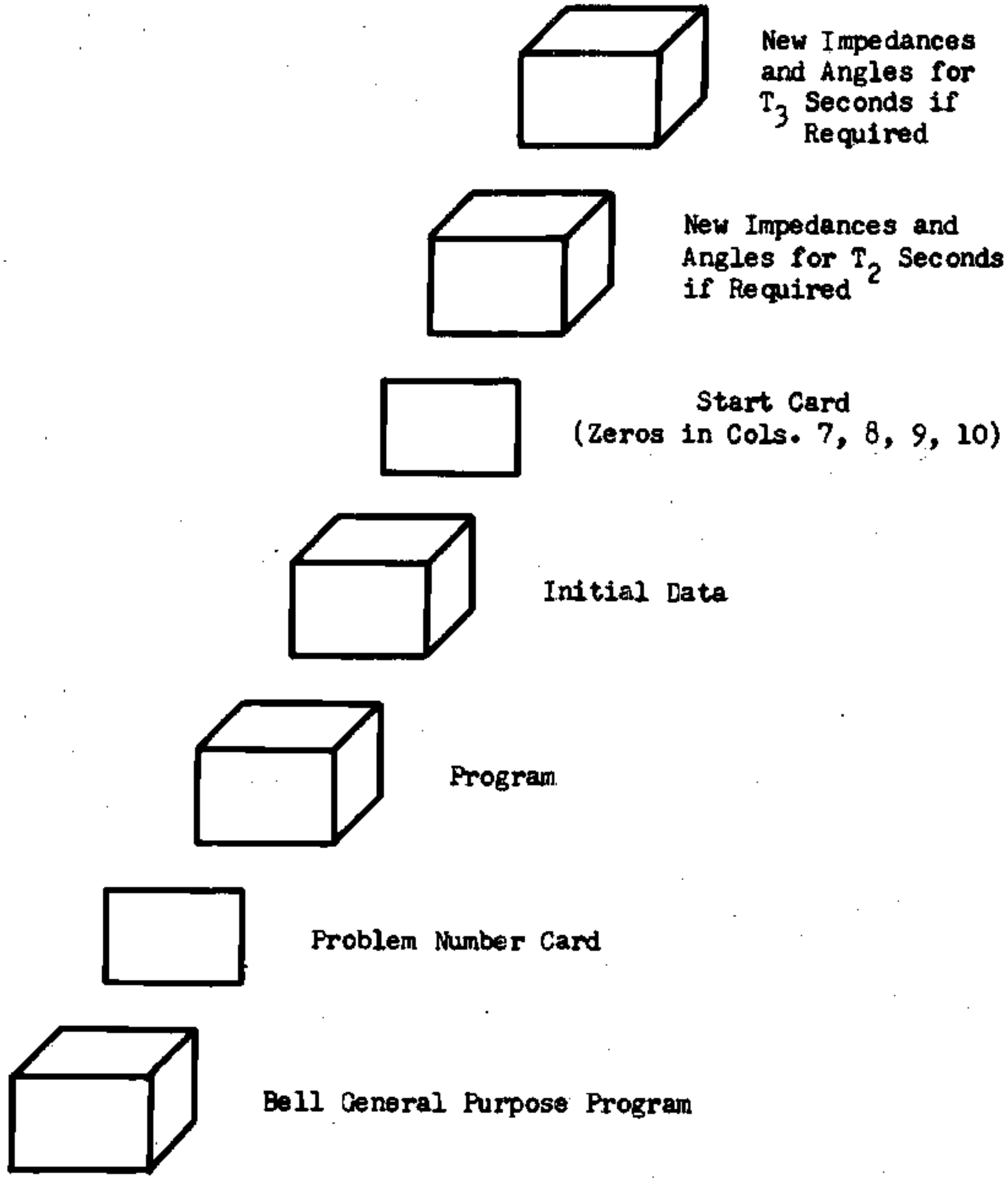

Fig. 6 Deck Arrangement for Digital Program

CHAPTER IV

RESULTS

To illustrate the application of the digital programs and to provide a comparison with the solution by the a-c network calculator, two systems were selected for study. The first system was a four machine problem taken from Reference (5) and the second system was a five machine problem taken from a section of the Georgia Power Company network.

The first system was studied primarily for checking and "debugging" the digital program. A complete longhand solution of this system was available as a check. The three cases considered were a three-phase fault with no circuit breakers opening, a three-phase fault with one circuit breaker opening, and a three-phase fault with two circuit breakers opening. The computer values checked sufficiently close to the reference values to assure the correctness of the program. A close agreement should be expected since both values were calculated by the same mathematical method. The maximum error in the values was 1.107 degrees. This error was caused by the differences in the number of digits carried through the solution.

