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Flexural stiffness of corrugated board

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Abstract

The top-load compression strength of a corrugated box depends on the flexural stiffness in both principal directions of the combined board as well as its edgewise compression strength. Flexural stiffness is the ability to resist bending. Differences in compression strength of A-, B-, and C-flute boxes is mainly due to the differences in flexural stiffness of these constructions. Flexural stiffness depends primarily on the modulus of elasticity and caliper of the liners and on the square of the combined board caliper. Methods of experimentally determining the flexural stiffnesses of corrugated board by means of beam tests are described. The commonly used, simple beam test underestimates the true flexural stiffness of corrugated board because of shear effects, although a correction may be applied by testing at two different spans. A specimen loaded at four points, on the other hand, does not suffer from the influence of shear and thus permits direct evaluation of flexural stiffness. A bench model of the four-point beam test, requiring only simple testing equipment, was explored in the interest of making beam testing feasible in the box plant. The bench model underestimated flexural stiffness by about 6%, on the average, because of creep during the test, but it may be useful for obtaining a relative measure of flexural stiffness in the box plant.

The compression strength of a corrugated box is of importance both as a partial indication of its warehouse stacking performance and as an overall measure of the quality of the fiberboard materials and conversion efficiency. A laboratory test on the box provides one means of evaluating its compression performance. For purposes of quality control and material specification at earlier stages in box manufacture, however, the box plant and board mill require meaningful test methods for evaluating the potential performance of liners, medium, and corrugated board.

A previous article[1] discussed the edgewise compression strength of corrugated board and its importance to top-to-bottom compression of the box. It was shown that the regular column crush test does not provide an accurate evaluation of this property of combined board. Improved estimates can be obtained with a necked-down column specimen or, as shown by subsequent unpublished work, by means of a rectangular column specimen whose loading edges are reinforced by dipping in wax.

A second type of material property important to box strength is the flexural stiffness of the corrugated board. The present article is concerned with this property, with emphasis on methods of measurement of flexural stiffness of combined board.

Portions of the underlying studies performed at The Institute of Paper Chemistry were conducted on behalf of the Fourdrinier Kraft Board Institute in conjunction with its basic research program. The latter organization has requested that the results of these studies on flexural stiffness of their material be made available to the industry in the interests of advancing basic research on container performance.

Importance of flexural stiffness

As discussed in greater detail in Reference (1), the side and end panels of an R.S.C. box will usually bow outward or inward when subjected to top-to-bottom compression, provided the box is not extremely short. Bending of the panels limits their load-carrying ability (on a load per unit width basis) over the central region of each panel. The portions of the panel adjacent to the vertical edges, on the other hand, remain essentially plane and, therefore, are capable of resisting higher load intensity. These differences in load-carrying ability across a box panel are illustrated in Figure 1 in terms of an idealized load profile around the box perimeter.

Failure of the combined board at the vertical edges in edgewise compression triggers box failure and accounts for the importance of edgewise compression strength. Nonetheless, the central region of each panel makes a significant contribution to the total box load. Inasmuch as the behavior of these central regions reflects the bending characteristics of the combined board, the analysis of box compression strength involves consideration of the flexural stiffness of corrugated board.

Flexural stiffness is the capacity of a structural member to resist bending. In terms of the simple beam illustrated in Figure 2, flexural stiffness is essentially the ratio of load to the deflection produced by the load, that is,

$$\text{Flexural stiffness} = D = (1/48)(P/Y)\, L^3. \qquad (1)$$

The greater the flexural stiffness, the greater is the load required to produce a given deflection. If the beam is comprised of a material possessing identical properties in tension and compression, the flexural stiffness is equal to $E\ I$,

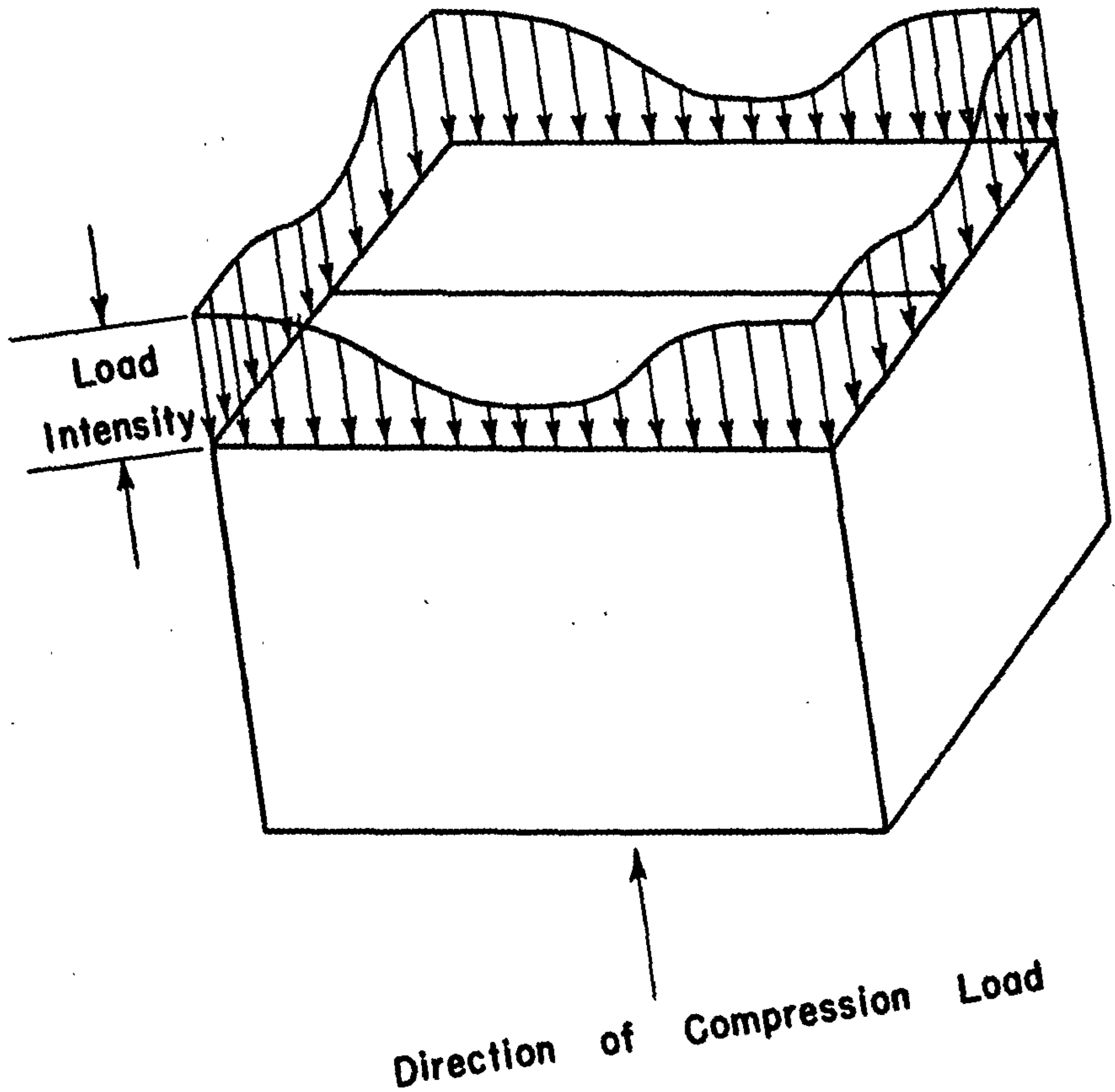


Figure 1. Distribution of compression load around the perimeter of a box

where E is the modulus of elasticity of the material and I is the moment of inertia of the cross-section of the beam. If the cross-section is rectangular and continuous, its moment of inertia is $b\ h^3/12$ (see Figure 2). For corrugated board the moment of inertia is proportional to the caliper of the liners and to the square of the combined board caliper, approximately. With sheet materials such as corrugated board, flexural stiffness is usually expressed on a unit width basis and has units of $\text{lb.-in.}^2/\text{in.}$, or simply, lb.-in.

