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MECHANICAL PROPERTIES OF A SANDWICH CORE STRUCTURE
FABRICATED FROM STEEL CANS

A THESIS
Presented to
The Faculty of the Division of Graduate Studies and Research

By
Terry Cl. Domm

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MECHANICAL PROPERTIES OF A SANDWITH CORE

STRUCTURE FABRICATED FROM STEEL CANS

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Date approved by Chairman: 11 May 1972
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</tr>
<tr>
<td>K</td>
<td>Constant</td>
<td></td>
</tr>
<tr>
<td>L</td>
<td>Principal core direction</td>
<td></td>
</tr>
<tr>
<td>L'</td>
<td>Length of shear specimen, inches</td>
<td>inches</td>
</tr>
<tr>
<td>L</td>
<td>Constant</td>
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<tr>
<td>m</td>
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<td>P</td>
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<td>q</td>
<td>Uniform load density, lb./in.²</td>
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- **r**: measured displacement, inches
- **T**: principal core direction
- **t**: plate thickness, inches
- **t₁**: thickness of upper face plate, inches
- **t₂**: thickness of lower face plate, inches
- **u**: length of loaded area in x direction, inches
- **u_c**: core displacement in x direction, inches
- **V**: volume, in.³
- **v**: width of loaded area in y direction, inches
- **v_c**: core displacement in y direction, inches
- **W**: weight, pounds
- **W**: principal core direction
- **w**: displacement in z direction, inches
- **w_c**: core displacement in z direction, inches
- **x**: rectangular coordinate
- **y**: rectangular coordinate
- **z**: rectangular coordinate
- **α**: deflection factor
- **γ**: shear strain, in./in.
- **ε**: compressive strain, in./in.
- **η**: distance to load center in y direction, inches
- **ν**: Poisson's ratio
- **ξ**: distance to load center in x direction, inches
- **ρ**: density, lb./ft.³
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<td>( p )</td>
<td>( a/b )</td>
</tr>
<tr>
<td>( \sigma_c )</td>
<td>compressive stress, ( \text{lb./in.}^2 )</td>
</tr>
<tr>
<td>( \sigma_u )</td>
<td>ultimate stress, ( \text{lb./in.}^2 )</td>
</tr>
<tr>
<td>( \sigma_y )</td>
<td>yield stress, ( \text{lb./in.}^2 )</td>
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<tr>
<td>( \sigma_z )</td>
<td>normal stress in ( z ) direction, ( \text{lb./in.}^2 )</td>
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<tr>
<td>( \tau )</td>
<td>shear stress, ( \text{lb./in.}^2 )</td>
</tr>
<tr>
<td>( \phi )</td>
<td>angle between loading plane and surface plates in shear test, degrees</td>
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SUMMARY

One of the most pressing problems facing our environment today is that of solid waste disposal which would include discarded cans which are among the largest contributors to the problem. It would be desirable to recycle the waste cans, but before recycling becomes practical uses must be found for the scrap cans. It is possible to fabricate a honeycomb core material from the steel cans which could be used in sandwich form in various structural applications.

In this research, a fabrication process was developed using resistance welding to join the steel cans to each other. Techniques of fabrication were developed to maximize the bond strength.

In order to determine the feasibility of using the fabricated core in structural applications, certain mechanical properties of the core material were determined. Experimental stress-strain data was obtained in shear and flatwise compression. From the experimental data values of shear modulus in the principal directions, shear yield strength, stabilized compressive modulus, ultimate compressive strength, and crush strength of the core were determined. Properties found experimentally were compared to those of commercially available cores.

A theory for deflection of a rectangular, sandwich panel, simply supported on all four edges and loaded by a uniform load
over a rectangular area, was developed, a computer program was written to solve the resulting set of seven simultaneous equations in double infinite series form and seven unknowns. Deflection values were calculated for a typical panel using the dimensions and properties of the fabricated core material. The results were compared to those for a solid steel plate having the same maximum deflection, and the sandwich panel was shown to possess an excellent strength to weight ratio.

On the basis of mechanical properties determined in the research the fabricated core was shown to be promising as a structural material in a sandwich form. If an economical production method can be developed, the core material could provide a needed market for recycled steel cans.
CHAPTER I

INTRODUCTION

An increasing public concern for the environment has created a need for investigation into means of disposal and recycling of many items, among which is the common tinned steel can. Tin plate and tin free steels are used in more than sixty billion cans annually. The key to success in a recycling operation lies with the market for the item being reclaimed. It is possible to fabricate core sections for sandwich type structures from reclaimed cans. The use of cans as core material represents possible new applications of used cans in various structural applications, which by creating new markets for used cans would assist environmental clean-up efforts. In order to determine the feasibility of using cans as core material, core properties must first be determined. In an attempt to evaluate the core, the investigation described in this thesis was carried out to determine several of the major mechanical properties of the material.

Sandwich Structures

A sandwich core structure consists basically of two face plates separated by and attached to a core as shown in Figure 1. In general the face plates are much thinner and possess greater strength and stiffness than the low density core. Core sections are
Figure 1. Sandwich Panel.
commercially available in a variety of cell configurations, cell sizes, densities, and materials. The most common cell sizes are quite small, usually ranging from one-eighth of an inch to one-half of an inch. Cores made of a variety of materials to include metals, plastics, and paper have been developed.

Many applications of sandwich material can be found in the aerospace industry due to the outstanding weight to strength ratio it possesses. The advantages of this material form have not gone unnoticed in other types of application however. Sandwich material has been utilized in the construction of pre-fabricated shelters, exterior curtain walls, partitions, and doors [1]. Other diverse applications include desk tops, boats, scaffolding, house trailer flooring, and cargo containers among others.

The sandwich structure has several physical properties which make it very desirable in structural application. Those properties which particularly apply to the type of core section under study are as follows [2,3]:

1. high strength to weight ratio
2. low thermal conductivity
3. low acoustical transmission
4. uniform energy absorption capability
5. directionalization of fluid flow.

In the form of a sandwich panel, the core structure is capable of the highest strength to weight and rigidity to weight ratios obtainable using ordinary design methods [4]. The sandwich structure
acts as an "I" beam. The face plates carry tensile and compressive bending stresses as do the beam flanges while the core takes the shear stresses and stabilizes the face plates as does the web of the beam.

Thermal resistance is a function of core thickness, density, and material. Basically, in the sandwich form the panel acts as a good insulator due to the low conducting area and entrapped air spaces. It has been suggested that sandwich structures utilizing low conductive core members and having a vacuum inside be used for insulation purposes [5].

Sandwich panels can be used as sound barriers because of transmission losses as the sound passes through the material. By using perforated or porous face plates sandwich panels may be employed as sound absorbers.

Sandwich cores also possess very desirable energy absorption characteristics. When loaded beyond the ultimate compressive stress in a direction parallel to the cell axes, the core will crush at a practically constant stress level.

**Historical Background**

The idea of using hollow cylindrical bodies in a structural application was suggested as early as August, 1923 when a patent was granted to Axel E. Alander [6]. His idea consisted of cylindrical bodies with flanged ends placed side by side with their axes approximately parallel. The bodies were to be held in place by an
adhesive bond between the flanges. The resulting structure was to be used in wall construction.

Also in the same year Vincent V. Pittman invented a building slab of a plastic material in which cans were embedded between reinforcing meshes [7]. Some of the cans were arranged with their axes normal to the slab surfaces while others were placed in the plane of the slab with open ends adjoining.

In 1966 Maurice Norman considered the possibility of constructing partition panels of a tubular core construction [8]. His prefabricated panel consisted of two face plates joined to a core of cylindrical tubular members with longitudinal axes parallel to the planes of the face sheets. The tubes were secured to the face plates.

Still another approach was that of Fred M. Walker who was concerned with reuse of reclaimed tinned metal waste pieces [9]. He developed a method of making a multilayered metal sheet from tinned cans. The cans were cut and flattened so that they could be arranged in an interlocking overlapping configuration.

Although there are many ideas suitable for using recycled tinned steel cans in structural applications, there is little evidence that any of these ideas were actually applied or in fact was economically feasible. One exception is a forty foot high geodesic dome being constructed to house a particle-physics chamber for the National Accelerator Laboratory in Batavia, Illinois [10]. The translucent dome will be built of honeycomb panels made of empty beer and soda cans sandwiched between plastic sheets. After removal
of can tops and bottoms and oven drying the cans are glued to the surface sheets with an epoxy-resin adhesive. Aluminum side channels are attached using the same adhesive. Each panel is an equilateral triangle with sides slightly in excess of ten feet and a thickness of five inches. Tests have shown these panels to be extremely stiff, even though the cans are not bonded to each other.

