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A SCIENTIFIC ANALYSIS
OF BANDSAW BLADE TOOTH CONFIGURATIONS
FOR MEAT AND BONE CUTTING

A THESIS

Presented to
the Faculty of the Graduate Division
by
Robinson P. Ramirez

In Partial Fulfillment
of the Requirements for the Degree
Doctor of Philosophy in the School
of Mechanical Engineering

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May 1963
A SCIENTIFIC ANALYSIS
OF BANDSAW BLADE TOOTH CONFIGURATIONS
FOR MEAT AND BONE CUTTING

Approved:

P. Vidosic

Thomas W. Jackson

R. B. Belser

Date Approved by Chairman: May 24, 1969
DEDICATION

This work is dedicated to my wife, Nellie, who has endured much inconvenience during its progress. Her selfless devotion to me and our four children has been of primary significance in the accomplishment of the work.
ACKNOWLEDGMENT

This study was made possible through the foresight and perseverance of Mr. E. A. Anderson, president of Southern Saw Service, Inc., Atlanta, Georgia. He was the sole financial contributor to the entire program which was initiated almost six years ago. I am forever indebted to him for this and for his undivided interest during the development of the various phases implicated.

My special thanks go to Mr. Aubrey Sievers, Engineering Assistant at Southern Saw Service, who in a dedicated manner gave of his time (outside of his normal working hours) to this investigation in order that several important deadlines would be achieved.

I am grateful to Dr. J. P. Vidosic (the thesis advisor) for his patience and support, and to Drs. R. B. Belser and T. W. Jackson (members of the thesis reading committee) for their comments.

It is fitting to give a share of credit to my wife, who was instrumental in various facets of the investigation where help was needed and where no high degree of technical competence was required. I can never be thankful enough for her part in this study.

I wish to express my sincere thanks to Mrs. Sherry Redmon (typist), and Mrs. Martha Shoemaker (draftsman), who toiled many hours with me in preparing the final draft of this analysis. Because of existing time limitations, much of this material had to be done outside of usual working hours. Also I appreciate Mr. Frank S. Longshore (editor of the Technical Information Section, Experiment Station.), and Mr. James Ed Garrett,
Photo Lab Head, who exerted their influence to expedite this work.

In addition, I acknowledge my dependence upon God (through Jesus Christ my Lord) who provided the inspiration, guidance, and health, which led to the culmination of this analysis.
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Bandsaws, as separate and distinct tools in the saw family, did not come into existence until the early eighteenth century. From that time until now they have undergone a variety of modifications in order to meet the demands of advancing technology. The primary changes made were related to tooth size in order to accommodate the specific medium to be cut. Unfortunately, the changes wrought did not come about as a result of scientific analyses, with perhaps a few exceptions. The usual approach would be to take a handful of bands with different tooth configurations (size included) and to try them in cutting a particular medium. The final choice of band with suitable tooth shape was made on the bases of the ease with which it cut and of the quality of the surface which underwent cutting. This procedure has been acceptable, in general, and is probably still acceptable for a variety of situations. However, the method is not necessarily adequate for the cutting needs of certain materials.

The work presented here deals with the bandsaw cutting of soft meats, frozen meats, and bone. This is done scientifically by first making a visual interpretation and then a mathematical representation of the cutting mechanisms involved during bandsaw cutting of each of the three solids in question. Once this is done it is followed by the development of mathematical relationships to measure four important factors which are: (1) meat and bone smear formation (waste materials which become deposited on the freshly cut surfaces), (2)肉和骨的废料生成，(3) meat and bone waste...
development (typical of sawdust resulting from a wood cutting process), (3) power consumption, and (4) useful tooth sharpness life. The last phase of this investigation is the experimental part. The test results are used in conjunction with the mathematical relationships in order to obtain a relative measure of the four factors just mentioned.

The reasons for examining the last three factors are reasonably self-explanatory. The smear problem is not particularly self-evident, however. Smear on the surface of meat cuts is objectionable because it detracts from its best possible appearance. For saleability, then, bandsaw meat cuts usually require a surface scraping before being placed in public meat counters.

A thorough search showed that there is no literature on bandsawing meat and bone. However, a significant amount of work on cutting with sharp-edged tools has been published since 1851. The experimental phases reported upon in these papers deal with cutting metallic materials only. The great difference in properties and in sub-microscopic construction between metals and the three media involved in the current work, does not wholly invalidate existing literature with regard to subject analysis. Metallic chip formation as viewed by Merchant\(^2,3\) is considered an appropriate basis for the study involving bone and/or frozen meat orthogonal sawing. In this work the force reaction at the tool-workpiece interface is used beneficially whereas Merchant's work does not involve its use. Merchant also concerns himself with a continuous chip but the chips resulting from bone or frozen meat sawing are discontinuous and segmental. The difficulty here is overcome by allowing 15 to 25 bandsaw teeth to be in action coinstantaneously. Moreover, the bandsawing of bone on frozen meat
entails shearing the two kerf (side) surfaces of the chip segments in addition to the shear plane surface. Despite the basic differences mentioned, the resulting shear angle equation is the same as that for metallic orthogonal cutting with single point tools. The equation is $2\phi = 90 + \alpha_1 - \gamma$. Experimental verification was possible for bone cutting at velocity ratios (feed velocity over band velocity in fpm) between $\frac{1}{600}$ and $\frac{8}{9600}$. The rapidly thawing condition of the frozen meat made physical proof impossible at this time. This work included the introduction and use of the dynamic shear strengths of the three media studied.

The mechanism of bandsawing soft meat was isolated partly by making a geometrical study of teeth in action against fairly flexible materials. Dozens of experimental cuts were also taken at velocity ratios from $\frac{100}{9600}$ to $\frac{100}{1200}$ and at a band speed of 15 inches per minute. The meat surfaces were examined microscopically during all these tests. From the combined study evolved only one clear-cut concept, i.e., that the meat was being severed largely by means of simple shear at the band teeth leading edges (primary cutting edges) and along the kerf (side) surfaces (secondary cutting edges). It was learned that tensile behavior begins to take precedence over shearing action at velocity ratios outside the range previously mentioned.

From a qualitative point of view certain results were found to hold satisfactorily for the three cutting media examined in the current paper. They are:

(1) Any increase in rake angle from $-15^\circ$ to $+5^\circ$ causes band and feed forces to drop, thereby lessening specific energy requirements

(2) The difference in input requirements between case 1 bands (no
side rake angle), and cases 2 and 3 bands (positive or negative side rake angle) is significant only at low and medium velocity ratios (approximately two-thirds of ranges tested). Ranges of $\frac{v_f}{v_B}$ examined are:

(a) soft meat $- \frac{100}{9600}$ to $\frac{100}{1200}$ (fpm)
(b) bone $- \frac{1}{9600}$ to $\frac{1}{1200}$
(c) frozen meat $- \frac{10}{9600}$ to $\frac{10}{800}$

(3) Changes in velocity ratio do not alter the specific energy requirements significantly.

(4) Approximately eight-tenths of the total energy expended is absorbed by the various shearing actions. One-tenth is dissipated in friction and the remainder is expended in momentum change, deformation, storage (residual form), losses, etc.

(5) Energy requirements for frozen meat cutting may be two to five times as much as that for soft meat. Bone cutting takes approximately twenty times as much energy to cut as frozen meats.

(6) Bands with side rake angles (either positive or negative) are markedly sensitive (forcewise) to velocity ratio changes.

(7) Bands with side rake angles use up to one-seventh as much energy as bands without side rake angles at high band speeds (> 7000 fpm). The ratio drops to one-half for lower band speeds (2000 fpm).

(8) The specific energy of the operator is less than 2 percent (much less for soft meat cutting).

Tooth sharpness life was studied on the basis of tooth flank wear (ignoring tooth edge radius). Wear life criteria were established for the three media cut, as follows: Wear life of a band is reached after cutting 1200 square inches of bone cross-section, or 30,000 square inches of...
or 18,000 square inches of soft meat (with about 5 percent bone in terms of surface area cut). Tests show that a small amount of bone present inside soft meats can reduce the life of a band substantially.

The use of Taylor's equation for a portion of the study (bone cutting) shows that bands with negative rake angles are more sensitive to random external changes than those with positive rake angles (between -5° and +5°).

Tests were carried out to determine the effect of rake angle and velocity ratio on the formation of smear. Smear was measured on the basis of two theoretical interpretations. Data obtained and plotted indicated that both interpretations were in fair agreement with each other. The majority of test results suggested that smear formation decreases with velocity ratio and that it decreases as the side rake angle rises from zero in either a positive or a negative direction. A small change of rake angles (between -10° and +10°) has no readily apparent effect on smear formation.

Waste development during a cutting process was also studied for the same variables and ranges stipulated in the previous paragraph. It was found that a change in velocity ratio had no measurable effect on the amount of waste developed, and that side rake angles cause less waste than no side rake angle at all. However, it appears that a positive side rake angle generates less waste than a negative one.
NOMENCLATURE

$\alpha_1$  tooth back rake angle

$\alpha_2$  positive side rake angle

$\alpha_3$  negative side rake angle

$\alpha_3'$ sometimes referred to as $\{-\}$ $\alpha_2$

$\beta_1$  end cutting edge angle

$\beta_2$  side cutting edge angle

$\theta_1$  end relief angle

$\theta_2$  side relief angle

$\gamma_1$  $90 + \theta_1 - \alpha_3$

$\gamma_2$  $90 - \gamma_1 = \alpha_3 - \theta_1$

$\beta$  $90 - \theta_1 - \alpha_2$

$t$  band or tooth thickness

$t_1$  shallow depth of cut, \{ in direction of $v_f$\}

$t_2$  large " " " \{ in direction of $v_f$\}

$v_B$  band velocity (feet per minute)

$v_f$  feed velocity (velocity at which cutting medium is fed into the bandsaw, in a direction \perp to the $v_B$, in fpm)

$\theta$  \(\tan^{-1}\frac{v}{v_B}\)

$N$  number of active teeth (engaged in cutting at any instant)

$l$  height of medium engaged in cutting

$p$  \(\frac{N}{l}\) - band pitch (number teeth/unit band length)

$P$  \(\frac{1}{p}\) - tooth pitch (inches per tooth spacing)

$\psi$  distance a tooth cuts into the cutting medium per height (l) of medium engaged.
\( T \)  
\( A_c \)  
\( v_R \)  
\( d \)  
\( t_l \)  
\( A_i \)  
\( \ell_c \)  
\( d \)  
\( t \)  
\( w_c \)  
\( b \)  
\( \psi \)  
\( v_c \)  
\( v_{Bn} \)  
\( v_{c/Bn} \)  
\( A_{sp} \)  
\( A_{sk} \)  
\( F_v' \)  
\( F_H' \)  
\( F_v \)  
\( F_H \)  
\( N_1 \)  
\( f_1 \)  
\( \mu_{m_1} \)  
\( \mu_{m_2} \)  
\( \mu_B \)  

- time of cut - secs.
- area of meat cut \( (l \times \psi_c', \text{in inches squared}) \)
- resultant velocity of \( v_B \) and \( v_f \) (fpm)
- depth of cut in direction \( \perp \) to \( v_R \) (inches)
- depth of cut in direction \( \perp \) to \( v_B \) (inches)
- instantaneous area of cut \( (v_f \times \ell) \) (square in/sec.)
- discontinuous chip depth (inches)
- depth of cut in direction \( \perp \) to \( v_R \) (inches)
- band thickness (inches)
- width of chip (inches)
- discontinuous chip length (inches)
- shear plane angle (degrees)
- velocity of the chip (fpm)
- velocity of the medium w.r.t. the band (same as \( v_R \)) (fpm)
- velocity of the chip w.r.t. cutting medium (fpm)
- shear plane area (square inches)
- kerf surface area of chip segment (square inches)
- total vertical reaction force of cutting medium on band (oz.)
- total horizontal reaction force of cutting medium on band (oz.)
- same as \( F_v' \), less band side frictional forces (oz.)
- same as \( F_H' \), less band side frictional forces (oz.)
- normal force on tooth backland (oz.)
- frictional force on tooth backland (oz.)
- coefficient of friction of soft meat and steel band
- coefficient of friction of frozen meat and steel band
- static coefficient of friction of frozen meat and steel band
\( N_2 \)  
normal force on tooth topland (oz.)

\( f_2 \)  
frictional force on tooth topland (oz.)

\( R \)  
resultant force of \( F_v \) and \( F_H \) (oz.)

\( \theta \)  
\( \tan^{-1} \frac{F_v}{F_H} \)

\( f_3 \)  
alxial frictional force of cutting medium on band sides (oz.)

\( f_4 \)  
transverse frictional force of cutting medium on band sides (oz.)

\( a_s \)  
area of cutting medium sheared per unit time - (square inch/sec)

\( a_c \)  
cross-sectional area cut - (square inch/sec)

\( s_1 \)  
primary tooth shearing edge (inches)

\( s_2 \)  
secondary tooth shearing edge (inches)

\( s_1' \) and \( s_2' \)  
same as above except projected on a plane perpendicular to the \( v_R \) (inches)

\( f_s \)  
shear force along \( v_R \), per tooth - oz.

\( f_{s_1} \)  
shear force along \( v_R \), per tooth, contributed by \( s_1 \) (oz.)

\( f_{s_2} \)  
shear force along \( v_R \), per tooth, contributed by \( s_2 \) (oz.)

\( P_s \)  
bandsaw power input (oz. in.)

\( P_o \)  
operator power input (oz. in.)

\( V \)  
volume of material removed per unit time (in.\(^3\)/sec.)

\( (SE)_B \)  
specific energy from bandsaw in oz. in. per sq. inch of medium surface cut.

\( (SE)_o \)  
specific energy from operator (oz. in. per square inch)

\( (SE)_f \)  
specific energy of friction

\( (SE)_m \)  
specific energy of momentum

\( (SE)_s \)  
specific energy of shear

\( \tau_s \)  
frozen meat or bone shear stress (ounces per square inch)

\( p \)  
line load in shear during soft meat cutting (ounces per inch)

\( A_s \)  
surface area smeared (square inch)
$S$ percentage area smeared

$W$ meat wastes (ounces)

$w$ waste material (square inches per ounce)

$T$ tool life in minutes

$H$ tooth height (inches)

$k$ band kerf (inches)

$\varphi$ shear plane angle

$S_c$ shear force on bone chip along shear plane (oz.)

$N_c$ normal force on bone chip perpendicular to shear plane (oz.)

$W_c$ chip width (inches)

$\lambda_c$ chip segment length (inches)

$b$ chip segment depth (inches)

$R_c$ resultant $S_c$ and $N_c$ (oz.)

$R_l$ resultant of $N_l$ and $f_l$ (oz.)

$\frac{dF_v}{d\varphi}$ differential of the vertical cutting force with respect to the shear plane angle

$F_{H_w}$ horizontal feed force worn band teeth (oz.)

$F_{V_w}$ vertical feed force on worn band teeth (oz.)
CHAPTER I

INTRODUCTION

Historical Background

Although bandsaws date back to prehistoric times and circular saws to about 1777, the development of practical bandsaws had to await the development of strip steel of uniform quality and hardness which could be joined into a continuous belt-like band which was just as strong and rigid at the joint as at any part of the band. Although bandsaw patents were issued in England as early as 1808, the use of bandsaws was restricted to a few centralized water power and belt-driven assemblies until the arrival of small portable power sources such as electric and gasoline motors.

