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Alvin Lowe, Jr.
HYDRAULIC RESISTANCE IN TWO-PHASE FLOW AND ITS DEPENDENCE ON THE FROUDE NUMBER

A THESIS
Presented to
the Faculty of the Graduate Division

by
Alvin Lowi, Jr.

In Partial Fulfillment
of the Requirements for the Degree
Master of Science in Mechanical Engineering

Georgia Institute of Technology
December, 1955
HYDRAULIC RESISTANCE IN TWO-PHASE FLOW AND ITS DEPENDENCE ON THE FROUDE NUMBER

Approved:

Date Approved by Chairman: 14 Dec. 1955
To my wife,

GUILLERMINA GERARDO LOWI,

whose devotion is a constant source
of inspiration.
ACKNOWLEDGMENT

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SUMMARY

Some recent Soviet research on hydraulic resistance in the flow of air-water mixtures purports to show a very general functional dependence of this resistance on the Froude modulus. The author, S. I. Kosterin, specifically states that the deviation of the resistance for the two-phase flow in horizontal pipes from the flow resistance law of a homogeneous fluid is a unique function of the Froude number and the volume fraction of gas in the mixture. The effect of diameter is said to be reflected in the Froude number where the diameter is used as the characteristic length.

The objective of this research was to examine the Kosterin correlation for its validity and applicability. Based on the definitions of Kosterin, the data of Lockhart and Martinelli, and Gazley, are reduced to the Kosterin form to show experimental similarity, and to attempt a direct correlation for comparison with the results in question. The correlation of Lockhart and Martinelli was examined in order to demonstrate any compatibility with Kosterin's separate method of treating the same kind of data.

The results of the effort to correlate experimental data using the parameters given by Kosterin were inconclusive. Resistance parameters calculated from the data were several times greater in magnitude than any predicted by Kosterin.

The Correlation of Lockhart and Martinelli, although not perfect, was found lacking in dependence on the Froude number.
Experimental verification of Kosterin's work was found to be necessary.

Although the utility of the Froude modulus is doubtful as a significant parameter for correlating two-phase flow resistance data in general, some indication exists that it may influence the transition between stratified and slugging flow and thus serve as an index of such transition.
CHAPTER I

INTRODUCTION

The prevalence of systems involving the simultaneous flow of gases or vapors with liquids in circular pipes gives rise to the need for understanding the many facets of the flow mechanisms involved. A principal one among these is the rule governing the pressure variation during this so-called two-phase flow.

The flow of homogeneous fluids — meaning fluids of a single phase and uniform composition — that completely fill the conduit in which they are flowing is a comparatively well ordered problem. Laws for the prediction of pressure drop in this case are well established and give reliable results in practice; but the occurrence of two phases during flow requires extensive modification to the ideas employed in this development. An example of this departure showing the modified treatment in the handling of the integration of momentum equation in two-phase flow is given by Gladden, Goglia and Ward (1).

The hydrodynamics of two-phase flow has been the subject of considerable research in the past fifteen years. Foster, Gresham, and Kyle (2), in their recent survey of the technical literature on two-phase flow, give an excellent and complete history of research on the subject with summaries of the major contributions in the field. One may conclude from this work that the theoretical explorations contribute little to the understanding of experimental results, a prominent discrepancy in seeking appropriate
correlation parameters. Until 1949, when Lockhart and Martinelli (3) proposed their empirical correlation, no generally useful correlation was available.

In 1949, S. I. Kosterin (4) published the results of some experimental studies made in Soviet Russia on hydraulic resistance and flow structure in the flow of air-water mixtures in horizontal and inclined tubes of various diameters. He purports to show a functional dependence of hydraulic resistance in the horizontal tubes on the Froude modulus (5). Since this is the only known work of its kind an inquiry was made into the validity and reproducibility of Kosterin's claims.

The objective of this research was to analyze the work of Kosterin (4) in an attempt to reveal, first, the obscured experimental details and, second, the validity of the purported correlation. The procedure proposed for the investigation was outlined in three steps.