**Objective**

The object of this study is to fabricate a sandwich core material from steel cans and to determine certain mechanical properties of the core as mentioned previously. To accomplish the objective techniques of fabrication were developed and an attempt was made to obtain the strongest section possible. The method of fabrication used was resistance spot welding. Shear modulus, shear yield strength, stabilized compressive modulus, ultimate compressive strength, and crush strength of the core material were determined experimentally.
CHAPTER II

FABRICATION

Before core properties could be determined it was necessary to develop a method of core fabrication. Initially a means of attaching the cans to each other was selected. Then techniques of fabrication were developed to maximize the strength of the bond between the cans and to give the best core material possible. Fabrication cost was an important design consideration.

The core was constructed from number 303 tinned steel cans manufactured by the Continental Can Company. Average can dimensions

![Can Dimensions]

Figure 2. Can Dimensions
after removal of both ends are shown in Figure 2. Thickness of can sides was 0.007 inches. A sample of core material after fabrication is shown in Figure 3.

Method of Bonding

There are a great many possible methods available by which to attach the cans to each other. Among the most promising possibilities were mechanical fasteners, adhesive bonding, and welding. Each of these possibilities represents a broad range of possibilities in itself. The use of mechanical fasteners such as nuts and bolts or rivets was limited by the small working space available inside the can. Major modification of existing equipment would be required to utilize this method. Adhesive bonding might require spacers between the cans at each bond or removal of the rims since the can rims prevent the cylindrical sides from touching each other.

Resistance welding was the means chosen for joining the cans. This method of metal fabrication is in common use in industry today. Existing welding equipment can be employed without modification since only the electrodes themselves need to reach inside the cans. No spacer is required between the cans as the electrodes force the can sides together during the welding process. The greatest disadvantage of this method is the requirement that the metal surfaces be free of paint or other non-conducting coatings in order for electrical contact to be achieved between the electrodes.

Core Configuration

If the rims were removed from the cans each cell would remain
Figure 3. Fabricated Core Material.

Figure 4. Fabrication Apparatus.
cylindrical in shape upon fabrication into a core. It is possible to arrange the cans so that each can is joined to either four or six adjacent cans in the core. The latter arrangement is a stronger section as it has a higher density and stiffness.

When the rims are not removed, the cans are deformed upon welding their sides together. As in the case of cylindrical cells, each cell can be joined to four or six others as illustrated in Figure 5. The typical cell will approach a square or hexagonal shape as shown. The hexagonal cell shape was chosen since it is the strongest possible configuration without requiring rims to be removed.

![Square Cell](image1)

![Hexagonal Cell](image2)

Figure 5. Core Configurations.
Welding Apparatus

The welder used for the fabrication was a Miller Model Lectro portable spot welder as shown in Figure 4. Standard twelve inch electrodes were used having a rated output of 4550 amperes and an open circuit rated voltage of 1.6 volts. A built in electronic timer allows the weld time period to be varied from two to sixty cycles based on sixty cycles per second without variation from weld to weld. Tip pressure is preset and uniform for all welds.

It was desired that the weld seams be as uniform as possible. As described above weld time and tip pressure can be held constant and consequently the spot weld itself will be nearly constant from weld to weld. Each seam consists of a row of ten welded spots on one-quarter inch intervals. In order to be able to uniformly space the spots along each seam and to repeat the same spacing for each seam a welding jig was designed and constructed. Portions of the jig may be seen in Figure 4. Basically the jig consists of a vertical work table, a holding device to hold the work piece during welding, and an indexing assembly to locate the welder with respect to the table. Figures 6 and 7 show the indexing assembly and holding device respectively.

As was stated previously the cans are deformed into a hexagonal shape upon welding. It was found that the stiffness of the electrodes was insufficient to pull the can sides together with the necessary pressure for a good weld. To remedy this problem a clamp was constructed to hold the can sides together during welding. The clamping
Figure 6. Indexing Assembly.

Figure 7. Holding Device.
arrangement is shown in Figure 8. Two clamps are used, one on each end of the seam being welded.

Weld Strength Tests

A series of tests was conducted to determine the optimum weld to be used in construction of the honeycomb core. The tests were designed to investigate the effect on weld strength due to variation of the parameters present in the welding process. The variables considered were weld time, electrode tip area of contact, and surface preparation. In addition yield strength as a function of the number of welds in a seam was investigated.

Surface preparation investigated was of two types. One method tested was to remove the tin coating from the surfaces to be welded by means of fine abrasive cloth. The other method utilized was to apply the welder twice at each spot. It was intended that the first application would burn the tin off the steel, allowing the second to form a good bond.

The test specimens were prepared from metal cut from the same type of tinned steel cans as were to be used in the actual core construction. Each specimen consisted of two metal strips with the ends overlapped and welded. Tests were performed on the Instron Universal Testing Machine-Floor Model TT. The test specimens were clamped with Instron Type 10F Wedge-Action Jaws and the welds were pulled apart at a rate of 0.02 inches per minute. Tensile loads were recorded directly on a strip recorder built into the testing machine from signals received from a Type F load cell. Loading on the welds
Figure 8. Clamping Arrangement.

Figure 9. Seam Detail.
was basically shear, but there was some peeling evident. Since the purpose of the tests was to determine relative strength and each specimen was loaded in the same manner, the peeling was considered insignificant. Strength values obtained were not true shear strengths, however.

Results of Weld Strength Tests

Results are shown in Figures 10 through 14. In Figures 10 through 13 the yield load, the load at which failure occurred, is plotted against weld time. The following results are noted:

(1) The trend was for relatively constant or slightly increased weld strength with increased weld time. This is expected since a longer weld time allows more time for diffusion at the weld.

(2) Yield load values are higher for larger tip areas and scatter of the data points was reduced. Although an attempt to match electrode tip surfaces was made, deflection of the welder electrodes prevented contact from being made over the entire tip area. Actual weld area was less than tip area in both cases, but the larger tip area produced a larger weld area.

(3) Yield load for specimens from which the tinned layer had been removed from welded surfaces by means of abrasive cloth was higher for low weld times, but decreased for higher times.

(4) Yield load values for applying the welder twice are slightly less than for one application. Weld time shown is the sum of both applications.

In actual application weld time is restricted to low values.
Figure 10. Weld Strength.

Figure 11. Weld Strength with 5/32 Inch Electrode Tip Area.
Figure 12. Weld Strength with Surface Cleaned.

Figure 13. Weld Strength with Two Applications of the Welder.
Tip Diameter: $\frac{3}{8}$ in.
Thickness: 0.007 in.
Feed Rate: 0.02 in./min.

Figure 14. Seam Strength.
because of material thickness. Low weld times minimize burn through and prevent the electrodes from bonding to the cans. Since yield load does not increase appreciably with increase in weld time, the lowest possible time giving a full strength weld each time it was initiated was selected to minimize burn through. The value chosen was fifteen cycles. The additional increase in yield load for de-tinned surfaces was not enough to justify the additional effort required in material preparation. Larger electrode tip areas and one application of the welder gave the greatest yield loads.

Multiple Welds

The relationship between the number of welds in a seam and total yield load may be seen in Figure 14. It was anticipated that the total yield load would be less than the sum of the individual welds and the tests confirmed it. All welds are not loaded uniformly causing the seam to fail at a lesser load than if all welds were acting simultaneously.

Each seam consisted of ten welded spots on one-quarter inch intervals. Details of the seam may be seen in Figure 9.

Bonding of Surface Plates

In order to obtain experimental data it was necessary to bond metal plates of various thicknesses to the core. Bonding was accomplished by means of an epoxy adhesive. Since the core material is generally used in a sandwich form, these techniques may also be employed in preparation of structural material.

Cold rolled steel was chosen as the material for the surface
plates to avoid lengthy surface preparation required of aluminum. Metal plates were necessary to achieve the stiffness required in the experiments. The surface to be bonded was first rubbed with a very fine abrasive cloth to remove oxides and slightly roughen the surface. The surface was then washed with xylene followed by acetone to remove all contaminates. Both the surface plates and the edges of the core were prepared in this manner.

