A SIMPLIFIED PHOTOELASTIC TECHNIQUE

A THESIS

Submitted in partial fulfillment
of the requirements for the Degree
of Master of Science in Aeronautical Engineering

by

Jerry Hoffer

Georgia School of Technology
Atlanta, Georgia
1941

A SIMPLIFIED PHOTOELASTIC TECHNIQUE

Approved:

[redacted]

[redacted]

[redacted]

Date Approved by Chairman May 24, 1941

ACKNOWLEDGMENTS

The author wishes to express his appreciation to Professor Montgomery Knight who made many helpful suggestions on the writeup of this report, to Dr. J.E. Boyd who made clear to the author many of the photoelastic principles used, to Professor A.M. Schwartz who made many suggestions on the design of the equipment, and to Mr. W.C. Slocum who aided in constructing the apparatus.

TABLE OF CONTENTS

A SIMPLIFIED PHOTOELASTIC TECHNIQUE

SUMMARY

A simple polariscope incorporating the method of reflection for its polarizing action was built and used to analyze several Bakelite beams. The object was to devise some simple method or technique that would enable this inexpensive polariscope to yield valuable stress information. The author did this and found that with the proper handling there was no reason why an operator should not obtain satisfactory results through the use of such a polariscope. Information available through such use was (a) the boundary stress, (b) the maximum shearing stress, and (c) the direction of the planes of principal stress for any point in a model. The author furthur believes that this equipment may be used for a study of stress concentrations in fittings.

INTRODUCTION

The application of photoelastic principles to commercial problems in stress analysis is at present beset by difficulties. The greatest of these difficulties is a lack of general knowledge of the subject. Workers in fields that could use photoelasticity are largely blind to the vast wealth of information at their fingertips. This is a result of the condition that most workers in photoelasticity are skilled scientists interested in pure science or the complete breakdown of a specific problem.

Consequently, their writings have been of a highly theoretical or specialized nature and, as the complete problem of analysis is long and tedious, these papers have discouraged many prospective users of the method.

Another difficulty that has beset the worker in photoelasticity is that of equipment and its cost. All writers examined by the author have used a spot light source with an optical system for holding the rays parallel while passing through the model, then another optical system to collect the rays and project an image of the model on a screen. Also, quarter-wave plates are used to circularly polarize the light and two types of light source - a white and a monochromatic - must be available. While a system utilizing these means unquestionably is essential for a complete analysis, the author has developed a system that permits the determination of useful information while requiring only inexpensive and easily obtainable materials.

DISCUSSION

Photoelasticity is derived from the two words Phot-, a Greek word for light and Elasticity which refers not to any elastic nature of light but to the Theory of Elasticity - a theory based on the study of the equilibrium of perfectly elastic bodies under the action of external forces. In order to refresh the reader's knowledge of the phenomenon, a few brief statements will be made about the photoelastic effect.

The Photoelastic Effect

Some transparent, homogeneous, single-refracting materials, such as celluloid and Bakelite, become double refracting when they are subjected to deforming loads, and this double refraction will vary with a change in the strain condition.[1]

When plane-polarized light is passed through such a loaded material, with the plane of polarization at 45 degrees to the axis of the tension in the specimen, it leaves the specimen as two beams of plane-polarized light, one beam vibrating in a plane inclined at 90 degrees to the other, Figure 1, Section DD. Also, the rates of advance of the two beams - while in the material - are at different velocities, which cause these vibrations to be out of phase with each other after they leave the material. Since the white source light consisted of many vibrations of different wave lengths and as all vibrations are retarded approximately the same amount, some will interfere destructively and others will interfere constructively. Hence colors are formed. In short, Alexander[2] states that the deforming load has produced a temporary redistribution of molecules that makes the stressed material behave like a crystal. In the direction of a tensile stress the material is elongated and the particles are far apart; similarly, in the direction of a compressive stress the molecules are forced closer together, Figure 2. As a result of the change "a wave polarized in the direction of the tension is accelerated with respect to the wave polarized at right angles to the tension."[3] Consequently, the difference in phase of the two plane polarized rays of light will depend upon the

---

[1] N. Alexander, <u>Photoelasticity</u>, Rhode Island State College, 1936. 16 pp.

[2] Ibid.

[3] L.N.G. Filon, <u>Photoelasticity for Engineers</u>, Cambridge: University Press, 1936. 35 pp.

stress condition in the material and its thickness. In the diagrams of Figures 1 and 3 it is seen that the magnitude of the vector at Section DD, which depends upon the stress condition, will also determine the amplitude of the vibration emerging from the analyzer for this amplitude is a vector component of the vibration at Section DD.

When a non-optical, white-light system - such as used by the author - is employed, the interference patterns are quite complicated. For this type work "calibration curves" or color graphs (Figures 10, 11, and 12) are used.

