HYDRODYNAMICS OF INITIAL MIXING ZONES
OF WASTEWATER DISCHARGES

Final Technical Report
EPA Grant Number: R 826216

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Executive Summary

The successful development of a scanning laser system for three-dimensional Laser-Induced Fluorescence (3DLIF) imaging in large-scale turbulent stratified flows is described. The objectives were to obtain experimental data on the physics of turbulent mixing processes in buoyancy-modified flows typical of industrial and municipal wastewater discharges. The data will be used to refine the mathematical models of dilution and mixing zones used by EPA, and to test the hypothesis that the end of the near field mixing zone is caused by turbulence collapse under the influence of gravity forces. The data consist primarily of laser-induced fluorescence (LIF) images that are converted to tracer concentration fields. Image processing techniques are then used for visualizing the results in three dimensions. Refractive index matching is used to enable use of the system in density-stratified environments.

The experimental system consists of two laser beam scanners, a high-speed video camera, a computer for image acquisition, and a separate computer for system timing and control. The scanners are two orthogonal galvanometer mirrors that are driven by an analog waveform supplied by a data acquisition card in the control computer. This card controls the overall timing of the system by also sending a synchronized digital signal to control the camera image exposure and image acquisition. The camera is capable of frame rates up to 260 frames per second. Images are written directly to a computer hard disc system in real time, which avoids memory limitations on experimental duration.

The system is described in detail and its performance is verified by comparing experimental results to well-studied flow for which much information is available. The flows are a round jet in a homogenous, stationary environment, and vertical round buoyant jets in flowing stratified and unstratified environments.

Example applications to various complex flow problems are described. The flows are all typical of wastewater discharges into the environment. All are from multiport diffusers. First is a produced water outfall in California. This consists of high-momentum jets discharging into a flowing, stratified environment. Next is the Boston tunneled outfall, which is has multiport risers. Again, the discharge is into a stratified flow. Field data exist for this case and comparisons will be reported at a later date. Next is discharge from a thermal power plant. This is a shallow water discharge in which the mixing is dominated by the momentum of the jets. Finally, generic experiments on multiport diffusers are reported. The receiving waters are flowing and stationary. In all cases, the general characteristics of the flow are a rapid increase in dilution within the mixing zone. The dilution eventually levels off where the self-induced turbulence collapses under the influence of buoyancy forces. This marks the end of the near field.

These experiments and analyses are continuing and will be reported in detail in future publications.
1. INTRODUCTION

1.1 Background

There are thousands of wastewater discharges into rivers, lakes, estuaries, and coastal waters in the United States and around the world. These span a huge range of flow rates from small municipal and industrial discharges of a few gallons per day up to the Boston Sewage Outfall which has a capacity of more than a billion gallons per day. The environmental impacts of contaminants contained in these discharges are highly dependent on the way in which they are released. For example, discharge as a high velocity jet from a diffuser results in turbulence which causes efficient mixing. This results in a rapid reduction in the concentrations of toxics and confines the impact of the discharge to a small area. Any risk to human health and ecosystems is significantly reduced.

These effects are recognized as a part of toxics control by the EPA and other regulatory agencies, by allowing a “mixing zone” around the discharge (U.S. EPA, 1991). The mixing zone is an “allocated impact zone” where numeric water quality criteria can be exceeded as long as acutely toxic conditions are prevented. An example is the requirements for the mixing zone for coastal water municipal wastewater discharges contained in Section 301(h) of the Clean Water Act (U.S. EPA, 1994). Within the mixing zone, the “initial dilution” of the discharge occurs, and (in this case) the mixing zone is called a “zone of initial dilution” (ZID). The ZID includes the bottom area and water column and is a regularly shaped area surrounding the diffuser of dimension equal to the water depth, $H$. Most states also make use of a mixing zone. The zone may be defined as a length, an area, or a volume; it may be expressed as a width or cross-section excess for streams or rivers, or a surface area for lakes, estuaries, and coastal waters.

Water quality criteria apply at the boundary of the mixing zone, rather than within the mixing zone itself. The U.S. EPA maintains two water quality criteria for allowable concentrations of toxic discharges: The Criterion Maximum Concentration (CMC), and the Criterion Continuous Concentration (CCC). The purpose of the CMC is to protect against acute or lethal (short term) effects, and the purpose of the CCC is to protect against chronic (long term) effects. The CCC must be met at the edge of the mixing zone. Details of the various mixing zones and the four alternatives allowed for prevention of lethality to passing organisms are given in U.S. EPA (1991).

Prediction of water quality at the edge of a mixing zone is therefore often required. For example, it is a vital part of the National Pollutant Discharge Elimination System (NPDES) permitting process. This requires prediction of turbulent mixing processes, and reliable prediction is essential for rational and appropriate choice of wastewater treatment. Consider, for example, the San Diego sewage outfall, where modeling of initial mixing played a major role in the granting of a waiver from secondary treatment resulting in savings of hundreds of millions of dollars. The mathematical model used, RSB, was developed by the Principal Investigator of this project under a previous EPA grant.
Mixing zones generally consist of two distinct regions, commonly called the near and far fields. This is illustrated in Figure 1.1 for the case of a wastewater discharge from a multiport diffuser into a stratified water environment flowing with a speed \( u \). The main physical mechanism responsible for mixing the discharge and reducing the concentrations of contaminants contained within the discharge is turbulence. There is a distinction between the two fields: one lies in the source of this turbulence, and the other lies in the discharge itself. Turbulence induced by the high source exit velocity, or the buoyancy of the discharge, or both. In the near field, the turbulence responsible for mixing is that naturally present in the receiving environment, for example, turbulence due to boundary shear in a flowing river.

The difference between the near and far fields is illustrated by the photographs shown in Figure 1.2. These are discharges from a round nozzle into a coflowing turbulent stream. In Figure 1.2a the release is iso-kinetic, i.e. at the same velocity as the ambient flow. The mixing is therefore entirely due to the ambient turbulence, and so is entirely in the far field. In Figure 1.2b the source discharge velocity is higher than the ambient velocity. The excess source momentum flux generates shear that causes turbulence. The mixing is therefore initially in the near field. This self-induced turbulence decays with distance from the source, and eventually ambient turbulent mixing dominates. The mixing then moves into the far field.

Other terminology is sometimes used in the context of mixing zones. Examples are Regulatory Mixing Zones, Hydrodynamic Mixing Zones, and the Zone of Initial Dilution (ZID). The Hydrodynamic Mixing Zone is synonymous with the near field. A Regulatory Mixing Zone is specified by some environmental regulatory agency; water quality regulations must be met at the boundary of this zone. The Regulatory Mixing Zone may or may not coincide with the near or far fields. Indeed, a future issue is to reconcile regulatory definitions and concepts of mixing zones, which have not kept up with recent advances in our understanding of the hydrodynamics of mixing zones.
Most real discharges are modified by buoyancy effects and are more complex than the simple situations of Figure 1.2. A wide range of flow possibilities exist as shown in the examples sketched in Figure 1.3. Complications arise due to interactions between discharges from multiple ports, arbitrary discharge angles, free surface and bottom boundary interactions, and the effects of ambient density stratification.

These discharges can be classified as buoyant jets. A buoyant jet is a jet whose density differs from the receiving fluid density so that both momentum and buoyancy affect the flow dynamics. For example, sewage with approximately the density of fresh water discharged into denser coastal ocean waters, and thermal wastewater discharged into colder rivers or lakes. The dilution of a buoyant jet results from entrainment of ambient water along its trajectory. It is difficult to predict this dilution, however, because of the complexity and variety of the hydrodynamic process involved. Understanding the mechanics of buoyant jets for various flow conditions, is essential for the design of wastewater diffusers. For these buoyancy-modified flows, the dilution is ultimately limited by collapse of the turbulence due to buoyancy forces (which also marks the end of the near field). The conditions leading to this collapse are not well understood. This study will focus on the effect of cross flows and ambient stratification on the mixing of buoyant jets in configurations typical of wastewater discharges and the collapse of the near field turbulence.

1.2 Mathematical Models

In order to predict mixing of buoyant jets under various conditions, mathematical models are commonly used. These will be described further in Section 2.2. The models fall into three general types. The first were entrainment models, where the rate of entrainment into the jet or plume is computed from local conditions. Entrainment models work fairly well with relatively deep receiving waters, but not in shallow waters where the supply of entraining water may be limited and mixing at the boundaries may be significant. These problems are overcome to some extent by semi-empirical models based on dimensional
analyses. These models classify the flows according to the relative magnitudes of characteristic length scales, and employ asymptotic solutions based on experimental data. Length scale models can be unreliable when applied near transitions in flow behavior, and are not well suited to arbitrarily shaped density stratifications and non-uniform ambient velocities. Finally, Computational Fluid Dynamics (CFD) models, in which the averaged Navier-Stokes equations are solved with turbulence closure assumptions, are beginning to appear. These are difficult to apply to complex geometries, however, particularly with buoyancy effects, and CFD models have not yet made significant inroads into engineering predictions of mixing zones. Predictions of these mathematical models often show wide discrepancies, indicating a lack of understanding of the basic physical processes occurring in mixing zones.

1.3 Experimental Studies

Experimental data are required, to develop these models further, particularly for buoyant jets in flowing, density-stratified currents. Experimental work has long played a major role in understanding such flows. Dilutions have usually been inferred by tracer concentrations measured by single-point techniques such as conductivity probes. This makes it tedious, or even impossible, to measure the whole concentration field. In addition, the mixing processes depend on unsteady turbulent eddies that result in temporally fluctuating concentrations. Instantaneous concentration fields cannot be captured with point measurements.

The advent of Laser-Induced Fluorescence (LIF) in the 1970s was a substantial advance as it enabled capturing the entire concentration field in a fraction of a second. These methods are discussed further in Section 3.4.1. LIF has now become widely used and is a well-established technique. It can capture the whole tracer concentration field in a plane instantaneously, and this ability to obtain high spatial and temporal resolution measurements of instantaneous scalar concentration fields has proven very useful to understanding the mechanics of turbulent mixing processes. Most of these studies have been planar (PLIF), however, in which information is obtained from images in a two-dimensional plane. Even relatively simple jet and plume flows are inherently three-dimensional, however, and PLIF cannot reveal this three-dimensionality.

To overcome these difficulties, three-dimensional LIF (3DLIF) systems have recently begun to appear (Section 3.4.1). The laser sheet is swept through the flow at high speed and images are captured with a synchronized camera and saved. Through suitable post-processing and calibration, the three-dimensional concentration field can then be obtained. Early systems were limited to short duration experiments in small areas due to image storage capacity and low camera sensitivity. These obstacles have gradually been overcome by recent advances in instrumentation, especially opto-electronics, low-light high-speed cameras, high-speed scanning mirrors, image capture and processing techniques, and fast mass storage devices. Whereas it may only have been possible a few years ago to use a dedicated high-speed video system at very high cost, it is now possible to develop even better systems for much lower costs.

Another difficulty with the use of LIF in stratified flows is refractive index variations arising from density variations that cause random fluctuations in laser intensity. This has
limited most studies to situations with small density differences, and a small region of interest. These limitations can be overcome by the use of refractive index matching in which liquids of different density but equal refractive index, are used. This is discussed in Section 3.2.

1.4 Objectives of the Present Study

Predictions of the mathematical mixing zone models often show wide discrepancies, indicating a lack of understanding of the basic physical processes that cause mixing. Particularly lacking is our ability to reliably predict turbulence collapse due to buoyancy forces, and additional mixing caused by, for example, internal hydraulic jumps. The flows of interest are inherently three-dimensional.

The objective of this study is to develop a 3DLIF system and apply it to investigate the characteristics of buoyant jet flows typical of wastewater discharges under various ambient conditions. Refractive index matching will be used for stratified environments. The system is capable of obtaining whole 3D field measurements and there is no doubt that it will be more effective in revealing the flow structure and the complex turbulent flow processes than previous techniques.

In this report, the development and some applications of the 3DLIF system for stratified buoyant jet flows are described. Typical buoyant jet problems are discussed in Chapter 2 along with the analytical techniques frequently used. The 3DLIF system is described in Chapter 3, and its development in Chapter 4. The system validation, by applying it to previously well-studied problems of round jets and buoyant jets in stratified and unstratified crossflows, is presented in Chapter 5. In Chapter 6 new applications of the system to several flows typical of diffuser discharges are presented. Finally, discussions and conclusions are presented in Chapter 7.
2. **Typical Buoyant Jet Problems**

2.1 Introduction

In general, buoyant jet behavior depends on three main classes of parameters: discharge parameters, environmental parameters, and geometric parameters. The discharge parameters include the nozzle size and geometry, initial jet velocity, and effluent density. The environmental parameters include ambient flows, turbulence level, and density stratification. The geometric parameters arise because of interactions with the free surface, solid boundaries, or adjacent jets. In engineering applications, the discharge and geometric parameters can be completely or partially controlled by the designer; the environmental parameters are determined by the location of the discharge.

Consider the discharge from a round nozzle as shown in Figure 2.1. The basic parameters are the nozzle diameter $d$, the initial jet velocity $u_0$, the ambient flow, the jet density $\rho_j$, and the receiving fluid density $\rho_a$.

![Figure 2.1 Basic definitions for a buoyant jet](image)

Following Fischer et al. (1979), an alternate set of parameters are:

- **Volume flux**: 
  \[ Q = \frac{\pi}{4} d^2 u_0 \]  

- **Momentum flux**: 
  \[ M = u_0 Q = \frac{\pi}{4} d^2 u_0^2 \]  

- **Buoyancy flux**:
  \[ B = g \frac{\rho_a - \rho_0}{\rho_0} Q = g \frac{\rho_a - \rho_0}{\rho_0} \frac{\pi d^2}{4} u_0 \]  

- **Modified gravity**:
  \[ g'_b = g \left( \frac{\rho_a(0) - \rho_0}{\rho_a(0)} \right) \]  

- **Buoyancy frequency**:
  \[ N = \sqrt{\frac{g}{\rho_0}} \frac{d\rho_j}{dz} \]  

where $g$ is the acceleration due to gravity.
Buoyant jet flows such as these have been extensively studied over the past fifty years or so. The methods of analysis can be broadly classified into two types: Mathematical models and dimensional analysis combined with length scale arguments. Mathematical models are discussed in Section 2.2, then we discuss the application of dimensional analysis and length scale approaches to various situations in Section 2.3. This is only a brief review of the many studies on buoyant jets under different conditions. For reviews of multiple buoyant jets see Roberts et al. (1939), Roberts and Snyder (1993), and Daviero (1997).

2.2 Mathematical Models

Mathematical models of buoyant jets flows fall into three general types.

First were entrainment models based on an integral analysis, for example Fan and Brooks (1969). The jet flow is typically assumed to be self-similar, and the axial velocity, concentration and density deficit are assumed to have Gaussian distributions. Together with the assumption that the density difference is small compared to absolute densities (the Boussinesq approximation), the conservation equations can be integrated in a control volume. To close this system of equations, the entrainment assumption is commonly used. This assumption, originally proposed by Morton, Taylor and Turner (1956), is widely used for engineering and geophysical flows such as jets, mixing layers, and density currents on a slope. For jets in stagnant ambient fluids, it assumes that the increase in volume flux is due to entraining flow with a velocity $u_e$ at the jet radius where $u_e = \alpha \cdot u_m$, $\alpha$ is an entrainment coefficient and $u_m$ is the centerline velocity. For flowing ambients, $\alpha$ is assumed to be some function of the local centerline velocity, the ambient velocity, the angle between them, and local density differences.

There are two types of entrainment models: Lagrangian and Eulerian. Lagrangian models follow a plume element along its trajectory. An example is UM, which is available in the U.S. EPA interfaces PLUMES (Baumgartner et al., 1994) and Visual PLUMES (Frick et al., 2000). The equations for conservation of mass, momentum, and energy are solved at each time step, giving the dilution along the plume trajectory. The growth of each element is determined by an entrainment hypothesis. For multiport manifolds the flows begin as round buoyant jets issuing from one side of the diffuser and can merge to a plane buoyant jet. The current is assumed to be perpendicular to the diffuser with a magnitude equal to the component perpendicular to the diffuser. The model output consists of plume characteristics along its trajectory such as centerline dilution, width, and centerline height. Eulerian models are similar, but solve the conservation equations numerically on a fixed grid. Entrainment models work fairly well with relatively deep receiving waters, but not in shallow waters where the supply of entraining water may be limited and boundary mixing may be significant.

