A MODEL STUDY OF A LATERALLY LOADED PILE

A THESIS

Presented to

The Faculty of the Graduate Division

by

Michael James Grandolfi

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Civil Engineering

Georgia Institute of Technology

September, 1964

A MODEL STUDY OF A LATERALLY LOADED PILE

Approved:

Date approved by Chairman: August 28, 1964

ACKNOWLEDGMENTS

The author expresses sincere appreciation to Dr. Aleksandar S. Vesic who provided guidance, encouragement, and friendly assistance while serving as advisor for this research. Gratitude is also extended to Professor G. F. Sowers and Dr. C. E. Stoneking, members of the reading committee, for their helpful criticisms on the text.

To my wife, Dorothy, I express deep gratitude for her encouragement in the preparation of the text.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

SUMMARY

The purpose of this research was to experimentally check the validity of some existing theories of laterally loaded pile analysis and to determine the accuracy to which these methods predict pile stresses and displacements. This was accomplished by observing the stresses and displacements of a model pile subjected to four different conditions of head displacement.

A hollow model aluminum pile 0.87 inches square with SR-4 electric strain gages applied inside along its length was embedded 46 inches in a bentonite clay. A loading device was designed to apply to the pile at the soil line pure lateral load, pure moment, translation with no rotation, and rotation with no translation. Stress, translation, and rotation were recorded for successive load increments for each of the four displacement conditions investigated. Before each test the bentonite was removed, completely remolded, and replaced around the pile. After each pile test vane shear determinations were made at various depths for control purposes. Periodic unconfined compression determinations and plate load tests for various plate diameters were made.

Test results prove for the first time that the theoretical pile head constants k_t, t, and ρ are true independent constants as presented by Vesic. The constants may be predicted, by using the appropriate equations, from laboratory tests with sufficient accuracy for engineering calculations. Deflections and rotations at the soil line are predicted fairly closely by the theory of elastic subgrade. Good moment

curve shape and depth to the point of maximum moment are predicted by the same theory using a constant modulus or subgrade reaction K. Magnitude of maximum bending moment, however, is predicted with less accuracy and does not increase linearly with load.

A loaded plate of a diameter equal to that of the pile seems to give a value of E_s which, when introduced into the appropriate equation for K and used with the elastic subgrade theory, results in good correlation with measured values of slope, deflection, and moment.

Tests of this nature should be expanded to determine the effect of pile shape and surchage loading. Other soil types, larger piles, and more reliable strain gage installation techniques should be investigated. Data of this nature must be accumulated before the conclusions reached in this study may be expanded to cover piles and soils different than those used in this research.

CHAPTER I

INTRODUCTION

Statement of the Problem

The function of a pile, similar to that of any foundation structure, is to transmit load to a soil stratum at an intensity which the stratum can sustain. Structurally a pile is like a column in that loads are most economically supported when applied axially to the member. Pile-soil action resulting from this load condition has been investigated at great length. The problem of pile action due to non-axial or lateral loads is more complicated because of the indeterminate nature of the resisting forces.

The lateral resistance of a vertical pile depends mainly on the following factors: stiffness of adjacent soil, stiffness of pile, degree of pile head fixity, and character of loading. Because the problem involves a three-dimensional analysis of elastic and non-elastic elements, an exact mathematical solution has not been obtained. The greatest obstacle to an exact solution has been the difficulty in expressing the complicated soil stress-deformation characteristics.

Background

Consideration of vertical loads was the principal design criterion for piles in the early days of foundation design. Later, batter piles were utilized to transmit inclined loads into the soil. Batter piles were driven into the soil inclined in order to place the longitudinal pile axis in line with the load direction. The large axial strength of piles was mobilized to support non-vertical loads when installed in this manner. Batter piles are an expedient method of dealing with forces from structures such as retaining walls, dams, locks, and piers. Increased emphasis has recently been created for vertical pile structures to resist large lateral loads. A solution of this problem has become extremely important with regard to construction of drilling platforms in the Gulf of Mexico. Because of the large lateral loads and limitations of construction techniques, it is not possible to use piles with sufficient batter to resist lateral loads with the horizontal component of axial pile resistance. Therefore, vertical and slightly battered piles must be designed to resist lateral loads of large magnitude. The great expense involved has forced engineers to seek more rational design methods and has increased interest and research in laterally loaded piles.

Purpose of the Research

There generally are two main questions that arise in the design of laterally loaded piles. The first is the magnitude of the bending moment and shear forces; the second is the magnitude of lateral displacement. We must have a reliable theory for the bending of laterally loaded piles in order to obtain answers to these questions. From this theory it should be possible to determine bending moments and shear forces of a laterally loaded pile group under different conditions of loading and displacement of the individual pile heads. The first unknown is the mode of distribution of the group load to the individual piles. The second unknown, the deformation and stress of an individual pile subjected to various head displacement conditions, is the subject of this thesis.

The purpose of this research is twofold. The first aim is to experimentally check the validity of some existing theories of laterally loaded pile design. The second aim is to determine the accuracy to which existing theories predict stresses and displacements.

CHAPTER II

THEORY

Most attempts to describe laterally loaded pile action have utilized elastic theory, obtaining solutions through the use of methods which have been developed for beams on elastic foundations.

The soil is commonly assumed to behave as a series of closely spaced springs. This assumption, first proposed by Winkler (1) in 1867, assumes the ratio of contact pressure to the deflection is the same at every point of the pile and may be expressed as:

$$\frac{p}{y} = k = \text{constant} \qquad (1)$$

where k is the coefficient of soil reaction in pounds per cubic inch, p is the soil resistance per unit length of the pile in pounds per inch, and y is the horizontal deflection of a vertical pile in inches.

The soil stiffness for a particular problem may also be expressed by the modulus of soil reaction which is related to the coefficient of soil reaction by:

$$K = kD \qquad (2)$$

where K is the modulus of soil reaction in pounds per square inch, and D is the pile diameter in inches.

The analysis of a pile subjected to a lateral load is most successfully approached by means of the well known differential equation for the bending of beams:

$$EI\left(\frac{d^4y}{dx^4}\right) = \text{load} = -\ Ky \tag{3}$$

where E is Young's modulus for the pile in pounds per square inch, I is the moment of inertia of the pile in inches^4, and x is the depth below the soil surface measured along the pile in inches.

Various assumptions have been made regarding the variation of the modulus of soil reaction with depth. In 1932 Titze (2) presented a solution with K varying at an exponential function with depth. The resulting equations, while far too complex for practical use, served as the basis of later work. Chang (3) in 1937, Timoshenko (4) in 1941, and Hetényi (5) in 1946 have presented workable solutions and coefficients for solving the problem with K assumed constant with depth. Solutions and coefficients for solving the problem with K varying linearly with depth were presented by Rifaat (6) in 1935 and Reese and Matlock (7) in 1956. Difference equations have been used by Palmer (8) in 1948 and Gleser (9) in 1953 to solve the problem with K varying at random with depth. Full-scale field tests, however, are necessary for a complete solution.

The above analyses have assumed K as a linear function of pile deflection. Typically the soil reaction-pile deflection relationship is not linear. This non-linear relationship has been the subject of much recent analytical study and research (10)(11)(12)(13).

In analyzing the problem of distribution of load among the individual piles of a group, Hrennikoff (14) in 1950 and Vesic (15) in 1956 presented somewhat similar approximate design methods. Using this approach, they defined pile constants based on the assumption the load carried by a pile is proportional to the movement of the pile head, or top.

Following are the constants as derived by Vesic. They will be developed in detail, as the derivations are not contained in the literature in English.

A pile head subjected to a pure translation is shown in Figure 1a. In order to accomplish a lateral displacement y in inches, a lateral force T in pounds and a moment M in pound-inches are needed. The slope ϕ, in radians, of the pile with respect to a vertical remains zero. Within the range of admissible loads a linear relationship is assumed between T, M, and y. This relationship may be expressed as:

$$T = k_t \, y \tag{4}$$

$$M = Tt = t \, k_t \, y \tag{5}$$

where k_t is the coefficient of lateral reaction of a fixed head pile in pounds per inch, and t is the first characteristic length of a pile in inches.

A pure rotation of a pile head is possible only when the pile is subjected to a moment and lateral force as shown in Figure 1b. In a manner similar to that used for the fixed head case the following expressions may be written:

$$T = k_m \, \Phi \tag{6}$$

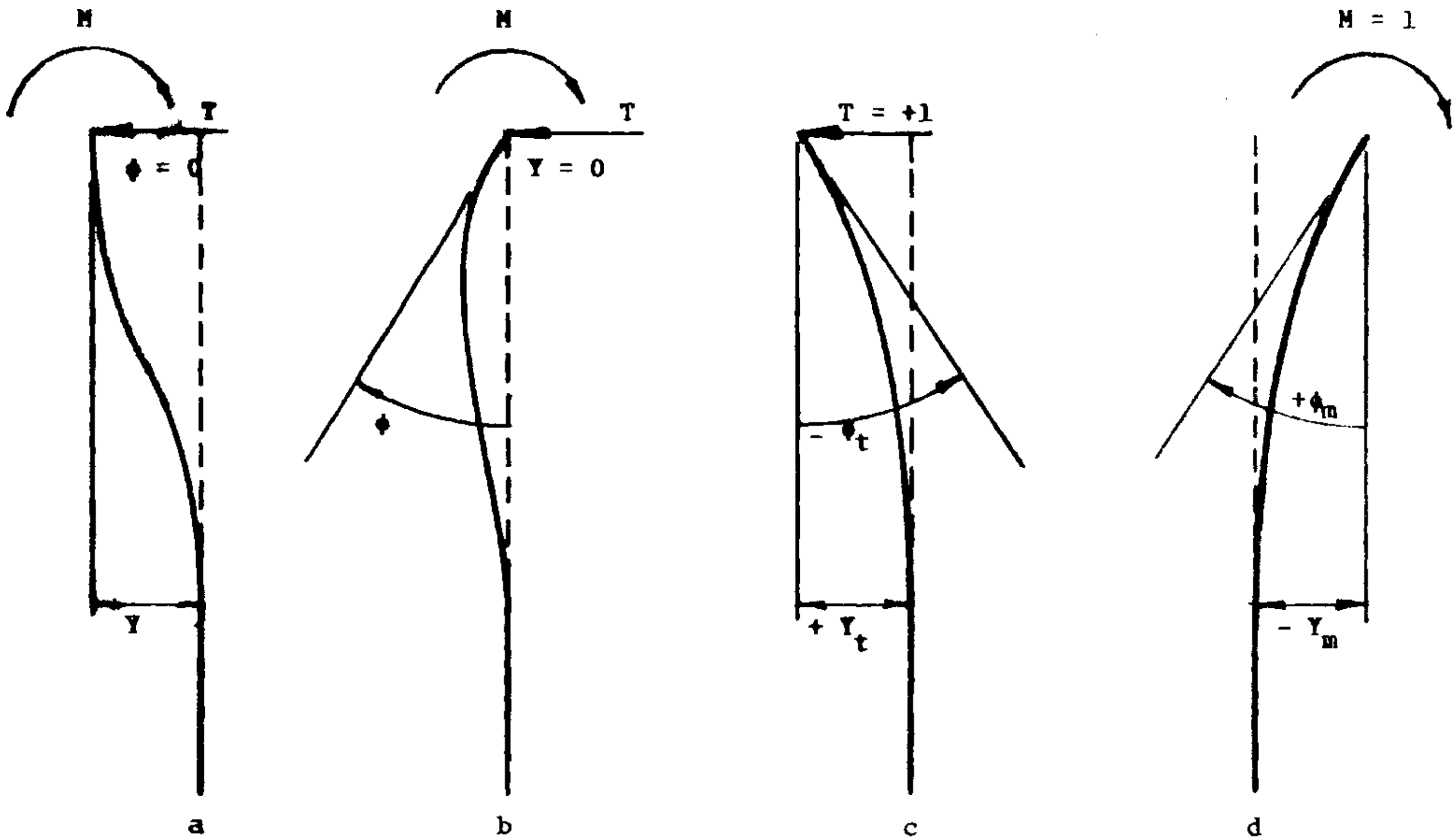


Figure 1. Conditions of Pile Head Loading

$$M = s\ T = s\ k_m\ \phi \tag{7}$$

where k_m and s are constants similar to k_t and t. Among k_t, t, k_m, and s, only three quantities can be proven independent. Vesic has selected k_t, t, and ρ as independents:

$$\rho = \frac{s}{t} \tag{8}$$

where ρ, the ratio of the lengths s and t is dimensionless. There are simple methods of experimentally determining s or t. From a consideration of Figures 1c and 1d and the reciprocal theorem (see Appendix for derivation) it can be shown that:

$$s = \frac{y_t}{\phi_t} \tag{9}$$

$$t = \frac{\phi_t}{\phi_m} = \frac{y_m}{\phi_m}$$

where y_t and ϕ_t are the deflection and rotation, respectively, of a free head pile test as depicted in Figure 1c and y_m and ϕ_m are the deflection and rotation, respectively, of a pile loaded with a pure moment as shown in Figure 1d.

According to the above theory, k_t, t, and ρ may be determined from a combination of results from the free head and pure moment cases shown in Figures 1c and 1d. The theory also states that the same constants may be determined from the fixed head and pure rotation cases depicted in Figures 1a and 1b. Therefore, if they are true constants as the theory predicts, they should check when determined experimentally from the two independent groups of pile tests described above. In this study, a model pile will be used, and a test performed for each of the four head conditions shown in Figure 1 in an attempt to verify the constants.

Vesic has attempted to predict these constants from physical soil properties. Using the theory of subgrade reaction and considering K as defined by equation 2, Vesic reports for K constant with depth:

$$k_t = 1.41\ K^{3/4}\ (EI)^{1/4} \tag{12}$$

$$t = 0.750\ \left(\frac{EI}{K}\right)^{1/4} \tag{13}$$

$$\rho = 2.00 \tag{14}$$

and for K increasing linearly with depth:

$$k_t = 1.07\ n_h^{3/5}\ (EI)^{2/5} \tag{15}$$

$$t = 0.920\ \left(\frac{EI}{n_h}\right)^{1/5} \tag{16}$$

$$\rho = 1.64 \tag{17}$$

where n_h is a soil constant in pounds per cubic inch. Terzaghi (16) in 1955 presented values of K and n_h in terms of the compressive strength of clays and the density of sand.

Vesic (17)(18) in 1961 suggested the equation:

$$K = 0.65 \left(\frac{E_s\ D^4}{EI}\right)^{1/12} \left(\frac{E_s}{1-u_s^2}\right) \tag{18}$$

where E_s and u_s are the Young's modulus in pounds per square inch and Poisson's ratio of the soil, respectively.

Expressions 12 through 17 have been developed for piles of infinite length. According to Barber (19) piles may be treated as infinitely long if:

$$\frac{K\ D\ L^4}{EI} \tag{19}$$

or

$$\frac{n_h\ L^5}{EI} \tag{20}$$

is greater than 500. In these expressions L is the pile length. Applying these conditions, Vesic (20) reports piles may be considered infinitely long when:

For square piles

$$\text{in sand } \frac{l}{D} > 2.1 \left(\frac{E}{n_h\ D}\right)^{1/5} \tag{21}$$

$$\text{in clay } \frac{l}{D} > 2.54 \left(\frac{E}{KD}\right)^{1/4} \tag{22}$$

For round piles

$$\text{in sand} \quad \frac{l}{D} > 1.9 \left(\frac{E}{n_h \quad D}\right)^{1/5} \qquad (23)$$

$$\text{in clay} \quad \frac{l}{D} > 2.22 \left(\frac{E}{K \quad D}\right)^{1/4} \qquad (24)$$

In 1937 Biot, utilizing Fourier's Series, solved the problem of bending of beams on an elastic isotropic solid. Vesic (21) has extended Biot's solution yielding for the fixed head case:

$$t = 0.824 \left(\frac{E \ I}{E_s}\right)^{1/4} \qquad (25)$$

$$k_t = 0.963 \ D \ E_s \left(\frac{EI}{E_s \ D^4}\right)^{1/5} \qquad (26)$$

$$\rho = 1.88 \qquad (27)$$

CHAPTER III

EQUIPMENT

Figure 2a shows a view of the test equipment. The apparatus may be considered as five component parts: container, soil, pile, electronic equipment, and loading assembly.

The soil was contained in a corrugated steel cylinder 36 inches in diameter and 50 inches deep. A polyethylene membrane covered the entire assembly, except during testing, to prevent escape of moisture. The commercial bentonite selected as the foundation medium had been used in other research in the Soil Mechanics Laboratory of the Georgia Institute of Technology. Bentonite is a highly colloidal sensitive clay which, when mixed with water, forms a thixotropic gel and swells to several times its original volume. Since bentonite has the ability to re-gel after remolding, it is desirable in programs involving repeated testing. This material exhibits elastic stress-strain characteristics which make the concept of constant soil modulus of reaction more realistic than with the more inelastic clays found in nature. The clay had a water content of approximately 500 per cent and a unit weight of approximately 75 pounds per cubic foot.