EQUIPMENT

Polariscope

The polariscope, Figure 4, used for the author's work consisted of three main parts: a light source, a polarizer, and an analyzer. The light source was a No. 2 Photoflood bulb mounted in the center of an 11 inch aluminum mixing bowl. A 1/2 inch opening was provided around the edge of the bowl, and the "can" that held the bulb was partially cut away so as to provide a circulation of air to cool the bulb. Just in front of the reflector a 12 inch circular, translucent disk was mounted to serve as a diffuser for the light. This disk was made by painting a thin coat of white pigmented airplane dope on a piece of glass.

The method of polarizing the light at both the polarizer and the analyzer was that of reflection. In the polarizer the light source and the reflecting mirror were arranged so that the angle of incidence and the angle of reflection were each 57 degrees. This entire unit, comprising light source and mirror, was mounted in a 12 inch 114 degree elbow which, in turn, was attached to the table in a manner that left it free to rotate about the axis of the polarized beam between the polarizer and the analyzer.

The analyzer, Figure 5, was fundamentally a duplicate of the polarizer, the differences being that in this case the light source was omitted and that instead of the elbow being mounted directly on the table, an intermediate platform was used. This platform was provided so that the analyzer could be removed from the table thus giving the operator greater freedom to mount his model.

Tension Clamps

The clamp rig used to hold the tension pieces was made from 1 inch by 1/4 inch and 1 inch by 1 inch steel bar. The photograph, Figure 6, shows the details of construction and Figure 7 shows the clamps in use. For the upper end, two pieces of the 1/4 inch by 1 inch stock were arranged with a bolt to form a vise. This vise was then pin attached to the 1 inch by 1 inch member. The lower clamp consisted of another vise with an extra piece of linkage added to free the test piece of any undue bending moments. The faces of the vises were knurled for better gripping action.

CONSTRUCTION OF THE MODEL

The first step in constructing the model was to pick out a suitable piece of photoelastic material. For the following tests Bakelite BT 48-306, manufactured by the Bakelite Corporation, New York City, was chosen. This material was not the most desirable for photoelastic work because of its perishable nature. However, when the writer began his experiments it was the only available material that had come to his attention. The perishability of this Bakelite material becomes apparent in the form of edge effect, i.e., after a fresh edge is cut, it apparently dries out and produces a residual stress. Thus, whenever a model was to be tested. it had to be freshly cut. To select a section of the Bakelite sheet for a new model the entire sheet had to be examined in the polariscope to insure the exclusion of any areas containing residual stresses.

After an area was selected, a scriber was used to outline the model roughly on the Bakelite sheet. The model was then roughed out with a hand hack saw. In this roughing out process about 1/4 inch of excess material was left around the model. This was for two reasons: (1) to prevent the heat from friction of the hack saw blade from reaching into the model proper, and (2) to leave a margin to finish the model accurately in the milling machine. All roughing out was done under oil to help keep the Bakelite cool and to keep down the harmful Bakelite dust.

The next step was to mill off this excess material and accurately finish up the model. For this work a milling machine was used. All cutting was done under oil and with a very low cutting speed. The purpose was to insure furthur the fact that the model would be free of residual stresses when tested.

I Immediately upon being finished the model was placed in the room with the polariscope and allowed to stand for about 1/2 hour so as to come to room temperature.

ANALYSIS

The photoelastic method of stress analysis is based upon an experimental fact which states that in a model of uniform thickness the fringe order is constant at all points at which the difference between the principal stresses, p and q, is constant.[3]

These color fringes are brightest when the planes of the polarizer and analyzer are at 45 degrees to the direction of the principal stresses. This fact is best illustrated by the diagram of Figure 3. Here the vector along the 1-1 plane is the magnitude of the polarized beam. This is also shown in Figure 1, Section CC. When this beam of light enters the model, it is resolved into two components perpendicular to each other and parallel to the directions of principal stress in the model.[4] Thus, the original vector in the 1-1 plane resolved into components in the P-P and Q-Q planes as in Figure 1, Section DD, and Figure 2. Of these vectors along the P-P and Q-Q plane only those components which are parallel to the analyzer ultimately present themselves for observation, and the brightness of the observed light depends upon the magnitude of these components. Referring to Figure 3,

$$b = a \cos \Theta$$

$$c = b \cos \Phi$$

$$c = a \cos \Theta \cos \Phi$$

$$\Phi = 90 - \Theta$$

---

[3] Max M. Frocht, "The Place of Photoelasticity in the Analysis of Statically Indeterminate Structures," Carnegie Institute of Technology, Engineering Bulletin, 1938, 9 pp.

[4] E. E. Weibel, Lecture and Laboratory Course in Photoelasticity, University of Michigan, 1934, 4 pp.

$$\cos \Phi = \cos (90 - \Theta) = \sin \Theta$$

$$c = a \cos \Theta \sin \Theta$$

$$= \frac{a}{2} \sin 2 \Theta$$

or c is a maximum for $\Theta = 45^{\circ}$ and for any angle greater or less than 45° between the planes of the polarizer and analyzer and the planes of principal stresses, the observed color will be reduced in brilliance.

The condition at Section DD depends upon the double refracting properties of the material and, as previously cited, varies somewhat directly with the strain condition. Hence, for any point under observation, the brilliance of the color will be an expression for the strain condition in the model at that point. Since the stresses are usually critical at the edge of a model, the method explained here will be as applied only to the edge of the model.

