VISCIOUS CLUTCH APPLICATION OF MECHANICALLY STORED ENERGY TO A DAMPED OSCILLATORY SYSTEM

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VICIOUS CLUTCH APPLICATION OF MECHANICALLY STORED ENERGY
TO A DAMPED OSCILLATORY SYSTEM

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NOMENCLATURE

\( b_1, b_2, b_3 \) time constants \((\text{sec}^{-1})\)
\( c \) damping coefficient \((\text{in-lbf-sec/rad})\)
\( c_0 \) constant of integration \((\text{rad/sec})\)
\( c_1, \ldots, c_8 \) constants of integration \((\text{radians})\)
\( d \) clutch groove depth \((\text{inches})\)
\( h \) clutch groove clearance \((\text{inches})\)
\( h_A, h_W \) heat transfer coefficient \((\text{BTU/hr-ft}^2-\text{°F})\)
\( \lambda \) characteristic length \((\text{inches})\)
\( m \) time constant \((\text{sec}^{-1})\)
\( n \) number of clutch grooves
\( q_1, q_2 \) time constant \((\text{sec}^{-1})\)
\( r \) intermediate variable \((\text{sec}^{-2})\) and radius \((\text{inches})\)
\( s \) intermediate variable \((\text{sec}^{-3})\)
\( t \) time \((\text{sec})\)
\( t_1 \) time at maximum basket displacement \((\text{sec})\)
\( t_2 \) time of clutch engagement for coupled solution \((\text{sec})\)
\( t_3 \) time of clutch engagement
\( u \) clutch temperature rise \((\text{°F})\)
\( v \) characteristic velocity \((\text{in/sec})\)
\( x \) depth of clutch engagement \((\text{inches})\)
\( A \) intermediate variable \((\text{sec}^{-1})\)
\( A_A \) area of clutch exposed to air \((\text{ft}^2)\)
\( A_i \) area of surface \(i\)th clutch groove \((\text{in}^2)\)
\( \Lambda_w \) area of clutch exposed to water (\( \text{ft}^2 \))

\( \Lambda_i \) intermediate variable (\( \text{sec}^{-1} \))

\( \Lambda_2 \) intermediate variable (\( \text{sec}^{-2} \))

\( \Lambda_3 \) intermediate variable (\( \text{sec}^{-3} \))

\( B \) intermediate variable (\( \text{sec}^{-1} \))

\( \text{BRKDOWN} \) minimum motor shaft speed (\( \text{rad/sec} \))

\( C_h \) heat capacity (\( \text{BTU/} ^\circ\text{F} \))

\( E \) variance (\( \text{rad}^2 \))

\( F \) particular solution of \( \theta_2(t) \) (radians)

\( F_1 \) force (lbf)

\( H \) basket displacement (radians)

\( I_F \) flywheel moment of inertia (in-lbf-sec\(^2\)/rad)

\( I_I \) basket moment of inertia (in-lbf-sec\(^2\)/rad)

\( I_W \) moment of inertia of basket contents (in-lbf-sec\(^2\)/rad)

\( K \) spring constant (in-lbf/rad)

\( N \) gear ratio (rad/rad)

\( P \) average power through motor shaft (hp)

\( P_c \) average clutch power dissipation (hp)

\( R \) Reynolds number

\( T \) torque through clutch (in-lbf)

\( T_{\text{max}} \) maximum torque through clutch (in-lbf)

\( T_1 \) torque transmitted through surfaces 1 (in-lbf)

\( T_2 \) torque transmitted through surfaces 2 (in-lbf)

\( TQ \) \( T \) intercept of motor-torque curve (in-lbf)

\( V \) clutch constant (in-lbf-sec/rad)

\( V_s \) lower bound on clutch constant (in-lbf-sec/rad)
$V_1$, upper bound on clutch constant (in-lbf-sec/rad)

$W$, motor work per cycles (in-lbf)

$W_c$, clutch dissipated work per cycle (in-lbf)

$\alpha$, slope of motor torque curve (in-lbf-sec/rad)

$\mu$, viscosity (lbf-sec/in$^2$)

$\rho$, clutch fluid density (lbf/in$^3$)

$\tau$, period (sec)

$\omega_1$, frequency of uncoupled motion (sec$^{-1}$)

$\omega_2$, frequency of coupled motion (sec$^{-1}$)

$\theta$, basket displacement (radians)

$\theta_{\text{max}}$, maximum basket displacement (radians)

$\theta_{\text{min}}$, basket displacement at time zero (radians)

$\theta_1$, uncoupled basket displacement (radians)

$\theta_2$, coupled basket displacement (radians)

$\theta_3$, start-up basket displacement (radians)

$\phi$, driving clutch member displacement and displacement of $I_w$ (radians)

$\phi_1$, uncoupled driving member displacement (radians)

$\phi_2$, coupled driving member displacement (radians)

$\phi_{\text{max}}$, driving member velocity at time zero (rad/sec)

$\phi_{\text{max}}$, lower bound on $\dot{\phi}_{\text{max}}$ (rad/sec)

$\phi_{\text{max}}$, upper bound on $\dot{\phi}_{\text{max}}$ (rad/sec)
SUMMARY

Consideration of wear and complexity of design led to the proposal that a viscous clutch be substituted for the dry friction clutch which was used in a previous analysis of an agi-basket drive mechanism for a vertical axis, automatic clothes washing machine. Values of horsepower loss in the clutch, maximum transmitted torque, time of clutch release, and physical constants of the clutch are analytically determined to obtain the proper amplitude and period of oscillation of the agi-basket. Gear ratios and spring constants of the mechanism are then adjusted to minimize motor power requirements. An experimental investigation is conducted to compare a previously used model of the oscillating system with alternative models, and values of damping and inertia characteristic of the system are determined.
CHAPTER I

INTRODUCTION

In an effort to increase the load capacity of the vertical axis, automatic clothes washing machine, the idea of the agi-basket has been advanced. The conventional washing machine oscillates a central agitator in a stationary tub to provide the cleansing motion and when this configuration is enlarged to gain a larger wash load, the washing action is diminished at the periphery of the tub due to the distance from the agitator. If the tub is oscillated with an attached agitator to improve washing at the periphery, the increase in oscillating mass requires a heavy motor and transmission. To reduce this inertia, a light-weight perforated plastic tub, the agi-basket, can be agitated within the conventional tub. The agitator is eliminated and the washing motion provided by paddles on the inside of the basket.

The conventional bellcrank transmission used on agitator machines was found to be incapable of dealing with the inertial loads. Another attempt was made in which a large motor was connected through a spring to the basket and was run in one direction and then reversed to give the desired motion.
All later attempts have used a spring to store energy at zero velocity and reverse the motion of the basket. In a design currently under evaluation, the basket rebounds off springs at the last few degrees of either end of its stroke. A continuously running motor applies torque through either of two electric clutches at some point after zero velocity. There is a separate clutch and drive train for each direction of motion.

Horn [1] describes an analytical study in which an agi-basket is attached directly to a rotational spring. The energy dissipated in the washing action is restored by coupling the basket to a flywheel during one direction of oscillation and the flywheel speed is maintained by an electric motor. A dry friction clutch is engaged to transmit energy from the flywheel to the basket and released at a proper time to produce the desired amplitude and period.

Since the flywheel speed is greater than the basket speed, the clutch slips continuously. This requires that the clutch be large in order to effect a reasonable wear life. Also, the mechanism requires a device to apply a large normal force to the clutch surfaces in order to transmit the necessary torque. For these reasons it was decided to carry out a similar investigation to that done by Horn [1], but using a viscous clutch in place of the dry friction clutch. This research is devoted to this analysis and an experimental evaluation of some assumptions made in
this and earlier work [1].

The proposed mechanism is as shown in Figure 1. As before, the agi-basket is attached permanently to the spring. The clutch is situated between the basket and a gear reduction unit which, in turn, is coupled to the flywheel and motor. The motor runs continuously during the agitation.
Figure 1. Proposed Mechanism
CHAPTER II

GOVERNING EQUATIONS

The purpose of this analysis is to give evidence that the proposed mechanism will provide a suitable washing action with a practical electric motor output and to specify clutch, spring, and gear ratio design parameters which will minimize power requirements.

Clutch

The proposed viscous clutch consists of two grooved discs with plane surfaces facing and provision for changing the fluid filled clearance between the two, Figure 2. Assuming laminar flow and no edge effects, the force transmitted through surfaces (1) is

\[ F_i = A_i \mu \frac{(r_i - \frac{W}{2})(\phi - \theta)}{h} \]

where \( A_i \) is the area of the surface (1) undergoing shear and \( \mu \) is the absolute viscosity of the fluid.

\[ A_i = 2\pi (r_i - \frac{W}{2})x \]

and the torque transmitted through surfaces (1),
Figure 2. Clutch Detail
\[ T_1 = \sum_{i=1}^{n} P_i \left( r_i \frac{w}{2} \right) \]

Making the substitutions,

\[ T_1 = \sum_{i=1}^{n} 2\pi \mu x \left( r_i \frac{w}{2} \right)^3 \left( \frac{\phi - \delta}{h} \right) \]

If surfaces (2) are thought of as two continuous parallel plates, then

\[ dT_2 = 2\pi \mu \left( \frac{r^2}{(d-x)} \right) dr \]

\[ T_2 = \int_{r_2 - \frac{w}{2}}^{r_2 + \frac{w}{2}} 2\pi \mu r^3 \left( \frac{\phi - \delta}{(d-x)} \right) dr \]

\[ T_2 = \left[ \left( r_2 + \frac{w}{2} \right)^4 - \left( r_1 - \frac{w}{2} \right)^4 \right] \mu \left( \frac{\phi - \delta}{(d-x)} \right) \]

Summing the contributions from surfaces of type (1) and (2), the total transmitted torque becomes

\[ T = \pi \mu \left( \frac{\phi - \delta}{h} \right) \left\{ \frac{2x}{h} \sum_{i=1}^{n} \left( r_i - \frac{w}{2} \right)^3 + \frac{1}{2(d-x)} \left[ \left( r_n + \frac{w}{2} \right)^4 - \left( r_1 - \frac{w}{2} \right)^4 \right] \right\} \]

for positive values of \( x \). If

\[ V = \pi \mu \left\{ \frac{2x}{h} \sum_{i=1}^{n} \left( r_i - \frac{w}{2} \right)^3 + \frac{1}{2(d-x)} \left[ \left( r_n + \frac{w}{2} \right)^4 - \left( r_1 - \frac{w}{2} \right)^4 \right] \right\} \]  \hspace{1cm} (1) \]

then
To check the validity of the laminar flow assumption take the Reynold's number for a fluid between plates moving one with respect to another [2]

$$Re = \frac{\rho v \lambda}{\mu}$$

where $\lambda$ is a characteristic length, $v$ is a characteristic velocity, and $\rho$ is the density of the fluid. Since a high Reynold's number is characteristic of turbulent flow the highest possible values of $v$ and $\lambda$ will be used to show that the flow is laminar. Taking the dimensions of a typical clutch to be

- $r_n = 5 \text{ in.}$
- $d = 0.210 \text{ in.}$
- $r_1 = 2.5 \text{ in.}$
- $h = 0.010 \text{ in.}$
- $n = 26$ grooves
- $x = 0.200 \text{ in.}$ engaged

then $W = 0.10 \text{ inches}$ (Figure 2). A typical value of $V$ is about 110 lbf-in-sec (see Chapter IV). Substituting these dimensions and $V$ into equation (1)

$$\mu = \frac{110}{\pi} \frac{1}{40[1601]+50[611]}$$
\[ \mu = 3.71 \times 10^{-4} \text{ lbf-sec/in}^2 \]

or

\[ 0.143 \text{ lbm/in-sec} \]

A typical fluid might have a density of about

\[ \rho = 0.035 \text{ lbm/in}^3 \]

A maximum characteristic velocity as seen between the clutch surfaces would be the relative velocity of the surfaces and this relative velocity is greatest at \( r_n \). Therefore,

\[ v = r_n (\dot{\phi} - \dot{\delta}) \]

The largest \((\dot{\phi} - \dot{\delta})\) encountered (Chapter IV) is

\[ (\dot{\phi} - \dot{\delta}) \approx 12 \text{ rad/sec} \]

The cross-section of the flow region is a slot 0.10 inches by 0.01 inches. To maximize \( l \), let

\[ l = w = 0.10 \text{ inches} \]

Then,
\[ R_e = \frac{(60 \text{ in/sec})(0.10 \text{ in})(0.035 \text{ lbm/in}^3)}{0.143 \text{ lbm/in-sec}} \]

\[ R_e = 1.47 \]

which is far below the transition value of about 2300 for turbulent flow.