The model pile was extruded aluminum tubing, 0.87 inches square, with a wall thickness of 0.055 inches, a total length of 55 inches, and a Young's modulus of 10.3×10^6 pounds per square inch. The pile bottom was fitted with a recessed plug assembly to give a point support with no

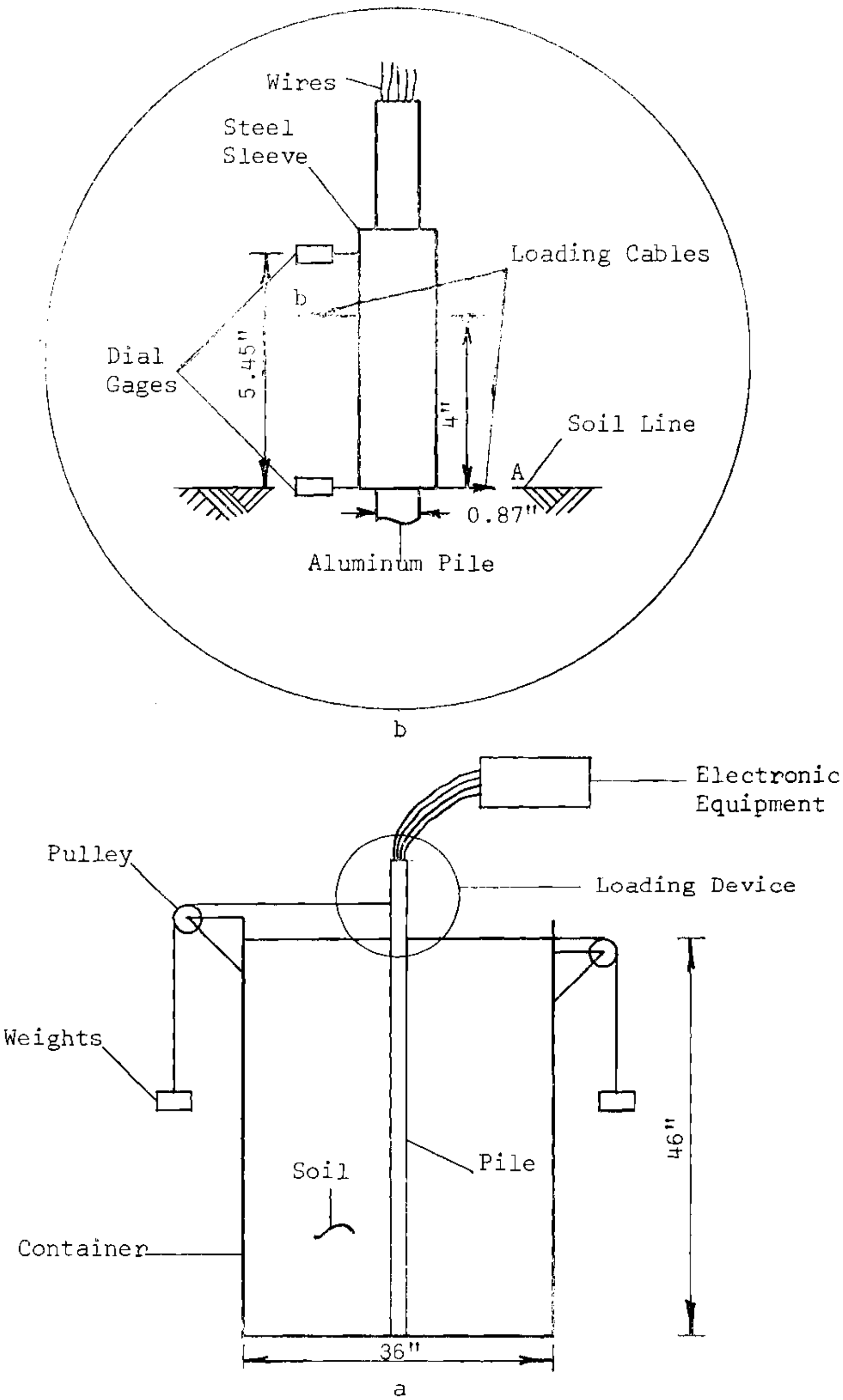
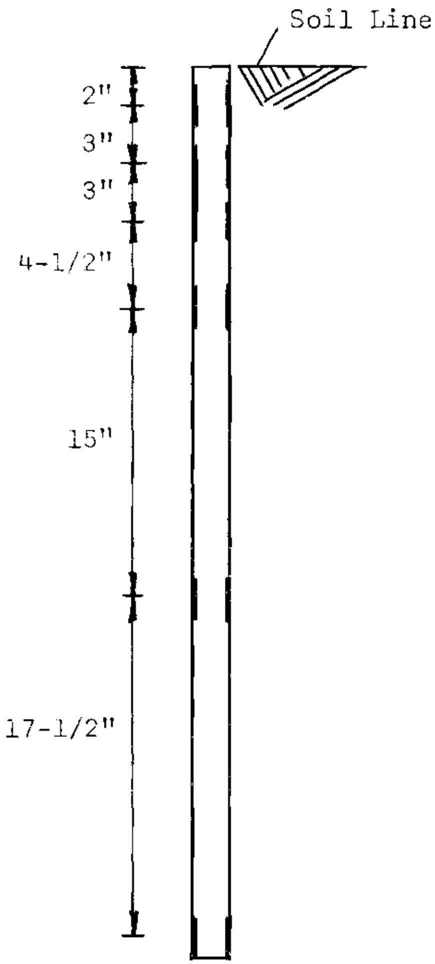


Figure 2. Test Equipment

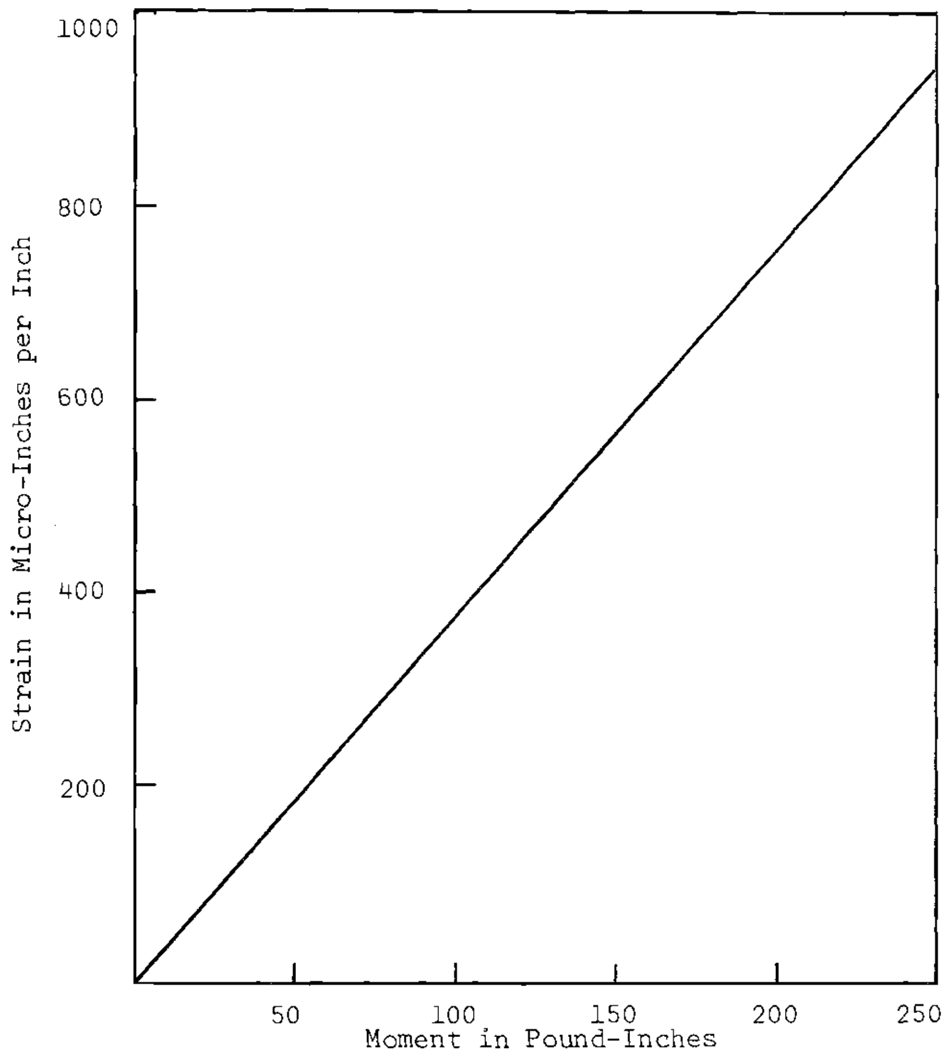
moment capacity to hold the pile during testing and while the tank was being filled with soil. An attempt was made to measure pile bending strains with type A-1 electric strain gages, manufactured by the Baldwin-Lima-Hamilton Corporation. The gages were placed in pairs on opposite sides of the pile bending axis in the six positions shown in Figure 3a. The gages were placed inside the pile section, without damaging or cutting the pile, and the lead wires run through the pile center and out the top. In order to compensate for temperature variations, a compensating gage was placed in the pile in such a manner as to be unaffected by bending. A Baldwin Type L strain indicator with switching and balancing capacity was used to measure strains. Calibration charts were utilized to convert strains to bending moment. The pile was placed horizontally and loaded as a beam, and the known moment and measured strain plotted as shown in Figure 3b to produce the calibration chart for each gage location.

Liquid vinyl plastic was then brushed on the pile to prevent moisture from entering through the bottom, and to prevent chemical action between the pile and soil. Granular silica gel, an anhydrous gelatinous silica, was placed inside the pile to absorb moisture. The anulus at the pile top was sealed around the lead wires completely sealing the inside of the pile.

In this study it was desired to load the pile at the soil level with moments and lateral forces. The loading device shown in Figure 2b was developed for this purpose. A lateral load at the soil level was accomplished by placing weights on cable a. A moment at the soil level was developed by placing weights on cable b. The moment is



A. Gage Locations on Pile



B. Strain-Moment Calibration Curve

Figure 3. Strain Gage Location and Calibration Curve

realized by the force in cable b acting through a four-inch moment arm to the soil level. The lateral pile deflection at the soil surface was measured with the lower dial gage, which was sensitive to 0.001 inches. A similar dial was placed 5.45 inches above the soil line. If no bending of the pile takes place between the two gages, the readings may be utilized in calculating the pile slope at the soil line. In order to prevent bending between the dial gages, a rigid sleeve six inches long was made from 0.250 inch thick steel and fitted tightly over the pile as shown.

Standard testing devices were used for unconfined compression and plate load soil tests. A miniature vane was also used for shear tests. The device contained two vanes, each two inches deep and two inches wide.

CHAPTER IV

PROCEDURE

Previous research at Georgia Tech by Martin (22) indicated that five days were needed for bentonite to re-gel to full strength after remolding. It was therefore decided to run pile tests five days after the bentonite had been remolded and placed into the container.

Before each of the pile loading tests the bentonite was removed from the container and remolded. After the pile had been positioned within the soil container, the bentonite was placed, by hand, in layers of approximately one and one half inch thickness. In order to obtain a homogeneous medium with no air pockets, each soil layer was thoroughly kneeded and tamped by hand.

The first test was run with a lateral load only, as shown in Figure 1[illegible]. Cable b was removed and the lateral load T applied through cable a. Loads were applied in ten pound increments. Lateral pile movement ceased after three minutes, at which time dial gages were read, strain readings taken, and dial gages read again. An additional lateral movement of 0.005 inches was recorded while making readings for the last two load increments only. A periodic second check of strain reading indicated no appreciable change with time.

All soil tests were made from soil unaffected by pile movement. Vane shear tests were made at depths of 5, 20, and 34 inches below the soil surface. These tests were run after each of the four pile tests as a soil strength control. Unconfined compression tests were made on undisturbed samples from depths 5 and 34 inches below the soil line.

Test two was performed with a moment only, applied to the pile as shown in Figure 1d. This load condition was obtained by loading cables a and b simultaneously with equal weights. The applied moment was equal to the cable b load time the four-inch moment arm to the soil line. There was no applied lateral load, as a result of the equal and opposite loads in the cables. Loading was applied in ten pound increments, and readings taken as during test one. Vane shear determinations were made after testing.

A pure translation as shown in Figure 1a was applied to the pile for test three. Utilizing equation 9 and data from tests one and two, t may be calculated. An examination of equation 5 shows that t also is the ratio of moment to lateral load in the fixed head, or pure translation case. In this test, loads were applied to the bottom cable in 20 pound increments. Since t was known, it was possible to predict the necessary bottom cable load needed to maintain a slope of zero at the soil line. During testing, loads were applied simultaneously on the cables with only slight adjustments necessary in cable b loads to maintain a zero slope. During testing it was discovered that through error in dial reading, the slope was not zero, and therefore load increments two through six are not representative of a fixed head pile. After testing, the usual vane shear determinations were made, and three undisturbed samples obtained for triaxial shear tests.

Test four was a pure rotation applied to a pile as shown in Figure 1b. Utilizing data from tests one and two, s may be determined.

Equation 7 shows that s is also the ratio between moment and lateral load in the pure rotation case. Therefore, the ratio of loads on cables a and b was determined before testing. Again, loads were applied simultaneously on the cables with only minor weight adjustment necessary to maintain zero lateral deflection. The usual procedure for dial and strain readings was adhered to. After testing, vane shear determinations were made. Unconfined compressive data were obtained from the five-inch depth, and plate load tests performed on the soil surface with plate diameters of two, three, and five inches.

CHAPTER V

DISCUSSION OF RESULTS

Soil Tests

After each pile test, vane shear determinations were made. The results are presented in Figure 3. The plots indicate the shear strength was essentially constant for the four pile tests at corresponding depths. The increase of strength with depth is probably due to friction on the vane shaft and not indicative of an actual increase of strength.

Figures 5 and 6 exhibit plots of unconfined compression tests performed on 1.4 inch diameter samples taken at the 5 and 35 inch depths, respectively. The data indicate no increase in strength with depth. In each case, the secant modulus of deformation of the soil was 33 pounds per square inch. It may be noted that the modulus was calculated at one-half of the failure load. This procedure has been presented by Skempton (23), Terzaghi (24), and others.

Figures 7, 8, and 9 show results of plate load tests on circular plates of 2, 3, and 5 inch diameters, respectively. Ultimate values may not be used from these tests because tilting and sliding of the plates took place as failure was approached. The initial portion of the curves, however, can be utilized to calculate the Young's modulus of the soil from the theory of elasticity (25):

$$E_s = \frac{.785\ p\ d}{s}\ (1 - \mu_s^2) \tag{28}$$

where p is the soil pressure in pounds per square inch, d is the plate diameter in inches, and u_s is the Poisson's ratio of the soil which may be assumed as 0.50 for saturated clays. E_s values were calculated for each test at one-half the failure load, and the results presented in Figure 10. E_s varies with the size of the loaded area. In order to find the E_s applicable to the model pile, the chart can be entered with the pile diameter of 0.87 inches. Using this procedure, an E_s of 34 pounds per square inch is obtained and will be used in later calculations.

Pile Tests

Test 1--Free Head

The results from the free head pile test are shown in Figures 11, 12, and 13. In this and the following pile tests, line A was drawn through the measured points. Line B shows values computed by equations 25 through 27, utilizing $E_s = 34$ pounds per square inch as selected from Figure 10. Line C is based on the theory of elastic subgrade, utilizing a constant $K = 13.5$ pounds per square inch calculated from equation 18. Within the testing range, s may be considered a constant with a value of 10.4 inches as shown in Figure 11. Values from lines B and C are 133 per cent and 152 per cent, respectively, of the measured s. A comparison of measured and predicted values for all pile tests is listed in Table I in the Appendix.

Test 2--Pure Moment

Figures 14, 15, and 16 exhibit results of a pile loaded as shown in Figure 1d. A straight line may be drawn through most of the measured values in Figure 14. The fact that the straight line does not pass through the origin is an indication of testing error. The computed value of t is this case is 5.6 inches. Lines B and C do not plot close to the experimental values; however, their slopes, t, are a fairly close 131 per cent and 141 per cent, respectively, of the measured value. As indicated by Figure 15, line C gives a fairly close 80.5 per cent correlation while line B gives values of 53.5 per cent of experimental data. Lines B and C both exhibit poor correlation with the experimental moment-deflection curve in Figure 16.

Test 3--Pure Lateral Translation

Results of a fixed head pile test are presented in Figures 17, 18, and 19. Figure 17 shows a constant load-deflection curve up to the 70 pound load, at which point the bending stress was 11,000 pounds per square inch in the aluminum pile. Therefore, within the range of permissible working loads, k_t is a constant. Lines B and C have values of 50 per cent and 58 per cent, respectively, of the measured k_t. Excellent correlations of 99 per cent and 107 per cent of the measured t value were obtained in this case. Lines B and C are fairly close to the moment-deflection plot in Figure 19. Both k_t and t appear to be constants which can be predicted with good accuracy. t not only is a constant throughout the testing range, but appears to be a true constant, since it was predicted within 24 per cent by two independent pile tests of different loading nature as listed in Table I.

Test 4--Pure Rotation

Results are presented in Figures 20, 21, and 22 of a pile loaded as shown in Figure 1a. Within the range of permissible loading, k_m is a constant as shown in Figure 20. Lines B and C values are 45.2 per cent and 56.2 per cent of the experimental k_m. The straight line plot of Figure 21 shows that s is a constant. Predictions of 134 per cent and 154 per cent of the measured s are shown by lines B and C. Test 4 proves that s is a true constant, because the value from Test 3, under different loading conditions, is within 0.96 per cent of the Test 4 value. An excellent straight line plot is exhibited between moment and slope in Figure 22. Values of line C are 93.5 per cent of the measured slope, while line B values are 67.5 per cent.

