VERTICAL ASCENT ANALYSIS OF THE LIFTING AISRCREW

A THESIS

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of the requirements for the Degree
of Master of Science in Aeronautical Engineering

by

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Preface

Symbols Used

$T = \text{Thrust force.}$

$\rho = \text{Air density.}$

$\Gamma = \text{Total circulation.}$

$B = \text{Number of blades.}$

$c = \text{Blade chord.}$

$\alpha = \text{Angular velocity of rotor.}$

$r = \text{Distance of element from rotor axis.}$

$R = \text{Length of blade (rotor radius).}$

$c_l = \text{Airfoil section lift coefficient.}$

$a = \text{Slope of lift curve} = \frac{d c_l}{d \alpha}$

$\alpha = \text{Blade angle of attack.}$

$\phi = \text{Induced angle.}$

$W = \text{Total downwash.}$

$\gamma = \text{Angle of relative wind.}$

$w = \text{Induced flow or downwash.}$

$V = \text{Air velocity.}$

$\sigma = \text{Rotor solidity.}$

$x = \frac{r}{R} = \text{ratio of element distance to radius.}$

$\lambda = \frac{\nu}{\alpha R} = \text{ratio of air velocity to rotor tip velocity.}$
\( \varphi' = \varphi + \delta' = \text{Total downwash angle (or down flow).} \)

\[
C_T = \frac{T}{\frac{1}{2} \pi R^2 \rho^2 \bar{R}^2}.
\]

\[
C_0 = \frac{Q}{\frac{1}{2} \pi R^2 \rho \bar{R}^2 R^3}.
\]

Subscripts:
- \( t \) = tip conditions.
- \( o \) = static conditions.

\[ T_o = \frac{C_T}{\sigma^2}. \]

and \[ Q_o = \frac{C_0}{\sigma \bar{R}^3}. \]

All angles having a subscript \( \sigma \) are \( \frac{\text{Angle}}{\sigma} \).

Example: \( \theta_o = \frac{\theta}{\sigma} \).
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Summary

An extension of the theoretical and experimental investigation, as given by Montgomery Knight and E. A. Hofner in N.A.C.A. Technical Notes No. 626, to cover the vertical ascent of the lifting airscrew is presented in this paper. The theory is based on Glauert's analysis with modifications as recommended by Mr. Knight. Thrust and torque expressions for both constant-pitch and constant-incidence airscrews are developed.

The constant-incidence expression is in terms of the blade parameters only. Either the static thrust values of the blades or the blade parameters may be used to determine the rotor characteristics in vertical ascent when using the constant-pitch (constant-circulation) theory.

Experimental test data were obtained in the wind tunnel for verification of the theory.

Introduction

Increasing interest in the development of the gyroplane and helicopter has shown the need for a simple theoretical analysis for the determination of rotor characteristics in vertical motion. Glauert, Reference 1, has presented a paper developing a theory for the performance of a helicopter in vertical ascent.

During the last six years a general study of rotating wing aircraft has been conducted at the Daniel Guggenheim School of Aeronautics, Georgia School of Technology. Financial assistance for this research has been
provided by the Georgia State Engineering Experiment Station. The study has included both experimental tests in the nine foot wind tunnel and the development of a suitable theoretical analysis. In the first published report, Reference 2, on the lifting airscrew the elimination of the solidity factor as a separate parameter in rotor calculations was indicated as being both feasible and practical. A second portion of this general study will cover the ground effect on the lifting airscrew and is now in preparation.

The present investigation is a continuation of the general study and develops the theory for the case of vertical ascent. The vortex theory, based on Glauert's assumptions, Reference 1, and further developed by Knight and Heafner, Reference 2, has been used in the theoretical portion of this investigation.

Discussion

Basic Assumptions

The following simplifying assumptions were made in the development of the theory and are identical with those made in Reference 2.

1. The number of blades may be taken as infinite.
2. All angles associated with the induced velocity are small, thus permitting the substitution of the angle for the sine and tangent of the angle and unity for the cosine.
3. Rotational and radial components of velocity and tip effects may be neglected.
4. The slipstream contraction may be neglected.
**Induced Velocity**

The induced velocity may be developed by two different methods. In the one case the thrust and torque coefficients are calculated from the known blade parameters. The general expressions developed can then be simplified for special cases and approximations. By use of the second method developed in this paper the rotor characteristics in vertical ascent can be obtained in terms of the static thrust coefficients. This latter method gives a simplified expression for induced velocity and is similar to the method used in Reference 3.

**First Method:** The first method to be considered will be that of using the known parameters of the rotor and determining from them the rotor coefficients. The induced velocity relationship due to the vortex field as determined in Reference 2 is:

\[
\frac{d\theta}{dz} = 2\omega
\]

\[= \frac{\Gamma}{Bd} \quad \text{Inside the cylinder.} \quad (1)
\]

In order to derive a general expression for the circulation, \(\Gamma\), as a function of the blade profile characteristics, consider an element of a blade of length \(dr\) at a distance \(r\) from the axis of rotation, as shown in Fig. 3. The thrust on this element will then be:

\[
dT = \rho \frac{\Gamma}{B} \omega r dr
\]

\[= \frac{\rho}{2} C_2 \omega^2 r^2 c dr, \quad \text{or} \]

\[
\Gamma = \frac{\rho \omega \omega c}{2} \quad C_4. \quad (2)
\]
The assumption that \( C_t \) varies linearly with \( \alpha \) places the restriction on the theory that the expressions developed will hold only over the straight portion of the lift curve.

Now, referring to Fig. 1 for the rotor element:

\[
C_t = a \alpha
\]

\[= a(\theta - \varphi').\]

where\[\varphi' = (\varphi + \delta)\]

and\[\varphi = \tan^{-1} \frac{w}{\Delta r}\]

\[\varphi = \tan^{-1} \frac{V}{\Delta r}.\]

Substituting \( C_t \) in Eq. 2, the circulation becomes

\[
\Gamma = \frac{B_5}{2} \Delta r \alpha (\theta - \varphi').
\]

The distance between successive turns of the vortex helix is:

\[
d = \frac{2\pi r}{B} \tan \varphi'
\]

\[= \frac{2\pi r}{B} \tan (\varphi + \delta)
\]

\[= \frac{2\pi r}{B} \frac{\tan \varphi + \tan \delta}{1 - \tan \varphi \tan \delta}.
\]

Assuming that \( \varphi \) and \( \delta \) are small, then the product of their tangents
will equal zero approximately.

Therefore:

\[ d = \frac{2 \pi r}{B} \left( \frac{w}{\alpha r} + \frac{v}{\alpha r} \right) \quad (4) \]

Substituting the values of \( \alpha \) and \( d \) obtained above into Eq. 1,

\[ 2w = \frac{B \pi}{2} \eta \alpha a (\theta - \gamma') \]

\[ \frac{B \pi}{2} \frac{2 \pi (w + v)}{B \alpha} \]

\[ = \frac{a}{\eta} \frac{B \pi}{2} \eta \alpha^2 r (\theta - \gamma') \]

\[ w(w + v) = \frac{a}{\eta} \frac{B \pi}{2} (\theta - \gamma') \eta^2 r . \]

Let

\[ \sigma = \frac{B \pi \alpha^2}{2 \alpha^2} \]

\[ = \frac{B \pi \alpha}{2 \alpha} \]

the solidity factor.