The first step in the analysis was to make a tension calibration curve, this curve was a plot of stress in PSI vs. COLOR for a tension specimen, the planes of the polarizer and analyzer having been set at 45° to the axis of the specimen so as to obtain brilliant colors. This tension piece was made out of the same sheet of material as the model and was tested at about the same time, for different batches of Bakelite vary slightly in the treatment received during and after manufacture. Consequently, the optical properties of the Bakelite may vary.

To eliminate the possibility of a peculiar stress distribution in the tension piece it should be about four times as wide as thick and not less than five widths in length.

The tension piece was clamped, as shown in Figure 8, and a dead weight used to load it. The load was increased in increments of about 30 PSI. At each interval of loading, the PSI load and the color were recorded. These data were then plotted to obtain the tension calibration curve such as Figures 10, 11, and 12. The ordinate scale used for plotting the colors was arbitrarily selected so that the PSI vs. COLOR plot as derived from the tension test would be a straight line. Such a selection of color scale is permissible for when a polariscope without an optical system for keeping the light rays parallel while they pass through the model is used, the general diffusion of light rays around the particles in the photoelastic material--especially for the higher order color cycles--results in some colors being reinforced and others cancelled. Hence, the color observed through the polariscope depends not only on the stress condition and thickness of the model but also on the manner in which the light rays penetrate the model.

At the edge of a model the only stress is a tensile or compressive stress parallel to the edge of the model. Hence, if the plane of polarization is set at an angle of 45° to the edge of a loaded model, the polariscope will yield the brightest possible image of the photoelastic color at that point. Thus, to make an analysis of a point, the polarizer was set in this manner and the model slowly loaded up, an observer being on duty to record the color and the cycle of the color that was ultimately formed at the point in question. With this color information the calibration curve is consulted and opposite the observed color the tensile or compressive stress at the point could be read.

The author has tested several families of beams by this method and photographs of two tests in the same family are presented in Figures 13 and 14.

In Figure 13 the observed color in the outer fibers was second order blue. From the calibration curve for this model, Figure 10, second order blue corresponds to a stress of (360 ± 10) PSI. This 1.028 inch by .268 inch beam was of a 6 inch span and was loaded at two points, each one inch from a support. The loads were equal, hence the center four inches of the beam were loaded in pure bending only. A calculation with the flexural stress formula for this point A yields a value of 372 PSI, a difference in values of (3.2 ± 3.0) per cent. Frocht[5] in his investigations of beams determined that with the most careful analysis his results had an average discrepancy of 2.65 per cent. Hence, considering the equipment, the (3.2 ± 3.0) per cent discrepancy between the calculated and photoelastic value of stress is reasonable.

Figure 14 was the same beam but with a different load. Here the photoelastic value of the stress at the outer fivers was (325 ± 10) PSI against 318 PSI by the flexural formula, a difference of (2.2 ± 3.0) per cent.

A further extension of the information obtainable with this polariscope can be made to yield the maximum shearing stress at any point in a model. The shearing stress at any point is equal to 1/2 (p-q) where p and q are the principal stresses. The value of (p-q) may be obtained for any point by rotating the polarizer and analyzer as a unit until the color is brightest at the point in question, then determining the color. Generally, it is wise to unload and reload the model so as to be able to watch the colors at the point and make certain to which cycle the final

---

[5] Frocht, _op. cit._

color belongs. With this knowledge of the color at the point, (p-q) instantly becomes available by means of the tension calibration curve, and to evaluate the shearing stress requires only that we apply the relationship $f_s = 1/2\ (p-q)$.

Coker and Filon[6] also demonstrate that if the plane of polarization lines up with the plane of principal stress, complete extinction of color results. Such regions are visible through use of the equipment described here and can be photographed or drawn from observation. Figures 13 and 14 show these areas as heavy black bands through the neutral axis.

The photoelastic equipment described here may also be used to study stress concentrations at load points or other critical sections of a model as long as the stress gradient is not so great as to make the bands indistinguishable. Figure 15, a photograph of two Lucite blower blades one of which was permanently deformed by a centrifugal load, could be used for a study of stresses in a blower blade.

CONCLUSIONS

From the results of this investigation the following conclusions seem justified:

(1) That a workable polariscope incorporating the method of reflection for its polarizing action can be built from inexpensive and easily obtainable materials.

(2) That using such a polariscope in conjunction with the methods

---

[6] E. G. Coker and N. G. Filon, A Treatise on Photo-Elasticity, (Cambridge: The University Press, 1931).

explained in this thesis enabled the author to obtain the following information about the stress at any point in a model:

(a) The boundary stress,

(b) The maximum shearing stress,

(c) The directions of the planes of principal stresses.

(3) That to obtain accurate results the tension calibration test and the model test should be made at the same temperature.

(4) That a Bakelite model should be tested within two hours after it is cut.

BIBLIOGRAPHY

Alexander, N., Photoelasticity, Rhode Island State College, 1936. pp 1-44.