Moment Curves

Figures 23 through 28 are plots of moment along the embedded length of the pile for each load increment of Test 1. In each case line A is the measured moment, while line B is the moment predicted by elastic subgrade theory. In the calculations, K was taken as 13.5 pounds per square inch, and considered constant with depth as well as deflection. An examination of the plots shows measured moment less than predicted moment for the 10 and 20 pound loads. At the 30 pound load maximum predicted and measured moments are equal, while measured values exceed predicted values for load increments of 40 pounds and larger. This is excellent proof that K is not a "soil constant" but varies with deflection as well as other factors. Figure 29 is a plot showing the variation of K as loads and deflections increase. A value of K was chosen for each load increment of Test 1 in order that the calculated and measured moment values would be equal. Similar, but not exact plots may be obtained by equating deflections or slopes. K is large for small deflections and small for large deflections. Therefore, selecting a single value of K for calculations is an approximation. Although methods have been presented to utilize a K variable with deflection, they are necessary only when such refinement is justified.

Generally calculations based on K constant with depth show good correlation for overall moment curve shape with measured values. Measured and calculated depths of maximum moment agree very well. Generally, measured values show the depth of maximum moment increasing with increasing load. Theoretical values of depth of maximum moment remain constant.

CHAPTER VI

CONCLUSIONS

The following conclusions pertain only to an aluminum pile 0.87 inches square, embedded in a cohesive soil and statically loaded.

1. The independent values of k_t, t, and ρ are constants as presented by Vesic.

2. These constants may be predicted from laboratory tests with the necessary accuracy for engineering calculations by using the appropriate equations.

3. Deflections and rotations at the soil line are predicted fairly closely by the theory of elastic subgrade.

4. Good moment curve shape is predicted using K constant with depth. However, the moments do not increase linearly with load.

5. A loaded plate of diameter equal to the pile seems to give a value of E_s that may be used in equation 18 for K, resulting in good correlation with measured values of slope, deflection, and moment.

CHAPTER VII

RECOMMENDATIONS

Additional experimental studies of laterally loaded piles are necessary to increase the basic knowledge of the subject.

Cylindrical as well as square pile shapes should be studied in order to observe the shape factor influence.

The effect of surcharge loading on pile displacements should be studied.

Pile tests should be made with other types of soils. Possibly a K increasing with depth is valid for some clay deposits as well as for sand.

Larger size piles should be used for testing purposes in order to accomplish better and more reliable strain gage installation.

Methods should be developed to produce more accurate bending moment curves that could be used to obtain soil pressures from double differentiation. Soil pressure variation with deflection could then be studied.

APPENDICES

APPENDIX I

ADDITIONAL DERIVATIONS

Consider a pile subjected to a lateral force $T = +1$ (Figure 1c), producing displacement $+ y_t$ and rotation $- \phi_t$. Consider the same pile with the application of a moment $M = + 1$ (Figure 1d) inducing in the pile displacement $- y_m$ and rotation $+ \phi_m$. If the pile and soil exhibit linear deformation, then according to the reciprocity theorem:

$$\phi_t = y_m \tag{29}$$

If lateral translation is pure (Figure 1a) we have:

$$- T \phi_t + M \phi_m = 0$$

or:

$$\frac{M}{T} = t = \frac{\phi_t}{\phi_m} \tag{30}$$

According to equations 29 and 30 we can write:

$$y = T y_t - M y_m = T \left(y_t - \frac{\phi_t{}^2}{\phi_m}\right) \tag{31}$$

If the rotation is pure (Figure 1b) we may write:

$$T\ y_t - M\ y_m = 0$$

or:

$$\frac{M}{T} = s = \frac{y_t}{\phi_t} \tag{9}$$

In a manner similar to that used to obtain equation 31 we can write:

$$\phi = - T\ \phi_t + M\ \phi_m = T\left(- \phi_t + y_t\ \frac{\phi_m}{\phi_t}\right) \tag{32}$$

From 31 and 33, and taking into account 4 through 7, we find:

$$\frac{k_m}{k_t} = \frac{\phi_t}{\phi_m} = \tau \tag{10}$$

or:

$$k_m = k_t\ t \tag{33}$$

Relations 6 and 7 become:

$$T = k_t\ \tau\ \phi \tag{34}$$

$$M = \rho\ k_t\ \tau\ \phi = \rho\ T\ \tau \tag{35}$$

Vesic arrives at final expressions by using the superposition principle and relations 4, 5, 34, and 35 to obtain:

$$T = k_t \, (y + t\phi) \tag{36}$$

$$M = k_t \, t \, (y + \rho t \phi) \tag{37}$$

APPENDIX II

NOTATION

k = Coefficient of subgrade reaction (lb./in.^{3}).

p = Soil pressure between the pile and soil, per unit length of pile (lb./in.).

y = Horizontal deflection of the pile (in.).

K = Modulus of subgrade reaction (lb./in.^{2}).

D = Pile width (in.).

E = Young's modulus for the pile (lb./in.^{2}).

I = Moment of inertia of the pile (in.^{4}).

x = Depth below the soil surface (in.).

T = Lateral force applied to pile (lb.).

M = Moment applied to pile (lb.-in.).

ϕ = Slope of the pile (radians).

k_t = Coefficient of lateral reaction of a pile (lb./in.).

t = First characteristic length of a pile (in.).

k_m = Coefficient of moment reaction of a pile (lb.).

s = Second characteristic length of a pile (in.).

ρ = Ratio of lengths s and t, dimensionless.

n_h = Constant of lateral pile reaction in sand (lb./in.^{3}).

E_s = Young's modulus for soil (lb./in.^{2}).

μ_s = Poisson's ratio of the soil.

L = Pile length (in.).

d = Plate diameter (in.).