Then

\[ w^2 + \frac{v}{\eta} \sigma (\theta - \gamma') \eta^2 r R = 0 . \]

Dividing through by \( \eta R \) and remembering that

\[ \lambda = \frac{v}{\eta R} \]

\[ = \gamma c \]

\[ \frac{w^2}{\eta R} + \lambda w - \frac{a}{\eta} \sigma (\theta - \gamma') \eta^2 r = 0 . \]
Dividing through by \( \Delta \mathcal{R} x^2 \)

\[
\frac{w^2}{J^2 \mathcal{R}^2 x^4} + \lambda \frac{w}{\mathcal{R} x^2} - \frac{a}{b} \sigma (\theta - \theta') \frac{1}{x} = 0
\]

\[
\theta^2 + \frac{1}{x} \theta - \frac{a \sigma}{b x} (\theta - \theta') = 0 . \tag{5}
\]

Since \( \theta' \) contains the unknown, \( \theta \), its components are substituted:

\[
\theta^2 + \left( \frac{\lambda}{x} + \frac{a \sigma}{b x} \right) \theta - \frac{a \sigma}{b x} (\theta - \theta') = 0
\]

\[
\theta = \frac{1}{2} \left[ -\frac{\lambda}{x} - \frac{a \sigma}{b x} + \sqrt{\left( \frac{\lambda}{x} + \frac{a \sigma}{b x} \right)^2 + 4 \frac{a \sigma}{b x} (\theta - \theta')} \right] .
\]

Dividing both sides by \( \sigma \):

\[
\frac{\theta}{\sigma} = \frac{1}{2x} \left[ -\lambda \sigma - \frac{\sigma}{b} + \sqrt{\left( \lambda \sigma + \frac{\sigma}{b} \right)^2 + 4 \frac{\sigma}{b} (\theta \sigma - \theta') x} \right] . \tag{6}
\]

Since

\[
\lambda = \frac{v}{\mathcal{R} A}
\]

and

\[
\tau = \frac{v}{\mathcal{R} \mathcal{R}}
\]

therefore,

\[
\theta = \frac{\lambda \sigma}{x}
\]

and

\[
\frac{\theta}{\sigma} = \frac{\lambda \sigma}{x}
\]

Substituting into Eq. 6:

\[
\frac{\theta}{\sigma} = \frac{1}{2x} \left[ -\lambda \sigma - \frac{\sigma}{b} + \sqrt{\left( \lambda \sigma + \frac{\sigma}{b} \right)^2 + 4 \frac{\sigma}{b} (\theta \sigma - \frac{\lambda \sigma}{x}) x} \right] . \tag{7}
\]
This equation is a general expression for the induced flow angle in terms of the radius variable $X$, $\Theta_0$, and $A_0$; $\Theta$ being effectively a constant for a given blade or airfoil. This is the equation for the constant incidence rotor, where $\Theta_0 = \text{constant}$.

The case of constant pitch can be developed in the same manner by making the proper substitutions for $\Theta$ and $\theta'$ to obtain the tip conditions. However, it was found that the following method used to calculate the constant-pitch condition proved simpler and gave the same equations.

**Second Method:** The development of the induced velocity by the second and simpler method will now be considered for constant-pitch blades.

Substituting Eq. 4 into Eq. 1 and holding $\Delta$ constant:

$$\frac{d\tau}{dz} = \frac{\tau}{\beta d}$$

$$= \frac{\tau}{B \frac{2\pi (w + V)}{B \Delta}}$$

$$= \frac{\tau \Delta}{2\pi (w + V)}$$

$$= 2w;$$

$$\frac{\tau \Delta}{4\pi} = w(w + V)$$

$$= W_0^2.$$

where $W_0$ refers to the downflow for static conditions due to the lift obtained from the blade. Now dividing both sides of the equation by $\Delta^2 R^2$ and solving for $\tau'$:

...
\[ \phi_e (\phi_e + \lambda) = \phi_e^2 \]

\[ \phi_e^2 + \lambda \phi_e - \phi_e^2 = 0 \]

\[ \phi_e = \frac{1}{2} \left[ -\lambda \pm \sqrt{\lambda^2 + 4 \phi_e^2} \right] \]

\[ \phi_{e_0} = \frac{1}{2} \left[ -\lambda_0 \pm \sqrt{\lambda_0^2 + 4 \phi_{e_0}^2} \right]. \quad (8) \]

In Reference 2 for static conditions is found:

\[ \left[ \phi_x = \frac{\sigma_{x_0}}{\sigma_x} (\theta - \varphi) \right]_0 \]

Since the case of constant pitch is being considered, the following substitutions can be made:

\[ (\theta = \frac{\phi_x}{x})_0 \]

\[ (\varphi = \frac{\phi_x}{x})_0 \quad \text{(Reference 2)} \]

Substituting and solving for \( \sigma_{e_0} \):

\[ \left[ \frac{\sigma_{e_0}}{\sigma} = \frac{\lambda_0}{2} (\theta_0 - \phi_e) \right]_0 \]

\[ \left[ \frac{\sigma_{e_0}}{\sigma} = \frac{\lambda_0}{2} (\theta_0 - \phi_e) \right]_0 \]

\[ = T_{e_0}. \quad \text{(Reference 2)} \]
Therefore:

\[ \gamma_{\phi_0} = \frac{\tau_{\phi_0}}{\nu} \]

Eq. 8 now becomes:

\[ \gamma_{\phi} = \frac{1}{2} \left[ \lambda_{\phi} \pm \sqrt{\lambda_{\phi}^2 + \tau_{\phi}} \right] \]

When

\[ \tau_{\phi} = 0 \]

\[ \gamma_{\phi} = 0 \]

therefore the expression is:

\[ \gamma_{\phi} = \frac{1}{2} \left[ -\lambda_{\phi} + \sqrt{\lambda_{\phi}^2 + \tau_{\phi}} \right] \]  \hspace{1cm} (9)

Thus, for constant pitch, the induced velocity angle has been expressed in terms of the static thrust, \( \tau_{\phi} \), and the velocity parameter, \( \lambda_{\phi} \).

Thrust

Glauert, Reference 1, showed that the thrust produced by a rotor may be expressed as follows:

\[ T = \frac{P}{2} \pi R^3 \omega^2 R^2 C_T \]  \hspace{1cm} (10)

Now in terms of the rotor parameter:

\[ T = \frac{P}{2} B C \int_0^R C_4 \omega^2 r^2 \, dr \]

Multiplying by \( \frac{P}{R^3} \) and substituting for \( C_4 \).
Substituting the value of \( T \) obtained in Eq. 11 into Eq. 10 and solving for \( C_T \):

\[
C_T = \frac{\frac{2}{3} Bca R^3 \int_0^\infty (\theta - \varphi') x^3 dx}{\frac{P}{2} \pi R^2 \Omega^2 R^2}
\]

\[
C_T = \frac{Bc}{R^2} a \int_0^\infty (\theta - \varphi') x^2 dx.
\]

\[
T_\sigma = \frac{C_T}{\alpha_1^2}
\]

\[
= \frac{\alpha_1}{\alpha_2} a \int_0^\infty (\theta - \varphi') x^2 dx.
\]

With the above equation for \( T_\sigma \) it is possible to obtain the complete equation by merely substituting the induced angle equations for either case, i.e., constant incidence or constant pitch.