UM3 is a three-dimensional Lagrangian entrainment model available in the interface Visual PLUMES (Frick et al., 2001). The equations for conservation of mass, momentum, and energy are solved at each time step along the plume trajectory. The flows begin as round buoyant jets issuing from one side of the diffuser and can merge to
a plane buoyant jet. The model output consists of plume characteristics along its trajectory, such as centerline dilution, width, and centerline height.

RSB is available in the Visual PLUMES interface and also as a separate Windows program RSBWIN. The model is described in Roberts (1999), where a version (called NRFIELD) that was recorded to use long time series of oceanographic data was used. RSB is based on the extensive experiments on multiport diffusers in density-stratified currents of arbitrary direction of Roberts et al. (1989). It is a length scale model that uses semi-empirical formulations based on the relative magnitudes of the dominant length scales of the problem. The model output consists of the plume characteristics (dilution, rise height, and wastefield thickness) at the end of the near field.

Second were semi-empirical models based on dimensional analysis and length scale arguments that overcome these problems to some extent. These models classify the flows according to the relative magnitudes of characteristic flow length scales, and employ asymptotic solutions based on experimental data. Examples include RSB for marine sewage discharges (Roberts, 1999). Another well-known model is CORMIX (Jirka and Akar, 1991, Jirka and Doneker, 1991) that congregates several routines to analyze the geometry and dilution in the mixing zones. CORMIX classifies the flow based on the relative magnitudes of length-scales computed from the input information. It may also employ an Eulerian entrainment model, CORJET, to obtain numerical predictions of plume behavior. Other algorithms are then used to predict further near field mixing processes, if any. RSB and CORMIX are also available from the U.S. EPA. Length scale models can be unreliable when applied near transitions in flow behavior, and are not well suited to arbitrarily shaped density stratifications and non-uniform ambient velocities.

Finally, turbulence models attempt to derive a constitutive equation between the Reynolds stress tensor and the deformation tensor of the fluid, enabling closure of the Navier-Stokes equations. The theories of eddy viscosity and mixing length are two examples. There are different levels of complexity among the turbulence models presented in the literature and thorough reviews are given by Rodi (1982) and Rodi (1993). These are difficult to apply to complex geometries, however, particularly with buoyancy effects, and CFD models have not yet made significant inroads into engineering predictions of mixing zones.

2.3 Dimensional Analysis and Length-Scale Analyses

2.3.1 Single buoyant jet in homogeneous stagnant ambient

Consider a single buoyant jet in a stagnant homogeneous ambient. Following Wright (1980) and others, the following length scales can be defined.

\[
l_e = \frac{Q}{M^{1/2}}
\]

\[
l_M = \frac{M^{3/4}}{B^{1/2}}
\]
$l_Q$ is the length over which nozzle geometry affects flow field. $l_M$ is the measure of the distance above the nozzle where the flow becomes a plume. The flow behaves like a jet when $y$ is much smaller than $l_M$ and like a plume when $y$ is much greater than $l_M$. Two asymptotic solutions for $z << l_M$ (simple jet) and $z >> l_M$ (simple plume) can be obtained. The ratio of $l_Q$ to $l_M$ is defined as a Richardson number $Ri$. For round buoyant jets, $Ri$ can be expressed as (Fischer et al. 1979):

$$ Ri = \frac{l_Q}{l_M} = \left( \frac{\pi}{4} \right)^{1.4} \frac{1}{F} \quad (2.8) $$

where $F$ is the densimetric Froude number:

$$ F = \frac{u_q}{\sqrt{g'd}} \quad (2.9) $$

From dimensional analysis, it can be shown that the dilution and trajectory of fully turbulent buoyant jet flows (independent of Reynolds number) are given by:

$$ S_m \frac{l_Q}{l_M} = f \left( \frac{z}{l_M^2}, \frac{l_Q}{l_M} \right) \quad (2.10) $$

$$ \frac{x}{l_M} = f \left( \frac{z}{l_Q}, \frac{l_Q}{l_M} \right) \quad (2.11) $$

Similar results were first obtained by Rawn, Bowerman and Brooks (1960). Eq. 2.11 applies to a horizontal buoyant jet; the trajectory of a vertical buoyant jet is a straight line. For $l_Q/l_M << 1$ (i.e. $F >> 1$), the effect of nozzle geometry can be neglected, and Eqs. 2.10 and 2.11 become:

$$ S_m \frac{l_Q}{l_M} = f \left( \frac{z}{l_M} \right) \quad (2.12) $$

$$ \frac{x}{l_M} = f \left( \frac{z}{l_Q} \right) \quad (2.13) $$

This is usually the case for wastewater discharges, so the most important source parameters are the source momentum and buoyancy fluxes, $M$ and $B$. For $l_M >> 1$ only the source buoyancy flux is important and the so-called plume equation applies. Fischer et al. (1979) review the dimensional analysis of a single buoyant jet and present asymptotic solutions; more experimental data and analyses can be found in Chen and Rodi (1980).
2.3.2 Single buoyant jet in unstratified crossflow

Buoyant jets in cross flows are more complex. Pairs of trailing vortices give rise to kidney shape concentration profiles with two concentration maxima on both sides of centerline and possible bifurcation. In addition to the length scales introduced above, two more length scales can be defined (Wright, 1984):

\[ l_m = \frac{M^{1/2}}{u} \]  \hspace{1cm} (2.14)

\[ l_b = \frac{B}{u^3} \]  \hspace{1cm} (2.15)

Wright (1984) has classified the flow into four limiting cases: MDNF (momentum-dominated near field), MDFF (momentum-dominated far field), BDNF (buoyancy-dominated near field), BDFF (buoyancy-dominated far field).

For a momentum-dominated jet, the only characteristic length scale is \( l_m \) and all properties of the flow can be described in terms of \( z/l_m \). The jet is in the near field (MDNF) if \( z/l_m \ll 1 \). It is only slightly bent over and is assumed to be almost unaffected by the cross flow. The jet is in the far field (MDFF) if \( z/l_m \gg 1 \). The behavior of a bent over jet is assumed to be the same as a cylindrical buoyant thermal at the same vertical rise height. For a buoyancy-dominated jet, the only characteristic length scale is \( l_b \), and all properties of the flow can be described in terms of \( z/l_b \). It is in the near field (BDNF) if \( z/l_b \ll 1 \) and the far field (BDFF) if \( z/l_b \gg 1 \). The trajectory and dilution equations can be deduced from dimensional analysis (Fischer et al., 1979). This technique has been confirmed in experimental work by Hoult et al. (1969), Chu & Goldberg (1974) and Wright (1977). The results were summarized by Wright (1984) as shown in Table 2.1.

<table>
<thead>
<tr>
<th>Flow Regime</th>
<th>Trajectory, ( z = f(x) )</th>
<th>Centerline dilution, ( S_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>MDNF</td>
<td>( \frac{z}{l_m} = 2.3 \left( \frac{x}{l_m} \right)^{1/2} )</td>
<td>( \frac{S_0Q}{ul_m^2} = 0.2 \left( \frac{z}{l_m} \right) )</td>
</tr>
<tr>
<td>MDFF</td>
<td>( \frac{z}{l_m} = 1.9 \left( \frac{x}{l_m} \right)^{1/3} )</td>
<td>( \frac{S_0Q}{ul_m^2} = 0.25 \left( \frac{z}{l_m} \right)^2 )</td>
</tr>
<tr>
<td>BDNF</td>
<td>( \frac{z}{l_b} = 2.5 \left( \frac{x}{l_b} \right)^{3/4} )</td>
<td>( \frac{S_0Q}{ul_b^2} = 0.12 \left( \frac{z}{l_b} \right)^{5/3} )</td>
</tr>
<tr>
<td>BDFF</td>
<td>( \frac{z}{l_b} = 1.7 \left( \frac{x}{l_b} \right)^{3/3} )</td>
<td>( \frac{S_0Q}{ul_b^2} = 0.25 \left( \frac{z}{l_b} \right)^2 )</td>
</tr>
</tbody>
</table>
Integral models have been applied to predict the behavior of a buoyant jet in a cross flow. Most of them use versions of the integral analysis developed by Morton et al. (1956). Eulerian models include Hirst (1972), Chu and Goldberg (1974), Wood (1993), and Chu and Lee (1996). Lagrangian models which follow the motion of a turbulent element or "dominant eddy" include Frick (1984), Chu (1985), and Lee and Cheung (1990). Larsen et al. (1990) investigated the behavior of buoyant jets in cross flows with both an integral model and a $k - \varepsilon$ turbulence model. The aim of his study was to verify the validation of $k - \varepsilon$ model for buoyant jets in cross flows.

2.3.3 Single buoyant jet in stratified stagnant ambient

Density stratification suppresses vertical motions and mixing. Sketches of the behavior of a horizontal pure jet and a horizontal buoyant jet are shown as Figure 2.2. The pure jet (Figure 2.2a) behaves initially as though in a uniform ambient fluid, but at some distance from the nozzle the ambient stratification causes it to collapse vertically. It then spreads sideways and the thickness of the jet becomes approximately constant. The buoyant jet (Figure 2.2b) rises to some terminal level then spreads horizontally.

![Figure 2.2 Schematics of jets in stagnant stratified ambient fluid](image)

Two more length scales can be defined for this case:

$$l_p' = \frac{M^{1/4}}{N^{1/2}} \quad (2.16)$$

$$l_b' = \frac{B^{1/4}}{N^{3/4}} \quad (2.17)$$

Works prior to the 1980s are reviewed by Fisher et al. (1979). A thorough study of round buoyant jets discharging into linearly density-stratified stagnant fluids was done by Wong and Wright (1988). Their semi-empirical formulae of vertical round buoyant jets in a linearly stratified fluid are given in Table 2.2.
Table 2.2 Characteristics of round buoyant jets in a linearly stratified fluid (Wong & Wright, 1988).

<table>
<thead>
<tr>
<th></th>
<th>Plume-like ( \left( \frac{l_m}{l_b} &lt; 1 \right) )</th>
<th>Jet-like ( \left( \frac{l_m}{l_b} &gt; 1 \right) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height of rise, ( z_m )</td>
<td>( \frac{z_m}{l_b} = 4.5 ) (3.8*)</td>
<td>( \frac{z_m}{l_m} = 3.6 ) (3.8*)</td>
</tr>
<tr>
<td>Spreading layer thickness, ( h_e )</td>
<td>( \frac{h_e}{l_b} = 1.5 )</td>
<td>( \frac{h_e}{l_m} = 1.4 )</td>
</tr>
<tr>
<td>Minimum dilution, ( S_m )</td>
<td>( \frac{S_m Q N^{5/4}}{B^{3/4}} = 0.8 )</td>
<td>( \frac{S_m Q N^{1/2}}{M^{3/4}} = 0.68 )</td>
</tr>
</tbody>
</table>

*Values given by Fischer et al. (1979)

Roberts and Matthews (1987) did extensive work on horizontal pure jets and jets with low buoyancy in a linearly stratified fluid. They suggested that there is little or no influence of stratification on the jet until the collapse distance is reached. They also determined the thickness of the spreading layer, the collapse distance and the average dilution:

\[
\frac{h_e}{l_b} = 1.5
\]

\[
\frac{h_e}{l_m} = 1.4
\]

\[
\frac{S_m Q N^{5/4}}{B^{3/4}} = 0.8
\]

\[
\frac{S_m Q N^{1/2}}{M^{3/4}} = 0.68
\]

2.3.4 Single buoyant jet in stratified crossflow

The typical flow pattern of a buoyant jet in a flowing stratified fluid is illustrated in Figure 2.3. With ambient density stratification and cross flows, the problem is more complex, and there are fewer studies. Early studies for buoyant jets in a stratified cross flow

![Figure 2.3 Schematics of a jet in flowing stratified fluid](image)
flow were based on integral models, including Fay et al. (1970), Slawson and Csanady (1971), and Schwartz and Tulin (1972). Briggs (1975) summarized the observational studies and presented two relations for the observed maximum rise height, $z_m$, of a momentum driven jet and a buoyancy driven jet in a stratified cross flow:

For a momentum driven jet:  
\[ z_m \propto \left( \frac{M}{uN} \right)^{1/3} \]  \hspace{1cm} (2.21)

For a buoyancy driven jet:  
\[ z_m \propto \left( \frac{B}{uN^2} \right)^{1/3} \]

Schatzmann (1979) presented an integral model to predict the spreading and rising of a round plume in a stratified crossflow. Luti and Brzustowski (1977) developed a model to describe the characteristics of two-dimensional plume in a stratified cross flow. A classic work was done by Wright (1984) using dimensional analysis and length scales arguments. In addition to the length scales introduced above, one more length scale was defined by Wright (1984).

\[ l_a = \frac{u}{N} \]  \hspace{1cm} (2.22)

Similar to the analysis of buoyant jets in a uniform cross flow, Wright classified the flow into four limiting cases: MDNF, MDFF, BDNF, BDFF. He obtained expressions for rise height and maximum dilution at the terminal rise height for these four cases as shown in Table 2.3. The coefficients for rise height were experimentally determined; no coefficient for dilution was given.

**Table 2.3 Relations for buoyant jets in stratified cross flow (Wright, 1984).**

<table>
<thead>
<tr>
<th>Flow regime</th>
<th>Rise height</th>
<th>Dilution*</th>
</tr>
</thead>
<tbody>
<tr>
<td>MDNF</td>
<td>$z_m = 2.8 \left( \frac{l_m}{l_m} \right)^{1/2}$, $z_m = 1.75 \left( \frac{l_m}{l_m} \right)^{1/3}$</td>
<td>$\frac{S_n Q}{u l_m^2} = C_1 \left( \frac{z_m}{l_m} \right)$</td>
</tr>
<tr>
<td>MDFF</td>
<td>$z_m = 2.2 \left( \frac{l_m}{l_m} \right)^{1/3}$, $z_m = 1.5 \left( \frac{l_m}{l_m} \right)^{1/3}$</td>
<td>$\frac{S_n Q}{u l_m^2} = C_2 \left( \frac{z_m}{l_m} \right)^2$</td>
</tr>
<tr>
<td>BDNF</td>
<td>$z_m = 3.5 \left( \frac{l_m}{l_m} \right)^{3/4}$, $z_m = 3.0 \left( \frac{l_m}{l_m} \right)^{3/4}$</td>
<td>$\frac{S_n Q}{u l_b^2} = C_3 \left( \frac{z_m}{l_b} \right)^{5/3}$</td>
</tr>
<tr>
<td>BDFF</td>
<td>$z_m = 2.3 \left( \frac{l_m}{l_b} \right)^{3/3}$, $z_m = 1.85 \left( \frac{l_m}{l_b} \right)^{2/3}$</td>
<td>$\frac{S_n Q}{u l_b^2} = C_4 \left( \frac{z_m}{l_b} \right)^2$</td>
</tr>
</tbody>
</table>

* Dilution is the minimum dilution at the terminal rise height.
Turbulence models have been developed, for example, by Hwang and Chiang (1999). A $k-\varepsilon$ turbulence model was used to compute the flow field of a round buoyant jet discharging vertically into a density stratified cross flow. It was concluded that the ambient stratification prohibits the development of the plume radius and reduces the mixing rate as well the plume rise, and a wave-like trajectory is formed for the case of higher density stratification and higher jet-to-cross flow velocity ratio.
3. EXPERIMENTAL SYSTEM

3.1 Introduction

The main objective of this study is to develop a 3DLIF system for the study of stratified buoyant jets. The facility is located in the Environmental Fluid Mechanics Laboratory of the School of Civil and Environmental Engineering, Georgia Institute of Technology. It is housed in a specially built darkroom to eliminate ambient light. A schematic depiction of the experimental configuration is shown in Figure 3.1. The various elements of the system are described below.

![Diagram of 3DLIF system](image)

Figure 3.1 Schematic depiction of 3DLIF system

3.2 Towing Tank and Filling System

The towing tank is 6.10 m long by 0.91 m wide by 0.61 m deep. The front and left walls of the tank are glass. The front consists of two three-meter long glass panels to enable long duration tows with unobstructed views. The right, rear walls, and floor are steel painted with black epoxy paint to resist corrosion and to reduce reflections. The tank is equipped with a towing system that is used to simulate ambient currents. The towing carriage rides on two precision one-inch diameter rails that run the length of the tank. The carriage is propelled via a chain and gear system by a 130 volt Bodine motor connected in series to a Penta-Drive DC motor speed control device. The Penta-Drive enables variation of the speed and direction of the towing carriage along the tank.

The effluent, a mixture of water, salt, and fluorescent dye, is supplied from a reservoir by a rotary pump (see Section 3.3). Most experiments are conducted with a negatively buoyant jet discharged near the water surface. This is more convenient to model than a positively buoyant plume discharged from the tank floor, particularly for towed experiments. This modeling technique is permitted if the variations in the fluid density
throughout the flow field are small and are only important in the buoyancy forces (the Boussinesq assumption). This assumption is valid for all the cases considered here.