APPENDIX III

ILLUSTRATIONS

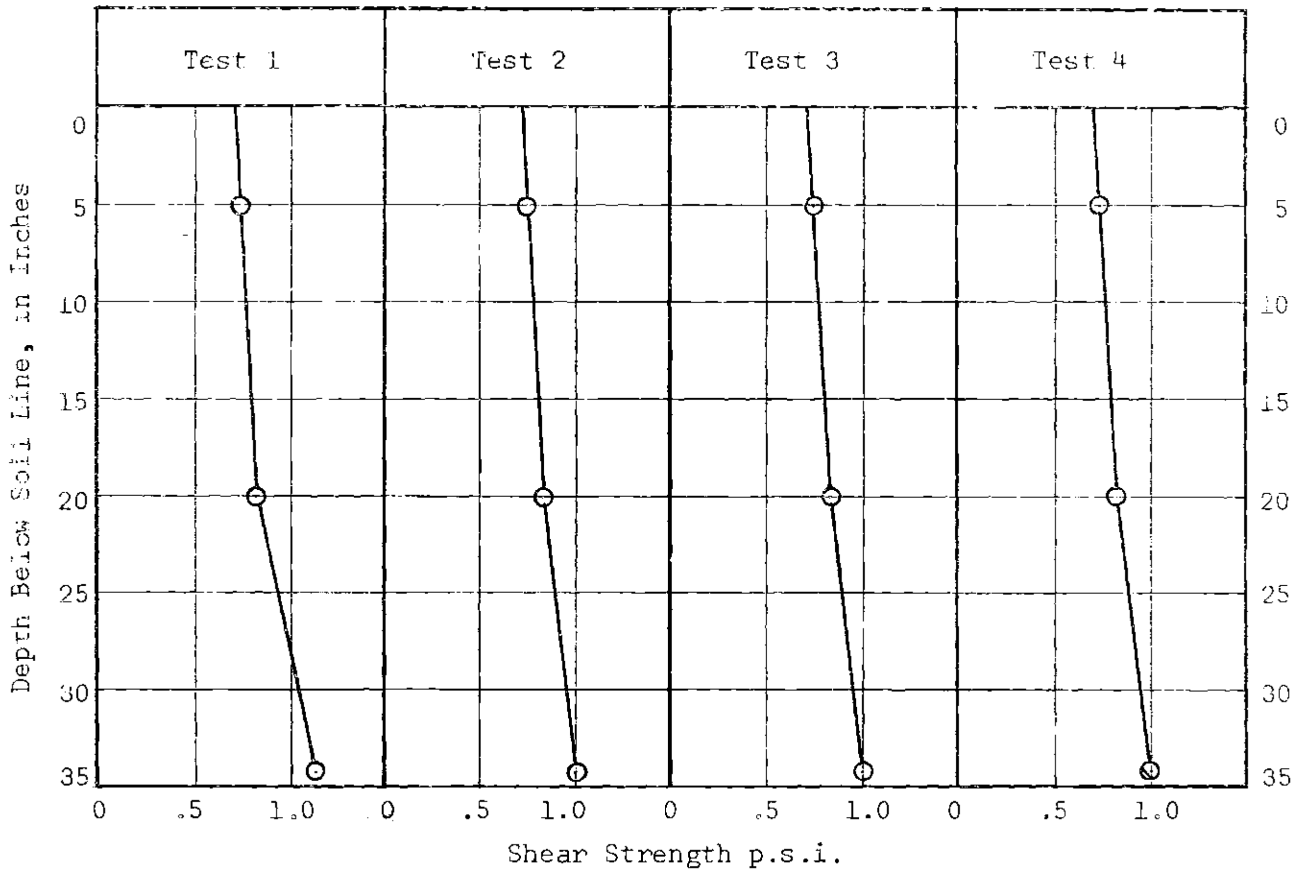


Figure 4. Vane Shear Tests

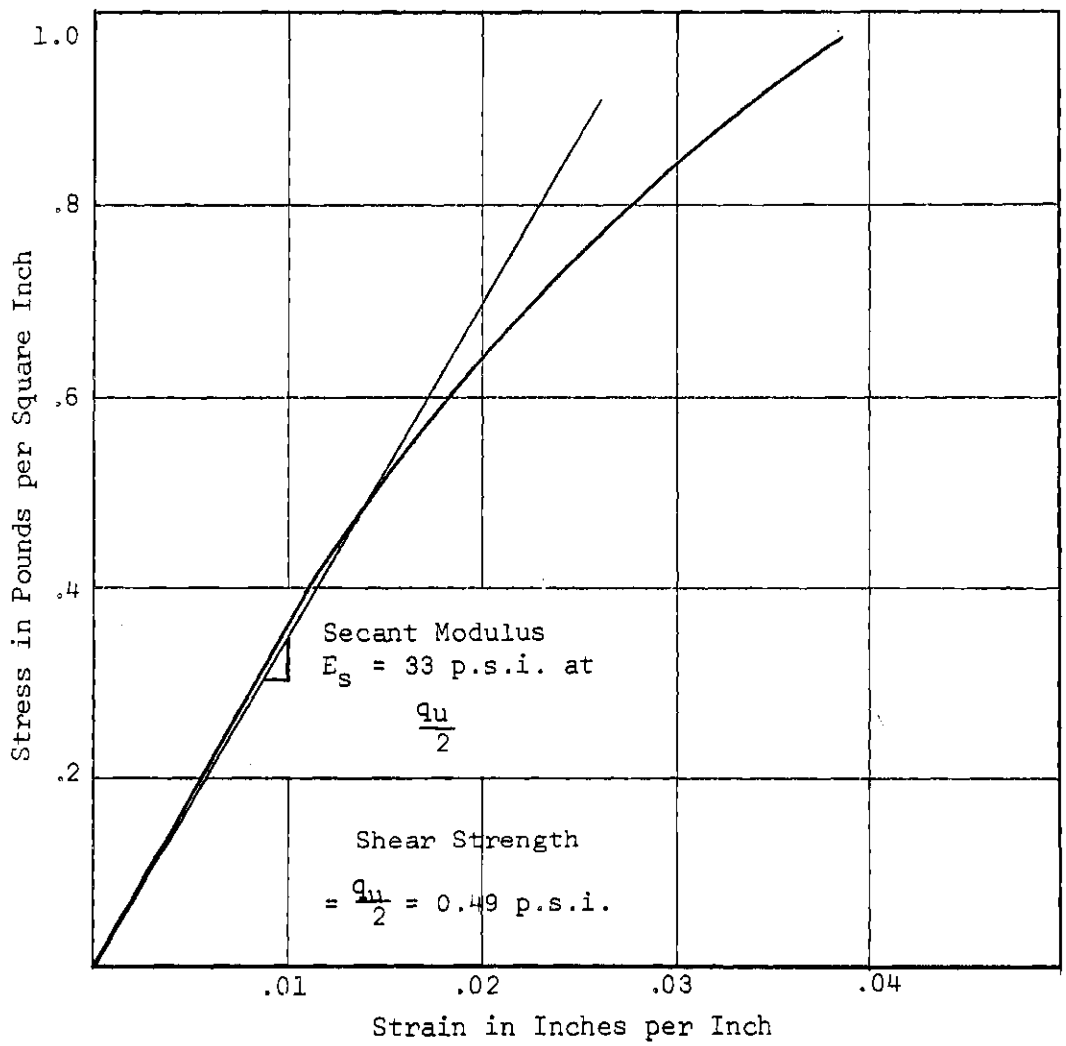


Figure 5. Unconfined Compression Test Depth 5 Inches Below Soil Line

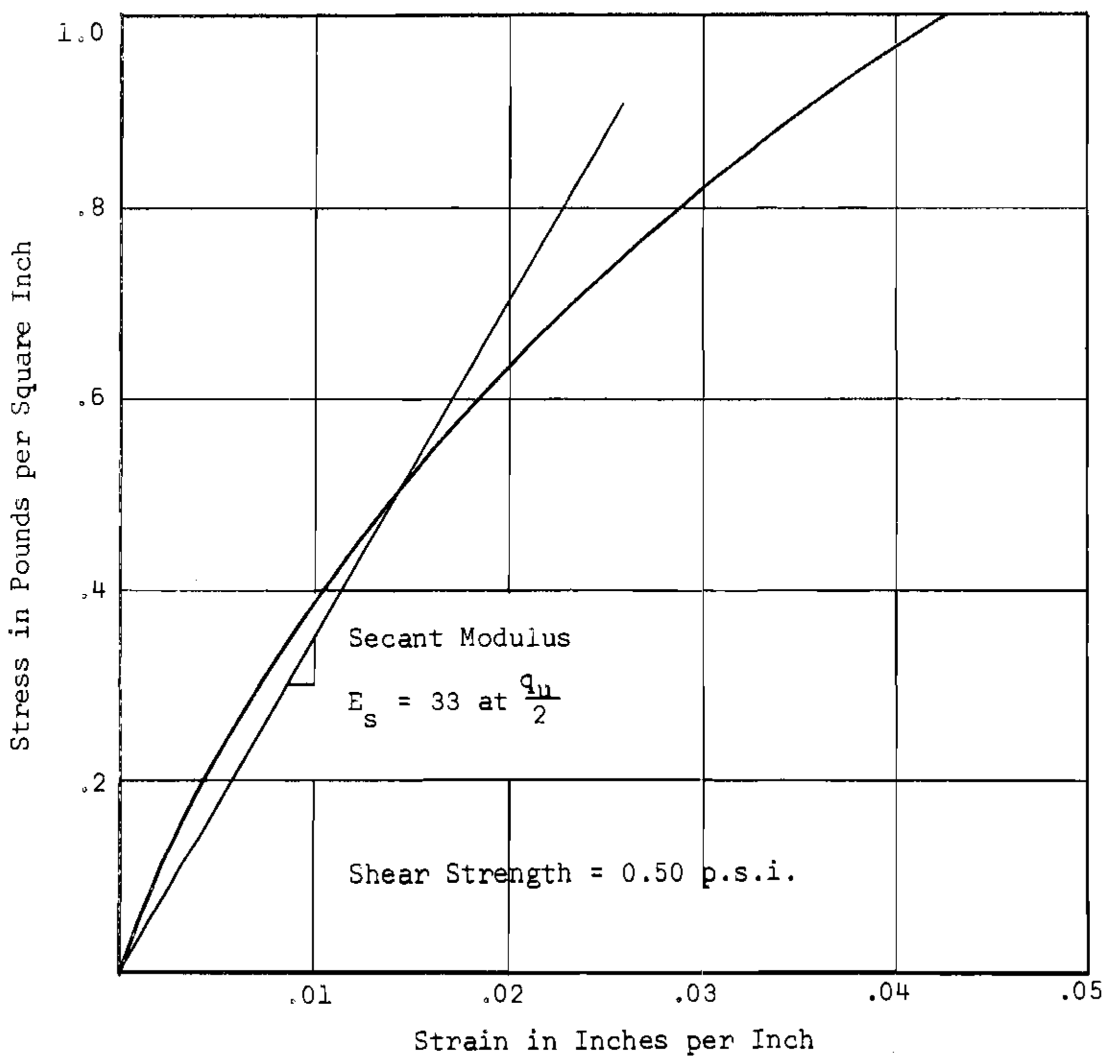


Figure 6. Unconfined Compression Test Depth 35 Inches Below Soil Line

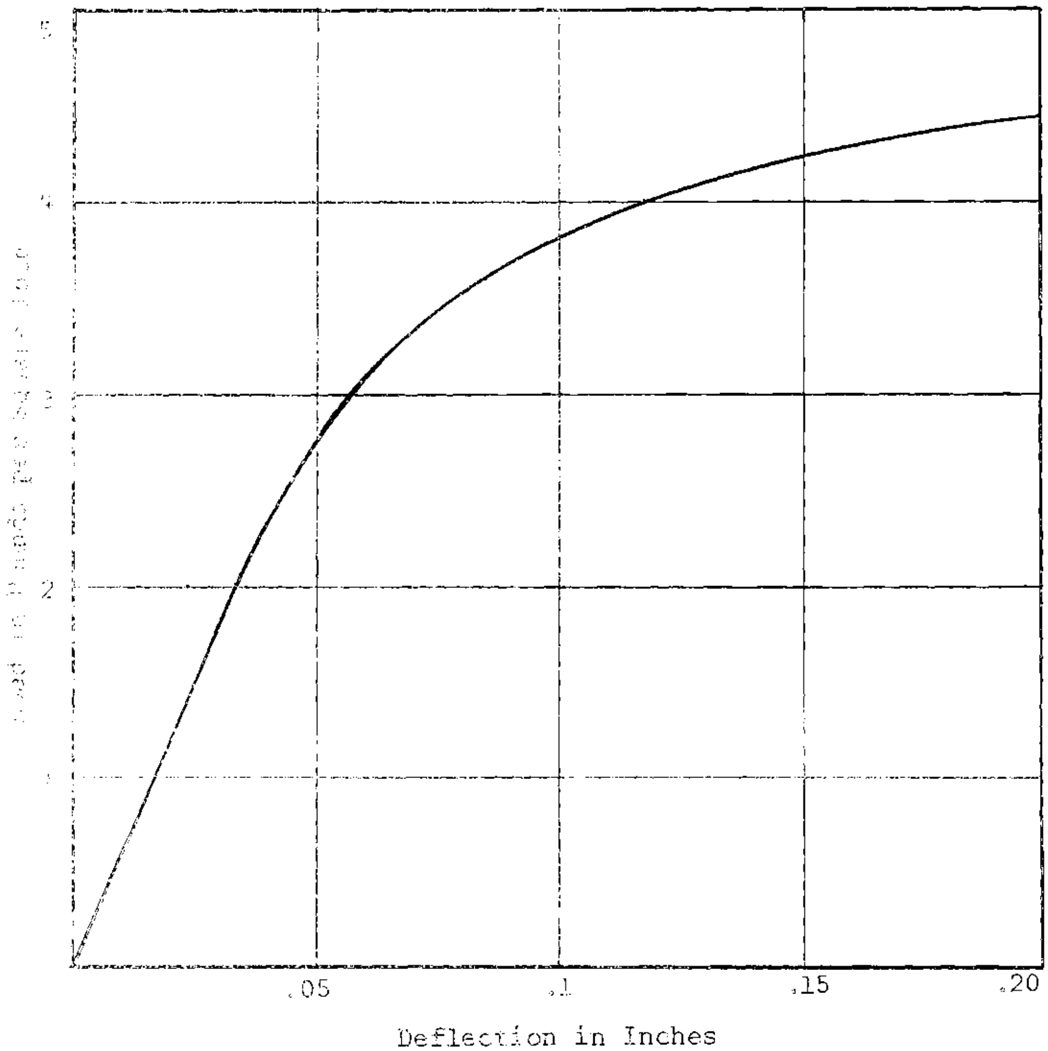


Figure 7. Plate Load Test--Two Inch Diameter

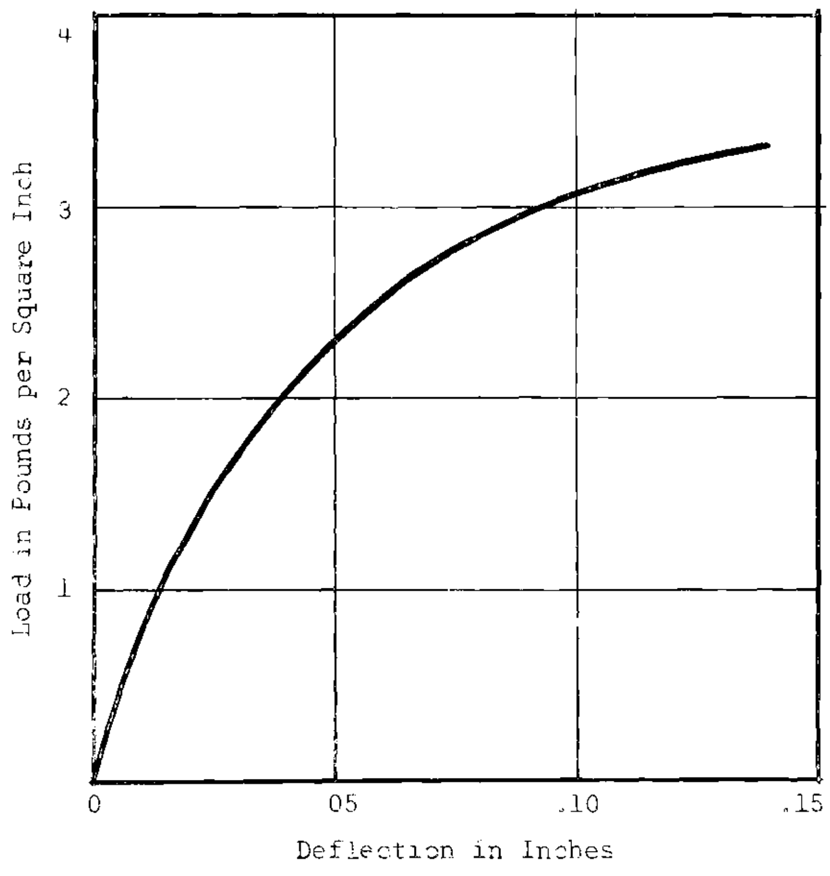


Figure 8. Plate Load Test--Three Inch Diameter

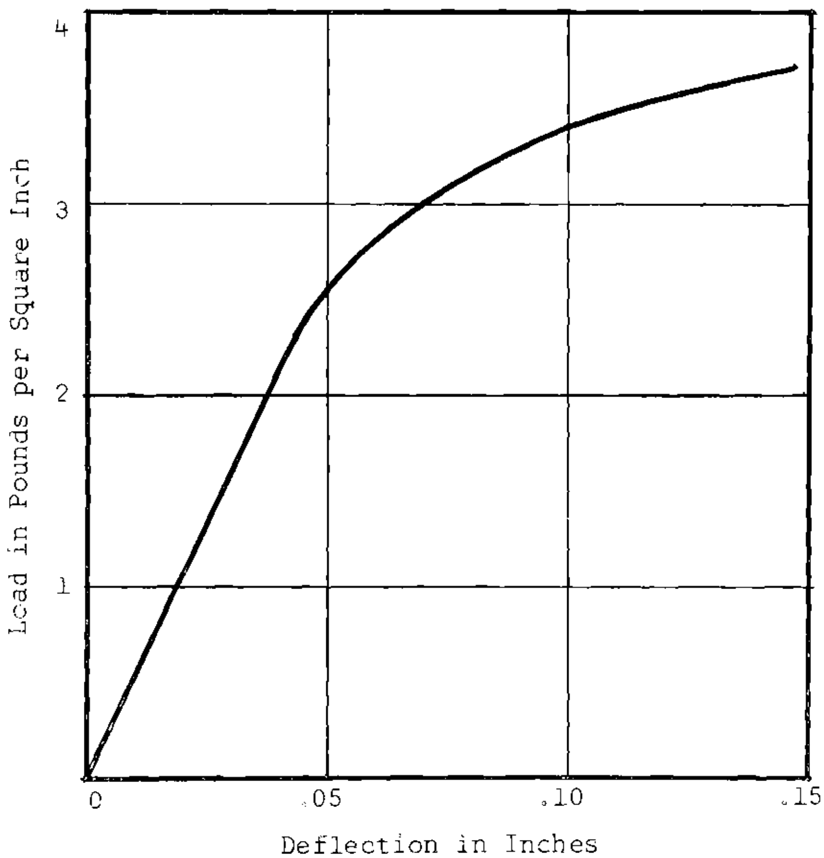


Figure 9. Plate Load Test--Five Inch Diameter

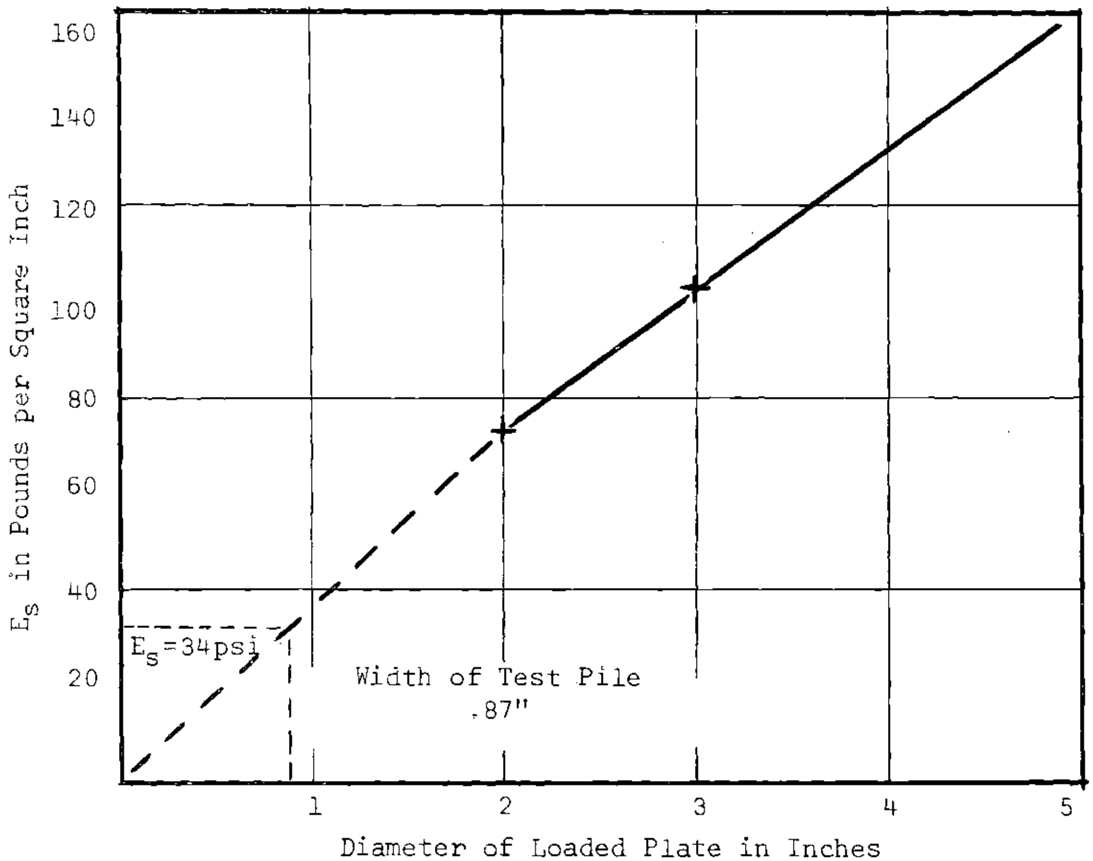
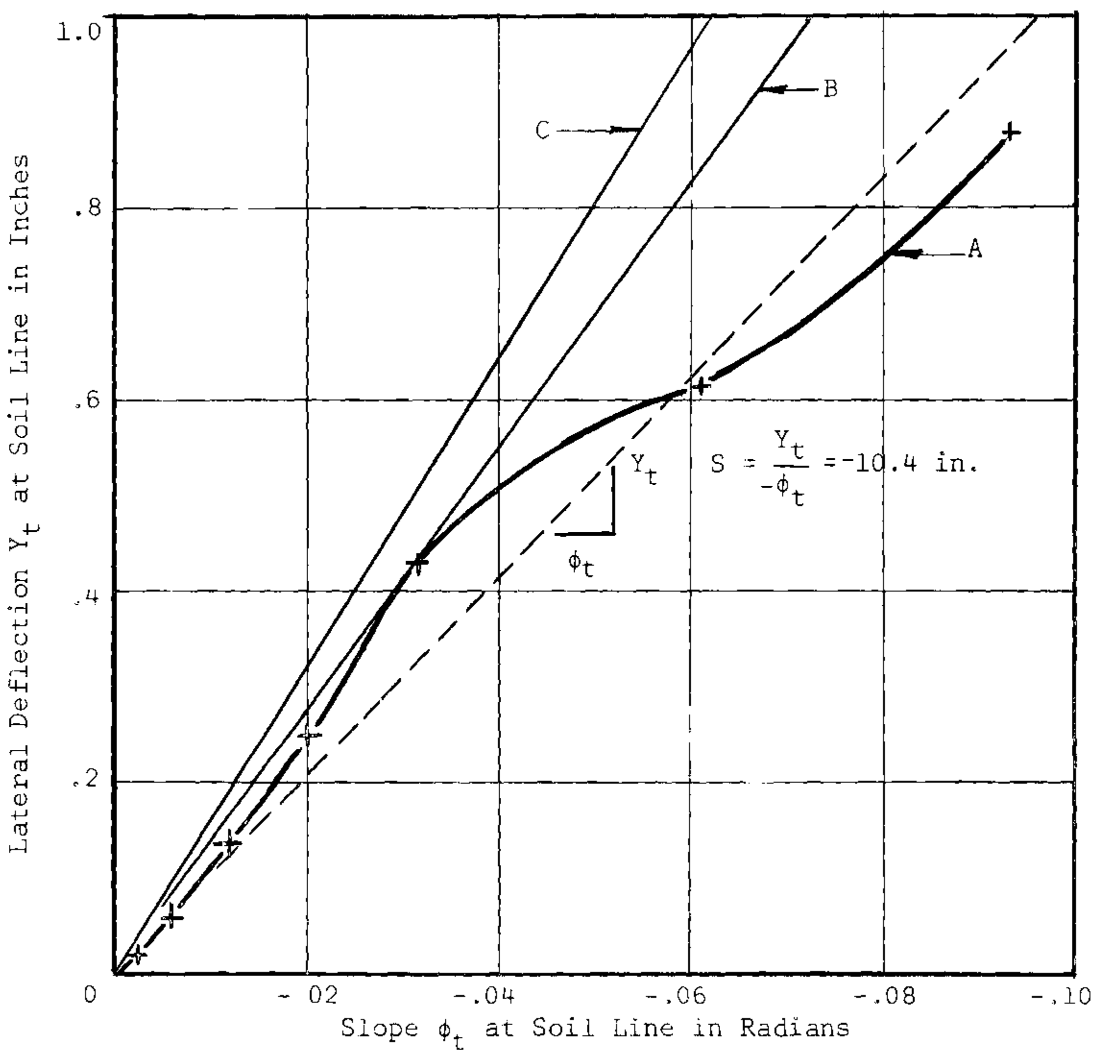
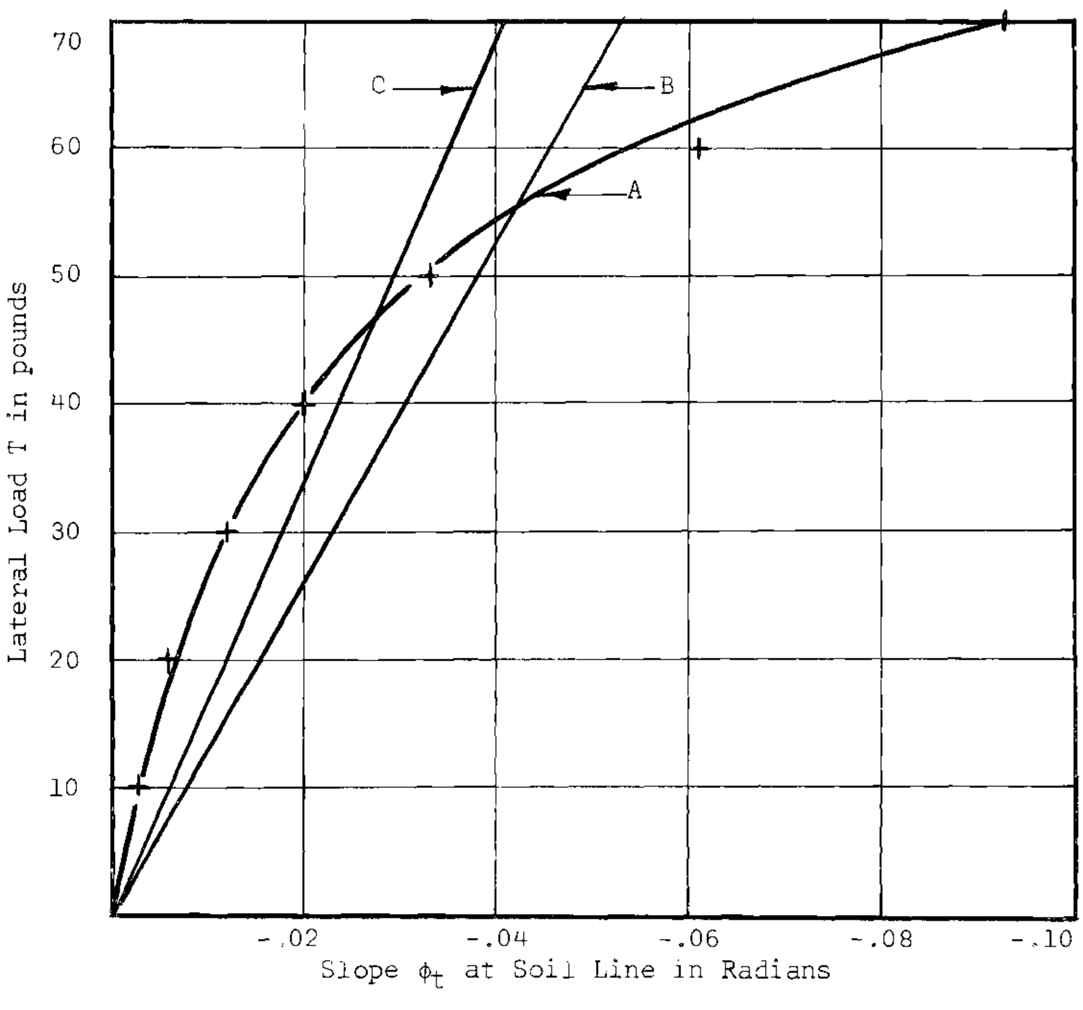