It is assumed for the system studied that \( V(t) \) is a step function of time, Figure 3, \( V \) being approximately zero when the clutch is disengaged.

This, of course, is accomplished by manipulation of the clutch engagement depth, \( x \). \( V \) can be made as small as desired during disengagement by bringing the plates farther apart.

**Definitions**

Each agitation cycle of the agi-basket consists of two parts: (1) uncoupled motion, and (2) coupled motion (clutch engaged). To aid in the analysis, zero time will be at the zero velocity point immediately after coasting, Figure 4, and will also be the time of clutch engagement, while \( t_3 = t_2 \) is the time of clutch disengagement. \( \theta \) is the angular displacement of the basket and \( \dot{\theta} \) is the angular velocity of the flywheel member of the clutch. Subscripts (1) and (2) indicate uncoupled and coupled motion, respectively. In other words,
Figure 3. Clutch Parameter vs. Time
Figure 4. Two Stages of Motion
\[ \begin{align*}
\theta(t) &= \theta_1(t) \text{ if } t_3 < t < 0 \\
\dot{\theta}(t) &= \theta_2(t) \text{ if } 0 < t < t_2 \\
\phi(t) &= \phi_1(t) \text{ if } t_3 < t < 0 \\
\dot{\phi}(t) &= \phi_2(t) \text{ if } 0 < t < t_2
\end{align*} \]

Notice that,

\[ \begin{align*}
\theta_{\min} &= \theta_1(0) = \theta_2(0) \\
\theta_{\max} &= \theta_1(t_1) \\
\phi_{\max} &= \phi_1(0) = \phi_2(0)
\end{align*} \]

The clutch is engaged at \( t = 0 \) not only to simplify the analysis but to represent a desirable design feature since the change in direction of basket motion that occurs at time zero is easily detected. It is also assumed that the criteria for clutch release is displacement. That is, the clutch releases at \( \theta = \theta(t_2) \).

See Figure 5 for a schematic of the system. Notice that for description the mechanism is divided between the clutch members into a driving side and a driven side. A reduction gear assembly is included for reasons to be discussed
later. To keep the flywheel inertia requirements small and in turn keep the flywheel diameter small, assume that the gear assembly is between the clutch and the flywheel. Then the angular velocity of the flywheel and motor shaft is $\omega_1$, where $N$ is the reduction gear ratio.

The basket motion specified by one washing machine manufacturer [3] is periodic, rotational with a peak to peak displacement of $270^\circ$ and a period of 1.50 seconds or an arc of $196^\circ$ with a period of 1.07 seconds or an interpolation between these limits. Horn [1] chose an arc of $270^\circ$ and a period of 1.40 seconds, a sufficient set of constraints considering the previous criteria.

**Uncoupled Motion**

Unlike the dry-friction clutch, the transmitted torque for a viscous clutch is a function of slip speed. Therefore, the equations of motion of the drive side and of the driven side are coupled when the clutch is engaged, making the analysis more complex. The agi-basket model used in previous analysis [1] consists of a mass anchored to its support by a spring and a damper, Figure 5. This model will be used in order that comparisons can be made. In Chapter V discrepancies between this model and the real system will be discussed and an alternative model introduced.

The differential equation for the uncoupled basket motion is
Figure 5. System Schematic
\[ I_T \ddot{\theta}_1 + C \dot{\theta}_1 + K \theta_1 = 0 \]  

assuming negligible friction in the disengaged clutch and supporting bearings (an allowable assumption due to the large damping component). Where \( I_T \) is the moment of inertia of the agi-basket and the driven side of the clutch, \( c \) is the damping coefficient corresponding to the energy dissipated in the washing action, and \( k \) is the angular spring constant for the particular spring being used, and a dot indicates differentiation with respect to time. Solving for \( \theta_1 \),

\[
\theta_1(t) = e^{-q_1 t} [c_7 \cos \omega_1 t + c_8 \sin \omega_1 t] \tag{4}
\]

where

\[
q_1 = \frac{c}{2I_T}, \quad \omega_1 = \frac{1}{2I_T} \sqrt{4I_T K c^2}
\]

and \( c_7 \) and \( c_8 \) are constants of integration. Also,

\[
\dot{\theta}_1(t) = e^{-q_1 t} [-q_1 c_7 \cos \omega_1 t - \omega_1 c_7 \sin \omega_1 t - q_1 c_8 \sin \omega_1 t + \omega_1 c_8 \cos \omega_1 t]
\]

By definition, at \( t = 0 \),
\[ \dot{\theta}_1(t) = 0 \]

\[ \omega_1 c_8 - q_1 c_7 = 0 \]  \hspace{1cm} (5)

Since \( \dot{\theta}_1 = 0 \) at \( \sin \omega_1 t = 0 \) or in other words at

\[ \omega_1 t = \ldots, -2\pi, -\pi, 0, \pi, 2\pi, \ldots \]

and \( t = t_1 \) at the zero velocity (\( \dot{\theta}_1 = 0 \)) point previous to \( t = 0 \) (see Figure 4),

\[ \omega_1 t_1 = -\pi \]

\[ t_1 = -\frac{\pi}{\omega_1} \]

It is desired that the total arc be \( \frac{3\pi}{2} \) radius, therefore,

\[ \theta_1 \left( \frac{-\pi}{\omega_1} \right) - \theta_1(0) = \frac{3\pi}{2} \]

From equation (4),

\[ -c_7 e^{-q_1 \left( \frac{-\pi}{\omega_1} \right)} - c_7 = \frac{3\pi}{2} \]  \hspace{1cm} (6)

\[ c_7 = \frac{-3\pi}{2} / \left( 1 + e^{-\frac{\pi}{\omega_1}} \right) \]
Inserting $C_7$ into equation (5),

$$C_8 = \frac{q_1}{\omega_1} \left( \frac{-3\pi}{q_1 \pi} \right) \frac{1+e}{\omega_1}$$

Notice that

$$\theta_{\text{min}} = C_7$$

$$\theta_{\text{max}} = C_7 + \frac{3\pi}{2}$$

In the previous analysis [1], a 0.5 hp split-base electric motor was used as the source of mechanical power. The motor torque versus speed was approximated by a straight line (Figure 6) with the relationship

$$T_{\text{motor}} = a(N_\phi^*) + T_Q$$

where $T_Q$ is the $T_{\text{motor}}$ intercept and $a$ is the slope of the straight line approximation. Then the no load running speed is

$$\left( \frac{-T_Q}{a} \right)$$

Note that below the breakdown speed, BRKDOWN, motor torque decreases with decreasing motor speed and the straight line
Figure 6. Motor Torque Curve
does not provide a satisfactory fit to the real curve. For the sake of meaningful comparisons, the same motor-torque approximations will be used here and checks will be made to insure that \( N\dot{\phi} \) is never smaller than BRKDOWN. Here,

\[
TQ = 431 \text{ in-lbf}, \quad \alpha = -2.2 \text{ in-lbf-sec/rad},
\]

\[
\text{BRKDOWN} = 168 \text{ rad/sec}
\]

The differential equation for the motion of the uncoupled flywheel is

\[
I_F(\ddot{N}\dot{\phi}_1) = \alpha(N\dot{\phi}_1) + TQ
\]

Solving for \( \dot{\phi}_1(t) \) we have

\[
N\dot{\phi}_1(t) = c_0 e^{\frac{\alpha}{I_F t} - \frac{TQ}{\alpha}}
\]

(7)

Since \( \dot{\phi} \) is maximum at \( t = 0 \),

\[
c_0 = \frac{TQ}{\alpha} + N\dot{\phi}_{\text{max}}
\]

**Coupled Motion**

The torque transmitted through the clutch is

\[
T = V(\dot{\phi}_2 - \dot{\phi}_2)
\]

where \( V \) is as defined in equation (1). The differential
equation for the coupled agi-basket and flywheel are respectively,

\[ I_T \ddot{\phi}_2 + c \dot{\phi}_2 + K\phi_2 = V(\dot{\phi}_2 - \dot{\delta}_2) \]  
(8)

\[ I_F(N\ddot{\phi}_2) = \frac{T_{\text{motor}}}{N} \]  
(9)

where \( \frac{T}{N} \) is the torque at the clutch reflected across the gear reduction to the flywheel. Making the proper substitutions, equation (9) becomes

\[ I_F(N\ddot{\phi}_2) = \alpha N\dot{\phi}_2 + TQ \frac{V}{N}(\dot{\phi}_2 - \dot{\delta}_2) \]  
(9a)

Solving equation (8) for \( \dot{\phi}_2 \) and differentiating with respect to time,

\[ \dot{\phi}_2 = \frac{I_T}{V} \ddot{\phi}_2 + \frac{C+V}{V} \dot{\phi}_2 + \frac{K}{V} \phi_2 \]  
(10)

\[ \ddot{\phi}_2 = \frac{I_T}{V} \dddot{\phi}_2 + \frac{C+V}{V} \ddot{\phi}_2 + \frac{K}{V} \dot{\phi}_2 \]  
(11)

Substituting equations (10) and (11) into equation (9a) and simplifying,
This is a third order, linear differential equation in one variable. To solve, let

\[
\begin{align*}
\theta_2 \cdot A_1 \theta_2 + A_2 \theta_2^2 + A_3 \theta_2^3 &= \frac{\text{TQV}}{I_F I_T N} \\
\end{align*}
\]

for which the characteristic equation is

\[
\lambda^3 + A_1 \lambda^2 + A_2 \lambda + A_3 = 0
\]

To solve this cubic equation let,

\[
\begin{align*}
\tau &= \frac{1}{3} (3A_2 - A_1^2) \\
\sigma &= \frac{1}{27} (2A_1^3 - 9A_1A_2^2 + 27A_3)
\end{align*}
\]

and,
Then the solutions for $\lambda$ are:

$$\lambda_1 = A + B \cdot \frac{A_1}{3}$$

$$\lambda_2 = \frac{-A + B}{2} \cdot \frac{A_1}{3} + \frac{A - B}{2} \cdot \sqrt{-3}$$

$$\lambda_3 = \frac{-A + B}{2} \cdot \frac{A_1}{3} - \frac{A - B}{2} \cdot \sqrt{-3}$$

It is obvious that there are two general cases for the solution of $\theta_2$.