**Constant Incidence;** - Substituting Eq. 7 into Eq. 12:

\[
T_\sigma = a \int_0^\infty (\theta_\sigma - \frac{\Delta \varphi}{x} - \frac{1}{2} x \left[ (\lambda_\sigma + \frac{g}{2}) + \sqrt{(\lambda_\sigma + \frac{g}{2})^2 + 4 \lambda (\theta_\sigma - \frac{\Delta \varphi}{x})^2} \right]) x^2 dx
\]
Upon integrating:

\[
T_0 = \alpha \left\{ \frac{\theta}{3} - \frac{\Lambda \theta}{2} + \frac{1}{4} \left( \Lambda_\sigma + \frac{a}{B} \right) + \frac{\left[ E - 6 \frac{a}{B} \theta_\sigma \right] \left[ E + 4 \frac{a}{B} \theta_\sigma \right]^{\frac{3}{2}}}{120 \left( \frac{a}{B} \right)^2 \theta_\sigma^2} \right\} (13)
\]

Where

\[
E = \left[ \left( \Lambda_\sigma + \frac{a}{B} \right)^2 - 4 \left( \frac{a}{B} \right) \Lambda_\sigma \right] .
\]

**Constant Pitch:** The thrust on each element of the blade is:

\[
dT = \rho \left( \frac{f'}{B} \right) W dr = \rho \left( \frac{f'}{B} \right) \omega r dr = \rho \left( \frac{f'}{B} \right) \omega R^2 dx .
\]

\[
T = \rho \left( \frac{f'}{B} \right) \omega R^2 \int_0^x dx = \frac{\rho \left( \frac{f'}{B} \right) \omega R^2}{2} .
\]

From Eq. 10

\[
C_T = \frac{\rho \left( \frac{f'}{B} \right) \omega R^2}{\frac{1}{2} \pi R^2 \omega \cdot \frac{R^2}{2}} = \frac{f'}{B \pi R^2 \omega} .
\]
and for the entire rotor

\[ C_r = \frac{\Gamma}{\pi R^2 \Omega} \]
\[ T_\sigma = \frac{C_r}{\sigma} \]
\[ = \frac{\Gamma}{\pi R^2 \Omega \sigma} \]
\[ = T_{\sigma_0}. \]

If \( \Omega \) is held constant

\[ T_\sigma = T_{\sigma_0} \quad \text{for constant circulations.} \quad (14) \]

This means that the thrust is now considered to be the independent variable. However, if it is desired to calculate the thrust coefficient for constant-pitch blades using the blade parameters, it is possible to develop the equation by use of this simplified induced angle equation, as follows:

Remembering that for the constant-pitch rotor

\[ \theta_\sigma = \frac{\theta_{\sigma e}}{x} \]

and

\[ \psi_\sigma = \frac{\psi_{\sigma e}}{x} \]

From Eq. 12

\[ T_\sigma = a \int \left( \frac{\theta_{\sigma e}}{x} - \frac{\psi_{\sigma e}}{x} \right) x^2 dx \]
\[ = \frac{a}{2} \left( \theta_{\sigma e} - \psi_{\sigma e} \right) \]

Thus:

\[ T_\sigma = \frac{a}{2} \left( \theta_{\sigma e} - \psi_{\sigma e} - \lambda_\sigma \right). \quad (15) \]
Now, substituting for \( \sigma_c \), the expression obtained in Eq. 9 and solving for \( \sigma_c \): (Note: Care must be taken in observing the sign changes when both sides of the equation are squared or the correct root will not be obtained.)

\[
\sigma_c = \frac{a}{2} \left[ \theta_{\sigma_c} + \frac{\sigma_c}{2} - \sqrt{\frac{\sigma_c^2 + T_o}{2}} - \lambda_o \right]
\]

Substituting

\[
T_o = T_{o_0}
\]

as was proven in Eq. 14 and rearranging:

\[
\frac{2T_o}{a} + \frac{\lambda_o}{2} - \theta_{\sigma_c} = -\frac{\sqrt{\lambda_o^2 + T_o}}{2}
\]

\[
\left( \frac{4T_o}{a} + \lambda_o - 2 \theta_{\sigma_c} \right)^2 = -\left( \lambda_o^2 + T_o \right)
\]

\[
\frac{16T_o}{a^2} + \frac{8T_o \lambda_o}{a} - \frac{16T_o \theta_{\sigma_c}}{a} + T_o - 4 \theta_{\sigma_c} \lambda_o + 4 \theta_{\sigma_c}^2 + 2 \lambda_o^2 = 0
\]

\[
\frac{T_o}{a^2} + \left( \frac{\theta_{\sigma_c}}{a} - \alpha \theta_{\sigma_c} + \frac{a^2}{16} \right) T_o + \frac{a^2}{4} \left( \theta_{\sigma_c}^2 - \theta_{\sigma_c} \lambda_o + \alpha^2 \lambda_o \right) = 0
\]

\[
T_o = \frac{a}{2} \left[ \theta_{\sigma_c} - \frac{\sigma_c}{2} - \frac{a}{16} + \sqrt{\left( \frac{\sigma_c}{2} + \frac{a}{16} \right)^2 - \frac{\theta_{\sigma_c} a^2}{8} - \frac{1}{2} \lambda_o^2} \right].
\]

**Torque**

The expression for the torque produced by a rotor is also given by Glauert, Reference 1:

\[
Q = \frac{c}{2} \pi R^2 a^2 R^3 C_q
\]

The total torque is the summation of three separate torques: the induced torque, the minimum profile torque, and the profile variation torque.
Each of these components will be considered separately and the torque coefficient will be developed for each component. The total torque coefficient, $C_T$, will then equal the sum of the three component coefficients.

Induced Torque:

In terms of the rotor parameters the induced torque due to an element of the rotor blades is:

$$Q_i = \frac{F}{2} B c \int_0^R c_2 (\tan \varphi') \Omega^2 r^3 dr$$

Multiplying the right side by $\frac{R'}{R'}$, substituting for $c_2$, and assuming the induced angle, $\varphi'$, to be small so that $\varphi' = \tan \varphi'$.

$$Q_i = \frac{F}{2} B c R' \int_0^1 a (\theta - \varphi') \varphi' \Omega^2 \frac{x^3}{3} dx.$$ (18)

Substituting the value of $Q_i$ obtained above, Eq. 18, into Eq. 17 and solving for $C_{Q_i}$:

$$C_{Q_i} = \frac{F}{2} B c R' \int_0^1 a (\theta - \varphi') \varphi' \Omega^2 \frac{x^3}{3} dx$$

$$C_{Q_i} = \frac{F}{2} \pi \Omega^2 R^2$$

$$Q_i = \sigma a \int_0^1 \varphi' (\theta - \varphi') x^3 dx.$$ (19)

Minimum Profile Torque:

Let $\delta = C_{Q_{MIN}}$. Then in the same manner as for the induced torque coefficient the expression for the minimum profile torque coefficient is developed:

$$Q_\delta = \frac{F}{2} B c \int_0^R \delta \Omega^2 r^3 dr$$
\[ Q_x = \frac{p}{2} Bc R^2 \int \delta x^3 dx \]

\[ C_{Qx} = \sigma \delta \int x^3 dx. \]  

(30)

Profile Variation Torque:

Assume \( \Delta C_d = \epsilon C^3 \).