Figure 10. Secant Modulus from Plate Load Tests



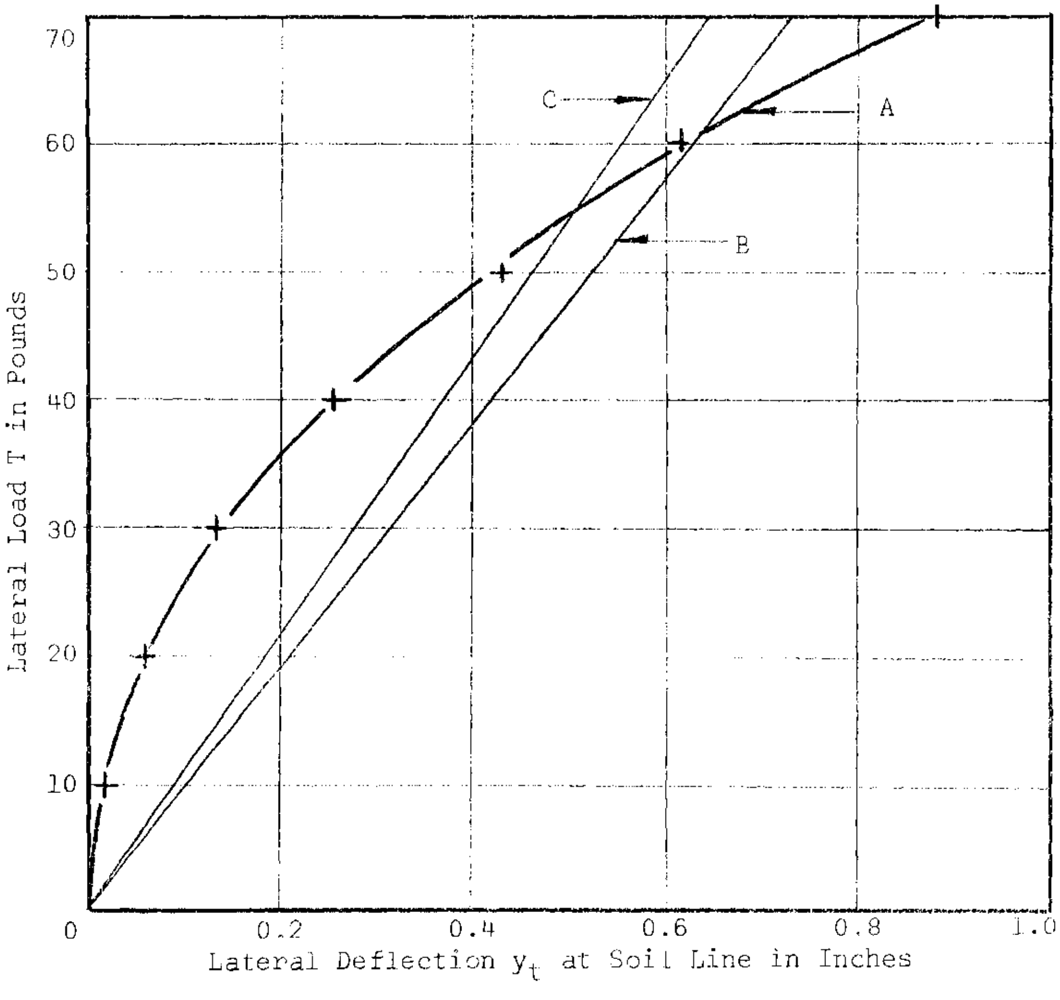
A. Measured
B. Computed by Equations 25-27 with E_s = 34 p.s.i.
C. Computed by Equations 12-14 with K = 13.5 p.s.i

Figure 11. Pure Lateral Load Pile Test



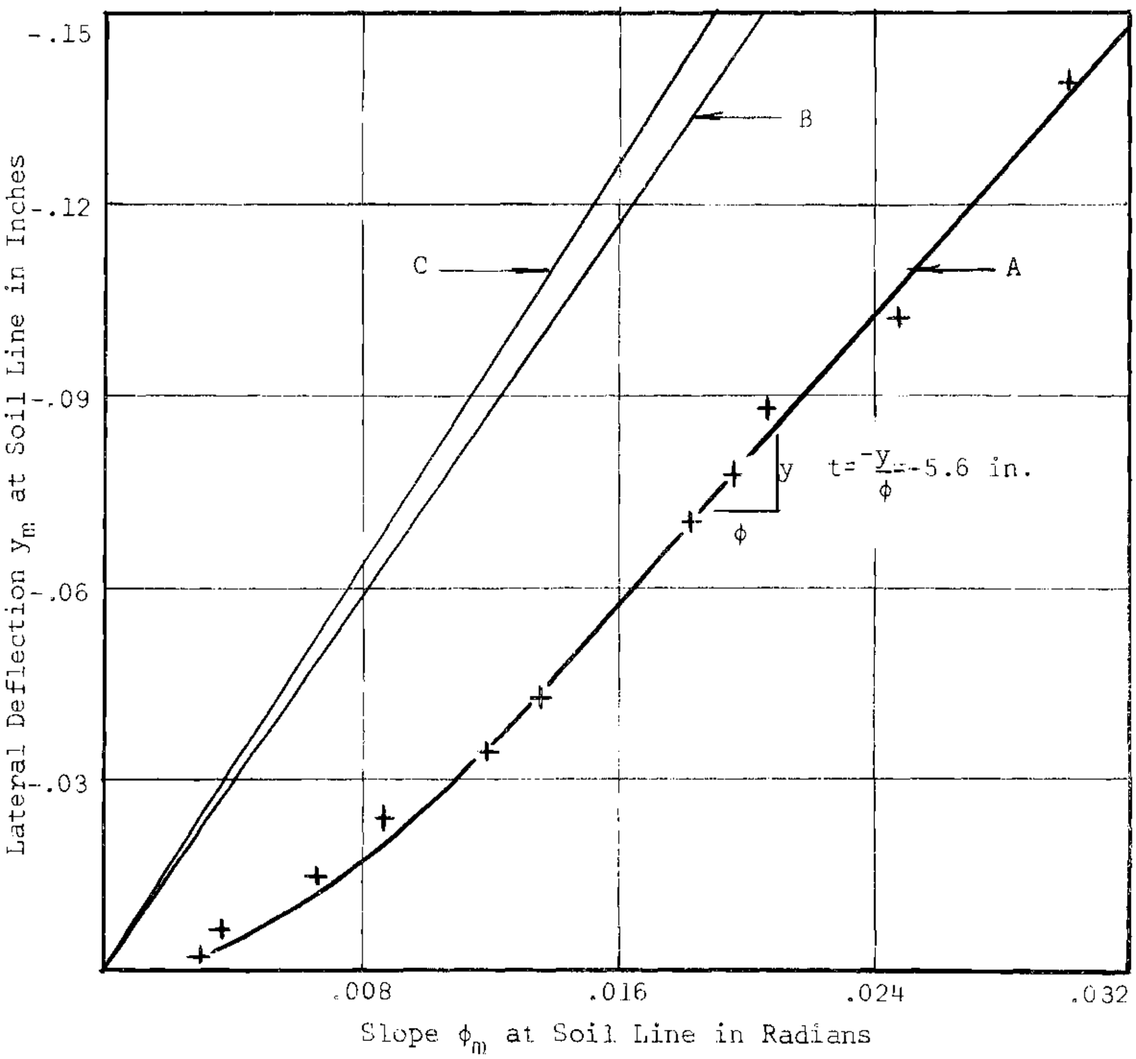
A. Measured.
B. Computed by Equations 25-27 with E_s = 34 p.s.i.
C. Computed by Equations 12-14 with K = 13.5 p.s.i.

Figure 12. Pure Lateral Load Pile Test



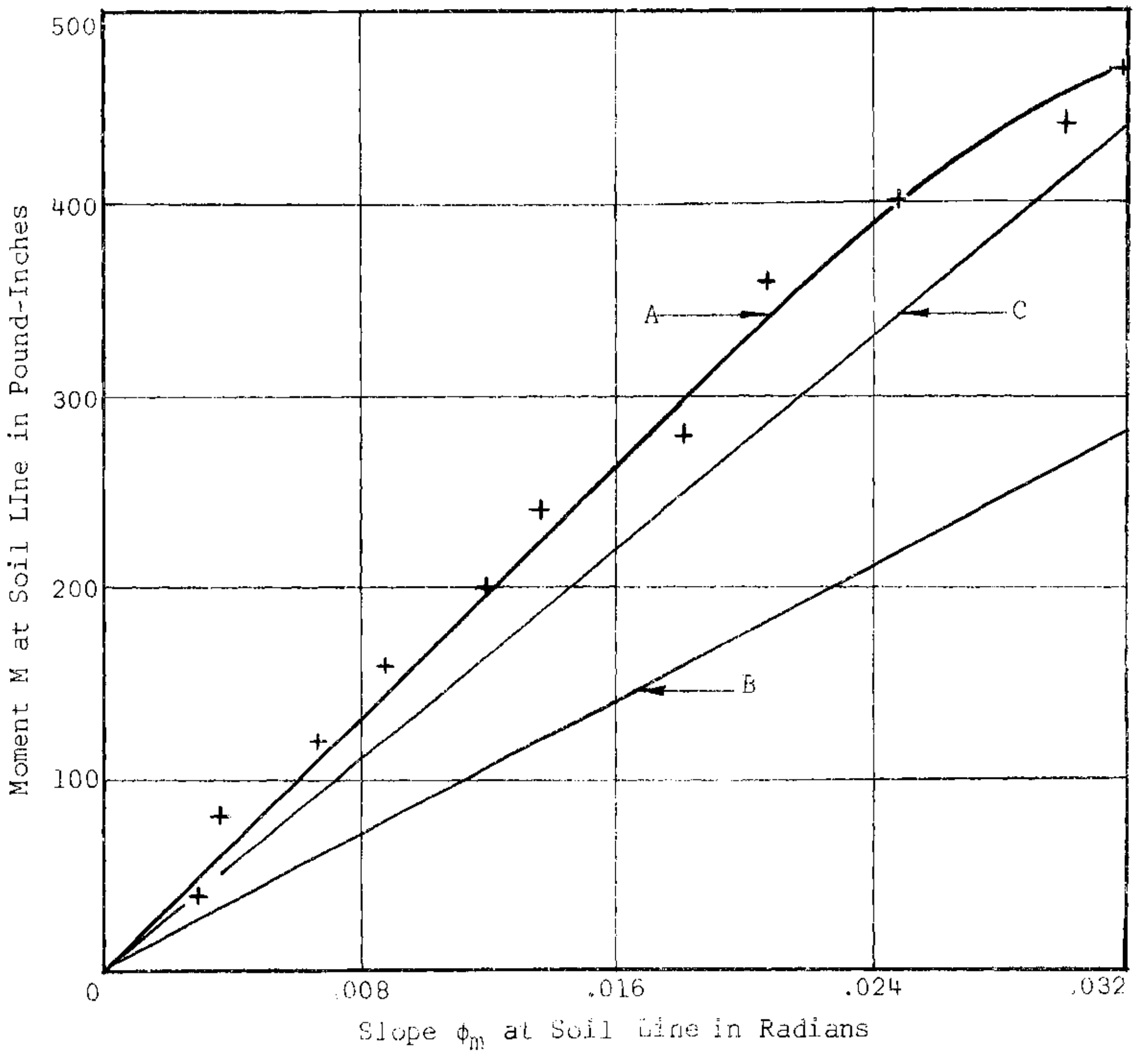
A. Measured.
B. Computed by Equations 25-27 with E_s = 34 p.s.i.
C. Computed by Equations 12-14 with K = 13.5 p.s.i.