**Underdamped**

If $s^2 + \frac{r^2}{27} > 0$, then $A$ and $B$ are real numbers and there will be two imaginary and one real root of $\lambda$. The corresponding solution for $\theta_2^*$ where superscript star indicates underdamped case is,

$$\theta_2^*(t) = C_1^* e^{mt} + e^{-q_2^2t} (C_2^* \cos \omega_2 t + C_3^* \sin \omega_2 t) + F \quad (12)$$

where

$$m = A + B \cdot \frac{A_1}{3}, \quad q_2 = \frac{A + B}{2} \cdot \frac{A_1}{3}, \quad \omega_2 = \frac{\sqrt{3}}{2} (A - B)$$
\[ F = \frac{TQV}{1 + \frac{T}{NA}} \]

and \( c^*, c^*_2, c^*_3 \) are constants of integration.

**Overdamped**

If \( \frac{S^2}{4} + r^3 > 0 \) then \( A \) and \( B \) are complex conjugates and \( \lambda \) has three real solutions. In this case \( \theta'_2 \), where prime indicates overdamped solution, is

\[ \theta'_2(t) = c_1^* e^{b_1 t} + c_2^* e^{b_2 t} + c_3^* e^{b_3 t} + F \quad (13) \]

where

\[ b_1 = A + B - \frac{A^2}{3} \]
\[ b_2 = \frac{A + B}{2} \left( \frac{A}{3} + \frac{\sqrt{3}}{2} (A - B) \right) \]
\[ b_3 = \frac{A + B}{2} \left( \frac{A}{3} - \frac{\sqrt{3}}{2} (A - B) \right) \]

**Initial Conditions**

1. \( \dot{\theta}_2 = 0 \) \( \theta = 0 \) t = 0 from the definition of \( t \)

2. \( \dot{\theta}_2 = \theta_{\text{min}} \) \( \theta = 0 \) t = 0 from the definition of \( \theta_{\text{min}} \)

3. \( \dot{\theta}_2 = V \) \( \phi_{\text{max}} \) \( \theta = 0 \) from equation (8), condition 1. and 2. and the definition of \( \phi_{\text{max}} \)

Solving the three linear simultaneous equations.
resulting from the initial conditions, the constants of integration are,

\[ c_1^* = \frac{(F-\theta_{\text{min}})(q_2^2 I_T + \omega_2^2 I_T) - V_d^*}{I_T(m^2 - q_2^2 - \omega_2^2 - 2q_2 m)} \]

\[ c_2^* = \sqrt{\omega_2^2 + c_1^*} \]

\[ c_3^* = \frac{V_d^*}{K} \frac{1}{I_T} \theta_{\text{min}} + c_1 m^2 + c_2 (q_2^2 - \omega_2^2) \]

\[ c_1^* = \frac{(\theta_{\text{min}} - F)(b_2 b_3^2 - b_3 b_2^2) + (b_3^2 - b_2^2) \left( \frac{V_d^*}{I_T} \phi_{\text{max}} - \frac{1}{K} (\theta_{\text{min}} - F) \right)}{b_1 (b_2^2 - b_3^2) + b_2 (b_3^2 - b_1^2) + b_3 (b_1^2 - b_2^2)} \]

\[ c_2^* = \frac{(\theta_{\text{min}} - F)(b_3 b_1^2 - b_1 b_3^2) + (b_1^2 - b_3^2) \left( \frac{V_d^*}{I_T} \phi_{\text{max}} - \frac{1}{K} (\theta_{\text{min}} - F) \right)}{b_1 (b_2^2 - b_3^2) + b_2 (b_3^2 - b_1^2) + b_3 (b_1^2 - b_2^2)} \]

\[ c_3^* = \sqrt{\omega_2^2 + c_2^* - c_3^*} \]

**Underdamped** \( \dot{\phi}_2 \)

Differentiating equation (12) twice with respect to time and substituting into equation (10) we have \( \dot{\phi}_2^* \) for the underdamped case,

\[ \dot{\phi}_2^*(t) = c_4^* e^{mt} e^{-q_2^*} (c_5^* \cos \omega_2 t + c_6^* \sin \omega_2 t) + \frac{FK}{V} \] (14)
where the constants of integration are

\[ c_4^* = c_1 \left( \frac{K (c+V) q_2 c_2 + I_T q_2 c_2 - I_T \omega_2^2 c_2 + 2 I_T q_2 \omega_2 c_2^* (c+V) \omega_2 c_3^*}{V} \right) \]

\[ c_5^* = \frac{1}{V} (K c_3^* - (c+V) q_2 c_3^* + I_T q_2 c_3^* - I_T \omega_2^2 c_3^* + 2 I_T q_2 \omega_2 c_3^* (c+V) \omega_2 c_2^*) \]

\[ c_6^* = \frac{1}{V} (K c_3^* - (c+V) q_2 c_3^* + I_T q_2 c_3^* - I_T \omega_2^2 c_3^* + 2 I_T q_2 \omega_2 c_3^* (c+V) \omega_2 c_2^*) \]

**Overdamped \( \dot{\phi}_2 \)**

Differentiating equation (13) twice with respect to time and substituting into equation (10) we have \( \dot{\phi}_2 \) for the overdamped case,

\[ \dot{\phi}_2(t) = c_4 e^{b_1 t} + c_5 e^{b_2 t} + c_6 e^{b_3 t} + \frac{K}{V} \]

(15)

where

\[ c_4^* = \frac{1}{V} (I_T c_1^* b_1^2 + (c+V) c_1^* b_1 + K c_1^*) \]

\[ c_5^* = \frac{1}{V} (I_T c_2^* b_2^2 + (c+V) c_2^* b_2 + K c_2^*) \]

\[ c_6^* = \frac{1}{V} (I_T c_3^* b_3^2 + (c+V) c_3^* b_3 + K c_3^*) \]
CHAPTER III

NUMERICAL SOLUTIONS

At this point solutions are available for $\theta_1$, $\theta_2$, $\dot{\theta}_1$, and $\dot{\theta}_2$ provided we know the clutch parameter $V$, the maximum flywheel speed $N_{\max}^\phi$, the reduction ratio $N$, and the spring rate $K$. To construct a picture of the entire motion, the time of clutch release, $t_2$ for $\theta_2$ and $t_3$ for $\theta_1$ must be known so that the transition from coupled to uncoupled motion can be made. $K$ and $N$ are given assumed values at the beginning of the numerical analysis. These variables ($K$ and $N$) are design parameters and will be used later in an effort to minimize power requirements. The damping coefficient $c$ and the basket inertia $I_T$ as used by Horn [1] and specified by a manufacturer [3] are

$$c = 28.8 \text{ in-lbf-sec}$$
$$I_T = 4.44 \text{ in-lbf-sec}^2$$

Values of $V$, $\dot{\theta}_{max}$, $t_2$ and $t_3$ will be found using the iterative methods of bisection and Newton-Raphson, and appropriate constraints. Three nested loops are employed, Figure 7.
Figure 7. Nested Loops
Loop A

An increase in $V$, equation (1), whether by using a more viscous fluid, decreasing clearances or increasing the area of the working surfaces, would be expected to produce a higher acceleration of the coupled basket, Figure 8. The time of clutch engagement, effectively $t_z$, would then decrease. The period is

$$\tau = t_2 - t_3 \tag{16}$$

Recall from Page 14,

$$\tau = 1.40 \text{ seconds}$$

The bisection method [4] is used to arrive at a value of $V$ which satisfies the criterion of period. In these iterative steps, a variable subscripted with $i$ indicates the value that the variable takes on after the $i$'th iteration and zero indicates the initial value.

Choose $V_{S_0}$ and $V_{L_0}$, the initial upper and lower limits on $V_i$, respectively, so that

$$V_{S_0} \leq V \leq V_{L_0}$$

Note that $V_{S_0}$ must be reasonably "close" to $V$ or $\theta_2$ will never reach a sufficiently large displacement or velocity to
Figure 8. Effect of $V$ on $\tau$ (Period)
complete the cycle with the specified arc \( \left( \frac{3\pi}{2} \right) \) radius. In this case loop C would not converge.

Begin the iteration by setting

\[
V_i = \frac{V_{s_i} + V_{L_i}}{2}
\]

Evaluate \( \tau_i = t_{2_i} - t_{3_i} \) (\( t_{2_i} \) and \( t_{3_i} \) from loop C). If

\[|\tau_i - \tau| < \Delta \tau\]

where \( \Delta \tau \) is error in period, take

\[V = V_i\]

and stop. \( \Delta \tau \) is taken as 0.005 seconds in this program.

Smaller \( \Delta \tau \) will give greater accuracy at the expense of more computer time. Otherwise, if

\[\tau_i < \tau\]

then \( V_i \) is too large. Since it is known that

\[V < V_i\]

set
\[ V_{L_{i+1}} = V_i \]

\[ V_{S_{i+1}} = V_i \]

and \( V \) will be contained in the interval \([V_{S_{i+1}}, V_{L_{i+1}}]\). If

\[ \tau_i > \tau \]

then \( V_i \) is too small. Set

\[ V_{L_{i+1}} = V_{L_i} \]

\[ V_{S_{i+1}} = V_i \]

In either case, we have reduced the interval in which we know \( V \) to exist by one-half.

**Loop B**

In order to calculate \( \tau \) from equation (16) it is necessary to have \( t_2 \) and \( t_3 \) which cannot be found without a value for \( \dot{\phi}_{\text{max}} \). (\( \dot{\phi}_{\text{max}} \) is necessary to calculate the constants of integration for coupled solutions.) In this way all three nested loops are interdependent. An obvious set of constraints is that the solutions of \( \dot{\phi}_1 \) and \( \dot{\phi}_2 \) be equal at both the time the clutch engages and the time the clutch disengages, Figure 9. In other words, the velocity of the
Figure 9. Velocity of Driving Clutch Member Versus Time
clutch driving member \( \dot{\phi} \) should not have discontinuities where the solution changes from the coupled case \( \dot{\phi}_2 \), to the uncoupled case \( \dot{\phi}_1 \) or from \( \dot{\phi}_1 \) to \( \dot{\phi}_2 \).

\[ \dot{\phi}_1(t_3) = \dot{\phi}_2(t_2) \quad (17) \]

\[ \dot{\phi}_1(0) = \dot{\phi}_2(0) \quad (18) \]

By using the same value of \( \dot{\phi}_{\text{max}} \) in calculating the constants of integration for \( \dot{\phi}_1(t) \) and \( \dot{\phi}_2(t) \) we satisfy the constraint given by equation (18) since \( \dot{\phi}_{\text{max}} \) is the clutch driving member speed \( \dot{\phi} \) at \( t = 0 \).