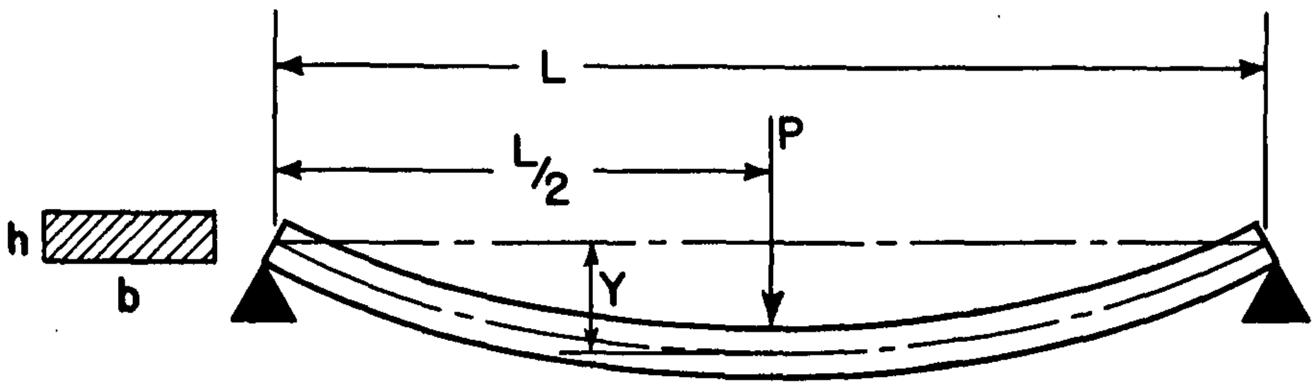


Figure 2. Three-point beam test

It is important to note that flexural stiffness depends on (a) material properties (i.e., E) and (b) cross-section geometry. A high flexural stiffness of corrugated board is achieved by (a) using liners and medium which are stiff in tension and compression and (b) maintaining as large a combined board caliper as possible. With identical components, A-flute board has a substantially higher flexural stiffness than B-flute board because of the difference in combined board caliper. Conversion operations such as printing which crush the combined board caliper detract from its potential flexural stiffness.

[Strictly speaking, flexural stiffness is defined as the ratio of bending moment to the resulting curvature[2], rather than in terms of load and deflection of a laterally loaded beam as given by Equation (1). Inasmuch as the present article is concerned mainly with experimental determination of combined board flexural stiffness and this is accomplished by testing specimens as beams, it appears adequate to define stiffness as in Equation (1).]

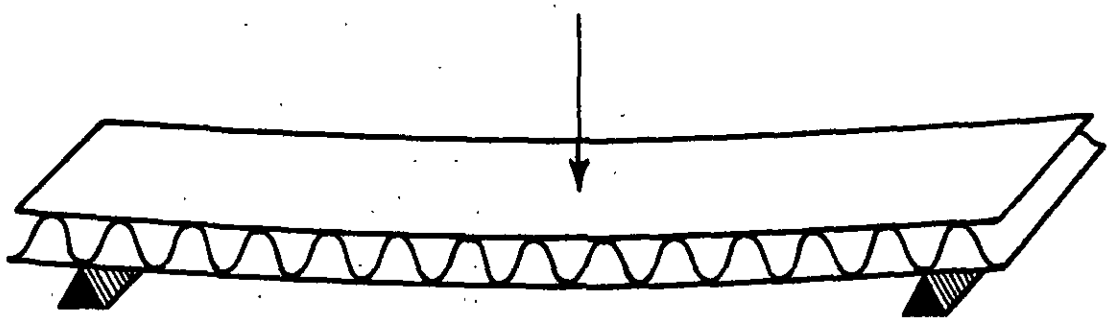
Returning to consideration of box compression strength, investigators at The Institute of Paper Chemistry have developed the following type of equation relating top-load strength to material properties and perimeter for boxes whose panels are not of extremely short depth and therefore bow:

$$P = a\, P_m^{\,b}\, (\sqrt{D_x D_y})^{1-b}\, Z^{2b-1} \quad (2)$$

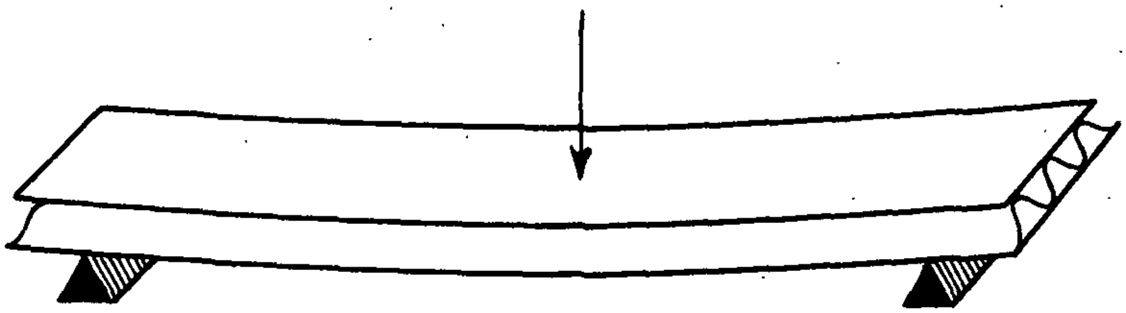
where P = box strength, P_m = edgewise compression strength of combined board in direction parallel to box load, D_x = flexural stiffness of combined board in machine direction, D_y = flexural stiffness in cross-machine direction, Z = perimeter, and a and b are empirical constants.

It may be noted that the box compression equation involves flexural stiffness in the machine direction (namely, D_x) as well as in the cross-machine direction (D_y). This follows from the fact that when the box panel bows, it has curvatures in both principal directions of the board. Machine-direction flexural stiffness, D_x, may be determined from a specimen tested as a beam whose length (or span) is parallel to the machine direction of the board, as illustrated in Figure 3a. A specimen for evaluating cross-machine stiffness is shown in Figure 3b. The machine direction stiffness of corrugated board generally is substantially greater than the cross-machine stiffness; the greater cross-sectional area in the cross-machine direction (because of the flute cross-section) is more than offset by the higher modulus of the liners in the machine direction.

The significance of flexural stiffness to top-load box compression strength is evident when comparing vertical flute boxes fabricated from the same



(a) Machine Direction



(b) Cross - Machine Direction

Figure 3. Orientation of specimen for determination of flexural stiffness of corrugated board

components in A-, B-, and C-flute. The edgewise compression strengths of such boards are very nearly equal; the small differences which appear may be attributable to the modest differences in amount of medium (draw factor), number of glue lines and other geometrical factors affecting the stability of the liners and flute side walls. The minor differences in edgewise compression strength between the several flute sizes do not account for the fact that the top-load box compression of A-flute boxes is appreciably greater than that of corresponding C-flute boxes, which in turn is greater than for corresponding B-flute boxes. There are, as noted earlier, major differences in flexural stiffness for the several flute sizes because of the differing calipers of the combined board: A greater than C greater than B. Thus, it is flexural stiffness which accounts for the difference in the box strength of A-, B-, and C-flute constructions.

Three-point beam test

One of the most common methods of measuring flexural stiffness is by means of the simple beam test, as illustrated in Figure 2. This test setup may also be designated the "three point" test because the specimen is loaded at three points along the span--at each end and at the middle. Usually the load is applied by a testing machine and a curve of load *versus* deflection is obtained. The load, P, and the mid-span deflection, Y, at some point in the initial straight-line portion of the curve (or alternatively the slope P/Y of the curve) is substituted in Equation (1) to give an estimate of the elastic flexural stiffness, D, of the specimen.

The three-point beam test suffers from one major disadvantage. The deflection of the specimen is the result of two effects: a portion of the deflection is due to flexure and the remainder is due to shear. Accordingly, the stiffness calculated from Equation (1) is not solely the flexural stiffness but rather is a combination of flexural stiffness and shear stiffness of the specimen. If the calculated stiffness is interpreted as being the true flexural stiffness of the specimen, it will result in underestimation of the true flexural stiffness.

This ambiguity of the three-point beam test becomes very apparent with corrugated board when a given sample is tested at various spans, L. Figure 4 shows a graph of apparent flexural stiffness as calculated from Equation (1) at various spans for a 200-lb. series, A-flute construction. In the machine direction, for which the shear effect is particularly severe, the apparent flexural stiffness varies from about 45 lb.-in. at a span of four inches to 166 lb.-in. at a 26-inch span. The curve suggests that even higher values of apparent stiffness would be obtained at spans greater than 26 inches.