The first power bandsaws were in large installations and were used predominantly for sawmill work and furniture making. With the advent of small and light power sources, the economical adaptation of bandsaws to a wide range of industries came a certainty. Concurrently, the development of refrigeration, rapid transportation, and mass marketing created a need for mass production in meat cutting, consequently, in the early part of the twentieth century the meat cutting bandsaw was developed to satisfy this gap.

Since its inception, the bandsaw has undergone relatively minor changes with regard to tooth configuration. Foremost in the list of changes has been tooth size, followed by a tooth inclination angle called "rake" angle. These have been modified again and again experimentally
until the evolution of bandsaws of the following types: those for cutting soft woods, those for hardwoods, cross-grain saws, soft and/or hard metal bandsaws, bandsaws for plastics, saws for timber, those for cutting small and intricate shapes in wood, saws for meat and bone cutting, and saws for many other purposes.

Two criteria are usually applied toward the development of these specialized saws: (1) ease of cutting, by both the operator and the prime mover of the bandsaw, and (2) quality of cut or surface finish. It can be readily seen that there is no special difficulty in deciding which of several saw-tooth configurations cuts with greater ease during the process of trying them out. Nor is it a costly venture to determine which saw-tooth shape gives a smooth cut. Simple visual observations will reveal these aspects by trial cutting, even to a fairly untrained experimenter. The result is that the choice of tooth shapes for existing bandsaws has been based largely on poorly planned and controlled field tests.

The general process of cutting with sharp-edged tools has received its reasonable share of attention. The first scientific examination of the cutting process, by Cocquillhat, was published in 1851. He made some torque measurements on a drill while the work piece was being rotated and pressed against the drill. Other investigators followed, concerning a variety of such related phenomena as the influence of cutting fluids, and the forces required to remove material of given dimensions. In 1864 Joessel reported a study of the influences of tool geometry. By 1905 many theoretical and experimental analysis had been made and published. Contributions had been made on chip formation by Time in 1870, Tresca in 1873, and Mallack in 1881; on measuring cutting forces
by Smith (1882), Haussner (1892), Zvorykin (1893), and by at least half a dozen other interested experimenters. Between 1905 and 1925, approximately, a great majority of the progress made was in securing empirical relationships and/or information.

The further development of such things as photography, microscopy, photoelasticity, hardness testers, and dynamometers of various designs gave great impetus toward solving important problems in the area of cutting with sharp-edged tools. A host of important works by dozens of investigators have been published during the last thirty-five years or so. These will not be listed here because of their large number and because much of the material is only remotely related to the work being reported here. Only those articles which are immediately relevant will be referred to in the bibliography.

Despite the innumerable studies made in an attempt to measure and to understand the variables involved during a cutting process, fundamental relationships accounting for variables, heretofore assumed negligible or otherwise unknown, are still waiting to be determined.

A very important but unfortunate situation regarding this brief review of the progress of studies made in tool cutting processes is that perhaps ninety-eight percent of the work has been done for metallic materials as the media of cutting. As a consequence, many important assumptions necessary to obtain relationships or data were based on various phenomenological aspects of metallic material behavior under mechanical loading conditions only. This situation is unfortunate because the investigations of current work are concerned with the cutting of bone and meat. These media are very different from metals, with regard to nature
and texture. Existing literature, however, is important as a foundation for this work.

The Generic Problem and Its Definition

One object of this work is to interpret the actual soft meat, frozen meat, and bone bandsaw cutting processes scientifically, i.e., both qualitatively and quantitatively. In order to do this, preliminary bandsaw cutting tests were run. They revealed that the process generates a discontinuous type of chip, with either frozen meat or bone as the cutting medium. This cutting process, then, is further studied on the basis of a chip's being formed in a manner similar to that which occurs in metal cutting. Merchant\[2,3\] has a reasonably clear-cut and simple approach to a continuous chip formed during an orthogonal cutting process of a metallic material. An important difference is that this work takes into account the force reaction between the back land of the cutting tooth and the workpiece. Both Merchant and all previous investigators have neglected this force and because it would complicate the solution and because it is a force virtually impossible to isolate and not easy to measure. In this investigation, subject force is isolated somewhat by cutting back the back land of the cutting tooth. Dynanometer force readings with and without the back land cut-back, yields this force approximately and its attendant frictional effect on the back land. Moreover, initial contact pressure is determined as the minimum feed force required between tooth tip and workpiece, to initiate cutting action. Less than this minimum force will only cause the teeth to slide on the medium and/or to deform it without cutting into it. A second difference between this work and Merchant's is that the process of bandsawing involves the shearing
of kerf (side) surfaces, simultaneously with the shearing that takes place along the shear plane. Merchant's work accounts for the shearing along the shear plane only, whereas both surfaces are considered herein. A third and very important difference is that Merchant's chip is a continuous one, but the chip resulting from either bone or frozen meat cutting is not only discontinuous but segmental. In the work presented here, the forces measured from the dynamometer are assumed to be exerted by a continuous chip, in spite of the fact that the chips are not continuous. This is nearly true because there are from 15 to 25 bandsaw teeth in action concomitantly. In addition, the number of chip segments that can form theoretically may be as few as 100,000 per sec. at very low band speeds ($v_B = 600$ fpm), and as many as 1,500,000 per sec. at very high rates. Thus, recorder values plotted must be nothing less than average forces which approach the magnitudes of actual forces.

Studies have been made of the process of discontinuous chip formation. The first attempt was by Piispanen in 1946. Field and Merchant extended this analysis in 1949, but three years later Cook, Finnie, and Shaw approached the problem from an entirely different aspect. They claimed the discontinuous chip formation to be analogous to that of a periodic extrusion, rather than to the original concept of simple shear. Subsequently, Lee made some closely related theoretical studies of the discontinuous chip formation during the early stages of the process. Perhaps the most recent work in this realm was done by Okushima and Hitomi, published in November, 1961. The majority of the work just related in brief falls outside the province of this paper since it deals explicitly with various approaches to express the mechanism of discontinuous
chip formation. The current work assumes the formation in question to take place by virtue of simple shear. The author feels that this assumption is quite adequate, as it pertains to frozen meats and bone cutting.

The bandsaw cutting of soft meats is an entirely different affair from the above processes, primarily because there is no chip formed. Preparatory tests and studies of the surfaces cut and of the cutting process itself, were made. They showed that shearing is definitely the mechanism of consequence here. The meat is virtually sheared by the sharp cutting edges of the band teeth, upon mutual contact and a fair amount of initial deformation. During the time that cutting edges are deforming the meat, the contact pressures are rising until minimum pressure for severing is reached, at which time the meat becomes sheared. Tensile and compressive stresses are somewhat involved during initial deformation, but because the deformation process is of an indeterminate character, subject stresses are assumed to be ineffective. Observations show, however, that the tensile stresses are quite important at ultra-high velocities of feed largely. Under such cutting conditions the meat surfaces are very ragged and torn -- a very undesirable state for meat surfaces, from the standpoint of the meat market. Bandsaw-cut meats very often sell more quickly because of their attractive appearance.

The presumed mechanisms of cutting soft and frozen meats, as well as bone, are studied with respect to the overall force reaction picture. Dynamic shear stresses are obtained experimentally and used in connection with forces and assumed sheared areas to compare with actual dynamometer forces. This serves as an indication of the soundness of the interpretations given to the cutting mechanisms studied.
A second objective of this study is to examine four important factors immediately associated with the cutting processes just described. They are: (1) cutting efficiency (least power consumption); (2) meat and bone smear development (macerated specks of meat, bone, and fat which become aggregated and adhere to freshly cut meat surfaces, imparting an unsightly appearance to these); (3) meat and bone wastes (analogous to sawdust from wood cutting); and (4) useful tooth sharpness life. These four factors have become significant problems in very recent years because of existing competitive and economic trends. Prior to that, the only two of these four areas receiving any degree of attention have been cutting efficiency and tooth sharpness life. These were solved handily by making the tooth edges reasonably hard, and, as stated before, by trying out various tooth configurations and quickly deciding which one cuts with the greatest apparent ease. These methods are not particularly effectual for today's competitive economy.

In this work, the aforementioned four factors are examined with regard to tooth configuration, feed speed, and band speed. In order to determine the least power consumption, specific energy relations are derived for important force -- velocity combinations, and comparisons are made. Tool sharpness life is studied for bone cutting, by means of standard Taylor's relationship between speed of feed and band tooth longevity. This same study is not carried out with frozen meats and soft meats because of the prohibitive costs of the large amounts of these materials needed to perform such tests. Progressive tooth flank wear is viewed on the basis of cross-sectional area of material cut. All three cutting media are involved here.
There is no literature on meat or bone cutting with bandsaws or any other tool. It seems certain that work in this area has been done by other scientists but their results have not been published.
CHAPTER II

CUTTING MECHANISMS

Tooth Geometry and Bandsaw Kinematics

The three general types of tooth configurations studied are shown in Figures 1, 2, and 3. For convenience of reference they are called case 1, case 2, and case 3, in the same order as the figures. The profiles of the three are exactly alike in contour as well as in size and in lateral set. Case 1 has its top rake angle as the only dimensional parameter. It is typical of the tooth used widely for meat cutting. Cases 2 and 3 have a top rake angle and a side rake angle as their size parameters. The difference between case 2 and 3 is that their side rake angles are opposite in direction, with respect to each other. Therefore, case 2 has its tooth apex (cutting tip) in a direction agreeing with the set (away from the band center), whereas the tooth apex leans toward the band center for case 3.

The elements of major importance in these teeth are the cutting edges themselves. Case 1 has three cutting edges (Figure 29) which are \( a'd' \) (primary cutting edge), \( a'^j' \) (secondary cutting edge) along the tooth profile starting at the apex, and again \( a'^j' \) repeats itself along the other side of the tooth (also secondary cutting edge). Case 2 has \( a'c' \) (primary cutting edge) and \( a'^j \) (secondary cutting edge), and case 3 has \( d'b' \) (primary cutting edge) and \( d'^j_2 \) (secondary cutting edge), as their two cutting edges each. The derivations for these various lengths are given in Appendix C.
Figure 1. Tooth Geometry (Case 1).

(a) PERSPECTIVE VIEW

(b) THREE VIEWS

$\theta_1$ - END RELIEF ANGLE

$\alpha_1$ - BACK RAKE ANGLE

$\beta_2$ - SIDE CUTTING EDGE ANGLE
Figure 2. Tooth Geometry (Case 2).
Figure 3. Tooth Geometry (Case 3).
Figure 4 shows all the band kinematics terms necessary for this work.

**Cutting Soft Meats**

Meats that are soft are in an unfrozen state (40-60°F). The meats chosen for these tests were reasonably firm in consistency and of such a texture that their fibrous nature is quite uniform and in one direction.

In order to interpret the mechanism of cutting, mathematically, it was imperative that many trial tests and observations be made. Cuts were examined microscopically during these tests. This resulted in the discovery that the meats were being sheared regardless of tooth configuration, as long as the cutting edges remained reasonably sharp. Appendix D, Figure 34 shows a meat surface after it was sliced carefully with a knife. Figure 36, on the other hand, exposes the meat surface after being cut with a sharp bandsaw. For the time being the paper clip and the region below it, as present in the latter figure, should be ignored. However, the relatively clean, smooth, and undamaged surfaces of the meats in these two pictures should be noticed. The appearance of the meat surfaces look as if they were severed by means of tension (see Figure 37). This cut was also made with a sharp bandsaw but at an excessive feed rate (> 100 fpm). A similar surface is manifest when the bandsaw velocity is considerable (> 9600 fpm). These pictures are typical of the dozens of observations made in order to verify this finding. Continued scrutiny discloses, in addition, that a sharp-edged band with burrs on the cutting edges does more tensile tearing than cutting by shear as it would without the burr. Burrs, of course, are usually present along cutting edges which have been sharpened by filing or by grinding. However, burrs resulting from filing
\( v_B \) = Velocity of band (fpm)
\( v_f \) = Velocity of cutting feed
\( v_R \) = Resultant velocity of band with respect to the medium being cut.

\[
P = \frac{1}{p} = \text{linear pitch (distance between teeth)} \quad (1)
\]

\[
p = \frac{N}{I} = \text{band pitch (number of teeth per inch of band contact length)} \quad (2)
\]

\[
x_f = P \tan \theta \quad (3)
\]

\[
x_R = P \sin \theta \quad (4)
\]

\[
\theta = \tan^{-1} \frac{v_f}{v_B} \quad (5)
\]

Figure 4. Bandsaw Kinematics and Equations.
tend to fall off very shortly after a band is put in operation. The bands used in this work had their burrs removed before testing.

The band and feed velocities used in the current analysis (see Table 4) are both lower and higher than those encountered in practice. Nevertheless, they are below the values which cause the meats to be cut by tension instead of by shear. Figure 5 displays cutting (shearing) action of bandsaw teeth on soft meats. The primary and secondary cutting edges which do the shearing have been described already in the previous section. Note that the lack of absolute rigidity of the meat causes it to bulge into the opening between teeth (gullet). Such bulging is fairly small because of the pulling action of the meat by the teeth ahead, provided the feed velocity is not excessive (> 100 fpm).

Having ascertained to a fair degree that the mechanism of cutting soft meats with bandsaws occurs almost entirely by shearing action, this statement is now postulated as the first and perhaps the most important assumption in the mathematical analysis which is to characterize this panorama of cutting activity. A second assumption is that all cutting edges are shearing the meat simultaneously. This may introduce an error as high as ten percent when cutting with the case 1 band, at very high velocities of feed. The error diminishes with a drop of feed velocity, since the primary cutting edge and at least one of the two secondary cutting edges must shear simultaneously if the meat is to be cut. The third cutting edge (secondary) becomes as high as ten percent of the total shear length when the feed velocity is high, and it decreases with a lessening of the feed velocity. However, since it is expected that a part of this third shear edge performs some shearing, the actual error should be less.
Figure 5. Meat Cutting Action.
This second assumption is probably no more in error than ten percent when applied to cases 2 and 3 because their only two cutting edges meet at the tooth apex and should cut simultaneously. Figure 29 (Appendix B) exhibits these two cutting edges at key cutting positions on the tooth.

A third assumption is that all shearing action approaches that of a simple direct shear.

The complexion of forces present during the cutting of soft meats is exemplified by Figure 6. The components $N_1$ and $f_1$ come from the total shearing force acting at the shear edges. The $N_2$ and $f_2$ forces result from meat pressure against the back land of the tooth. The frictional forces on the band sides are denoted by $f_3$ and $f_4$. Figure 7 is a polygon of these forces in equilibrium. The force relationships resulting from these are:

**Longitudinal feed force:**

$$F_v = -N_2 \sin \theta_1 + f_2 \cos \theta_1 + N_1 \cos \alpha_1 + f_1 \sin \alpha_1 + f_3$$  \hspace{1cm} (6)

**Horizontal feed force:**

$$F_H = N_2 \cos \theta_1 + f_2 \sin \theta_1 - N_1 \sin \alpha_1 + f_1 \cos \alpha_1 + f_4$$  \hspace{1cm} (7)

or

$$F_v = N_2 (\mu_1 \cos \theta_1 - \sin \theta_1) + N_1 (\mu_1 \sin \alpha_1 + \cos \alpha_1) + f_3$$  \hspace{1cm} (8)

$$F_H = N_2 (\mu_1 \sin \theta_1 + \cos \theta_1) + N_1 (\sin \alpha_1 - \mu_1 \cos \alpha_1) + f_4$$  \hspace{1cm} (9)

For the case $f_3 = f_4 = 0$, the equations are easily solved for $N_1$ and $N_2$. Values of $f_3$ and $f_4$ are determined by way of a cutting dynamometer. By determinants:
Figure 6. Equilibrium During Meat Cutting Action.