The second system was studied to give a comparison of the digital computer swing curves with those from the network calculator. Figure 7 shows the section of Georgia Power System used. The generating schedule, the bus loads, and the line impedances for this system are given in Tables 9 - 11. The disturbance considered was a

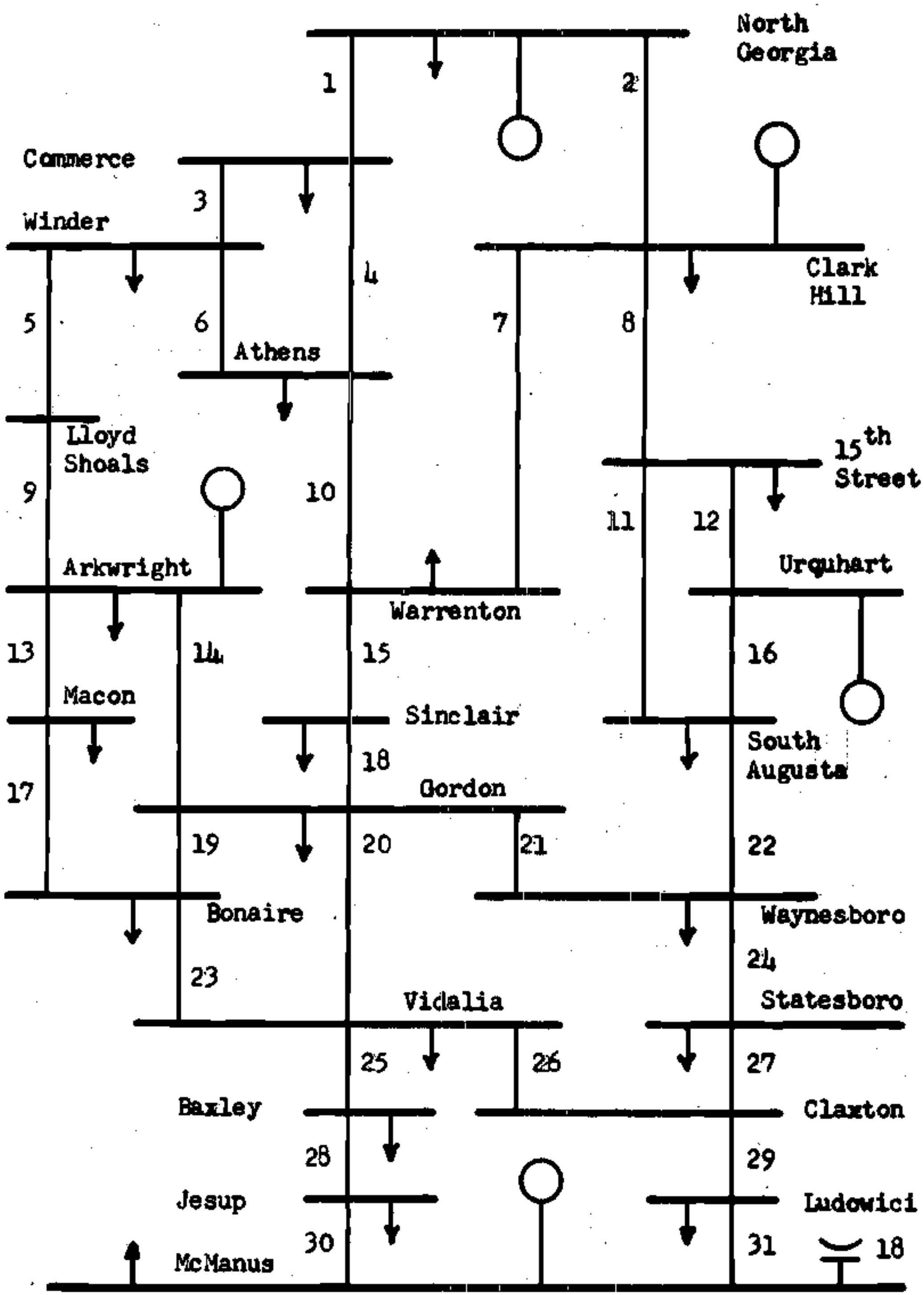

Fig. 7 Section of Georgia Power Company Network that was used for System Two (Numbers Indicate Line Designations)

three-phase fault on a radial line at the Commerce bus with the circuit breaker opening in 0.2 seconds. The angular difference between each generator and the Arkwright generator are shown in Figures 8 - 11. The results of the two methods were in agreement, with a maximum error of 4.5 degrees or 4.3 per cent. An error of this magnitude could be expected since the digital computer solution was a mathematical solution based on fixed input data, whereas the network calculator solution was based on a combination of instrument readings and mathematical calculations.