(1) The definitions of Kosterin were examined to establish theoretical consistency and to discover any thermodynamical or mechanical restrictions their use implies. Then, using basic concepts and other information given by Kosterin (6), the approximate environmental conditions were established in order to ascertain the exit conditions and to make comparisons with other experimental data.

(2) Reliable data available from the literature which most nearly matches the data Kosterin could have had were selected (by virtue of conditions ascertained in step (1) above). All such data were reduced to the Kosterin form and an attempt was made to correlate it directly.

(3) Subject to a correspondence of definitions and thermodynamic conditions, the compatibility of the proposed correlation of Lockhart and
Martinelli (3) with Kosterin's results (4) was determined.

A review of the literature revealed few specific references to Froude number studies of hydraulic resistance. Hom-Ma (7) examined the Froude number influence on head loss in the free surface flow of water in rectangular flumes. Although his results may have little connection with two-phase flow in closed conduits, one might suppose that some qualitative analogy exists for stratified flow (8). In this respect, the important free surface flow phenomena hydraulic jump and slugging have definite dependencies on the Froude modulus. Rouse (9) has shown that the depth ratio of a hydraulic jump is theoretically a unique function of Froude number when the characteristic dimension is the upstream depth. This relationship is in close accord with the experimental evidence of Bakhmeteff and Matzke (10). Rouse further shows that the influence of this Froude number on the energy losses occurring in jumps predominates over the viscous shear effects.
CHAPTER II

THE PHYSICAL SYSTEM

Since this research is concerned with a comparison of the works of Kosterin (4) and Lockhart and Martinelli (3), the hypothetical system described here is one which probably approaches closest conditions common to both experiments. Categorically, it is a system wherein a gas and a liquid flow in the same direction in a common conduit without exchange of mass (11).

The physical system might consist of a length of bare, horizontal, smooth tubing provided with static pressure taps at each end. Air and water are metered separately and introduced through a calming section prior to passage through the test length. The flow discharges at ambient conditions at exit from the test section. Means are provided for such heating, cooling, and recirculation as necessary to insure that temperature equilibrium exists between the phases and the surroundings so that an isothermal flow is effectively maintained. Consequently, the air would be preserved saturated with water vapor and the absorption and evolvement of air by the water would be kept to a minimum. Thus, inter-phase mass transfer is practically eliminated.

No consideration appears necessary to qualify the flow structures (6, 8) which may occur. Therefore, it may be assumed that any or all patterns may exist.
Definitions and Environmental Analysis.—Analysis of Kosterin's definitions (4) shows substantial conformity with current local usage. Subject to the translation from the Russian and a transformation of nomenclature, his key definitions will be stated here for later reference. Particular note is made wherever a definition implies thermodynamical or mechanical restrictions not explicitly given by the author.

The chief resistance parameter, $\psi$, called the "corrected coefficient of resistance", represents the deviation of the two-phase flow resistance from the resistance law for a homogeneous fluid. $\psi$ is defined as the ratio of Weisbach friction factors of the two-phase mixture to that for a homogeneous fluid taken at the Reynolds modulus $Re^*$; or

$$\psi = \frac{f_m}{f_{Re^*}}$$  \hspace{1cm} (1)

The Reynolds number $Re^*$, defined by the equation

$$Re^* = \frac{h \, \bar{w}_m}{\pi \, D \, \mu_L}$$  \hspace{1cm} (2)

is actually fictitious. Note that the liquid phase, while flowing simultaneously with the gas phase, is not conventionally described by this
Reynolds number. Since neither phase always completely fills the pipe, the tube diameter is not a characteristic dimension. Nor is Re* the mixture Reynolds number, as Kosterin implies, since the viscosity of the liquid is used.*

The mixture friction factor is defined by a Weisbach type equation,

\[ f_m = 2 \left( \frac{\Delta P}{\Delta l} \right)_m \frac{D}{\rho_m u_m^2} \]  

(3)

The "delivered mixture density", \( \rho_m \), is defined as

\[ \rho_m = \rho_d \rho_g + (1 - \rho_d) \rho_l \]  

(4)

where \( \rho_d \) is the "delivered volumetric concentration of gas." This definition implies that the properties of both phases are independent of the radial coordinate.