Coker, E.G., and Filon, L.N.G., A Treatise on Photoelasticity, Cambridge: University Press, 1931. pp 416-432.

Filon, L.N.G., Photoelasticity for Engineers, Cambridge: University Press, 1936. 35pp.

Fried, Bernard, and Weller, Royal, "Photoelastic Analysis of Two and Three Dimensional Stress Systems", Ohio State University, Engineering Bulletin No. 106, July, 1940. pp 1-8.

Frocht, Max M., "A Photoelastic Investigation of Shear and Bending Stresses in Centrally Loaded Beams," Carnegie Institute of Technology, Engineering Bulletin, 1937. 2 pp.

______, "The Place of Photoelasticity in the A nalysis of Statically Indeterminate Structures," Carnegie Institute of Technology, Engineering Bulletin, 1938. pp 3-14.

Frocht, Max M., and Riggs, Norman C., Strength of Materials, The Ronald Press Company, 1938. pp 365-380.

Hausmann, Erich, and Slack, Edgar P., Physics, D. Van Nostrand Company, 1935, pp 724-729.

Weibel, E.E., Lecture and Laboratory Course in Photoelasticity, University of Michigan, 1934. pp 3 & 20-25.

TABLE I

Data for Tension Calibration Curves

1-7-41 T 23½ C
Model .272 by .507

PSI	Color
84	Yellow
120	Red
171	Blue
230	Green
266	Yellow Green
302	Red
338	Red
392	Light Green
448	Green
520	Rose
556	Rose Green
594	Green
630	Pale Green
666	Rose
700	Rose
734	Pale Green
775	Pale Green

Model showed a residual straw color after being unloaded.

1-16-41 T 24 C
Model .238 by .750

PSI	Color
28	Clear
70	Faint Yellow
140	Yellow
168	Orange
196	Red
252	Blue
280	Blue Green
308	Green
336	Yellow Green
364	Light Orange
392	Red
448	Purple
485	Green
532	Light Green
588	Light Red
616	Red
644	Light Red
730	Light Green
780	Pale Green
840	Pink
925	Light Green

TABLE II

Data for Tension Calibration Curves
3-23-41 T 23 C

Model .257 by 1.075		Model .258 by .940		Model .252 by .930	
PSI	Color	PSI	Color	PSI	Color
24	Frosty	27	Clear	28	Clear
42	Frosty	48	Frosty	49	Frosty
78	Frost Yellow	110	Frost Yellow	92	Frost Yellow
113	Pale Yellow	130	Pale Yellow	113	Pale Yellow
132	Pale Straw	172	Yellow	156	Yellow
151	Yellow	188	Orange Yellow	177	Yellow Orange
187	Orange Red	214	Orange Red	195	Blue
223	Blue	260	Blue	221	Blue
296	Blue Green	337	Yellow Green	263	Rich Blue
332	Yellow Green	378	Yellow	306	Green
404	Pale Red	420	Rose Red	348	Yellow
440	Red Purple	461	Red	392	Rose
477	Green	502	Rose Green	434	Red
513	Bright Green	544	Bright Green	477	Green
565	Rose Green	585	Light Green	518	Bright Green
621	Red	627	Rose Green	562	Rose Green
695	Rose	667	Rose Red	605	Red
730	Green	780	Green	648	Rose
767	Light Green	834	Light Green	690	Green
804	Rose Green	875	Rose Green	735	Green
840	Rose	917	Rose	776	Rose