The epoxy adhesive used was American Cyanamid FM 123-2. The epoxy is in sheet form and a weight of 0.085 pounds per square foot was selected.

The surfaces to be bonded were clamped securely together with a load of approximately 50 lb./in.² applied to the surface plates. Clamping was accomplished by means of large C-clamps.

The epoxy must be cured at a temperature of 225°F. to 250°F. for a period of sixty to ninety minutes. The samples were cured in an electric oven. Temperature was monitored by means of a Thermo Electric Minimite which measures temperature directly using a copper-constantan thermocouple.

Core Thickness

Thickness measurements were made with a vernier caliper over two samples of the fabricated core material. Thickness of the core varied from 4.414 inches to 4.425 inches and from 4.412 inches to 4.423 inches at interior points measured on the two samples. Measurements on the outside edges of the core sections varied from 4.408 inches to 4.412 inches and 4.403 inches to 4.406 inches respectively.
As can be seen, the edges of the core were not as thick as the interior points. With the exception of the outside row of cans, average core thickness was 4.419 inches. The average edge thickness was 4.408 inches.

**Core Density**

Density in pounds per cubic foot is calculated by

\[ \rho = \frac{1728 \ W}{V} \]

where

- \( W \) is the weight of the sample, pounds force
- \( V \) is the volume of the sample, in.\(^3\)

A typical core sample weighing 2.94 pounds provided the data used in the density calculation. Calculated dimensions based upon the can dimensions were used to obtain the volume. The volume was calculated in this manner to minimize dimension discrepancies caused by edge effects.

\[ W = 2.94 \text{ lbf.} \]

\[ V = (24.97 \text{ in.})(9.375 \text{ in.})(4.41 \text{ in.}) = 1030 \text{ in.}^3 \]

\[ \rho = \frac{1728 \times 2.94}{1030} = 4.9 \frac{\text{lb.}}{\text{ft.}^3} \]
CHAPTER III

SHEAR EXPERIMENTAL INVESTIGATION

Purpose of Experimentation

The principal strength of a honeycomb core lies in its shear strength. Consequently, the mechanical properties of the core which provide a measure of shear strength are of primary interest. Tests were conducted upon the core material to obtain stress-strain characteristics for the core.

Shear Properties

The shear properties of the core consist of shear modulus, yield strength, and ultimate strength. All three properties are directly obtainable from load-deformation data of the core. Only the shear modulus and yield strength were determined since the load capacity of the testing machine used for the experiment was insufficient to reach the ultimate strength.

The shear modulus is normally defined as the initial slope of the stress-strain curve for the core. It is obtained by drawing a tangent to the stress-strain curve at the origin of the curve. Since the core is orthotropic, the properties show directional characteristics. Separate load-deformation data was taken in each of the two principal directions, allowing the shear properties to be obtained for each direction. Principal directions, L and W, are
Yield strength is the stress at which yielding of the core begins. Since the exact point at which the elastic range ends is impossible to determine, yield strength is normally defined in terms of a certain value of yield strain or permanent set. Yield stress defined in this manner is called offset yield strength. The standard offset of 0.2 per cent was used in the investigation.

Shear Test Specimens

Specimens were prepared for the study of shear properties in the TL and TW planes. Core sections were cut with edges parallel to the principal planes as shown in Figures 16 and 17. The core sections were made as large as possible in order to minimize edge effects. Edge effects are a major consideration due to the large cell size. The maximum specimen dimensions were limited by the physical dimensions of the testing machine as well as its load capacity. The specimens were made as long as possible along the direction of loading, L', in order to minimize the compressive load component as illustrated in Figure 18. To obtain an adequate portion of the stress-strain diagram for determination of the offset yield strength, the width of the specimens was restricted by the limited load capacity of the testing machine. Thickness of the core is the can thickness.

Rigid steel plates were bonded to the core sections with epoxy adhesive as previously described. A knife edge was machined on the loading edge of each plate as shown in Figure 17. The plates were positioned relative to the core such that the line of action passed
Figure 15. Principal Directions.
Figure 16. Shear Specimen for Loading in TW Plane.  Figure 17. Shear Specimen for Loading in TL Plane.
Figure 18. Shear Test Specimen and Apparatus for Attachment to Loading Machine.
through diagonally opposite corners of the core. Loading plate thickness was designed to give a plate stiffness greater than 600,000 lb. in$^2$/inch of width per inch of core thickness. This stiffness value is suggested by Reference [11]. A plate thickness of 0.375 inches was used in the experiment.

**Experimental Equipment**

The samples were loaded by means of an Instron Universal Testing Instrument. Instron Type F Load Cell with a maximum capacity of 10,000 pounds was used to measure compressive loads. Displacement measurements were made with an Instron Strain Cage Extensometer Model G-51-12. Load-displacement curves were plotted by the strip recorder mounted in the testing machine.

**Experimental Procedure**

A compressive load was applied to the knife edges of the loading plates by the testing machine. The upper and lower loading blocks were designed to distribute the load uniformly along the edge and consequently across the core width. The load was continuously applied at a constant rate of 0.05 inches per minute. An extensometer was attached to the mounts in such a way as to measure relative displacement between the two loading plates at the center of the core. Load-displacement curves were recorded for each test specimen.

**Discussion of Results**

Stress-strain curves for the core in the two principal directions are plotted in Figures 19 and 20. Shear properties are
Figure 19. Stress-Strain Diagram for Shear Load in TL Direction.
Figure 20. Stress-Strain Diagram for Shear Load in TW Direction.
summarized in Table 1.

The experimentally determined values can be compared with existing data for hexagonal aluminum honeycomb core material. Typical published shear properties for hexagonal core material having a density similar to the test samples is listed in Table 2 [12]. Several factors which make only an order of magnitude analysis possible must be considered in making the comparison. The cell size and material thickness are substantially larger for the tested specimens and the material is different. Although both cores have hexagonal cells, Figure 21 illustrates the difference in orientation of the cells. Comparing shear modulus values for similar densities it is found that the experimental values are of the same order of magnitude as the published values. The published values show a ratio of shear modulus values for the principal directions as

\[
\frac{G_{TL}}{G_{TW}} \approx 2.5 \quad [13].
\]

Experimental values showed that the shear modulus did not differ substantially between the principal planes. The comparison of cell arrangement and bonding points indicates that the difference in values should be much less pronounced for the test samples than for the conventional hexagonal core. Yield strength values also appear to be of the correct order of magnitude when compared to published ultimate strength values. Yield strength would normally be about
Table 1. Shear Properties of the Core Material

<table>
<thead>
<tr>
<th>Density (lb./ft.)</th>
<th>Loading Plane</th>
<th>Shear Modulus (lb./in.$^2$)</th>
<th>Shear Yield Strength (0.2 per cent offset) (lb./in.$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.9</td>
<td>TL</td>
<td>$4.23 \times 10^4$</td>
<td>36.8</td>
</tr>
<tr>
<td></td>
<td>TW</td>
<td>$3.18 \times 10^4$</td>
<td>42.8</td>
</tr>
</tbody>
</table>

Table 2. Shear Properties of Hexagonal Honeycomb Material

<table>
<thead>
<tr>
<th>Material</th>
<th>Thickness (in.)</th>
<th>Density (lb./ft.$^3$)</th>
<th>Shear Modulus (lb./in.$^2$)</th>
<th>Ultimate Strength (lb./in.$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>TL</td>
<td>TW</td>
</tr>
<tr>
<td>Al 5052</td>
<td>0.001</td>
<td>4.5</td>
<td>70,000</td>
<td>340</td>
</tr>
<tr>
<td>Al 5052</td>
<td>0.002</td>
<td>4.3</td>
<td>66,000</td>
<td>320</td>
</tr>
<tr>
<td>Al 5052</td>
<td>0.003</td>
<td>4.2</td>
<td>65,000</td>
<td>310</td>
</tr>
<tr>
<td>Al</td>
<td>0.003</td>
<td>3.96</td>
<td>42,300</td>
<td>238</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>TL</td>
<td>TW</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 21. Comparison Between Test Core Cell Arrangement and Conventional Hexagonal Honeycomb.
50 to 70 per cent of the ultimate strength. Experimental values are somewhat lower at about 20 per cent of published ultimate strengths.