It would seem that a high value of \( \dot{\phi}_{\text{max}} \) would cause an increased drop in \( \dot{\phi}_2 \) from \( t = 0 \) to \( t_2 \), Figure 10, since the torque transmitted through the clutch would be larger. Recall equation (2). It would also seem that a high value of \( \dot{\phi}_{\text{max}} \) would produce a smaller gain in \( \dot{\phi}_1 \) from \( t_3 \) to \( t = 0 \) since motor torque decreases as motor speed approaches the no-load running speed giving a lesser acceleration of the flywheel (Figure 6). In other words, if a value for \( \dot{\phi}_{\text{max}} \) is chosen too high the calculated \( \dot{\phi}_1(t_3) \) will be larger than the calculated \( \dot{\phi}_2(t_2) \).

Using the bisection method again, choose \( \dot{\phi}_{\text{max}}^{s_0} \) and \( \dot{\phi}_{\text{max}}^{L_0} \), the upper and lower limits of \( \dot{\phi}_{\text{max}}^1 \) such that
Figure 10. Effect of $\dot{\phi}_{\text{max}_1}$ on $\dot{\phi}_1$ and $\dot{\phi}_2$. 
\[ \dot{\phi}_{\text{max}}_{s_0} \leq \dot{\phi}_{\text{max}} \leq \dot{\phi}_{\text{max,LO}} \]

For \( \dot{\phi}_{\text{max,LO}} \) we may take the no load running speed reflected to the driving clutch member, \( \frac{T_a}{N\alpha} \), as this imposes an upper limit on \( \dot{\phi} \). Begin by setting
\[ \dot{\phi}_{\text{max,1}} = \frac{\dot{\phi}_{\text{max}} + \dot{\phi}_{\text{max,1i}}}{2} \]

Evaluate \( \dot{\phi}_1(t_3) \) and \( \dot{\phi}_2(t_2) \) using \( \dot{\phi}_{\text{max,1}} \) in equations (7) and (14) or (15).

If
\[ |\dot{\phi}_1(t_3) - \dot{\phi}_2(t_2)| < \Delta \dot{\phi} \]
take
\[ \dot{\phi}_{\text{max}} = \dot{\phi}_{\text{max,1}} \]

as a reasonably good approximation and stop. 0.5 rad/sec is used for \( \Delta \dot{\phi} \) here but smaller values can be employed with correspondingly greater accuracy and increased computer time cost. Otherwise, if
\[ \dot{\phi}_1(t_3) - \dot{\phi}_2(t_2) > 0 \]

then $\dot{\phi}_{\text{max}_i}$ is too large and $\dot{\phi}_{\text{max}_i}$ can become the next upper limit. Set

$$\dot{\phi}_{\text{max}_{i+1}} = \dot{\phi}_{\text{max}_i}$$

$$\dot{\phi}_{\text{max}_{S_{i+1}}} = \dot{\phi}_{\text{max}_i}$$

If

$$\dot{\phi}_1(t_3) - \dot{\phi}_2(t_2) < 0$$

set

$$\dot{\phi}_{\text{max}_{i+1}} = \dot{\phi}_{\text{max}_i}$$

$$\dot{\phi}_{\text{max}_{S_{i+1}}} = \dot{\phi}_{\text{max}_i}$$

Loop C

In order for the disengaged basket to reach a maximum displacement, $\theta_{\text{max}}$ and repeat the uncoupled motion phase, the clutch should be released at a time $t_2$ for $\theta_2$ such that

$$\theta_2(t_2) = \theta_1(t_3)$$  \hspace{1cm} (19)

$$\dot{\theta}_2(t_2) = \dot{\theta}_1(t_3)$$  \hspace{1cm} (20)
for a \( t_3 \) which must also be determined for \( \theta_1 \). To show this is true consider bringing together curves of \( \theta_2 \) and \( \theta_1 \) at \( t_2 \) and \( t_3 \), respectively, Figure 11. At this time in the agitation cycle the clutch releases and there is a transition in the solution from \( \theta_2(t) \) to \( \theta_1(t) \). If this change in solutions is to accurately represent events in the real system, two criteria must be met. The displacement of the basket must coincide for both solutions and the velocity of the basket must coincide for both solutions. In other words, the velocity and acceleration must be finite at the time of disengagement. (Since it is assumed that the clutch releases instantaneously the accelerations of \( \theta_1 \) and \( \theta_2 \) do not match at this point.) The problem is then one of finding \( t_2 \) on the curve \( \theta_2(t) \) and \( t_3 \) on the curve \( \theta_1(t) \) such that equations (19) and (20) hold.

The iterative process used to find \( t_2 \) and \( t_3 \) is Newton's method for systems of equations [4]. Letting,

\[
t_2 = x, \quad t_3 = y
\]

and

\[
\theta_2(t_2) - \theta_1(t_3) = f(x,y)
\]

\[
\dot{\theta}_2(t_2) - \dot{\theta}_1(t_3) = g(x,y)
\]
Figure 11. Clutch Release
After choosing an approximation $x_o, y_o$ to the roots of equations (19) and (20) use the recursion formulas,

$$x_{i+1} = x_i - \frac{\frac{\partial g}{\partial y} - \frac{\partial f}{\partial y}}{\frac{\partial f}{\partial x} - \frac{\partial g}{\partial x}} \Delta t \quad x=x_i, y=y_i$$

$$y_{i+1} = y_i - \frac{\frac{\partial f}{\partial x} - \frac{\partial g}{\partial x}}{\frac{\partial f}{\partial y} - \frac{\partial g}{\partial y}} \Delta t \quad x=x_i, y=y_i$$

to generate successive values of $x_i, y_i$. Making the substitutions,

$$t_{2i+1} = t_{2i} + \frac{\frac{\partial^2 \theta_1(t_3)}{\partial t_3^2}(\theta_2(t_2) - \theta_1(t_3)) + \frac{\partial \theta_1(t_3)}{\partial t_3} \frac{\partial \theta_2(t_2)}{\partial t_2} - \frac{\partial \theta_1(t_3)}{\partial t_3} \frac{\partial \theta_2(t_2)}{\partial t_2}}{\frac{\partial^2 \theta_2(t_2)}{\partial t_2^2} - \frac{\partial \theta_2(t_2)}{\partial t_2} \frac{\partial \theta_1(t_3)}{\partial t_3}} \Delta t \quad t_2 = t_{2i}, t_3 = t_{3i}$$

$$t_{3i+1} = t_{3i} + \frac{\frac{\partial^2 \theta_2(t_2)}{\partial t_2^2}(\theta_2(t_2) - \theta_1(t_3)) + \frac{\partial \theta_2(t_2)}{\partial t_2} \frac{\partial \theta_2(t_2)}{\partial t_2} - \frac{\partial \theta_2(t_2)}{\partial t_2} \frac{\partial \theta_1(t_3)}{\partial t_3}}{\frac{\partial^2 \theta_1(t_3)}{\partial t_3^2} - \frac{\partial \theta_2(t_2)}{\partial t_2} \frac{\partial \theta_1(t_3)}{\partial t_3}} \Delta t \quad t_2 = t_{2i}, t_3 = t_{3i}$$
The rate of convergence of this process is a great deal faster than that of the previous bisection method, but care must be taken that the Jacobian \( \frac{\partial f}{\partial x} \frac{\partial g}{\partial y} - \frac{\partial g}{\partial x} \frac{\partial f}{\partial y} \) does not vanish or become so small as to vanish due to round off error. It should be mentioned also that \( x_0, y_0 \) must be very good approximations to the roots or this method will not converge at all. This requires changes to the initial approximations of \( t_2, t_3 \) every time a system parameter is varied significantly. The test,

\[ |t_{2i+1} - t_{2i}| < \Delta t \]

and

\[ |t_{3i+1} - t_{3i}| < \Delta t \]

terminates loop C if true. The value of \( \Delta t \) used here is 0.0001 seconds.

**Power Determinations**

Now that complete solutions are available for \( \Theta_2 \) and \( \phi_2 \) it is possible to calculate the average power requirements at the motor \( P \) and the average power dissipated by the viscous clutch, \( P_c \). See Figure 12. The remaining power, \( P - P_c \), is dissipated by the damping on the basket, that is, \( P - P_c \) is the power that goes into washing clothes. The work
Figure 12. Power Flow
done per cycle by the motor-flywheel assembly is

\[ W = \int_{0}^{t_2} f_2 T \, dt \]  \hspace{1cm} (21)

where torque \( T \) is as defined by equation (2)

\[ T = V(\phi_2 - \dot{\phi}_2) \]  \hspace{1cm} (2)

Note that this \( W \) is the work done per cycle by the motor since any energy removed from the flywheel by the clutch is restored to the flywheel by the motor before the cycle is completed. In other words, the flywheel merely stores energy and lessens the maximum power draw from the motor. See Figure 13.

Substituting equation (2) into equation (21),

\[ W = V \int_{0}^{t_2} (\phi_2^2 - \dot{\phi}_2^2) \, dt \]

The work dissipated by viscous damping in the clutch per cycle is

\[ W_c = \int_{0}^{t_2} (\dot{\phi}_2 - \dot{\phi}_2) T \, dt \]

or

\[ W_c = V \int_{0}^{t_2} (\phi_2^2 - \dot{\phi}_2^2)^2 \, dt \]
Figure 13. Effect of Flywheel on Power Requirements
Exact solutions for $W$ and $W_c$ would be very difficult to obtain but numerical methods of integration provide sufficiently accurate results. The trapezoid rule [4] is used here (Figure 14). Let

$$f(t) = V(\dot{\phi}_2(t) \dot{\theta}_2(t))$$

Then

$$W = \int_{0}^{t_2} f(t) dt$$

and summing the area of the trapezoids in Figure 14,

$$W = \sum_{i=0}^{n-1} \frac{f(\frac{i}{n}t_2) - f(\frac{i+1}{n}t_2)}{2} \cdot \frac{t_2}{n}$$

$$= \frac{t_2}{2n} [f(0) + f(t_2) + 2 \sum_{i=1}^{n-1} f(\frac{i}{n}t_2)]$$

$W_c$ is calculated in the same manner with

$$f_c(t) = V(\dot{\phi}_2(t) \dot{\theta}_2(t))^2$$

Since the duration of a cycle $T$ is known, $P$ and $P_c$ can be calculated.
Figure 14. Numerical Integration
\[ P = \frac{W}{t} \]

\[ P_c = \frac{W_c}{t} \]

Keep in mind that \( P \) and \( P_c \) are average power values and instantaneous values may be quite different.