(Note: The value of the profile variation constant, \( \epsilon \), is different from the \( \epsilon \) used in Reference 2. To obtain the previous constant, divide the \( \epsilon \) used in this report by \( C^3 \). For simplification purposes \( \epsilon \) is assumed to be a constant in this report. However, in a portion of the study at this institution there was an indication that \( \epsilon \) could not be assumed constant and that it varied slightly with \( T \) and \( \lambda \).)

Again the coefficient is developed in the same way as above:

\[ Q_\epsilon = \frac{p}{2} Bc \int C^2 \delta x^3 dr \]

\[ = \frac{p}{2} Bc \int C^2 R^2 \int C^2 \delta \int (\theta - \psi)^2 x^3 dx \]

\[ C_{Q_\epsilon} = \sigma \epsilon \int (\theta - \psi)^2 x^3 dx. \]  

(21)

Therefore, the total torque coefficient is now the sum of Eqs. 19, 20, and 21:

\[ C_Q = \sigma \delta \int (\theta - \psi) x^3 dx + \sigma \delta \int x^3 dx + \sigma \epsilon \int (\theta - \psi)^2 x^3 dx. \]  

(22)

Changing this coefficient into a new form which eliminates \( \sigma \) as an independent parameter the following expression is obtained:
Again, this is a general equation from which it is possible to calculate the torque characteristics for either the constant-incidence or the constant-pitch rotor blades.

**Constant Incidence:** Substituting the expression obtained for the induced angle, Eq. 7, into the total torque coefficient equation, Eq. 23:

\[ Q_\sigma = \frac{d}{d\sigma} \int_0^1 \theta^3 dx + \frac{1}{\lambda} \int_0^1 \left( \theta - \frac{\sigma}{\theta} \right) \theta^3 dx + \frac{1}{\lambda} \int_0^1 \left( \theta - \frac{\sigma}{\theta} \right)^3 \theta^3 dx. \]  

Integrating:

\[ Q_\sigma = \frac{d}{d\sigma} + a \left\{ \frac{\theta \lambda_\sigma - \sigma q}{\theta} + \lambda - \frac{q}{\theta} \left( \frac{E - 6 \theta_\lambda}{E + 4 \theta_\lambda} \right)^{\frac{3}{2}} - \frac{\theta_\lambda^2}{4} \right\} \]

where

\[ E = \left( \lambda + \frac{q}{\theta} \right)^2 - 4 \left( \frac{q}{\theta} \right) \lambda. \]  

(24)
**Constant Pitch (Constant Circulation):** It is possible to obtain the total torque coefficient expression for this case by making the proper substitutions for $\theta_p$ and $\theta_p'$ in the development of Eq. 23. However, the results are the same as obtained in the following manner:

**Minimum Profile Torque:**

The minimum profile torque will be the same as that calculated for constant incidence since the induced angle does not enter into the expression.

$$C_{q_0} = \sigma \int_0^s (x^3) \, dx$$  \hspace{1cm} \text{from Eq. 20}

$$Q_{q_0} = \frac{\sigma}{\sigma v}$$

**Induced Torque:**

Considering $\rho^2 = \text{constant}$ and the thrust coefficient as the independent variable:

$$dQ_i = \left( \frac{w'}{W} \right) r \, d\tau$$

where

$$w' = R \sigma v'$$

but

$$d\tau = \rho \left( \frac{R}{\rho} \right) W \, dr$$

therefore,

$$dQ_i = \rho \left( \frac{R}{\rho} \right) R^3 v' x \, dx$$

Multiplying by $\rho$ and integrating, the induced torque of the entire rotor is:

$$Q_i = \rho R \int_0^s R^3 v' x \, dx$$
\[ Q_s = \frac{\rho}{2} \rho' R^3 \sigma \]
\[ Q_{\sigma s} = \frac{\rho}{2} \rho' R^3 \sigma \]
\[ = \frac{\rho}{2} \rho' R^3 \sigma \]

But

\[ T_{o} = \frac{\Gamma}{\pi \sigma R^2} \]

Therefore,

\[ Q_{o s} = \frac{T_{o}}{\sigma} \rho' \]
\[ = T_{o} \rho' \sigma \]
\[ = T_{o} \left( \frac{\sigma_{o}}{\sigma} + \lambda \sigma \right) \]  (25)

where

\[ \sigma_{o} = \frac{1}{2} \left[ -\lambda \sigma + \sqrt{\lambda^2 + T_{o}^{-1}} \right] \]  Eq.9

Profile Variation Torque:

In like manner the profile variation torque is developed, assuming \( \sigma \) to be a constant:

\[ dQ_{\sigma} = \frac{\rho}{2} c e C_{2} W^{2} r \, dr \]

where

\[ C_{2} = \frac{2 \Gamma}{BV_{c}} \]
\[ = \frac{2 \Gamma}{\sigma r B c} \]

\[ dQ_{\sigma} = \frac{\rho}{2} c e \frac{4 \Gamma^{1/2}}{\lambda^{1/2} B^{2} c^{1/2}} - \sigma^{2} r^{2} \, dr \]  (Single Blade)
Now, summing up the component parts of the torque, the total torque coefficient for the case of constant pitch is the following simple expression:

\[
Q_\sigma = \frac{\sigma}{\sigma^2} + T_{\sigma_o} \frac{\varphi_\sigma'}{\sigma^2} + 2 \varepsilon T_{\sigma_o}^2 .
\]  

(27)
Experimental Data

In order to obtain experimental verification of the theory developed in this report a series of tests were conducted in the 9-foot closed-return wind tunnel at this institution. From these data, curves were plotted and compared with the theoretical results obtained.

Equipment: A five-foot diameter rotor was used in this test. The rotor consisted of three blades of two inch chord with an N.A.C.A. 0015 airfoil section. The blades, rotor, balance, and model support are completely described in Reference 2 since the same equipment was used for this investigation.

The model position for the static thrust tests was such that the axis of rotation of the rotor was at right angles to the wind tunnel axis and in a horizontal plane. This position was necessary so that there would be no flow in the tunnel which would give other than static readings. In order to simulate vertical motion it was necessary to rotate the model through 90° (in the horizontal plane) into the upwind position. (Fig. 4). In this position the interference due to the torque tube support and its fairing was negligible. However, the exposed portion of the torque tube gave a slight tare value which was subtracted from each reading. Vertical ascent and descent were determined by the setting of the blade angle with respect to the air flow.

The equipment as described in Reference 2 has been altered slightly in order to obtain a more accurate rotational speed. In place of the induction motor with belt drive a one HP salient pole synchronous motor was used to drive the rotor in this series of tests. The rotor was directly driven by means of a vertical drive shaft between two gear boxes. The box at the top...
had a two to one reduction and the one at the bottom a one to one ratio. The latter was connected to the motor by means of a drive shaft with two universal joints. Since the change in line frequency was negligible, the rotational speed of the model rotor was 900 r.p.m., this being a constant regardless of the load. Blade settings were only carried up to 12° due to overloading of the motor beyond this angle. In order to reduce the high starting torque of this motor it was necessary to place a resistance in one phase of the three phase line.