The sample in the TW plane was unloaded after being loaded into the plastic range. As the sample is loaded again the slope is the same as for the initial elastic line. The slope of the reloading portion of the curve equals the shear modulus and acts as a check on the tangent drawn through the origin.

The cans along the knife edges in the TW sample showed signs of buckling in the TW plane. Figure 22 shows the deformation of the cans. Since the cans on the edges are not as rigid as the interior cans, it is expected that failure will occur here first and not uniformly in all cans. The buckling produced indentations inclined at approximately 45 degrees to the loading plates as would be expected for shear loading. The buckling is typical of the instability found in the webs of beams due to shearing forces. The Wagner beam, a thin web with stiffeners between two substantial flanges, is exemplary of this type of failure [14].

Loading was not pure shear, but also included a compressive component. The sample was made as long as possible in the L' direction to minimize compressive effects. The angle between the applied load and the loading plates was approximately 10 degrees and the effect of the compression component was considered negligible. Only the shear component was used in computing the stress values.

Edge effects are an important factor in a core with such a large core size. As was stated previously, the core samples were made as large as possible within capabilities of the testing machine.
Figure 22. Buckling of Core in Shear.

Figure 23. Stabilized Compressive Specimen.
available to minimize edge effects. The data obtained gives a good
ingestation of the shear properties of the core and is sufficient to
evaluate the feasibility of using the core material in structural
applications.

The epoxy adhesive bond was analyzed to determine what
percentage of the total displacement could be attributed to the
epoxy instead of the core itself. The elastic modulus of the epoxy
is about 650,000 psi. and the shear modulus was estimated at 50
per cent of the elastic modulus or 325,000 psi. Total bonded area is
on the order of 1.5 square inches. Nominal epoxy thickness is 0.014
inches and there are two bonded surfaces. Since the bond is cured
under a sizeable pressure a conservative value of total adhesive
thickness is assumed to be 0.01 inches for both bonded surfaces.
Solving Hooke's Law for shear to obtain the total displacement in the
adhesive yields a maximum value of 0.0002 inches or 5 per cent
of the total core deflection at a load of 10,000 pounds. Use of
adhesive bonding in preparation of shear specimens is consistent
with accepted practice. Any error attributable to the adhesive would
be present to some degree in all available test data.
CHAPTER IV

COMPRESSIVE EXPERIMENTAL INVESTIGATION

Purpose of Experimentation

In addition to the shear properties, the compressive mechanical properties of a core material are important in determining the value of the core in structural design. Together the shear and compressive properties give all the information normally required in order to evaluate the core. Compression is one of the most common types of loading to which a core is subjected. Since honeycomb cores are relatively weak under compressive loading, the compressive properties often represent the limiting conditions of the core. Tests were conducted upon the core material in order to obtain the principal compressive properties of the core.

Compressive Properties

Compressive properties of the core consist of compressive strength, compressive modulus, and crush strength. All three properties were obtained from load-deformation data taken during tests on the core.

Compressive strength is the ultimate compressive strength of the core when subjected to a load in the T direction. It is common practice to stabilize the core by one of two methods: bonding face plates to the core with an adhesive or casting the edges of the core
in a plastic dip resin. The stabilization prevents local crushing at the core edges. Alternately, the tests may be made using an unstabilized or bare core. Due to simplicity of the latter method, it is often used in acceptance testing. Values of ultimate strength obtained by both methods are found in the literature and are about the same [15].

The compressive modulus is defined as the slope of the initial straight line portion of the stress-strain diagram. Since the core material is nearly always used in the form of a sandwich panel, the stabilized compressive modulus is more meaningful than that obtained using a bare core section. In the remainder of this study compressive modulus will refer to the stabilized compressive modulus.

If a honeycomb core is loaded beyond its ultimate strength, plastic deformation occurs. Honeycomb cores exhibit the property of continuing to deform with the application of a relatively constant or uniform load as shown in Figure 24. The load per unit cross sectional area required for uniform crushing is defined as the crush strength. Crush strength is always determined using a bare core.

**Compressive Test Specimens**

Specimens were prepared for the study of compressive properties of the core. Both stabilized and bare core specimens were used. The core specimens were made as large as possible as determined by the testing machine dimensions to minimize edge effects. Figure 25 shows the specimen size and orientation. Thickness of the specimen was determined by the can thickness. The stabilized specimens were
Figure 24. Typical Load-Deflection Diagram.

Figure 25. Compression Specimen Dimensions.
prepared by adhesively bonding face plates to the core. Face plates were of cold-rolled steel one-eighth of an inch thick. The adhesive used was Scotch-Weld 2155B/A which is a two part general purpose structural adhesive that cures at room temperature and contact pressure. The face plates and core were cleaned carefully prior to bonding with abrasive cloth and acetone. Figure 23 shows a stabilized compressive specimen.

Experimental Equipment

The compressive specimens were loaded by means of a Titus Olson Plastiversal Testing Machine which has a capacity of 50,000 pounds in compression. Displacement was measured using an Instron Strain Gage Extensometer Model G-51-12. Displacement was recorded from the extensometer on a Sanborn Recorder Model 350-1100B. Corresponding loads were displayed on a dial on the testing machine and were marked on the strip displacement recording at regular load intervals. A self-aligning spherical loading block was used to insure that the load was distributed uniformly across the cross sectional area of the specimens.

Experimental Procedure

A compressive load was applied to the core specimens by the testing machine as shown in Figure 26. To obtain load-deformation data the load was applied through the spherical loading block. The load was applied continuously at a constant rate of movement of the upper loading head. The loading rate used was 0.026 inches per
Figure 26. Compression Test Apparatus
minute. The deflection data was measured by an extensometer and recorded by means of a preamplifier and strip recorder. The extensometer was attached to the central portion of the length of the core as shown in Figure 23. Load readings were made from the dial on the testing machine and were marked on the strip recording of the deflection at intervals of 500 pounds. The procedure gives approximately twenty data points between zero and the ultimate load. For crush strength determination the spherical loading block was removed and the load was applied to bare core specimens by the upper loading plate alone. Crush load readings were made from the dial on the testing machine.

**Discussion of Results**

Stress-strain diagrams for compressive loading of the stabilized core samples are presented as Figures 27 and 28. Compressive properties are summarized in Table 3. The ultimate strength and crush strength values obtained are in excellent agreement with each other. The stress-strain diagrams for the stabilized cores are found to be very similar in shape and value as would be expected. Slight irregularities are found in the curves just prior to the ultimate strength. It is noted that both diagrams exhibit the irregular points. The core reached the plastic range prior to reaching the ultimate strength and some buckling was evident in this range. The scattered portion of the diagrams is believed caused by the buckling of the core specimens. The stabilized compressive modulus shows wider variation than the other properties. Some
Figure 27. Stress-Strain Diagram for Compression Load, Stabilized Run 1.
Figure 28. Stress-Strain Diagram for Compression Load, Stabilized Run 2.
Table 3. Compressive Properties of the Core Material

<table>
<thead>
<tr>
<th>Run</th>
<th>Type</th>
<th>Ultimate Strength</th>
<th>Crush Strength</th>
<th>Compressive Modulus</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>lb./in.$^2$</td>
<td>lb./in.$^2$</td>
<td>lb./in.$^2$</td>
</tr>
<tr>
<td>1</td>
<td>Stabilized</td>
<td>91.2</td>
<td>-</td>
<td>9,889</td>
</tr>
<tr>
<td>2</td>
<td>Stabilized</td>
<td>92.2</td>
<td>-</td>
<td>11,750</td>
</tr>
<tr>
<td>3</td>
<td>Bare</td>
<td>90.2</td>
<td>40.7 - 42.2</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>Bare</td>
<td>93.7</td>
<td>37.4 - 42.7</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>Bare</td>
<td>92.7</td>
<td>40.3 - 43.6</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>Bare</td>
<td>93.0</td>
<td>34.4 - 40.7</td>
<td>-</td>
</tr>
</tbody>
</table>
variation is to be expected due to the complex nature of the core material.

As was stated previously the core specimens were constructed as large as possible within the limits of the testing machine in order to minimize edge effects. The specimens were typical specimens containing seven complete cells. The A. S. T. M. Standards specify that for large-celled cores a minimum specimen size is one large enough to include only one complete cell [16]. Since the actual test specimens were substantially larger than the minimum required in the literature, it is assumed that edge effects in the compression testing are negligible.