The parameter \( C \) is dimensionally "the volume of gas per unit volume of mixture." It is consistent with the customary notion of thermodynamic quality if defined as

\[ C = \frac{U_g}{U_g + U_l} \]  

(5)

where \( U \) is volume. This requires the experimental achievement of making an accurate measurement of the volume of each phase at a given section of

---

*The viscosity of a heterogeneous two-phase mixture is a property which lacks utility at this time. There is a complete absence of information of this quantity from the literature.
pipe simultaneously. Lacking a reasonable explanation for this feat, it was assumed that the volumes in Equation (5) were replaced by the volume rates of flow,

\[ v_g = \frac{W_g}{\rho_g} \]  \hfill (6)

and

\[ v_l = \frac{W_l}{\rho_l} \]  \hfill (7)

Now, \( C \) may be written

\[ C = \frac{v_g}{v_g + v_l} \]  \hfill (8)

and this form permits easy determination of \( C_d \) from elementary data. However, its use implies the theoretical restriction that the average velocities of the phases are equal.

The Froude number is defined in the customary manner

\[ F_r = \frac{U_m^2}{g D} \]  \hfill (9)

where \( U_m \) is the "superficial velocity of the mixture" defined by the steady flow continuity equation for the mixture

\[ U_m = \frac{W_m}{\rho_m A} \]  \hfill (10)

As usual

\[ A = \frac{\pi D^2}{4} \]  \hfill (11)
The general statement of continuity implies

\[ W_g + W_A = W_m \]  

(12)

In determining the environment (pressure and temperature at pipe exit) in which Kosterin's experiments with flow resistance \((k)\) were performed, the assumption was made that the conditions were the same for the study of flow structures \((6)\). Then, a method for this purpose results from examining Equation (10) with respect to Fig. 1 (12). The combination of equations \((4)\), \((10)\), and \((11)\) leads to the expression

\[ U_m = \frac{4 W_m}{\pi D^2 \left[ C_d \rho_g + (1-C_d) \rho_l \right]} \]  

(13)

which is the analytical expression for the lines of constant total mass flow of Fig. 1 in terms of the parameters \(W_m\), \(D\), \(t\), and \(P_d\). The limiting values of \(C_d\) from Fig. 1 (that is, \(C_d\) equal to unity for all air and \(C_d\) equal to zero for all water) and their intercepts with the constant \(W_m\) lines give all the information needed to calculate the densities of the saturated water and the saturated air at exit conditions. The temperature is then determined from a table of properties of saturated water as a function of temperature such as Keenan and Keyes \((13)\). The approximate exit pressure may be calculated from the delivered density of the saturated air using the Gibbs-Dalton Law \((14)\) and the ideal gas equation of state

\[ P = \rho_g R_g T \]  

(14)

where \(T\) is the absolute temperature previously determined and \(R_g\) is the specific gas constant for dry air. The \(W_m\) lines are practically asymptotic
to the condition of all gas flow \((C_d = 1)\); therefore, use of the condition \(C_d = 0.9\) is made to calculate the air density. The execution of this procedure shows that the exit temperature and pressure prevailing in Kosterin's experiments \(\left(\text{4, 6}\right)\) are 210° F and 15.153 psia, respectively.*

**Experimental Data Analysis.**—An ample body of data is available from Lockhart and Martinelli \(\left(\text{15}\right)\), and Gazley \(\left(\text{16}\right)\), which originated from experiments similar in most respects to the one described in Chapter II. These are data for the isothermal flow of air and water in smooth tubes of 1.017 and 2.065 inches internal diameter having a test length of approximately twenty feet. The only significant discrepancy lies in the temperature where its average value was about 60° F compared to 210° F for Kosterin.

Essentially, these data consist of simultaneous readings of the mass flow rates of the gas and liquid, temperature, average static pressure, and the static pressure loss of the two-phase mixture per unit length of test section. These quantities provided the necessary information to calculate the quantities of Kosterin \(\left(\text{4}\right)\) via the definitions set forth previously.** The reduced data appears in Tables 3 and 4 of Appendix A.