The trend evident in Figure 4 is explainable by the following consideration. As the span is increased, the flexural stresses in the specimen corresponding to a given applied load also increase, but the shear stresses remain constant. Therefore, the portion of the total deflection which is attributable to flexure increases with increase in span while the portion associated with shear remains constant. At long spans the shear deflection becomes an insignificantly small fraction of the total deflection, and the latter is essentially deflection caused by flexure. At long spans, therefore, the apparent stiffness of the specimen approaches the true flexural stiffness.

The foregoing considerations indicate that long test spans are required for an accurate determination of flexural stiffness, particularly in the machine direction. For the 200-lb. series sample illustrated in Figure 4, a span of over 26 inches in the machine direction and at least 10 inches in the cross-machine direction would be necessary. In some instances, a long test span becomes prohibitive. If the beam specimen must be procured from the panel of a box, the test

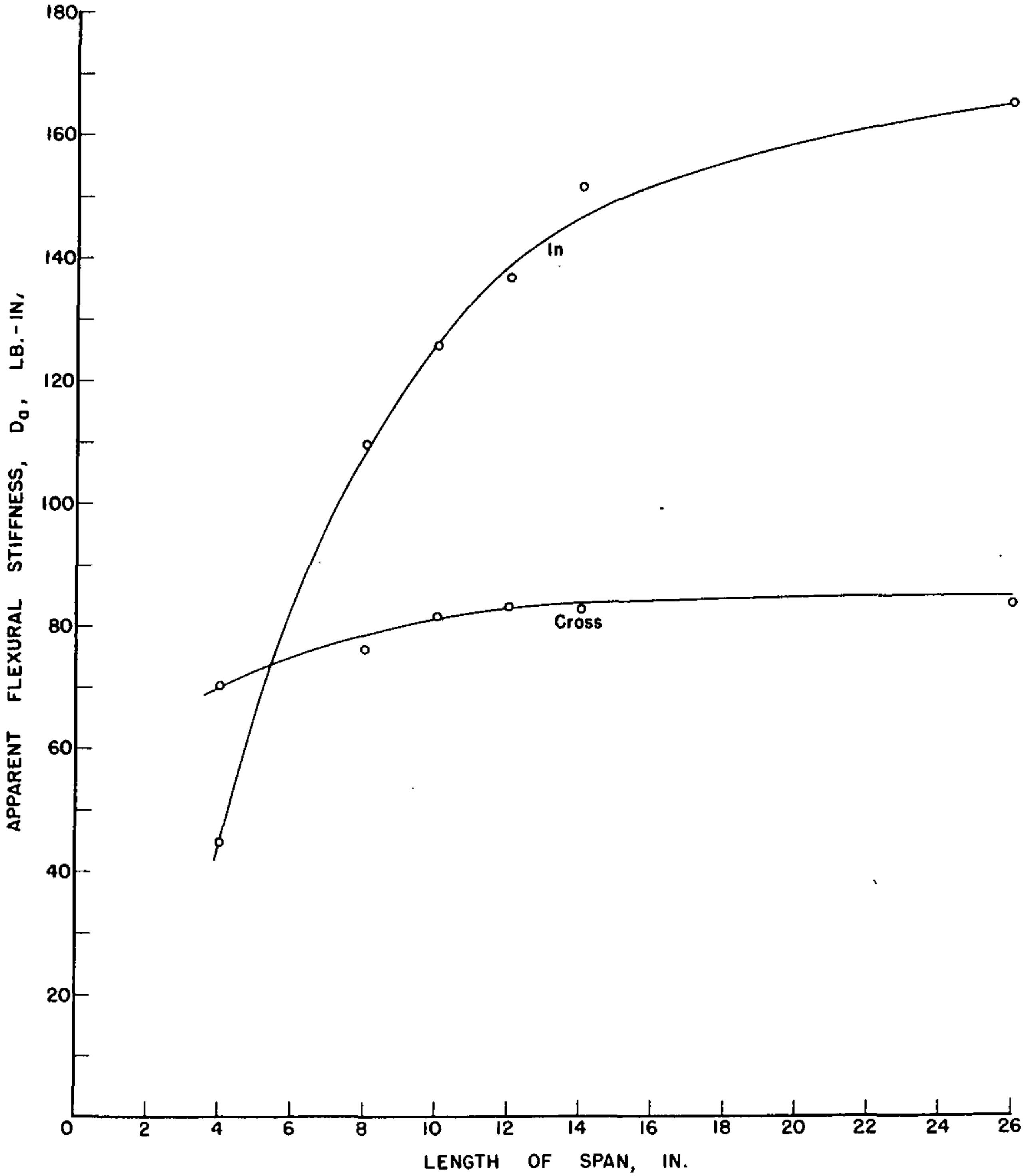


Figure 4. Effect of span on apparent flexural stiffness determined from three-point beam test

could not even be performed for average size boxes. Moreover, the longer the test span, the more sensitive must be the load-measuring apparatus of the testing machine because proportionately lower loads will be required in the test.

Although it may be possible to establish a correlation between true flexural stiffness and apparent stiffness at some conveniently short span, the accuracy of the correlation would be dependent on the shear effect remaining constant; at best the latter assumption would require checking from time to time.

The testing difficulties discussed above may be alleviated by employing a correction based on a theoretical relationship between true flexural stiffness and apparent flexural stiffness which accounts for shear effects[3]. A sample of board is tested at two spans, L_1, and L_2, giving two values of apparent stiffness, D_{a1}, and D_{a2}, respectively. The true flexural stiffness, D, may be estimated by the following equation:

$$D = D_{a1} D_{a2} (L_1^2 - L_2^2)/(D_{a2} L_1^2 - D_{a1} L_2^2) \qquad (3)$$

Of course, this approach doubles the amount of testing as compared with a single long span. It should be remarked that the underlying theory of Equation (3) was developed for sandwich materials having continuous cores rather than for corrugated board.

Four-point beam test

It has long been recognized by materials testing personnel that the inherent disadvantage of the three-point beam test can be avoided by using an alternate type of test setup. The latter method is known as the four-point beam or pure-moment beam[4-7]. The loading arrangement is diagrammed in Figure 5. The advantage of this beam test is that flexural stresses, but not shear stresses, act over the central span, L. The central span, therefore, is in a state of pure flexure (no shear) and measurement of load and deflection, Y, at the middle of the central span enable calculation of the true flexural stiffness, D, by means of the following equation:

$$D = (1/16)(P/Y)(L^3/b)(a/L) \qquad (4)$$

where b is the width of the specimen. Exact location of the applied loads is largely arbitrary, except that the two outer spans, a, must be equal. It is important to note that the deflection of the central span is measured relative to the inner supports with the test arrangement illustrated in Figure 5.

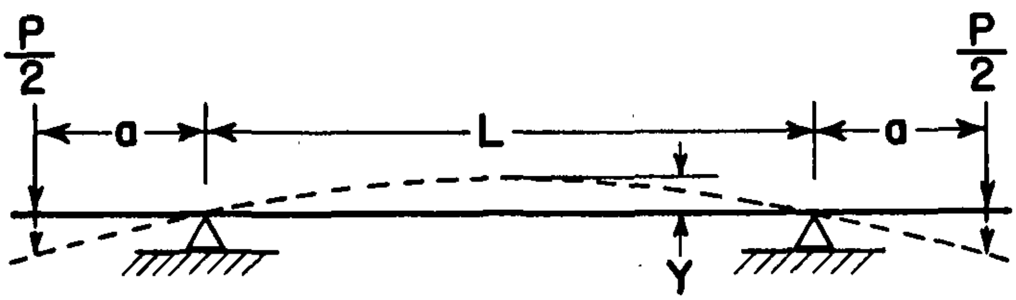


Figure 5. Four-point beam test

A test apparatus was constructed at The Institute of Paper Chemistry utilizing the principle of four-point loading. Figure 6 is a photograph of the upper and lower loading assemblies positioned on a corrugated board specimen. Figure 7 shows the apparatus in a testing machine during the flexure test on combined board. In the foreground of Figure 7, partly obscuring the specimen and loading assemblies, is a deflectometer which will be described later.

The lower support assembly is a standard accessory of the type available from testing machine manufacturers. Its loading anvils have 1/8-inch radius of curvature and may be spaced for a central span ranging from two to 12 inches.