The towing tank filling system is shown in Figure 3.2. Along the length of the towing tank at the intersection of the rear wall and the floor is a 1.5 inch diameter influent diffuser with 0.125 inch diameter ports spaced 2.5 inch apart. This diffuser is used to evenly distribute the inflow along the towing tank. The towing tank is filled using solutions stored in two completely mixed 500-gallon storage tanks (Tank A and Tank B) connected in series. Both Tank A and Tank B are equipped with centrifugal pumps. The pumps recirculate the tanks keeping them well-mixed in addition to pumping to the towing tank. The tanks A and B are filled with tap water and treated with sodium thiosulfate to remove any residual chlorine that would quench the fluorescence. When an unstratified environment was created, the towing tank was filled from Tank B only. In this case, the filling rate is unimportant.

![Figure 3.2 Schematic diagram of the filling and stratification system.](image)

Creation of a stratified environment is more complex and uses the two-tank filling method (Daviero, 1997). The procedure begins by filling Tank A with a more dense liquid (either a salt or sugar solution) and Tank B with a less dense liquid (either fresh water or an ethanol solution). The solution is Tank A is pumped into tank B at a flowrate of $Q_A$ and from Tank B into the towing tank at a flowrate of $2Q_A$. As time proceeds, the density of the solution in Tank B increases so that each subsequent layer of fluid is heavier than the previous layer. The new layers slide under the previous ones, lifting them. The filling must be done slowly to minimize mixing between the layers. Filling typically takes several hours. After the towing tank has settled, the stratification is linear with the density at the surface equal to the solution density in Tank B and the density at the tank floor equal to that of Tank A. The strength of the stratification was measured either by traversing the fluid with a conductivity probe or by removing samples of the ambient fluid at various depths. The densities of the samples were measured with a
calibrated Troemner Specific Gravity Scale to an accuracy of ±0.1σ, where σ = (ρ-1)*1000, with the density ρ in g/cc.

3.3 The Source Discharge System

The source discharge system is shown in Figure 3.3. Various model diffusers or nozzles can be used. The effluent, stored in a 40-liter source tank, is usually a mixture of salt, fresh water, and fluorescent dye. Sodium thiosulfate was added to the effluent in the source tank to remove any residual chlorine. The effluent is pumped at a constant flowrate from the source tank, and the flowrate is measured by one of two precision rotameters that are connected in parallel.

![Figure 3.3 Schematic of the source discharge system](image)

3.4 LIF System

3.4.1 Introduction

The advent of Laser-Induced Fluorescence (LIF) in the 1970s enabled capturing the entire tracer concentration field in a plane in a fraction of a second. LIF is non-intrusive and overcomes the disadvantages of traditional probe-based techniques. It is an excellent technique for visualizing and measuring scalar tracer concentrations. LIF has now become widely used and is a well-established technique. Many studies have been reported since the earliest ones by Owen (1976) and Dimotakis et al. (1983) including Dimotakis et al. (1983), Sreenivasan and Meneveau (1986), Papanicolaou and List (1988), Papantoniou and List (1989), Prasad and Sreenivasan (1990), Ferrier and Roberts (1993), Houcine et al. (1996), and Webster et al. (2001).

In a typical LIF experiment, a fluorescent dye such as Rhodamine 6G, Rhodamine B, or Fluorescein, is added to the flow. A laser sheet illuminates the flow and excites the fluorescent tracer. The intensity of the emitted fluorescent light is a function of the dye concentration and the intensity of the incident laser light. The relationship between the emitted light intensity and dye concentration is linear if the laser power or the dye concentration are not high enough to saturate the sensor. This relationship is usually obtained by calibration. A CCD camera is used to record the emitted light intensity as a gray scale image. The dye concentration field is then obtained from the image using the calibration results. Typically, a PLIF system includes three key parts: 1) Optical system and camera; 2) Image capturing and processing system; 3) Software to enable flow visualization and to extract quantitative information from the images.
The main characteristics of the fluorescent dyes can be found in Bertman (1971) and are summarized in Table 3.1. An evaluation of these dyes for LIF was given by Arcoumanis et al. (1990). Usually, the fluorescent dye selected depends on the laser wavelength. The laser wavelength should be as close as possible to the maximum absorption wavelength of fluorescent dye (see Table 3.1). In this study, we use Rhodamine 6G because the wavelength of the Argon ion laser here is 514 nm.

<table>
<thead>
<tr>
<th>Dye</th>
<th>Absorption Spectrum (nm)</th>
<th>Emission Spectrum (nm)</th>
<th>Chemical Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lower</td>
<td>Upper</td>
<td>Max</td>
</tr>
<tr>
<td>Rhodamine B</td>
<td>460</td>
<td>590</td>
<td>550</td>
</tr>
<tr>
<td>Rhodamine 6G</td>
<td>460</td>
<td>560</td>
<td>530</td>
</tr>
<tr>
<td>Fluorescein</td>
<td>430</td>
<td>520</td>
<td>490</td>
</tr>
</tbody>
</table>

The two dimensional LIF (PLIF) technique is now well established and has become a standard experimental tool in experimental fluid mechanics. The flows considered here are three-dimensional, however. Different techniques are required for capturing and processing 3D data as discussed below.

To overcome these difficulties, three-dimensional LIF systems have recently begun to appear. In these applications, the laser sheet is swept through the flow at high speed; images are captured with a synchronized camera and saved. Through suitable post-processing and calibration, the three-dimensional concentration field can then be obtained. The first application was possibly Kychakoff et al. (1987) who studied a laminar premixed flame by scanning the plane of illumination through the flow with an oscillating mirror and obtaining images with an intensified array. Turbulent mixing in homogeneous jets was studied by Winter et al. (1987), and Prasad and Sreenivasan (1990) who mapped the three-dimensional tracer concentration field by sweeping the sheet through the flow by a rotating mirror and capturing images with a framing camera. More recently, Deusch (1998) studied turbulent mixing, velocity, and velocity gradient using a multi-patch 3D image correlation approach.

These systems were limited to short duration experiments in small areas due to image storage capacity and low camera sensitivity. As the area illuminated increases, the laser intensity decreases, necessitating either higher laser power or increased fluorescent dye concentrations. Increasing dye concentrations, however, causes further problems in that laser attenuation by the dye increases substantially and the calibration becomes nonlinear. Furthermore, maximum image storage is often limited by expensive computer memory. These obstacles have gradually been overcome by recent advances in instrumentation, especially opto-electronics, low-light high-speed cameras, high-speed scanning mirrors, image capture and processing software, and fast mass storage devices. The speed of writing to hard disc has increased rapidly while the cost has fallen dramatically. It is now
possible to write images to disc in real time, providing (almost) unlimited storage capacity. Whereas it may only have been possible a few years ago to use a dedicated high-speed video system at very high cost, it is now possible to develop even better systems for much lower costs.

3.4.2 Scanning mirror system and high speed camera

Several methods have been used to extend from 2D to 3D imaging. These include holography, or multiple cameras with different viewing angles. These methods have been used by several researchers in 3D Particle Image Velocimetry (PIV) such as Meng and Hussain (1991), Sinha and Kuhlman (1992), Prasad and Adrian (1993), and Fabry et al (1997). These techniques require expensive optical systems and powerful computational processors (Brücker, 1995). A more convenient method is to use scanning laser sheets. The laser sheet samples the flow in successive planes, providing a 2D image in each plane. If the scanning frequency is high enough, frozen 3D flows can be reconstructed from the sets of 2D images.

Three types of scanning systems are shown in Figure 3.4. The type shown in Figure 3.4a was used by Merkel (1994), Goldstein and Smits (1994), Yip et al. (1988), Prasad and Sreenivasan (1990), and Maas et al. (1994). An oscillating mirror scans the laser beam and a cylindrical lens changes the beam to a sheet. For the type shown in Figure 3.4b, used by Guezennec et al (1994), Delo and Smits (1997), and Brücker (1997), a rotating drum is used. Several tiny mirrors are attached around it spirally at different heights. A major disadvantage of these two types is the non-uniformity of the laser sheet. This results in over-exposure in the central part of the flow image and under-exposure at the edges. The size of the illuminated flow is also limited.

The scanning system used in this study is shown in Figure 3.4c. Two orthogonal scanning mirrors make uniform intensity laser sheets, and the frequency and amplitude of the sheets can be easily varied. Because the laser sheet sweeps through the flow very quickly, it is necessary to use a high speed camera. The oscillating mirrors must be synchronized with the camera so that an image is obtained for each laser sweep.

Figure 3.4 Types of scanning mirror systems.
We use a two-axis scanning mirrors system with the mirror and camera timing and synchronization controlled by a computer. A schematic depiction of the system is shown in Figure 3.5. A National Instruments Multifunction I/O Board (Model PCI-MIO-16E-4) provides two analog signals to control the motion of the two mirrors and a TTL digital signal to trigger the camera. This board was installed in the "scanning controller computer" and is programmed in LabView. The analog signal is converted at a resolution of 12 bits from a digital file. The I/O Board is the "control center" that synchronizes the laser beam and image acquisition. It sends a TTL signal to the frame grabber board to begin image acquisition. Simultaneously, the same TTL signal is sent to the I/O board to begin sending a triangle analog signal (Figure 3.6). This triangle analog signal is used to drive one of the scanning mirrors and move the beam vertically. The camera is exposing while the beam moves up and down (one sweep). Then a step analog signal is sent to drive the other mirror and move the beam a small distance horizontally. The cycle then begins again with another TTL signal that downloads the previous frame and begins the next exposure. This is repeated so that multiple vertical "slices" through the flow are obtained. In between slices, there is a 300 ns delay to enable the grabber board to download the previous frame. The beam returns to the starting point and the cycle starts again after a predetermined number of "slices". A great advantage of this system is that the sheet height, number of slices and their separation, and scanning frequencies are fully controllable. A LabView program sets all parameters for scanning mirrors with the user-friendly interface shown in Figure 3.7.
The optical system is shown in Figure 3.8. The laser and optics are mounted on a 48 inches by 48 inches square breadboard. The laser is a Lexel 95-4 Argon-Ion laser with nominal maximum power in all-line mode about 4 W. We use it in single line mode in the green line (514 nm) with a power of about 2 W. The two orthogonal fast galvanometer mirrors are made by Cambridge Technology. The mirrors are located at the focal point of a large plano-convex lens 350 mm in diameter with a focal length of 940 mm. The laser beam is therefore always refracted parallel to the tank axis after it passes through the lens, resulting in a collimated laser sheet in the tank. In practice, it is not possible to maintain the beam origin exactly at the lens focal point because the beam location moves slightly relative to the lens as the mirrors scan. This movement is quite small relative to the focal length, however, so deviations from parallel are small and can be neglected.

The fluorescent dye is Rhodamine 6G, which has peak absorption at about 530 nm and peak emission at about 560 nm (Table 3.1). Characteristics of this dye are given in Ferrier et al. (1993). Small amounts of the dye are added so that dye concentrations in the diluted jet are low, typically around 10 µg/L.

The high-speed camera is a Dalsa CA-D6. This digital camera provides output in 8-bit resolution, i.e., a gray scale with 256 levels. The resolution (number of active pixels) is 532 by 516. The LVDS (Low Voltage Differential Signal, also known as EIA-644) data format enables high data transmission rates over long cable lengths. The maximum
frame rate of this camera is 260 frames per second, which gives a maximum data rate of about 71 MHz. This high data rate is achieved by using four taps, each capable of 25 MHz. For the experiments reported here, frame rates of 100 or 200 frames per second were used. The camera is externally triggered by a TTL signal from the National Instruments I/O Board. It has a high-gain A/D converter to enable use with low fluorescence light levels. Even with the high gain, the noise level is still quite low. For the experiments reported here a Fujinon CCTV camera lens of 25 mm focal length and f/0.85 aperture was used. A long pass orange filter (Schott glass 530) is placed over the camera lens to pass only the fluoresced light and eliminate the laser scattered light. The CCD camera is attached to the tow carriage and moves with it so that the discharge appears to be stationary relative to the camera.

The frame grabber board is a Bitflow RoadRunner. Video Savant software is used to control image capturing and saving in the “image acquisition and processing computer” (Figure 3.5). This computer has four high-speed hard discs that are controlled by an Adaptec Ultra160 SCSI card. This system allows streaming images to disc in real time at up to 260 frames per second. The total size of the SCSI drives is 32.4 GB, so we are not limited by computer memory. This allows long experimental durations.

The laser and image acquisition are controlled and synchronized as follows. The I/O Board sends a TTL signal to the frame grabber board to initiate frame capture, and the frame grabber board in turn sends an LVDS signal to the camera to begin image acquisition (exposure). Simultaneously, the I/O Board begins sending an analog voltage to move the vertical (z) mirror. The beam makes one sweep down and back while the camera is exposing (i.e. the shutter is “open”). A voltage is then sent to the horizontal (y) mirror to move the beam a small distance horizontally. The cycle then begins again with another TTL signal that downloads the previous frame, clears the camera buffer, and begins the next exposure. This is repeated so that multiple vertical “slices” through the flow are obtained. After a predetermined number of “slices” the beam returns to the starting point and the cycle starts again. For example, 20 slices at 200 frames per second yields an effective sample rate, at which the whole sequence of images through the flow is captured, of 10 Hz.

There are trade-offs between the height of the area imaged, the camera frame-rate, and the dye concentration. As the height is increased, the laser intensity at any point decreases, and as the frame rate increases the exposure time decreases. Both of these reduce the light level reaching the camera. Increasing dye concentrations can compensate for this, but for quantitative work, it is desirable to keep the dye concentrations below about 50 µg/l if possible. Beyond these levels, the light output as a function of dye concentration becomes non-linear and attenuation of the laser by the dye increases rapidly (Ferrier et al, 1993). We maximized the light reaching the camera by using a fast (f/0.85) lens and a sharp cutoff filter (Schott Glass 530) that passes most of the fluoresced light while blocking the scattered laser light. For example, a dye concentration of 120 µg/l causes pixel saturation with a laser power of 2 W, a sheet height of 230 mm, and a frame rate of 100 frames per second.
Quantitative scalar concentration data are obtained by calibration. We capture images of a tall cylinder containing known amounts of dye placed in the towing tank, similar to the procedure of Ferrier et al. (1993). A pixel-by-pixel calibration in which corrections for lens luminance variation (vignetting) and individual pixel response is then done. Finally, the images are corrected for attenuation due to clear water, dye, salt, and ethanol, using the methods of Daviero et al. (2001). The multiple “slices” through the flowfield are then regenerated, using three-dimensional image processing software, into a three-dimensional image of the flow. The time of scanning through the flow is sufficiently short to “freeze” the larger turbulent scales.

3.4.3 3D visualization techniques

Flow visualization can provide valuable insight into turbulent flows. Obtaining high quality images is the first important step. In order to investigate the mixing or other characteristics of turbulent flows, quantitative data is also obtained from images.

Concentration fields are obtained from each image directly. The data is then used to reconstruct the 3-D image by visualization techniques. Other properties can also be obtained such as the mean concentration distribution, the gradient ($\nabla c$) in three directions, the dissipation rate ($\nabla c \cdot \nabla c$), and other statistical properties.

More visualization techniques can also be applied. One may explore the 3-D data obtained from experiments using popular visualization software such as IBM Data Explorer, IRIS Explorer. The technique of color mapping that has been used in PLIF can show the concentration distribution and eddy motion clearly. Contouring is very good to show the iso-surfaces of jets. Although what one obtains from 3-D imaging is a series of data in x-y plane (Figure 3.9), the variations of concentration or velocity in the z-direction can also be investigated. In order to investigate the relationship between concentration and velocity, one may combine these two kinds of data in one image. Because the data are time-dependent, animation technique can be applied. Either the 3-D animation of the whole data or slices animated in either direction can be investigated. Some of these techniques were used by Kychakoff et al (1987), Yip et al (1988), Wu and Hesselink (1988), Patrie (1994), and Delo and Smits (1997).