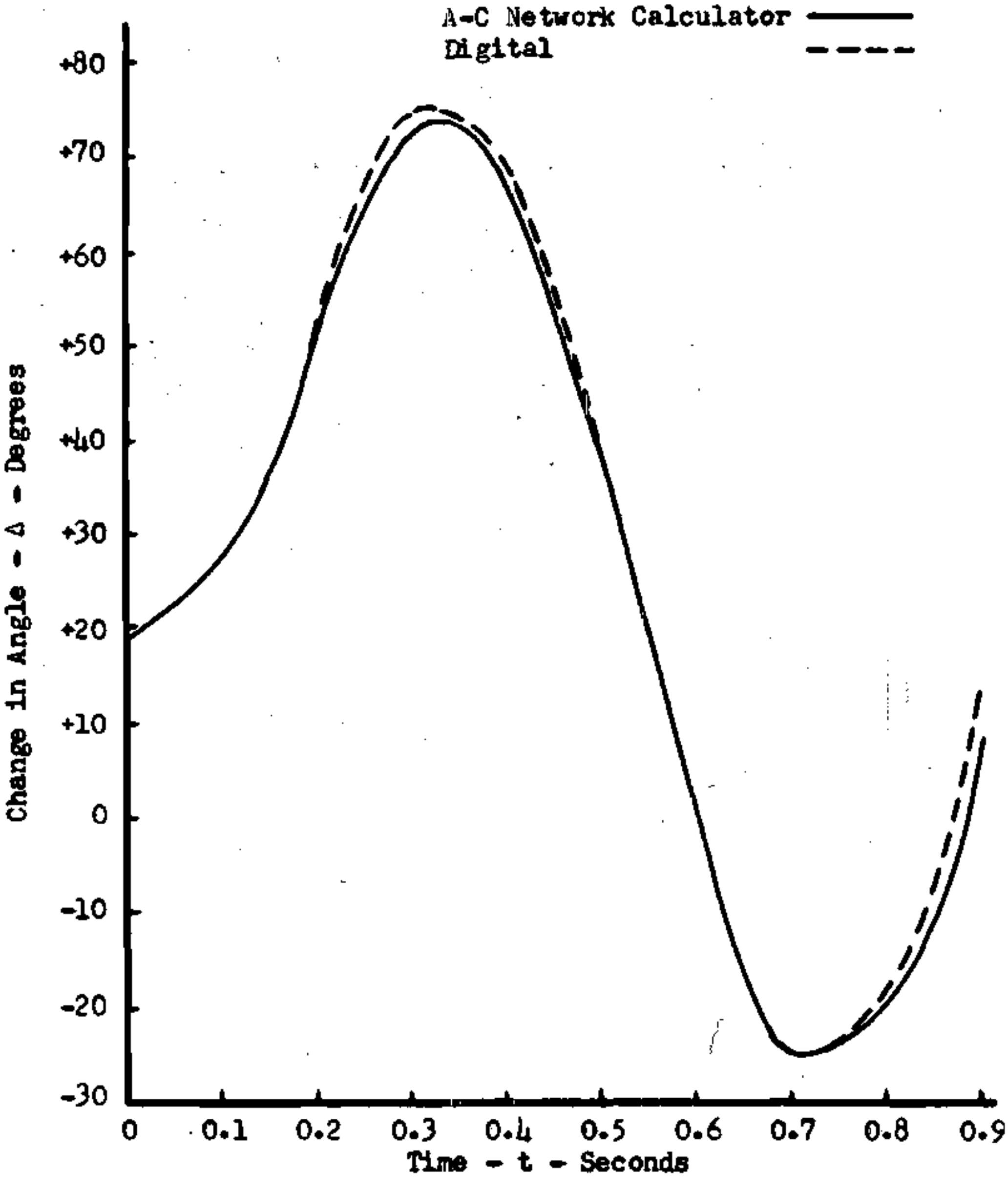

Fig. 8 Change in North Georgia's Rotor Angle with Respect to Arkwright as a Function of Time

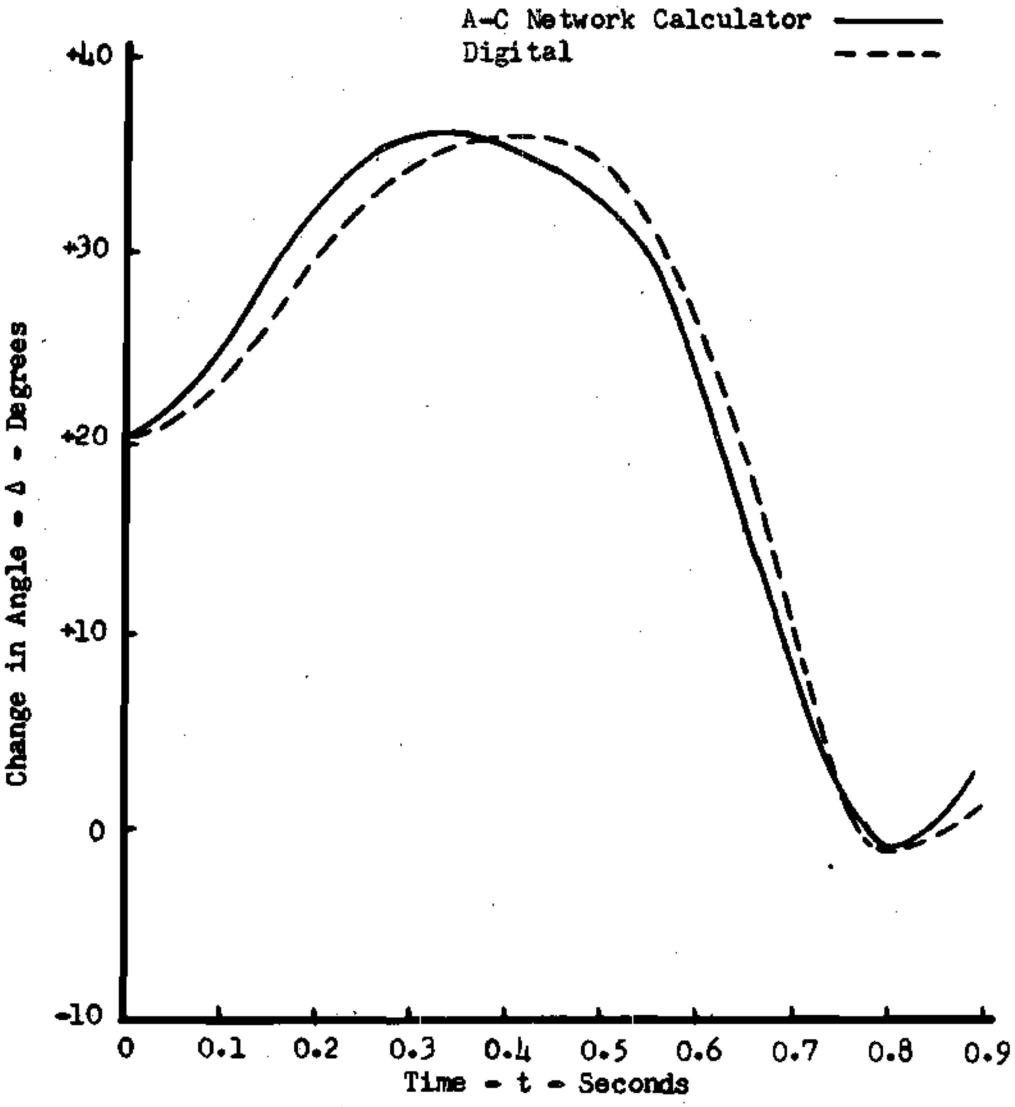

Fig. 9 Change in Clark Hill's Rotor Angle with Respect to Arkwright as a Function of Time