**Investigation of the Compatibility of the Lockhart and Martinelli Proposed Correlation \(\left(\text{3}\right)\) with the Kosterin Results \(\left(\text{4}\right)\).**—In order to advance a comparison of these two works, the Lockhart and Martinelli correlation

---

*See Appendix B, Summary of Computations, Items 1 through 4.

**See Appendix B, Summary of Computations, Items 5 through 19.
is reviewed briefly stating the postulates and definitions employed.

The development of the generalized parameters of Lockhart and Martinelli followed two basic postulates, namely:

(1) the static pressure drop for the liquid phase equals that for the gas phase regardless of flow structure provided no appreciable radial pressure gradient exists, and

(2) the volume of the liquid phase plus the volume of the gas phase equals the total volume of the pipe at any instant.

By algebraic deduction they propose to show the physically significant dimensionless variables based on the above postulates and then prove the significance by successfully correlating random experimental data.

The use of dynamics or dimensional analysis is avoided completely in attempting to bypass the complex microscopic phenomena accompanying two-phase flow.

From Postulate (1), the two-phase pressure drop is assumed expressible as a function of either the gas phase or liquid phase pressure drops when these phases are flowing alone in the pipe. In terms of the liquid phase, then,

$$\left( \frac{\Delta P}{\Delta l} \right)_m = \Phi_L \left( \frac{\Delta P}{\Delta l} \right)_L$$  (15)

where $\Phi_L$ is a dimensionless parameter.

A new dimensionless variable $\chi$ is defined such that

$$\chi^2 = \frac{\left( \frac{\Delta P}{\Delta l} \right)_L}{\left( \frac{\Delta P}{\Delta l} \right)_G}$$  (16)
Finally, it is postulated that subject to experimental verification, 
$\Phi_\lambda$ is a function of $X$. Subsequent treatment of experimental data on 
$\Phi_\lambda$ and $X$ show a correlation effective within plus 20 and minus 30 
percent of a faired curve.*

If the flow is considered for the case where both the gas and 
liquid phases are turbulent (17), then it can be shown that

$$X^2 = \left(\frac{\rho_g}{\rho_l}\right) \left(\frac{\mu_l}{\mu_g}\right)^{0.2} \left(\frac{W_l}{W_g}\right)^{1.8}$$ (17)

Similar expressions are available for the cases where the liquid is 
viscous and gas is turbulent, liquid is turbulent and gas is viscous, and 
both phases are viscous. The criteria proposed to establish turbulent or 
viscous flow in either phase are noted as arbitrary and tentative.

To establish the compatibility of the Lockhart and Martinelli work 
with the Kosterin proposal, the parameters $\Phi_\lambda$ and $X$ were examined 
for Froude number dependency. Assuming the correlation $\Phi_\lambda$ versus $X$ 
is unique there remained only to show the dependence of $X$ on the Froude 
number. The first two factors of Equation (17) are seen to depend only 
on the thermodynamic state. The third factor may be expressed as

$$\frac{W_l}{W_g} = \frac{\rho_l}{\rho_g} \left[ \frac{1-Gd}{C_d} \right]$$ (18)

*The term "faired curve" evidently means a line drawn by 
inspection which best fits the data plotted.
which results from a combination of definitions, Equations (6), (7), and (8). Equation (18) shows a complete lack of Froude number dependence; therefore, $X$ and likewise $\Phi_\lambda$ are independent of the Froude number.
CHAPTER IV

RESULTS

The analysis of definitions used by Kosterin shows substantial conformity with current practice; however, a number of thermodynamic restrictions were implicitly demanded through the use of some. They, mainly, require the assumptions of (1) one dimensional, steady, isothermal flow, (2) equal phase velocities, and (3) thermodynamic equilibrium.