Figure 7. Forces on the Band Segment.
\[ N_2 = \frac{F_v K_2}{K_1 K_2} = \frac{F_v K_4 - F_H K_2}{(K_1 K_4 - K_2 K_3)} \]  

(10)

and

\[ N_1 = \frac{K_1 F_v}{K_3 F_H} = \frac{F_H K_1 - F_v K_3}{(K_1 K_4 - K_2 K_3)} \]  

(11)

where:

\[ K_1 = \mu_1 \cos \theta_1 - \sin \theta_1 \]

\[ K_2 = \mu_1 \sin \alpha_1 + \cos \alpha_1 \]

\[ K_3 = \mu_1 \sin \theta_1 + \cos \theta_1 \]

\[ K_4 = \sin \alpha_1 - \mu_1 \cos \alpha_1 \]

An appropriate and convenient meat property to use in connection with the forces is its shear strength. For this purpose the "line load \((p)\)" is used. It is the resistance to shear per unit of cutting edge. The equation defining this term (Appendix E), is as follows:

\[ p = \frac{N_1}{s_1 + s_2} = \frac{N_1}{\frac{\cos \alpha_2}{\sin \theta_1} + \frac{\sin \theta}{\cos (\alpha_1 + \theta)}} \]  

(12)

where:  \( N_1 = \) total load perpendicular to shear edges

\( s_1 + s_2 = \) total length of the cutting edges
This value is useful for determining the shear strength of the meat per unit length of cutting edge at various feed velocities, band velocities, and tooth geometries. This is a property not readily used nor found in reference materials because the nature of the cutting condition it represents is seldom met in practice.

Cutting Bone and Frozen Meats

Despite the fact that all remarks made in this section are directed to both bone and frozen meats (unless otherwise specified), the term cutting medium is used for simplicity. The reason for this is that the cutting process for bone is virtually a mirror image of that for frozen meats.

After an extensive series of trial tests and observations of the cutting process with the media in question, it was decided that the mechanism of cutting involved here is that of shear. The waste material carried out by the band was examined carefully and found to be segments of chips which were generated during cutting action.

Chapter I gives a brief historical background of the formation of metallic chips. It also describes the important differences between the current study and past studies of the metallic chip formation with single point cutting tools.

Figure 5 shows the theoretical paths which bandsaw teeth traverse during the process of cutting fairly rigid media, as are bone or frozen meats. Cutting tests used to cut subject media are made with case 1 type bands only. This is done in order to approach the state of orthogonal cutting. The process of cutting frozen meats or bone with case 2 and 3 bandsaws is very complex because there can be as many as three shear
planes intersecting each other simultaneously. A mathematical inter­
terpretation of this situation does not appear possible because of the brittle
nature of the cutting media and the discontinuous character of the chip
formation. This phase, therefore, becomes a lengthy problem in itself
and falls outside the scope of the current analysis.

Figure 41 is representative of the chip formation during a pre­
sumed orthogonal cutting process of the medium by a handsaw tooth. As
stated previously, the mechanism of chip formation is assumed to be that
of simple shear along the shear plane and along kerf surfaces (shaded
area in Figure 41), in this work. The mathematical analysis detailed in
Appendix G is based on the following assumptions:

(1) Shearing occurs cointstantaneously along the shear plane and
along the kerf surfaces.

(2) There is no interdependence between the normal and shear
stresses at the shear plane. This has been much in dispute recently.
Competent experimenters have expressed their opinions in support as well
as in opposition to it. The author bases his argument largely upon the
fact that most materials seem to have very nearly the same flow stress
capacity whether in tension or compression.

(3) The forces from the tool on the chip and those from the work­
piece on the chip, are equal, opposite, and colinear.

(4) There are no bending force reactions from the chip.

(5) Shear plane lies in the direction of maximum shear stress.

Figure 42 shows two force circles. Number 1 concerns itself with forces
acting on the chip only, whereas, number 2 accounts for those forces act­
ing on the tool. The following equations for the horizontal and vertical
feed forces, are obtained from these circles:

\[
F_v = -N_2 \sin \theta_1 + f_2 \cos \theta_1 + \frac{S_c \cos (\alpha_1 - \gamma_1)}{\cos (\phi - \alpha_1 + \gamma_1)}
\]  
\[
F_H = N_2 \cos \theta_2 + f_2 \sin \theta_1 - \frac{S_c \sin (\alpha_1 - \gamma_1)}{\cos (\phi - \alpha_1 + \gamma_1)}
\]  

(13)  

(14)

The property considered useful in this chip forming force system should be the shear strength, unquestionably. This is true because stresses causing failure are basically of a shear type. The stress equation developed in Appendix H, is:

\[
\tau_s = \frac{N_1 \cos (\phi - \alpha_1 + \gamma_1) \sin \phi}{\cos \gamma_1 P \sin \theta (2b + w_c)}
\]  

(15)

This equation is used to determine the dynamic shear strength of the medium at various feed velocities, band velocities, and tooth geometries. The underlying theory here is that the instantaneous shear stresses operating at the shear and kerf surfaces are equal to the total shear load on these surfaces (in direction of shear plane), divided by the total area of these surfaces. This relation yields the following form when integrated with equation 14 (Appendix G):

\[
F_H = N_2 \cos \theta_1 + f_2 \sin \theta_1 - \frac{\tau_s P \sin \theta (2b + w_c) \sin (\alpha_1 - \gamma_1)}{\sin \phi \cos (\phi - \alpha_1 + \gamma_1)}
\]  

(16)

After applying the principle of minimum energy, equation 16 reduces to equation 17 (Appendix G), or:
\[ 2\varphi = 90 + \alpha_1 - \gamma \]  

(17)

This equation proves to be reasonably satisfactory as attested to it by experimental data examined later.
CHAPTER III

FACTORS OF IMPORTANCE IN MEAT AND BONE CUTTING

Introduction

The four factors which have recently become important in meat and bone cutting are: (1) smear development, (2) waste formation, (3) power consumption, and (4) tooth sharpness life. In this chapter, each factor is studied with regard to the effect which feed velocity, band velocity, and tooth rake angle have on each.

Smear Development

During the process of meat cutting the meat surfaces become covered with unsightly waste products called smear. The connotation implied by the term "smear" is that the meat surfaces become smeared with avalanches of waste materials, instead of being clean and eye-appealing after being freshly cut. The importance of this problem is now reasonably evident. Extra energy and time must be devoted to the meat after it is cut by the butcher in order to improve its show-case salability. This results in a higher priced product or less profit for the meat market.

The formation of smear involves basically two steps:

(1) Meat and bone masses become virtually macerated from the blows of dozens of tooth cutting edges.

(2) The macerations become aggregated (piled-up) and deposited on the meat surfaces in a random manner.

Figure 39 shows a bandsaw-cut soft meat surface. The smeared areas and
smear lumps are readily discernible. They cover the granular texture of
the meat. Figure 34 shows a meat surface which has been sliced with a
sharp butcher knife. Notice its relatively clean surface compared to that
of the previous picture. Figure 36 has a bandsaw cut surface, part of
which has been cleaned off with a paper clip. Notice the smear piled up
around the clip and to one side of it. Figure 38 shows smeared surfaces
on frozen meats.

Since it is the teeth which chop up the meat and scrape the surfaces,
it would seem that the least amount of contact between the teeth and meats
(after initial cutting contact) would result in least smear particle for-
mation, accumulation, and deposition. Tooth contact frequency (band ve­
city) is therefore expected to affect smear formation. Since tooth edges
behave as scrapers and choppers, certainly the rake angles also are ex­
pected to affect smear development.

Smear formed from the chopping-up and scraping of meats has been
emphasized up to this point. However, smear very often consists largely
of fatty granules which are too small to be seen with the naked eye. Thus
meats with high fat content are more susceptible to smear formation. In
addition, bone chip segments are another constituent of smear material.

This work involves two approaches for measuring smear. The first
one relates the amount of surface area covered with smear to the total
surface area cut and examined. The appropriate expression is:

\[
\%S = \frac{A_s (\text{in.}^2)}{A_c (\text{in.}^2)} \times 100
\]

(18)

The difficulty in using this method is that very often it is not easy to
designate all smeared areas accurately. Sometimes a region is very heavily smeared and another lightly smeared. The area lightly smeared may not be chosen as a smeared surface because the smear may be sufficiently light to be ignored.

The second method for measuring smear relates the weight of the smear formed on the surfaces, to the surface area of the meat in question. A mathematical formulation of this method is:

\[
S = \frac{W_s \text{ (ounces)}}{A_c \text{ (in.}^2)}
\]  

The weakness to this approach is that the intensity of smear on a meat surface is very seldom uniform. As a consequence the figures may be somewhat misleading. Smear is measured here by both methods just described in order to minimize this possibility.

Waste Formation

When bandsawing, a certain amount of the bulk material is removed by the teeth, carried away and deposited as waste (quite like sawdust in wood sawing). Some of this waste product is deposited on the meat surfaces as smear, and most of it is scattered along the path of the bandsaw. It is not difficult to see that meat wastes are a costly proposition, especially in meat markets where thousands of pounds of meats are sawed into small cuts each week.

The majority of meat wastes consists of shavings and scrapings of meat, fatty granules, and bone chips which are carried away by the bandsaw teeth. The remaining waste becomes deposited as smear on meat surfaces.
A theoretical expression of the volume of waste is obtained by multiplying the surface area cut by the bandsaw kerf. The kerf is the maximum width of the saw teeth at their tips. Equations 20 and 21 represent the volume and the weight of meat wastes, respectively.

\[ V(\text{in.}^3) = A_c k \]  
(20)

\[ w(\text{ounces}) = A_c k \rho \]  
(21)

and where:

- \( \rho \) = density
- \( A_c \) = area cut (in.\(^2\))
- \( k \) = band kerf (in.)

An acceptable measure for wastes is the number of square inches of meat surface cut per ounce of waste material, or

\[ w = \frac{A_c}{N} = \frac{A_c}{A_c k \rho} = \frac{1}{k \rho} \left( \text{in.}^2 \right) \]  
(22)

This equation is reasonably true for frozen meats but not for soft meats. Frozen meat is rigid and is cut by the bandsaw by means of chip segment removal. On the other hand, soft meat may be firm but is certainly lacking in rigidity, so there is no assurance that all cutting edges will cut simultaneously. It is more likely that just enough cutting edges will cut the meat in order to allow the band teeth to pass on. This amount of cutting edges is less than what is in contact with the meat at any time. The above equation is more applicable to soft meat, in the following form:

\[ w = \frac{A_c}{W} \left( \text{in.}^2 \right) \]  
ounces \]  
(23)
Power Consumption

Power consumption has become important in current times because of the increased costs in electricity and man power. From the point of view of man power, increased production requirements are prone to fatigue a bandsaw operator quite readily. Of course, the type of bandsaw as well as operating conditions are of some consequence here. Energies for cutting action come from the operator and from the bandsaw.

This work entails the unravelling of the existing energy consuming operations during a bandsaw cutting process, and expressing them in the form of useful equations. The energy is consumed in deforming or shearing, in changing inertial conditions, in overcoming surface energy (separation energy), and in slipping (rubbing action).

Input Energies

Energy is expended by the bandsaw and by the operator. The operator exerts a horizontal force as he feeds the cutting medium into the band at a certain rate. The band exerts a vertical (downward) force on the medium at a prescribed rate. The power required of the bandsaw and that of the operator are defined by equations 24 and 25, respectively.

\[ P_B = F_v v_B \text{ (ounce-feet per minute)} \]  \hspace{1cm} (24)

\[ P_o = F_H v_f \text{ (ounce-feet per minute)} \] \hspace{1cm} (25)

Material Removal

The volume of material removal is represented by equation 26.

\[ V = k d v_R \left( \text{in.}^3 \right) \text{ (min.)} \] \hspace{1cm} (26)
This equation is less in error when used for frozen meats and bone than for soft meats. The rigidity of the frozen meats and bone causes virtually the entire chip undergoing shear, to be removed. In soft meat cutting only an unestablished amount of V is really removed. Nevertheless, the term V is used for all the cutting media involved in this work, for convenience and for simplicity of reference.

**Specific Energy**

This term is generally defined as the energy in question per cubic inch of material removed or as the power involved in an isolated process divided by the volume rate of material removal. In this work it is more significant to define the specific energy with respect to surface area cut.

\[
SE = \frac{\frac{P}{A_c}}{\text{in.}^2} \quad (27)
\]

**Input Specific Energy**

Again this term involves input from both bandsaw and the operator.

\[
(SE)_B = \frac{P_B}{V} = \frac{F_h v_B}{d v_R} \quad (28)
\]

\[
(SE)_o = \frac{P_o}{V} = \frac{F_H v_f}{d v_B} = \frac{F_H \tan \theta}{d} \quad (29)
\]

\[
d = P \sin \theta \quad \therefore (SE)_o = \frac{F_H}{P \cos \theta} \quad (30)
\]

**Output Specific Energy**
The energies put into the cutting process are consumed in friction (from rubbing action), in surface separation, changing inertia of masses (chip segments), in the shearing, and in deformation. The corresponding relationships are now developed.

1. Friction specific energy (see Figure 6)

\[
\text{SE}_{f_2} = \frac{f_2 \cos(\theta_1 - \theta) v_R}{d v_R} = \frac{f_2 \cos(\theta_1 - \theta)}{d} \tag{31}
\]

\[
\text{SE}_{f_1} = \frac{f_1 \sin(\alpha_1 + \theta) v_R}{d v_R} = \frac{f_1 \sin(\alpha_1 + \theta)}{d} \tag{32}
\]

\[
\text{SE}_{f_3} = \frac{f_3 v_B}{d v_R} \tag{33}
\]

\[
\text{SE}_{f_4} = \frac{f_4 v_f}{d v_R} \tag{34}
\]

2. Specific energy of momentum (Appendix I)

\[
\text{SE}_m = \frac{F_H v_f + F_V v_B}{d v_R} \tag{35}
\]

where \( F_H \) and \( F_V \) are defined by equations I-6 and I-7, in Appendix I.

3. Shear specific energy (for frozen meat and/or bone)

\[
\text{SE}_{s_1} = \frac{S_c v_c/m}{d v_R} \tag{36}
\]
but from equation 15, the expression becomes,

\[
\text{SE}_{s_1} = \frac{T_s v_R \cos \alpha_1}{v_R \sin \varphi \cos (\alpha_1 - \varphi)} \left[ \frac{t}{\cos \theta} + 2b \right]
\]  
(37)

(4) Shear specific energy (for soft meats)

\[
\text{SE}_{s_2} = \frac{p(s_1 + s_2 + s_3)}{d}
\]  
(38)

\[s_3 = 0\] for sharp bands

See Figures 29 and 30 (d'c)

(5) Surface specific energy

\[
\text{SE}_{s} = \frac{2 \gamma s}{d v_R}
\]  
(39)

where \(\gamma\) = surface tension in ounces/lineal inch

\(s = \) length of separation in inches

It was attempted to obtain \(\gamma\) for the various types of interfaces made during cutting but with no success. The interfaces in question are the following:

(a) soft meat and soft meat

(b) soft meat and steel

(c) soft meat and air

The same combinations may be made for bone and for frozen meats.