The extensometer was attached to the central portion of the core specimens for measurement of strain. The displacement measured was over approximately one-third of the specimen length. The A. S. T. M. Standards specify the gage length be less than two-thirds of the length [17]. The use of the extensometer gives much more accurate results than measuring head movement to obtain strains. It has been shown that measurement of head movement yields compressive modulus values with wide variation and values as little as one-tenth of the correct value as well as greater proportional limit loads [18].

A rough comparison may be made between experimentally determined values and those in the literature for small celled, aluminum, hexagonal cores [19]. Values in the literature show that compressive strength is nearly the same for stabilized and bare specimens and that crush strength is approximately one-half of the value of the ultimate
strength. The same correlation is exhibited by the experimental values. Published values for cores of similar densities are substantially higher, on the order of 500 pounds per square inch for ultimate strength and 150,000 pounds per square inch for compressive modulus compared to an experimental ultimate strength of about 90 psi and compressive modulus of about 10,000 psi [20]. The smaller actual strength and modulus values of the specimens tested are probably due to differences in cell size, material, and cell orientation as discussed in Chapter III. The experimental values are found to be of the same order of magnitude as the values in the literature.
CHAPTER V

DEFLECTION THEORY OF A RECTANGULAR,
SIMPLY SUPPORTED SANDWICH PLATE

A deflection theory is developed in this chapter for the case of a rectangular sandwich plate simply supported on all four edges and uniformly loaded over a rectangular portion of the surface. This case is one of the most common encountered in design with sandwich panels. Data is presented for a typical design application using the fabricated core. The deflection data is compared to a solid plate of the same material to demonstrate the advantage of the core structure.

Theoretical Analysis

The theory is based upon work done by Dr. Milton E. Raville at the Forest Products Laboratory [21]. The analytical work of Dr. Raville was based upon several assumptions as follows:

(1) The sandwich core is composed of an orthotropic core bonded to isotropic surface plates.

(2) The surface plates carry all loading in the plane of the plate allowing core normal and shear stresses in the plane and shear stresses perpendicular to the plane to be neglected.

(3) Small deflection theory may be used for the surface plates.
Figure 29. Sandwich Plate Nomenclature.
By summing forces over a differential element of the core it is possible to derive the equations of equilibrium of the core. Using Hooke's Law and stress-strain relationships the equilibrium equations can be written in terms of displacements $u_c$, $v_c$, and $w_c$ of the core in the $x$, $y$, and $z$ directions respectively. Directions $x$ and $y$ lie in the plane parallel to the face plates and bisecting the core thickness. The core displacements are assumed to be of the following form [22]:

\[
\begin{align*}
    u_c &= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} f_1(z) \cos \frac{\pi mx}{a} \sin \frac{\pi ny}{b} \\
    v_c &= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} f_2(z) \sin \frac{\pi mx}{a} \cos \frac{\pi ny}{b} \\
    w_c &= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} f_3(z) \sin \frac{\pi mx}{a} \sin \frac{\pi ny}{b}
\end{align*}
\]  

which may be shown to satisfy the boundary conditions for a plate simply supported at the edges [23]. Solving for functions $f_1$, $f_2$, and $f_3$ by use of the equilibrium equations and substituting the results back into the displacement relationships yields the core displacements in terms of seven arbitrary constants, $A_{mn}$ through $L_{mn}$.
An additional restraint equation is required for the core displacement expressions to satisfy the equilibrium equations.

\[ G_{xz} \frac{M_{mn}}{F_{mn}} + G_{yz} \frac{N_{mn}}{K_{mn}} = \frac{\delta a}{\eta c} \frac{E_{a}}{c_{mn}} \]
The summation of forces and moments on a differential element of the surface plates gives the equilibrium equations for the plates. These equations are then expressed in terms of the core displacements by equating displacements at the core-surface plate interface. The assumption is made that the displacement in the z direction is constant through the plate thickness and that displacements in the face plates in the x and y directions vary linearly through the thickness of the plates. Substitution of expressions for core displacements into the equations of equilibrium for the face plates gives six simultaneous equations with seven unknown constants. These six equations can be solved with the aid of restraint equation (7).

The six simultaneous equations as derived by Dr. Raville are as follows:

\[
\frac{E_t}{1-v^2} \left[ \frac{\partial^2 u_c}{\partial x^2} + \left( \frac{1+v}{2} \right) \frac{\partial^2 u_c}{\partial y^2} + \frac{1}{2} \frac{\partial}{\partial x}(v^2 v_c) \right]_{z=-\frac{c}{2}} = -\tau_{xz} \quad (8)
\]

\[
\frac{E_t}{1-v^2} \left[ -\frac{\partial^2 v_c}{\partial y^2} + \left( \frac{1-v}{2} \right) \frac{\partial^2 v_c}{\partial x^2} + \frac{1}{2} \frac{\partial}{\partial y}(v^2 v_c) \right]_{z=-\frac{c}{2}} = -\tau_{yz} \quad (9)
\]

\[
\frac{E_t}{12(1-v^2)} \left[ \frac{\partial}{\partial z} \left( \nu^2 v_c \right) \right]_{z=-\frac{c}{2}} = q + (\sigma_z)_{z=-\frac{c}{2}} + \frac{1}{2} \left( \frac{\partial^2 x_{xz}}{\partial x^2} + \frac{\partial^2 y_{xz}}{\partial y^2} \right) \quad (10)
\]
The intensity of lateral loading, \( q \), is replaced by a double Fourier sine series expansion describing the loading. For the case of a uniform loading over a rectangular area located at any point on the plate \( q \) may be expressed as

\[
q = \frac{16P}{\pi^2uv} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \sin \frac{mn\pi}{2a} 
\]  

(14)
where $P$ is the total load on area $uv$ [24]. The loading is shown in Figure 30. Substituting for $q$ in equation (10), expressing core stresses in terms of core displacements, and simplifying yields

$$\frac{P c}{a} \left( \left( m + \frac{2}{2} \rho^2 \right) \left( -A_{mn} \left( \frac{1}{6} + \frac{t_{1}}{2c} \right) + B_{mn} \left( \frac{1}{4} + \frac{t_{1}}{2c} \right) - C_{mn} \left( \frac{1}{2} + \frac{t_{1}}{2c} \right) \right) + \right)$$

$$+ \left( m + \frac{2}{2} \rho^2 \left( \frac{F_{mn}}{2} - H_{mn} \right) \right) + \left( 1 + \nu \right) \frac{K_{mn}}{2} - \frac{L_{mn}}{2} \right) =$$

$$= - \frac{G_{xz} a^2 (1 - \nu^2)}{\pi^2 E t_1 c} F_{mn}$$

$$\frac{P c}{a} \left( \left( m + \frac{2}{2} \rho^2 \right) \left( -A_{mn} \left( \frac{1}{6} + \frac{t_{1}}{2c} \right) + B_{mn} \left( \frac{1}{4} + \frac{t_{1}}{2c} \right) - C_{mn} \left( \frac{1}{2} + \frac{t_{1}}{2c} \right) \right) + \right)$$

$$+ \left( m + \frac{2}{2} \rho^2 \left( \frac{F_{mn}}{2} - H_{mn} \right) \right) + \left( 1 + \nu \right) \frac{K_{mn}}{2} - \frac{L_{mn}}{2} \right) =$$

$$= - \frac{G_{xz} a^2 (1 - \nu^2)}{\pi^2 E t_1 c} K_{mn}$$
Figure 30. Loading Location.
\[
(m^2 + n^2 \rho^2)^2 (A_{mn} - B_{mn} + C_{mn}) - \frac{12E a^4 (1-\nu^2)}{\pi E t_{3c}} (-\nu A_{mn} + 2B_{mn}) + \quad (16)
\]

\[
+ \frac{6G a^3 (1-\nu^2)}{\pi E t_{3c}} m F_{mn} + \frac{6G a^3 (1-\nu^2)}{\pi E t_{3c}} n p K_{mn} =
\]

\[
= \frac{192F a^3 (1-\nu^2)}{\pi E t_{3c} u v m n} \sin \frac{m u}{a} \sin \frac{n v}{b} \sin \frac{m u}{2a} \sin \frac{n v}{2b}
\]

\[
\frac{m n c}{a} (m^2 + n^2 \rho^2) \left[ A_{mn} \left( \frac{1}{6} + \frac{t_2}{2e} \right) + B_{mn} \left( \frac{1}{4} + \frac{t_2}{2e} \right) + C_{mn} \left( \frac{1}{2} + \frac{t_2}{2e} \right) \right] -
\]

\[
- \left[ \frac{m^2}{2} + \frac{(1-\nu) n^2 \rho^2}{2} \right] \left( \frac{m u}{2} + H_{mn} \right) - \frac{(1+\nu) m n p}{2} \left( \frac{k_{mn}}{2} + L_{mn} \right) =
\]

\[
= \frac{G a^2 (1-\nu^2)}{\pi E t_{c} e} \frac{F_{mn}}{m n}
\]
The seven simultaneous equations can be solved for the unknown constants and the results for $A_{mn}$, $B_{mn}$, and $C_{mn}$ can be substituted into the expression for $w_c$, equation (6), to give deflection of
the sandwich panel in the z direction.