![Figure 3.9 Image planes and coordinate system](image)
Some researchers have developed techniques to derive velocity fields from 3-D LIF images. Mass et al (1994) developed a method to calculate the displacement of a cube of 193 voxel volume and the relative displacement of the corners of this volume at each time step. From this, velocities, strain tensors and rotation vectors were determined. The voxel is a primary three-dimensional cell. Each face of the voxel is perpendicular to on the the coordinate x-y-z axes. Same method was also used by Deush (1998). This is a type of correlation technique. An algorithm based on least square fitting is used to minimize the residue of gray value differences of transformation parameters for a small volume. Another method was developed by Dahmi et al (1991, 1992). The velocity field is extracted from the conserved scalar transport equation. Both methods provided a good way to investigate fine scale structures. However, they need a very high spatial resolution. For example, in Dahm et al. (1991), the in-plane spatial resolution was $\Delta x = \Delta y = 116 \, \mu m$, the spatial resolution between successive planes was $\Delta z = 220 \, \mu m$. A very small volume of $30 \, mm \times 30 \, mm \times 1.0 \, mm$ flow was investigated in their study.
4. System Development

4.1 Introduction
The LIF images must be corrected before quantitative information can be extracted. These include corrections for pixel response and lens brightness variation, attenuation, and magnification. These corrections are described in this Chapter along with the refractive index matching technique. Validation experiments to verify the experimental procedures are described in Chapter 5.

4.2 Camera and Lens Correction
In this study, we use a Fujinon CCTV camera lens of 25 mm focal length, f0.85. A "standard image" (Figure 4.1) was obtained of a uniform white board illuminated by a uniform light source. The gray scale of such an image would be uniform for a perfect lens; however, it is brighter at the center and darker at the edges, a phenomenon known as vignetting. In Figure 4.1, the lowest intensity (at the corner) is about 15% lower than the highest intensity (at the center). This image is used to correct for the non-uniform brightness variation. In addition, a black level image (Figure 4.2) was obtained by covering the lens. This image is also not uniform. Four thin bright bands can be seen in

![The standard image and its intensity variation](image1)

![The black-level image and its intensity variation](image2)
the standard and black-level images. This is because the Dalsa CA-D6 camera has four taps, each corresponding to a 133 by 516 region in the image, in order to achieve the very high data transmission rate needed. The bright bands are about 10 pixels in width. In Figure 4.2, the peak intensity in the band is about 10 and elsewhere about 5.

To correct for these lens and camera deviations, the black level image is first subtracted from each raw image and also the standard image. The new raw images are then divided by the new standard image, pixel by pixel, and the final images are then multiplied by a scale factor. The images are corrected by:

\[
I_c(i,j) = C \cdot \frac{I_r(i,j) - I_b(i,j)}{I_s(i,j) - I_b(i,j)}
\]

(4.1)

where \(I_c(i,j)\) is the corrected pixel value, \(I_r(i,j)\) is the raw image pixel value, \(I_s(i,j)\) is the standard image pixel value, \(I_b(i,j)\) is the black-level image pixel value. \(C\) is a scale factor, and \((i,j)\) are the pixel indices.

To test this procedure, a white board image was taken with a uniform light source. The test image (Figure 4.3a and b) is not same as the standard image (Figure 4.1), although they look similar. The light source intensity was lower and the distance between the camera and board was farther. The raw image and the results after the correction procedures were applied are shown in Figure 4.4.
4.3 Refractive Index Matching

Density variations cause refractive index variations. This can severely hamper the application of LIF in stratified environments: Even though the variations in refractive index may be slight, the combination of the long beam path length and small internal waves are sufficient to cause significant laser intensity variations. These variations propagate through the system, causing the laser sheet to wobble and produce pseudo-turbulent fluctuations. Any variations in refractive index must be minimized in order to obtain quantitative information. This can be done by refractive index matching as reviewed by Daviero et al. (2001). The dramatic improvement in image quality obtained by using refractive index matching is illustrated in Figure 4.4.

![Figure 4.4](image)

We use the technique of Daviero (1997) with solutions of salt and ethanol. Daviero established that a constant refractive index can be successfully created in a linearly stratified fluid. His technique can be used in large-scale facilities (volumes of 4000 L and greater and optical path lengths of several meters) at reasonable cost.
The general procedure for creating a refractive index matched stratified environment is:

1. Choose the water depth, $\Delta z$ and therefore the volume of the towing tank.

2. Select the desired buoyancy frequency, $N$, and determine the required density difference $\Delta \rho_e$ over the water depth from:

$$\Delta \rho_e = \Delta z \cdot N^2 \cdot \frac{\rho_0}{g},$$

where $\rho_0$ is the density of the effluent.

3. Determine the appropriate concentrations of the ethanol and salt solutions using Figure 4.5 and Eqs. (4.6), (4.7) or Eqs. (4.8), (4.9).

$$C_s(salt) = 5813.7n - 7749.7$$

$$C_s(ethanol) = 16164n - 21546$$

$$C_s(salt) = 898.24\Delta \rho_e - 0.0896$$

$$C_s(ethanol) = 2497.8\Delta \rho_e - 0.5434$$ (4.9)

4. Fill tank A with the appropriate concentration salt solution to a volume equal to one half of the towing tank volume.

5. Fill tank B with the appropriate concentration ethanol solution to a volume equal to one half the towing tank volume. Since there is a significant increase in temperature when ethanol is added to water, the ethanol must be allowed to cool to room temperature before continuing.

6. Stratify the tow tank using the procedure outlined in Section 3.2.

The tank is allowed to rest for about two hours after it is filled. This allows internal waves to dampen and any density discontinuities generated during the filling process to become linear by molecular diffusion. In general, the longer the tank is allowed to rest the better the results of the refractive index matching procedure will be. However, it should be noted that the ethanol begins to evaporate from the upper surface of the tank, causing noticeable refractive index deviations in that region, after approximately three hours.
4.4 Attenuation Correction

The laser beam decreases significantly in power as it traverses the water. This attenuation is greater if there is salt, ethanol or fluorescent dye present. Ethanol, in particular, greatly increases the attenuation. This attenuation can be corrected using the technique below (Daviero et al., 2001).

Discussions of attenuation can be found in Koochesfahani and Dimotakis (1985), Walker (1987), Van Cruyningen et al. (1990), Ferrier et al. (1993), Daviero et al. (2001). The laser power decays exponentially as:

\[ I = I_0 \cdot e^{-a(x-x_0)} \]  

(4.10)

Where \( I \) is the laser intensity at location \( x \), \( I_0 \) is the laser intensity at location \( x_0 \), and \( a \) is the attenuation coefficient. Attenuations due to the various components are additive (Daviero, 2001), and the attenuation coefficient can be computed from:

\[ a = a_w + 0.000124 \cdot C_{\text{Salt}} + 0.000547 \cdot C_{\text{Eth}} + 0.00023 \cdot C_{\text{Rhod}} \]  

(4.11)

where \( C_{\text{Salt}} \) and \( C_{\text{Eth}} \) are the solute concentrations of salt and ethanol in g/L, and \( C_{\text{Rhod}} \) is the Rhodamine concentration in \( \mu \)g/L. \( a_w \) is the clear water attenuation coefficient for filtered water; the values of the other attenuation coefficients are taken from Daviero et al. (2001). The clear water attenuation coefficient \( a_w \) varies daily from about 0.0011 to 0.0045 cm\(^{-1}\) and should be measured before each experiment. This is done by obtaining images of a Lucite cylinder containing a known concentration of Rhodamine 6G dye at two different locations in the tow tank. \( a_w \) is then determined from Eq. (4.10).

Because of attenuation, the LIF images obtained in the experiments must be corrected during post processing. A detailed procedure for attenuation correction was discussed in Ferrier et al. (1993). All laser beams are assumed to be parallel and horizontal. The correction starts from the pixel at the left-top corner of each image and proceeds pixel by pixel for each line according to:

\[ I_i^c = \frac{I_i}{\sum_{j=1}^{N} e^{-a_j(x-x_j)}} (i = 1, 2, ..., N) \]  

(4.12)

Where, \( I_i^c \) is the corrected intensity, \( I_i \) is the original intensity, \( N \) is the pixel number along the laser beam, \( \Delta x \) is the distance in cm between two pixels, \( a_j \) is the attenuation coefficient for each particular pixel, determined from Eq. 4.11. It should be noted that \( a_j \) is locally variable, i.e. \( a_j \) is calculated from the concentration of salt, ethanol, or fluorescence dye at the \( j \)th pixel. To test this correction algorithm, an LIF image was taken of a known uniform dye concentration. The raw and corrected images are shown in Figure 4.6. For the corrected image, the lens correction was also done using Eq. 4.1 before the attenuation corrections were applied.
4.5 Depth of Field

In a 2DLIF system, there is only one laser sheet and the camera can be focused on the plane of this sheet. In a 3DLIF system, multiple images are taken but the camera can only focus on the center image, as shown in Figure 4.7. The images behind or in front of the center image are out of focus. To test the degree to which this causes defocusing of the off-center images, images were taken of a target moved from the back of the towing tank to the front with the camera focused on the center. The distance from the central image to the camera was 240 cm. The farthest image from the camera was 280 cm and the nearest image was 200 cm. Some images are shown in Figure 4.8. As can been seen, the images remain in acceptable focus at least up to ±19.0 cm from center. In the experiments here, the largest distance that the laser sheet sweeps is less than ±15.0 cm, therefore, the focus problem can be neglected.
The magnification of the image varies with the distance of the image from the camera. As shown in Figure 4.9, $L-L_c$ is the distance from the central image, where negative distance means images are behind the central image. This variation is accounted for in post processing. Before each experiment, a scale of the central image is obtained. The magnification scales for other images are then determined by simple geometry:

$$S = \frac{L}{L_c} \cdot S_c$$  \hspace{1cm} (4.13)

where, $S$ is the scale for any image, $S_c$ is the scale in the center, $L$ is the distance from the camera to the image, and $L_c$ is the distance from the camera to the central image.
4.6 Gray Scale Intensity Calibration

A linear relationship between camera gray scale level and dye concentration is expected (Ferrier et al., 1993) but the coefficient of proportionality must be determined experimentally. A Lucite cylinder 28.0 cm long by 5.0 cm wide by 50.0 cm deep was used for this. In a 3DLIF system, the coefficient varies with distance from the camera. This variation is caused by two main factors. The plano-convex lens that is used to refract the laser beam parallel to the tank axis is not perfect, so the heights of different laser sheets are not identical. The center laser sheet is the tallest and the heights of the sheets on each side are smaller. The other factor is that the distance between the camera and the laser sheet varies. This causes increased attenuation of the fluoresced light as it passes through the water.

As can be seen from the typical calibration curve for the center sheet shown in Figure 4.10a, the calibration result from the center laser sheet is the same as that from a 2DLIF system. Figure 4.10b shows the variation of the calibration slope in each laser sheet of a 3DLIF system. The variation is not large; if the average slope is used for all images, the error would be less than 5%. In this study, however, the individual slope from each laser sheet is used rather than the average.
4.7 Turbulent Scales
In a 3DLIF system, the three dimensional images are actually reconstructions of multiple images obtained sequentially through the flow. For the image to be considered a good approximation of the flow field, the spatial and temporal resolutions must be chosen appropriately. The fields should be obtained rapidly enough that they represent a frozen image of the flow.

The turbulent scales range from the integral scales down to the Kolmogorov scales. The integral scales are the largest, and have the lowest frequencies; the Kolmogorov scales are the smallest, and have the highest frequencies. In this study, the camera is usually set at 100 or 200 frames per second. The number of total slices is usually 20 or 40 and the distance between adjacent slices is typically 5 to 10 mm. Typical jet velocities are 0.2 to 1.0 m/s and cross flow velocities is 10 to 50 mm/s. For example, if 100 frames per second speed is used and the number of slices is 20, five 3D images are taken per second. The effective sample rate is therefore 5Hz, which is usually too slow to freeze the Komogorov scales. The larger scales can be captured, however, as will be discussed in Section 4.10.

4.8 Image Processing and Data Analysis Software
Video Savant software is used to control the image capturing and saving (see Section 2.4.2). It can only provide instantaneous raw LIF images, however, and the capability of this software was not sufficient for the specific objectives of the present research. Software, named TFLOOK, was therefore developed to cope with the LIF calibration, correction, animations, and 2D and 3D LIF image analysis. TFLOOK is a 32-bit Windows program with a user-friendly interface written in Visual C++ 6.0. The main functions of this software are described below.

1) Display Images
The original images are stored in Video Savant raw image format. There is a file header of twelve bytes before the image data. The first four bytes of the header are the image width, the second four bytes are the image height, and the third four bytes are the total number of images in the file. The image data is saved using line first order, one byte corresponds to one pixel, and there are no spaces between frames. TFLOOK can read these image files and display them. If the image file includes more than one image, TFLOOK can play them as a movie.

2) Basic Image Information
TFLOOK can track mouse position and show the gray scale intensity of the image at that position. It can select a profile with the mouse and show the gray scale distribution along this line graphically. It can also select an area of the image and show the histogram of the selected area. It can convert a gray scale image to a pseudo-color image; users can use the default palette or define a specific palette.

3) Calibration and correction
The calibration images are saved in several files, each file corresponding to one value of dye concentration. TFLOOK can obtain the 3DLIF calibration from these files.
least squares algorithm is used to fit a straight line that passes through zero. The calibration results are saved as slopes for each slice. The camera and lens corrections (Section 4.1) are applied before calibration. TFLOOK can also do attenuation corrections on each image using the algorithms described in Section 4.2.

4) 2DLIF Analysis

2DLIF analyses include time averaging, profile extraction, and computation of standard deviations. The results can be saved and sent to Microsoft Excel or Tecplot for plotting or further analysis. The standard deviation of the concentration fluctuations is calculated from:

\[
\sigma(i,j) = \sqrt{\frac{\sum_{i=1}^{N} [C(i,j) - \overline{C(i,j)}]^2}{N}},
\]

(4.11)

where \(\sigma(i,j)\) is the standard deviation of the concentration fluctuations at point \((i,j)\), \(N\) is the number of images being processed, \(C(i,j)\) is the instantaneous concentration at point \((i,j)\), and \(\overline{C(i,j)}\) is the average concentration.

5) 3DLIF Analyses

3DLIF analyses include time averaging, vertical or horizontal plane extracting, cross-section extracting, 3D reconstruction and visualization. Slices extracted from 3D data can be sent to 2DLIF analysis functions for further analysis. Tecplot is used to reconstruct the 3D images.

Other functions such as extraction of buoyant jet centerline trajectory, minimum dilution, etc., were also developed as needed during this study.

4.9 Towing Tank Carriage Speed Control Calibration

The towing tank carriage speed is varied with the speed control setting. It was calibrated by timing the passage of the carriage between two points a known distance apart. The calibration curve is shown as Figure 4.11. The range of speeds possible is 10 mm/s to 50 mm/s. Although higher speeds are possible, they cannot be used because of the limited tank length.

![Figure 4.11 Towing tank carriage speed control calibration](image-url)
5. SYSTEM VALIDATION

5.1 Introduction

This study consists of two parts: developing the 3DLIF system and applying it to studies of buoyant jets under different ambient conditions. Before application to new experimental conditions, the system must be verified by comparing its results with known ones. The flows chosen for this validation are a pure jet in a uniform stagnant fluid and vertical buoyant jets in stratified and unstratified cross flows. The results from these experiments are discussed below.

5.2 Homogeneous round jet

The simple pure round jet in a uniform fluid has been extensively studied, and much data are available for comparison. The parameters for this experiment are:

- Nozzle diameter: 4.0 mm
- Jet initial velocity: 0.67 m/s
- Source concentration: 350 µg/L
- Camera speed: 100 frames per second
- Number of slices: 40
- Distance between slices: 5.0 mm
- Duration of recording: 60 s
- Attenuation coefficient in fresh water: $\alpha_w = 0.0035 \text{ cm}^{-1}$

Some results are shown in Figure 5.1 to 5.5. Figure 5.1 is reconstructed 3D images for time-averaged and instantaneous data. In these two images, one quarter was removed to show the inner concentration distribution with pseudo-color. Figure 5.2 shows vertical concentration profiles of the jet. Figure 5.2b shows the concentration distribution in a cross-section at $x = 18 \text{ cm}$, where $x$ is distance from the nozzle. The visualizations of Figure 5.1 and Figure 5.2 successfully reconstructed the pure jet and clearly visualized the concentration distribution in three direction planes. The familiar concentration distribution is apparent in these figures.

![Figure 5.1 3D images of a simple jet (unit of concentration is µg/L)](image-url)
The mean concentration decay along the jet centerline is shown in Figure 5.3. It agrees very well with the semi-empirical equation (Fischer et al., 1979), $\frac{C}{C_0} = \frac{5}{d/x}$, where $C$ is the concentration along the centerline, $C_0$ is the source concentration, $d$ is the nozzle diameter, and $x$ is distance from the nozzle.