**Torque Calculation**

The torque transmitted through the clutch is

\[ T(t) = V(\phi_2(t) - \dot{\phi}_2(t)) \tag{2} \]

therefore the maximum torque \( T_{\text{max}} \) can be found from equation (2) if \( t \) is found that maximizes \((\phi_2(t) - \dot{\phi}_2(t))\). \( \phi_{\text{max}} \) occurs at \( t = 0 \) and \( \dot{\phi}_2(0) = 0 \) by definition. Since \( \dot{\phi}_2 \) is always positive, \( T_{\text{max}} \) occurs at time zero, or

\[ T_{\text{max}} = V \cdot \phi_{\text{max}} \]

This numerical work was accomplished with a computer program written in Fortran IV language and run on the Univac 1108 machine. A flow chart, Figure 15, and a copy of the main program, Appendix A, are included as tools for further investigation. A fourth loop, loop D, is shown in which the reduction ratio \( N \) is increased by an increment of 1.0 to some value \( N_{\text{max}} \). This has been included in the flow chart to
Figure 15. Flow Chart for Main Program
\[ \dot{\phi}_{\text{max}} < \text{BRKDFWN} \quad \text{No} \quad \text{stop} \]

\[ \dot{\phi}_{\text{max}} = \dot{\phi}_{\text{max}} \quad \text{(B)} \]

\[ |\tau_i - \tau| < 10^{-3}? \quad \text{No} \quad \text{End Loop A} \]

\[ |\tau_i > \tau? \quad \text{Yes} \]

FIND P, P_c

FIND T_{\text{max}}

N = N + 1.0

\[ N > N_{\text{max}} \quad \text{No} \quad \text{(D)} \]

\[ N > N_{\text{max}} \quad \text{Yes} \quad \text{stop} \]

\[ V_s = V \]

\[ V_L = V \quad \text{(A)} \]

Figure 15 (concluded)
illustrate the steps taken to evaluate the effects of the parameters N and K on the system operation. A similar loop was used for spring constant K while holding N constant.
CHAPTER IV

RESULTS OF ANALYSIS

The numerical methods of the previous chapter yield various power requirements, maximum torque at the clutch, clutch constant, limits of basket motion and flywheel speed, time of clutch disengagement, and the appropriate constants for evaluating the equations of motion. A sample of the computer output is included at the end of Appendix A. Through the analysis, values of

\[ I_F = 1.00 \text{ in-lbf-sec}^2 \]
\[ I_T = 4.44 \text{ in-lbf-sec}^2 \]
\[ C = 28.8 \text{ in-lbf-sec} \]

have been used for the inertias of the flywheel and basket and the damping coefficient. These are the same values used by Horn [1] so that comparisons are valid. A larger flywheel inertia would result in a smaller drop in flywheel speed when the clutch is engaged. The flywheel inertia used gave a drop in speed of about 5 percent of \( N_{\text{max}} \). In any future analysis it may be safe to assume a constant flywheel speed, especially if \( I_F \) were increased. Spring constant \( K \) and
reduction ratio $N$ are the independent variables.

**Plots**

In order to minimize the number of steps for each iteration (see Figure 7) initial approximations for $V$, $\dot{\phi}_{\text{max}}$, $t_2$, and $t_3$ were arrived at as experience with the program was gained. Since the clutch constant $V$ appeared to be relatively linear with respect to reduction ratio $N$, Figure 16, initial values for $V_i$ and $V_g$ (Chapter III, loop A) were replaced by linear equations dependent on $N$ for constant $K$.

Figure 17 is a plot of power at the motor shaft $P$ and power dissipated in the clutch $P_c$ versus $N$ for $K = 150$ in-lbf/rad. As might be expected $P-P_c$, the power dissipated in the basket due to washing action, remains nearly constant, while $P_c$ decreases with increasing $N$. Since the speed of the flywheel side of the clutch approaches the speed of the basket side as $N$ increases while transmitted torque remains fairly constant, this is not surprising. There is, however, an upper limit on $N$. For a large $N$ the difference in velocities of clutch members will be insufficient to drive the basket to a displacement and velocity large enough so that the specified arc and period are achieved. In this case there is no solution for the time of clutch disengagement, $t_2$ and $t_3$.

A plot of $P$ versus $K$ for $N$ constant at 16.0 rad/rad, Figure 18, reveals that a small $K$ minimizes $P$. However, making $K$ small lengthens the coasting or uncoupled time for
Figure 16. Clutch Constant Versus Gear Ratio

for
\[ K = 150 \text{ in.lbf./rad.} \]
\[ T = 1.40 \text{ sec.} \]
Figure 17. Power Versus Gear Ratio

Power at Motor $P$

Power Dissipated in Clutch $P_c$

for

$K = 150 \text{ in.lbf./rad.}$

$T = 1.40 \text{ sec.}$
Figure 18. Clutch Constant and Power Versus Spring Constant

for
\[ N = 16.0 \]
\[ T = 1.40 \text{ sec} \]
the basket and a larger $V$ is required to complete the cycle within the specified period. From Figure 18 it can be seen that there is some practical lower limit to $K$ so that $V$ is not prohibitively large. A large $V$ would necessitate a large clutch, extremely small clearances, or excessively high fluid viscosities. This power draw $P$ at the motor is of primary concern since it determines whether or not the use of the viscous clutch or the dry friction clutch is justified.

The power dissipated in the clutch $P_c$ results in an increase in the temperature of the clutch fluid until the heat transfer rate from the working fluid to the clutch environment is equal to $P_c$. The rise in temperature would decrease $V$ which is directly proportional to viscosity. In Figure 19, the period $\tau$ is plotted against $V$ for three combinations of gear ratio and spring rate. Loop A in the computation scheme shown in Figure 7 provides successive values of $V$ and the resulting $\tau$ for the plots in Figure 19. Within loop A, however, the clutch release displacement, $\theta_2(t_2)$ is adjusted by varying $t_2$ to give the proper arc, while in the real system the clutch release is fixed and the arc will vary. Table 1 shows that $\theta_2(t_2)$ remains fairly constant with respect to $V$ so that the plots in Figure 19 are representative of the effect of temperature on the period of oscillation during warm-up.
Figure 19. Period Versus Clutch Constant for Warm-Up
### Table 1. Main Program Results

<table>
<thead>
<tr>
<th>K (in. lbf./rad.)</th>
<th>N</th>
<th>P (hp.)</th>
<th>P_C (hp.)</th>
<th>V (in. lbf. sec./rad.)</th>
<th>Φ_{max} (rad./sec.)</th>
<th>Φ(t_2) (rad./sec.)</th>
<th>T_{max} (in. lbf.)</th>
<th>t_2 (sec.)</th>
<th>Θ_2(t_2) (rad.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>150.0</td>
<td>14.0</td>
<td>0.5551</td>
<td>0.3212</td>
<td>76.1</td>
<td>13.75</td>
<td>12.9</td>
<td>1047</td>
<td>0.723</td>
<td>4.160</td>
</tr>
<tr>
<td>150.0</td>
<td>16.0</td>
<td>0.4872</td>
<td>0.2566</td>
<td>98.5</td>
<td>12.05</td>
<td>11.3</td>
<td>1107</td>
<td>0.719</td>
<td>4.150</td>
</tr>
<tr>
<td>150.0</td>
<td>18.0</td>
<td>0.4363</td>
<td>0.2086</td>
<td>128.3</td>
<td>10.73</td>
<td>10.2</td>
<td>1381</td>
<td>0.716</td>
<td>4.139</td>
</tr>
<tr>
<td>125.0</td>
<td>16.0</td>
<td>0.4137</td>
<td>0.1871</td>
<td>118.3</td>
<td>12.1</td>
<td>11.4</td>
<td>1132</td>
<td>0.613</td>
<td>4.211</td>
</tr>
<tr>
<td>112.0</td>
<td>16.0</td>
<td>0.3701</td>
<td>0.1505</td>
<td>163.3</td>
<td>12.1</td>
<td>11.3</td>
<td>1952</td>
<td>0.527</td>
<td>4.202</td>
</tr>
</tbody>
</table>
Warm-Up

To maintain a more nearly constant clutch fluid temperature, the clutch assembly could be exposed directly to the water in the agi-basket as in Figure 1. For example, consider the case of \( N = 16.0 \) and \( K = 112 \text{ in-lbf/rad} \). From Table 1

\[
P_c = 0.150 \text{ horsepower}
\]

\[
= 382 \text{ BTU/hr}
\]

For this heat input the water temperature will increase 1.3°F during a 20 minute agitation if no heat is transferred out of the water. Assume that the convective heat transfer coefficient for the clutch housing and water is about

\[
h_w = 800 \frac{\text{BTU}}{\text{hr-ft}^2\cdot\circ\text{F}}
\]

Then

\[
P_c = h_w A_w U_w
\]

where \( A_w \) is the area of the clutch housing exposed to water and \( U_w \) is the temperature difference between the clutch housing surface and the water. For the clutch example in Chapter I, a typical housing might be 7 inches in radius with
an exposed perimeter 4 inches in height. Then $A_w$ is

$$A_w = \pi\left(\frac{7}{12}\right)^2 + 2\pi\left(\frac{7}{12}\right)\left(\frac{4}{12}\right)$$

$$= 2.29 \text{ ft}^2$$

Substituting these values into equation (22)

$$382 \text{ BTU/hr} = 800 \text{ BTU/hr-ft}^2 \cdot F^\circ \cdot 2.29 \text{ ft}^2 \cdot U_w$$

$$U_w = 0.209 \text{ F}^\circ$$

Temperature variations in the washing water will then have more effect on period than $U_w$. Also the initial temperature of the water may vary due to different washing requirements.

If this is the case the clutch may be fined and exposed to air. Now the area exposed for cooling can be

$$A_A = 2\pi\left(\frac{7}{12}\right)^2 + 2\pi\left(\frac{7}{12}\right)\left(\frac{4}{12}\right)$$

Take the heat transfer coefficient to be about

$$h_A = 8 \text{ BTU/hr-ft}^2 \cdot F^\circ$$

and the heat capacity of the clutch, about

$$C_h = 8 \text{ BTU/F}^\circ$$
Assuming that the ambient air temperature remains constant, then

\[ U_a(t) = \frac{P}{h_A A} \left( 1 - e^{-\frac{h_A A t}{C h}} \right) \]

where \( U_a(t) \) is the temperature difference between the air and the clutch. In Figure 20, \( \tau \) (period) is plotted against time for Dow Corning 200 Fluid [S], a silicon oil with a low viscosity-temperature gradient.

**Start Up**

If the mechanism is able to start without external assistance then a sufficient torque must be applied to the basket for a sufficient time to bring the basket to the clutch release displacement (\( \theta_2(t_2) \)). If this displacement is not reached, the basket will stall at some lower basket displacement since the clutch will never disengage. Suppose the motor and flywheel were brought up to the no load running speed, \( -\frac{T_Q}{\alpha} \) (Figure 6), and the clutch engaged. If subscript \( 3 \) denotes start up,

\[ I_T \ddot{\theta}_3 + C_2 \dot{\theta}_3 + K \theta_3 = V(\dot{\phi}_2 - \dot{\phi}_2) \]

where the solution is the same as for \( \dot{\phi}_2 \) and \( \dot{\phi}_2 \) except that the initial conditions have changed.
Figure 20. Period Versus Time for Warm-Up
Initial Conditions

\[ \phi_3 = \frac{-TQ}{Na} \quad \theta_3 = 0 \quad \dot{\theta}_3 = 0 \quad \ddot{\theta}_3 = 0 \quad \epsilon_3 = 0 \quad \epsilon_3 = 0 \]

However, these initial conditions affect only the constants of integration.