The experimental environment of Kosterin's study of hydraulic resistances, based on the presumption that experimental data of the same nature is used in his study of flow structures, was found to be about 210° F, exhausting to an atmosphere at 15.153 pounds per square inch, absolute. This result lacks reliability, however, as the determination of the exit pressure required that the difference of two large numbers yield the density of the gas, a comparatively small number.* These differences were often less than zero. This effect might easily rise from quite small experimental and graphical errors.

The similarity of experiments is close between Kosterin, Lockhart and Martinelli, and Gazley even though the temperature of the former differs from the latter two by 150° F. This temperature discrepancy apparently has little effect since discrete points from the data of the match closely their corresponding points on Fig. 1.

*See Appendix B, Summary of Computations, Item 3(c).
The reduced data of Table 3 failed to provide the means for reproducing Kosterin's Froude number functions. Lack of sufficient data at any given Froude number prevented their construction directly. Failure accompanied several attempts to correlate the data to auxiliary parameters with intentions of making a graphical transformation to the $\psi - \text{Fr-C}_d$ surface. The usefulness of the reduced data was seriously limited due to the lack of a uniform distribution of runs over the whole range of values of $C_d$. Over half of the data reduced was for values of $C_d$ greater than 0.98.

It is of interest to note that values of $\psi$ were frequently encountered (see Table 3) which were several times greater in magnitude than any predicted by Kosterin, Figure 2.

The Lockhart and Martinelli Proposed Correlation was shown to lack any dependence whatever on the Froude modulus. Thus, the Lockhart and Martinelli correlation can not be put into the form suggested by Kosterin for correlating data for two-phase flow.
CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

The following conclusions appear justifiable based on the preceding arguments:

(1) The data of Lockhart and Martinelli, Gazley, and the presumed data of Kosterin have similarity.

(2) The Proposed Correlation of Lockhart and Martinelli contradicts the claim by Kosterin of a functional dependence of hydraulic resistance on the Froude modulus in two-phase flow in pipes.

(3) Those values of \( \phi \) that recur at several times the magnitude of any predicted by Kosterin, seriously question the validity of his results.

(4) The results of Hom-Ma (7) indicate no appreciable dependence of hydraulic resistance on Froude number in the open channel, tranquil flow of water.

(5) The correlation of Lockhart and Martinelli is known to be incomplete so that conclusive authentication of Kosterin's claims must be left to actual experimentation where the Froude modulus is a controlled parameter.

Recommendations for the future study of the Froude modulus as a significant parameter in two-phase hydrodynamics includes its use as a transition criterion between stratified and slugging flow (8). Gazley (18) indicates that the formation of slugs following stratified two-phase flow
occurs upon the breaking of the interfacial surface waves, a phenomenon wherein the Froude number plays an important role. Actual observation of slug formation during stratified flow indicates that slugs may be caused for reasons other than the breaking of surface waves. The appearance of some slugs draws a close likeness to transient hydraulic jumps. Thus, the Froude number may yet be significant as an influential parameter on hydraulic resistance in pipes for the restricted cases of transitory stratified and slugging flow should an analogy with open channel flow be exhibited.

The analytical approach to the subject of hydraulic resistance in two-phase flow is still highly perplexing. A more realistic correlation of experimental data is needed and, in this respect, the recommendation is made that more study be given to the development of dynamic criteria characterizing two-phase flow. Further attention might be directed toward the evaluation of viscosity as a property of a heterogeneous fluid.
### APPENDIX A