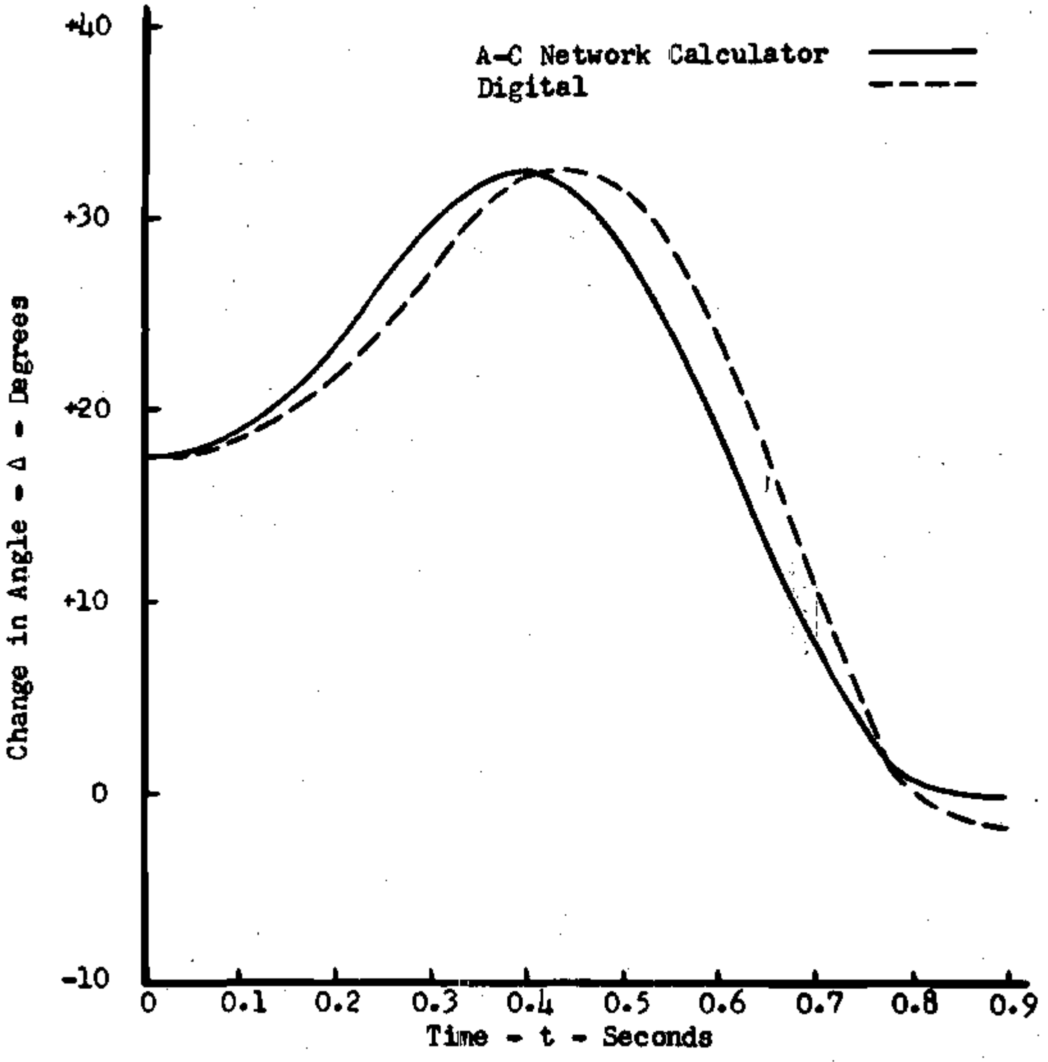

Fig. 10 Change in Urquhart's Rotor Angle with Respect to Arkwright as a Function of Time