The precision of the measurements was as follows:

- Thrust...................... ± 1 per cent
- Torque....................... ± 1 per cent
- Minimum torque............. ± 2 per cent
- Rate of rotation.......... ± 0 per cent
- Blade incidence angle... ± 0.05°

**Reduction of Data:** - The following formulae were used to reduce the experimental data of the rotor tests into coefficient form for comparison with the theory:

\[
T_\sigma = \frac{2T}{\rho \pi R^2 \alpha \Delta^2 \sigma^2}
\]

\[
Q_\sigma = \frac{2Q}{\rho \pi R^3 \alpha \Delta^3 \sigma^3}.
\]

\[
Q'_\sigma = Q_\sigma - \frac{5}{4 \sigma^2}.
\]

\[
\lambda = \frac{V}{\Delta \frac{R}{2}}.
\]
The torque coefficient used for comparison with the theory is the summation of the induced and profile variation torque, therefore, it was necessary to subtract the minimum profile torque to obtain the curves shown.

The inclination of the airstream due to the jet boundary was not considered since the axis of the airstream had only a very small angle (0.30°) as was calculated in Reference 2. The lowest values of $\lambda$ were those obtained with the tunnel motor stopped. The flow was then that set up by the rotor in the tunnel.

**Analysis of Parameters**

In the theoretical computations the slope of the lift curve, $\alpha$, was assumed to be a straight line whose value was taken as 5.75. This value was obtained from standard airfoil tests conducted in this laboratory, Reference 2. However, it will be observed, (Fig. 6), that the theoretical values are higher than the experimental curves in the large incidence and low vertical velocity region. This is due to the actual curvature of the lift curve. The experimental curves for vertical descent give the impression that the blades might be stalled. A theoretical investigation disclosed that the stall should occur only at high angles of incidence or at very high descent speeds and low angles of incidence.

The profile drag variation coefficient was obtained from Reference 2 and a value of 0.06 was used. On some previous performance runs on a single-bladed rotor the experimental curves gave reason to suspect that this value was not strictly a constant but a function of the velocity and the angle of incidence (or thrust). No attempt was made in this investigation to ascertain and analyze that phenomenon. However, it might account for some of the
The minimum profile drag coefficient was obtained from rotor tests in this wind tunnel. It was found that the coefficient obtained from these tests at zero degrees incidence was higher by about six per cent (0.072 to 0.076) than for the tests made in Reference 2, although the same equipment was used. This difference was accounted for in a slight alteration made to the root section of the blades in order to strengthen them.

Comparison of Theory and Experiment

Thrust: - The experimental curves and the theoretical points for $T_o$ vs. $\lambda$ are plotted in Fig. 6. The theoretical points are calculated using the constant-incidence theory. The agreement is very close for the smaller angles of incidence, but a divergence appears for the higher angles of incidence. This would seem to indicate that the slope of the lift curve used for calculating the theoretical points was not exactly that of the blades used in the tests. Also, at high angles of incidence and low values of $\lambda$ there is a greater divergence between the theoretical and the experimental points. This is probably due to the curvature of the lift curve at high angles of attack, as was mentioned previously.

Torque: - The experimental curves and theoretical points for $Q_o$ vs. $\lambda$ are plotted in Fig. 7. The minimum profile torque was eliminated, thus allowing the curve for $0^o$ incidence to pass through the origin at $\lambda = 0$. Since the minimum profile torque is a constant and is dependent entirely upon the profile of the blade, it could be omitted in the comparison of the theory with experiment. Again, the theoretical and experimental results differ in much the same manner as for the thrust. The divergence in this
case might be attributed not only to the improper value of the slope of the lift curve but to the assumption that the profile variation coefficient is a constant. This parameter has already been discussed above.

**General:** Due to the assumptions that: (1) the total induced angle is small so that the tangent of the angle can be assumed equal to the angle, and (2) the inclination of the lift and drag forces on the blades is small so that the sine of the inclination angle is equal to zero, a slight error may exist in the theory. However, since this error was found to be small, it was neglected. For high angles of incidence and high vertical velocities these assumptions would not hold and may be partially the cause for the divergence shown.

Theoretical curves were calculated using the simplified constant-pitch theory. Since all the tests were conducted with constant-incidence blades, higher theoretical values were obtained in theory, if the value of $\Theta_c$ was equal to the angle of incidence. However, it was found that by using the following empirical relationship very close checks in the thrust and torque were obtained, as shown in Figs. 8 and 9:

$$\Theta_c = \frac{2}{3} \Theta.$$

Using the constant-pitch theory gave close checks for the experimental $0^\circ$ incidence curve. It must be remembered that in this case the blade does have constant pitch and the theory should agree.

Fig. 10 gives a comparison of the theoretical and experimental constant-incidence polars. It will be noted that there is a very close agreement between these two curves. In Fig. 11 a comparison of constant-pitch theory, using the empirical relation mentioned above, and constant-incidence test
results gives a very close check at \( \lambda = 0 \). The tendency of the theory to follow the experimental points into the vertical descent region will be noted (i.e., curves back toward a positive \( \Phi' \) in the negative \( \tau \) region). The theoretical curves were carried as far as possible, since when carried further the radical in the formula became imaginary. A point of interest in these curves is that for constant pitch all curves pass through the origin, while for the case of constant-incidence, only the \( \lambda = 0 \) curve will pass through the origin. The explanation of this is that using a constant-incidence blade, the thrust is the total of the thrust along the blade which, under certain conditions, is both positive and negative at the same time and, therefore, the induced and profile variation torque will not be zero when the total thrust is equal to zero.

Figs. 12 and 13 are given in this paper merely to show the trend of the experimental curves in the vertical descent region as was obtained during the tests. The formulae developed and used in this report break down for the case of positive angles in vertical descent by giving values having no relation to the experimental curves. However, for negative angles of incidence, the constant-incidence theory did hold and gave extremely close checks. The zero degrees incidence curve could only be calculated using the constant-pitch theory. From the experimental curves in vertical descent, positive angles of incidence, it would appear that the blades are stalled and in a portion of that region this is true. However, as was mentioned before, from theoretical considerations this is not true for the whole range of positive angles in vertical descent. In the opinion of the author a further study and investigation of vertical descent would be a valuable contribution to a thorough understanding of helicopter motion.
Conclusions

The following conclusions were obtained from this investigation:

1. The theoretical analysis, as developed by M. Knight, can be applied to vertical ascent of the helicopter rotor by the addition of the velocity parameter, \( \lambda \), as presented in this report.

2. The experiments verified the theoretical development within the limits of the assumptions made in the theory for constant-incidence.

3. Using the simplified constant-pitch theory with an empirical constant applied to the angle of incidence, very close agreement was obtained with the constant-incidence tests. A broader study would be necessary to determine whether this empirical constant is generally valid.