Taking the example of \( N = 16.0 \),
\( K = 112 \text{ in-lbf/rad (Appendix A)} \)

\[ b_1 = -0.81833 \]
\[ b_2 = -2.02242 \]
\[ b_3 = -43.25145 \]
\[ F = 13.836 \text{ radius} \]

where the solution is of the form
\[ \theta_2(t) = C_1 e^{b_1 t} + C_2 e^{b_2 t} + C_3 e^{b_3 t} + F \]

Then,
\[ \theta_2(\infty) = F \]
\[ \theta_2(\infty) = 13.836 \text{ radians} \]

For the example above,

\[ \theta_2(t_2) = 4.2 \text{ radians} \]

Therefore,

\[ \theta_2(\infty) > \theta_2(t_2) \]

and start-up is easily achieved. Figures 21, 22, 23 and 24 are plots of basket displacement, basket velocity, and driving member velocity for two combinations of reduction ratio and spring rate.
Figure 21. Basket Displacement Versus Time

K = 160 in-lbf./rad.
N = 16
Figure 22. Clutch Member Velocities Versus Time
Figure 23. Basket Displacement Versus Time

K = 112 in.lbf./rad.
N = 16
Figure 24. Clutch Member Velocities Versus Time
CHAPTER V

EXPERIMENTAL METHODS

The agi-basket model which has been used until the present is shown schematically in Figure 25(a). The damping in this case is assumed to be between the tub inertia and the support. Since there is no velocity dependent interaction between the tub and the support the placement of the damping in this model lacks physical significance. An experiment is conducted to determine whether this model or another shown schematically in Figure 25(b) is capable of representing basket motion and to determine the appropriate values of damping and inertia representative of the agi-basket. The model in Figure 25(b) has two inertias $I_T$ and $I_w$ connected by a dashpot $C$ and anchored to a support at $I_T$ by a spring with rate $K$. The inertia, $I_w$, signifies the contents of the basket (water and clothes). In this model all damping is considered to be between the basket and its contents.

In reality the system is much more complex than either of these models. Water and clothes trapped between paddles are partially carried along with the basket and separate regions of circulation are present at a distance from the paddles as seen from the operation of the experimental equipment. These rigid body models, however, provide a basis
Figure 25. Agi-Basket Models
for analysis.

**Equipment**

The apparatus used in this experiment is shown in Figure 26. A conventional vertical axis washing machine tub was used for the basket and a conventional central agitator is fixed to the center to provide paddles. (An outer container and a perforated basket were not used in order to simplify construction.) The tub diameter is 20.50 inches, the depth is 13.25 inches and it is filled with 108.5 lbm of water. The empty agi-basket weighs 20.03 pounds. Towels are used to represent the wash load. A displacement transducer is fixed to the bottom of the basket through a fork and pin and the basket is supported at the top by a tapered roller bearing and a bushing. A 6.5 inch diameter drive pulley is also fixed to the basket. A weight applies a torque to the basket by a wire attached to the drive pulley and a release mechanism between the tub and the bushing support serves as a trigger for an oscilloscope. The transducer is incorporated in a voltage divider providing a signal for the oscilloscope. A camera attachment records the displacement-time curve produced on the oscilloscope.

The spring in both agi-basket models in Figure 25 has been replaced by a constant torque due to the weight (if the inertia of the weight is added to the tub inertia $I_T$) for this experiment. It was found that the constant torque
simplifies the analysis of data since the torque applied by a spring is dependent upon displacement. Since the spring is external to the basket and its rate determined only by the designer, this substitution is justified.

**Procedure**

Six towels weighing 0.75 pounds each were found to provide an agitation representative of the cleaning action in a vertical axis washing machine when a weight of 20.0 lbf was used to accelerate the tub. The transducer was calibrated by turning the basket through 360° by ten degree increments. Linearity was observed throughout this range.

For the real agi-basket the motion is reversed every 0.7 seconds, therefore the data was recorded for 0.7 seconds after release and read every 0.1 seconds. It is, after all, the transient response of the tub that yields the system variables $C$, $I_T$, and $I_W$ and at a longer time from release the steady-state response becomes dominant.
CHAPTER VI

ANALYSIS OF DATA

The data from this experiment is a plot of basket angular displacement versus time for an agi-basket accelerated by a constant torque $T$. Schematics for the one mass and two mass models with damping are shown for this situation in Figure 27 a and b.

A third system is included (Figure 27c) to facilitate comparison of the other two models. For each model a solution is found for the displacement of the inertia $I_T$, that is

$$\theta_{IC} = \theta_{IC}(t,I_T,C)$$

$$\theta_{ICI} = \theta_{ICI}(t,I_T,C,I_w)$$

$$\theta_I = \theta_I(t,I_T)$$

Values of inertia and damping ($I_{T_1}, C_1, I_{T_2}, \ldots$, etc.) are found that give the best correlation between the particular solution and the experimental curve at the points where the data is read. The models are then judged by the degree of correlation.

The criterion for judging the degree of correlation between the function $\theta_{IC}$, $\theta_{ICI}$, or $\theta_I$ and the data points
Figure 27. Models of Agi-Basket Used in Experiment
$H_0, H_1, \ldots, H_7$ is the variance, defined by

$$E_{IC}(I_T, C) = \sum_{i=0}^{7} [\theta_{IC}(t_i, I_T, C) - H_i]^2 \quad (23)$$

$$E_{ICI}(I_T, C, I_W) = \sum_{i=0}^{7} [\theta_{ICI}(t_i, I_T, C, I_W) - H_i]^2 \quad (24)$$

$$E_I(I_T) = \sum_{i=0}^{7} [\theta_I(t_i, I_T) - H_i]^2 \quad (25)$$

where $t_i = 0.1(i)$ seconds.

**One Mass with Damping**

The differential equation for the one mass system with damping is

$$I_T \dddot{\theta}_{IC} + C \dot{\theta}_{IC} = T$$

with initial conditions

$$\theta_{IC} = 0 \quad \theta = 0 \quad t = 0$$

$$\dot{\theta}_{IC} = 0 \quad \dot{\theta} = 0 \quad t = 0$$

for which the solution is

$$\theta_{IC}(t, C, I_T) = \frac{I_T}{C_1^{1/2}} e^{-\frac{C}{C_1 T} t} + \frac{T}{C} t$$
To minimize the variance $E_{IC}$, set

$$\frac{\partial E_{IC}}{\partial C} = 0 = -2 \sum_i \left[ \theta_{IC}(t_i, I_T, C) \cdot H_i \right] \frac{\partial \theta_{IC}(t_i, I_T, C)}{\partial C}$$

$$\frac{\partial E_{IC}}{\partial I_T} = 0 = -2 \sum_i \left[ \theta_{IC}(t_i, I_T, C) \cdot H_i \right] \frac{\partial \theta_{IC}(t_i, I_T, C)}{\partial I_T}$$

and solve these two simultaneous equations for $I_T$ and $C$. Newton's method for simultaneous equations is used where initial values are chosen for $C$ and $I_T$. Denoting these initial values $C_0$ and $I_{T_0}$ and each iterative value $C_i$ and $I_{T_i}$, each successive value is generated by

$$C_{i+1} = C_i + \begin{vmatrix} \frac{-\partial E_{IC}}{\partial C} & \frac{\partial^2 E_{IC}}{\partial C \partial I_T} \\ \frac{-\partial E_{IC}}{\partial I_T} & \frac{\partial^2 E_{IC}}{\partial I_T \partial C} \end{vmatrix}^{-1} \begin{vmatrix} \frac{\partial^2 E_{IC}}{\partial C \partial I_T} \\ \frac{\partial^2 E_{IC}}{\partial I_T \partial C} \end{vmatrix} (C_{i} - C_{i-1})$$

$$I_{T_{i+1}} = I_{T_i}$$

$$C_{i+1} = C_i + \begin{vmatrix} \frac{\partial^2 E_{IC}}{\partial C \partial I_T} & \frac{\partial^2 E_{IC}}{\partial I_T \partial C} \\ \frac{\partial^2 E_{IC}}{\partial I_T \partial C} & \frac{\partial^2 E_{IC}}{\partial I_T^2} \end{vmatrix}^{-1} \begin{vmatrix} \frac{\partial^2 E_{IC}}{\partial C \partial I_T} \\ \frac{\partial^2 E_{IC}}{\partial I_T \partial C} \end{vmatrix} (I_{T_i} - I_{T_{i-1}})$$

$$C = C_i$$

$$I_T = I_{T_i}$$
The iteration terminates when

\[ |C_{i+1} - C_i| < \Delta C \]

and

\[ |I_{T_{i+1}} - I_i| < \Delta I_T \]

For this analysis

\[ \Delta C = 0.001 \text{ in-lbf-sec/rad} \]
\[ \Delta I_T = 0.001 \text{ in-lbf-sec}^2/\text{rad} \]

The minimized \( E_{IC} \) is obtained from the final \( I_{T_i} \) and \( C_i \) by equation (23).

Two Mass System with Damping

The differential equations for the two mass system
shown in Figure 27 are

\[ \ddot{I}_T^0 + \ddot{I}_W^0 = T \]  \quad (26)

\[ \ddot{I}_W^0 = C(\dot{\delta}_l^{ICI} - \dot{\phi}_l^{ICI}) \]  \quad (27)

with initial conditions

\[ \theta_l^{ICI} = 0 \text{ at } t = 0 \]

\[ \delta_l^{ICI} = 0 \text{ at } t = 0 \]

\[ \dot{\phi} = 0 \text{ at } t = 0 \]

Differentiating equation (27) with respect to time and rearranging

\[ \frac{\ddot{I}_W^0}{C} + \dot{\phi}_l^{ICI} - \dot{\phi}_l^{ICI} = 0 \]

Combining with equation (26)

\[ \ddot{\phi}_l^{ICI} + C(\frac{1}{T} + \frac{1}{W})\dot{\phi}_l^{ICI} = \frac{TC}{T + \frac{1}{W}} \]

This equation can easily be solved for \( \ddot{\phi}_l^{ICI} \)

\[ \ddot{\phi}_l^{ICI} = A'e^{qt} + \frac{T}{T + \frac{1}{W}} \]
where \( A' \) is a constant of integration and

\[
q = -C\left(\frac{1}{T} + \frac{1}{I_W}\right)
\]

Substitute this solution for \( IC1 \) into equation (26)

\[
\ddot{\theta}_{IC1} = Ae^{qt} + \frac{T}{I_T + I_W}
\]

Integrating twice with respect to time and applying the initial conditions

\[
\theta_{IC1}(t, I_T, C, I_W) = \frac{A}{q^2} e^{qt} + \frac{Tt^2}{a(I_T + I_W)} - \frac{A}{q} + \frac{A}{q^2}
\]

where \( A = \frac{I_W^T}{I_T(I_T + I_W)} \)

Using Newton's method, as in the previous model, \( I_T, C, \) and \( I_W \) are found which minimize \( E_{IC1}(t, I_T, C, I_W) \). Again the iteration is terminated by the test,

\[
|I_{T_{i+1}} - I_{T_i}| < \Delta I_T
\]

\[
|C_{i+1} - C_i| < \Delta C
\]

\[
|I_{W_{i+1}} - I_{W_i}| < \Delta I_W
\]
where $\Delta I_T$ and $\Delta C$ are as given for the first model and

$$\Delta I_W = 0.001 \text{ in-lbf-sec}^2/\text{rad}$$

The minimized $E_{I_C I}$ is then evaluated from equation (24).