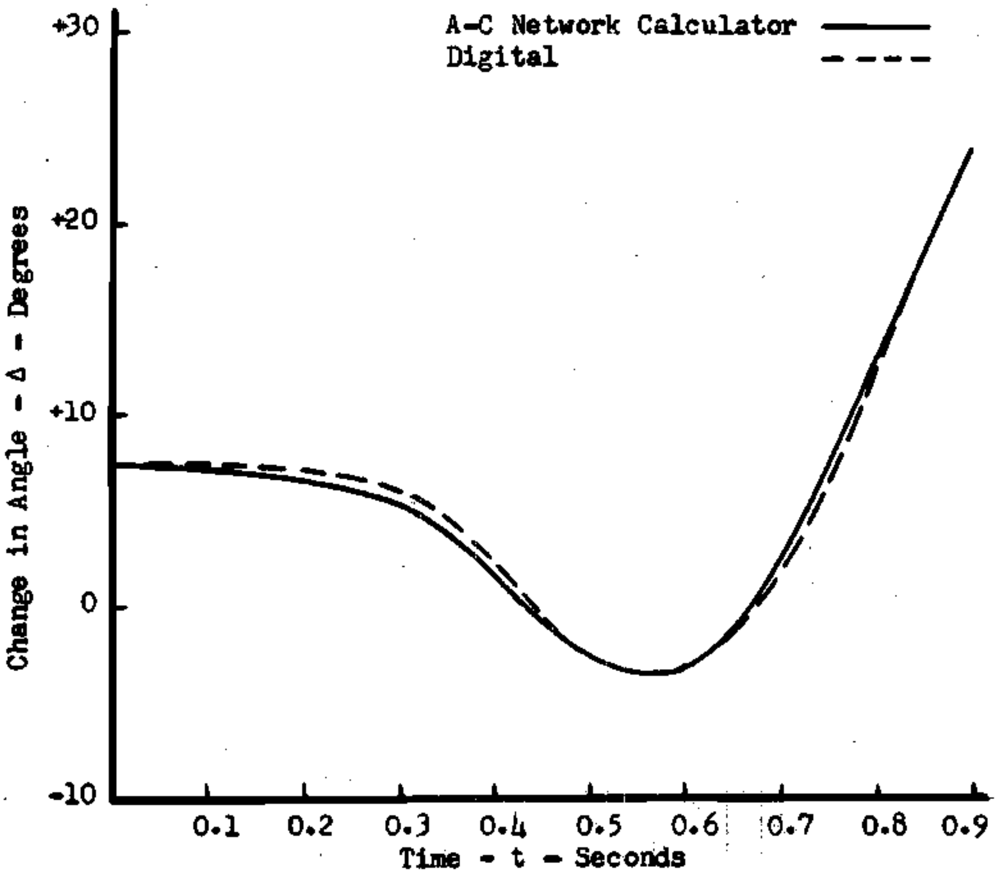

Fig. 11 Change in McManus' Rotor Angle with Respect to Arkwright as a Function of Time

CHAPTER V

COMPARISON OF THE METHODS

Overall time.--The time required for a network calculator stability study will depend upon the available man-power to perform the calculations between intervals. For the five machine problem in system two, the time for the network calculator solution was forty minutes using two trained engineers, and the time for the computer solution was one hour. The time for the computer solution was divided into three parts: the time on the network calculator to obtain the initial conditions and impedances, the time on the card punch machine, and the time on the IBM 650 digital computer. The actual computer time was approximately nine minutes.

Man-power.--The man-power used in the network calculator solution is composed of highly-trained engineers of the power company conducting the study. Of course, theoretically, highly-trained engineers are not required for the calculations but, practically, they are the persons who are required to perform them. The man-power used in the digital computer solution consists of a network calculator operator to read the initial conditions and impedances and a semi-skilled IBM machine operator to punch the cards and put them in the computer.

For the five machine problem in system two, the number of man-hours to perform the network calculator solution was one and one-third. This man must be highly trained. The number of man-hours to perform the digital solution was one-third of a man-hour of a highly-trained engineer and two-thirds of a man-hour of a semi-skilled operator.

Data.--In both methods of analysis, the initial conditions must be set up on the network calculator. For the five machine system problem, 175 readings had to be taken on the network calculator including the angles which must be set at each interval. In the digital method, only 65 readings were necessary and these were taken with no interruptions. If a fifteen machine system was analyzed, the number of readings would be approximately the same for both methods. For machine systems greater than fifteen, the digital method would require more readings.

The data readings for the digital study can be performed by the operators of the network calculator. The power company requesting the information would have to have a representative to decide the location of the disturbances and the system operating conditions. In lieu of the above, the power companies could reserve part of their regular board study time to obtain the data. In this way, the system would not have to be simulated on the network calculator for the stability study alone.

The effect of relay opening time can be studied by the digital method with no additional data. The only change in the input data is a change in the card containing the number of time intervals prior to the circuit breaker opening. To accomplish this with the network calculator, the solution must be returned to the conditions preceding the first relay opening time and the computations repeated from that point.

The effect of a change in time interval can be easily studied by the digital method with no additional data. The only change in the digital program is a change in the acceleration constant. The complete study would have to be repeated on the a-c network calculator.

A study of changes of division of generation between plants from the stability viewpoint usually will not require entirely new input data. There will be a small change in the impedances with changes in generation, but the change is small and can usually be neglected. The additional data required for the digital computer would be new values for voltages, rotor angles, and initial powers of the generators. The complete study would have to be repeated on the a-c network calculator.

As the number of stability studies increases, for the same system and generating schedule, the number of sets of impedances which must be measured on the network calculator decreases proportionally. The pre-disturbance impedances will be the same for a given system and generating schedule. The only additional data for the digital computer stability study will be new sets of post-disturbance impedances. The complete study would have to be repeated on the a-c network calculator.

During the swing curve calculations, additional information may be obtained on the a-c network calculator by taking readings of voltage, current, and power flow. This information can be used to set and check relays and to determine the performance of any element whose identity is not maintained in the equivalent circuit.

At present, the programs written in this thesis will not give this information.

CHAPTER VI

CONCLUSIONS

The digital computer solution of the stability problem is practical.

Highly-trained engineers are not required to perform the lengthy calculations that are required in the network calculator solution.

Likelihood of human errors that are possible in the step-by-step calculation will be greatly reduced by the use of the digital computer.