4. The theory as presented in this paper does not hold for the case of positive angles of incidence in vertical descent.

5. A more complete investigation of the vortex ring regime and vertical descent would be a valuable contribution to the helicopter theory.
References


# TABLE I

**HELICOPTER ROTOR TESTS**

\( \theta = 0^\circ \)

3-bladed rotor, \( \sigma = 0.0636, \quad \theta_0 = 0.000, \quad \frac{\Sigma}{\sqrt{\sigma^2}} = 0.755 \)

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>( T_\sigma )</th>
<th>( Q_\sigma )</th>
<th>( Q'_\sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.000</td>
<td>0.755</td>
<td>0.000</td>
</tr>
<tr>
<td>0.03</td>
<td>-0.241</td>
<td>0.769</td>
<td>0.005</td>
</tr>
<tr>
<td>0.06</td>
<td>-0.673</td>
<td>0.510</td>
<td>-0.245</td>
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<tr>
<td>0.09</td>
<td>-1.940</td>
<td>-0.322</td>
<td>-1.077</td>
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<tr>
<td>0.12</td>
<td>-3.078</td>
<td>-1.695</td>
<td>-2.450</td>
</tr>
</tbody>
</table>

# TABLE II

**HELICOPTER ROTOR TESTS**

\( \theta = 2^\circ \)

3-bladed rotor, \( \sigma = 0.0636, \quad \theta_0 = 0.549, \quad \frac{\Sigma}{\sqrt{\sigma^2}} = 0.755 \)

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>( T_\sigma )</th>
<th>( Q_\sigma )</th>
<th>( Q'_\sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.253</td>
<td>0.800</td>
<td>0.045</td>
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<tr>
<td>0.03</td>
<td>-0.075</td>
<td>0.739</td>
<td>-0.016</td>
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<tr>
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<td>-0.680</td>
<td>0.431</td>
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<tr>
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<td>-1.633</td>
<td>-0.502</td>
<td>-1.257</td>
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<tr>
<td>0.12</td>
<td>-2.675</td>
<td>-1.995</td>
<td>-2.750</td>
</tr>
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</table>

# TABLE III

**HELICOPTER ROTOR TESTS**

\( \theta = 4^\circ \)

3-bladed rotor, \( \sigma = 0.0636 \quad \theta_0 = 1.097, \quad \frac{\Sigma}{\sqrt{\sigma^2}} = 0.755 \)

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>( T_\sigma )</th>
<th>( Q_\sigma )</th>
<th>( Q'_\sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.726</td>
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<tr>
<td>0.03</td>
<td>0.314</td>
<td>1.019</td>
<td>0.255</td>
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<td>0.06</td>
<td>-0.288</td>
<td>0.600</td>
<td>-0.155</td>
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<tr>
<td>0.09</td>
<td>-1.165</td>
<td>-0.363</td>
<td>-1.118</td>
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<tr>
<td>0.12</td>
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</tr>
</tbody>
</table>
### TABLE IV
**HELICOPTER ROTOR TESTS**

\( \theta = 6^\circ \)

3-bladed rotor, \( \sigma = 0.0636 \) \( \Theta_{\sigma} = 1.646, \frac{\delta}{\mu / \sigma^2} = 0.755 \)

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>( T_{\sigma} )</th>
<th>( Q_{\sigma} )</th>
<th>( Q'_{\sigma} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>1.350</td>
<td>1.840</td>
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<td>0.03</td>
<td>0.896</td>
<td>1.850</td>
<td>1.095</td>
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<tr>
<td>0.06</td>
<td>0.368</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>0.12</td>
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<td></td>
</tr>
</tbody>
</table>

### TABLE V
**HELICOPTER ROTOR TESTS**

\( \theta = 8^\circ \)

3-bladed rotor, \( \sigma = 0.0636 \) \( \Theta_{\sigma} = 2.195, \frac{\delta}{\mu / \sigma^2} = 0.755 \)

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>( T_{\sigma} )</th>
<th>( Q_{\sigma} )</th>
<th>( Q'_{\sigma} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>2.060</td>
<td>2.850</td>
<td>2.095</td>
</tr>
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<td>1.575</td>
<td>2.670</td>
<td>1.915</td>
</tr>
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<td>0.833</td>
<td>2.004</td>
<td>1.249</td>
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<tr>
<td>0.09</td>
<td>-0.028</td>
<td>0.853</td>
<td>0.098</td>
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<tr>
<td>0.12</td>
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</tr>
</tbody>
</table>

### TABLE VI
**HELICOPTER ROTOR TESTS**

\( \theta = 10^\circ \)

3-bladed rotor, \( \sigma = 0.0636 \) \( \Theta_{\sigma} = 2.744, \frac{\delta}{\mu / \sigma^2} = 0.755 \)

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>( T_{\sigma} )</th>
<th>( Q_{\sigma} )</th>
<th>( Q'_{\sigma} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>2.780</td>
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<td>3.205</td>
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<tr>
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<td>2.367</td>
<td>3.963</td>
<td>3.208</td>
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<td>1.553</td>
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<td>2.787</td>
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<td>0.09</td>
<td>0.669</td>
<td>2.405</td>
<td>1.650</td>
</tr>
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<td>0.12</td>
<td>-0.265</td>
<td>0.702</td>
<td>-0.053</td>
</tr>
</tbody>
</table>
TABLE VII
HELICOPTER ROTOR TESTS

$\theta = 12^\circ$

3-bladed rotor, $\sigma = 0.0636$, $\theta = 3.292$, $\delta_{\sigma} = 0.755$

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$T_\sigma$</th>
<th>$Q_\sigma$</th>
<th>$Q'_\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>3.380</td>
<td>5.260</td>
<td>4.505</td>
</tr>
<tr>
<td>0.03</td>
<td>2.980</td>
<td>5.320</td>
<td>4.565</td>
</tr>
<tr>
<td>0.06</td>
<td>2.355</td>
<td>5.135</td>
<td>4.380</td>
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<td>0.09</td>
<td>1.491</td>
<td>4.210</td>
<td>3.455</td>
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<tr>
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<td>0.470</td>
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<td>1.907</td>
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TABLE VIII
HELICOPTER THEORY

Constant Pitch

3-bladed rotor, $\sigma = 0.0636$, $\alpha = 5.75$, $\epsilon = 0.06$, $\delta_{\theta} = 0.755$

$\theta = 0^\circ$

$\theta = 0.000$

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$T_\sigma$</th>
<th>$Q_\sigma$</th>
<th>$Q'_\sigma$</th>
<th>$Q''_\sigma$</th>
<th>$Q_{\sigma}$</th>
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<tr>
<td>0.00</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.755</td>
</tr>
<tr>
<td>0.03</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.755</td>
</tr>
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<td>-0.146</td>
<td>0.950</td>
<td>-0.696</td>
<td>0.659</td>
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<tr>
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<tr>
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TABLE IX
HELICOPTER THEORY

Constant Pitch

3-bladed rotor, $\sigma = 0.0636$, $\alpha = 5.75$, $\epsilon = 0.06$, $\delta_{\theta} = 0.755$

$\theta = 2.9^\circ$

$\theta = 0.731$

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$T_\sigma$</th>
<th>$Q_\sigma$</th>
<th>$Q'_\sigma$</th>
<th>$Q''_\sigma$</th>
<th>$Q_{\sigma}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.803</td>
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<td>0.442</td>
<td>1.197</td>
</tr>
<tr>
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<td>0.342</td>
<td>0.299</td>
<td>0.014</td>
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<td>0.978</td>
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<td>-0.277</td>
<td>0.013</td>
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<td>0.491</td>
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<tr>
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<td>-2.254</td>
<td>-3.415</td>
<td>0.610</td>
<td>-2.905</td>
<td>-2.050</td>
</tr>
</tbody>
</table>
**TABLE I**

HELIICOPTER THEORY

Constant Pitch

3-bladed rotor, $\sigma = 0.0636$, $a = 5.75$, $c = 0.06$, $\frac{d}{\sigma_r} = 0.755$

$\theta = 5.3^\circ$ $\theta_{\sigma} = 1.464$

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$T_\sigma$</th>
<th>$Q_{\sigma_r}$</th>
<th>$Q_{\sigma_c}$</th>
<th>$Q_{\sigma_r}'$</th>
<th>$Q_{\sigma_c}'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
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<td>2.077</td>
<td>2.532</td>
</tr>
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<td>1.450</td>
<td>0.305</td>
<td>1.755</td>
<td>2.510</td>
</tr>
<tr>
<td>0.06</td>
<td>0.920</td>
<td>1.052</td>
<td>0.102</td>
<td>1.154</td>
<td>1.909</td>
</tr>
<tr>
<td>0.09</td>
<td>0.092</td>
<td>0.132</td>
<td>0.001</td>
<td>0.133</td>
<td>0.388</td>
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<td>-0.863</td>
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<td>0.089</td>
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</table>