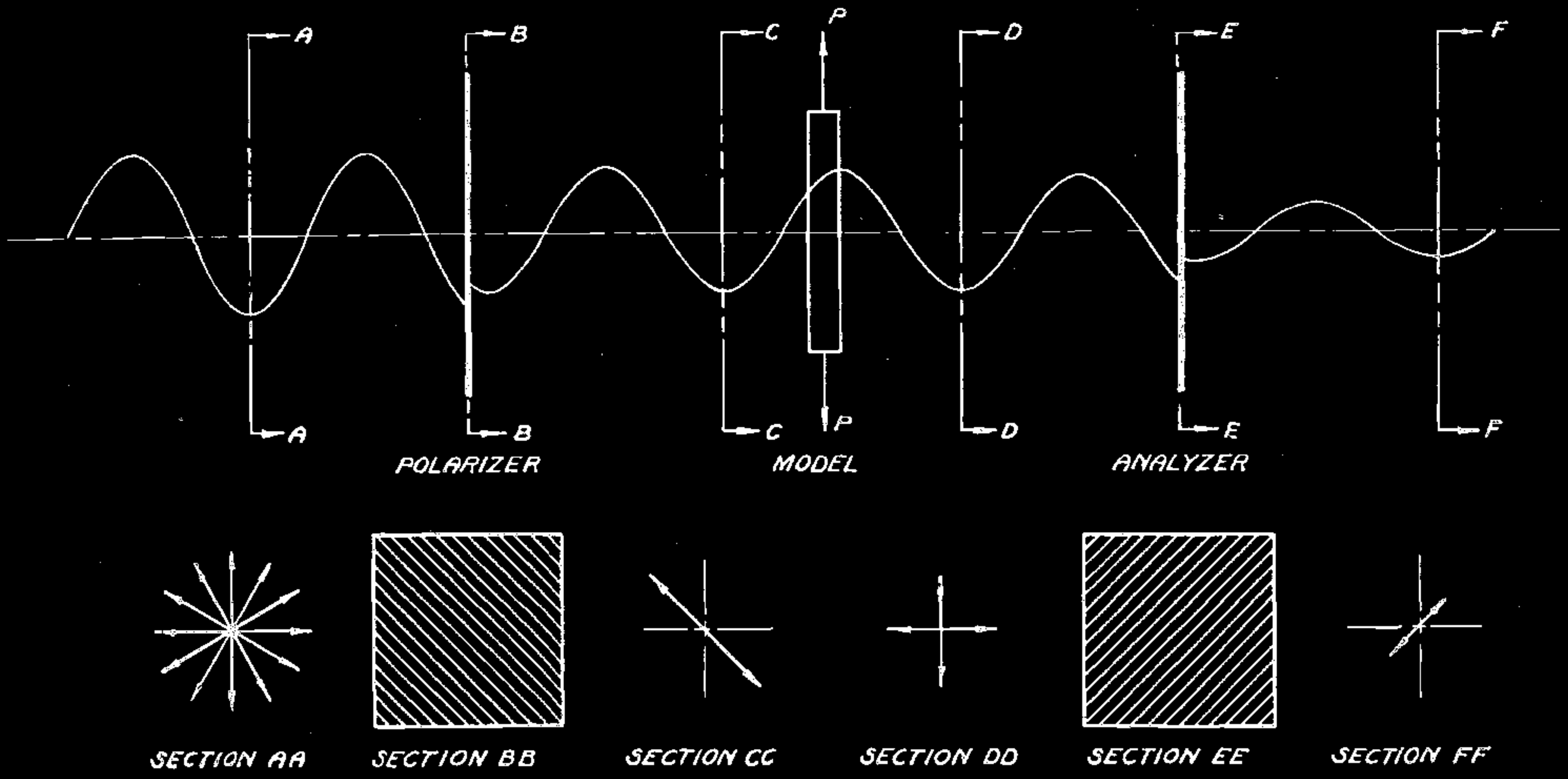
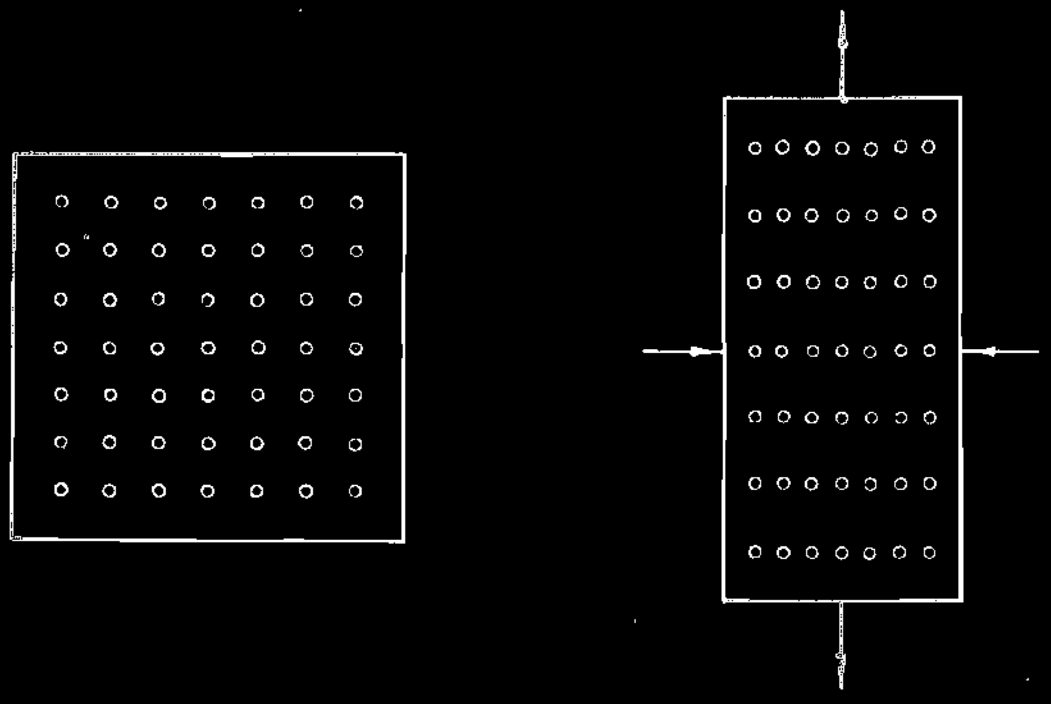