Effects of various relay opening times and different time intervals can be easily studied by the digital method.

A study of favorable division of generation between plants from the stability viewpoint is simpler with the digital computer than with the a-c network calculator.

For the same system and generating schedule, as the number of stability studies increases the number of measurements on the network calculator needed for the digital solution decreases proportionally.

At present, the a-c network calculator gives a greater amount of information on a study than does a corresponding digital study.

The a-c network calculator solution gives the power company representatives a better understanding of the performance of the system than does the digital computer solution.

CHAPTER VII

RECOMMENDATIONS

Computer time can be reduced by writing a new program in basic IBM 650 machine language using the program and flow diagram of this thesis.

A more exact solution of the stability problem is possible by modifying the existing program to include exciter response, saturation, effects of flux decay, and other refinements.

A reduction of time in the digital solution may be possible by calculating digitally the driving point and transfer impedances. The necessary system data could also be used in the digital solutions of other power system problems such as load flows and short circuit calculations.

Consideration should be given to the possibility of making an investigation of methods of including induction machines in existing digital programs.

APPENDIX

Table 1. Seventeen Machines Stability Program

Address	Order	Address	Order
001	+2901901848	041	+9600003000
002	+2900900849	042	+8600853006
003	+2900900152	043	+9600001000
004	+9800001000	044	+9600002000
005	+9100001000	045	+8000000008
006	+2900900831	046	+9800005000
007	+9100001000	047	-1854901854
008	+2900900814	048	+8301017004
009	+8101017001	049	+9800006000
010	+9800002000	050	-1853901853
011	+9100001000	051	+8217017007
012	+2900900797	052	+9800007000
013	+8101017002	053	+8101017003
014	+7201209853	054	+9800008000
015	+9800003000	055	+9100111000
016	+2900900852	056	-1170797797
017	+7201209854	057	+9100111000
018	+9100010000	058	+2187797797
019	+7201135850	059	+8101017008
020	+9100010000	060	+8600848009
021	+7201153851	061	+8600849011
022	+9200010000	062	+9800010000
023	+7217215860	063	+9100111000
024	+9200010000	064	+1831797831
025	+7217505880	065	+8101017010
026	+9800004000	066	+8000000017
027	+9300010000	067	+9800009000
028	+2850135857	068	+9100111000
029	+9300010000	069	+1814797797
030	+3857860857	070	+9100101000
031	+9300100000	071	+3797902797
032	+1880851858	072	+9100111000
033	+9300010000	073	+1831797831
034	-1858153858	074	+8101017009
035	+0354858858	075	-1848901848
036	+2857858859	076	+8000000017
037	+1859852852	077	+9800011000
038	+8600854005	078	+7217797814
039	+9100001000	079	+7201209855
040	+7201852797	080	+7201209856

Table 1. Seventeen Machines Stability Program
(Continued)

Address	Order
081	+9800012000
082	+9400011000
083	+7000215231
084	+8600855013
085	+9600004000
086	+9800014000
087	+9400011000
088	+7000505521
089	+8600856015
090	+9600004000
091	+8000000016
092	+9800013000
093	-1855901855
094	+8417017012
095	+9800015000
096	-1856901856
097	+8417017014
098	+9800016000
099	+1848901848
100	-1849901849
101	+8000000002
102	+9800017000
103	+9200111000
104	+1153831153
105	+8201017017
106	+1152210152
107	+7300152169
108	+9800018000
109	+8600205019
110	+8600204020
111	+8600206021
112	+8600208022
113	+8000000023
114	+9800019000
115	-1205901205
116	+8000000002
117	+9800020000
118	+1849901849
119	-1204901204
120	+8000000002
121	+9800021000
122	+7201207205
123	+1204901204
124	-1206901206
125	+8000000018
126	+9800022000
127	-1208901208
128	+8000000002
129	+9800023000
130	+0000000000

Table 2. Ten Machine Stability Program

Address	Order
001	+2901901390
002	+2900900391
003	+2900900319
004	+9800001000
005	+9100001000
006	+2900900380
007	+9100001000
008	+2900900360
009	+8101010001
010	+9800002000
011	+9100001000
012	+2900900340
013	+8101010002
014	+7201398253
015	+9800003000
016	+2900900252
017	+7201398254
018	+9100010000
019	+7201300250
020	+9100010000
021	+7201320251
022	+9200010000
023	+7210401260
024	+9200010000
025	+7210501270
026	+9800004000
027	+9300010000
028	+2250300650
029	+9300010000
030	+3650260650
031	+9300100000
032	+1270251651
033	+9300010000
034	-1651320651
035	+0354651651
036	+2650651652
037	+1652252252
038	+8600254005
039	+9100001000
040	+7201252340
041	+9600003000
042	+8600253006
043	+9600001000
044	+9600002000
045	+8000000008
046	+9800005000
047	-1254901254
048	+8301010004
049	+9800006000
050	-1253901253
051	+8210010007
052	+9800007000
053	+8101010003
054	+9800008000
055	+9100111000
056	-1350340340
057	+9100111000
058	+2370340340
059	+8101010008
060	+8600390009
061	+8600391011
062	+9800010000
063	+9100111000
064	+1340380380
065	+8101010010
066	+8000000017
067	+9800009000
068	+9100111000
069	+1360340340
070	+9100101000
071	+3340902340
072	+9100111000
073	+1340380380
074	+8101010009
075	-1390901390
076	+8000000017
077	+9800011000
078	+7210340360
079	+7201398255
080	+7201398256