**TABLE II**

HELIICOPTER THEORY

Constant Pitch

3-bladed rotor, $\sigma = 0.0636$, $a = 5.75$, $c = 0.06$, $\frac{d}{\sigma_r} = 0.755$

$\theta = 8^\circ$ $\theta_{\sigma} = 2.195$

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$T_\sigma$</th>
<th>$Q_{\sigma_r}$</th>
<th>$Q_{\sigma_c}$</th>
<th>$Q_{\sigma_r}'$</th>
<th>$Q_{\sigma_c}'$</th>
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</thead>
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<tr>
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<td>1.544</td>
<td>4.942</td>
<td>5.697</td>
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<td>1.106</td>
<td>4.564</td>
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<td>2.363</td>
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<td>1.375</td>
<td>0.050</td>
<td>1.325</td>
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</table>

**TABLE XII**

HELIICOPTER THEORY

Constant Incidence

3-bladed rotor, $\sigma = 0.0636$, $a = 5.75$, $c = 0.06$, $\frac{d}{\sigma_r} = 0.755$

$\theta = 0^\circ$ $\theta_{\sigma} = 0.000$

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$T_\sigma$</th>
<th>$Q_{\sigma_r}$</th>
<th>$Q_{\sigma_c}$</th>
<th>$Q_{\sigma_r}'$</th>
<th>$Q_{\sigma_c}'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>1.035</td>
<td>-0.742</td>
<td>0.256</td>
<td>-0.486</td>
<td>0.269</td>
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<tr>
<td>0.03</td>
<td>0.397</td>
<td>-0.092</td>
<td>0.032</td>
<td>-0.060</td>
<td>0.695</td>
</tr>
<tr>
<td>0.06</td>
<td>-0.322</td>
<td>-0.075</td>
<td>0.026</td>
<td>-0.049</td>
<td>0.706</td>
</tr>
<tr>
<td>0.09</td>
<td>-1.001</td>
<td>-0.702</td>
<td>0.242</td>
<td>-0.460</td>
<td>0.295</td>
</tr>
<tr>
<td>0.12</td>
<td>-1.679</td>
<td>-1.961</td>
<td>0.677</td>
<td>-1.284</td>
<td>-0.529</td>
</tr>
</tbody>
</table>
### TABLE XIII
HELIKOPTER THEORY
Constant Incidence
3-bladed rotor, \( \sigma = 0.0636, \ a = 5.75, \ \epsilon = 0.06, \ \frac{f}{\sigma^2} = 0.755 \)

\[ \theta = 4^\circ \]
\[ \theta_{\sigma} = 1.097 \]

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>( T_{\sigma} )</th>
<th>( Q_{\sigma^2} )</th>
<th>( Q_{\sigma\epsilon} )</th>
<th>( Q_{\sigma} )</th>
<th>( Q_{\sigma} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.840</td>
<td>0.414</td>
<td>0.101</td>
<td>0.515</td>
<td>1.270</td>
</tr>
<tr>
<td>0.03</td>
<td>0.365</td>
<td>0.265</td>
<td>0.032</td>
<td>0.297</td>
<td>1.052</td>
</tr>
<tr>
<td>0.06</td>
<td>-0.283</td>
<td>-0.201</td>
<td>0.026</td>
<td>-0.175</td>
<td>0.580</td>
</tr>
<tr>
<td>0.09</td>
<td>-1.179</td>
<td>-1.311</td>
<td>0.186</td>
<td>-1.125</td>
<td>-0.370</td>
</tr>
<tr>
<td>0.12</td>
<td>-2.237</td>
<td>-3.335</td>
<td>0.621</td>
<td>-2.714</td>
<td>-1.959</td>
</tr>
</tbody>
</table>

### TABLE XIV
HELIKOPTER THEORY
Constant Incidence
3-bladed rotor, \( \sigma = 0.0636, \ a = 5.75, \ \epsilon = 0.06, \ \frac{f}{\sigma^2} = 0.755 \)

\[ \theta = 9^\circ \]
\[ \theta_{\sigma} = 2.198 \]

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>( T_{\sigma} )</th>
<th>( Q_{\sigma^2} )</th>
<th>( Q_{\sigma\epsilon} )</th>
<th>( Q_{\sigma} )</th>
<th>( Q_{\sigma} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>2.168</td>
<td>1.714</td>
<td>0.671</td>
<td>2.385</td>
<td>3.149</td>
</tr>
<tr>
<td>0.03</td>
<td>1.656</td>
<td>1.639</td>
<td>0.431</td>
<td>2.070</td>
<td>2.825</td>
</tr>
<tr>
<td>0.06</td>
<td>0.983</td>
<td>1.277</td>
<td>0.318</td>
<td>1.495</td>
<td>2.238</td>
</tr>
<tr>
<td>0.09</td>
<td>0.144</td>
<td>0.368</td>
<td>0.113</td>
<td>0.481</td>
<td>1.231</td>
</tr>
<tr>
<td>0.12</td>
<td>-0.328</td>
<td>-1.294</td>
<td>0.206</td>
<td>-1.988</td>
<td>-0.339</td>
</tr>
</tbody>
</table>

### TABLE XV
HELIKOPTER THEORY
Constant Incidence
3-bladed rotor, \( \sigma = 0.0636, \ a = 5.75, \ \epsilon = 0.06, \ \frac{f}{\sigma^2} = 0.755 \)

\[ \theta = 12^\circ \]
\[ \theta_{\sigma} = 3.292 \]

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>( T_{\sigma} )</th>
<th>( Q_{\sigma^2} )</th>
<th>( Q_{\sigma\epsilon} )</th>
<th>( Q_{\sigma} )</th>
<th>( Q_{\sigma} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>3.651</td>
<td>3.732</td>
<td>1.895</td>
<td>5.627</td>
<td>6.382</td>
</tr>
<tr>
<td>0.03</td>
<td>2.111</td>
<td>3.824</td>
<td>1.442</td>
<td>5.268</td>
<td>6.021</td>
</tr>
<tr>
<td>0.06</td>
<td>2.432</td>
<td>3.651</td>
<td>0.994</td>
<td>4.645</td>
<td>5.400</td>
</tr>
<tr>
<td>0.09</td>
<td>1.627</td>
<td>3.607</td>
<td>0.613</td>
<td>3.620</td>
<td>4.375</td>
</tr>
<tr>
<td>0.12</td>
<td>0.696</td>
<td>1.702</td>
<td>0.373</td>
<td>2.075</td>
<td>2.850</td>
</tr>
</tbody>
</table>
### TABLE XVI

**HELIÇOPTER THEORY**

**Constant Pitch - Second Method**

\[ \sigma = 0.0636, \quad \varepsilon = 0.06, \quad \frac{\sigma}{\varepsilon} = 0.755 \quad T_{\sigma} = 2.0 \]