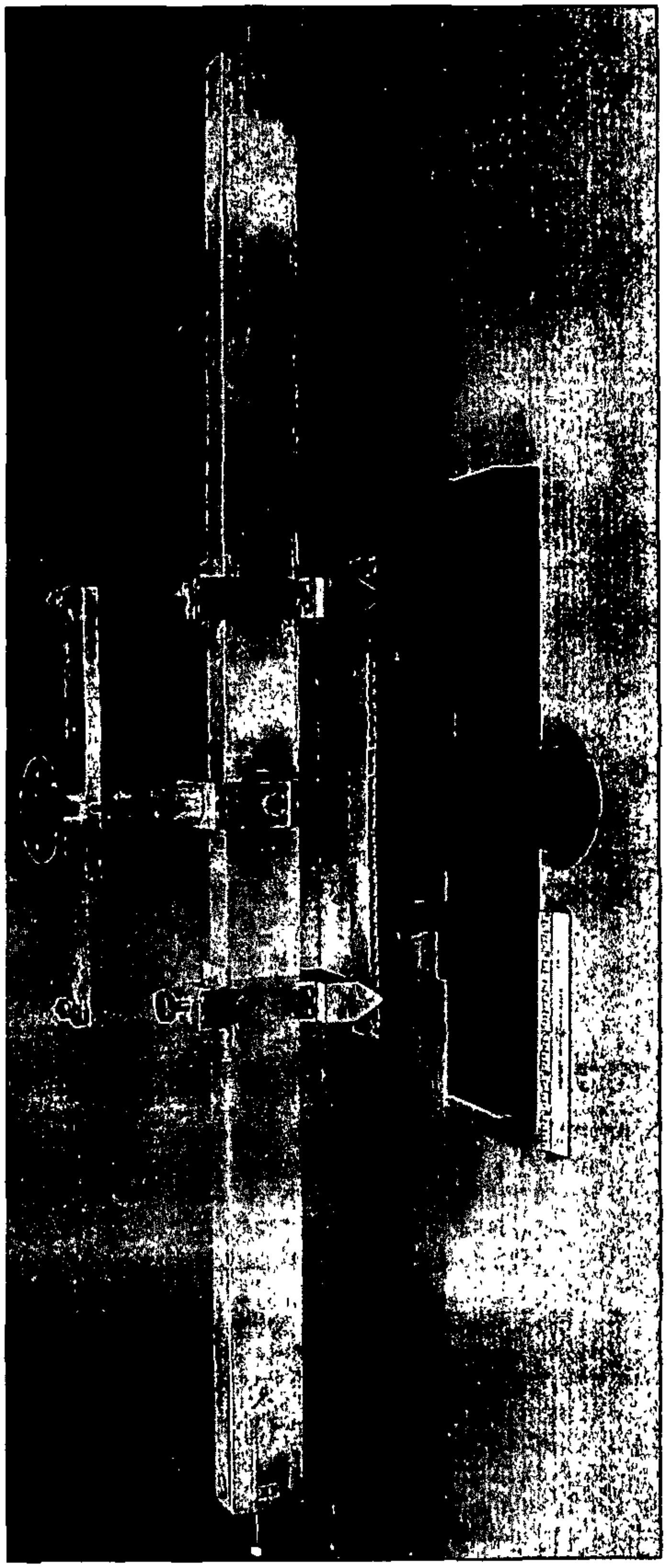


Figure 6. Apparatus for four-point loading of corrugated board

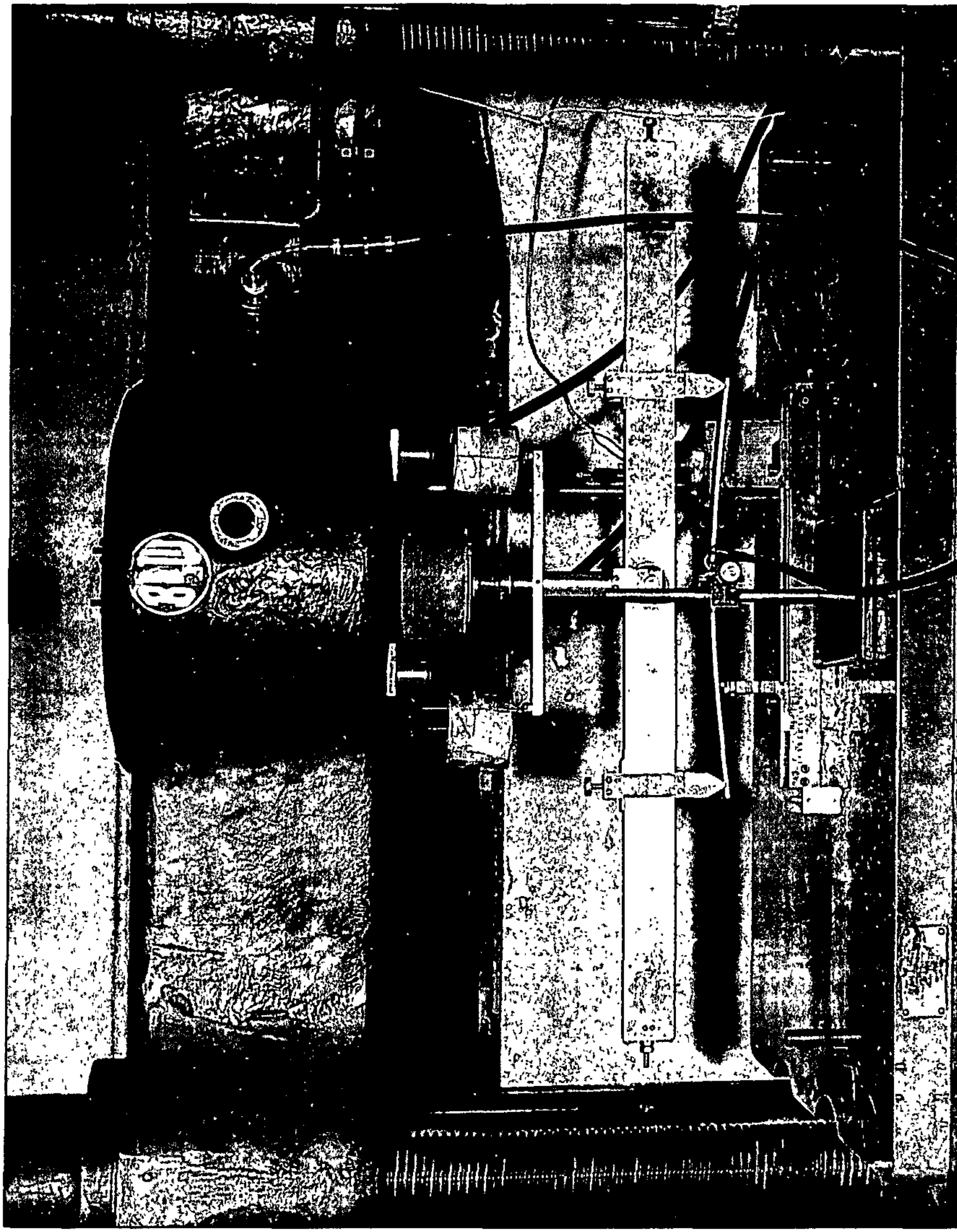


Figure 7. Test set-up for determination of flexural stiffness by four-point beam method

The upper loading assembly was constructed to offer a number of features believed to be desirable in a research apparatus. First, the horizontal bar to which the 1/8-inch radius loading anvils are attached is pivoted on a low-friction bearing; this equalizes the loads applied to the specimen and accommodates initial lack of straightness of the specimen. Second, as may be seen in the photographs, for research purposes provision was made for testing a wide range of central and outer spans. Third, the assembly was constructed of aluminum to minimize the inertia of the part (important for load equilibration) and for ease in handling. Regarding the latter, the upper loading assembly is suspended and counterbalanced from the cross-head of the testing machine (see Figure 7); the assembly is abutted lightly against an air-cell in the crosshead which measures the total applied load during the test.

Mid-span deflection is measured by means of a specially-constructed, low-force deflectometer, shown in Figure 8. This deflectometer consists of a cam-operated, linear differential transformer. All moving parts have low inertia, low friction, and low return spring force. The force required to actuate the deflectometer is about 0.01 pound, which is substantially less than in commercially available deflectometers with which the investigators are acquainted. A low-actuating force is required because, otherwise, a significant shear load may be imposed on the central span of the specimen by the deflectometer. The shear load attributable to the low-force deflectometer described above is less than 1% of the applied loads causing flexure in a typical test and, therefore, is believed to be negligible. Calculations indicate that the small restraint to bending caused by the deflectometer can be expected to introduce less than 1% error in flexural stiffness in a typical test on corrugated board.