Table 2. Ten Machine Stability Program
(Continued)

Address	Order
081	+9800012000
082	+9400011000
083	+7000401410
084	+8600255013
085	+9600004000
086	+9800014000
087	+9400011000
088	+7000501510
089	+8600256015
090	+9600004000
091	+8000000016
092	+9800013000
093	-1255901255
094	+8410010012
095	+9800015000
096	-1256901256
097	+8410010014
098	+9800016000
099	+1390901390
100	-1391901391
101	+8000000002
102	+9800017000
103	+9200111000
104	+1320380320
105	+8201010017
106	+1319397319
107	+7300319329
108	+9800018000
109	+8600393019
110	+8600392020
111	+8600394021
112	+8600396022
113	+8000000023
114	+9800019000
115	-1393901393
116	+8000000002
117	+9800020000
118	+1391901391
119	-1392901392
120	+8000000002
121	+9800021000
122	+7201395393
123	+1900901392
124	-1394901394
125	+8000000018
126	+9800022000
127	-1396901396
128	+8000000002
129	+9800023000
130	+0000000000

Table 3. General Breakdown of Computer Space for Seventeen Machine Program

Addresses	Purpose
001 - 130	Program
131 - 134	Not Used
135 - 151	Input Data
152	Output Data
153 - 169	Input and Output Data
170 - 210	Input Data
211 - 214	Not Used
215 - 503	Input Data
504	Not Used
505 - 793	Input Data
794 - 796	Not Used
797 - 876	Computations
877 - 879	Not Used
880 - 896	Computations
897 - 899	Not Used
900 - 2000	Bell General Purpose Program

Table 4. Input Data Locations for Seventeen Machine Program

Addresses	Input Data
135 - 151	Voltages
153 - 169	Initial Rotor Angles
170 - 186	Output Powers
187 - 203	Accelerating Constants
204	B-3
205	B-4
206	B-5
207	B-6
208	B-7
209	B-8
210	Δt
215	$+Z_{11}$
216 - 231	$-Z_{12}, -Z_{13}, \cdots, -Z_{1-17}$
232	$-Z_{21}$
233 - 248	$+Z_{22}, -Z_{23}, \cdots, -Z_{2-17}$
249	$-Z_{31}$
250 - 265	$-Z_{32}, +Z_{33}, \cdots, -Z_{3-17}$
266	$-Z_{41}$
267 - 282	$-Z_{42}, -Z_{43}, \cdots, -Z_{4-17}$
283	$-Z_{51}$
284 - 299	$-Z_{52}, -Z_{53}, \cdots, -Z_{5-17}$
300	$-Z_{61}$
301 - 316	$-Z_{62}, -Z_{63}, \cdots, -Z_{6-17}$
317	$-Z_{71}$
318 - 333	$-Z_{72}, -Z_{73}, \cdots, -Z_{7-17}$
334	$-Z_{81}$
335 - 350	$-Z_{82}, -Z_{83}, \cdots, -Z_{8-17}$
351	$-Z_{91}$
352 - 367	$-Z_{92}, -Z_{93}, \cdots, -Z_{9-17}$
368	$-Z_{10-1}$

Table 4. Input Data Locations for Seventeen Machine Program (Continued)

Addresses	Input Data
369 - 384	$-Z_{10-2}, -Z_{10-3}, \cdots, -Z_{10-17}$
385	$-Z_{11-1}$
386 - 401	$-Z_{11-2}, -Z_{11-3}, \cdots, -Z_{11-17}$
402	$-Z_{12-1}$
403 - 418	$-Z_{12-2}, -Z_{12-3}, \cdots, -Z_{12-17}$
419	$-Z_{13-1}$
420 - 435	$-Z_{13-2}, -Z_{13-3}, \cdots, -Z_{13-17}$
436	$-Z_{14-1}$
437 - 452	$-Z_{14-2}, -Z_{14-3}, \cdots, -Z_{14-17}$
453	$-Z_{15-1}$
454 - 469	$-Z_{15-2}, -Z_{15-3}, \cdots, -Z_{15-17}$
470	$-Z_{16-1}$
471 - 486	$-Z_{16-2}, -Z_{16-3}, \cdots, -Z_{16-17}$
487	$-Z_{17-1}$
488 - 503	$-Z_{17-2}, -Z_{17-3}, \cdots, +Z_{17-17}$
505 - 521	Phase Angles for Machine One
522 - 538	Phase Angles for Machine Two
539 - 555	Phase Angles for Machine Three
556 - 572	Phase Angles for Machine Four
573 - 589	Phase Angles for Machine Five
590 - 606	Phase Angles for Machine Six
607 - 623	Phase Angles for Machine Seven
624 - 640	Phase Angles for Machine Eight
641 - 657	Phase Angles for Machine Nine
658 - 674	Phase Angles for Machine Ten
675 - 691	Phase Angles for Machine Eleven
692 - 708	Phase Angles for Machine Twelve
709 - 725	Phase Angles for Machine Thirteen
726 - 742	Phase Angles for Machine Fourteen
743 - 759	Phase Angles for Machine Fifteen
760 - 776	Phase Angles for Machine Sixteen
777 - 793	Phase Angles for Machine Seventeen