Energies of deformation and residual energies are not readily expressible mathematically.
Band Sharpness Life

This factor is important because as the bandsaw teeth get worn (lose their sharpness) the energies required of the operator and of the bandsaw increases. There are other side effects that may be as important as this or more, depending upon the medium being cut. In bone cutting the main concern is that feed forces may rise to the point of making the cutting process unmanageable. The same argument applies to frozen meats. When cutting soft meats, however, straightness of cutting is probably more important than the relatively small increase in forces resulting from wear of the band teeth. Straight cutting is imperative because expensive cuts of meat may be easily ruined from crooked cutting and will have to be sold at substantially reduced prices. Band sharpness life is defined here as the amount of material necessary to be cut (in terms of cross-sectional area), to cause the band teeth to become dull by wearing of the flank. Consequently, a bandsaw is dull when operator feed forces have reached a proportion of the order of several times the initial feed forces required for cutting frozen meats or bone. In soft meat cutting a bandsaw is assumed dull when it deviates from straight cutting as much as 1/2 inch along a 6-inch long cut, at a fairly rapid rate (50 fpm) at a normal bandsaw velocity (2500-3500 fpm).

The type wear studied here concerns loss in sharpness at the primary cutting edge by means of gradual attritional wear of the flank. (See Figure 8).

Generally, tool life decreases as cutting speed rises in conventional cutting operations. Such processes are normally represented by Taylor's expression, $VT^n = C$, where:

\[ (41) \]
\[ V = \text{cutting speed in fpm} \]

\[ T = \text{tool life in minutes} \]

\[ n = \text{exponent depending on tool and work piece} \]

\[ c = \text{a constant defined as the cutting speed required to give} \]
\[ \text{a tool life of one minute} \]

This relation is used here for bone cutting only. \( V \) is not the cutting speed but the feed speed and \( T \) is the bandsaw life in square inches of cross-section of the medium cut, instead of in minutes. The bandsaw speed is not used because a rise of band speed lessens the amount of material engaged and the contact forces drop. The opposite effect is encountered in other type cutting operations than bandsawing. Thus Taylor's relationship would not satisfy. In addition, bandsaw speeds are more or less standardized for meat and bone cutting operations (2200-3500 fpm) and industrial bandsaws generally have a fixed cutting speed.

The cutting life is not measured in minutes as in Taylor's equation because bandsaw cutting is not continuous like a lathe operation is. Instead it is continual or intermittent in character. As such, the cross-sectional area cut is used since it is a reasonable measure of bandsaw sharpness life. In metal cutting \( V \) is defined as the cutting speed at which a tool would fail after 20 minutes of operation. For meat and bone cutting, however, re-definition of this term is essential. For the current work the following definitions are reasonable:

(a) \( V \) is the feed speed at which the band would cut 1200 square inches of bone during its life.

(b) \( V \) is the feed speed at which the band would cut 18,000 square inches of soft meat during its life.
(c) $V$ is the feed speed at which the band would cut 30,000 square inches of frozen meat during its life. These values were chosen on the basis of experimental data plotted in Figure 19, and on the life criterion deemed satisfactory for each of the three cutting media.

It was stated that Taylor's equation is used for bone cutting only. This is true because in order to obtain a minimum amount of data from soft and frozen meat cutting, it would require a tremendous amount of the media to be cut. Such a venture is too costly to attempt in the laboratory. Tests of the type in question are virtually impossible in meat markets even though the amounts of materials cut there are unlimited. It is not possible to regulate or vary the feed velocity of a given operator. Neither can band velocities be varied because bandsaws operate at only one speed. Alternatively, the determination of progressive flank wear relative to the area cut is the only other possibility. Another tool-life criterion to be evaluated concerns the increase of feed load and specific energy with flank wear. This is a convenient measure because it changes almost linearly with area cut.

With data of the type mentioned it is hoped that useful generalizations will result.

In order to compute the increase of force on the wear flank, the following equations are used, as determined from Figure 8.

$$
\sum F_v = f_3 - N_2 \sin \theta_1 + f_2 \cos \theta_1
+ N_1 \cos \alpha_1 + f_1 \sin \alpha_1 - F_v = 0
$$

(42)
Both equations apply to the worn teeth, whereas they are just as good for sharp teeth where \( N_3 = 0 \) and \( f_3 = 0 \). In order to obtain \( N_3 \) and \( f_3 \) from the above equations it is necessary to assume that \( N_1 \) and \( N_2 \) do not change for a given depth of cut with either a sharp or a worn band. Consequently, when worn band forces \( (F_v, F_H) \) are measured with a dynamometer, and \( N_1 \) and \( N_2 \) determined from the equations and forces \( (F'_v, F'_H) \) measured for a sharp band, the values are substituted in the above relations. The only unknowns \( N_3 \) and \( f_3 \) are then solved for.
Figure 8. Forces on Sharp Band and Worn Band Teeth. (Orthogonal Cutting).
CHAPTER IV
EQUIPMENT AND EXPERIMENTAL PHASES

Cutting Process

Soft Meats

Dozens of preliminary cutting trials were made in an effort to appreciate the cutting process. Meats were frozen in dry ice and then cut by bandsaw into appropriate rectangular pieces of various sizes. The width was four inches, the length four inches to six inches, and the height two inches to three inches. These specimens were allowed to thaw out before they were used for testing. Each condition of cutting \( v_B, v_f, \) and band tooth was tested four times. The vertical and horizontal component forces in action were measured as reaction forces on a strain gauge dynamometer (see Figure 9). The force signals registered on a Sanborn Dual Channel Recorder. Typical record charts are shown in Appendix S. A standard Butcher Boy bandsaw machine had to be modified for band speed limits between 600 fpm to 14,200 fpm, and a carriage feed set-up had to be added to it for feed speeds between 1 fpm to 125 fpm.

A spring loaded device (Appendix T) was used to separate the two parts of the meat being cut in order to minimize frictional contact between the freshly cut surfaces and the band sides.

The dynamic shear stresses were determined by recording the dynamometer forces resulting when cutting meat with a special band of \( \alpha_1 = 0^\circ, \) and \( \alpha_3 = 40^\circ. \) Details of the test bands used are listed in Appendix A.
Figure 9. Bandsaw-Dynamometer Set-Up.
Frozen Meats

The technique just described for soft meat cutting was also used for frozen meats. Most of the details given for soft meat cutting also apply here except for those which follow. Two dry ice chests were used. One was for storing the specimens to be tested and the other was used to control the steady state temperature at which the specimen was to be cut. Trial tests showed that the specimen should not be warmer than 15° F, or else there would be partial thawing-out during cutting. It was also learned that the meat temperature could be as low as -40° F without causing a measurable change in reaction forces. Thus the frozen specimens cut were at a temperature between 0° F and -25° F.

Bone Cutting

The details just described for cutting frozen meat and soft meat are basically the same as for bone cutting with the exception of the points listed below. The bone specimens were prepared according to Appendix A. Cuts were made across slabs of different vertical dimensions. It was found that the forces were proportional to this dimension provided it exceeded one inch. Less than one inch thickness caused appreciable signal fluctuation from the dynamometer even though the average value of the recorded forces still indicated a direct proportionality with thickness. All cuts were made on one and one-fourth to three inch deep specimens. Bone shear angles were obtained by direct measurement though a microscope angular grid reticle of one degree increments.

A reasonably reliable technique had to be developed for quickly stopping the band during a cutting test. This is necessary for obtaining specimens where the shear angle is evident and can be measured. A fairly
simple approach was possible for accomplishing this work. The band was tightened a fraction of that which is usually required for field cutting operations. In order to bring the band to a sudden halt from a steady state cutting situation, a piece of hard wood was jammed instantly into the moving bandteeth. The band stops moving and the drive pulley which is slipping on the band soon comes to a stop. The bandsaw power is cut off a fraction of a second before the wood is thrust onto the band. Microscopic observations of the shear angles disclose that the method is quite reliable.

Smear Measurement

As previously stated, smear deposits were measured in two different ways. In one method the rectangular surfaces were examined and a piece of clear plastic with parallel line grid work (1/10 inch apart) was placed over it. The area smeared was measured by counting the squares of the gridwork within the smeared regions. The second method involved scraping the freshly cut meat surface and weighing the scrapings. Care was taken not to over scrape the surfaces because the metal edges of the scraper can readily scrape pieces of meat off the bulk material.

Wastes

The meat specimen is weighed before it is cut. After cutting the surfaces are scraped clean and the specimen is weighed again. The difference between initial and final weights is assumed wasted during the cut. The cross-sectional area of the cut surface is measured and recorded. Waste is also computed by means of the theoretical relationship (equations 14-17) developed in the last chapter, and compared with actual values measured.
Power Consumption

This item is determined for the various power consuming factors of importance described in the previous chapter. The equations listed there are used with the various parameters measured according to this chapter. Frozen meat and bone chip dimensions are measured by a toolmaker's microscope and by a Nikon multistage comparator. Both of these have an accuracy of ± 0.0001 inch. Band velocities were checked by measuring the band pulley rpm's with a tachometer.

Band Sharpness Life

Bands with only three different \( \alpha_1 \) angles were tested in soft meats, frozen meats, and bone. These angles were -5\(^\circ\), 0\(^\circ\), and +5\(^\circ\). All other dimensional parameters are the same as those stipulated previously in this chapter. The band velocity for all flank wear tests was 2400 fpm but the feed velocity for bone varied between 1 to 5 fpm, but for soft meats it was 50 fpm and for frozen meats it was 10 fpm. Trial runs showed that these velocities were measured within 5 percent of error.

Cross-sections of cuts were measured by laying the cut surface on a graph paper and tracing the outline. By adding the number of the squares the cumulative areas of cut were obtained. Wear flanks were measured with a toolmaker's microscope by aligning the microscope hairs with the leading edge of the wear flank and traversing the microscope table until the cross-hair matched the trailing edge of the wear flank. The difference between the initial and final readings of the micrometer gave the flank wear length. The sizes of the specimens were different and greatly varied in order to make use of all the medium tested.
When obtaining dynamometer forces on worn bands the velocities were varied between 2400 fpm and 9600 fpm for these tests.

**Miscellaneous**

In order to isolate the forces $N_1$ and $f_1$, approximately, the tooth backland is relieved until the end relief angle ($\theta_1$) is between 55° and 80°. When the band cuts, forces $N_1$ and $f_1$ become negligible and only $N_2$ and $f_2$ are operative and easily solved for. Cutting with normal (unmodified) bands and using the latter corresponding values of $N_2$ and $f_2$, is sufficient to compute for $N_1$ and $f_1$ (equations 8 and 9).
CHAPTER V

DISCUSSION OF RESULTS

Mechanism of Soft Meat Cutting

Microscopic observations of the meat surfaces cut by various bandsaws, band speeds, and feed speeds attest to the fact that shear is the basic mechanism of cutting. Comparison of Figures 34, 36, and 37 shows the surface appearance of three cutting conditions involved. The first figure shows the statically sheared surface with a sharp butcher knife. The other two figures show surfaces resulting from bandsaw cutting. The first one of these is a typically sheared surface. The other looks as though the surface had been pulled apart (as in tension) from the original bulk material. This last picture is characteristic of how the meats get severed when the band velocity is approximately 9600 fpm or more, notwithstanding the velocity of feed. The same tearing or pulling effect occurs when the feed velocity is about 150 fpm at a band velocity less than 1400 fpm, or at a feed velocity of 75 fpm and a band velocity less than 600 fpm. These last two speed combinations are such that the teeth gullets are totally jammed, with the meat being fed into the band. In these cases it is very likely that cutting is being done by the entire tooth configuration rather than by the immediate region of the tip. Apparently, the overcrowding of the primary and secondary shear edges results in partial tensile tearing. This argument does not substantiate the tearing which occurs at the very high band speeds because the tearing occurs regardless of feed speed. At these very high band speeds the depth of material
engaged by the band teeth is of the order of 0.001 inch and less. Since
the sharpness radii of these bands are of the same order of magnitude, it
is expected that the cutting edge is relatively dull with respect to the
depth of cut. Thus, tearing (tensile type) becomes the primary mechanism
of cutting. It is not believed that the time element on material proper­
ties or material behavior are factors of consequence, as expected for crys­
talline materials.

Cutting Forces

No dynamometer force readings were made for velocity ratios below
\[ \frac{100}{9600} = \frac{v_f}{v_B}, \]
and above \[ \frac{100}{1200}. \] At the high velocity ratios the shearing
action had a tendency to become a tearing (from tension) action and caused
undesirable fluctuation of forces. At the low velocity ratios the force
readings were close to zero for case 2 and 3 (type bands). It was diffi­
cult to distinguish between the low cutting forces and those transmitted
to the dynamometer from the dynamic characteristics of the bandsaw machine.
Case 1 type bands, however, registered discernible forces even at the very
low velocity ratios. This is attributable to the fact that case 1 type
bands make cutting contact with their primary cutting edges whereas con­
tact is made only by part of the primary cutting edges of case 2 and 3
type bands.

The salient points to be had from Figures 10 through 12 are:

Case 1.

(a) The horizontal forces (operator feed forces) do not seem
to change with velocity ratio. This is easily chargeable to the fact
that a change in velocity ratio changes the overall cutting edge length
by a very small percentage. This effect is apparently more sensitive
to the vertical cutting forces since they do change significantly.
Figure 10. Force Components from Soft Meat Cutting. (Case 1).
(b) A rise of rake angle from a magnitude of $-15^\circ$, in a positive direction, minimizes both horizontal and vertical component forces.

(c) The vertical forces are one and one-half to two times as big as the feed forces for band 1 ($\alpha_1 = -15^\circ$), whereas they are 10 to 15 times as great for band 7 ($\alpha_1 = +15^\circ$). Thus band 7 requires the operator to exert a horizontal feed force of about $\frac{1}{10}$ the force which the band exerts during cutting. Band 1 requires the operator to push with a force almost as great as the force applied by the band itself. It is obvious that band 7 is more desirable for the operator to use in cutting soft meats.

Cases 2 and 3.

(a) Both the horizontal and vertical components of force generally rise at a rapid rate with an increase in velocity ratio. This is as expected because any small change in velocity ratio results in an appreciable increment of the length of the cutting edge in action. The horizontal projection of the tooth - meat contact area is also affected measurably.

(b) Bands 10, 13, 16, 19, 22, and 25 are characterized by showing almost no change in horizontal reaction. These bands as well as that of case 1 (band 7), have a large positive rake angle ($\alpha_1 = +15^\circ$). This is so because the force component $N_1$ (Figure 6) rotates with $\alpha_1$ resulting in a general lowering of the horizontal feed force.

(c) There appears to be a very minor difference in vertical force reaction ($F_v$) among bands 8 through 25. This is because $F_v$ relies largely upon the cutting edge length which in turn is a function of $\alpha_1$ and $\alpha_2$ (or $\alpha_3$). For the range of testing rake angle values the $\alpha_2$'s can
Figure 11. Force Components from Soft Meat Cutting. (Case 2).

**NOTE:** $v_f = 50$ fpm

$f_a = f_4 = 0$

$\frac{v_f}{v_B}$ (FEED TO BAND VELOCITY RATIO IN LOG SCALE)
Figure 12. Force Components from Soft Meat Cutting. (Case 3).

NOTES: \( v_f = 50 \text{ fpm} \)
\[ f_3 = f_4 \]
be said to produce virtually the same effect as corresponding \( \alpha_3 \)'s.

(d) Case 2 and 3 bands are markedly sensitive to velocity ratio changes, except for those bands listed in (b) above. The qualifications here are the same as those at (a) above.

(e) All bands record very small band forces (Fv) and feed forces (Fh) for high band velocities. As a matter of fact the forces are almost negligibly small. This is caused by the relatively small amount of tooth penetration.

Cases 1, 2, and 3

(a) At low band velocities (1200 fpm) Fv's are about the same for all 25 bands because tooth engagement is virtually the same for all three cases.