Method of Solution

A numerical solution for the constants in equations (14) through (19) and (7) was accomplished with the aid of the digital computer. The computer was required due to the size of the system of equations and the number of series terms to be considered. The method of solution employed was Gauss-Jordan reduction with the maximum pivot strategy [25]. To prevent possible error due to a diagonal element which is very small in relation to the other coefficients in the line, the magnitude of the coefficients on the diagonal should be as great as possible. The maximum pivot strategy modifies the Gauss-Jordan reduction method by allowing row and column interchanges to maximize the diagonal elements. The method produces less round off error. Additionally, double precision was used in the calculations to minimize error which is likely due to the large differences in the magnitude of the coefficients in the coefficient matrix. The computer program written in Fortran IV is presented in Appendix E with sample input and output.

Since the set of simultaneous equations to be solved are in terms of infinite series, the coefficients found must be summed together until the effect of additional series terms can be considered negligible. The results were calculated for values of m and n from one to ten. Values of w were printed out for each change in m or n and it was observed that changes became negligible very rapidly.
Program Check

Several checks were performed to assure that the computer program was functioning as expected. The Gauss-Jordan reduction subroutine was checked by using input for which a solution to the system of equations was known. Included was data with very small diagonal elements designed to test the ability of the subroutine to minimize error for this case.

If the variables $u$, $v$, $\xi$, and $\eta$ are set equal to $a$, $b$, $a/2$, and $b/2$ respectively, the problem becomes one of a uniformly loaded plate over the entire surface, the particular case studied by Dr. Raville [26]. In Reference [27] calculated results are presented for several actual sandwich plates using the solution of Dr. Raville. There are several possible sources of difference between the published values and those computed with the computer program. Material properties were not given for the core and facings in the referenced data and had to be assumed. The number of series terms considered in the two methods was different. Dr. Raville had made the additional simplifying assumptions that the flexural stiffness of the face plates is negligible and the modulus of elasticity of the core is infinite in the z direction. The computed results of the program should be more accurate due to fewer simplifying assumptions. Comparison of results for one of the sandwich panels studied in the literature gave values of 0.0247 inches for the computer program and 0.0270 inches for the reference data [28]. In view of the possible sources of difference, the computer program results are considered accurate.
A final check consisted of computing values for varying x and y values across the plate. The deflection profiles plotted from the results were of the shape expected showing zero deflection at the boundaries and maximum deflection at the center for the case of a load applied at the center.

**Theory Results**

In order to demonstrate the deflection characteristics of a sandwich plate using the fabricated core, several sets of data were computed. Core properties used were from experimental results. An arbitrary panel size of ten feet by twenty feet was chosen as was a face plate of cold-rolled steel, one-sixteenth of an inch thick. Data was obtained for two cases, a uniform loading over the entire surface of one pound per square inch and a point load at the center of 10,000 pounds. Deflection of the sandwich panel is shown in Table 4.

Several observations should be made about the chosen panel. The loads applied are quite large. A load of forty pounds per square foot or 0.278 pounds per square inch is normally considered a design floor load for dwellings. This load gives a maximum deflection for the panel of 2.59 inches. The unsupported dimensions of the panel are also quite large. The deflections calculated are larger than would actually be experienced in most applications since the edges would probably be at least partially fixed.

A comparison was made between the sandwich panel and a solid steel plate. For a solid rectangular plate uniformly loaded over the
Table 4. Deflection Data for a Rectangular Sandwich Panel
Simply supported on All Edges.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>w (Uniform Load)</th>
<th>w (Point Load)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>120.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>20.0</td>
<td>120.0</td>
<td>4.7401</td>
<td>4.6802</td>
</tr>
<tr>
<td>40.0</td>
<td>120.0</td>
<td>8.1138</td>
<td>8.6101</td>
</tr>
<tr>
<td>60.0</td>
<td>120.0</td>
<td>9.3242</td>
<td>10.563</td>
</tr>
<tr>
<td>60.0</td>
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<td>0.0</td>
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<td>60.0</td>
<td>40.0</td>
<td>5.3754</td>
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<tr>
<td>60.0</td>
<td>80.0</td>
<td>8.4090</td>
<td>7.0290</td>
</tr>
</tbody>
</table>
entire surface and simply supported on all edges maximum deflection is expressed by

\[ w_{\text{max}} = \frac{q a^4}{E t^3} \]

where

- \( q \) is the uniform load, psi.
- \( a \) is plate width, in.
- \( E \) is modulus of elasticity of the material, psi.
- \( t \) is plate thickness, in.
- \( \alpha \) is a factor based on \( b/a \)

From Roark \( \alpha \) is found to be 0.1110 for the plate [29]. Setting \( w_{\text{max}} \) equal to the maximum deflection of the sandwich panel it is found that a solid plate of cold-rolled steel with a thickness, \( t \), equal to 0.14 inches would be required for the same deflection. The weight of the sandwich panel is 1380 pounds of which the core weighed only 360 pounds. Correspondingly the weight of the solid steel plate required for the same maximum deflection was 3690 pounds for a ratio of sandwich panel weight to steel plate weight of 0.37. Normally stiffeners would be used with the steel plate to reduce weight, but they would be required very close together to give support as the steel plate becomes thinner. This problem does not present itself in the sandwich core where the core stiffens even thin face plates to prevent local deformation and failure.
In the case of a concentrated load at the center of a rectangular plate simply supported at all edges the maximum deflection becomes

\[ w_{\text{max}} = \frac{\alpha P a^2}{D} \]

where

- \( P \) is the concentrated load, lb.
- \( D \) is flexural rigidity defined as \( D = \frac{E t^3}{12(1-v^2)} \)

where \( v \) is Poisson's ratio for the material.

Timoshenko gives \( \alpha \) for this case as 0.01620 and the thickness of a steel plate required to equal the maximum deflection of the sandwich panel is 0.43 inches [30]. The weight of the steel plate required is 3500 pounds giving a ratio of the weights equal to 0.394.

It is evident that the core structure possesses the excellent strength to weight ratio which is expected of sandwich cores. The sandwich panel of considerable size and under substantial load conditions showed excellent stiffness in bending, especially if the thinness of the face plates is considered. It achieved the same results as a solid steel plate weighing approximately three times as much.

The possibility of using the deflection theory as a check of the experimentally determined shear modulus values was examined. If
computed values of deflection were obtained using the experimental properties and were compared to actual measured deflections for a panel of the same dimensions and under the same loading, the computed and measured deflections should be equal. In practice the corresponding deflection changes for a sizeable change in shear modulus were very small. Possible differences in deflection between computed and measured values due to simplifying assumptions in the theory make such small differences meaningless.
CHAPTER VI

CONCLUSIONS

In the course of this study a honeycomb core material was fabricated from tinned-steel cans and mechanical properties of the core were determined. The purpose was to evaluate the possible use of such a core in structural applications. The values of the mechanical properties determined support, from the standpoint of strength, the conclusion that the fabricated core in sandwich form is an excellent structural material. The mechanical properties determined were well in line with published values of honeycomb cores which are already in use in a variety of structural applications. A deflection analysis showed that a sandwich structure utilizing the fabricated core was capable of high loads with relatively small deflections. Compared to other materials the sandwich panel proved to have an excellent strength to weight ratio. The fabricated core possesses the same qualities which have made honeycomb cores one of the most promising material forms. Its insulating, sound absorbing, and energy absorbing properties further enhance its desirability. Already, sandwich structures conserve valuable natural resources by requiring less material. If in addition the core material is made of reclaimed material the conservation will be even greater.