Table 5. Output Data Locations for Seventeen Machine Program

Address	Output Data
152	Time
153	Rotor Angle for Machine One
154	Rotor Angle for Machine Two
155	Rotor Angle for Machine Three
156	Rotor Angle for Machine Four
157	Rotor Angle for Machine Five
158	Rotor Angle for Machine Six
159	Rotor Angle for Machine Seven
160	Rotor Angle for Machine Eight
161	Rotor Angle for Machine Nine
162	Rotor Angle for Machine Ten
163	Rotor Angle for Machine Eleven
164	Rotor Angle for Machine Twelve
165	Rotor Angle for Machine Thirteen
166	Rotor Angle for Machine Fourteen
167	Rotor Angle for Machine Fifteen
168	Rotor Angle for Machine Sixteen
169	Rotor Angle for Machine Seventeen

Table 6. General Breakdown of Computer Space for Ten Machine Program

Addresses	Purpose
001 - 130	Program
131 - 299	Not Used
300 - 309	Input Data
310 - 318	Not Used
319	Output Data
320 - 329	Input and Output Data
330 - 339	Not Used
340 - 349	Computations
350 - 359	Input Data
360 - 369	Computations
370 - 379	Input Data
380 - 391	Computations
392 - 398	Input Data
399 - 400	Not Used
401 - 600	Input Data
601 - 649	Not Used
650 - 652	Computations
653 - 899	Not Used
900 - 2000	Bell General Purpose Program

Table 7. Input Data Locations for Ten Machine Program

Addresses	Input Data
300 - 309	Voltages
320 - 329	Initial Rotor Angles
350 - 359	Output Powers
370 - 379	Accelerating Constants
392	B-3
393	B-4
394	B-5
395	B-6
396	B-7
397	Δt
398	B-8
401 - 410	Impedances for Machine One*
411 - 420	Impedances for Machine Two*
421 - 430	Impedances for Machine Three*
431 - 440	Impedances for Machine Four*
441 - 450	Impedances for Machine Five*
451 - 460	Impedances for Machine Six*
461 - 470	Impedances for Machine Seven*
471 - 480	Impedances for Machine Eight*
481 - 490	Impedances for Machine Nine*
491 - 500	Impedances for Machine Ten*
501 - 510	Phase Angles for Machine One
511 - 520	Phase Angles for Machine Two
521 - 530	Phase Angles for Machine Three
531 - 540	Phase Angles for Machine Four
541 - 550	Phase Angles for Machine Five
551 - 560	Phase Angles for Machine Six
561 - 570	Phase Angles for Machine Seven
571 - 580	Phase Angles for Machine Eight
581 - 590	Phase Angles for Machine Nine
591 - 600	Phase Angles for Machine Ten

*Impedances go into addresses in same sequence as shown in Table 4, page 34.

Table 8. Output Data Locations for Ten Machine Program

Address	Output Data
319	Time
320	Rotor Angle for Machine One
321	Rotor Angle for Machine Two
322	Rotor Angle for Machine Three
323	Rotor Angle for Machine Four
324	Rotor Angle for Machine Five
325	Rotor Angle for Machine Six
326	Rotor Angle for Machine Seven
327	Rotor Angle for Machine Eight
328	Rotor Angle for Machine Nine
329	Rotor Angle for Machine Ten

Table 9. Generating Schedule for System Two
200 MVA Base

	Terminal			Internal		
Location	Voltage in Volts	Megawatts	Megavars	Voltage in Volts	Angle in Degrees	Megawatts
North Georgia	105	150	70	119	53.3	152
Clark Hill	105	100	75	133	54.8	103
Urquhart	103	150	92	109.5	51.8	152
Arkwright	105	173	82	118.5	34.4	177
McManus	105	132	72	113.5	41.6	132

Table 10. Load Schedule for System Two
200 MVA Base

Location	Megawatts	Megavars
15th Street	70.7	49.6
South Augusta	19.7	10.3
Waynesboro	25.6	2.6
Statesboro	27.2	2.6
Ludowici	12.5	0
Jesup	10.9	6.1
Baxley	18.9	5.3
Vidalia	36.8	3.6
Gordon	51.7	29.3
Warrenton	25.1	10.4
Athens	49.0	1.3
Commerce	29.1	0
Winder	30.9	10.7
Bonaire	57.9	13.9
Macon	68.1	25.3
North Georgia	48.2	32.2
Arkwright	33.5	7.8
McManus	65.3	22.1
Sinclair	16.2	10.7
Clark Hill	21.2	14.1

Table 11. Line Impedances for System Two
200 MVA Base

Line	Resistance in Per Cent	Reactance in Per Cent	Capacitance in Per Cent
1	8.4	26.0	3.5
2	44.6	147.0	4.3
3	5.2	16.0	0
4	9.2	27.6	0
5	23.6	76.5	2.2
6	10.6	32.1	2.0
7	16.8	53.8	0
8	4.6	15.6	0
9	10.2	49.5	1.3
10	27.2	81.5	3.1
11	1.8	5.2	0
12	2.0	7.8	1.9
13	1.8	5.5	1.5
14	11.0	33.2	0
15	16.0	48.8	1.4
16	1.6	7.0	0
17	13.0	28.0	0
18	19.0	30.9	0
19	19.4	66.0	0
20	39.2	77.0	6.1
21	37.4	112.2	0
22	10.6	31.6	1.0
23	32.8	118.0	5.7
24	21.8	65.5	3.3
25	28.6	48.8	0
26	22.8	43.8	0
27	7.8	23.3	0
28	26.0	44.4	1.4
29	15.4	46.5	1.9
30	32.0	56.8	1.2
31	18.8	52.1	1.3

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