To investigate the self-similar behavior of the jet, six mean concentration profiles are shown in Figure 5.4. Three of these were extracted from the vertical center plane and the other three were extracted from the horizontal center plane at the same distance. The distances of all profiles are far beyond $6d$ downstream from the jet nozzle. In other words, all profiles were obtained in the Zone of Established Flow (ZEF). As expected, the mean concentration field is symmetric about the centerline. Also, the semi-empirical curve of Fischer et al. (1979), $\frac{C}{C_m} = \exp\left[-\left(\frac{r}{b_c}\right)^2\right]$, is shown in Figure 5.4. $C_m$ is the maximum concentration at that cross-section, $b_c$ is the half-width of the concentration field. From this experimental results, $b_c = 0.134x$; the coefficient 0.134 is slightly higher than the average (0.127) of several experimental data sets reported by Fischer et al.
(1979), but falls within the range of quoted values (0.101 < \( b_c < 0.156 \)). The data agrees very well with the Gaussian curve with the coefficient 0.134. Figure 5.5 shows six profiles of the concentration standard fluctuations. As in Figure 5.4, three of these were extracted from the vertical center plane and the other three from the horizontal center plane. The standard deviation was calculated by Eq. (4.11), and was normalized by \( C_m \). The profiles are symmetric about the centerline and collapse onto a self-similar shape. The peak value is displaced off the centerline and its location is at approximately \( r_lx = 0.1 \). The results agree closely with those obtained by Webster et al. (2001).

These results verify that reliable quantitative data can be obtained with the 3D imaging system developed here. While traditional PLIF systems can only image one plane, the new 3DLIF system can obtain whole images of a flow, from which much more information can be extracted.

For this particular flow, the largest turbulent scale \( L \) is the width of the jet and the largest velocity scale \( U \) is the centerline velocity. From the empirical formulas in Fisher et al. (1979):

\[
L = 0.214x \\
U = 6.2 \frac{u_0 d}{x}
\]

where, \( u_0 \) is the jet initial velocity, \( d \) is the nozzle diameter. Then, the largest time scale \( T \) is:

\[
T = \frac{L}{U} = \frac{0.214x}{6.2 \frac{u_0 d}{x}} = 0.035 \frac{x^2}{u_0 d}
\]

For example, let \( x = 25 \text{ cm} \), we have: \( L = 5.35 \text{ cm} \), \( U = 6.65 \text{ cm/s} \), \( T = 0.82 \text{ s} \). According to Roberts et al. (2001), the dissipation rate of a single pure jet discharging into a uniform stagnant flow can be calculated as:
\[ \varepsilon = 48 \left( \frac{u_0}{d} \right) \left( \frac{x}{d} \right)^4 \]  

(5.4)

For \( x = 25 \text{ cm} \), we get \( \varepsilon = 2.37 \text{ cm}^3/\text{s}^3 \). The Kolmogorov scales can then be obtained,

\[ \eta = \left( \frac{v^3}{\varepsilon} \right)^{1/4} = 0.25 \text{ mm}, \quad \tau = \left( \frac{v}{\varepsilon} \right)^{1/3} = 0.06 \text{ s}, \quad \nu = \left( \varepsilon \eta \right)^{1/4} = 4.0 \text{ mm/s}. \]

In this experiment, the spatial resolution is \( \Delta x = \Delta y = 0.1 \text{ cm}, \Delta z = 0.5 \text{ cm} \), the time to capture one image is \( 0.4 \text{s} \). It is clear that the largest scales can be captured but the Kolmogorov scales cannot be.

5.3 Vertical Buoyant Jets in Crossflow

5.3.1 Unstratified crossflow

Two experiments were done are of a vertical round buoyant jet into unstratified and stratified crossflows. The experiments are performed as shown in Figure 3.1, in which a more dense effluent is discharged downwards. The results are reported here as inverted, i.e. as a positively buoyant effluent discharging upwards. This is allowable because the relative density difference between the effluent and receiving water is small and is therefore significant only for buoyancy forces and not inertia forces (the Boussinesq assumption).

For both experiments, the nozzle diameter \( d \) was 0.42 cm, the current speed \( u \) was 4.0 cm/s, and the effluent flowrate \( Q \) was 6.31 cm³/s. For the unstratified experiment, the density difference between the effluent and ambient was 14.7 kg/m³, and for the stratified experiment was 16.3 kg/m³. For the stratified experiment, the density profile was approximately linear with a buoyancy frequency, \( N = \sqrt{-\frac{g}{\rho_0} \frac{d\rho}{dz}} = 0.46 \text{ s}^{-1} \) where \( g \) is acceleration due to gravity, \( \rho_0 \) is a reference density, taken as 1,000 kg/m³, and \( d\rho/dz \) is the vertical density gradient. This yields the following parameters: Jet Reynolds number, \( Re = 1900 \), buoyancy flux, \( B = g(\Delta \rho/\rho)Q = 91.0 \text{ cm}^4/\text{s} \) (unstratified), and 101 cm³/s (stratified), and momentum flux, \( M = u_0Q = 287 \text{ cm}^3/\text{s}^2 \) where \( u_0 \) is the jet velocity. Relevant length scales for the flow (Wright, 1984) are: \( l_b = B/u^3 = 1.42 \text{ cm} \) (unstratified) = 1.58 cm (stratified), and \( l_m = M^{1/2}/u = 4.24 \text{ cm} \). For the stratified flow two further length scales are \( l_m' = M^{1/4}/N^{1/2} = 6.1 \text{ cm} \) and \( l_b' = B^{1/4}/N^{3/4} = 5.5 \text{ cm} \). Both the buoyancy and momentum fluxes are therefore important for this particular flow.

For both experiments the height of the laser sheet was 23 cm, the thickness about 2 mm, the horizontal distance between laser sheets was 0.87 cm, and twenty sheets were imaged per full sequence for a total width imaged of about 16.5 cm. The camera frame rate was 100 frames per second, and 2800 images were captured for 28 seconds. The source dye concentration was 600 µg/l and 800 µg/l in the unstratified and stratified cases, respectively. The camera saturated near to the nozzle so no quantitative data was obtained there.
The conditions and parameters for this experiment are as follows.

- Nozzle diameter: 4.0 mm
- Jet initial velocity: 0.25 m/s
- Source concentration: 800 µg/L
- Camera speed: 100 frames per second
- Number of slices: 20
- Distance between slices: 5.0 mm
- Attenuation coefficient in fresh water: $\alpha_w = 0.0011 \text{ cm}^{-1}$
- Duration of recording: 35 s
- Cross Flow Velocity: 40 mm/s
- Density difference at nozzle level: 0.028 g/cm$^3$
- Reynolds number: $R_e = 1004$

The length scales can be calculated from Eqs. (1.6), (1.7), (1.12), (1.13)

\[ l_Q = 0.35 \text{ cm}, \quad l_M = 2.85 \text{ cm}, \quad l_m = 2.22 \text{ cm}, \quad l_b = 1.35 \text{ cm} \quad (5.5) \]

For this case, buoyancy flux is more important than momentum flux.

Figure 5.6 shows 3D time-averaged and instantaneous images of the outer surface reconstructed from the 2D slices. Figure 5.6a shows the familiar shape and trajectory of a buoyant jet.

![Figure 5.6](image)

**Figure 5.6** Outer surface of a buoyant jet in an unstratified cross flow

The variation of tracer concentration through a vertical plane through the jet centerline is shown in Figure 5.7. Concentrations are shown as pseudo-color. Animated movies have also been made to show the temporal variation of the instantaneous outer surface of the jet and instantaneous variations in center-plane concentrations.
Once the 3D data is obtained, a great amount of information can be extracted from the data set. In addition to the center plane or any plane off the center, cross sections of the jet can be extracted. Figure 5.8a shows a series of vertical cross sections that show the familiar kidney shape. One of these sections is shown in Figure 5.8b as 2D iso-concentration contours. It is clear that the maximum concentration occurs on both sides off the center plane. The nozzle was not exactly parallel with the laser sheet for this experiment, causing slight asymmetry.
According to Wright (1984), the trajectory and centerline dilution relations of two limiting cases (momentum dominated flow and buoyancy dominated flow) follow the relations given in Table 1.1. For this experiment, a fitted curve for the trajectory 
\[ \frac{z}{l_b} = 2.35 \left( \frac{x}{l_b} \right)^{0.54} \]
was obtained. Compared with Wright's result, this result is closest to the BDFF result. The power value of 0.67 obtained by Wright (1984), is 20% higher than the value obtained here.

The jet trajectory extracted from the center plane is shown in Figures 5.9 and 5.10. According to Wright (1984), the trajectory and centerline dilution relations of two limiting cases (momentum dominated flow and buoyancy dominated flow) follow the relations given in Table 1.1. For this experiment, a fitted curve for the trajectory 
\[ \frac{z}{l_b} = 2.35 \left( \frac{x}{l_b} \right)^{0.54} \]
was obtained. Compared with Wright's result, this result is closest to the BDFF result. The power value of 0.67 obtained by Wright (1984), is 20% higher than the value obtained here.

The centerline dilution is shown in normalized form in Figure 5.11. The results were fitted with the empirical curve 
\[ \frac{S_0 Q}{u l_b^2} = 0.20 \left( \frac{z}{l_b} \right)^{2.13} \]
The dilution result is also close to Wright's BDFF result, but the exponent value 2.13 is 6.5% higher. The buoyant jet flow in this experiment is close to buoyancy dominated flow and the results obtained here are in the far field. The minimum dilution occurs on the sides, off the center plane. To compare the minimum and centerline dilution, \( \frac{x}{l_b} \) is used rather than \( \frac{z}{l_b} \). The results
were fitted with the curve \( \frac{S_0 Q}{u_b l_b^2} = 1.29 \left( \frac{x}{l_b} \right)^{1.15} \) and the minimum dilution relation \( \frac{S_m Q}{u_b l_b^2} = 1.85 \left( \frac{x}{l_b} \right)^{0.97} \) was obtained as shown in Figure 5.12. The exponent for minimum dilution is smaller than that of centerline dilution. Initially, the minimum and centerline dilutions are very close. Farther downstream, the minimum dilution is less than the centerline dilution. The minimum ratio of \( S_m \) to \( S_0 \) is 0.75.

5.3.2 Stratified crossflow

The conditions and parameters for the stratified crossflow experiment were:

- Nozzle diameter: 4.0 mm
- Jet initial velocity: 0.25 m/s
- Source concentration: 800 µg/L
- Camera speed: 100 frames per second
- Number of slices: 20
- Distance between slices: 5.0 mm
- Attenuation coefficient in fresh water: \( \alpha_w = 0.0030 \, \text{cm}^{-1} \)
- Duration of recording: 35 s
- Cross Flow Velocity: 40 mm/s
- Density difference at nozzle level: 0.026 g/cm³
- Buoyancy frequency: \( N = 0.32^{-1} \)
- Reynolds number: \( Re = 1004 \)

The length scales, calculated from Eqs. 1.6, 1.7, 1.12 to 1.15, 1.20 are:

\[
\begin{align*}
    l_Q &= 0.35 \, \text{cm}, & l_M &= 2.96 \, \text{cm}, & l_m &= 2.22 \, \text{cm}, & l_b &= 1.26 \, \text{cm} \\
    l_m' &= 5.27 \, \text{cm}, & l_b' &= 7.03 \, \text{cm}, & l_a &= 12.5 \, \text{cm}
\end{align*}
\]

(5.6)

For this case, buoyancy flux is more important than momentum flux. The refractive index matching technique (see Section 3.3) was applied in this experiment.

The density profile of the ambient flow is shown in Figure 5.13.
Three-dimensional visualizations of time-averaged and instantaneous outer surfaces of the jets are shown in Figure 5.14. In these figures, the surface threshold level is set just above zero. The unstratified jet shows a familiar shape and trajectory; the stratified jet shows flattening near its terminal rise height. Because the vertical mixing is suppressed, the jet shown in Figure 5.14 is wider and flatter than that shown in Figure 5.6. The jet in the density-stratified flow reaches a terminal rise height after which it levels off.

Figure 5.14 Outer surface of a buoyant jet in a stratified crossflow

Tracer concentrations through the vertical center plane are shown with pseudo-color in Figure 5.15. Animated movies were made the instantaneous variations of the outer jet surface jet and the center plane concentrations. Several cross sections through the jet were extracted as shown in Figure 5.16. The kidney shape of cross sections is less evident than that of the unstratified case. Figure 5.16b shows iso-concentration contours of a cross section at a horizontal distance of 22 cm from the nozzle.

Figure 5.15 Center plane of a buoyant jet in a stratified cross flow
The mean jet trajectory is shown in Figure 5.17.

5.3.3 Comparison of stratified and unstratified flows.

To investigate the effect of stratification on the buoyant jets, the centerline trajectory, centerline dilution and minimum dilution of the buoyant jets are shown in Figures 5.18 to 5.20. As can be seen, the stratification has no effect for $x/l_b < 10$, but for $x/l_b > 10$ the stratification causes the jet to level off and dilution decreases. The dilution becomes constant beyond $(x/l_b > 25)$, presumably due to turbulence collapse. The maximum height of rise was defined by Wright (1984) as the location of maximum mean concentration at the horizontal position of the maximum vertical dye penetration. According to this definition, the maximum rise height for this experiment is $z_m = 13.1$ cm.

This value is very close to the value 13.4 cm obtained from $z_m = 2.3 \left( \frac{l_b}{l_0} \right)^{2/3}$ (see Table 1.3). This supports the view that the buoyancy flux is more important in this experiment. Because of the image view limitation, the equilibrium height of rise, $z_e$ (see Figure 1.3),
could not be determined. These data will be reported further in future papers. This information should be of great value in improving mathematical plume models.

We have also studied a horizontal buoyant jet directed normal to the crossflow (i.e. with a three-dimensional trajectory). Comparisons of the variation of dilution with distance between the horizontal and vertical jet are shown in Figure 5.20.
6. APPLICATIONS

6.1 Introduction
The 3DLIF system has been applied to several mixing studies typical of wastewater discharges. Four of these are summarized below. Other studies are ongoing. These and the others will be reported in more detail in subsequent journal articles.

6.2 Scale Model Laws
Three of the experiments below are scale-model tests of particular configurations. The fourth is generic experiments. The experiments were conducted in the configuration shown in Figure 3.1. All images and results shown here are inverted to represent the actual situation, which is a positively buoyant discharge ascending upwards.

The scale model tests are based on equality of the jet densimetric Froude number between model and prototype. The jet densimetric Froude number is defined as:

\[ F_j = \frac{u_j}{\sqrt{\frac{g}{\rho_0 d}}} \]  

(6.1)

where \( u_j \) is the jet velocity, \( g \) the acceleration due to gravity, \( \Delta \rho \) the density difference between the effluent and receiving water, \( \rho_o \) the effluent density, and \( d \) the nozzle diameter. The jet densimetric Froude number is equal between the model and prototype and the model is undistorted.

Using subscripts \( m \) to denote the model, \( p \) the prototype (i.e. the full scale outfall), and \( r \) the ratio of prototype-to-model, we have, therefore:

\[ F_{jm} = F_{jp} \]  

(6.2)

The length-scale ratio, \( d_r = d_p/d_m \), and the model is undistorted so that all lengths are scaled in the same ratio. For example, the ratio of the spacing between the risers, \( s_p/s_m \), and the ratio of water depths, \( H_p/H_m \) are also equal to \( d_r \).

All other ratios are then determined uniquely by the density difference ratio \( \Delta \rho/\rho_o r \) and the length-scale ratio, \( d_r \). For example, the ratios of the jet exit velocity and the ambient current speed are given by:

\[ u_{jr} = u_r = \left( \frac{\Delta \rho}{\rho} \right) \left( \frac{d_r}{\rho_o} \right)^{1/2} \]  

(6.3)

where \( u_a \) is the ambient current speed. The ambient density profile is scaled according to:

\[ \left( \frac{\rho_a(z) - \rho_a(0)}{\rho_a(0)} \right)_r = \left( \frac{\Delta \rho}{\rho_o} \right)_r \]  

(6.4)
where \( \rho_0(z) \) is the ambient density at height \( z \). For further discussion of this scaling, see Roberts and Snyder (1993).

This scaling also ensures equality of other ratios. For example, an important dimensionless parameter for line diffusers is another Froude number (Roberts, 1979):

\[
F = \frac{u_j^3}{b}
\]  

(6.5)

where \( b = g \Delta \rho \frac{Q_T}{\rho_0 L} \) is the buoyancy flux per unit diffuser length, \( Q_T \) is the total effluent flowrate, and \( L \) the diffuser length. This Froude number is also the same in model and prototype, and the source volume, momentum, and buoyancy fluxes are correctly modeled.