Table 1. Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Area of tube cross-section</td>
<td>in$^2$ or ft.$^2$</td>
</tr>
<tr>
<td>C</td>
<td>Volumetric concentration of gas</td>
<td>in or ft.</td>
</tr>
<tr>
<td>D</td>
<td>Tube diameter</td>
<td>lb$_m$/ft$^2$sec.</td>
</tr>
<tr>
<td>G</td>
<td>Mass velocity</td>
<td>°R</td>
</tr>
<tr>
<td>T</td>
<td>Absolute temperature</td>
<td>ft/sec</td>
</tr>
<tr>
<td>U</td>
<td>Average velocity</td>
<td>ft$^3$/sec</td>
</tr>
<tr>
<td>V</td>
<td>Volume rate of flow</td>
<td>lb$_m$/sec</td>
</tr>
<tr>
<td>W</td>
<td>Mass rate of flow</td>
<td>ft$^3$/sec</td>
</tr>
<tr>
<td>X</td>
<td>Lockhart and Martinelli parameter</td>
<td>ft$^3$</td>
</tr>
<tr>
<td>U</td>
<td>Volume</td>
<td>ft$^3$/in$^2$</td>
</tr>
<tr>
<td>f</td>
<td>Weisbach friction factor</td>
<td>°F</td>
</tr>
<tr>
<td>g</td>
<td>Acceleration of gravity</td>
<td>ft/sec$^2$</td>
</tr>
<tr>
<td>l</td>
<td>Length of test section</td>
<td>ft</td>
</tr>
<tr>
<td>p</td>
<td>Static pressure</td>
<td>lb$_m$/in$^2$</td>
</tr>
<tr>
<td>t</td>
<td>Temperature</td>
<td>°F</td>
</tr>
<tr>
<td>v</td>
<td>Specific volume</td>
<td>ft$^3$/lb$_m$</td>
</tr>
<tr>
<td>P</td>
<td>Lockhart and Martinelli parameter</td>
<td>ft$^3$/lb$_m$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Corrected coefficient of resistance</td>
<td>ft$^2$/lb$_m$</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Dynamic viscosity</td>
<td>lb$_m$/ft$^3$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density</td>
<td>lb$_m$/ft$^3$</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>A finite increment</td>
<td>in$^2$ or ft.$^2$</td>
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Table 1. Nomenclature (continued)

<table>
<thead>
<tr>
<th>Symbol</th>
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<tbody>
<tr>
<td>Re</td>
<td>Reynolds' modulus</td>
<td></td>
</tr>
<tr>
<td>Fr</td>
<td>Froude's modulus</td>
<td></td>
</tr>
<tr>
<td>Rg</td>
<td>Specific gas constant</td>
<td></td>
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</tbody>
</table>

Subscripts

- m: Two-phase mixture, two-phase flow
- g: Gas phase, two-phase flow
- l: Liquid phase, two-phase flow
- G: Gas phase flowing alone
- L: Liquid phase flowing alone
- d: Delivered, or at exit conditions

Superscript

- *: Denotes fictitious property
Table 2. Conversion Factors

1 metric ton = 1000 kilograms

1 kilogram = 2.2 pounds

1 pound = 16 ounces

1 meter = 3.32 feet

1 foot = 12 inches

1 pound force = 32.2 pounds mass feet per sec.$^2$

1 reyn = $\frac{1 \text{lb}}{\text{ft-sec}} = 6.72 \times 10^{-4}$ centipoise

Specific gas constant for dry air = 53.3 $\frac{\text{ft lb}}{\text{lb}_m \cdot \text{F}}$
Table 3. Reduced Data of Lockhart and Martinelli, Group Eight (15).

(Isothermal flow of air and water in a glass pipe of 1.017 inches internal diameter, 21.6 feet long)

\[ T = 60 \, ^\circ\text{F}, \quad p_d = 15.0 \, \text{psia} \]

<table>
<thead>
<tr>
<th>Run</th>
<th>( \dot{m} )</th>
<th>( C_d )</th>
<th>( \rho_m )</th>
<th>( U_m )</th>
<th>( F_r )</th>
<th>( f_m )</th>
<th>( Re^* )</th>
<th>( f_{Re^*} )</th>
<th>( \psi )</th>
</tr>
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<tr>
<td>lb(_m)/sec</td>
<td>lb(_m)/ft(^3)</td>
<td>ft/sec</td>
<td>x10(^{-4})</td>
<td></td>
<td></td>
<td></td>
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<tr>
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<td>0.75</td>
<td>15.7</td>
<td>32.3</td>
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<td>6.15</td>
<td>0.0200</td>
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<td>0.51</td>
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(Continued)
Table 3. Reduced Data of Lockhart and Martinelli, Group Eight (15)

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Table 4. Reduced Data of Gazley (16)

(Stratified isothermal flow of air and water in a 2.065 in. I.D. tube 20.0 feet long)

\[ T = 72^\circ F, \quad p_d = 14.7 \text{ psia} \]

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APPENDIX B

SUMMARY OF COMPUTATIONS

Environmental Study of Kosterin's Experiments.