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>( Q_{\sigma_{d}} )</th>
<th>( Q_{\sigma_{x}} )</th>
<th>( Q_{\sigma_{r}} )</th>
<th>( Q_{\sigma_{r}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>1.414</td>
<td>0.480</td>
<td>1.894</td>
<td>2.649</td>
</tr>
<tr>
<td>0.03</td>
<td>1.392</td>
<td>0.480</td>
<td>2.442</td>
<td>3.197</td>
</tr>
<tr>
<td>0.06</td>
<td>2.644</td>
<td>0.480</td>
<td>3.124</td>
<td>3.879</td>
</tr>
<tr>
<td>0.09</td>
<td>3.416</td>
<td>0.480</td>
<td>3.896</td>
<td>4.652</td>
</tr>
<tr>
<td>0.12</td>
<td>4.248</td>
<td>0.480</td>
<td>4.728</td>
<td>5.493</td>
</tr>
</tbody>
</table>

### TABLE XVII

**HELIÇOPTER THEORY**

**Constant Pitch - Second Method**

\[ \sigma = 0.0636, \quad \varepsilon = 0.06, \quad \frac{\sigma}{\varepsilon} = 0.755 \quad T_{\sigma} = 1.5 \]

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>( Q_{\sigma_{d}} )</th>
<th>( Q_{\sigma_{x}} )</th>
<th>( Q_{\sigma_{r}} )</th>
<th>( Q_{\sigma_{r}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.920</td>
<td>0.270</td>
<td>1.120</td>
<td>1.945</td>
</tr>
<tr>
<td>0.03</td>
<td>1.238</td>
<td>0.270</td>
<td>1.608</td>
<td>2.363</td>
</tr>
<tr>
<td>0.06</td>
<td>1.888</td>
<td>0.270</td>
<td>2.139</td>
<td>2.893</td>
</tr>
<tr>
<td>0.09</td>
<td>2.455</td>
<td>0.270</td>
<td>2.735</td>
<td>3.490</td>
</tr>
<tr>
<td>0.12</td>
<td>3.105</td>
<td>0.270</td>
<td>3.375</td>
<td>4.130</td>
</tr>
</tbody>
</table>

### TABLE XVIII

**HELIÇOPTER THEORY**

**Constant Pitch - Second Method**

\[ \sigma = 0.0636, \quad \varepsilon = 0.06, \quad \frac{\sigma}{\varepsilon} = 0.755 \quad T_{\sigma} = 1.0 \]

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>( Q_{\sigma_{d}} )</th>
<th>( Q_{\sigma_{x}} )</th>
<th>( Q_{\sigma_{r}} )</th>
<th>( Q_{\sigma_{r}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.500</td>
<td>0.120</td>
<td>0.620</td>
<td>1.375</td>
</tr>
<tr>
<td>0.03</td>
<td>0.783</td>
<td>0.120</td>
<td>0.908</td>
<td>1.663</td>
</tr>
<tr>
<td>0.06</td>
<td>1.159</td>
<td>0.120</td>
<td>1.279</td>
<td>2.034</td>
</tr>
<tr>
<td>0.09</td>
<td>1.574</td>
<td>0.120</td>
<td>1.684</td>
<td>2.449</td>
</tr>
<tr>
<td>0.12</td>
<td>2.013</td>
<td>0.120</td>
<td>2.133</td>
<td>2.888</td>
</tr>
</tbody>
</table>
### TABLE XIX

**HELICOPTER THEORY**

Constant Pitch - Second Method

\[ \sigma = 0.0636, \epsilon = 0.06, \frac{\sigma}{\sigma_0} = 0.755, \quad T_\sigma = 0.5 \]

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>( Q_\sigma )</th>
<th>( Q_{\sigma_0} )</th>
<th>( Q_{\sigma_0} )</th>
<th>( Q_\sigma )</th>
<th>( Q_{\sigma_0} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.177</td>
<td>0.030</td>
<td>0.237</td>
<td>0.962</td>
<td></td>
</tr>
<tr>
<td>0.03</td>
<td>0.331</td>
<td>0.030</td>
<td>0.361</td>
<td>1.116</td>
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</tr>
<tr>
<td>0.06</td>
<td>0.531</td>
<td>0.030</td>
<td>0.561</td>
<td>1.316</td>
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</tr>
<tr>
<td>0.09</td>
<td>0.750</td>
<td>0.030</td>
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<td>0.976</td>
<td>0.030</td>
<td>1.006</td>
<td>1.761</td>
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</tbody>
</table>

### TABLE XX

**HELICOPTER THEORY**

Constant Pitch - Second Method

\[ \sigma = 0.0636, \epsilon = 0.06, \frac{\sigma}{\sigma_0} = 0.755, \quad T_\sigma = 0.00 \]

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>( Q_\sigma )</th>
<th>( Q_{\sigma_0} )</th>
<th>( Q_{\sigma_0} )</th>
<th>( Q_\sigma )</th>
<th>( Q_{\sigma_0} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.000</td>
<td>0.755</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.03</td>
<td>0.000</td>
<td>0.755</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.06</td>
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<td>0.755</td>
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<tr>
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<td>0.755</td>
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<td></td>
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<tr>
<td>0.12</td>
<td>0.000</td>
<td>0.755</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 4. Three-Bladed Rotor Mounted on Wind Tunnel Balance
Figure 5. Rotor Model Mounted on Blade Incidence Jig.
Figure 8.
Helicopter Vertical Ascent
Thrust vs. Velocity for Constant Angles

Experimental Curve
x x Theoretical Points
Constant Inclination Theory

\( \theta = 12^\circ \)
\( \theta = 10^\circ \)
\( \theta = 8^\circ \)
\( \theta = 6^\circ \)
\( \theta = 4^\circ \)
\( \theta = 2^\circ \)
\( \theta = 0^\circ \)
Figure 7.
Helicopter Vertical Ascent
Torque vs. Velocity for Constant Angles

Experimental Curve
Theoretical Points
Constant Incidence Theory
Figure 8.
Helicopter Vertical Ascent
Thrust vs. Velocity for Constant Angles

--- Experimental Curve
X X Theoretical Points
Constant Pitch Theory
(θ = \( \frac{2}{3} \theta \))
Figure 9.
Helicopter Vertical Ascent
Torque vs. Velocity for Constant Angle

- Experimental Curve
- Theoretical Points
- Constant Pitch Theory
\( \theta = \frac{\theta}{3} \theta \)

\( \theta = 12^\circ \)
\( \theta = 10^\circ \)
\( \theta = 8^\circ \)
\( \theta = 6^\circ \)
\( \theta = 4^\circ \)
\( \theta = 2^\circ \)
\( \theta = 0^\circ \)
Figure 10.
Helicopter Vertical Ascent
Thrust vs. Torque for Constant Velocities

Experimental Curves
Theoretical Curves
Constant Incidence Theory
Figure 11.
Helicopter Vertical Ascent
Thrust vs. Torque for Constant Velocities

Experimental Curves
Theoretical Curves
Constant Pitch
Theory
Figure 12

Helicopter Vertical Ascent
Thrust vs. Velocity for Constant Angle

Experimental Curves

- X - Theoretical Points
  Constant Incidence Theory

- + - Theoretical Points
  Constant Pitch Theory

This is a graph showing the thrust vs. velocity for a helicopter during vertical ascent, with different constant angles and theoretical curves.
Figure 13
Helicopter Vertical Ascent
Torque vs. Velocity for Constant Angles

Experimental Curves
Theoretical Points
Constant Incidence Theory

Theoretical Points
Constant Pitch Theory