Figure 13. Pure Lateral Load Pile Test



A. Measured.
B. Computed by Equations 25-27 with E_s = 34 p.s.i.
C. Computed by Equations 12-14 with K = 13.5 p.s.i.

Figure 14. Pure Moment Pile Test

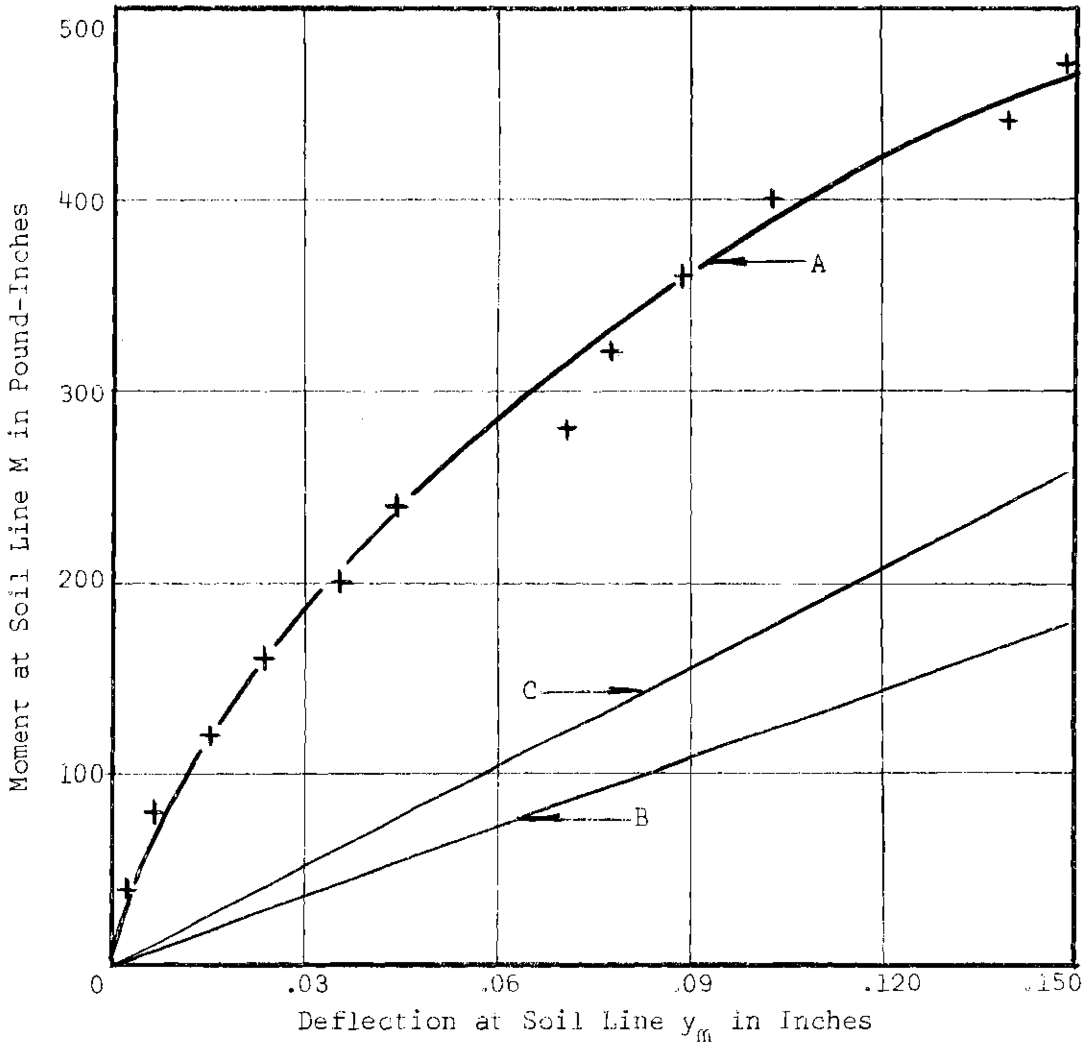


A. Measured.

B. Computed by Equations 25-27 with E_s = 34 p.s.i.

C. Computed by Equations 12-14 with K = 13.5 p.s.i.

Figure 15. Pure Moment Pile Test

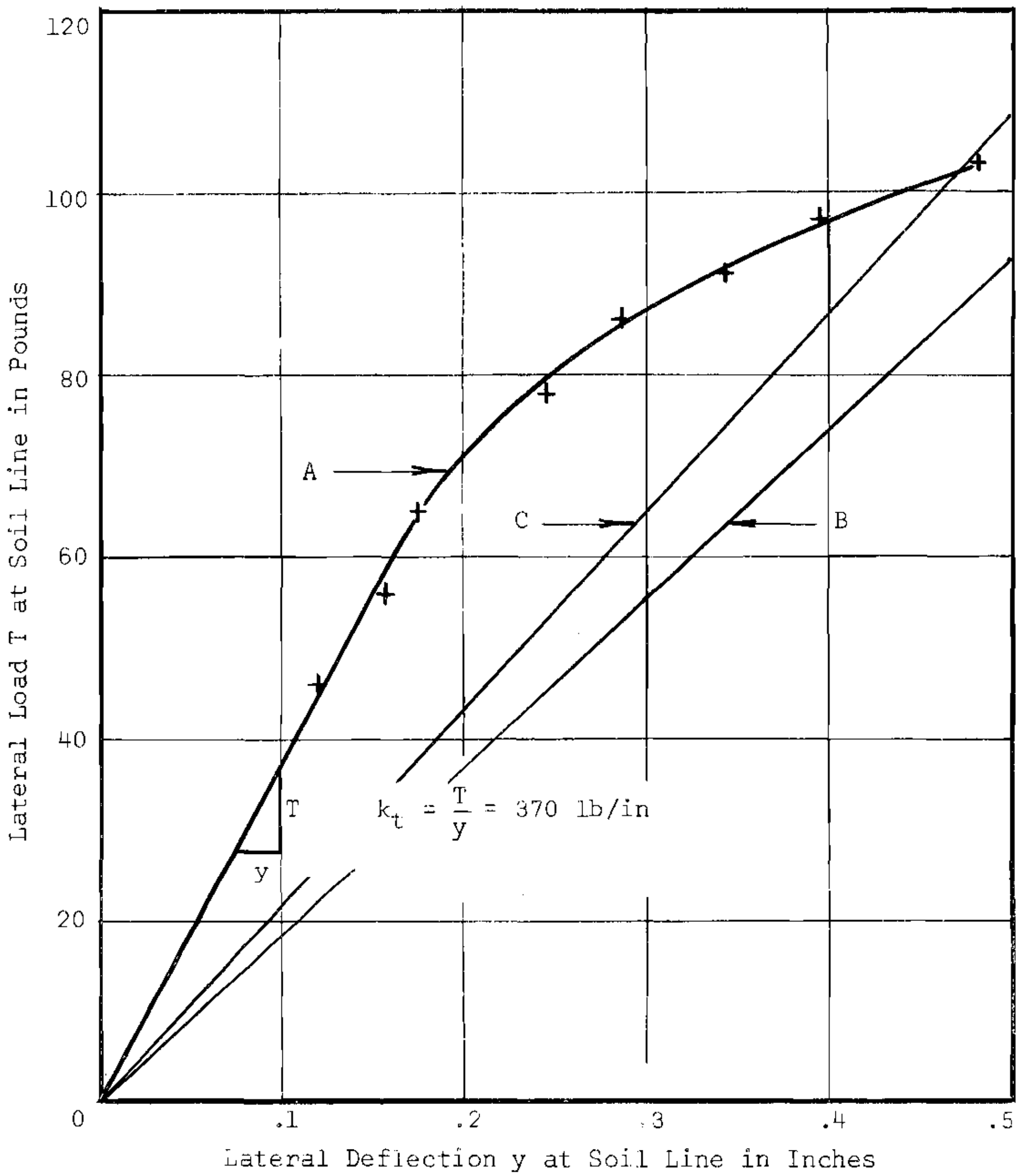


A. Measured.

B. Computed by Equations 25-27 with E_s = 34 p.s.i.

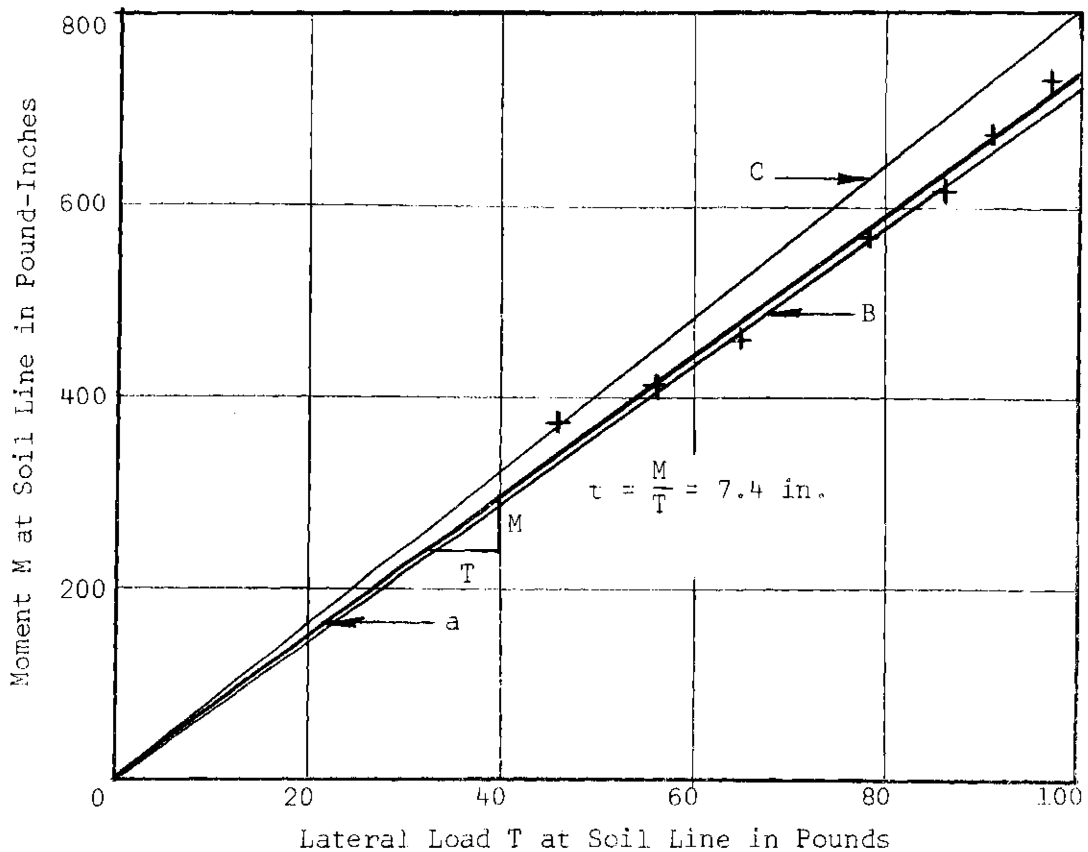
C. Computed by Equations 12-14 with K = 13.5 p.s.i.

Figure 16. Pure Moment Pile Test



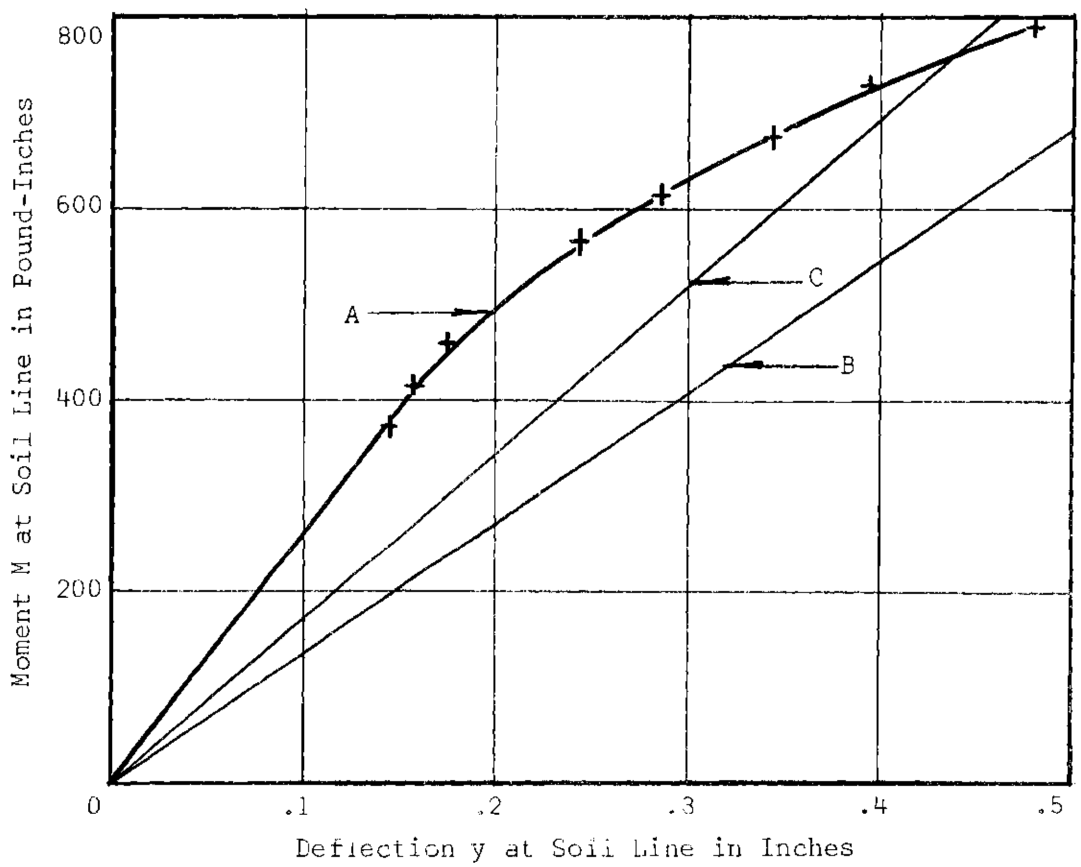
A. Measured.
B. Computed by Equations 25-27 with E_s = 34 p.s.i.
C. Computed by Equations 12-14 with K = 13.5 p.s.i.

Figure 17. Lateral Translation Pile Test



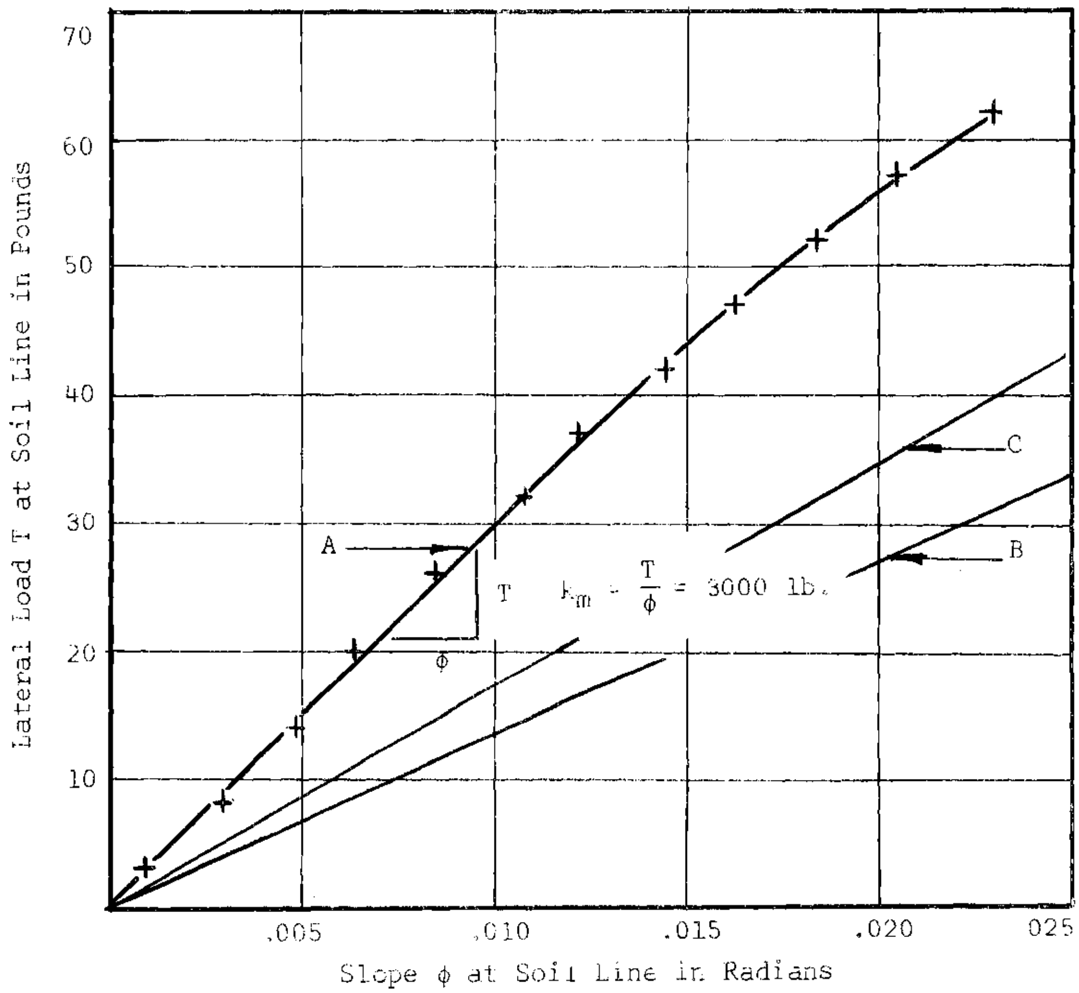
A. Measured.
B. Computed by Equations 25-27 with E_s = 34 p.s.i.
C. Computed by Equations 12-14 with K = 13.5 p.s.i.

Figure 18. Lateral Translation Pile Test



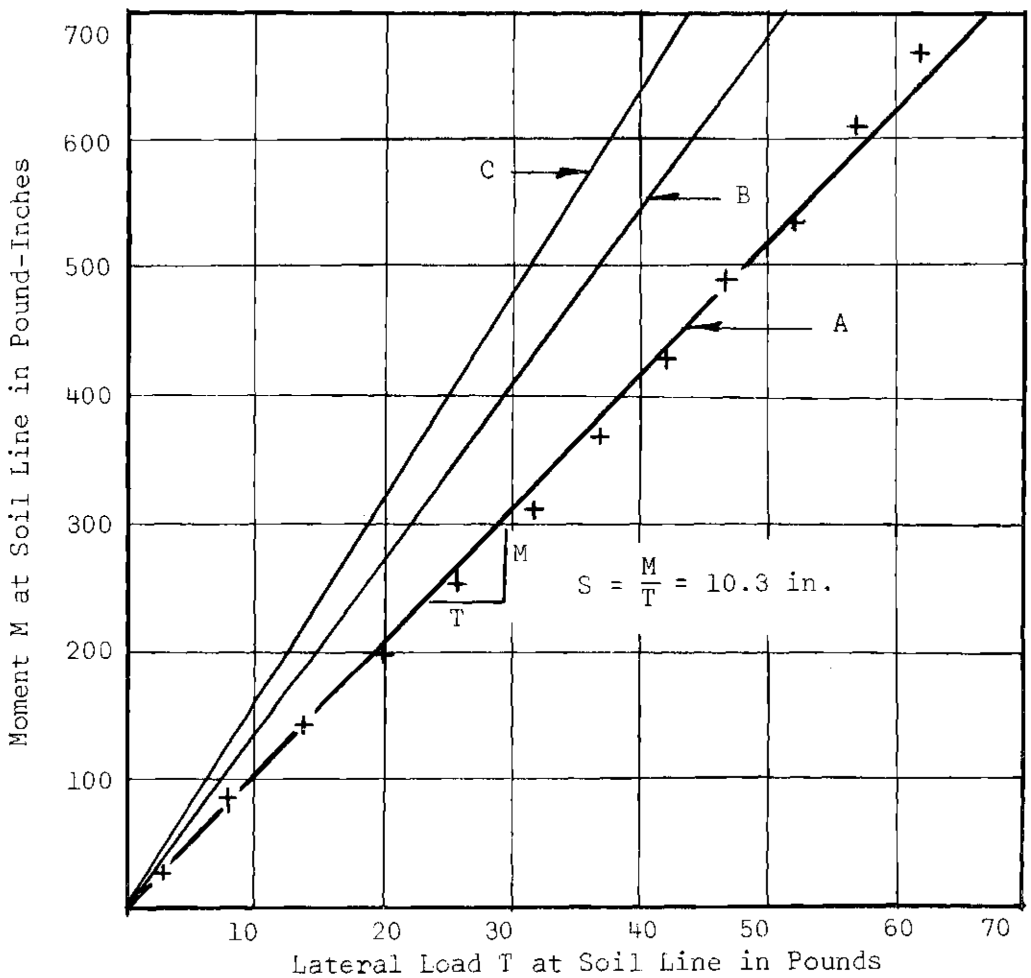
A. Measured.
B. Computed by Equations 25-27 with E_s = 34 p.s.i.
C. Computed by Equations 12-14 with K = 13.5 p.s.i.