(b) At medium band velocities (4800 fpm) the case 2 and 3 bands exhibited decidedly lower forces (both Fv and Fh) than case 1 bands, by as much as four to five hundred percent. The reason is that small changes of velocity ratio effect measurable changes of cutting edge length and horizontal tooth contact projected surface in case 2 and 3 bands but inconsequential changes in case 1 bands.

(c) Figures 45, 46, and 47, which plot forces (Fv and Fh) against band velocity serve to confirm all statements made for soft meat cutting with case 1, 2 and 3, type bands.

Specific Energy

Table 1 summarizes the entire specific energy complexion in soft meat cutting, graphically represented by Figures 48, 49, and 50, Appendix J.
Case 1. (type band)

(a) The specific energy input required from the band is 378 ounce inches per square inch at the lowest speed tabulated (2400 fpm). At double the speed the energy doubles but when the speed is quadrupled the specific energy required is almost quintupled to 1,740 ounce inches per square inch. This occurs from the fact that the band forces do not rise linearly as the band velocity drops (as ascertained in the previous section).

(b) From the standpoint of the operator the specific energy required nearly doubles from 2.4 to 4.5 ounce inches per square inch while quadrupling the band speed from 2400 to 9600 fpm. The reason for the rise instead of a drop is that the operator forces increase for bands of \( \alpha_1 \leq \alpha^o \) as a consequence of the direction shift in the \( N_1 \) component force (see Figure 6).

(c) It appears as though approximately 85% of the total specific energy is expended in the shear process and about 10% in friction. This is along the lines of expected values since frictional energy expended by \( f_3 \) and \( f_4 \) are not accounted for and may be charged to miscellaneous. As such, frictionally expended energy will rise to perhaps 12 to 13%.

Cases 2 and 3 (again these two cases are considered together because of the great similarity).

(a) Of great importance is the fact that the specific energy input remains between 200 and 234 ounces per square inch for all of these bands and all band speeds. This is significant because one may wish to gain the advantage of cutting at high band speeds without occasioning the usually expected high rise in specific energy as it happens in case 1 bands.
Table 1. Specific Energies in Soft Meat Cutting

<table>
<thead>
<tr>
<th>Band Case</th>
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<th>Ounce in/in.²</th>
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<th>Output</th>
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<td></td>
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<tr>
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<td>348</td>
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<td>&quot;</td>
<td>201</td>
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<td>1.20</td>
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Thus case 2 and 3 bands do not undergo this high rise in specific energy at the high band speeds because the increment of cutting edge length and the horizontally projected surface area of contact is quite insignificant.

(b) It seems that shear specific energy required for this type bands may be 5 percent or more, greater than for case 2 band. Actually, a drop in percentage was anticipated instead of a rise since case 2 and 3 type bands have less shear edge length in action than the corresponding case 1 band, for a specified cutting situation. In addition there is an apparent lessening of the frictional energy as predicted for higher speeds, because the rubbing contact area at the teeth is less for cases 2 and 3 than case 1 bands. At the low speeds the shear energy tends to approach 100 percent. This should not be so but there is no explanation for it, except that it could be ascribed to undetected error in recording $F_v$ and $F_h$.

Cases 1, 2, and 3.

(a) The most significant observation here is that the case 1 bands use up about 700 percent more specific energy to cut meat than do the case 2 and 3 bands at very high band velocities (9600 fpm). At low band velocities the number is about 200 percent. This is in agreement with the force arguments presented in the section previous to this one.

(b) The specific energy figures charged to the operator are relatively small and have the semblance of being negligible. That is not so however. A specific energy of 3 ounce-inches per square inch for the operator is a substantial quantity, especially since production cutting requires continual energy input from the operator. Physical fatigue can set in quite soon. Again it is seen that an increase in band speed from
2400 fpm to 9600 fpm drops the operator specific energy about 400 percent from a relatively low value to begin with, for case 2 and 3 bands. On the other hand, case 1 bands nearly double in energy requirement. Also at high speeds (9600 fpm) the case 1 bands have to have about 15 times the operator specific energy required for case 2 and 3 bands.

Mechanism of Frozen Meat Cutting

The existence of chips or chip segments is almost sufficient to validate the assumption that the cutting mechanism of frozen meats is caused by shear along a shear plane and along the kerf surfaces. Both sustain this idea. It has not been possible to confirm the fact that there is a shear plane, because the material thaws out during the machining process undertaken to reach these regions for microscopic observations. Also the cutting tool distorts the immediate areas desired to be examined.

Cutting Forces

Cutting forces were measured with the dynamometer, for feed and band velocity ranges of 10 to 75 fpm, and 800 to 9600 fpm, respectively. Points of interest:

(a) Figure 13 points out that both the horizontal and vertical components increase rapidly with a rise of the velocity ratio. This is a natural reaction to a depth of cut increase.

(b) The difference of forces (both Fv and Fh) is appreciable between bands 1 and 7 at high velocity ratios. At low velocity ratios the difference is of little consequence since penetration is very small. This difference is caused by a rotation of the tooth force $N_1$ (Figure 6) for band 1 which has a negative rake angle $(-15^\circ)$ and for band 7 which has a positive rake angle $(+15^\circ)$.
The vertical forces are about twice as big as the horizontal feed forces in band 1 ($\alpha_1 = -15^\circ$) but they are approximately five times as large in band 7 ($\alpha_1 = +15^\circ$). This shows that an increase $\alpha_1$ in the positive direction diminishes the feed force requirements of the operator and of the band also.

**Specific Energies**

Figure 14 is a review of the specific energy image resulting from cutting frozen meats. Points of value are:

(a) The specific energy input requirements drop with an increase of $\alpha_1$ in the positive direction starting with $\alpha_1 = -15^\circ$ (band 1), through $\alpha_1 = +15^\circ$ (band 7). This occurs since Fv drops similarly as discussed in the previous section.

(b) The input specific energies are of the same order of magnitude despite the great difference in velocity ratios. This may be chargeable to the fact that the forces decrease with a velocity ratio rise. This is a case where the product of a decreasing function and an increasing one tend to maintain constancy for a given range.

(c) The shear Sp. E. is approximately 88 percent of the total input energy for band 1 and it is about 72 percent for band 7. There appears to be a continuous drop in energy as $\alpha_1 = -15^\circ$ (band 1) increases in a positive direction toward $\alpha_1 = +15^\circ$ (band 7). This seems to agree with the theory because band 1 with its negative rake angle causes the shear angle to be smaller than the one which develops from band 7. Thus the length of the shear plane is greater for band 1 than band 7 and requires a greater percentage of the total cutting energy to produce shear.

(d) The friction specific energy plotted varies between three
Figure 13. Force Components in Frozen Meat Cutting.
Figure 14. Input and Output Energies in Frozen Meat Cutting.
and nine ounce inches per square inch. This does not account for the lateral friction from the kerf pressure, which is probably several times larger. Unaccounted for specific energy is from about 30 to 90 ounce inches per square inch. No doubt most of this energy goes into the various indeterminate processes that take place in cutting, and part of it may be attributed to experimental and theoretical errors. Incidentally, specific energy of momentum is about 0.15 to 3 ounce inches per square inch respectively, for the lower and higher velocities.

**Shear Plane Angle ($\psi$)**

Shear plane angles have been computed as a function of the rake angle $\alpha_1$ and the coefficient of friction, based on equation 17. This same equation has been accepted as satisfactory for both bone and frozen meats cutting, according to the assumptions and derivations made. Figure 15 shows a plot of equation 17 applied to both bone and frozen meats. Plotted on the same figure are bone cutting data. Remarks on the figure are:

(a) Generally it isn't difficult to see the vast difference between the theory and experimental data. This difference is greatest when $\alpha_1$ is the smallest negative value tested ($-15^\circ$), and it decreases as $\alpha_1$ rises in a positive direction. As a matter of fact, the best accord exists when $\alpha_1 = +15^\circ$ for the range tested.

(b) There is a marked difference in plotted data, with a change in velocity ratio. The greatest difference in $\psi$ for any one band and the four velocity ratios is about $9^\circ$. This may serve as an indication that there is a dependence of $\psi$ on $\frac{v_c}{v_B}$.

(c) There is no nicely behaved pattern of plotted data.
Figure 15. Shear Plane Angles (Bone).
There appears to be a tendency for the general slope of the experimental data to rotate counter-clockwise as \( \alpha \) increases and also when \( \frac{\nu_f}{\nu_B} \) rises. The significance here is that the lower the velocity ratio, the smaller is the shear angle for bands with negative rake angle. The reverse holds for bands with positive rake angle.

Since chip size is a relatively important part of this work, plots exhibiting them have been made and may be seen in Figures 53 and 54. Great care was taken in securing this data, with the hope of lessening error. Both curves show what the author has assumed as an average chip segment size for bone and for meat. There is a tendency for the chip segment to be more or less constant in size for the different velocity ratios in bone cutting. Although this statement cannot be reiterated for frozen meat cutting, the author has taken the liberty of assuming the "average value" as constant. This was done in order to have a value for \( b \) to use in determining \( \tau_s \) for bone and frozen meats (see next section).

The variability in chip sizes and the poor agreement experienced here between theory and practice may be readily charged to the following factors:

1. Band teeth waviness -- Though the bands tested had been ground and/or filed a sufficient number of times to secure desired tooth configurations, there always remained a tooth waviness of approximately 0.002 inch \( \pm 0.001 \) inch. This means that some teeth had a greater penetration than others. Some shear angle measurements have had a difference of as much as 4° between adjacent planes cut by successive teeth in a given band.

2. Tooth sharpness radii -- Though there were no facilities for
measuring this parameter, microscopic observations with fine gridwork reticles helped to establish that the radii were of the order of 0.001 inch and less. This tends to make many teeth points relatively dull at very high band speeds.

(3) Speculative nature of assumptions — All assumptions made have a dubitable nature about them — some more than others.

Mechanism of Bone Cutting

The first paragraph written in the previous section on mechanism of frozen meat cutting, may be replicated here with no discernible discrepancy. Thus the bone cutting process is also of a chip forming nature by means of shear along "shear planes." All the arguments presented in the last section with regard to frozen meat cutting generally apply to bone cutting, including the trends in forces and specific energies, with exception of the magnitudes quoted. As such, the only exposition made in this section is that which is immediately concerned with magnitudes. A brief comparison of Figure 16 with Figure 13, Figure 51 with Figure 52, and Figure 17 with Figure 14 will reveal the remarkable likeness in trends and relative proportions between bone and frozen meat data.

Cutting Forces

Bone cutting forces were measured with the dynamometer for feed and band velocity ranges of 1 to 5 fpm and 800 to 9600 fpm, respectively. See previous section for discussion. (Refer to Figure 8). In general, bone cutting forces (both $F_h$ and $F_v$) are about double those for general frozen meat cutting at corresponding cutting conditions.

Specific Energies

Refer to the previous section on specific energies, for discussion.
Figure 16. Force Components in Bone Cutting.
Figure 17. Input and Output Energies in Bone Cutting.
As a whole, the specific energies involved in bone cutting are 22 to 25 times as great as for frozen meat cutting. The energy of momentum is in the order of \( \frac{1}{6} \) percent of the shear specific energy. Comparatively however, the momentum specific energy in bone cutting is as high as 20 percent of the shear specific energy in frozen meats, and as much as 30 percent of the shear specific energy in some soft meat cutting processes.

Static and Dynamic Properties of Soft Meat, Frozen Meat, and Bone

Table 2 shows that the static strength properties are in each case lower than their respective dynamic strengths by as much as 24 percent. There is an urge to ascribe this rise in strength, to a size effect similar to that which operates in metallic materials. The author has not found information in this regard for the media tested. It is more likely that the large difference may be attributed to a somewhat more complex character of the cutting processes than assumed.

In the two media where chips are generated, the chip segment thicknesses \( b \) were used to establish the values of the properties. It was seen in a prior section that there was a great deal of variability in this dimension for replicate cutting conditions and that mean values for the thicknesses were chosen, nevertheless. It so happens that these mean values may be used or discarded as negligible in computations for determining \( g \) or \( r_\alpha \). It introduces an error not greater than 6 or 7 percent. Equation 15 shows \( b \) to be merely additive to a much larger term.

Figures 63, 64, and 65 plot these properties for the whole range of band velocities. The gradual rise in these properties with velocity is evident on these plots. An important point that shows up for the three
Table 2. Static and Dynamic Strength Properties

<table>
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<th>Medium</th>
<th>( v_c )</th>
<th>( v_B )</th>
<th>( p )</th>
<th>( \tau_s )</th>
<th>( \mu )</th>
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<td>1</td>
<td>9600</td>
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</table>

SM - Soft Meats
FM - Frozen Meats
BN - Bone

\( \nu_f = \) feed velocity (fpm)
\( \nu_B = \) band velocity (fpm)

\( p = \) shear line pressure (ounces/inch)
\( \tau_s = \) shear strength (psi)
\( \mu = \) coeff. of friction between medium and steel band.
media is that there seems to be an unexplained scatter of values at the high velocities. Perhaps waviness and shallow penetrations of high speed accentuate the significance of the tooth sharpness radii thereby complicating the cutting process.

At high band speeds, meat juice was trained at the band-meat faying surfaces to achieve stable readings. Thin frozen meat specimens were surrounded by slabs of dry ice to retard incipient melting tendencies.

Tooth Sharpness Life

It has been stated before that the testing techniques evaluated here are based on wear flank of teeth, and on measurement of increased forces with wear.

Taylor's Equation

This relationship is used for bone cutting only, as explained in Chapter III. Experimental data were plotted in Figure 18. The constants \( n \) and \( c \) for the equation \( VT^n = c \), are listed at the bottom left hand corner of the same Figure. It must be remembered that these values were based on a wear flank of 0.0165 inch for bone. At that time the band had cut 1200 square inches of bone, and according to Figure 19 the feed forces \( (N_3) \) had reached a high value and appear to begin to rise sharply (instead of remaining linear as they were at first).

The band with \( \alpha_1 = -5^\circ \) is more sensitive to extraneous disturbances than the one with a positive rake angle \( (\alpha_1 = +5^\circ) \), as it has a flatter slope. However, the difference in sensitivity is not great among the three bands tested. This shows up in Figure 66 also, where slope differences are negligible at the three velocity ratios tested on the dynamometer. Interestingly enough the same figure says that this
Figure 18. Sharpness Life of Band Saw Teeth in Bone Cutting.
Figure 19. Effect of Cutting Medium on Tooth Wear.
(Field Tests Data).

NOTE: The soft meat tests include 5% of bone.
badly worn band \((w = 0.015\text{"})\) will cut the bone at the low feed force of seven ounces if the velocity ratio is reduced to \(\frac{1}{2400}\). The same band would be resisted by an increase of 35 ounces at the high velocity ratio of \(\frac{5}{2400}\). In terms of energy this argument infers that the operator will succeed in cutting at the higher speed ratios and keep up production, but he would fatigue probably five times sooner than if he cuts at the lower speed ratios (see Figure 67). Another observation in the same figure is that the band ratio of energies which is approximately four to one when the band is sharp (zero wear flank), remains approximately four to one during the entire wear range of high to low velocity ratios.

Cutting Bone, Frozen Meats, and Soft Meats

The three media were cut at fixed velocity ratios (see Figure 19), and the band life was defined for each as follows: \((\alpha_1 = +5^\circ)\)

(a) Bone -- at 0.0165 inch wear flank, the band had cut 1200 square inches of bone and the feed forces were high and beginning to rise almost "exponentially."