FIGURE 1. PHOTOELASTIC EFFECT

JERRY HOFFER



UNSTRESSED SYSTEM OF PARTICLES

STRESSED SYSTEM OF PARTICLES

FIGURE 2

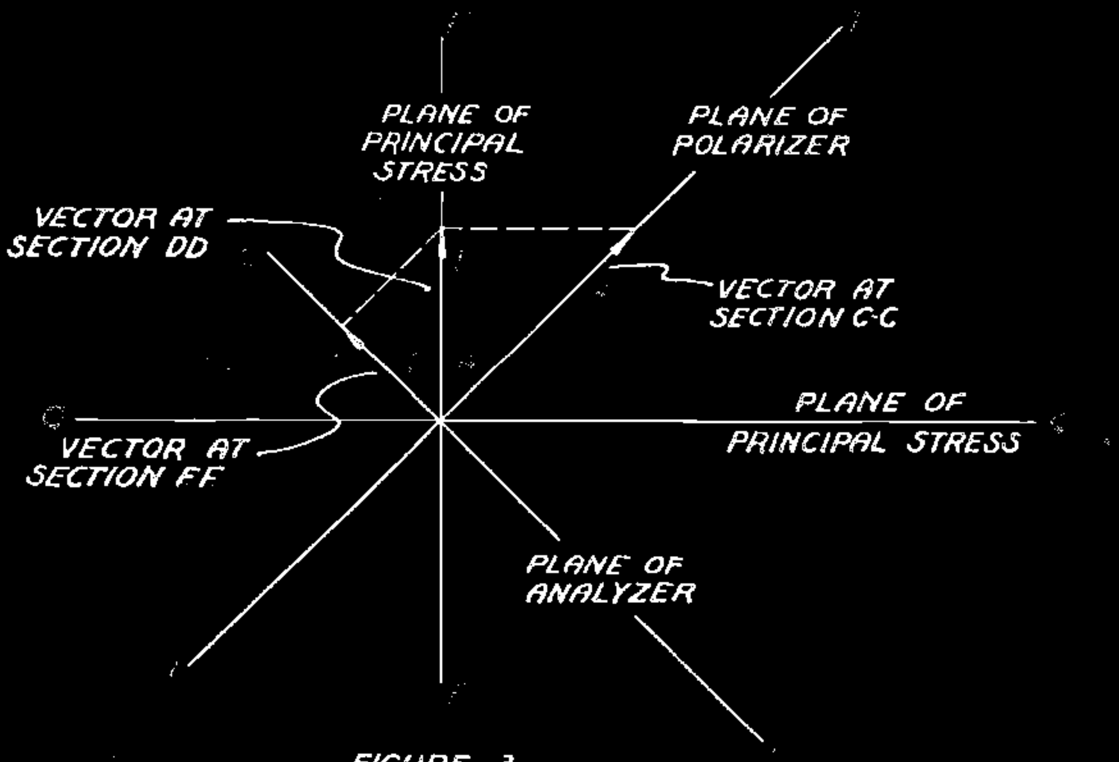
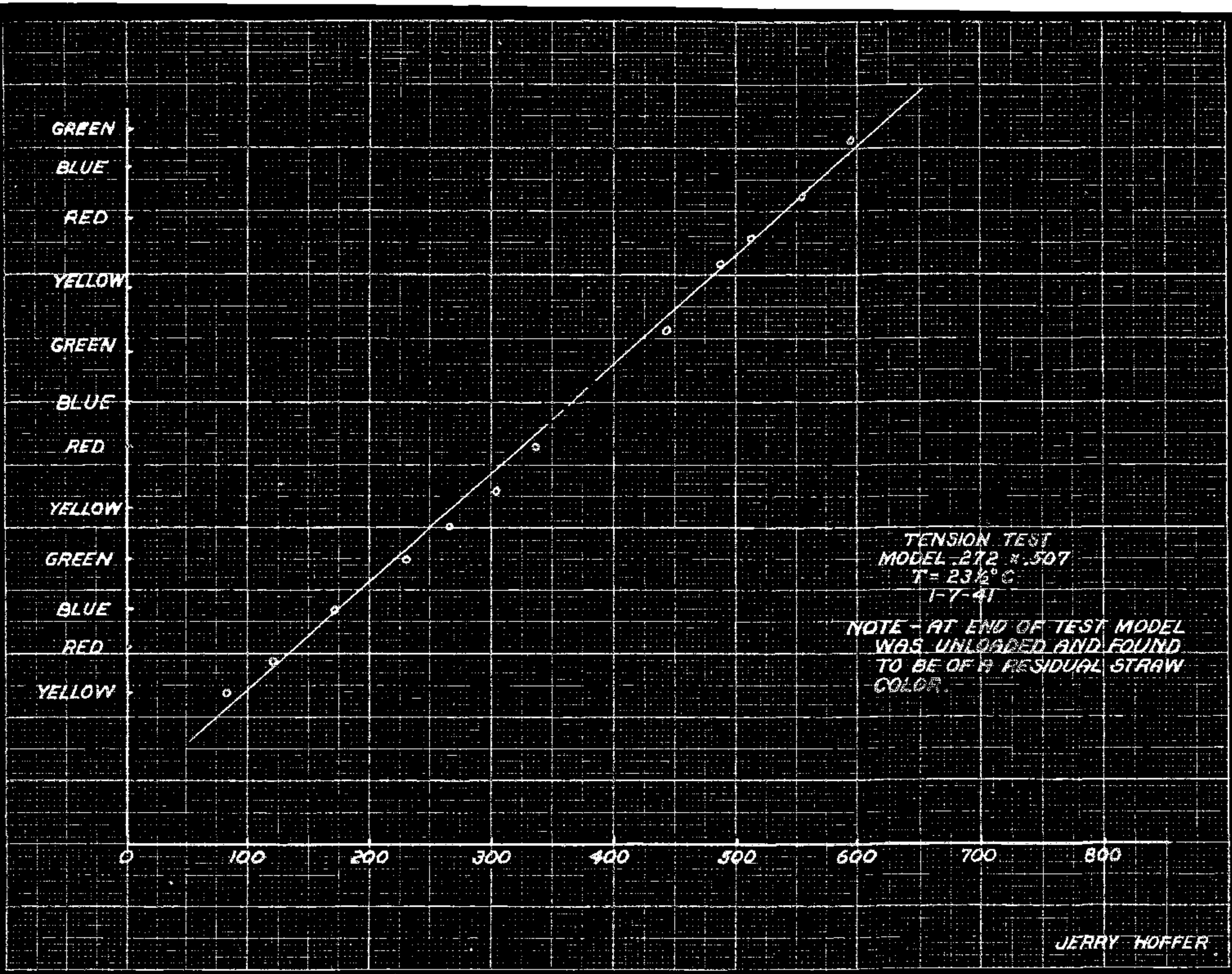