The effects of molecular viscosity are not correctly modeled, however. These are characterized by the jet Reynolds number, \( \text{Re} \):

\[
\text{Re} = \frac{u_j d}{\nu}
\]  

(6.6)

where \( \nu \) is the kinematic viscosity of the jet fluid. The Reynolds number in the model is much smaller than in the prototype. This is not a serious limitation, because, so long as the model Reynolds number is high enough that the jet quickly becomes turbulent after leaving the nozzle, the results are insensitive to the value of the Reynolds number. See Roberts and Snyder (1993) for further discussion.

6.3 Produced Water Diffuser

6.3.1 Introduction

Produced water, which is extracted along with oil in the drilling process, is often discharged back into the ocean through a submarine outfall. The outfall may include a diffuser with many ports from which the wastewater is discharged as high velocity jets. Produced water is less dense than seawater, resulting in a buoyancy force which causes the jets to ascend towards the water surface. The 3DLIF system was used in a scale model of a produced water outfall located in the Santa Barbara Channel near Carpinteria, California (Figure 6.1). For details of this outfall, see Washburn et al. (1999).

The outfall consists of a pipeline about 300 m long laid along the seabed terminating in a multiport diffuser. The mean water depth at the diffuser is about 12 m. The diffuser consists of seven Tee-shaped risers, each containing two opposed ports 29.5 mm in diameter that discharge horizontally in a direction perpendicular to the diffuser axis. The risers are spaced 3.5 m apart so the total diffuser length is 21 m. The ports are one meter above the seabed. The density of the produced water is 1011.5 kg/m\(^3\); it is therefore buoyant in the seawater, whose density ranges from 1023 to 1024 kg/m\(^3\). The average
daily flowrate is 0.0306 m$^3$/s. The diffuser is oriented approximately perpendicular to the local bathymetry (and therefore to the predominant currents, see below).

Extensive oceanographic measurements made in the vicinity of the diffuser are reported in Washburn et al. (1999). Currents are highly variable, with speeds ranging from zero to 0.47 m/s, although speeds are less than 0.1 m/s for 90% of the time. The average current speed is about 0.05 m/s. The current directions are predominantly parallel to the depth contours, i.e. along-isobath, although other directions can occur. The currents are influenced by the semidiurnal (M2) tide, although low frequency currents occur in which currents can be fairly steady for several days.

The density stratification varies seasonally. During summer, strong thermal stratification exists; during winter the stratification is weaker, and the water column can sometimes be homogeneous. Observed density differences over the water column ranged from zero to about 0.8 kg/m$^3$. Washburn et al. report plume simulations using the mathematical model RSB which indicate that the plume is often influenced by density stratification resulting in frequent plume submergence.

Based on these observations, a typical set of conditions was chosen for the physical model. The current speed was 0.05 m/s flowing perpendicular to the diffuser. The density stratification profile was linear, with the seawater density increasingly linearly from 1023.1 kg/m$^3$ at the water surface to 1024.0 kg/m$^3$ at the port depth of 11 m. This density difference is at the upper end of the range observed, and is chosen as a “worst case” scenario that minimizes dilution. The test conditions are summarized in Table 6.1.
The diameter of the model jets $d_m = 0.97$ mm, so the model scale $d_r = d_p/d_m = 30.5$. It is not necessary to maintain the same density difference between model and prototype. The density difference ratio ($\Delta \rho/\rho$, was chosen to be 0.25, i.e. density differences in the model were four times larger than in the prototype. The advantage of this is that it increases the value of the model jet Reynolds number causing the jets to become turbulent more quickly. The whole diffuser length (seven risers) was modeled.

The prototype and model parameter values are summarized in Table 6.1, including the values of the important dimensionless parameters, $F_j$ and $F$ (Eqs. 6.1 and 6.5). The jet Froude number $F_j = 53$. This is higher than typical for marine domestic sewage outfalls. This is because domestic sewage has a density close to that of fresh water, around 998 kg/m$^3$. A high jet Froude number signifies that source momentum flux is likely to play a significant role in the initial mixing of the produced water, whereas it is usually unimportant for domestic sewage discharges. The significance of the line Froude number $F$ is discussed in Roberts (1979) and Roberts, et al. (1989). It determines the importance of the current speed relative to the buoyancy of the source. For the present case, the value $F = 0.7$, suggest that the plume dynamics and mixing will be significantly affected by the current.

The tank was filled and stratified with refractive index matched fluids (salt and ethanol) using the methods of Section 4.3. The discharge was begun, and, after allowing time for the flow to establish, the laser scanner and the carriage movement were started. Forty images spaced approximately three mm apart were recorded for each sequence. The
sampling area encompasses the three middle risers, extending just beyond the outer two. In prototype dimensions the sampling volume is about seven meters wide and extends from three meters upstream of the diffuser to 17 m downstream. The images were acquired at 100 frames per second, so the sampling rate for one full sequence is 2.5 Hz.

The images were recorded for 45 seconds during which about one GB of data was captured. Images were then obtained of a calibration vessel placed in the tank containing known dye concentrations. These were used to calibrate the camera’s pixel response to the dye fluorescence and to convert the images obtained in the experiments to dye concentration according to the methods of Chapter 4. These data were then corrected, pixel-by-pixel, to account for attenuation of the laser due to clear water, salt, ethanol, and dye using the methods. Finally, the data were averaged to obtain the spatial distribution of average tracer concentration, and therefore dilution. The estimated accuracy of the concentration readings is ±10%.

6.3.2 Results

Image processing techniques are useful in showing the complex three-dimensional concentration distributions. The outer surface of the mixing field is shown in Figure 6.2. This figure was obtained by imaging an iso-concentration surface with a low threshold level. It can be seen that the upstream jets are rapidly swept downstream by the current. They then merge with those issuing downstream in the downstream direction to form a layer about 5 m thick. The bottom of the wastefield remains essentially at the level of the ports.

The internal concentration distributions are shown in Figures 6.2a and b. In these figures, the outer surface is made transparent and the concentration levels are shown as contour maps in various vertical planes that are color-coded according to the local concentration level. The levels are shown as \( \frac{c}{c_0} \), i.e. they are normalized to the source concentration \( c_0 \) with \( c \) the local tracer concentration. Thus the levels shown are the inverse of local dilution, \( S \).

Variations in the longitudinal plane through the centerline of the middle riser are shown in Figure 6.3a. Upstream of the diffuser, the individual upstream jets can be clearly seen as areas of high concentration; downstream of the diffuser the downstream jets can be similarly seen.

Figure 6.2 Rendering of outer surface of mixing zone. Diffuser is at \( x = 0 \).

The internal concentration distributions are shown in Figures 6.2a and b. In these figures, the outer surface is made transparent and the concentration levels are shown as contour maps in various vertical planes that are color-coded according to the local concentration level. The levels are shown as \( \frac{c}{c_0} \), i.e. they are normalized to the source concentration \( c_0 \) with \( c \) the local tracer concentration. Thus the levels shown are the inverse of local dilution, \( S \).

Variations in the longitudinal plane through the centerline of the middle riser are shown in Figure 6.3a. Upstream of the diffuser, the individual upstream jets can be clearly seen as areas of high concentration; downstream of the diffuser the downstream jets can be similarly seen.
Lateral concentration variations are shown in Figure 6.3b. These variations can perhaps be better seen in the three-dimensional representations shown in Figure 6.4, which are in the same planes as those of Figure 6.3b. Again, the upstream discharging jets can be clearly seen upstream of the diffuser by the high concentration levels in their cores. Farther downstream, at \( x = 5.7 \) m, the cores of the downstream discharging jets can be seen. Beyond this point, lateral mixing quickly results in the mixing layer becoming laterally homogeneous. The layer becomes essentially laterally homogeneous about 12 m downstream.

The variation of minimum dilution with distance is plotted in Figure 6.5. The minimum dilution is defined as the lowest value of dilution in a vertical plane located any distance from the source. The height where this minimum dilution occurs is also plotted. It can be seen that dilution initially increases rapidly with distance from the diffuser due to intense mixing in the near field. It begins to level off when the turbulence becomes affected by ambient density stratification, and reaches a limiting value of about 113 at a distance of about 15 m; this is also where the terminal rise height is reached. Beyond this distance little further mixing occurs. In keeping with the definition of near field (see Roberts et al., 1997, and discussion), the near field dilution is therefore 113, and the length of the near field is 15 m. At the end of the near field, the mixing due to the turbulence that is generated by the discharge is suppressed by the ambient stratification.

The near field results are summarized in Table 6.2.
6.3.3 Comparison with mathematical models

The results were compared with the predictions of the mathematical models UM3, RSB, and CORMIX which were described in Section 2.2. CORMIX used the subsystem CORMIX2 (Akar and Jirka, 1991) for submerged multiport diffusers, and an Eulerian entrainment model, CorJet, to obtain numerical predictions of plume behavior.

This diffuser is typical in that the ports are opposed and discharge from both sides of the pipeline. This does not require any further assumptions in RSB, which is based on
experiments with this discharge configuration (Roberts et al, 1989). Neither UM3 nor CORMIX directly accommodates this configuration, however. In UM3, the ports must all discharge in one direction. We followed the recommended approach (Frick, 2000), in which the ports are assumed to discharge horizontally from the downstream side of the diffuser with the port spacing reduced by half. This preserves the total diffuser length and the discharge per unit diffuser length. Input of the opposing port configuration into CORMIX results in the diffuser being approximated as an “alternating perpendicular diffuser” in which the ports become a single row of vertically discharging round buoyant jets with equivalent total momentum flux. For the present diffuser, the net source momentum flux in any direction is zero. This is true for RSB. In UM3, the total momentum flux is directed downstream, in CORMIX, the total momentum flux is directed vertically.

The predicted properties at the end of the near field are directly outputted by RSB. The near field predictions of UM3 were assumed to be those at the maximum plume rise height, where the program terminates. Similarly, the CORMIX near field predictions were assumed to be those at the end of the CorJet module, which also occurs at the terminal rise height. Although CORMIX continues beyond this point to a “near field region,” the predictions in this region were unrealistically high and were not used here.

The results are summarized in Table 6.2. In CORMIX, the wastefield thickness \( h_e \) was assumed to be equal to \( 4BV/\sqrt{2} \) where \( BV \) is the predicted Gaussian half-width of the plume.

<table>
<thead>
<tr>
<th>Near field property</th>
<th>Measured</th>
<th>RSB</th>
<th>Predicted</th>
<th>UM3</th>
<th>CORMIX</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dilution, ( S_n )</td>
<td>113</td>
<td>118</td>
<td>118</td>
<td>164</td>
<td></td>
</tr>
<tr>
<td>Length, ( x_n ) (m)</td>
<td>15.0</td>
<td>15.2</td>
<td>10.0</td>
<td>18.3</td>
<td></td>
</tr>
<tr>
<td>Height of dilution, ( z_m ) (m)</td>
<td>1.8</td>
<td>3.4</td>
<td>2.7</td>
<td>5.0</td>
<td></td>
</tr>
<tr>
<td>Wastefield thickness, ( h_e ) (m)</td>
<td>4.6</td>
<td>4.5</td>
<td>4.7</td>
<td>4.4</td>
<td></td>
</tr>
<tr>
<td>Height of wastefield top, ( z_e ) (m)</td>
<td>4.6</td>
<td>5.1</td>
<td>5.1</td>
<td>7.4</td>
<td></td>
</tr>
</tbody>
</table>

### 6.3.4 Discussion

Both RSB and UM3 predict the observed dilutions closely, within experimental limits. CORMIX, however, overestimates dilution considerably, by about 45%. The length of the near field is predicted closely by RSB, underestimated by UM3, and overestimated by CORMIX. All models reasonably predict the wastefield thickness. The height of the near field dilution is overestimated by all models, although UM3 is closest, and CORMIX overestimates it by a factor of 2.8. CORMIX also predicted that an upstream wedge of length 14.8 m would occur. This was not observed in the tests, where the jets
intruded only slightly upstream due to their high source momentum flux, and were then quickly swept downstream with no upstream wedge occurring.

The main reason for the overestimation by CORMIX is probably its approximation of the discharges as vertical jets. For this diffuser, the momentum flux of the individual jets is quite high, as reflected by the relatively large value of the jet densimetric Froude number, \( F_j \). In the actual diffuser, however, there is no net momentum flux as the momentum of the upstream jets cancels that of the downstream jets. CORMIX redirects the total momentum flux vertically, which is clearly a poor approximation in this case. UM3 directs the momentum downstream, and does not account for the upstream discharges; it can be seen that this is a good approximation for the case here. RSB is mainly intended for parameter ranges typical of ocean outfalls, in which the jet momentum is usually small. It does include the source momentum in its formulation, however, and it can be seen that it gives reliable results here.

The experiments revealed a complex mixing process. The jets discharging upstream were quickly swept downstream, where they began merging with the downstream jets. Lateral mixing caused the concentration profiles to quickly become laterally homogeneous. Initially, dilution increased rapidly with distance from the diffuser, but farther away the rate of increase of dilution slowed as the turbulence became affected by the ambient density stratification. The dilution eventually leveled off with a value of about 113 about 15 m from the diffuser. This leveling off defines the end of the near field, in which mixing is primarily due to turbulence and processes induced by the discharge itself. At the end of the near field this turbulence collapses under the influence of the ambient stratification. The thickness of the waste field at this point was about 5 m.

The results were compared to predictions of commonly used mathematical models of marine mixing zones: RSB, UM3, and CORMIX. The best predictions of the near field dilution and the length of the near field were by RSB. UM3 gave good predictions of the near field dilution, but CORMIX overestimated dilution and rise height substantially. CORMIX also incorrectly predicted an upstream intrusion that was not observed in the experiments. It is believed that this overestimation by CORMIX is due to its approximation of the diffuser as a row of vertically discharging jets of equivalent total momentum flux; this is clearly a poor approximation to the present situation.

6.4 Boston Outfall

6.4.1 Introduction

As part of the Boston Harbor clean-up process, a new offshore ocean outfall of unprecedented size has recently been constructed to discharge treated wastewater into Massachusetts Bay. This outfall is the final link in the proposed disposal scheme, shown conceptually in Figure 6.6, whose other main elements include a cross-harbor tunnel and a treatment plant on Deer Island. The wastewater may contain stormwater runoff up to a peak design flow of 1270 mgd (56 m³/s). To accommodate such a huge flow, the outfall is a deep rock tunnel with an internal diameter of approximately 24 feet (7.3 m) and a length of approximately 9 miles (14 km). The tunnel terminates in a diffuser. The diffuser length is 2008 m. The diffuser has 55 risers spaced 122 ft (37.2 m) apart.
Similar tunneled outfalls using multiport risers exist throughout the world and others are currently under consideration. Examples are the recently commissioned outfalls at Sydney, Australia, and that proposed for Hong Kong.

As part of the NPDES permit requirements, the Massachusetts Water Resources Authority (MWRA) is conducting field dye tracer experiments to verify that the diffuser performance is similar to that of the original physical model tests on which the diffuser design is based (Roberts and Snyder, 1993). These experiments were done in a towing tank with the dilution measured by extraction of samples at 100 points. Only time-averaged concentrations could be measured, and measurements could only be obtained at one distance downstream from the diffuser. Subsequently, 3DLIF experiments were done to further investigate the mixing processes, to aid in the design of the field tests, and to compare with the actual field data. An experiment in a density-stratified environment is reported here. Other experiments are underway and will be reported later.

6.4.2 Experiments

The model laws are the same as those used by Roberts and Snyder (1993) and discussed in Section 6.2. A stratified experiment was done for conditions similar to those of the field test of April 19 and 20, 2001. The model conditions corresponded to a flowrate of 390 mgd (17.1 m$^3$/s), a current speed 9 cm/s perpendicular to the diffuser, and a density profile that was linear over the water column with a density difference of 0.75 $\sigma_t$ from surface to bottom. The water depth was 30 m. In comparison, on April 19 and 20 the average current speed (computed from the drifter) was 9.3 cm/s. The flowrate varied from about 346 to 412 mgd, and the density profile (on 20 April) was around 1.0 $\sigma_t$ over the water column. All eight ports on each riser were assumed to be flowing, compared to the actual diffuser in which only five or six ports on each riser are discharging. This should not affect the results significantly.
6.4.3 Results

LIF images were obtained in forty vertical planes between two model risers. The fields extend slightly beyond the risers, for a prototype distance of about 40 m (the riser spacing is 37.2 m, or 122 ft). The horizontal extent of the images, and therefore of the dilution measurements, is about 90 m downstream from the risers. Refractive index matching was used to minimize refraction of the laser sheets due to density variations.