(b) Frozen Meats -- at 0.0080 inch wear flank, the band had cut 30,000 square inches of surface and the feed forces had almost quadrupled (as the high value), thereby increasing the fatigue rate to undesirable proportions.

(c) Soft Meats -- at 0.0076 inch wear flank, the band had cut 18,000 square inches of soft meats and its straightness cutting ability had reached its prescribed limit \(\frac{1}{8}\text{"}\) inch lateral deviation on a six inch long cut, at \(v_f = 50\text{ fpm}\) and \(2500 \leq v_B \leq 3500\text{ fpm}\). The test data was obtained in a meat market under fair control of the various parameters. Measurement of the areas cut may be in error as much as 10 percent, and
the feed velocities varied as much as 30 percent. The feed velocities are nominal values ±15 percent. All other measurements are within 5 percent of error.

Highlights:

(a) Band life is greater for frozen meats than for soft meat having approximately 5 percent bone. The significance of this is that a relatively small amount of bone within the soft meat will greatly shorten the life of the band. If soft meat were cut with no bone in it, the band would be expected to outlast the frozen meat cutting band.

(b) The band with largest positive rake angle (+5°) wears out the flank more rapidly than the one with smallest rake angle (-5°). The reason is that the larger the positive rake angles the weaker the teeth and the greater the wear rate. On the other hand the teeth with the larger positive rake angle can stand two to three times the amount of wear land and still cut straight within the limiting velocities involved here. The probable reason for this is that a large wear land on a positive rake angle tooth aids in shearing the meat in a direction normal to the wear land. Eventually the large wear land builds up sufficient resistance to feed, that it tends to deviate from straight cutting while seeking for a more stable state of cutting. The feed force picture (Figure 20) shows that these forces do not rise much. However, the relatively small increase is apparently sufficient to cause crooked cutting. The operator specific energy is thus expected to increase only slightly. (See Figure 67).

(c) In frozen meat cutting, again the feed forces must double or triple themselves only (see Figure 21) before they cause rapid operator
Figure 20. The Effect of Flank Wear on Feed Forces. (Soft Meat).
Figure 21. The Effect of Flank Wear on Feed Forces. (Frozen Meat Cutting).
fatigue. The same discussions just presented for soft meat cutting are applicable for frozen meats.

**Smear Formation**

Perhaps the more realistic way of measuring smear on soft meat is by means of equation. This is so because it is a reasonably true and indirect measure of the visual impression caused when a meat surface is examined for smear. Case 1, 2, and 3 bands show their results plotted in Figures 22, 23, and 24, respectively. Of the 14 bands represented by these three figures, only one band indicated an increase in smear formation with a rise in $v_p/v_B$ (Figure 23). All other bands resulted in a decreased smear formation with a velocity ratio rise. This behavior may be attributed to the three factors which follow:

1. The higher band speeds tend to cause the meat tensile forces to become more prominent than they are at lower speeds, therefore shearing effects are decreased.

2. The greater the band speed the more shallow the cut. This cutting condition approaches a scraping action by the primary cutting edges, resulting in a rise in smear formation.

3. A rise in band speed increases the incidence of contact between the meat surface and band teeth. This may be partly the cause for the added smear formation.

Whatever effect tooth configuration has on smear formation, it is not altogether evident. A good example of this is a band with positive rake angle (bands number 6, 12, and 21). At a high band velocity, case 1 band is next to the highest smear producer (Figure 22); in case 2 it is the highest smear producer (Figure 23); and in case 3 it is next to the
Figure 22. Smear Development in Cutting Soft Meats. (Case 1).
Figure 23. Smear Development in Cutting Soft Meats. (Case 2).
Figure 24. Smear Development in Cutting Soft Meats. (Case 3).
lowest smear producer. On the other hand, the same band appears to follow a consistent pattern at low band speeds, inasmuch as it is virtually next to the lowest smear producer in all three cases. If the same comparison is made of the other bands, it is found that only bands are concerned, no correlation between rake angles and speed ratios is readily obtainable from the data.

An important observation is that case 2 bands produced the least amount of smear and case 1 bands produced the greatest amount of smear at all velocity ratios. The governing factors here are not well understood, although it appears that the amount of tooth-meat contact is important. Case 3 bands have the least amount of cutting edges to engage the meats and case 1 bands have the greatest amount of cutting edges in action, for any given velocity ratio.

Smear development was measured according to equation 19 in order to compare with the foregoing results. Figures 57, 58, and 59 plot these results. These figures indicate no pattern with regard to rake angles $\alpha_1, \alpha_2,$ and $\alpha_3$. The one important similarity between the two methods of measuring smear is that they both show that case 2 bands are the least smear producers, while case 1 bands produce most smear for corresponding velocities.

Similar conclusions may be drawn for smeared frozen meat surfaces. (See Figures 60, 61, and 62)

Waste Development

Figures 25, 26, and 27 summarize the entire waste development picture for the bandsaw cutting of soft meats with case 1, 2, and 3 bands.
Figure 25. Meat Wastes During Process of Cutting Soft Meats. (Case 1).
Figure 26. Meat Wastes During Process of Cutting Soft Meats. (Case 2).
Figure 27. Meat Wastes During Process of Cutting Soft Meats. (Case 3).
A study of these figures results in the following three salient points of concern:

(1) Generally there seems to be no overall effect of the speed ratio on waste formation. The several cases where there are apparent increases or decreases in waste formation, cannot be readily associated with each other. The lack of consistency makes it impossible to separate the intimately related factors which may be operative here.

(2) Little correlation is possible between the rake angles and the velocity ratios for each particular bandsaw case. The only somewhat predominating situation is that bands 13, 27, and 22, 29 in cases 2 and 3, respectively, generally tend to be low waste producers. There is no clear-cut explanation for this behavior, other than the fact that increased rake angles make the cutting edges more knife-like.

(3) Case 2 bands produce the least waste and case 1 bands produce the most waste. Since this aspect is in reasonable consonance with smear formation, the arguments already presented on smear, appear adequate for waste development.

(4) Figure 28 shows that the previous statement is also true for frozen meat cutting. The basic difference lies in relative magnitudes of waste produced. The figure also shows the close agreement between theoretical and experimental values of developed wastes. The bands with no side rake angle gave waste values closer to theory than the case 2 and 3 bands.
Figure 28. Meat Wastes During Process of Cutting Frozen Meats. (Cases 1, 2, and 3).
CHAPTER VI

CONCLUSIONS

Soft Meat Cutting Mechanisms

(1) Shear is the basic mechanism whereby soft meats get cut at band velocities between 1400 fpm and 9600 fpm, and feed velocities not in excess of 100 fpm. Outside of these ranges and close to the upper and lower bounds, a combination of tension and shearing occurs.

(2) At low velocity ratios such as \( \frac{V_f}{V_B} = \frac{50}{9600} \) and lower, there is virtually no force resistance to the cutting process, by the band nor by the operator. This holds true for case 2 and 3 bands, i.e., bands having side rake angles.

(3) A rise of rake angles from a negative value (-15°) to a positive values (+15°) causes horizontal and vertical components of force to decrease.

(4) A band with a positive rake angle (+15°) requires an operator feed force 1/10 to 1/12 as great as the band force \( F_v \). The ratio is approximately one to two, for a negative rake band (-15°). This is true for bands with no side rake angle and velocity ratios of \( \frac{50}{1200} \) to \( \frac{50}{9600} \).

(5) At high velocity ratios of \( \frac{50}{2400} \) and greater, both vertical and horizontal force components for case 2 and 3 bands (having side rake angle), approach the corresponding force components of case 1 bands (no side rake angle). It seems therefore, that at the medium to low
feed velocities only, bands with side rake angles are significantly more efficient than those without a side rake angle.

(6) Bands with side rake angle are markedly sensitive (forcewise) to velocity ratio changes.

(7) At low band velocities (<1200 fpm) tooth penetration is about the same for bands with and without side rake angles (from -15° to +5°). Therefore, specific energy consumed by the vertical band force is also similar in magnitude for these same bands.

(8) Figures indicate that about 86 percent of the total input specific energy is consumed by the shear process and that approximately 10 percent is dissipated in friction.

(9) Case 1 bands (no side rake angle) use up about 700 percent more specific energy than cases 2 and 3 bands (side rake angle), at high band speeds. At low band speeds the percentage drops to 200.

(10) Specific energy required of the operator varies from a small fraction of 1 percent to a maximum of 2 percent of the total energy input.

**Frozen Meat Cutting Mechanism**

(1) Close observations during slow, medium, and rapid velocity conditions of the generated chip segments and surfaces cut, lead the author to believe that the process is primarily one of shear along the shear plane and along the kerf surfaces, rather than one of extrusion or tension. Also theory based on this premise seems to agree sufficiently well with data and apparently with sound practical considerations.

(2) Both horizontal and vertical components of force rise rapidly with an increase in velocity ratio, the forces always being larger for
bands with the largest negative rake angle (-15°) and smaller for bands with largest positive rake angle (+5°).

(3) Minor penetration of bands at low velocity ratios causes bands (all rake angles) to register \( F_H \) and \( F_V \) forces of the order of \( 10 \) percent or less, of what they would be at the high velocity ratios.

(4) The vertical forces are approximately twice as large as the feed forces in band 1 (\( \alpha_1 = -15° \)), but the proportion changes to five in band 7 (\( \alpha_1 = +15° \)). Thus an increase in \( \alpha_1 \) in the positive direction diminishes the feed force requirements from both the operator and the band.

(5) Both the input and shear specific energies decline steadily to approximately 60 percent their initial values if the initial values are for bands of negative rake angle (-15°), as the rake angle changes in the positive direction until it is +15°.

(6) Changes in velocity ratios do not change the specific energies of input and/or shear materially, for all bands (negative 15 to positive 15° rake angles).

(7) Approximately 85 percent of the total input energy appears ascribable to the shear process.

**Shear Plane Angles**

(1) The relation \( 2p = 90 + \alpha_1 - \gamma \), seems to be reasonably characteristic of the shear plane angle for both frozen meats and bone. The experimental verification for bone, is fair. No experimental validation was possible for frozen meats. The greatest degree of agreement was encountered when the rake angle equals zero to \( \alpha_1 = +15° \).
(2) Interdependence between $\varphi$ and the velocity ratios appears to be extant, since the data did show signs of variation with changes in velocity ratio.

(3) Laboratory data were not nicely behaved.

(4) The plot signifies that a decrease in velocity ratio results in a lessening of the shear angle.

Mechanism of Bone Cutting

(1) All remarks made in the prior section concerning the frozen meat mechanism of cutting hold reasonably true for bone cutting, from a qualitative point of view.

(2) In general the forces ($F_H$ and $F_V$) are about twice those involved in frozen meat cutting, at corresponding cutting conditions.

(3) The specific energies of shear and/or input are generally 22 to 25 times as great as for frozen meat cutting.

(4) Though the momentum specific energy is about $\frac{1}{2}$ percent of the shear specific energy in bone cutting, its magnitude is almost 20 percent and 30 percent of the shear specific energies involved in the cutting of frozen and of soft meats, respectively.

Coefficient of Friction

When coefficients of friction were established it was found that there was a tendency for the meat-metal interface to become dry quickly at higher band velocities. The remedy was to pour meat juices at the interfaces in action thereby causing $\mu$ to remain constant. The coefficients of friction between a steel band and the three media are:

$\mu_{\text{Bone}} = 0.103$, $\mu_{\text{soft meat}} = 0.314$, and $\mu_{\text{frozen meat}} = 0.131$. 
Tooth Sharpness Life

(1) In bone cutting it appears that 1200 square inches of cut surface may be an acceptable index for band life. At this time the wear flank magnitude is 0.0165 inch and the feed forces on the worn surface \( (N_3) \) are starting to rise exponentially or so.

(2) The band with \( \alpha_1 = -5^\circ \) has a Taylor exponent value of 3.8, compared to 2.51 and 2.01 for \( \alpha_1 = 0 \), and \( \alpha_1 = +5^\circ \), correspondingly. This implies that the \( \alpha_1 = -5^\circ \) band is more sensitive to miscellaneous effects and changes, than the others.

(3) In frozen meat cutting, an apparently reasonable wear criterion is 30,000 square inches of surface area cut. Wear flank should be approximately 0.0080 inch and operator feed forces are four times their initial (sharp band) value.

(4) Similarly, 18,000 square inches of meat surface area cut (with about 5 percent bone by cross-section surface cut) may be a satisfactory measure of limiting wear. Flank wear measured about 0.0075 inch and the band deviated \( \frac{1}{2} \) inch laterally along a 6 inch cut, at \( v_f = 50 \) fpm, and \( 2500 \leq v_B \leq 3500 \) fpm.

(5) The above data may have slightly over 10 percent error.

(6) Band life appears greater for frozen meats than for soft meats having at least 5 percent of bone by cross-sectional area cut. Bone tends to reduce the straightness cutting life of a band on soft meat.

(7) Bands with positive rake angle (+5°), flank wear more rapidly than the one with the -5° rake angle.
**Smear Formation**

1. Equation 18 is a better representation of smear formation than equation 19.

2. There does not seem to be very much difference in smear formation, when the rake angle ($\alpha_1$) is varied between plus and minus 10°.

3. A measurable smear effect is produced when the side rake angle ($\alpha_2$) is not zero. Apparently, smear formation is decreased as $\alpha_2$ rises in the positive direction. It is also lessened but to a smaller extent, as $\alpha_2$ rises in the negative direction.

4. The majority of the tests made indicate that smear formation increases with a drop of the velocity ratio.

5. The above statements seem to satisfy both soft meat and frozen meat cutting.

**Waste Development**

1. Within the ranges tested there is little evidence to show that a change in velocity ratio affects the amount of waste developed.

2. Bands with side rake angles (positive or negative) of about 10° cause less waste than those having no side rake angle.

3. A band with a positive side rake angle evidently generates less waste than one with a negative rake angle of equal magnitude.

4. The foregoing satisfies both soft meat and frozen meat cutting except that the effects are more pronounced in soft meat.
CHAPTER VII

RECOMMENDATIONS

The following list of problems for future programs is recommended:

(1) Determine the true interdependence between velocity ratios and the shear angle.

(2) Make a specific study of smear, accounting for cross-effects of the variables examined here as well as others.

(3) Make a specific study of each of the fields of waste, wear, smear, and cutting mechanisms in the same manner suggested in item two above.

(4) Examine the problems studied here but for ranges outside those already analyzed.

(5) Study the effects of meat surface damage by different cutting means, with the view in mind of establishing some criteria regarding rate of healing and subsequent pain responses in living matter.

(6) Make a theoretical study of the intersection of two or more shear planes, as the case is when cutting bone in a way other than orthogonally.

(7) If economically feasible, run $VT^N = C$ tests or other wear tests with frozen and soft meats.

(8) Determine the effect of quasi-permanent burrs (formed during grinding) on the mechanism of cutting meats and bone.