**One Mass System Without Damping**

The differential equation for the single mass without damping is

$$I_T \ddot{\theta}_I = T$$

with initial conditions

$$\theta_I = 0 \quad t = 0$$
$$\dot{\theta}_I = 0 \quad t = 0$$

The solution of the differential equation is

$$\theta_I(t, I_T) = \frac{T}{2I_T} t^2$$

To minimize $E_I$ set

$$\frac{\partial E_I}{\partial I_T} = -2\sum_i \left[ \theta_i(t_i, I_T) - H_i \right] \frac{\partial \theta_i(t_i, I_T)}{\partial I_T} = 0$$
The minimized $E_I$ is then calculated from equation (25).

A summary of these results can be found in Table 2. Values of $C$ and $I_T$ for this single mass with damping are in good correlation with $C$ and $I_T$ used in the analysis of the proposed drive mechanism (4.44 in-lbf-sec$^2$/rad and 28.8 in-lbf-sec/rad, respectively). As judged by the variance $E$, the two mass model with damping provides a more accurate displacement-time curve than either of the other models. More complex models might be expected to be in even closer agreement to experiment. While the analysis will not be carried out again with the two mass model here due to the complexity of the solutions in the coupled portion of basket motion (Chapter III) this is recommended for future research.
Table 2. Experimental Results

<table>
<thead>
<tr>
<th>MODEL</th>
<th>$E_{\text{rad}^2}$</th>
<th>$I_{T_{\text{in. lb sec}^2}}$</th>
<th>$C_{\text{in. lb sec}^2/\text{rad}}$</th>
<th>$I_{W_{\text{in. lb sec}^2/\text{rad}}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 MASS with DAMPING</td>
<td>0.00179</td>
<td>4.509</td>
<td>31.949</td>
<td></td>
</tr>
<tr>
<td>2 MASS with DAMPING</td>
<td>0.00029</td>
<td>2.197</td>
<td>55.234</td>
<td>30.318</td>
</tr>
<tr>
<td>1 MASS w/o DAMPING</td>
<td>0.08223</td>
<td>11.31</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX A

MAIN PROGRAM

C VISCOS CLUTCH APPLICATION
C
C IMPLICIT REAL (A-Z)
C COMPLEX *C
C
C WIT=IP(T)*X2+H(T)+P(T)
C LI=T-H(T)*X2
C
C PLAD (b, 100) K, C, JT, JF, BRKMIN, TO, ALFA, TAU
C 100 FORMAT (F8.0)
C
K=120.0
050 K=K+20.0
IF (K*6T. 180.0) STOP
T=15.0
300 N=N+1.0
IF (N*6T. 18.0) GO TO 50
C
JF=J=F(*1.0)
T=J+K
ALFA=ALFA+K
BRKMIN=BRKMIN./N.
C
W1=SQRT(4.0+JT-K*C)/C
B1=1.0/JT
TJ1=(-5.1555)/W1
C7=(1.5)*3.14159/(1.0+FXP((-1.0)*TJ1))
C6=(3.5)*W7/W1
C
THMIN=C7
THMAX=C7+3.14159
C
C BEGIN LOOP A
C FIND VALUE OF V TO GIVE PROPER TAU
V5=1.0*1-9A.0
V6=1.0*1=47.0
N=2.0
200 V=V5+V6/2.0
WHITE (6,140) V
140 FL=1.0-V3*V3
A1=1.0+F((1.0)+JT1*V-ALFA)/(JF*JT)
A2=1.0+F*V=W+ALFA*V)/(JF+JT)
A3=V-ALFA)/(JF*JT)
C
P=(1.0)*A1/A1/3.0
S=(3.0*1.0*3.0)-1.0*A2*A2*27.0*27.0/27.0
C
C TEST FOR F车 OF SQ UL ION OR THE TAE
Z=1.0
IF (15*Z/4.0+1.0*3/27.0)=LT. 0.0) GO TO 902
C
C EVALUATE COST TS ON 4 * POLYNOMIAL COEFF.
"B7 = T(1-5)/2.0, 4.5, 3.0, 1.0, 0.0, 0.0) 0 FD. 10
"C7 = T(1-5)/2.0, 3.0, 1.0, 0.0, 0.0, 0.0) 0 FD. 10
C 902 IF (b < 0.5) 105 ELSE (OVERFLOW))
Z=3
C C EVALUATE CONSTANTS FOR OVERPUMPED CASE
X=-5/2.0
Y=5*RT(-((S*S)/20.0)+(R**2/27.0))
AC=C*RT(C*PLX(-1.0))
BC=C*RT(C*PLX(+1.0))
C BI=REAL((C+AC)/-1.3.0)
BJ=REAL((C+AC)/-1.2.0)+CRT(1.3.0)*(AIMAG(AC-BC)/2.0)-A1/3.0
B3=REAL((C+AC)/-1.2.0)-5*RT(1.3.0)*(AIMAG(AC-BC)/2.0)-A1/3.0
F=(T0*V)/(JF*JT*A3)
C C BEGIN LOOP A
C C FIND VALUE OF PHMAX
C 601 CONTINUE
PHMAX=2*W**N=1.05
PHMAXL=TU/ALFA
N=0.0
201 PHMAX=(PHMAXL+PHMAXS)/2.0
C IF (7*GT. 2.0) GO TO 602
DELI=JT*(-n2)+1.22*222**2+N2*222*02*AM
C1=IF-TH1MIN=1.02*222*N2*JT+W2*222*JT)-2*V*PHMAX+W2*K+TH1MIN)
1 LEN1
C=TH1MIN-F=C1
C2=(K/JT)*TH1MIN=(V/JT)*PHMAX+C1*MEM*2+C2*(W2*222-W2*222))/(2.0*22)**
1 n2)
GO TO 603
C 602 DELI=21((n2**2)+1.3**2)+22((n2**2)-1.2**2)+23((n2**2)-1.2**2)+22((n2**2)-1.2**2)
C1=IF-TH1MIN=1.02*222*N2*JT+W2*222*JT)-2*V*PHMAX+W2*K+TH1MIN)
1 -(K/JT)*TH1MIN)/JEN1
C2=(TH1MIN-F)=C2-1
WHIT((n2,0090)+B1+B2+B3)+C1+C2+C3
089 FuncMAT (T3, 51415.6)
C C BEGIN LOOP C
C C FIND 12 AND T3 BY II, k, METHOD
C 603 CONTINUE
T3=0.0
T3=1.1(21)+3.1(159)/W1
W4=0.0
C 202 IF (7*GT. 2.0) GO TO 604
```plaintext
C END LOOP B
391 IF (*3+12) LT 0.085) GO TO 800
IF ((*3+12) .GT. 1.0) V=V
IF ( (*3+12) .LT. 1.0) V=V
M=H+1.7
IF ( (*3+12) .LT. 1.2-1) GO TO 903
GO TO 200
C END LOOP A
C CALCULATE POWER AND POWER LESS
C
800 W=0.*V+0.*T2+0.*T1+2.*V+0.*W(.1*T1)+2.*V+0.*W(.2*T1)+2.*V+0.*W(.3*T1)+
2.*V+0.*W(.4*T1)+2.*V+0.*W(.5*T1)+2.*V+0.*W(.6*T1)+2.*V+0.*W(.7*T1)+2.*V+0.*W(.8*T1)
GO TO 301
END
```

```
C WRITE (6,051) K
051 FORMAT (6X,K1,F4,1)
WRITE (6,103) M
103 FORMAT (6X,N=1,F4,1)
TH1MAX=V*PH1MAX
WRITE (6,791) T;GAX
791 FORMAT (6X,TR=AA2,F12,5)
C
WRITE (6,110) V
110 FORMAT (6X,V=1,F12,5)
WRITE (6,111) P;MAX
111 FORMAT (6X,P=1,F12,5)
WRITE (6,112) T;I13
112 FORMAT (6X,T=1,F12,5)
WRITE (6,113) TH1;X
113 FORMAT (6X,TH1=1,F12,5)
WRITE (6,780) T;I13
780 FORMAT (6X,T=1,F12,5)
WRITE (6,114) T;1
114 FORMAT (6X,T=1,F12,5)
WRITE (6,115) T;2
115 FORMAT (6X,T=1,F12,5)
WRITE (6,116) T;3
116 FORMAT (6X,T=1,F12,5)
WRITE (6,117) T;4
117 FORMAT (6X,T=1,F12,5)
WRITE (6,368) T;2R12
368 FORMAT (6X,T=1,F12,5)
WRITE (6,119) T;1
```

```
C END
```
114 FORMAT (7x, 'I &1=**F12.5)
115 WRITE (6,117) T
119 FORMAT (7x, 'T1=**F12.5)
C IF (T GT 2.0) GO TO 311
WRITE (6,118) T
120 FORMAT (3x,T1=**F12.5)
WRITE (6,119) T
121 FORMAT (7x, 'T2=**F12.5)
WRITE (6,122) T
122 FORMAT (7x, 'T2=**F12.5)
GO TO 311
C 311 WRITE (6,390) T
360 FORMAT (7x, 'F=**F12.5)
WRITE (6,391) T
361 FORMAT (7x, 'F=**F12.5)
WRITE (6,392) T
362 FORMAT (7x, 'F=**F12.5)
C 310 WRITE (6,123) T
123 FORMAT (7x, 'F=**F12.5)
WRITE (6,124) T
124 FORMAT (7x, 'F=**F12.5)
WRITE (6,125) T
125 FORMAT (7x, 'F=**F12.5)
WRITE (6,126) T
126 FORMAT (7x, 'F=**F12.5)
WRITE (6,127) T
127 FORMAT (7x, 'F=**F12.5)
WRITE (6,128) T
128 FORMAT (7x, 'F=**F12.5)
WRITE (6,129) T
C IF (T GT 2.0) GO TO 300
GO TO 390
390 WRITE (6,391)
391 FORMAT (7x,T1=**OVERPAMPED)
392 IF (T1 LT 360) GO TO 901
GO TO 300
901 WRITE (6,104)
104 FORMAT (7x,F0.1)
C 903 WRITE (6,106)
106 FORMAT (7x,F0.1)
GO TO 500
904 WRITE (6,107)
107 FORMAT (7x,F0.1)
GO TO 500
905 WRITE (6,108)
108 FORMAT (7x,F0.1)
GO TO 500
C DEFINE FUNCTIONS P(I) AND T(I)
FUNCTION TH(I)
FUNCTION: P(T)

IF (T > 1.0, 0)

1. P = C1*EXP(10*T) + C2*EXP(10*T) + C3*EXP(10*T)

IF (T > 2.0)

2. P = C1*EXP(10*T) + C2*EXP(10*T) + C3*EXP(10*T)

P = P

---

\[ P_{ik} = 0.7019 \]  \[ P_{i0} = 0.15058 \]