Figure 19. Lateral Translation Pile Test



A. Measured.
B. Computed by Equations 25-27 with E_s = 34 p.s.i.
C. Computed by Equations 12-14 with K = 13.5 p.s.i.

Figure 20. Pure Rotation Pile Test

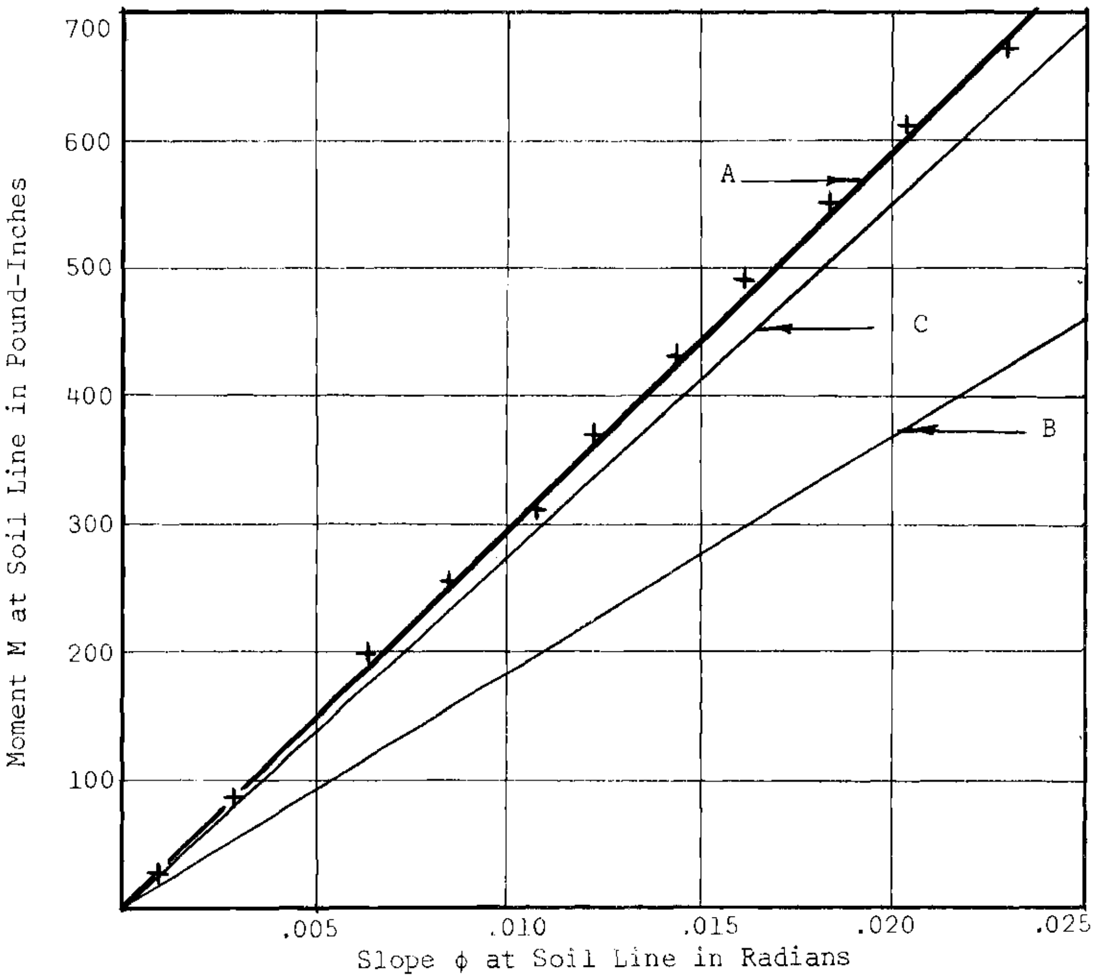


A. Measured.

B. Computed by Equations 25-27 with E_s = 34 p.s.i.

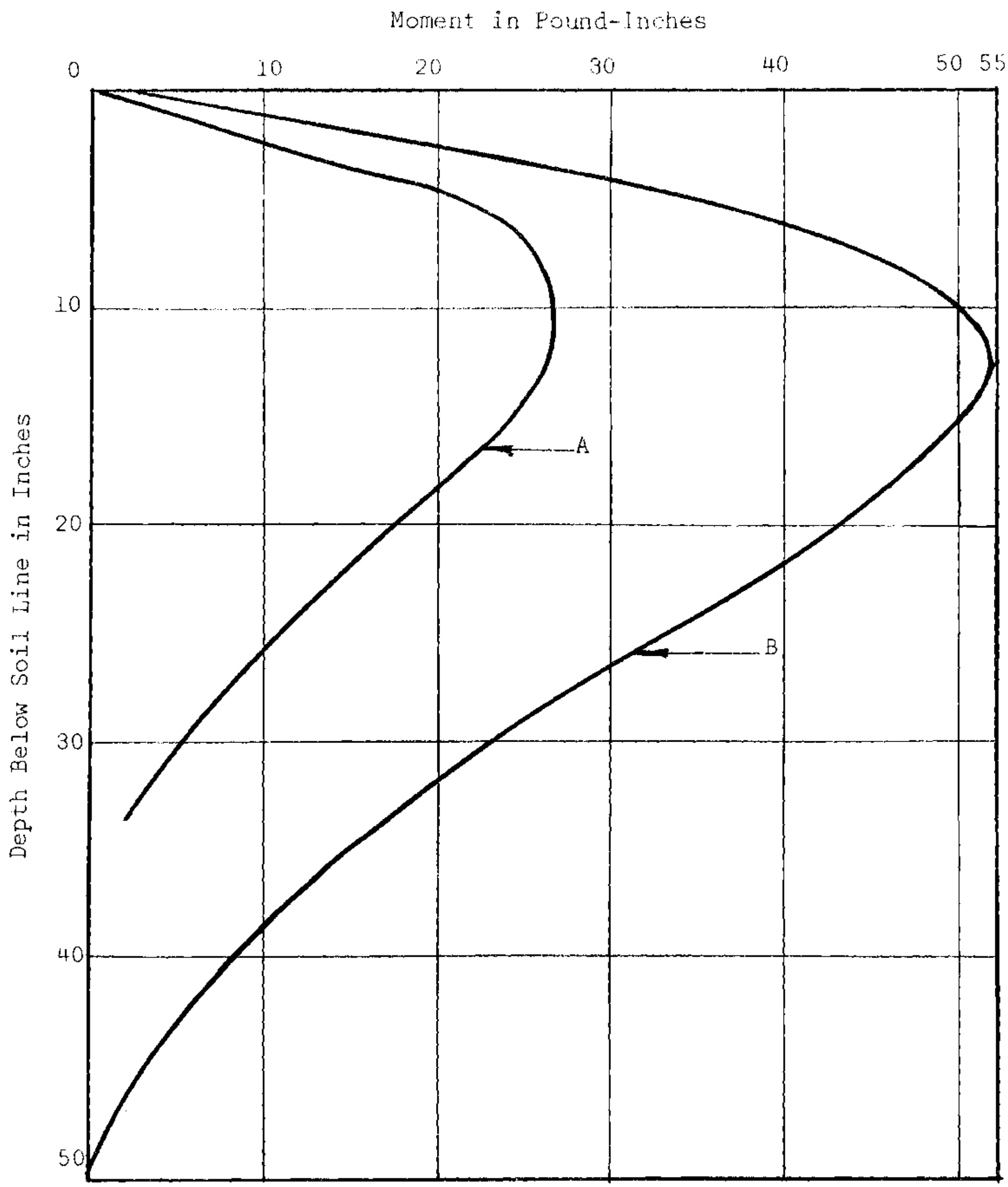
C. Computed by Equations 12-14 with K = 13.5 p.s.i.

Figure 21. Pure Rotation Pile Test



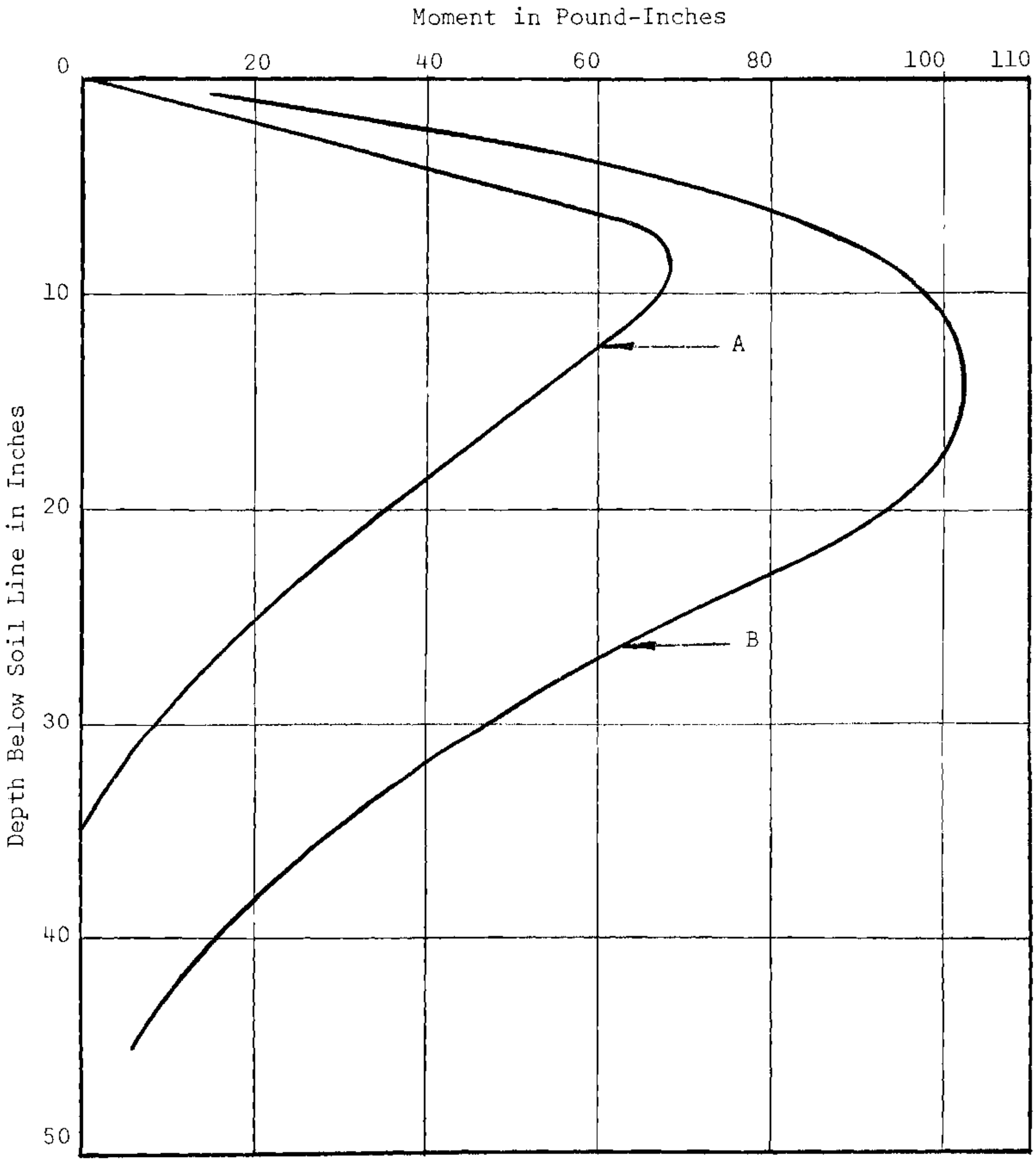
A. Measured.
B. Computed by Equations 25-27 with E_s = 34 p.s.i.
C. Computed by Equations 12-14 with K = 13.5 p.s.i.

Figure 22. Pure Rotation Pile Test



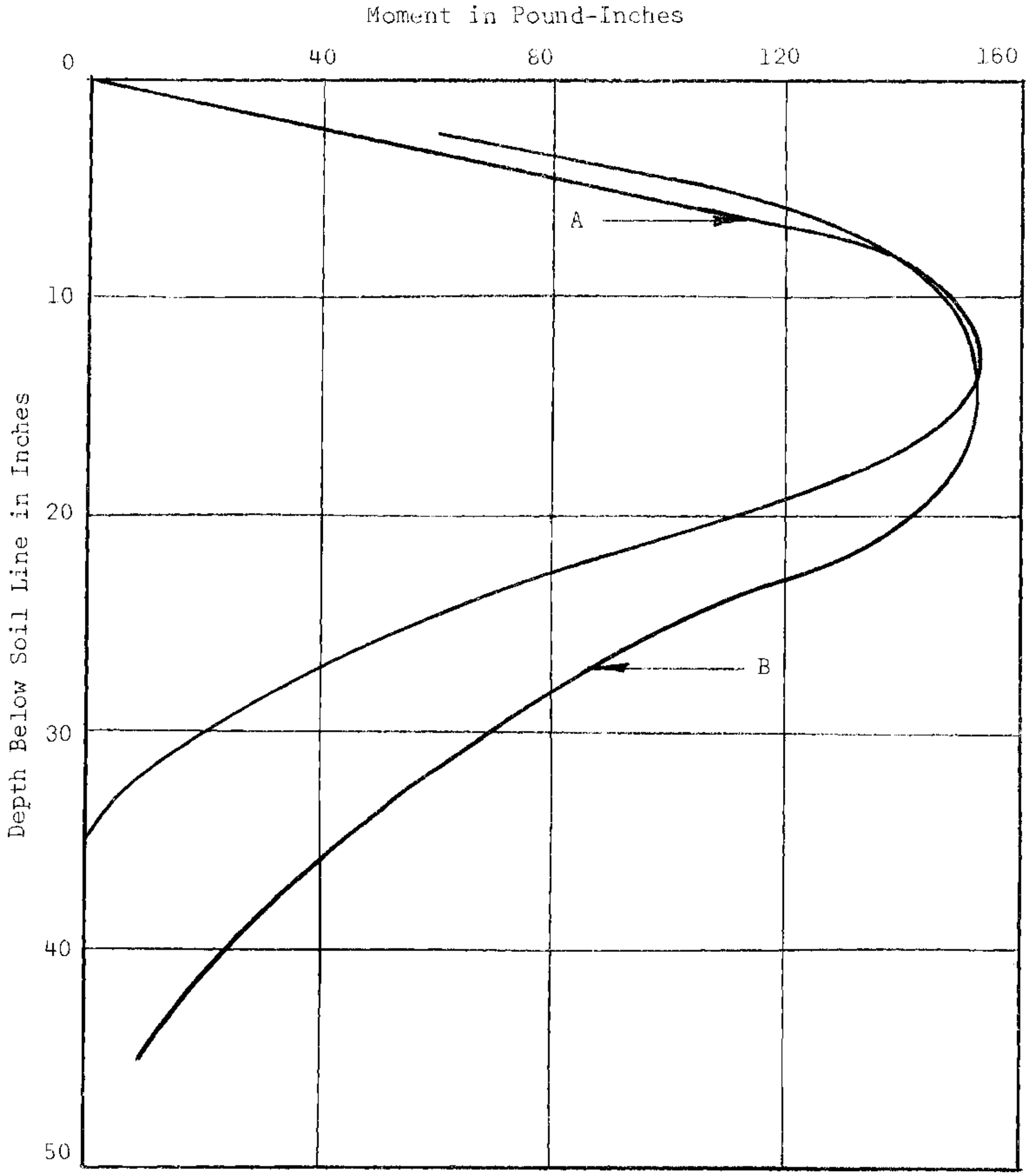
A. Measured.
B. Elastic Subgrade Theory

Figure 23. Moment Curves--Free Head Pile T = 10 Pounds



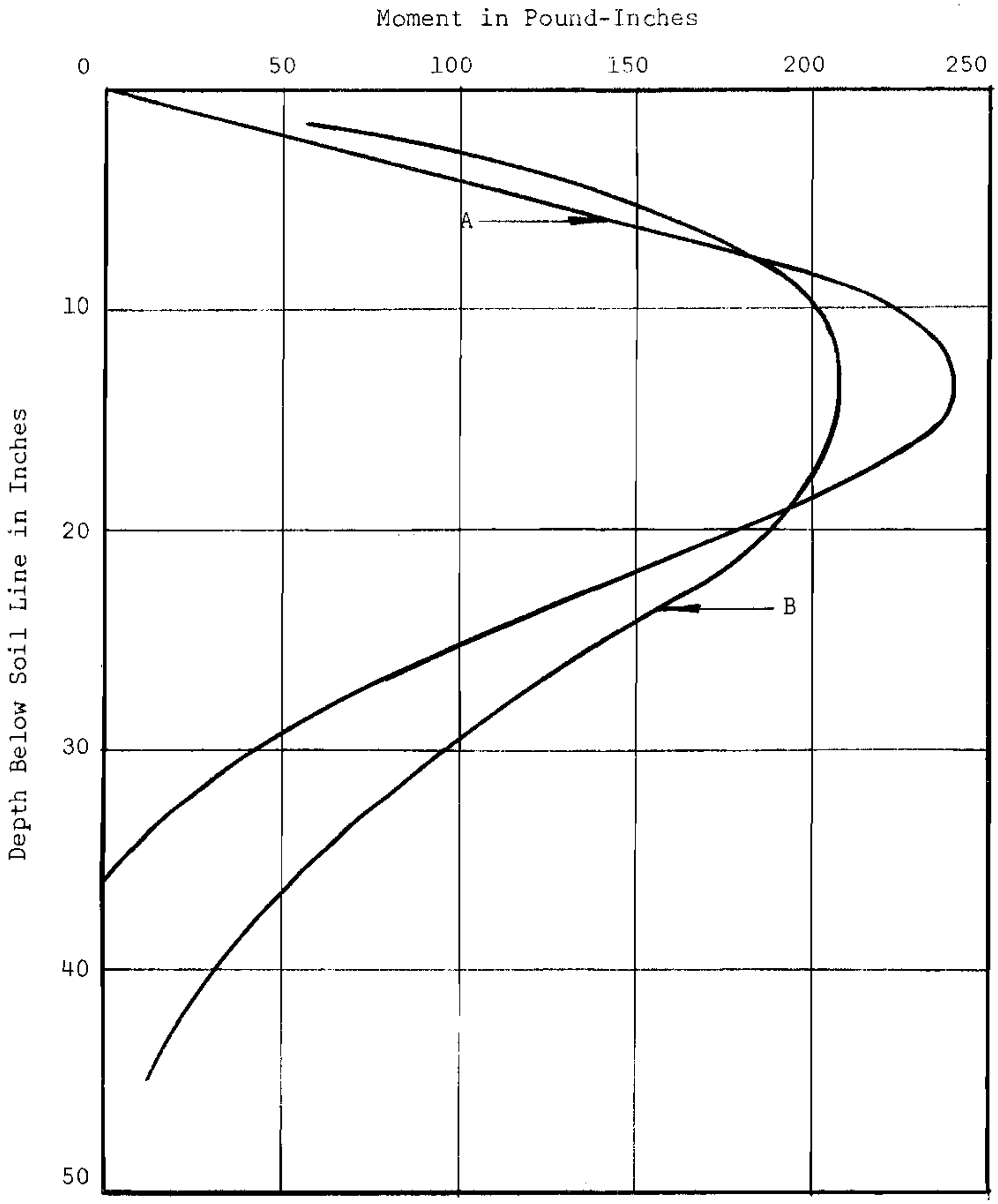
A. Measured.
B. Elastic Subgrade Theory.