The complex three-dimensional concentration distributions are shown in various ways in Figures 6.7 through 6.9. In Figure 1, the outer surface of the plumes and spreading layer are shown in gray; the flow is seen from various perspectives. The eight plumes from each riser merge as they are swept downstream by the current. The resulting plume then merges with those from the adjacent risers. This merging occurs about 17 m from the risers. The plumes then continue to mix laterally, forming a horizontal layer. For these conditions, the plumes overshoot their terminal rise height, and then fall back before leveling off. This flow is similar to an internal wave. The top of the layer is just below the water surface; the eventual thickness of the layer is about 20 m.

Variations in tracer concentrations (and therefore dilution) are shown in Figure 6.8. In these figures, the outer surface is semi-transparent. Tracer concentrations are shown as contour plots in vertical planes that are parallel (lateral profiles) or perpendicular (longitudinal profiles) to the diffuser axis. The concentration levels are shown color-
Figure 6.8 Tracer concentration variations downstream from the risers.

coded and are normalized as $c/c_0$ where $c_0$ is the source concentration. The local dilution is the inverse of the normalized concentration: $S = c_0/c$.

The cores of the individual plumes are still apparent as high tracer concentrations in the lateral profile (Figure 6.8a) through the risers ($x = 0$). Farther downstream, at $x \approx 13$ m, the plumes from the individual ports have partially merged. At $x \approx 26$ m, however, lateral mixing has erased these cores, and the layer is fairly homogeneous laterally. Some mixing continues beyond this point, and the layer thickens somewhat up until about 80 m from the risers. Two longitudinal profiles are shown in Figure 6.8b. These are at the midpoint between two risers, and through the center of one of the risers. Again, high concentrations are apparent in the merging plumes near to the riser.

The lateral variations in concentration are shown differently in Figure 6.9 as three-dimensional plots of the concentration distributions in vertical planes at various distances downstream from the risers (the same lateral profiles as shown in Figure 6.8a). Close to the risers, at $x \approx 13$ m, the high local peaks in the cores of the individual plumes are evident. At $x \approx 26$ m the plumes from adjacent risers are essentially fully merged laterally with only very slight evidence of them left. At $x \approx 40$ m and beyond, the layer is homogeneous horizontally. The ultimate dilution of about 100:1 is reached about 20 m from the risers.
6.4.4 Discussion

The length of the near field, i.e. the region of intense turbulent mixing, is about 26 m for these conditions. Field sampling closer to the risers would probably not result in useful data, as the tracer concentrations would fluctuate widely. The ultimate layer is quite thick, extending from near the water surface to more than 20 m depth. Beyond the near field region, the self-induced turbulence decays, and concentrations in the wastefield should not vary significantly in the horizontal directions.

The results show that the eight plumes from each riser merge, to form a plume that then merges with similar plumes from neighboring risers. This merging begins about 17 m downstream from the risers. After merging, transverse internal mixing erases lateral concentration gradients, and the wastefield becomes essentially laterally homogeneous about 26 m from the risers. At this distance the ultimate dilution of about 100:1 is reached. This marks the end of the near field, and little further dilution occurs beyond this point. The wastefield resembles an internal wave-like pattern and overshoots its ultimate rise height. The wastefield is more than 20 m thick in the vertical direction. Field sampling within the near field, i.e. less than about 26 m from the risers, would probably result in widely fluctuating results that would be difficult to interpret. Beyond this distance, however, wastefield characteristics do not vary rapidly so field sampling positioning is not critical.
6.5 Thermal Power Plant

6.5.1 Introduction

A power plant is currently under design that will take in water for cooling from San Francisco Bay and discharge it back into the bay. The temperature rise of the cooling water as it passes through the plant is about 20°F, and the allowable temperature rise in the receiving waters is 4°F. It is proposed to meet this requirement by discharging the heated water from submerged multiport diffusers that will result in cooling of the heated water by mixing with the colder bay water. The diffusers must result in an initial dilution of at least 5:1 in order to meet the allowable temperature rise.

The discharge will be through two outfalls. Each outfall will terminate in two diffusers about 100 feet long, each with 18 ports oriented upwards at an angle of 45° to the horizontal. The effluent is discharged from the ports as high velocity jets with high momentum flux. The discharge depth varies from 19 to 24 feet. Initial dilution estimates have been made using the U.S. EPA entrainment model UM in Visual PLUMES (Frick et al. 2000). Subsequently, questions were raised about the stability of the discharge due to the high source momentum flux, and the applicability of entrainment models in shallow water where the supply of entrainment water may be restricted.

In order to provide insight into the initial mixing processes of the discharge, and to measure the expected dilutions, scale model experiments were performed using the 3DLIF system to measure the three-dimensional evolution of the mixing processes induced by the diffuser and the expected temperature rise around the discharge. It is emphasized that the experiments were fully three-dimensional, i.e. the length of the diffuser was finite and less than the width of the test channel. The results of these experiments are summarized below and compared to mathematical model and analytical predictions.

6.5.2 Experiments

Each proposed diffuser has 18 ports spaced at 5 ft intervals discharging alternately to each side of the diffuser at an upwards angle of 45° to the horizontal. The diffuser length, end port to end port, is thus 85 ft. The port diameters are 10 inches and the flowrate per diffuser is 79,000 gpm (176 cfs), resulting in a nozzle exit velocity of 18 ft/s. For a temperature difference of 20°F the jet densimetric Froude number, \( F_J = 65 \). The model scale \( d_r \) was chosen to be 100:1. The ports were assumed to be 20 ft below the water surface, and to protrude a height of 3 ft above the bed. Two LIF experiments were done with zero ambient current speed.

6.5.3 Results

Photographs for flow visualization were obtained by adding blue dye to the effluent. Close-up and wide-angle side-view photographs are shown in Figure 6.10. The turbulent jets issuing from the nozzles are clearly visible. Following impaction with the water surface, the flow spreads horizontally. Initially, the spreading layer is still turbulent, resulting in additional mixing and further dilution. This also causes the layer thickness to
increase with distance from the diffuser until it appears to extend over the whole water depth (Figure 6.10b).

![a) Close-up of nozzle details.](image)

![b) Wide-angle view of spreading layer.](image)

**Figure 6.10 Photographs of the diffuser flowfield**

The three-dimensional nature of the effluent field and mixing processes that were obtained by 3DLIF are shown in various ways in Figure 6.11 through 6.14. In these figures, the distances are scaled to prototype dimensions. Temperature rises are inferred from $\Delta T = \Delta T_0 / S$, where $S$ is the local measured dilution and $\Delta T_0$ is the initial temperature rise, taken as 20°F. The water surface and the boundaries of the heated effluent field are shown as gray surfaces, which makes the individual jets visible. Local temperature rise is shown as false color contours in various planes.

Figure 6.11 shows perspective views of temperature rise in vertical planes. Figure 6.11a shows lateral profiles parallel to the diffuser axis at varying distances from the diffuser, and Figure 6.11b shows a longitudinal profile along the diffuser centerline. The three-dimensional nature of the flow is clearly evident. The surface dilution varies laterally, being lowest on the centerline, and increasing towards the layer edges. The dilution becomes more laterally homogeneous at greater distances from the diffuser due to lateral mixing and lateral spreading. The dilution also increases with distance from the diffuser beyond the jet impaction point due to further mixing in the turbulent spreading layer.

These features can also be seen in Figure 6.12 which is a perspective view of the topography of the bottom of the spreading layer, and Figure 6.13 which shows longitudinal profiles at various distances from the centerline. Five of the profile planes pass through the centers of jets, which can be clearly seen. The spreading layer is thicker at the centerline and thinner towards the edges. Although the layer extends over most of the water depth between about 70 and 150 feet from the diffuser, it becomes two-layered farther downstream due to buoyant surface spreading. This two-layered flow must always occur when the diffuser is of finite length, and the extent of the region where the effluent mixes over the depth is therefore limited.
The surface water temperature rise is shown in Figure 6.14 and the variation of surface dilution with distance from the diffuser along the diffuser centerline is shown in Figure 6.15. The lowest, or minimum, dilution (and therefore highest temperature rise), occurs where the jet centerline impacts the water surface, a distance of $x/H \approx \sqrt{2}$. The dilution at this point is about 6:1. Dilution then increases beyond this point up to about 9.5:1, at $x/H \approx 10$ from the diffuser. This is the near-field, or ultimate, dilution. The maximum temperature rise in the vicinity of the diffuser on the surface is therefore expected to be about $20/6 \approx 3.3^\circ F$; this occurs only over a small area, however (Figure 6.14).
The minimum dilution can be predicted by assuming the flow to behave as a free round jet between the nozzle and the water surface. The centerline dilution of a round jet $S_m$ is given by (Fischer et al., 1979):

$$S_m = 0.2 \frac{x}{d} \quad (6.6)$$
For a nozzle depth of 20 ft at an angle of 45°, the distance to the water surface is $20\sqrt{2} = 28.3$ ft, yielding a centerline dilution of 6.8, only about 13% higher than the measured value of 6:1.

![Figure 6.15 Variation of surface dilution with distance from the diffuser on the diffuser centerline.](image)

6.5.4 Discussion

Jirka (1982) introduced the concept of stability for diffuser flows with high momentum in shallow water. A stable effluent field consists of a buoyant surface layer that flows dynamically away from the diffuser with no reentrainment into the rising plumes; the flow field is therefore two-layered. An unstable field consists of a recirculating flow near the diffuser, or formation of an effluent mixture near the diffuser that extends over the whole water depth. This can cause reentrainment of mixed effluent into the rising plumes, leading to a reduction in dilution.

The criterion for stability depends on the source momentum and buoyancy fluxes and the receiving water depth. Jirka (1982) has suggested a criterion for stability:

$$\frac{m}{b^{2/3}H} \leq \frac{0.54}{(1 + \cos^2 \theta_0)^2}$$

where $m$ and $b$ are the buoyancy and momentum fluxes per unit diffuser length, $H$ is the water depth, and $\theta_0$ the nozzle angle to the horizontal. The ratio $\frac{m}{b^{2/3}} = l_m$, a length scale that approximates the distance over which the source momentum flux is important relative to the source buoyancy flux. Thus, the left hand side of Eq. 6.7 is a ratio of length scales; the flow is predicted to be unstable if this ratio is less than some critical value. This concept is built into the mixing zone model CORMIX.

For vertical discharges this approach is reasonable, as a recirculating zone with reentrainment can occur under conditions that are reasonably predicted by Eq. 6.7. For inclined jets with a horizontal momentum component, however, the definition of stability used by Jirka is slightly different: it is that the effluent mixture extends over the whole
Numerical dilution predictions were made using CORMIX. It was predicted that the flow would be unstable with a near-field dilution of 2.9, in other words, the water quality requirement would not be met. As can be seen from Figure 6.15, this underestimates the measured minimum dilution by a factor of two. The CORMIX prediction is apparently based on an equation recommended by Jirka (1982) (his Eq. 23) for unstable flows:

\[ \frac{S_e q}{b^{\gamma/3} H} = 0.5 \]  

(6.8)

This equation does not include the source momentum flux as a variable; it computes the entrained counterflow required to balance the outflow generated by an equivalent line source of buoyancy flux only, i.e. a line plume. In other words, the source momentum flux is assumed to play no role in the dilution process except to cause instability. Furthermore, the CORMIX analyses are two-dimensional, i.e. as if the flow occurs in a channel of finite width or a diffuser of infinite length. This neglects the strong three-dimensional effects that are evident here, and the supply of entraining water from the ends of the diffuser. This is a very crude approximation to the actual situation here, leading to the significant underestimation of the actual dilution. Finally, the dilution predicted by Eq. 6.8 is an average, or bulk, dilution, so the minimum surface dilutions would be even lower than the predicted value of 2.9.

Predictions were also made using the U.S. EPA model UM3 of Visual PLUMES. The predicted minimum surface dilution was 6.4, very close to the observed value. This is expected as the dilution can also be predicted by simple jet theory, Eq. 6.6.

**6.5.5 Conclusions**

Model tests were conducted of a diffuser proposed to discharge heated water from a power plant into San Francisco Bay. The model tests were fully three-dimensional in that the model diffuser was of finite length, allowing for end effects and the development of a three-dimensional initial mixing region and spreading layer. The dilutions were measured by a recently developed three-dimensional Laser-Induced Fluorescence system.

It was found the flow was strongly three-dimensional. Although the effluent mixed over the water depth, this occurred only in a limited region. A two-layer stratified flow developed towards the edges of the effluent field and farther downstream from the diffuser. The measured surface dilution varied laterally, being lower near the diffuser centerline and increasing towards the diffuser ends. The lowest dilution occurred where the jet centerline intersected the water surface. This dilution can be reasonably predicted by simple free jet calculations.

The lowest measured surface dilution is about 6:1. This would meet the allowed temperature rise of 4°F at the water surface. The surface dilution continues to increase.
with distance away from this point due to self-induced turbulence until an ultimate, or near-field, of about 9.5 is reached at a distance of about $x/H = 10$ from the diffuser.

Because the flow is three-dimensional, and particularly because the entraining water can be supplied from the ends, application of entrainment models such as UM3 should be reliable. Indeed, the prediction of UM3 was very close to the observed results. CORMIX is not applicable to this situation because it uses a two-dimensional analysis resulting in unrealistically low estimates of dilution. Two-dimensional analyses of diffusers with finite lengths should be used with caution until their limitations are better known.

6.6 Multiport Ocean Outfall Diffusers

6.6.1 Introduction

A series of experiments of generic experiments to investigate mixing of ocean multiport wastewater diffusers has been conducted. The diffusers used in all experiments consisted of Tee-shaped risers each with two ports.

6.6.2 Stationary ambient

Experiments in a stationary, unstratified environment are summarized in Table 6.3.

<table>
<thead>
<tr>
<th>Run</th>
<th>Flow rate $Q$ cm$^3$/s</th>
<th>Water depth $H$ cm</th>
<th>Jet velocity $u_j$ cm/s</th>
<th>Jet Re No. $Re$</th>
<th>Ambient density $\rho_a$ g/cc</th>
<th>Effluent density $\rho_e$ g/cc</th>
<th>No of risers $n$</th>
<th>Riser spacing $s$ go</th>
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<td>3</td>
<td>30.48</td>
</tr>
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<td>11</td>
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</tr>
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<td>17.26</td>
<td>552</td>
<td>1.0000</td>
<td>1.0244</td>
<td>15</td>
<td>3.81</td>
</tr>
</tbody>
</table>
Perspective views of the spreading layer in a line plume are shown in Figure 6.16. Fig. 6.16a shows a top view and Fig. 6.16b shows a bottom view. To get higher resolution of images, only flows on one side of risers were captured. The center of risers are set as $x = 0, y = 0$. A central slice that passes the centerline of one riser is set as $z = 0$. These rules were applied to all three-dimensional visualizations in this study. Fig. 1 shows clearly that discharges from individual ports merge together before they reach to the water surface. A line plume is a good assumption for this case.

Figure 6.17 shows isosurfaces of the spreading layer with longitudinal and lateral profiles for the same line plume conditions. Fig. 6.17a shows a longitudinal profile along a port centerline. There is a weak internal hydraulic jump right after the plume impacts the water surface. Fig. 6.17b shows lateral profiles parallel to the diffuser axis at varying distances downstream. Concentrations become laterally uniform beyond the hydraulic jump.

Figure 6.18 shows three-dimensional perspective views of the lateral concentration profiles. For $x/H \geq 0.8$, the profiles become laterally quite uniform. Because much more data can be obtained using the new system, the profiles are smoother than similar profiles shown in Roberts et. al (1989c).
Point plume conditions are shown in Figures 6.19 through 6.21. Figure 6.19 shows top and bottom perspective views of the spreading layer. To get higher image resolution, only one riser is included in the images and the range of view began 14 cm downstream from the diffuser. Figure 6.19 doesn't show any merging of neighbor risers before they reach the water surface.

Isosurfaces of the spreading layer with longitudinal and lateral profiles for the same point plume are shown in Figure 6.20. Figure 6.20a shows a longitudinal profile along the port centerline. Comparing with Figure 6.16a, Figure 6.20a shows a much stronger internal hydraulic jump. Figure 6.20b shows lateral profiles parallel to the diffuser axis at varying distances downstream. Similar to the line plume, the profiles become laterally uniform beyond the hydraulic jump. This can be seen more clearly in the three-dimensional...
profiles in Figure 6.21. When $x/H \geq 7.0$, the profiles become quite uniform. This distance is much longer than that for the line plume.