The greatest drawback to the use of welded steel cans for the core material lies in the fabrication process. After reclamation it
becomes necessary to sort and size the cans. Then the ends must be removed. The problem of sorting and end removal has been solved in at least one reported instance, the geodesic dome for the National Accelerator Laboratory [31]. Since the bonding process between the cans is resistance welding, electrical contact must be made between the electrodes through both cans. In order for this to be accomplished the paint and other coatings both inside and out must be removed prior to welding. A means must be developed to speed up the welding process. Finally a protective coating must be reapplied to prevent oxidation. It is believed that these fabrication problems are not insurmountable. The key to overall feasibility lies in fabrication techniques. In order for the material to be economically feasible a low cost method of fabrication must be developed to accompany the low cost of the reclaimed cans.
CHAPTER VII

RECOMMENDATIONS

In order to completely evaluate the core material further study is recommended in several areas. Study in these areas will give a more complete picture of the strengths and limitations of the fabricated core. It would be desirable to study the edge effects of the core material. Due to the large cell size edge effects should be considered where small panels are used. Shear tests on very large specimens where edge effects could be neglected would serve to give a more exact value for shear strength and modulus. An investigation of other loading cases would be of value. One case in particular, edgewise compression, is quite common for honeycomb core material in structural applications. In building construction for example, this is the principal load on wall sections. Various other properties such as acoustic properties, thermal properties, and energy absorption also warrant further study in order to completely analyze the material.

Other means of fabrication might also be the subject for further study. Excellent strength properties were shown with a welded bond between cans, but other methods could possibly give similar results. One method which rates consideration would be an adhesive bond between cans. Combinations such as welded spots at each end of the seam with adhesive bonding along the seam are also possible. There are many possibilities for fabrication other than the one chosen for study in this thesis.
APPENDIX A

INSTRUMENTATION AND EQUIPMENT FOR THE SHEAR TESTS

Loading of the specimens was performed by an Instron Universal Testing Instrument - Floor Model TT. The machine is a mechanical loading system which is capable of compressive loading at a constant rate of displacement of the loading member. The testing machine is equipped with a built in X-Y strip recorder enabling load-displacement curves to be recorded.

The load cell utilized was an Instron Type F Load Cell. It is capable of measuring loads up to 10,000 pounds in tension or compression.

Gore strain was measured by means of an Instron Strain Gage Extensometer, Model G-51-12. The gage length is one inch and measurements can be made up to 50 per cent strain or one-half inch. The extensometer has a maximum non-linearity for the calibrated range in use of 1/4 per cent and a maximum hysteresis of 0.3 per cent.
APPENDIX B

SAMPLE SHEAR CALCULATIONS

Since the core was loaded along the diagonal, only a portion of the load acts in shear as shown in Figure 31. The shear component alone is used in calculating the shear stresses.

The angle between the loading plane and surface plates, \( \varphi \), is

\[
\varphi = \tan^{-1} \frac{c}{L'}
\]

where

- \( c \) is the core thickness, inches
- \( L' \) is the length of the core, inches.

The shear component of load becomes

\[
F_x = F \cos \varphi
\]

where

- \( F \) is the total load, pounds

and the shear stress is found to be

\[
\gamma = \frac{F \cos \varphi}{L'b}
\]
Figure 31. Load Components.
where

\[ b \text{ is the specimen width, inches.} \]

Shear strain of the core is computed by

\[ \gamma = \frac{r}{c} \]

where

\[ r \text{ is the measured displacement, inches.} \]

Substituting the core dimensions as calculated from the can dimensions to minimize the effect due to distortion at the edges, \( \tau \) and \( \gamma \) can be expressed in terms of measured values \( F \) and \( r \).
Table 5. Shear Stress and Strain Relationship.

<table>
<thead>
<tr>
<th>Plane</th>
<th>L' (in.)</th>
<th>c (in.)</th>
<th>b (in.)</th>
<th>$\tau$ (lb./in.$^2$)</th>
<th>$\gamma$ (in./in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TW</td>
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<td>8.5</td>
<td>$4.67 \times 10^{-3}$</td>
<td>0.23</td>
</tr>
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<td>9.4</td>
<td>$4.2 \times 10^{-2}$</td>
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<td>HP</td>
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<td>10</td>
<td>7.0</td>
<td>10</td>
<td></td>
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<tr>
<td>----</td>
<td>----</td>
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<td>-----</td>
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<tr>
<td>9.8</td>
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<td>0.82</td>
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</tr>
<tr>
<td>9.6</td>
<td>0.68</td>
<td>6.9</td>
<td>0.65</td>
<td>0.82</td>
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<td>6.5</td>
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<td>0.65</td>
<td>0.82</td>
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<td>6.7</td>
<td>0.65</td>
<td>0.82</td>
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Table 6: Shear Test Data, TL plane.
Table 7. Shear Test Data, TW Plane.

<table>
<thead>
<tr>
<th>$r$ (in.)</th>
<th>$\gamma$ (in./in. x $10^4$)</th>
<th>$F$ (lb.)</th>
<th>$t$ (lb./in.$^2$)</th>
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<td>10060</td>
<td>45.9</td>
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Unloading:

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<tr>
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<th>$F$ (lb.)</th>
<th>$t$ (lb./in.$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>8000</td>
<td>36.5</td>
</tr>
<tr>
<td>0.015</td>
<td>34.5</td>
<td>6420</td>
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<tr>
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Reloading:

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<th>$F$ (lb.)</th>
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<td>4440</td>
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<tr>
<td>0.0102</td>
<td>23.5</td>
<td>4980</td>
<td>22.8</td>
</tr>
</tbody>
</table>
The compression specimens were loaded with a Titus Olson Plasti-
versal Testing Machine. The testing machine is capable of 50,000
pounds of load in compression and a constant rate of movement of the
movable loading head. Load is displayed on a dial marked in 100 pound
increments. Loading speed was measured using a dial indicator
marked in increments of 0.001 of an inch to measure head displacement
over a period of time. A spherical loading block served to uniformly
distribute the load over the specimen cross sectional area. Strain
measurements were made with the Instron Strain Gage Extensometer,
Model G-51-12 as described in Appendix A. The extensometer was
connected to the preamplifier section of a Sanborn Recorder, Model
35-1100B. Displacements were recorded continuously on the Strip
recorder.
APPENDIX D

SAMPLE COMPRESSION CALCULATIONS

Since load and displacement measurements were taken, it was necessary to convert them to stress and strain. Compressive stress is computed by

\[ \sigma_c = \frac{F}{L'b} \]

where

- \( F \) is the compressive load, pounds
- \( L' \) is core length, inches
- \( b \) is core width, inches.

The strain is found by

\[ \varepsilon = \frac{r}{c'} \]

where

- \( r \) is measured displacement, inches
- \( c' \) is length over which strain is measured, inches.

The following values were used in the experiment:

- \( L' = 9.45 \) inches
- \( b = 10.92 \) inches
- \( c' = 1.5 \) inches
Table 8. Stabilized Compression Test Data, Run 1.

<table>
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<th>( \varepsilon )</th>
<th>F</th>
<th>( \sigma_c )</th>
</tr>
</thead>
<tbody>
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<td>in. ( \times 10^3 )</td>
<td>in./in. ( \times 10^3 )</td>
<td>lb.</td>
<td>lb./in. ( ^2 )</td>
</tr>
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<td>0.0</td>
<td>0</td>
<td>0.0</td>
</tr>
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COMPUTER PROGRAM

MAIN PROGRAM

-RUN FORTN.52CI2015.80MM-TERRY.1,50
-FOR, IYS DEFL

C DEFLECTION IN A SIMPLY SUPPORTED, RECTANGULAR SANDWICH PLATE
C UNDER UNIFORM LOADING OVER A RECTANGULAR PORTION OF THE SURFACE