(9) Repeat tests in this dissertation with highly accurately manufactured bands having very small selected and constant tip radii on the leading edge of each tooth.
(10) Examine ploughing forces at ultra-high band speeds (9000 fpm) for degraded cutting tips of selected configuration.
APPENDIX A

TABLE OF BAND DETAILS
Table 3. Details of Bands Tested

<table>
<thead>
<tr>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>BN</td>
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<td>Case 3</td>
</tr>
<tr>
<td>α₁</td>
<td>α₂</td>
<td>α₃</td>
</tr>
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<td>-15</td>
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<td>-10</td>
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<td>7</td>
<td>+15</td>
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<td>26</td>
</tr>
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<td></td>
<td></td>
<td>27</td>
</tr>
</tbody>
</table>

General remarks:

a. Waviness average value is 0.002 inch

b. Band thickness is 0.022 inch

c. Set on all bands is 0.006 inch ± 0.002 inch

d. BN is "Band Number"
APPENDIX B

GEOMETRY FOR LENGTHS OF CUTTING EDGES
NOTES:
1. \( t_1 \) is depth of cut
2. This diagram covers all three cases of tooth configuration
3. \( t \) is band thickness

Figure 29. Geometry of Tooth Shear Edges.
NOTES:
\( t_1 = \text{Shallow depth of cut} \)
\( t_2 = \text{Heavy depth of cut} \)

Figure 30. Geometry of Shear Edges.
APPENDIX C

DEVELOPMENT OF GEOMETRICAL RELATIONS

\[ e'c' = e'b' + b'c' \]  \hspace{1cm} (C-1)

\[ e'b' = a'b' \sin \theta_1 \]  \hspace{1cm} (C-2)

\[ a'd' = \frac{t}{\sin(90 - \theta_1)} = b'c' = \frac{t}{\cos \theta_1} \]  \hspace{1cm} (C-3)

\[ e'a' = (e'c') \tan \alpha_2 \]  \hspace{1cm} (C-4)

According to the sine law:

\[ \frac{b'a'}{\sin \alpha_2} = \frac{c'b'}{\sin(90 - (\theta_1 + \alpha_2))} = \frac{t}{\cos \theta_1 \sin(90 - (\theta_1 + \alpha_2))} \]  \hspace{1cm} (C-5)

\[ e'b' = b'a') \sin \theta_1 = \frac{t \sin \alpha_2 \sin \theta_1}{\cos \theta_1 \sin(90 - (\theta_1 + \alpha_2))} \]  \hspace{1cm} (C-6)

But from Eq. (C-1) \( e'c' = \text{Eq. 4 + Eq. 3} \), or

\[ e'c' = \frac{t \sin \alpha_1 \tan \theta_1}{\sin(90 - (\theta_1 + \alpha_2))} + \frac{t}{\cos \theta_1} \]  \hspace{1cm} (C-7)

and from Eq. (C-4)

\[ e'a' = \tan \alpha_2 \left[ \frac{\sin \alpha_2 \tan \theta_1}{\cos (\theta_1 + \alpha_2)} + \frac{1}{\cos \theta_1} \right] \]  \hspace{1cm} (C-8)
\[
e^{'a'} = \tan \alpha_2 t \left[ \frac{\sin \alpha_2 \tan \theta_1 \cos \theta_1 + \cos \theta_1 \cos \alpha_2}{\cos (\theta_1 + \alpha_2) \cos \theta_1} \right]
\]

\[
\frac{\sin \theta_1 \sin \alpha_2}{\cos \theta_1} = \frac{t \sin \alpha_2}{\cos (\theta_1 + \alpha_2)}
\]

\[
c^{'a'} = \frac{e^{'a'}}{\sin \alpha_2} = \frac{t}{\cos (\theta_1 + \alpha_2)}
\]

**(C-9)**

**Note:** Eq. (C-8) used the following relationship:

\[
\sin \{90 - (\theta_1 + \alpha)\} = \cos (\theta_1 + \alpha)
\]

\[
a^{'d'} = b^{'c'} = \frac{t}{\cos \theta_1}
\]

**(C-10)**

**Case 3** \((\alpha_3 = (-) \alpha_2\))

\[
b^{'d'} = \frac{t}{\cos \gamma_2} = \frac{t}{\cos (\alpha_3 - \theta_2)}
\]

**(C-11)**

since

\[
\beta = 90 - \theta_2 - \alpha_2
\]

\[
\gamma_1 = 90 + \theta_2 - \alpha_3
\]

\[
\gamma_2 = 90 - \gamma_1 = \alpha_3 - \theta_2
\]

\[
\gamma_1 + \gamma_2 = (90 + \theta_2 - \alpha_3) + \alpha_3 - \theta_2 = 90^\circ
\]

\[
d^{'g'} = b^{'d'} \cos \alpha_3 = \frac{t \cos \alpha_3}{\cos (\alpha_3 - \theta_2)} = \frac{t}{\cos \theta_1 + \tan \alpha_3 \sin \theta_2}
\]

**(C-12)**
Case 1. \((\alpha_2 = 0 \text{ and } \alpha_3 = 0)\) From Case 2 Eq. (C-10) is

\[ a'd' = b'c' = \frac{t}{\cos \theta_2} \]

Length of Shear Edges on Shallow Cut \((t_1)\)

Case 1. \((\alpha_2 = \alpha_3 = 0)\)

(a) primary shear edge, \(a'd' = \frac{t}{\cos \theta_2}\) \hspace{1cm} (C-10)

(b) secondary shear edges,

\[ d'j_1 = \sqrt{(md')^2 + (j_1m)^2} \quad \text{md'} = kk' = t_1 \tan \beta_2 \]

\[ j_1m = \frac{t_1}{\cos \alpha_1} = \sqrt{\left(\frac{t_1 \tan \beta_2}{\cos \alpha_1}\right)^2 + \left(\frac{t_1}{\cos \alpha_1}\right)^2} \]

\[ d'j_1 = \frac{t_1}{\cos \alpha_1} \sqrt{\cos \alpha_1 \tan \beta_2}^2 + 1 \] \hspace{1cm} (C-13)

Case 2. \((\alpha_2 \neq 0)\)

(a) Primary shear edge

\[ a'h = \sqrt{(h'a')^2 + t_1^2} \quad h'a' = \frac{h'g'}{\sin \alpha_2} = \frac{1}{\tan \theta_1} \frac{t_1}{\sin \alpha_2} \]

\[ a'h = \frac{t_1}{\sin \alpha_2 \tan \theta_1} \sqrt{1 + \sin^2 \theta_2 \tan^2 \theta_1} \] \hspace{1cm} (C-14)

(b) Secondary shear edge

\[ d'j_1 = \frac{t}{\cos \alpha_1 \cos \beta_2} \sqrt{1 + \sin^2 \alpha_2 \tan^2 \theta_1} \] \hspace{1cm} (C-15)

Case 3. \((\alpha_3 = -\alpha_2)\)
(a) Primary shear edge

\[ nd' = \sqrt{t_1^2 + (h'd')^2} = \frac{t_1}{\sin \alpha_3 \tan \theta_1} \sqrt{1 + \sin^2 \alpha_3 \tan^2 \theta_1} \quad (C-16) \]

where

\[ h'd' = \frac{h'g'}{\sin \alpha_3} = \frac{1}{\sin \alpha_3} \frac{t_1}{\tan \theta_1} \]

(b) Secondary shear edge

\[ a'j_1 = \frac{t_1 \sqrt{1 + \sin^2 \alpha_3 \tan^2 \theta_1}}{\cos \alpha_1 \cos \beta_2} \quad (C-17) \]

t_1 is computed from tooth geometry and kinematics (see Figure 40), as follows:

\[ d = cd \sin \theta = P \sin \theta = t_1 \cos (\alpha_1 + \theta) \]

and finally

\[ t_1 = \frac{P \sin \theta}{\cos (\alpha_1 + \theta)} \]
APPENDIX D

MISCELLANEOUS PHOTOGRAPHS OF MEAT AND BONE SURFACES
Figure 31. Two Bone Chip Segments (Center).

Figure 32. Bone and Frozen Meat Chip Segments.

Figure 33. Bone Shear Plane Angles.
Commentary on Figures 31 through 39

Figure 31
The picture shows two bone chip segments in the center. The lighter segments surrounding it are from frozen meats (in a thawed out state). Note six or seven "blobs" of fat. (seven magnification, X7)

Figure 32
Here is an agglomeration of frozen meat and bone chips. X7

Figure 33
Orthogonal Cutting is involved. The difference in elevation is caused by tooth waviness. X7
Figure 34. Meat Surface Sliced with Butcher Knife.

Figure 35. Stretched-Out Slab of Meat.

Figure 36. Meat Surface (Bandsaw Cut).

Figure 37. Meat Surface (Bandsaw Cut at 9600 fpm).
Figure 34
Surface of a piece of meat which has been sliced with a sharp butcher knife.

Figure 35
Here is the surface of a piece of meat which is under tension in all directions of the plane of the picture. Note the fatty tissue stretching out between islands of lean meat. This tissue holds the meat together. X7

Figure 36
Surface of a piece of meat which has been cut with a sharp bandsaw. Note that the left hand surface area has been wiped clean with a paper clip. Smear waste is seen piled up on one side of the clip. X7

Figure 37
Surface of a piece of meat which has been sliced with a sharp bandsaw, but at very high band speeds (9600 fpm). X7
Figure 38. Smearred Surface of Frozen Meat (Bandsaw Cut).

Figure 39. Soft Meat Smearred Surface (Bandsaw Cut).
Figure 38

Smeared surface of a frozen piece of meat which has been cut with a bandsaw.

Figure 39

Smeared surface of a piece of meat cut with a bandsaw. (smear is the cloudy and lumpy regions).
APPENDIX E

LINE LOAD IN SOFT MEAT SHEARING

The line load which develops along ca (secondary shear edge) is defined as: (See Figure 40)

\[ p_1 = \frac{N_1}{s_2} = \frac{N_1 \cos (\alpha_1 + \theta)}{p \sin \theta} \]  \hspace{1cm} (E-1)

The line load along the primary shear edge \((s_1)\) is determined in the same direction \(N_1\), i.e.,

\[ p_2 = \frac{N_1}{s_1} = \frac{N_1 \sin \theta_1}{p \tan \theta \cos \alpha_2} \]  \hspace{1cm} (E-2)

since \(s_1 = \frac{t_1 \cos \alpha_2}{\sin \theta_1}\) and \(t_1 = \frac{d}{\cos \theta} = P \tan \theta\)  \hspace{1cm} (E-3)

However, under the condition of simultaneous shearing along \(s_1\) and \(s_2\), the average line load becomes:

\[ p = \frac{N_1}{P \left[ \frac{\tan \theta \cos \alpha_2}{\sin \theta_1} + \frac{\sin \theta}{\cos (\alpha_1 + \theta)} \right]} \]  \hspace{1cm} (12)

The above equation is acceptable for cases 2 and 3. It is readily adapted to case 1 by taking \(s_1 = \frac{t}{\cos \theta}\).
Figure 40. Tooth Cutting Edges and Line of Action.

where:

\[
\overline{ac} = s_2
\]

\[
P = \overline{cf}
\]
APPENDIX F

SEGMENTAL CHIP FORMATION DURING ORTHOGONAL CUTTING OF BONE OR FROZEN MEAT
Figure 41. Bone or Frozen Meat Chip Segment.

Figure 42. Force Circle Number 1, for Chip Segment.
Figure 43. Force Circle Number 2, for Tooth-Chip Interface and Tooth-Medium Interface.
APPENDIX G

CHIP MATHEMATICAL ANALYSIS

From diagram No. 1 (Figure 42)

Horizontal Components:

\[ + N'_1 \cos \alpha + f'_1 \sin \alpha = R_1 \cos (\alpha - \gamma_1) \]  \hspace{1cm} (G-1)

but \[ R_c = - R_1 \]

\[ N_c \sin \phi + S_c \cos \phi = R_c \cos (\alpha - \gamma_1) \]  \hspace{1cm} (G-2)

Divide (G-1) into (G-2) and get:

\[ - N'_1 \cos \alpha - f'_1 \sin \alpha = N_c \sin \phi + S_c \cos \phi \]  \hspace{1cm} (G-3)

Vertical Components:

\[ + N'_1 \sin \alpha - f'_1 \cos \alpha = R_1 \sin (\alpha - \gamma_1) \]  \hspace{1cm} (G-4)

\[ + N_c \cos \phi - S_c \sin \phi = - R_c \sin (\alpha - \gamma_1) \]  \hspace{1cm} (G-5)

Divide (G-4) into (G-5) and get:

\[ N'_1 \sin \alpha - f'_1 \cos \alpha = N_c \cos \phi - S_c \sin \phi \]  \hspace{1cm} (G-6)

\[ S_c = R_c \cos (\phi - \alpha + \gamma_1) \]  \hspace{1cm} (G-7)

\[ N_c = R_c \sin (\phi - \alpha + \gamma_1) \]  \hspace{1cm} (G-8)
also \[ \frac{N_c}{S_c} = \tan (\varphi - \alpha + \gamma_1) \] (G-9)

Now to determine \( F_v \) and \( F_H \) as functions of \( \theta \), \( \alpha \), and \( \gamma \): (Refer to circle diagram No. 2).

\[ \sum F_v = -N_2 \sin \theta_1 + f_2 \cos \theta_1 + N_1 \cos \alpha + f_1 \sin \alpha - F_v = 0 \] (G-10)

\[ \sum F_H = +N_2 \cos \theta_1 + f_2 \sin \theta_1 - N_1 \sin \alpha + f_1 \cos \alpha - F_H = 0 \] (G-11)

By the use of Equations (G-1) - (G-6) convert Equations (G-10) and (G-11) into the following form: (Note: \( N'_1 = -N'_1 \)
\[ f'_1 = -f_1 \]

\[ F_v = -N_2 \sin \theta_1 + f_2 \cos \theta_1 - R_1 \cos (\alpha_1 - \gamma_1) \] (G-12)

and

\[ F_H = N_2 \cos \theta_1 + f_2 \sin \theta_1 + R_1 \sin (\alpha_1 - \gamma_1) \] (G-13)

but because
\[ R_c = \frac{S_c}{\cos (\varphi - \alpha_1 + \gamma_1)} = -R_1 \] (G-14)

Equations (G-12) and (G-13) become:

\[ F_v = -N_2 \sin \theta_1 + f_2 \cos \theta_1 + \frac{S_c \cos (\alpha_1 - \gamma_1)}{\cos (\varphi - \alpha_1 + \gamma_1)} \] (13)

\[ F_H = +N_2 \cos \theta_1 + f_2 \sin \theta_1 - \frac{S_c \sin (\alpha_1 - \gamma_1)}{\cos (\varphi - \alpha_1 + \gamma_1)} \] (14)

also,
\[ R_1 = \frac{N_1}{\cos \gamma_1}, \] (G-15)
thus
\[ S_c = \frac{N_1 \cos (\phi - \alpha_1 + \gamma_1)}{\cos \gamma_1} \]  
\hspace{1cm} (G-16)

Integrating the dynamic stresses into Eq. 14 by means of Eq. 15, there results,
\[ F_H = N_2 \cos \theta_1 + f_2 \sin \theta_1 \]  
\hspace{1cm} (16)
\[ - \frac{\tau_s \rho \sin \theta (2b + W_c) \sin (\alpha_1 - \gamma_1)}{\sin \phi \cos (\phi - \alpha_1 + \gamma_1)} \]

Principle of Minimum Energy:

A minimum of external energy is required to shear the chip along the shear and kerf planes, simultaneously, where maximum shear stresses are induced. The shear plane assumes an orientation consistent with this principle.