The attributes of the four-point beam test are evident from a comparison of stiffness determinations at various spans. As an example, samples of the 200-lb.

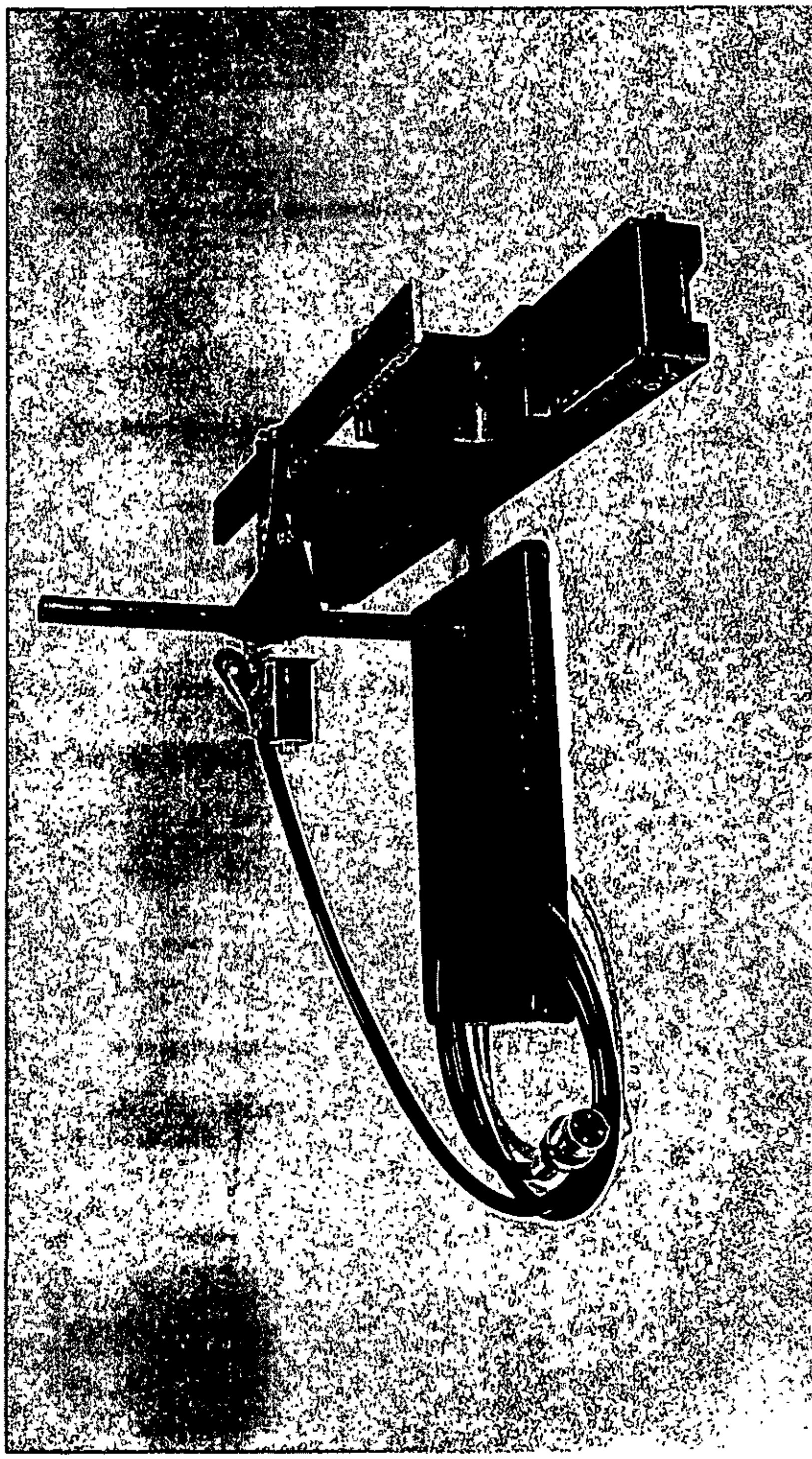


Figure 8. Low-force deflectometer used in four-point beam test

series, A-flute board (fourdrinier kraft liners, semichemical medium) mentioned earlier with regard to three-point beam tests were also tested by the four-point method at central spans of 4, 8, 10, and 12 inches. The outer span, a, in each instance was one-half of the central span. Ten specimens were tested at each span with both the three- and four-point methods and in all tests the loading rate was selected to give a maximum unit strain rate of 0.0025 in./in./min.

The average experimental flexural stiffnesses from the three- and four-point beam tests are listed in Table 1 for both principal directions of the combined board. The data are shown graphically in Figure 9 as a function of span. The curves for the three-point test are the same as in Figure 5. It may be seen that the machine direction, four-point flexural stiffness remained essentially constant at all four spans, with an average value of 194 lb.-in., indicating that the effect of shear was eliminated from the stiffness determination. In contrast, at the 26-inch span the three-point stiffness was only 166 lb.-in. and apparently a much greater span would be required to completely eliminate the shear influence.

Table 1--Effect of Span on Flexural Stiffness Determined
by Four-Point and Three-Point Beam Tests
(200-lb. series, A-flute, corrugated board)

Span, in.	Flexural Stiffness, lb.-in. — Machine Direction, Four-Point	Machine Direction, Three-Point	Cross-Machine Direction, Four-Point	Cross-Machine Direction, Three-Point
4	196	45	80.0	70.3
8	194	110	88.2	76.3
10	194	126	90.4	81.8
12	193	137	88.6	83.8
14	--	152	--	83.0
26	--	166	--	84.0
Av.	194	--	86.8	79.9

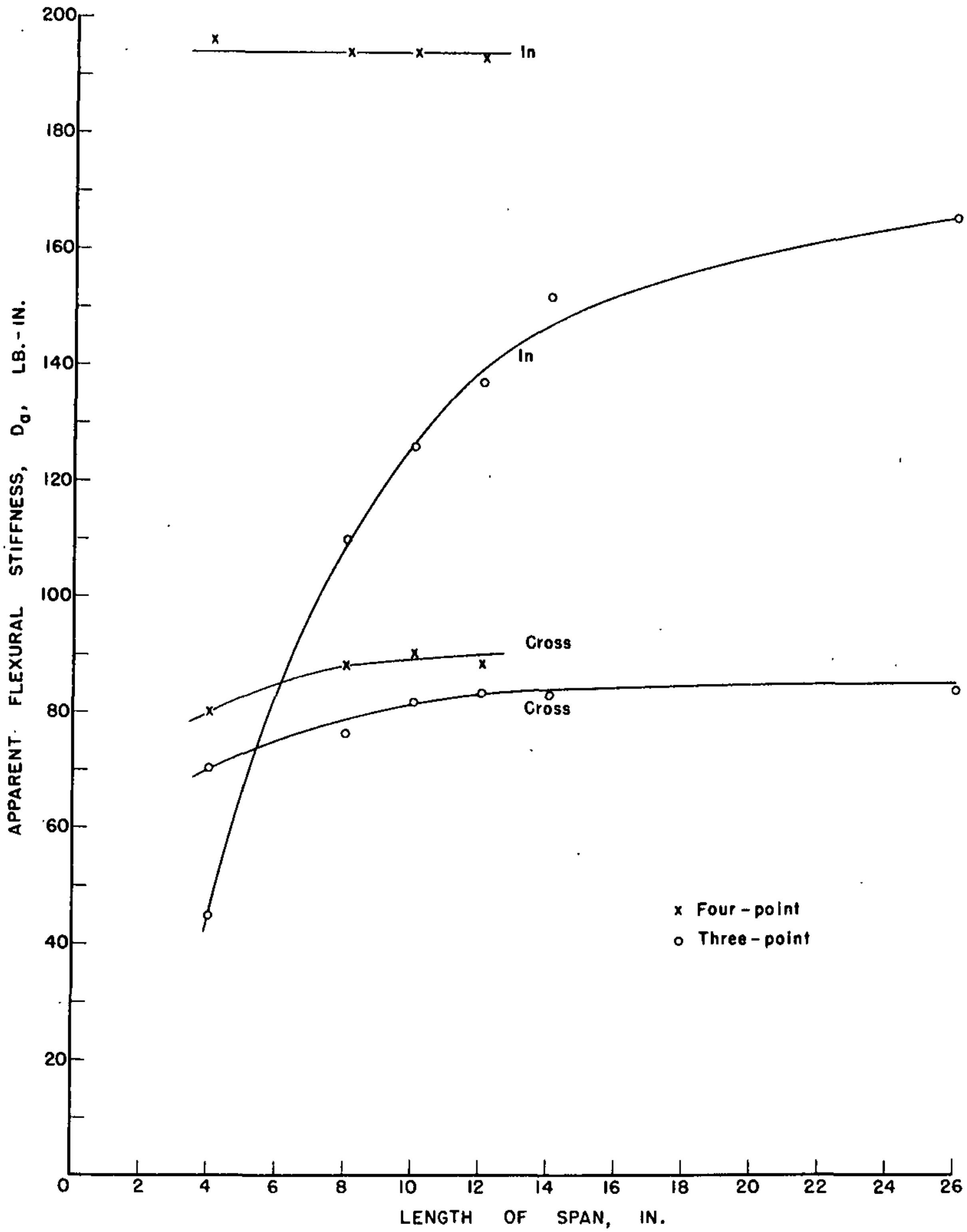


Figure 9. Effect of span on apparent flexural stiffness from three- and four-point beam tests