Figure 6.20 Isosurfaces of spreading layer with longitudinal and lateral profiles

Figure 6.21 Lateral concentration in profiles downstream from risers (MDUS17, $s/H = 4.01$)

Following Fisher et al. (1979), Wright (1980) and Daviero (1997), a set of length scales associated with buoyant jets is shown in Table 6.4.
Table 6.4 Length scales associated with buoyant jets

<table>
<thead>
<tr>
<th></th>
<th>Without stratification</th>
<th>With stratification</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Point plume</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( l_p )</td>
<td>( \frac{Q_i}{M^{1/2}} )</td>
<td>( \frac{M^{1/4}}{B^{1/4}} )</td>
</tr>
<tr>
<td>( l_m )</td>
<td>( \frac{M^{1/4}}{B^{1/4}} )</td>
<td>( \frac{B^{1/4}}{N^{3/4}} )</td>
</tr>
<tr>
<td><strong>Line plume</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( l_q )</td>
<td>( \frac{g}{m} )</td>
<td>( \frac{m}{b^{2/3}} )</td>
</tr>
<tr>
<td>( l_m )</td>
<td>( \frac{m}{b^{2/3}} )</td>
<td>( \frac{b^{1/3}}{N} )</td>
</tr>
</tbody>
</table>

Point plume in Table 6.4 means the riser spacing is very large, i.e. \( s/H \gg 1 \), and line plume means the riser spacing is very small, i.e. \( s/H \ll 1 \). The parameters in Table 6.4 are defined as:

- Individual port discharge,
  \[ Q_i = \frac{\pi}{4} d^2 u_j \]  
  (6.9a)

- Momentum flux,
  \[ M = u_j Q_i \]  
  (6.9b)

- Buoyancy flux,
  \[ B = g^0 Q_i = g \frac{\rho_a - \rho_0}{\rho_0} Q_i \]  
  (6.9c)

- Buoyancy frequency,
  \[ N = \sqrt{\frac{g}{\rho_r} \frac{d \rho}{dz}} \]  
  (6.9d)

- Volume flux per unit length,
  \[ q = \frac{n_p Q_j}{n \times s} \]  
  (6.9e)

- Momentum flux per unit length,
  \[ m = u_j q \]  
  (6.9f)

- Buoyancy flux per unit length,
  \[ b = g^0 q \]  
  (6.9g)

where \( d \) = the port diameter, \( u_j \) = the port exit velocity, \( g^0 \) = the modified acceleration due to gravity, \( g \) = the acceleration due to gravity, \( \rho_0 \) = the density of the effluent, \( \rho_a \) = the density of the ambient water at the discharge depth, \( \rho_r \) = the density at height \( z \), \( N \) = buoyancy frequency, \( n \) = total number of risers, \( n_p \) = total number of ports per riser, \( s \) = the riser spacing.

The effects of momentum flux become negligible for certain conditions. Papanicolaou and List (1987, 1988) experimentally investigated round vertical turbulent buoyant jets in unstratified ambient flows. They found that the flow is a momentum driven jet when \( z/l^2_M < 1 \) and it is a buoyancy driven plume when \( z/l^2_M > 5 \), where \( z \) is the vertical distance from the port. Brooks (1980) numerically investigated horizontal round and slot buoyant jets. His results showed that the effects of the source momentum flux are negligible when \( z/l^2_M > 5 \) for a point plume and \( z/l_m > 4 \) for a line plume. For stratified ambient flows, Wong
and Wright (1988) concluded that the source momentum is negligible when \( l_m/l_B < 1 \) for point plume, and Wright et al. (1982) found that the source momentum is negligible when \( l_m/l_B < 0.1 \) for a line plume. So for the effects of source momentum to be negligible, we must make sure \( H/l_M > 5 \) for a point plume and \( H/l_m > 4 \) for a line plume in unstratified ambient flows, and \( l_m/l_B < 1 \) for a point plume and \( l_m/l_B < 0.1 \) for a line plume in stratified ambient flows.

For all conditions shown in Table 6.3, \( 5.76 \leq H/l_M \leq 42.89 \) when \( 0.11 \leq s/H \leq 0.82 \) and \( 1.20 \leq H/l_M \leq 4.08 \) when \( 1.64 \leq s/H \leq 4.9 \). In this study, the source momentum is negligible when the riser spacing is very small but it cannot be neglected when the riser spacing is very large.

The variation of dilution with downstream distance for point plume and line plume are shown in Figure 6.22 and Figure 6.23 respectively. In a line plume, the impact point at water surface, \( x/H = 0.2 \), is very close to the center of risers. The minimum dilution along plume centerline occurs at this point. As shown in Figure 6.16a, the internal hydraulic jump in a line plume is small. The dilution is a minimum at the impact point and then increases towards its ultimate value. The internal hydraulic jump has little effect on the dilution. In a point plume, the impact point, \( x/H = 2.0 \), is much larger. As shown in Figure 6.20a, the internal hydraulic jump is quite significant in a point plume. It increases dilution substantially. The dilution is also a minimum at the impact point, increases to a maximum, and then decreases towards its ultimate value.

The ultimate dilution for a line plume is shown in Figure 6.24. \( \frac{S_m q}{b^{1/3} H} = C_1 \) for \( s/H \ll 1 \):

\[
\frac{S_m q}{b^{1/3} H} = 0.49
\]  

which is very close to the result, \( C_1 = 0.51 \), obtained by Daviero (1997). Experiments in this study are insufficient to get the variation of dilution with variation of riser spacing when \( s/H > 1 \).
Figure 6.24 Ultimate dilution in line plumes

Figure 6.25 shows the ultimate dilution for a point plume. Dilutions shown in Figure 10 are much bigger than those in previous studies because of the effect of source momentum. For example, Daviero (1997) got:

$$\frac{S_m Q_j}{B^{1/3} H^{5/3}} = 0.25$$  \hspace{1cm} (6.11)

In Figure 6.25, normalized dilution decreases rapidly when the effect of source momentum becoming less, and it finally reaches a constant result when the effect of source momentum can be neglected. The variation of dilution with the effect of source momentum is given by:

$$\frac{S_m Q_j}{B^{1/3} H^{5/3}} = 1.15 \left( \frac{H}{l_m} \right)^{0.84}$$  \hspace{1cm} (6.12)
The length of the initial mixing region and thickness of spreading layer in plumes are easy to measure with the new 3D-LIF system. Figure 6.26 shows the spreading layer thickness in line plumes and Figure 6.27 shows the length of initial mixing region in line plumes.

When $s/H \ll 1$, we have:

$$\frac{h_e}{H} = 0.37$$

$$\frac{x_i}{H} = 1.14$$  \hspace{1cm} (6.13)

Daviero (1997) results are given by:

$$\frac{h_e}{H} = 0.35$$

$$\frac{x_i}{H} = 0.78$$  \hspace{1cm} (6.14)

Both studies used LIF images to determine $h_e$ and $x_i$. The spreading layer thickness is easier to determine, and both studies show very close results for $h_e$. As shown by Eqs. 6.13 and 6.14, there is a big difference for $x_i$. The result of $x_i/H$ obtained by Daviero (1997) is 32% less than that in this study. Daviero is the first researcher to measure $x_i$; he probably underestimated it. From all experiments in this study, $x_i/H$ should not be less than 1.0. Figure 6.28 shows a detail of the center plane of a riser in a line plume. Point “A” is right after the impact point at water surface. It is still in transition from the initial mixing region to far field. It is more reasonable to consider point B as the end of initial mixing region.

Figure 6.29 shows the spreading layer thickness in point plumes and Figure 6.30 shows the length of initial mixing region in point plumes. Because of the effect of source momentum, the results are not constant. Both $h_e$ and $x_i$ decreases linearly when the effect of source momentum becomes weaker.
Three-dimensional images of both line plumes and point plumes were reconstructed successfully. This is the first attempt to show such images using experimental methods. The three-dimensional images clearly show the structure of flows and concentration distribution. Ultimate dilution, the spreading layer thickness, and the initial mixing region were measured using the new system. Measurements of dilution and the spreading layer thickness of line plumes are close to previous studies. Measurements of the initial mixing region are larger than previous measurements by Daviero (1997). The point plume measurements show large differences because of the effect of source momentum.

The test results in this study will be compared in the future to mathematical models such as RSB.
6.6.3 Flowing ambient

The tests in a flowing ambient are summarized in Table 6.5.

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<th>Current speed</th>
<th>Jet velocity</th>
<th>Jet Re</th>
<th>Ambient density</th>
<th>Effluent density</th>
<th>No. of risers</th>
<th>Riser spacing</th>
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<tr>
<td></td>
<td>Q (cm³/s)</td>
<td>H (cm)</td>
<td>u_c (cm/s)</td>
<td>u_j (cm/s)</td>
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</tbody>
</table>

Top and bottom perspective views of the spreading layer in a line plume are shown in Figure 6.31. Similar to the unstratified stationary experiments, only flows on one side of risers were captured. The center of risers are set as x = 0, y = 0. A central slice that passes the centerline of one riser is set as z = 0. In addition, flows that are very close to the diffuser (x < 5 cm) don’t show in both figures. Image capturing started downstream, at x = 5 cm, in order to get the ultimate dilution. As shown in Figure 6.31, the plume is swept downstream by the current. The ratio of riser spacing to the water depth is s/H = 0.214. Therefore, the flows approximate a line plume.

Longitudinal and lateral concentration profiles are shown in Figure 6.32 along with isosurfaces of the spreading layer for the same line plume. Figure 6.32a shows a longitudinal profile along the port centerline, and Figure 6.32b shows lateral profiles parallel to the diffuser axis at varying downstream distances. Concentration at water surface is getting uniform along lateral direction after the plume hit the water surface. Figure 6.33 shows the lateral concentration profiles more clearly. When x/H ≥ 1.5, the concentration is quite uniform laterally.
Visualizations of point plumes are not shown here. So here don’t show visualizations of point plumes. For all conditions shown in Table 6.5, 9.77 \( \leq H/l_M \) \( \leq 17.14 \) when \( s/H = 0.214 \) and \( 1.45 \leq H/l_M \) \( \leq 2.79 \) when \( s/H = 3.0 \). Therefore, the source momentum is negligible when the riser spacing is very large.

The variation of dilution with downstream distance for line plume and point plume are shown in Figures 6.34 and 6.35. In a line plume, the impact point at water surface, \( x/H = 2.5 \). The dilution is a minimum at the impact point and then it increases towards its ultimate value. The internal hydraulic jump has little effect on the variation of dilution.
In a point plume, the impact point, $x/H = 6.0$, is much farther from the diffuser. The dilution is a minimum at the impact point, then increases to a maximum, and then decreases towards its ultimate value. $F = 0.26$ for the line plume and $F = 1.06$ for the point plume, which moves the impact points much farther downstream than in the unstratified stationary experiments.

Shown in Figure 6.36, Figure 6.37 and Figure 6.38, are the ultimate dilution, the length of the mixing region and the thickness of spreading layer as functions of Froude number:

$$F = \frac{u_c}{b} \quad \text{(6.14)}$$

where, $u_c$ is current speed, and $b$ is buoyancy flux per unit length.
Daviero (1997) found that currents have no effect on the ultimate dilution if $F < 0.1$. In this study, $F \geq 0.26$ for the line plume and $F \geq 1.06$ for the point plume, so all experiments have strong currents. Daviero (1997) also found the ultimate dilution is constant for line plume when $F \geq 0.3$. As shown in Figure 6.36, the ultimate dilution is not constant for line plume when $F > 0.3$. Only three experiments were carried out here, and to prove the result obtained by Daviero (1997), more experiments need to be done. For the point plume, only two experimental results are shown, therefore the dependence of ultimate dilution on Froude number cannot be determined.

Figure 6.37 shows the lengths of the mixing regions increase with increasing Froude number for both line plume and point plume. Figure 6.38 shows the thickness of the spreading layer for line plumes does not vary significantly with Froude number. The spreading layer thickness of the point plume appears to decrease with increasing Froude number.

Three-dimensional images of line plumes in stratified flowing ambient flows were successfully reconstructed. The images clearly show the flow structure and concentration distribution. Ultimate dilution, spreading layer thickness, and length of initial mixing region were measured, and the effects of currents investigated. Further tests are ongoing. The test results will be compared to mathematical models such as RSB.
7. DISCUSSION AND CONCLUSIONS

A scanning laser system has been successfully developed to enable three-dimensional Laser-Induced Fluorescence imaging in large-scale turbulent stratified flows. The performance of the system was verified by comparing experimental results to well-studied flow for which much information is available. The flows are a round jet in a homogenous, stationary environment, and vertical round buoyant jets in flowing stratified and unstratified environments.

Example applications to various complex flow problems are then described. The flows are all typical of wastewater discharges into the environment. All are from multiport diffusers. First is a produced water outfall in California. This consists of high-momentum jets discharging into a flowing, stratified environment. Next is the Boston tunneled outfall, which is has multiport risers. Again, the discharge is into a stratified flow. Field data exist for this case and comparisons will be reported at a later date. Next is discharge from a thermal power plant. This is a shallow water discharge in which the mixing is dominated by the momentum of the jets. Finally, generic experiments on multiport diffusers are reported. The receiving waters are flowing and stationary. In all cases, the general characteristics of the flow are a rapid increase in dilution within the mixing zone. The dilution eventually levels off where the self-induced turbulence collapses under the influence of buoyancy forces. This marks the end of the near field.

These experiments and analyses are continuing and will be reported in detail in future publications. Detailed analyses of these flows will be reported later. We plan to apply the system to a wide variety of jet and plume problems related to the discharge and mixing zones of wastewaters in the environment.
REFERENCES


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PUBLICATIONS RESULTING FROM THIS RESEARCH


APPENDIX: QUALITY ASSURANCE

Quality assurance has been ensured by following the procedures of the original Quality Assurance Statement. The criterion for LIF data acceptance is an agreement within about 20% with that obtained by other techniques. The measurements are made in a density-stratified towing tank under conditions typical of wastewater discharges into various hydrological environments. The measurements are of tracer concentrations by laser-induced fluorescence. A small amount of fluorescent dye, Rhodamine 6G, is added to the effluent. The laser causes the dye to fluoresce, and the fluoresced light is captured by a sensor and digitized. A long pass optical filter ensures that only the fluoresced light is passed, and not that scattered by particles in the flow field. Typical dye concentrations are of the order of 10 μg/l. The local tracer concentration is computed from the sensor response after corrections. The laser power at the laser head is monitored continuously throughout the experiment by an internal photocell whose analog output is digitized at 12 bits to ensure a steady power output during the experiment.

Fluid densities are measured by a Mettler Specific Gravity Balance to an absolute precision of 0.1 σ-units (0.0001 g/cc). The tank is stratified by a two-tank filling system and the stratification is measured by the conductivity probe whose height is measured with a vernier scale to a precision of 0.01 inches. Effluent flowrate is measured by a flowmeter that is calibrated volumetrically to an accuracy of ±3%. The tow speed is measured by timing the travel of the towing carriage over a measured distance.

Because this is an in-situ method of measurement, no samples from the flow field are obtained. The measurements are made in a non-intrusive manner, with no contact with the flow. No samples are stored for long periods; therefore, there is no custody or handling of samples.

The LIF system is calibrated by obtaining images of a triple-cell filled with three dye solutions of known concentration. A concentrated dye solution is first made by adding dye, whose weight is measured on a precision balance, to water whose volume is measured in calibrated measuring cylinders. Samples of lower concentrations are then made by sequential dilution in which the concentrated solution is diluted in larger volumes of water. All volume measurements are made by calibrated measuring cylinders. The solutions are stored in dark brown bottles to avoid any reduction in fluorescence by photochemical decay. The water is dechlorinated prior to adding the dye to avoid quenching by chlorine.

In order to obtain tracer concentration levels from the fluorescence, further data reduction is necessary. This consists of corrections for a number of effects including: spatial variation in laser sheet intensity; spatial attenuation of the laser sheet; spatial variation in lens response (vignetting); and noise, including random and electronic and background fixed pattern noise (BFPN). The attenuation of the beam as it passes through the tank is exponential. We have measured the attenuation coefficient as a function of ethanol, salt, and dye concentration by measuring the decrease in laser intensity as it passes through the 20 ft long towing tank. The sheet intensity varies spatially due to the non-linear sweep velocity of the beam, which causes the integrated amount of light received by the
sensor to vary spatially. This spatial variation is obtained by the triple-cell calibration images discussed above.

The data are cataloged and archived on Travan Tape cartridges, along with the experimental conditions for each experiment.

We have thoroughly tested the experimental system and verified it by comparisons with several well-studied flows: a round turbulent jet in a stationary environment, and vertical round buoyant jets in stratified and unstratified crossflows. The system reproduces these previous results to within about ±15%, and is therefore judged acceptable.