Figure 24. Moment Curves--Free Head Pile T = 20 Pounds



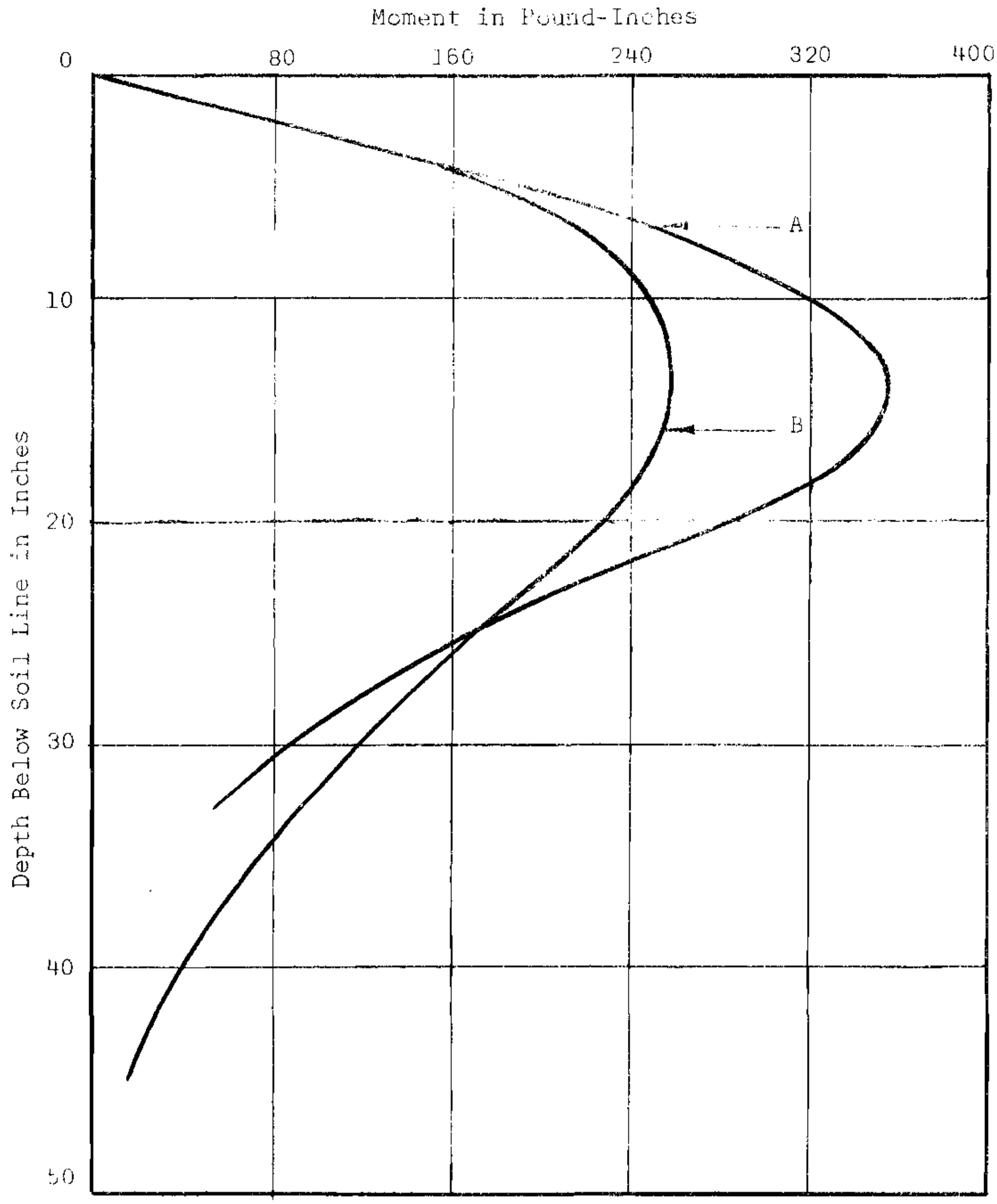
A. Measured.
B. Elastic Subgrade Theory.

Figure 25. Moment Curves--Free Head Pile T = 30 Pounds



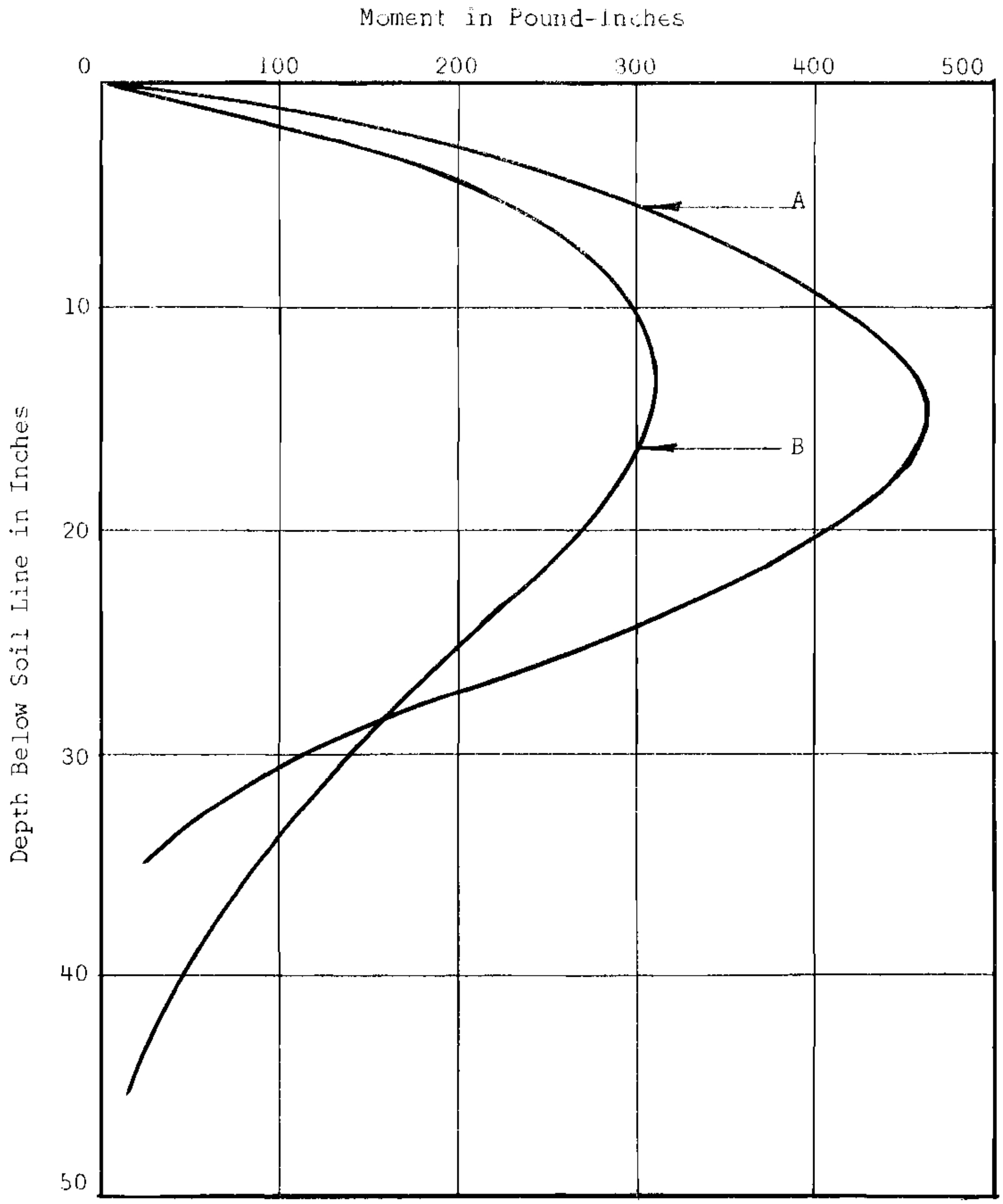
A. Measured.
B. Elastic Subgrade Theory.

Figure 26. Moment Curves--Free Head Pile T = 40 Pounds



A. Measured.
B. Elastic Subgrade Theory.

Figure 27. Moment Curves--Free Head Pile T = 50 Pounds



A. Measured.
B. Elastic Subgrade Theory.

Figure 28. Moment Curves--Free Head Pile T = 60 Pounds

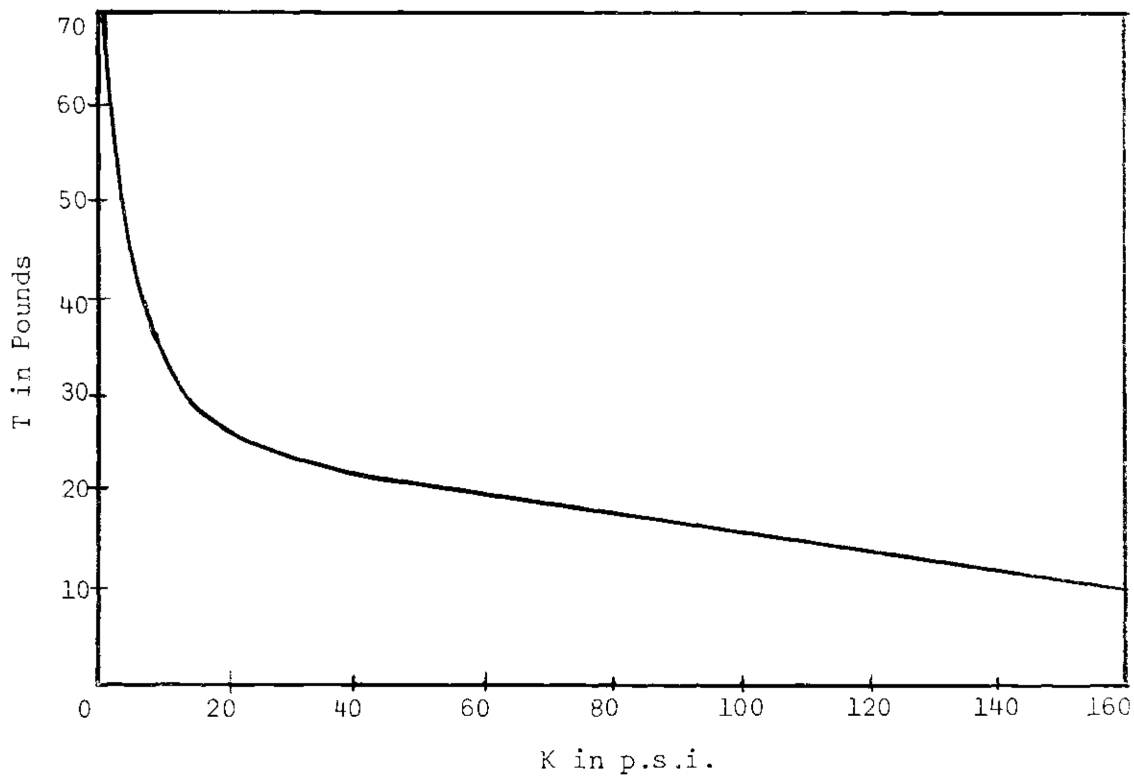


Figure 29. K Values Calculated to Give Moment Equal to Maximum Moment Measured for Each Load Increment of Free Head Pile Test

Table 1. Comparison of Theory and Experimental Results

	FROM MODEL TESTS			PREDICTIONS			
	Measured			Equations 25-27 E_s = 34 PSI		Equations 12-14 K = 13.5 PSI	
Pile Constant	Test Number	Test Value	Per Cent of Difference	Value	Per Cent of Measured	Value	Per Cent of Measured
K_t	III	370		185	50%	216	58%
t	III	7.4	24.2%	7.34	99%	7.94	107%
	II	5.6			131%		141%
	Av.	6.5			115%		122%
s	IV	10.3	0.96%	13.8	134%	15.88	154%
	I	10.4			133%		152%
	Av.	10.35			133.5%		153%
K_m	IV	3000		1355	45.2%	1700	56.5%
ρ		1.6		1.88	117.5%	2.0	125%

BIBLIOGRAPHY

References Cited

1. Timoshenko, S., Strength of Materials Part II, New York: Van Nostrand, 1941, p. 1.

2. Terzaghi, K., Theoretical Soil Mechanics, New York: John Wiley and Sons, Inc., 1943, p. 360.

3. Feagin, L., "Lateral Pile-Loading Tests," American Society of Civil Engineers Transactions, 1937, p. 272.

4. Timoshenko, S., op. cit., p. 1.

5. Hetenyi, M., Beams on Elastic Foundations, Ann Arbor, Michigan: The University of Michigan Press, 1946, p. 24.

6. Terzaghi, K., op. cit., p. 360.

7. Reese, L., and Matlock, H., "Non-Dimensional Solutions for Laterally Loaded Piles with Soil Modulus Assumed Proportional to Depth," Proceedings, Eighth Texas Conference on Soil Mechanics and Foundation Engineering, the University of Texas, 1956, Special Publication No. 29.

8. Palmer, L. A., and Thompson, J. B., "Horizontal Pressures on Pile Foundations," Proceedings, Third International Conference on Soil Mechanics and Foundation Engineering, Rotterdam, 1948, Volume 5, p. 156.

9. Gleser, S., "Lateral Load Tests on Vertical Fixed-Head and Free-Head Piles," Symposium on Lateral Load Tests on Piles, American Society for Testing Materials, Special Technical Publication No. 154, 1953, p. 75.

10. McNulty, J., "Thrust Loading on Piles," Proceedings, American Society of Civil Engineers, April 1956, Volume 82.

11. McClelland, B., and Focht, J., "Soil Modulus for Laterally Loaded Piles," Proceedings, American Society of Civil Engineers, October 1956, Volume 82.

12. Matlock, H., Ripperger, E., and Reese, L., "Recommendations Pertaining to the Design of Laterally Loaded Piles," Fourth Report to Shell Oil Company on Research Conducted by Engineering-Science Consultants, Austin, 1957.

13. Matlock, H., and Reese, L., "Foundation Analyses of Offshore Pile-Supported Structures," Proceedings, Fifth International Conference on Soil Mechanics and Foundation Engineering, 1961.

14. Hrennikoff, A., "Analysis of Pile Foundations with Batter Piles," Transactions, American Society of Civil Engineers, 1950, Volume 115, p. 351.

15. Vesic, A., "Contribution a L'etude des Fondations sur Pieux Verticaux et Inclines," Annales des Travaux Publiques de Belgique, 1956, No. 6.

16. Terzaghi, K., "Evaluation of Coefficients of Subgrade Reaction," Geotechnique, 1955, No. 4, p. 297.

17. Vesic, A., "Bending of Beams Resting on Isotropic Elastic Solid," Proceedings, American Society of Civil Engineers, 1961, Volume 87, No. EM2, p. 35.

18. Vesic, A., Discussion on "Generalized Solutions for Laterally Loaded Piles," Proceedings, American Society of Civil Engineers, 1961, Volume 87, No. SM3, p. 123.

19. Vesic, A., "Contribution a L'etude des Fondations," p. 19.

20. Ibid., p. 19.

21. Ibid., p. 20.

22. Martin, C. B., Model Studies of the Bearing Capacity of Pile Groups in a Saturated Clay, M. S. thesis presented to the Georgia Institute of Technology, 1957.

23. Skempton, A. W., "The Bearing Capacity of Clays," Building Research Congress, 1951, London, p. 183.

24. Terzaghi, K., and Peck, R. B., Soil Mechanics in Engineering Practice, New York: John Wiley and Sons, Inc., 1948.

25. Skempton, A. W., op. cit., p. 184.

OTHER REFERENCES

1. Chellis, R. D., Pile Foundations, New York: McGraw-Hill Book Company, Inc., 1951.

2. Sowers, G. B., and Sowers, G. F., Introductory Soil Mechanics and Foundations, 1st ed. New York: The Macmillan Company, 1951.