IMPLICIT DOUBLE PRECISION (A-H,0-Z)
COMMON Y(7,11),X(7,8)
REAL NU
SIN(X)=DSIN(X)
READ (5,100) XX,YY,Z,ML,NL
100 FORMAT (3F10.0)
READ (5,101) A,T1,T2
101 FORMAT (5F10.0)
READ (5,102) E,PR,EC,GXZ,IFY
102 FORMAT (D10.0, F10.0, 3D10.0)
READ (5,101) P,U,V,F,P,NU
WRITE (6,500) XX,YY,Z,ML,NL
500 FORMAT (1H1,13X,18H SYSTEM PARAMETERS ///
1 20H LOCATION OF DEFLECTION X=F7.3,3H IN / 26X,3H Y=F7.3,
2 3H IN / 26X,3H Z=F7.3,3H IN / 24H SERIES TERMS CONSIDERED *2X,
3 3H M=13 / 26X,3H N=13 /)
WRITE (6,501) A,T1,T2
501 FORMAT (17HPLATE DIMENSIONS,9X,3H A=F7.3,3H IN / 26X,3H B=,
1 F7.3,3H IN / 26X,3H C=F7.3,3H IN / 25X,4H T1=F7.3,3H IN /
2 25X,4H T2=F7.3,3H IN / 25X,4H T2=,F7.3,3H IN /)
WRITE (6,502) E,PR,EC,GXZ,IFY
502 FORMAT (17HPLATE PROPERTIES,9X,3H E=2PE11.2,4H PSI / 25X,
1 4H PR=0PF7.3 /25X,4H EC=2PE11.2,4H PSI /24X,5H GXZ=2PE11.2,
2 4H PS1=,F7.3,3H IN / 25X,5H GYZ=2PE11.2,4H PSI /)
WRITE (6,503) P,U,V,F,P,NU
503 FORMAT (5HLOAD,21X,3H P=,F7.3,3H LB / 26X,3H U=,F7.3,3H IN / 26X,
1 3H W=F7.3,3H IN / 16HLOAD LOCATION N11X,4H EP=F7.3,3H IN /
2 25X,4H MU=F7.3,3H IN /)
PRINT 1
1 FORMAT (1H0,18X,22H CALCULATED DEFLECTION / 3H0 M,3X,2H N,6X,
1 7H W (IN),5X,7H A(M,N),6X,7H B(M,N),6X,7H C(M,N) /)
RHO=A/78
PI=3.1415927
Q=1.0/6.0
Q1=(8.0*EC*A)/(PI*C)
T11=1/(2.0*PI)
T22=T2/(2.0*PI)
PR1=(1.0-PR)/2.0
PR2=1.0-(PR**2)
PR3=(1.0-PR)/2.0
PP=(192.0*(A**4)*PR2*Pl. )/( (PI**6)*E*(T1**3)*U*VV)
E1=((EC*(A**4)*PR2)/((P)**4)*E*C)
E11=E1/(T1**3)
E22=E1/(T2**3)
GX=(Gxz*(A**2)*PR2)/((P1**2)*E*C)
GX1=GX/T1
GX2=GX/T2
GX3=(GX1*A1)/(P1*T1)
GX4=(GX2*A1)/(P1*T2)
GY=(Gyz*(A**4)*PR2)/((P1**4)*E*C)
GY1=GY/T1
GY2=GY/T2
GY3=(GY1*A1)/(P1*T1)
GY4=(GY2*A1)/(P1*T2)
T10=(PI*EP)/A
T20=(PI*NU)/B
T3=(PI*U)/(2.0*A)
T4=(PI*VV)/12.0*B
T5=(PI*XX)/A
T6=(PI*YY)/B
W=G=U
Z1=2.0
Z2=2.0
DO 10 M=1,ML
DO 10 N=1,NL
CM=M
CN=N
C1=CM**2
C2=(CN**2)*RHO**2
C3=CM*CNRHO
C4=C1+C2
C5=(CM*PI*C)/A
C6=CN*RHO*PI*C/A
C7=C4*C6
C8=C4*C6
C9=PR3*C3
C10=C4**2
X1(1,1)=-C7*(O+T1)
X1(1,2)=C7*(O+T2)
X1(1,3)=-C7*(O+T1)
X1(1,4)=0.5*(C1+PR1*C2)+GX1
X1(1,5)=-(C1+PR1*C2)
X1(1,6)=C9/2.0
X1(1,7)=-C9
X1(2,1)=-C8*(O+T1)
X1(2,2)=C8*(O+T1)
X1(2,3)=-C8*(O+T1)
X1(2,4)=X1(1,6)
X1(2,5)=-C9
X1(2,6)=0.5*(PR1*C1)+C2+GY1
X1(2,7)=-(PR1*C1)-C2
X1(3,1)=C10+(48.0*E11)
X1(3,2)=-C10-(24.0*E11)
X1(3,3)=C10
X1(3,4)=6.0*GX3*CM
X1(3,5)=0.0
X1(3,6)=6.0*GY3*CNRHO
X1(3,7)=0.0
X1(4,1)=-(Q)
X1(4,2)=0.0
GAUSS-JORDAN REDUCTION WITH MAXIMUM PIVOT STRATEGY FOR SOLUTION OF SIMULTANEOUS EQUATIONS

FOR YES GAUSS2

SUBROUTINE GAUSS2
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
COMMON Y(7,1),X(7,8)
DIMENSION IRCW(7),JCOL(7)
EPS=1.0E-2
DO 18 K=1,7
KMI=K-1
PVOY =0.0
DO 11 I=1,7
DO 11 J=1,7
IF(K.EQ.1) GO TO 9
DO 8 ISCAN=1,KM1
DO 8 JSCAN=1,KM1
IF(I.EQ.IROW(ISCAN)) GO TO 11
8 IF(J.EQ.JCOL(JSCAN)) GO TO 11
9 IF(DABS(X(I,J)) .LE. DABS(PIVOT)) GO TO 11
PIVOT=X(I,J)
IROW(K)=I
JCOL(K)=J
11 CONTINUE
IF(DABS(PIVOT) .GT. EPS) GO TO 13
PRINT 2
2 FORMAT(12HSMALL PIVOT //)
RETURN
13 IROWK=IROW(K)
JCOLK=JCOL(K)
DO 14 J=1,8
14 X(IROWK,J)=X(IROWK,J)/PIVOT
X(IROWK,JCOLK)=1.0/PIVOT
DO 18 I=1,7
AIJCK=X(I,JCOLK)
IF(J.EQ.IROWK) GO TO 18
X(I,JCOLK)=-AIJCK/PIVOT
DO 17 J=1,8
17 IF(J.NE.JCOLK) X(I,J)=X(I,J)-AIJCK*X(IROWK,J)
18 CONTINUE
DO 20 I=1,7
IROWI=IROW(I)
JCOLI=JCOL(I)
20 Y(JCOLI,1)=X(IROWI,8)
RETURN
END

SAMPLE INPUT

60.0 120.0 0.0 10 10
120.0 240.0 0.41 0.0625 0.0625
30.0E6 0.27 30.0E6 4.23E4 3.18E4
28800.0 120.0 240.0 60.0 120.0

DATA 41
DATA 42
DATA 43
DATA 44
### SYSTEM PARAMETERS

**LOCATION OF DEFLECTION**
- \( x = 60.000 \, \text{IN} \)
- \( y = 120.000 \, \text{IN} \)
- \( z = 7000 \, \text{IN} \)

**SERIES TERMS CONSIDERED**
- \( M = 10 \)
- \( N = 10 \)

**PLATE DIMENSIONS**
- \( A = 120.000 \, \text{IN} \)
- \( B = 240.000 \, \text{IN} \)
- \( C = 1.410 \, \text{IN} \)
- \( T_1 = 0.063 \, \text{IN} \)
- \( T_2 = 0.063 \, \text{IN} \)

**PLATE PROPERTIES**
- \( E = 30.00 \times 10^6 \, \text{PSI} \)
- \( PR = 0.270 \)
- \( EC = 30.00 \times 10^6 \, \text{PSI} \)
- \( G_{12} = 42.30 \times 10^3 \, \text{PSI} \)
- \( G_{23} = 31.80 \times 10^3 \, \text{PSI} \)

**LOAD**
- \( P = 8000.00 \, \text{LB} \)
- \( U = 120.000 \, \text{IN} \)
- \( V = 210.000 \, \text{IN} \)

**LOAD LOCATION**
- \( \epsilon P = 60.000 \, \text{IN} \)
- \( \nu U = 120.000 \, \text{IN} \)

### CALCULATED DEFLECTION

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<th>( B(M,N) )</th>
<th>( C(M,N) )</th>
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BIBLIOGRAPHY

Literature Cited


4. Ibid.


17. Ibid.


20. Ibid.


22. Ibid., p. 6.


26. Raville, "Deflection and Stresses."

27. Lewis, "Deflection and Stresses - Experimental Verification of Theory."

28. Ibid.


Other References