The principle is now applied to Eq. 16, and the conditions stated are satisfied when
\[ \frac{d F_H}{d \phi} = 0 \]  
\hspace{1cm} (G-17)
\[ \frac{d F_H}{d \phi} = - \cos \phi \cos (\phi - \alpha_1 + \gamma_1) + \sin \phi \sin (\phi - \alpha_1 + \gamma_1) \]  
\[ \sin^2 \phi \cos^2 (\phi - \alpha_1 + \gamma_1) \]  
\hspace{1cm} (G-18)

changing signs by multiplying through by (-) gives:

\[ \cos \phi \cos (\phi - \alpha_1 + \gamma_1) - \sin \phi \sin (\phi - \alpha_1 + \gamma_1) = 0 \]

or
\[ \cos [\phi + (\phi - \alpha_1 + \gamma_1)] = 0 \]
and \[ 2\phi - \alpha_1 + \gamma_1 = 90^\circ \] (17)

This result is true, provided the parameters involved are constant with respect to the shear angle \( \phi \). The same result is realized by applying the principle of minimum energy to Equation 13.
APPENDIX H

DERIVATION OF DYNAMIC SHEAR STRESSES

Dynamic Stresses: (See Figure 40 in Appendix E)

These stresses are computed on the basis that the chip segment will shear along the shear plane and along the two kerf surfaces simultaneously. There are therefore three areas which shear instantly and at the same time, as expressed below.

\[ A = A_{sp} + 2A_{sk} \]  \hspace{1cm} (H-1)

\[ = \ell_c \omega_c + 2\ell_c b \]

\[ = \frac{dt}{\sin \phi \cos \theta_2} + \frac{2db}{\sin \phi} \]  \hspace{1cm} (H-2)

Now employing Eqs. (G-16) and (H-2), and the definition of pure shear, there results:

\[ \tau_s = \frac{S_c}{A} = \frac{N_1 \cos (\phi - \alpha_1 + \gamma_1) \sin \phi}{\cos \gamma_1 P \sin \theta (2b + \omega_c)} \]  \hspace{1cm} (15)
APPENDIX I

DETERMINATION OF SPECIFIC ENERGY OF MOMENTUM
ASSUMPTIONS: (a) Chip continuity
(b) Ignore small frictional effect at the interface between tooth land and medium
(c) The amount of material picked-up per unit time is the same as that lost. (nomass change)

Figure 44. Momentum Change in Bone Chips.
First, the force system resulting from the change of linear momentum is obtained, as follows:

\[ \sum F = \sum F_M + \sum F_c \]  \hspace{1cm} (I-1)

\[ \sum F_M = \frac{d}{dt} (M_M v_M) = v_M \frac{dM_M}{dt} \]  \hspace{1cm} (I-2)

and

\[ \sum F_c = \frac{d}{dt} (M_c v_c) = v_c \frac{dM_c}{dt} \]  \hspace{1cm} (I-3)

where \( V_M \) and \( V_c \) are taken as constant.

Now, since

\[ \frac{dM_M}{dt} = (-) \frac{dM_c}{dt} \]  \hspace{1cm} (I-4)

Eq. (I-1) becomes,

\[ \sum F = \frac{dM_c}{dt} (v_c \to v_M) \]  \hspace{1cm} (I-5)

or simply let

\[ \frac{dM_c}{dt} = \frac{dM}{dt} \]

Equation (I-5) is now expressed in terms of the familiar orthogonal axes components.

\[ \sum F_v = \frac{dM}{dt} \{ v_c \cos (90 - \alpha) - v_M \} \]  \hspace{1cm} (I-6)
and
\[ \sum F_H = \frac{dM}{dt} \left\{ v_c \sin (90 - \alpha_v) \right\} \] (I-7)

where:
\[ \frac{dM}{dt} = \rho \frac{A v_M}{g} = \rho \frac{d w_{v_c v_m}}{g} \] (I-8)

The total specific energy is the sum of the specific energies resulting from the two force components just derived.

Therefore,
\[ S.E. \) \_ \_ \_ M \_ \_ \_ = S.E. \) \_ \_ \_ H \_ \_ \_ + S.E. \) \_ \_ \_ V \_ \_ \_ \] (35)
\[ = F_H v_f + F_v v_b \]

This equation is satisfactory for frozen meat and bone cutting with the bandsaw. There is no simple way of determining a sufficiently accurate and useful relationship for specific energy of momentum for soft meat bandsawing.

The velocity relationships useful in the final form of the momentum change equation (I-5), are derived below:

\[ \rho = \alpha - \phi \]
where: \( V_c \) is the chip velocity.

\( V_{c/m} \) is the chip velocity with respect to the cutting medium.

\( V_m \) is the velocity of the cutting medium.

\( S = \alpha_1 - \phi < 90^\circ \)

\[
V_c = V_m + V_{c/m} \tag{I-9}
\]

\[
V_c \cos (\alpha_1 - \phi) = V_m \sin \phi \tag{I-10}
\]

\[
V_c \cos \alpha_1 = V_{c/m} \sin \phi \tag{I-11}
\]

and finally,

\[
V_m = V_{c/m} \frac{\cos (\phi - \alpha_1)}{\cos \alpha_1} \tag{I-12}
\]
APPENDIX J

EXPERIMENTAL DATA ON SOFT MEAT CUTTING
Figure 45. Force Components from Soft Meat Cutting. (Case 1).
Figure 46. Force Components from Soft Meat Cutting (Case 2).
Figure 47. Force Components from Soft Meat Cutting. (Case 3).

NOTES: $v_f = 5 \text{ fpm}$

$f_3 = f_4 = 0$
Figure 48. Energies Input and Output During Cutting of Soft Meats (Case 1).
Figure 49. Input and Output Energies in Soft Meat Cutting. (Case 2).
Figure 50. Input and Output Energies During Soft Meat Cutting. (Case 3).
APPENDIX K

EXPERIMENTAL DATA ON FROZEN MEAT AND BONE CUTTING
Figure 51. Force Components on Frozen Meat Cutting.

NOTES: \( v_f = 50 \text{ fpm} \)

\( f_3 = f_4 = 0 \)
Figure 52. Force Components in Bone Cutting.
APPENDIX L

CHIP SEGMENT SIZE AND RAKE ANGLE
Figure 53. The Effect of the Rake Angle on the Chip Segment Size.
(Bone Cutting).
Figure 54. The Effect of the Rake Angle on the Chip Segment Size.
(Frozen Meat Cutting).
APPENDIX M

OPERATOR SPECIFIC ENERGY
Figure 55. Specific Energy of the Operator in Soft Meat Cutting.
Figure 56. Specific Energy of the Operator in Cutting Frozen Meats.
APPENDIX N

SMEAR FORMATION
Figure 57. Smear Development in Soft Meat Cutting. (Case 1).
Figure 58. Smear Development in Soft Meat Cutting.
(Case 2).
Figure 59. Smear Development in Soft Meat Cutting. (Case 3).
Figure 60. Smear Development in Frozen Meats (Case 1).
Figure 61. Smear Development in Frozen Meats (Case 2).
Figure 62. Smear Development in Frozen Meats. (Case 3).
APPENDIX D

MEAT AND BONE CUTTING PROPERTIES
NOTE: $v_f = 50$ fpm

EACH POINT IS AN AVERAGE VALUE OF 2 TEST RUNS WITH EVERY BAND IN THE 3 GROUPS TESTED

- CASE 1
- CASE 2
- CASE 3

Figure 63. Line Pressure Developed in Shearing Soft Meat.
Figure 64. Average Shear Stress Developed in Cutting Frozen Meat with a Bandsaw.

NOTE: \( v_f = 15 \text{ fpm} \)
NOTE: $v_f = 2$ fpm

Figure 65. Shear Stress Developed in Cutting Bone with a Bandsaw.
APPENDIX P

WEAR FORCES AND OPERATOR SPECIFIC ENERGY DURING BONE CUTTING
Figure 66. The Effect of Flank Wear on Feed Forces. (Bone Cutting).
Figure 67. Specific Energy of the Operator in Bone Cutting.
APPENDIX Q

TABULATION OF BANDSAW CUTTING FORCES
Table 4. Cutting Forces on Bands

<table>
<thead>
<tr>
<th>BN</th>
<th>( \frac{v_f}{v_B} )</th>
<th>( F_v ) (ounces/inch)</th>
<th>( F_H ) (ounces)</th>
<th>( f_3 )</th>
<th>( f_4 )</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50/9600</td>
<td>8.54</td>
<td>5.42</td>
<td>0.31</td>
<td>0</td>
<td>SM</td>
</tr>
<tr>
<td>3</td>
<td>&quot;</td>
<td>9.30</td>
<td>3.84</td>
<td>0.32</td>
<td>0.12</td>
<td>SM</td>
</tr>
<tr>
<td>4</td>
<td>&quot;</td>
<td>9.50</td>
<td>2.99</td>
<td>0.45</td>
<td>0.14</td>
<td>SM</td>
</tr>
<tr>
<td>5</td>
<td>&quot;</td>
<td>9.85</td>
<td>2.17</td>
<td>0.61</td>
<td>0.13</td>
<td>SM</td>
</tr>
<tr>
<td>7</td>
<td>&quot;</td>
<td>10.30</td>
<td>0.44</td>
<td>0.31</td>
<td>0.02</td>
<td>SM</td>
</tr>
<tr>
<td>1</td>
<td>50/2400</td>
<td>10.61</td>
<td>4.20</td>
<td></td>
<td></td>
<td>FM</td>
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<tr>
<td>3</td>
<td>&quot;</td>
<td>9.90</td>
<td>2.83</td>
<td></td>
<td></td>
<td>FM</td>
</tr>
<tr>
<td>4</td>
<td>&quot;</td>
<td>8.95</td>
<td>2.07</td>
<td></td>
<td></td>
<td>&quot;</td>
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<tr>
<td>5</td>
<td>&quot;</td>
<td>8.41</td>
<td>1.43</td>
<td></td>
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</tr>
<tr>
<td>7</td>
<td>&quot;</td>
<td>7.45</td>
<td>0.725</td>
<td></td>
<td></td>
<td>&quot;</td>
</tr>
<tr>
<td>1</td>
<td>1/2400</td>
<td>6.71</td>
<td>4.56</td>
<td></td>
<td></td>
<td>Bone</td>
</tr>
<tr>
<td>3</td>
<td>&quot;</td>
<td>5.62</td>
<td>2.94</td>
<td></td>
<td></td>
<td>&quot;</td>
</tr>
<tr>
<td>4</td>
<td>&quot;</td>
<td>5.14</td>
<td>1.71</td>
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<td>&quot;</td>
</tr>
<tr>
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<td>&quot;</td>
<td>4.72</td>
<td>1.63</td>
<td></td>
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<td>&quot;</td>
</tr>
<tr>
<td>7</td>
<td>&quot;</td>
<td>3.96</td>
<td>0.77</td>
<td></td>
<td></td>
<td>&quot;</td>
</tr>
</tbody>
</table>

BN - band number  
SM - soft medium  
M - medium  
FM - frozen medium
APPENDIX R

DYNAMOMETER CONSTRUCTION AND SET-UP
NOTE: Numbers 1 through 8 denote SR-4 (A-7) strain gages on one octogonal ring.

Figure 68. Dynamometer Construction.
Figure 69. Dynamometer Set-Up with Bone Specimen Mounted.
APPENDIX S

MISCELLANEOUS DYNAMOMETER INFORMATION
NOTE: a - INITIAL CONTACT  
  b - FINAL CONTACT  
  v - 25 mm/sec (VELOCITY OF RECORDING PAPER)  
  δ - DEFLECTION OF RECORDER NEEDLE (cm)  
  K - RECORDER BASIC SENSITIVITY = 10 MICRO INCHES PER INCH  
  ε - DYNAMOMETER STRAIN IN MICRO INCHES PER INCH

\[
\frac{V_f}{V_B} = \frac{1}{600} \quad \frac{V_f}{V_B} = \frac{5}{2400}
\]

\[
F_v = 13.67 \text{ OUNCES/INCH} \quad F_v = 15.91 \text{ OUNCES/INCH DEPTH}
\]

\[
F_H = 3.70 \text{ OUNCES/INCH} \quad F_H = 4.18 \text{ OUNCES/INCH DEPTH}
\]

Figure 71. Record of Soft Meat Cutting.
\[
\frac{\nu_f}{\nu_B} = \frac{10}{1200} \\
F_v = 3.16 \text{ OUNCES/INCH} \\
F_H = 0.43 \text{ OUNCES/INCH}
\]

\[
\frac{\nu_f}{\nu_B} = \frac{150}{9600} \\
F_v = 7.12 \text{ OUNCES/INCH} \\
F_H = 0.86 \text{ OUNCES/INCH}
\]

Figure 70. Record of Bone Cutting.
Figure 72. Record of Frozen Meat Cutting.
Figure 73. Dynamometer Calibration Curves and Equations.

\[ F_x = 2.146 \varepsilon_x - 0.0781 \varepsilon_y \]
\[ F_y = 6.335 \varepsilon_y - 0.0575 \varepsilon_x \]
APPENDIX I

CUTTING SPECIMENS AND ACCESSORIES
SOFT MEAT AND FROZEN MEAT

<table>
<thead>
<tr>
<th>DIMENSION (in)</th>
<th>BONE</th>
<th>FROZEN OR SOFT MEAT</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>b</td>
<td>1</td>
<td>1-1/8</td>
</tr>
<tr>
<td>c</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

Figure 74. Meat and Bone Cutting Specimens.
Figure 75. Spring Loaded Suppressor Set-up.
BIBLIOGRAPHY

Literature Cited


Additional References


VITA

Robinson P. Ramirez was born on August 30, 1925 in Panama City, Panama. He received his primary and secondary education in Ancon School and Balboa High School, respectively.

Upon graduating from high school in June, 1945, he enrolled in the Canal Zone Junior College which he attended until he entered the United States Army Air Corps in March, 1946, where he served three years. He attended the University of Panama in the evenings during this period.

In March, 1949, he entered the Georgia Institute of Technology and graduated in 1952 with a degree of Bachelor of Science in Mechanical Engineering. He remained at Georgia Tech working toward a Master of Science in Mechanical Engineering, which degree was awarded in June, 1955. He continued his work toward the doctorate in the same institution.

His teaching career began in January, 1952, when he was employed by the School of Mechanical Engineering of the Georgia Institute of Technology as a Graduate Assistant. In 1953 he became an Instructor and a full-time faculty member, which position he holds at the present time.

Along with his regular teaching schedule and his studies he has had an appreciable variety of positions. In addition he is a member of the United States Air Force Reserve with the rank of Captain. During his undergraduate studies he worked as a co-operative student for one year at the Fly Ash Arrestor Corporation of Birmingham, Alabama, as chief draftsman. He also attended the University of Alabama (Birmingham branch) in the evenings during this period.
In 1952 he was employed for ten months by the Law-Barrow-Agee Engineering Laboratories of Atlanta, Georgia as a draftsman. In the summer of 1954 he worked as an Aircraft Design Engineer at Lockheed Aircraft Corporation, Marietta, Georgia. During the summer of 1955 he was employed as a Design Engineer by General Electric (Specialty Controls Department) in Waynesboro, Virginia. In 1957 he was appointed Research Associate at the Georgia Institute of Technology Experiment Station. From 1957 through 1962 he has done work as an Engineering Consultant for the following firms: General Motors Training Center; Southern Saw Service, Inc.; Georgia Power Company; and Atlanta Gas Light Company (all in Atlanta, Georgia); Cushman Motor Company (Illinois); General Motors Truck Division (Illinois and Michigan); Heco Crane Company (Dallas, Texas); General Motors Oldsmobile Division (Detroit, Michigan); Ford Motor Company (Detroit, Michigan); Perry-Nichols Law Firm (Miami, Florida); Baldwin-Lima-Hamilton Company (Lima, Ohio), and others.

He has been a member of the American Society of Mechanical Engineers, the Georgia Education Association, Pi Tau Sigma, and Tau Beta Pi; and he is an associate member of Sigma Xi.