---

n=16, 6

T1, T2, T3 = 1982, 45240

---

W = 0.32684

---

T = 12.358

---

T = 11.166

---

T = 9.862

---

T = 7.462

---

T = 6.162

---

T = 4.862

---

T = 3.639

---

T = 2.242

---

T = 1.045

---

T = 0.31921

---

T = 0.15058

---

T = 0.07919

---

T = 0.015058

---

T = 0.007919

---

T = 0.0015058

---

T = 0.0007919

---

T = 0.00015058

---

T = 0.00007919

---

T = 0.000015058
APPENDIX B

PLOTS

C PLOT OF DISPLACEMENT AND VELOCITIES

IMPLICIT REAL (A-Z)
INTEGER I

DIMENSION THETA(121), THETAF(121), PHIF(121), IBUFF(1200), T(121)

READ (5,201) Z, M, K, U, PHIMAX, THMINT, T2, T3

READ (5,201) Q1, W1, H, Q2, W2, F

READ (5,203) C1, C2, C3, C4, C5, C6

TO=431.0
ALFA=-2.2
JF=1.00
C7=THMINT
C6=Q1*C7/W1

IF (Z .LT. 2.0) GO TO 110
B1=M
B2=Q2
B3=W2

C

110 JF=JF+M
TO=TO+M
ALFA=ALFA+M

DO 104 I=1,121
T(I)=(I-1)*.02

100 T(I)=T(I)*2.5
THETA(I)=THETA(I)*.572958
THETAF(I)=THETAF(I)*(.572958)

102 IF (T(I) .GE. T2) GO TO 103
THETA(I)=TH2(I-T(I)-(T2-T3))
THETAF(I)=TH2F(I-T(I)-(T2-T3))

103 THETA(I)=TH1(I)-(T2-T3)
THETAF(I)=TH1F(I)-(T2-T3)

104 PHIF(I)=PHIF(F(I)+(T2-T3))
CALL PLOTS (1, UWF: 1200, 3)
CALL PLOT (1, UWF: 100, 3)
CALL AXIS (0.0, 0.0, 1, TIME: 0.0, 0.0, 0.0, 0.4)
CALL AXIS (0.0, 0.0, 1, THETA (DEG): 0.0, 0.0, 0.0, 0.4)
CALL PLOT (T(1), THETA(I) + 3)
DO 105 I = 2, 121
  105 CALL PLOT (T(I), THETA(I) + 2)
CALL PLOTS (1, UWF: 1200, 3)
CALL PLOT (12.0, 0.0, 0.3)
CALL AXIS (0.0, 0.0, 1, TIME: 0.0, 0.0, 0.0, 0.4)
CALL AXIS (0.0, 0.0, 1, VELOCITY (RPM): 0.0, 0.0, 90.0, 150.0, 50.0)
CALL PLOT (T(I), THETA(I) + 3)
DO 106 I = 2, 121
  106 CALL PLOT (T(I), THETA(I) + 2)
CALL PLOT (T(1), PHII(I) + 3)
DO 107 I = 2, 121
  107 CALL PLOT (T(I), PHII(I) + 2)
STOP

FUNCTION TH1(T)
TH1 = EXP(-Q1*T) * (C7 * COS(W1*T) + C8 * SIN(W1*T))
RETURN
C
FUNCTION TH2(T)
IF (Z < L. 2.0) TH2 = C1 * EXP(M*T) + EXP((-Q2)*T) * (C2 * COS(W2*T) + C3 * SIN(W3*T))
A + C5 * SIN(W2*T2) + EXP((-Q2)*T) * (-C2) * W2 * SIN(W2*T) + C3 * W2 * COS(W2*T))
IF (Z > L. 2.0) TH2 = C1 * EXP(B1*T) + C2 * EXP(B2*T) + C3 * EXP(B3*T) * F
RETURN
C
FUNCTION TH1F(T)
TH1F = EXP(-Q1*T) * (W1*C8 - Q1*C7) * COS(W1*T) - (W1*C7 + Q1*C8) * SIN(W1*T)
RETURN
C
FUNCTION TH2F(T)
A * B3 * T)
RETURN
C
FUNCTION PHIF(T)
PHIF = (T/Q/ALFA + PHIIAX) * EXP(ALFA/JF*T) - T/Q/ALFA
RETURN
C
FUNCTION PH2F(T)
IF (Z < L. 2.0) PH2F = C4 * EXP(M*T) * EXP((-Q2)*T) * (C5 * COS(W2*T) + C6 * A * SIN(W2*T)) + F * K/V
IF (Z > L. 2.0) PH2F = C4 * EXP(B1*T) + C5 * EXP(B2*T) + C6 * EXP(B3*T) + F * K/V
RETURN
END
APPENDIX C

TWO MASS WITH DAMPING

```
DIMENSION TH(50)
N=8

DO 106 I=1,N
READ (5,100) TH(I)
100 FORMAT (F6.0)
WRITE (6,106) I,TH(I)
106 FORMAT (12X,13X,6X,F8.5)

K=0
C=50.0
AW=30.0
AT=3.0

F=66.2

CONTINUE
IF (K .GE. 12) STOP
E=0.
EC=0.
EAW=0.
EAT=0.
ECC=0.
ECAW=0.
ECAST=0.
EAWAT=0.
EAWAT=0.
EATAT=0.

K=K+1

Q=(-C)*((1./AW+1./AT)
C1=AW*F/AT/(AT+AW)

DO 500 I=1,N
T=0.1*FLOAT(I-1)
EX=EXP(Q*T)

D=C1/Q*Q*(EX-1.)-C1*Q+F*Ex*T/2./(AT+AW)

DG=(-2.)*C1/(Q*Q)*(EX-1.)*C1/Q/T*EX*(EX+1.)
DC1=(EX-1.)/Q/0=T/Q

QC=Q/C
QAT=C/AT/AT
QA=C/AW/AW

C1AT=Aw*F/(2.*AT+AW)/((AT+AT+AT*AW)*2)
C1=Aw*F/((AT+AW)*2)

QC=Q/C
QA=Q/AT+Q/AT+(Q*Q)/((AT+AW)*2)

C1AT=Aw*F/(2.*AT+AW)/((AT+AT+AT*AW)*2)
```

`JCI1=0.0`
QCAT = 1./AW/AT
QCAT = 1./AT/AT
QATAT = (-2.) * C/(AT**3)
QAWAW = (-2.) * C/(AW**3)

C1ATAT = F * (6 * AT**4) * AT + 12 * (AT**4) * AT + AT + AT + AT**3
1 + 2 * AT/(AT + AW)**3
C1AWAT = (-2.) * F / (AW + AW)**3)
C1AWAW = C1AWAT

DCC = DQG * QC * QC
DCAW = DQG * CAW * QC + DQG * CAW
DQAT = DQG * QAT * QC + DQG * QAT

DAWAW = (DQG * QAW + DQG * QAW) * qAW + (DQG * CAW + DQG * CAW) * qCAW
1 + DQG * CAWAW + DQG * CAWAW * AT + (AT + AW)**3
DATAT = (DQG * QAT + DQG * QAT) * qAT + (DQG * CAW + DQG * CAW) * qCAW
1 + DQG * QATAT + DQG * QATAT * AT + (AT + AW)**3

C
S = T/I - D
E = E + S**2
C
EC = EC - 2.* S * DC
ECAW = EAW - 2.* S * DAW
EAT = EAT - 2.* S * DAT
ECC = ECC + 2.* (DC + DC - S * DCC)
ECAW = ECAW - 2.* (DC + DAW - S * DCAW)
EAT = EAT - 2.* (DC + DAT - S * DCAT)

C
500 CONTINUE
WRITE (6, 499) C, AW, AT, E
C
EAWA = EAWAT
EATC = EAT
EAWC = ECAW
C
Z = ECC * (EAWA + EATAT - EATAW + EAWAT) + ECC * (ECAW + EATAT - EATAW + EATC)
1 + EATC * (ECAW + EAWAT - EAWA + EATC)
C
DELT C = EAW * ECC * EAWAT + EATAW * EAWAT - ECC * EAWA - EATAW * EAWAT
1 - EATC * (ECAW + EATAT - EATAW + EAWA + EATC - ECC * EATAT * EATC)
1 - EATC * (ECAW + EATAT - EATAW + EAWA + EATC)
C
C = C + DELTA
AW = AW + DELTA
AT = AT + DELTA
C IF (DELTC .GT. 0.001) GO TO 401
IF (DELTAW .GT. 0.001) GO TO 401
IF (DELTAT .GT. 0.001) GO TO 401

C WRITE (6,501) C, AW, AT
501 FORMAT (6X*'FINAL RESULT' C='*F12.8,4X,*AW='*F12.8,4X,*AT='*F11.8)
STOP
END
APPENDIX D

ONE MASS WITH DAMPING

```
DIMENSION TH(I,50)
C
N=8
DO 100 I=1,N
READ*(5,7,E0,THT)
100 FORMAT (F6.0)
C
A=
C=30.0
F=66.2
K=0
C
400 CONTINUE
1+ (K .GT. 12) STOP
C
E=0.
EC=0.
E=0.
ECA=0.
EAA=0.
C
K=K+1
C
DO 500 I=1,N
T=0.1*FL*AT(I-1)
EX=EXP((I-C)/A-T)
C
D=A+F/C/C*(EX-1.)+F/C*T
DC=2.1*F/C*(EX-1.)*F*T/C/C*(EX+1.)
D=SF/C/C*(EX-1.)+F*T/A/C*EX
DC=0.1*F/C*(EX-1.)*F*T/A/C*EX
DC=0.1*F/C*(EX-1.)*F*T/A/C*EX
D=EX-1.1*F/T/A/C/EX
DA=EX-1.1*F/T/A/C/EX
C
S=TH(I)-T
EC=EX-2.*S
EA=EA-2.*S
EC=EC-0.5*(DC*DC-2.*DC)
EA=EA-0.5*(DA*DA-2.*DA)
C
500 CONTINUE
C
IF (K .GT. 12) C, E E
400 FORMAT (10x,C1,F12.8,F12.8,F12.8,F12.8)
C
FAC=CA
Z=EC*EA-ECA*EC
LT=EC*CA-ECA*CA/2
LT=EF*LC-EC*EA/2
C
CA=CA+ELC
A=A+ELT
```
IF (DELTA .GT. 0.001) GO TO 400
IF (DELTA .GT. 0.001) GO TO 400
C
WRITE (6,501) C, A
501 FORMAT (Fx, 'FINAL RESULT: C='F12.8, 'A='F12.8)
STOP
END
APPENDIX E

ONE MASS WITHOUT DAMPING

```
DIMENSION TH(50)
T(I)=0.1*FLOAT(I-1)
F=66.2

NEW
DEN=0.0
DO 106 I=1,N
  READ (5,106) TH(I)
  100 FORMAT (F5.0)
  THE=TH(I)
  DEN=DEN+THE
  WRITE (6,106) I,THE
  106 FORMAT (6,I16)
  NUM=0.0
  DO 700 I=1,N
    NUM=NUM+T(I)*T(I)
    700 WRITE (6,701) A,E
  A=F/2.0+NUM/DEN
    E=0.0
  DO 300 I=1,N
    G=0.5*T(I)*T(I)*F/A
  300 E=E+T(I)*G==2
  WRITE (6,701) A,E
  701 FORMAT (+/+/6X,+A==rF12.7,4X,rE==rF12.6)  
STOP
END
```
BIBLIOGRAPHY

Cited Literature


Other References