1. Determination of density of water in Kosterin's experiments.
   (a) Let $C = 0$, then $\rho_m = \rho_a$ and $W_m = W_a$.
   (b) From Fig. 1 for the 1 in. tube at $C = 0$, let $W_m = W_a = 0.162 \text{ lb/} \text{sec}$
       giving an intercept at $U_m = U_a = 0.195 \text{ ft/sec}$.
   (c) Equation (13) reduces to
       \[ U_a = \frac{h W_a}{\pi \rho_a}, \]
       and solving for $\rho_a$,
       \[ \rho_a = \frac{576 W_a}{\pi U_a} = \frac{576 \times 0.162}{3.14 \times 0.195} = 60.0 \text{ lb/ft}^3 \]

2. Determination of environment temperature neglecting pressure dependence
   of water density: from Keenan and Keyes (13) p. 31, the temperature
   corresponding to a density of saturated water of 60.0 \text{ lb/ft}^3 or
   \[ v_a = \frac{1}{\rho_a} = 0.0167 \text{ ft}^3/\text{lb_m} \]
   is $210^\circ \text{ F}$.

3. Determination of the density of saturated air from Figure 1.
   (a) Let $C = 0.9$ then where $\rho_a = 60.0, \text{ lb/ft}^3$, Equation (13) becomes
       \[ U_m = \frac{576 W_m}{\pi (0.9 \rho_a + 6)} \]
(b) From Fig. 1, where \( W_m = 0.162 \text{ lb/sec} \) the intercept with \( C = 0.9 \)
gives \( U_m = 4.92 \text{ ft/sec} \)

(c) Solving for \( P_g \) in Equation (13)

\[
P_g = \frac{576 \times 0.162}{3.1 \times 0.92 
\times 0.9} - 6.66 = 6.72 - 6.66
\]

\[
P_g = 0.06 \text{ lb/ft}^3 \text{ at exit conditions.}
\]

4. Determination of exit pressure from air density (assuming \( R_g = \frac{53.3}{60^o F} \) for dry air)

(a) Vapor pressure and density of the saturated water vapor, from

Keenan and Keyes, at 210 \( ^o F \) is 14.123 psia and 0.036 lb/ft\(^3\)
respectively.

(b) From Gibbs-Dalton Law of Gas Mixtures (in thermodynamic
equilibrium)

\[
P_g = P_{air} + P_{vapor}
\]

\[
P_{air} = P_g - P_{vapor} - (0.06 - 0.036) \text{ lb/ft}^3
\]

\[
P_{air} = 0.024 \text{ lb/ft}^3
\]

(c) From Equation (14) where \( t = 210 ^o F \)

\[
P_{air} = \frac{R_g \times T}{T_{air}} \frac{53.3 \times 670.0}{0.024 \times 144} = 1.03 \text{ psia}
\]

(d) From Gibbs-Dalton Law again for partial pressures, the exit
pressure is

\[
P_d = P_{air} + P_{vapor} = 14.123 + 1.03
\]

\[
P_d = 15.153 \text{ psia}
\]
Reduction of Data from Lockhart and Martinelli (15) [Data from run B-23 shown].--

5. Exit pressure

\[ P_{d} = P_{\text{avg}} - \frac{4}{2} \left( \frac{\Delta P}{\Delta l} \right) \]

\[ P_{d} = 15.5 - \frac{21.6}{2} \times 0.077 = 14.67 \text{ psia.} \]


(a) From Keenan and Keyes (13), saturated vapor pressure and specific volume are 0.285 psia and 1091.4 ft³/lbm.

(b) Density of gas is sum of densities of air and saturated vapor.