In the cross-machine direction, both types of tests gave more uniform results as the span was varied, although the four-point values were somewhat the higher of the two. In this particular sample, the stiffness from both types of tests fell off appreciably at the 4-inch span. Although this trend is suggestive of the shear effect, in the case of the four-point test, it is difficult to reconcile with the machine-direction data; the latter direction would be expected to be more sensitive to shear, but there is no evidence of it in these results. The data and other considerations indicate, however, that extremely short spans should be avoided.

Other exploratory studies with the four-point beam test indicated that the length of the outer span, a, relative to the central span, L, had no appreciable effect on flexural stiffness in the range a/L between 0.25 and 1.0. This ratio does influence the magnitude of the testing machine load and, therefore, may be selected for convenience. Width of a flexure specimen is a significant variable because of anticlastic curvature, i.e., the curvature across the specimen width and in the reverse sense to the curvature along the span. The loading anvils of the beam test apparatus interfere with the anticlastic curvature and make the specimen appear stiffer than it really is. The effect has been detected between one- and two-inch wide specimens and it is desirable, therefore, to use a narrow specimen. A width of one inch has been found to be satisfactory.

One of the most critical factors in flexure tests on corrugated board is the location of the loading anvils relative to the tips of the flutes in machine-direction specimens. When the anvil is located off the flute tip, the liner may be depressed into the void between flutes, thereby introducing a spurious increment to the measured deflection. In the four-point test setup illustrated in Figures 5 through 7, for example, local indentation of the lower liner detracts from the mid-span deflection and the specimen appears to be stiffer than it really is. Errors as great as 40% have been observed under these conditions. In three-point loading, liner deflection at the mid-span loading anvil increases the apparent deflection (when deflection is measured in terms of cross-head motion) and the specimen stiffness is underestimated. It is imperative, therefore, that the loading anvils be placed exactly on the flute tips of machine-direction beam specimens. An alternative or supplementary solution to this problem is the use of stress-spreaders in the form of a narrow plate between each anvil and the specimen, taking care that the plate does not interfere with the curvature. This technique, however, is usually not as convenient as simply adjusting the anvil spacing to coincide with the flute tips.

Comparison of three-point and four-point beam tests

Flexural stiffness in each principal direction was determined by both the three-point and four-point beam tests for a number of samples of combined board, as listed in Table 2. Beam specimens were taken from the panels of commercially manufactured boxes of the 200- and 275-lb. series in A-, B-, and C-flute constructions. All of the boxes were fabricated from fourdrinier kraft liners and semichemical mediums. The boxes represent the production of several box plants and, therefore, the properties of the components may differ significantly between samples.

Three-point beam tests were performed at two spans: six and fifteen inches, thereby permitting a correction for shear effects by means of Equation (3). The four-point specimens were tested with a six-inch central span and 1-1/2-inch outer spans. The specimen width was nominally one inch; the width of the cross-machine specimens was prepared to include exactly 3-1/2 flutes and the associated glue-lines. The loading anvils were placed on the flute-tips of machine-direction

Table 2--Comparison of Three-Point and Four-Point Beam Determinations of Flexural Stiffness

Flute	Series, lb.	Flexural Stiffness, lb.-in.				
		Three-point				
		6-inch Span	15-inch Span	Shear Corrected	Four-point	Diff., %[a]
			Machine Direction			
A	200	110[b]	152[c]	187	194	-3.6
A	275	120	221	263	250	+5.2
C	200	64.7	98.4	108	127	-15.0
C	275	68.1	130	158	150	+5.3
B	200	53.3	67.3	70.8	75.2	-5.9
B	275	73.1	91.4	95.9	108.4	-11.5
			Cross-machine Direction			
A	200	76.3[b]	83.0[c]	86.7	88.2	-1.7
A	275	126	152	158	160	-1.2
C	200	47.1	49.8	50.3	48.9	+2.9
C	275	63.2	85.1	91.1	89.5	+1.8
B	200	23.1	24.6[d]	25.1	27.8	-9.7
B	275	47.3	48.4	48.6	55.8	-12.9
					Av.[e]	6.4

[a]Based on four-point test results.
[b]Tested at 8-inch span.
[c]Tested at 14-inch span.
[d]Tested at 12-inch span.
[e]Without regard to sign.

specimens. A stress spreader was placed beneath the central anvil in the three-point test. Loads were applied at a rate corresponding to a flexure strain rate of 0.0025 in./in./min. in the liners at the most highly stressed location in the specimen. Ten specimens (one per box) were tested for each condition; mean values are listed in Table 2.

A comparison of the shear corrected three-point values and the four-point test results reveals that there was reasonably good agreement between these two types of tests. Considering all samples, the two methods of testing agreed to within 6.4%, on the average, although in individual instances the difference was as great as 15%. Noting the magnitudes and direction of the differences shown in Table 2, it appears that the corrected three-point determinations tended to be lower, in general, than the four-point values.

The coefficient of variation (standard deviation expressed as a per cent of the mean value) of the four-point beam test ranged from 1.9 to 8.7% for the samples listed in Table 2, with an average value of 4.8%. The standard error of the mean of a sample of ten specimens, therefore, is typically 1.5% of the flexural stiffness, indicating that the 95% confidence limits on the four-point stiffnesses are approximately 3% above and below the tabulated values. Assuming this same uncertainty in the shear-corrected, three-point stiffnesses (which is probably conservative), it would appear that only four of the twelve comparisons between three- and four-point stiffness in Table 2 represent significant differences. That is, in the majority of the comparisons there is probably no real difference between the results of these two ways of determining flexural stiffness.

The data of Table 2 illustrate that corrugated board is much stiffer in the machine direction than in the cross direction, as mentioned earlier. Calculations with these data reveal that the machine-direction stiffness was about 1-1/2 to 3 times greater than the cross-direction stiffness for these samples. This ratio reflects the higher modulus of elasticity of the machine direction of the liners relative to the cross direction. The flexural stiffnesses in the two principal directions apparently are equally important to top-load compression strength of vertical flute boxes [see Equation (2)]. It is interesting to speculate, therefore, on the effect of making a "squarer" sheet of liner: the flexural stiffnesses in the two directions would be more nearly equal, but would have the same net effect on box compression as the more highly directional liner of the same furnish. But a squarer sheet would be expected to lead to marked improvement in cross-direction edgewise compression strength of the combined board, with a consequent increase in top-load strength of the box (at the expense, however, of end-load strength).

Keeping in mind that the samples listed in Table 2 represent differing mill origin and thus differing liners and medium, it is evident from these data that flute size is a major factor in flexural stiffness of corrugated board. On the average, the four-point flexural stiffnesses of the C- and B-flute boards were 59 and 37%, respectively, of the A-flute stiffness, with only moderate variations due to series and direction. These percentages are very nearly in the same ratio as the square of the combined board caliper; on the average, the square of the C- and B-flute calipers were 61 and 38%, respectively, of the square of the A-flute caliper for these samples. This effect of caliper also emphasizes the importance of maintaining the flute height during corrugating and subsequent conversion operations.

Bench model four-point beam test

It has been pointed out that the three-point beam test suffers from the influence of shear. Although the situation may be alleviated by testing (a) a long span, or (b) two different spans and calculating a corrected value, neither of these two alternatives is particularly attractive from the standpoint of convenience.