FIGURE 3

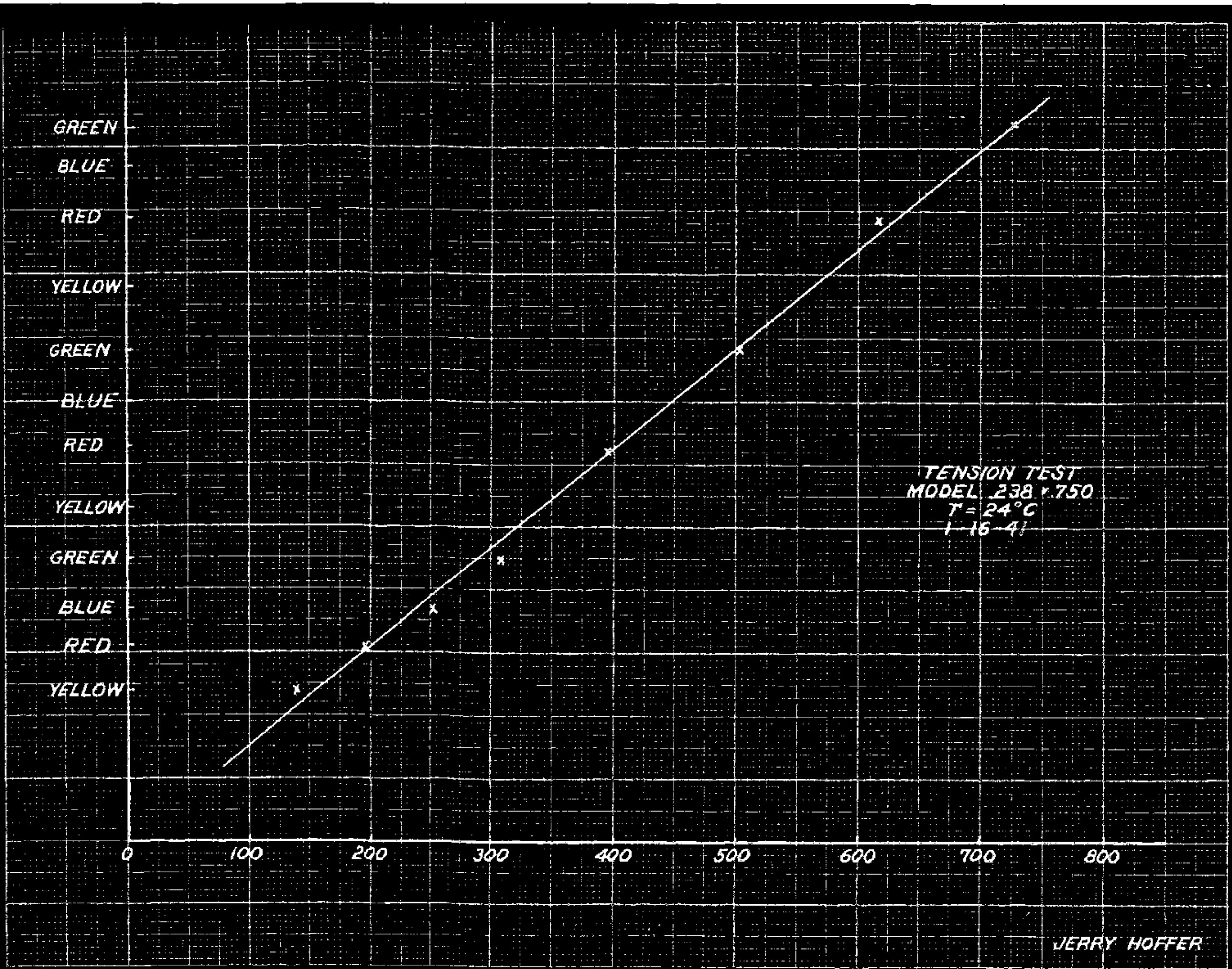
JERRY HOFFER

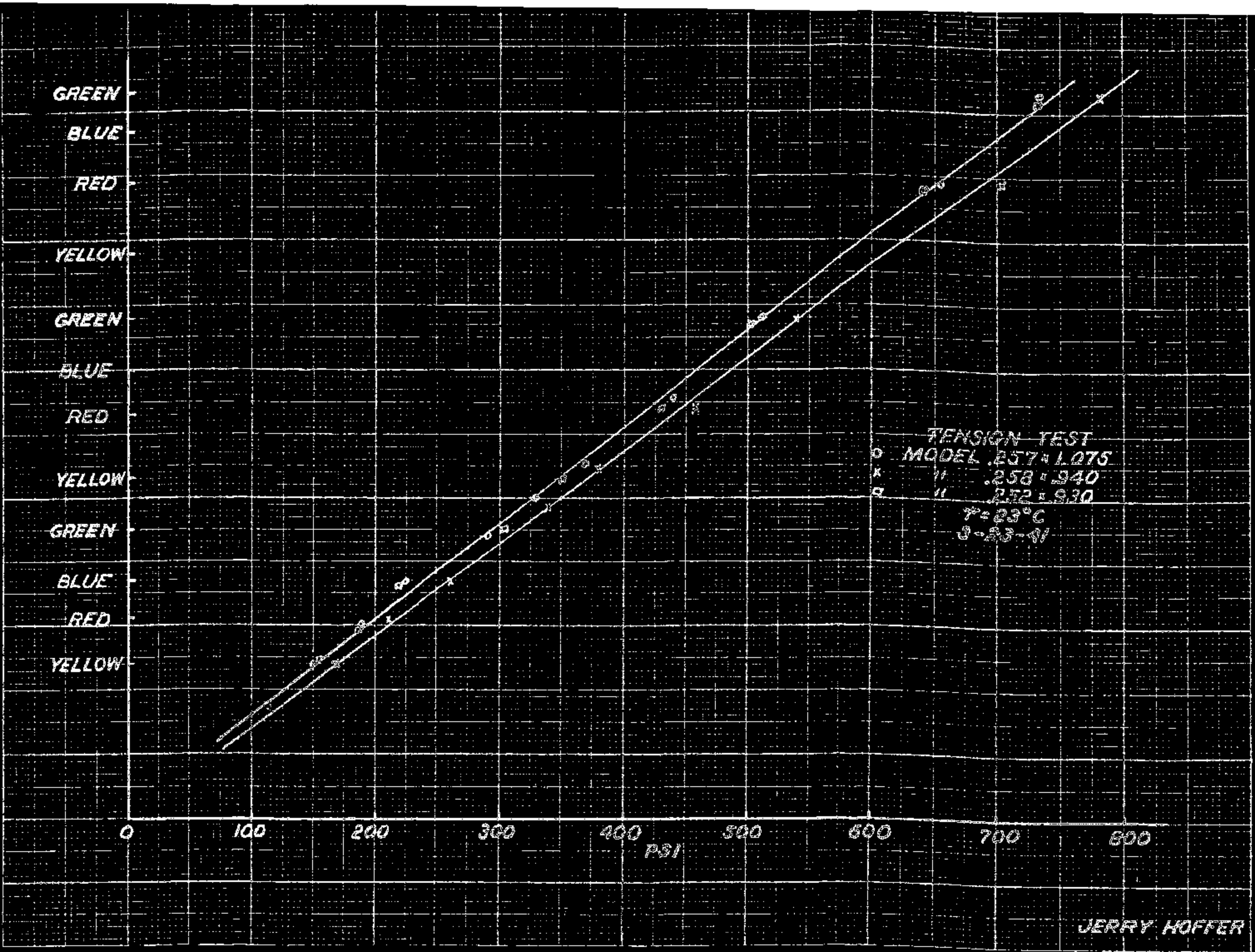


TENSION TEST
MODEL .272 x .507
T = 23½° C
1-7-41

NOTE - AT END OF TEST MODEL WAS UNLOADED AND FOUND TO BE OF A RESIDUAL STRAW COLOR.

JERRY HOFFER


GREEN
BLUE
RED
YELLOW
GREEN
BLUE
RED
YELLOW
GREEN
BLUE
RED
YELLOW
0
100
200
300
400
500
600
700
800
TENSION TEST
MODEL .238 x .750
T = 24°C
1-16-41
JERRY HOFFER


GREEN
BLUE
RED
YELLOW
GREEN
BLUE
RED
YELLOW
GREEN
BLUE
RED
YELLOW
0
100
200
300
400
500
600
700
800
PSI
TENSION TEST
○ MODEL .257 x 1.075
x " .258 x .940
□ " .252 x .930
T = 23°C
3-23-41
JERRY HOFFER