Using Equation (12)

\[ \rho_{g} = \frac{(P_{d} - P_{\text{vapor}})}{R_{g} T_{vapor}} + \frac{1}{V_{\text{vapor}}} \]

\[ \rho_{g} = \frac{(14.67 - 0.285) \times 146.2}{53.3 \times 523} + \frac{1}{1091.4} \]

\[ \rho_{g} = 0.075 \text{ lbm/ft}^{3} \]

7. Density of liquid phase at 63 °F and 14.67 psia (neglecting compressibility) from Keenan and Keyes (13), saturated liquid, is 62.3 lbm/ft³.

8. Mixture mass rate of flow from Equation (12)

\[ \bar{w}_{m} = \bar{w}_{l} + \bar{w}_{g} = 1.96 + 0.00073 = 1.96073 \text{ lb/sec} \]

9. Volume rate of flow of gas phase (or liquid phase), from Equation (6)

\[ \bar{v}_{g} = \frac{\bar{w}_{g}}{\rho_{g}} = \frac{0.00073}{0.075} = 0.00975 \text{ ft}^{3}/\text{sec.} \]
10. Delivered volume fraction of gas from Equation (8)

\[ C_d = \frac{V_g}{V_g + V_l} = \frac{0.00975}{0.00975 + 0.034} = 0.237 \]

11. Viscosity of liquid phase at 63 °F, from McAdams (19), linearly interpolated,

\[ \mu_l = \frac{2.6}{3600} = 7.22 \times 10^{-4} \frac{lbm}{ft \cdot sec} \]

12. Fictitious Reynolds number, from Equation (2), for \( D = 1.017 \) in., is

\[ Re^* = \frac{\frac{4 W_m}{\pi D \mu_l}} {\frac{W_m}{\pi D \mu_l}} \]

\[ Re^* = \frac{1 x 1.96073 x 12}{3.14 x 7.22 x 10^{-4}} \]

\[ Re^* = 4.15 \times 10^4 \]

13. Homogeneous flow friction factor at \( Re^* \), taken from Crane Company Chart (20) for smooth tubes, is 0.023.

14. Mixture density delivered, from Equation (4), is

\[ \rho_m = C_d \rho_g + (1-C_d) \rho_l \]

\[ \rho_m = 0.237 x 0.075 + 0.763 x 62.3 \]

\[ \rho_m = 47.7 \frac{lbm}{ft^3} \]

15. Tube area, from Equation (11) is

\[ A = \frac{\pi}{4} D^2 = \frac{3.14}{4} x (1.017)^2 = 0.785 x 1.03 = 0.809 \text{ in}^2 \]

\[ A = \frac{0.809}{\frac{1}{144}} = 0.00561 \text{ ft}^2 \]
16. Mixture superficial velocity, from Equation (10) is

\[ U_m = \frac{W_m}{\rho_m A} \]

\[ U_m = \frac{1.96073}{47.7 \times 0.00515} = 7.33 \text{ ft/sec} \]

17. Mixture friction factor, from Equation (3) is

\[ f_m = 2 \left( \frac{\Delta P}{\Delta L} \right)_m \frac{D}{\rho_m U^2_m} \]

\[ f_m = 2 \frac{0.077 \times 0.0846 \times 32.2 \times 1.41}{47.7 \times (7.33)^2} \]

\[ f_m = 0.0236 \]

18. Corrected coefficient of resistance, from Equation (1), is

\[ \psi = \frac{f_m}{f_{Re}^4} = \frac{0.0236}{0.0236} = 1.02 \]

19. Froude number, from Equation (9), is

\[ Fr = \frac{U^2_m}{gD} \]

\[ Fr = \frac{(7.33)^2}{32.2 \times 0.0846} = 19.8 \]
FIGURE 1. FLOW CONTINUITY CHART. For air and water in horizontal 1 and 2 in. ID tubes, from Kosteris (18).
FIGURE 2. FROUDE NUMBER FUNCTIONS OF KOSTERIN (4). For 1, 2, 3, and 4 inch tubes.

NOTE: Compressibility neglected.
BIBLIOGRAPHY


5. Ibid., Part 1, Figure 1.

6. Ibid., Part 2.


20. Crane Company, Technical Paper Number 409, May, 1942, Figure 5.