On the other hand, the four-point beam test described above imposes rather severe demands on the equipment needed to perform the test. Although the upper loading assembly can be simplified considerably from that shown in Figures 6 and 7, the testing machine and deflection measurement requirements are stringent. For example, the loads applied by the testing machine are generally less than five pounds. Thus, testing machines normally used for box compression or for combined board and component compression testing probably would not have the load sensitivity required for beam testing (either four-point or three-point). Although the magnitude of load applied to the beam specimen can be increased by shortening the central span and the outer spans of four-point specimens, this practice, when carried to extremes, can become detrimental to the test because of crushing and other effects localized at the anvils.

Regarding deflection measurement, the force required to actuate the deflectometer must be kept small relative to the applied load in order to assure pure bending (no shear) in the central span of the specimen. This requirement prevents the use of many commercially available deflectometers.

In order to make the four-point beam test feasible in the corrugating or box plant, a bench model of the test apparatus, requiring a minimum of specialized test equipment, has been proposed. A simple apparatus was constructed to explore this approach, utilizing dead-weight loading of the beam specimen and manual measurement of the mid-span deflection. A photograph of the apparatus is shown in Figure 10. The lower support assembly is quite similar to the commercial unit described earlier. In this exploratory study the anvils were spaced six inches apart. The upper loading assembly is constructed of magnesium rod and weighs about 0.2 pound. The upper anvil spacing is nine inches. During test, weights are placed on a platform at mid-length of the upper assembly.

A barrel micrometer with a conical point is attached to the lower support assembly for purposes of measuring mid-span deflection. Detecting contact between the micrometer spindle and the underside of the specimen is aided by a simple electrical circuit. A narrow strip of 0.3-mil aluminum foil is adhered to the underside of the specimen at mid-span by means of a thin pressure-sensitive tape. The foil and micrometer are connected into a circuit comprised of a low voltage dry cell and flashlight bulb. The bulb lights up when the micrometer tip contacts the foil. The micrometer may be read to the nearest 0.0001 inch.

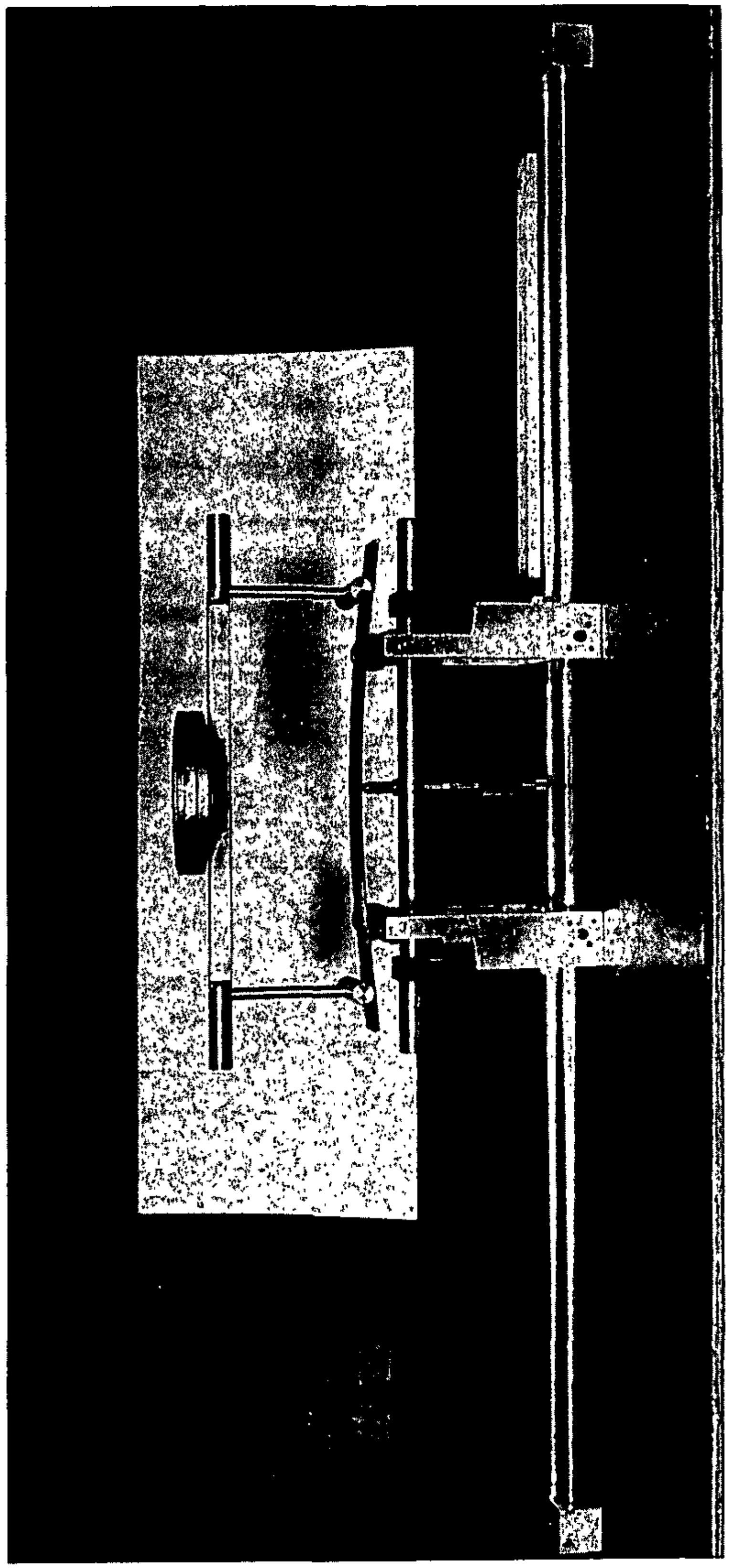


Figure 10. Bench model of four-point beam apparatus

To perform a flexure test, the specimen is placed on the lower anvils, taking care to align flute tips and anvils in the case of machine direction specimens. The upper loading assembly is then carefully placed onto the specimen, causing some small deflection of the specimen. A micrometer reading is taken. In terms of the load-deflection curve of the specimen that would be obtained from a testing machine, the initial loading described above corresponds to point A of Figure 11.

Next, a small weight, $\underline{W}$, on the order of one pound is gently placed on the platform of the upper assembly, bringing the specimen to point B on the curve of Figure 11, and the increment in deflection $\underline{Y}_1$ is measured with the micrometer.

Finally, a second weight, $\underline{W}$, equal to the previously applied weight, is added to the platform, bringing the specimen to point C in Figure 11, and the second increment of deflection, $\underline{Y}_2$, is measured. If the increments of deflection, $\underline{Y}_1$ and $\underline{Y}_2$, are equal, the operator knows that the test has been performed within the linear region of the load-deflection curve. Thereupon, the flexural stiffness of the specimen is determined by evaluating Equation (4) with $\underline{P} = 2\underline{W}$ and $\underline{Y} = \underline{Y}_1 + \underline{Y}_2$.

On the other hand, if $\underline{Y}_2$ is greater than $\underline{Y}_1$ (allowing for the precision of the micrometer measurements), the operator is made aware that the second weight application exceeded the proportional limit of the load-deflection curve, and the test must be repeated using lesser weights. In the study reported here, one pound weights were suitable for A-flute board and half-pound weights for B-flute board.

The bench model was compared with the more elaborate testing machine four-point apparatus by determining the flexural stiffness of a number of samples of A-, B-, and C-flute combined board in the 200- and 275-lb. series. The testing machine determination was performed at a loading rate corresponding to a maximum

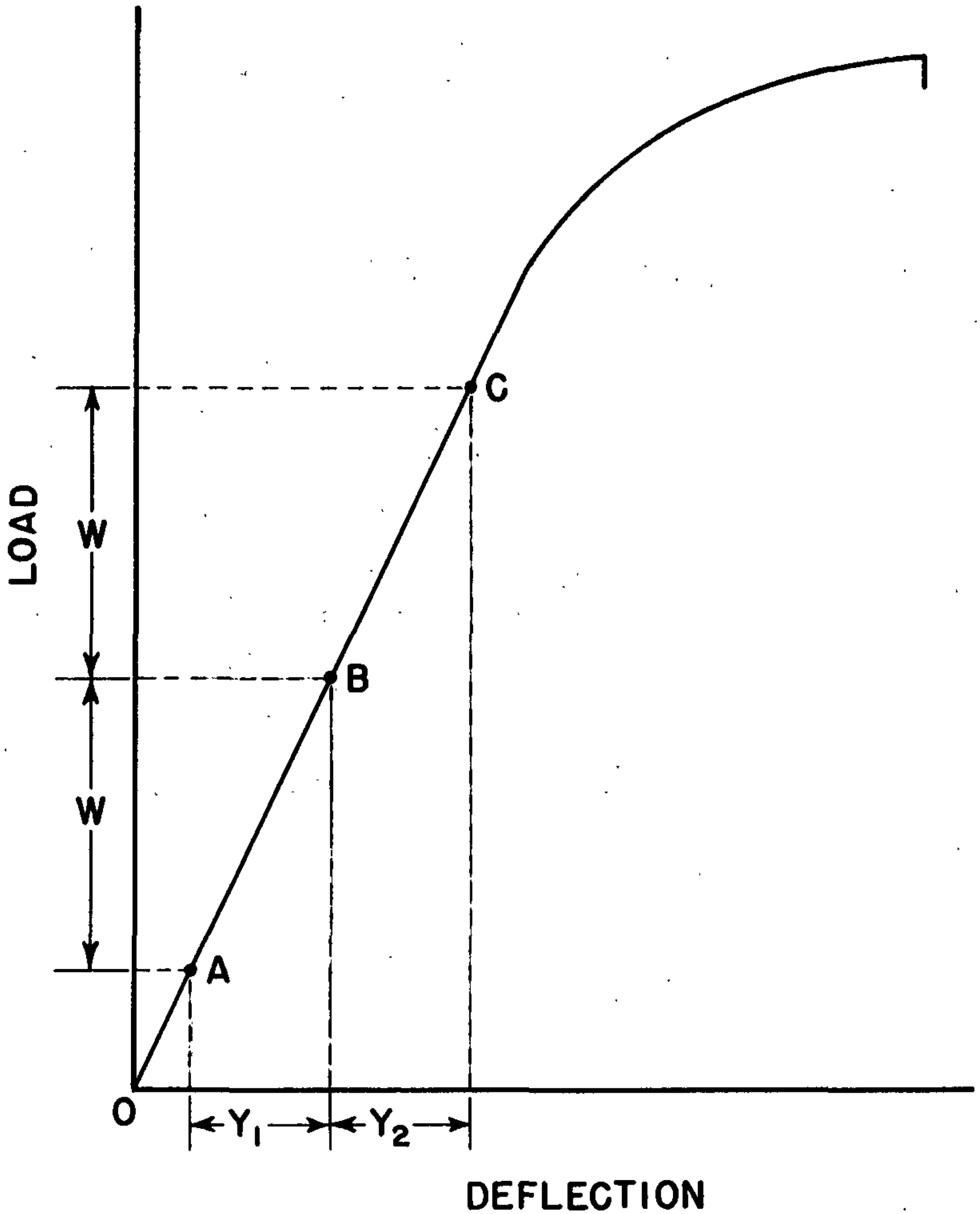


Figure 11. Representative load-deflection curve of four-point beam specimen showing loads and deflection applied in bench test

strain rate of 0.0025 in./in./min. in the specimen, requiring typically one-half minute to pass beyond the linear region of the load-deflection curve. The time interval between load applications in the bench model test was about one minute, during which time the micrometer was set and read; thus, the total time of loading one specimen was about three minutes.

Ten specimens were tested for each condition; the average value of flexural stiffness of each sample is listed in Table 3. It may be seen that in each instance the bench model gave lower values of flexural stiffness than the testing machine beam apparatus. The differences were greatest in the machine-direction specimens with an average difference of 8.4%, as contrasted with 3.4% in the cross direction.

It is believed that creep in the beam specimen during the bench-model test caused the test result to be lower than the testing machine determination. Creep is the continued deformation that occurs in a material even though the applied stress remains constant. It was frequently noted during the bench model tests that the specimen continued to deflect while the micrometer was being read, as evidenced by the detecting light going out. Accordingly, it may be assumed that the specimen experienced creep at all times during the test. This increase in deflection has the effect of lowering the apparent flexural stiffness, inasmuch as the latter is essentially the ratio of load to deflection of the specimen.

It appears from this exploratory study, therefore, that this version of the bench model, four-point beam test can be expected to underestimate the flexural stiffness of corrugated board because of the time factor in performing the test. The method may be used for a relative value of flexural stiffness, although it would require care in maintaining a uniform time of reading deflection from test to test. It is believed that the limitations of the bench model test

Table 3--Comparison of Bench Model and Testing Machine Determinations of Flexural Stiffness

Flute	Series, lb.	Flexural Stiffness, lb.-in. Testing Machine	Bench Model	Diff., %[a]
		Machine Direction		
A	200	196	180	-8.2
A	275	319	296	-7.2
B	200	75.2	65.8	-12.5
B	275	108.4	97.5	-10.1
C	200	127	111	-12.6
C	275	150	150	0.0
			Av.:	-8.4
		Cross-machine Direction		
A	200	84.7	81.9	-3.3
A	275	160	159	-0.9
B	200	27.8	26.7	-4.0
B	275	55.8	55.3	-0.9
C	200	60.5	55.2	-8.8
C	275	89.5	87.3	-2.5
			Av.:	-3.4
			Composite Av.:	-5.9

[a]Based on testing machine value.

may be overcome by devising a deflection measuring apparatus which will indicate deflection immediately upon application of load to the specimen.

Summary

1--The top-load compression strength of vertical flute, corrugated boxes which are not extremely short depends on (a) the edgewise compression strength of the combined board, (b) the flexural stiffnesses of the combined board in both principal directions, and (c) the box perimeter.

2--Flexural stiffness is the ability of the combined board to resist bending. This property depends primarily on the modulus of elasticity and caliper of the liners and on the square of the combined board caliper.

3--The differences in top-load compression strength of A-, B-, and C-flute boxes made from the same components and having the same dimensions is attributable mainly to the differences in flexural stiffness, the latter reflecting the differing combined board caliper of these flute constructions.

4--A common method of determining flexural stiffness is by testing a specimen of material as a centrally-loaded (i.e., three-point) beam. The test results from this type of beam test, however, reflect both the flexural stiffness and the shear rigidity of the material. Because of the low shear rigidity of corrugated board (particularly in the machine direction), the three-point beam test underestimates the true flexural stiffness of the board.

5--The error in determination of flexural stiffness by the three-point test can be minimized by (a) testing long specimens or (b) testing specimens at two different spans and calculating a corrected value.

6--A study was made of four-point beam testing, which does not involve shear effects.

7--Determinations of flexural stiffness of corrugated board by the four-point method and the shear-corrected, three-point method agreed to within 6.4% on the average, indicating that essentially the same test results can be obtained by either method. The four-point test is more direct because only one span is required.

8--To simplify the equipment required for determination of flexural stiffness in the box plant, a bench model of the four-point beam test apparatus was constructed and studied. This simple apparatus utilizes dead-weight loading and measurement of deflection by means of a micrometer.

9--Flexural stiffness determined by the bench model was about 6% lower, on the average, than that obtained with a testing machine. This difference appeared to be due to creep in the specimen during the period of testing. The bench model may be useful, however, for relative determinations of flexural stiffness, provided the testing time is standardized.

References

1--McKee, R. C., Gander, J. W., and Wachuta, J. R. Edgewise compression strength of corrugated board. Paperboard Packaging 46, no. 11:70-6(Nov., 1961).

2--Timoshenko, S. Strength of materials, Part I., p. 95, New York, D. Van Nostrand Co., 1955.

3--Boller, K. H. Buckling loads of flat sandwich panels in compression, Madison, Wis., USDA, Forest Service, Forest Products Laboratory, Report No. 1525-A, p. 11, Feb., 1947.

4--American Society for Testing Materials. Method of test for flexural strength of concrete. Standards C-78-59, 1961.

5--Libove, C. and Batdorf, S. B. A general small-deflection theory for flat sandwich plates, Washington, D. C., National Advisory Committee for Aeronautics, TN 1527, April, 1948.

6--Kuenzi, E. W. Flexure of structural sandwich construction, Madison, Wis., USDA, Forest Service, Forest Products Laboratory, Report No. 1829, Dec., 1951.

7--Marks, L. S. Mechanical engineer's handbook, p. 426, New York, McGraw-Hill Co., 1951.