AN ELECTROMECHANICAL CALCULATOR FOR
HORIZONTAL PATTERNS OF MULTI-ELEMENT ANTENNAS

A THESIS
Presented to
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Master of Science in Electrical Engineering

by
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AN ELECTROMECHANICAL CALCULATOR FOR
HORIZONTAL PATTERNS OF MULTI-ELEMENT ANTENNAS

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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>CHAPTER</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. THE PROBLEM AND DEFINITIONS OF TERMS USED</td>
<td>1</td>
</tr>
<tr>
<td>The Problem</td>
<td>1</td>
</tr>
<tr>
<td>Statement of the Problem</td>
<td>1</td>
</tr>
<tr>
<td>Importance of the Study</td>
<td>1</td>
</tr>
<tr>
<td>Definitions of terms used</td>
<td>2</td>
</tr>
<tr>
<td>Antenna</td>
<td>2</td>
</tr>
<tr>
<td>Element</td>
<td>2</td>
</tr>
<tr>
<td>Array</td>
<td>2</td>
</tr>
<tr>
<td>Differential Gear</td>
<td>2</td>
</tr>
<tr>
<td>Radiation Pattern</td>
<td>3</td>
</tr>
<tr>
<td>Organization of remainder of the thesis</td>
<td>3</td>
</tr>
<tr>
<td>II. REVIEW OF THE LITERATURE</td>
<td>4</td>
</tr>
<tr>
<td>Literature on mechanical antenna calculators</td>
<td>4</td>
</tr>
<tr>
<td>Literature on electromechanical antenna calculator</td>
<td>5</td>
</tr>
<tr>
<td>Literature on electronic antenna calculator</td>
<td>5</td>
</tr>
<tr>
<td>III. REVIEW OF DIRECTIONAL ANTENNA THEORY</td>
<td>7</td>
</tr>
<tr>
<td>Derivation of radiation pattern equation</td>
<td>7</td>
</tr>
<tr>
<td>Equation for number of parameters involved</td>
<td>10</td>
</tr>
<tr>
<td>Calculation of a simple two-element antenna pattern</td>
<td>10</td>
</tr>
<tr>
<td>IV. THE THEORY OF OPERATION</td>
<td>11</td>
</tr>
<tr>
<td>General Layout</td>
<td>11</td>
</tr>
<tr>
<td>The Drive Mechanism</td>
<td>11</td>
</tr>
<tr>
<td>Antenna Position Control</td>
<td>14</td>
</tr>
</tbody>
</table>
CHAPTER | PAGE
--- | ---
The Cosine Generator | 14
Antenna Spacing Control | 15
Phase Shifting Transformer and Current Phase Control | 15
Antenna Current Ratio Control | 16
The Reference Antenna | 16
An Analogy | 16
Details of the Electromechanical Calculator | 16
V. CONSTRUCTION OF THE CALCULATOR | 19
Phases of Construction | 19
Sources of Material | 19
The Cosine Generator | 19
Antenna Spacing Control | 19
Antenna Position Control | 19
Antenna Phasing Control | 19
One Antenna Calculator Unit | 26
The Five-Antenna Calculator | 26
Rear View | 26
Front View | 29
The Power Panel | 29
VI. THE CALCULATOR IN OPERATION | 30
Calibration | 30
The Current Ratio Dials | 30
The Current Phase Dials | 31
The Antenna Position Dials | 31
LIST OF TABLES

<table>
<thead>
<tr>
<th>TABLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. An Analogy</td>
<td>16</td>
</tr>
<tr>
<td>II. Comparison of Results</td>
<td>36</td>
</tr>
<tr>
<td>III. Computation of 2, 3, 4, and 5 element broadside arrays</td>
<td>50</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>FIGURE</th>
<th>DESCRIPTION</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Plan View of an Antenna Configuration Having k Antennas.</td>
<td>8</td>
</tr>
<tr>
<td>2.</td>
<td>Block Diagram of the Electromechanical Antenna Pattern Calculator.</td>
<td>12</td>
</tr>
<tr>
<td>3.</td>
<td>Block Diagram of a Unit Antenna Calculator.</td>
<td>13</td>
</tr>
<tr>
<td>4.</td>
<td>Electrical Circuit Diagram of the Electromechanical Calculator.</td>
<td>17</td>
</tr>
<tr>
<td>5.</td>
<td>Photograph of a Single Parallax Computer.</td>
<td>20</td>
</tr>
<tr>
<td>6.</td>
<td>Photograph of the Cosine Generator.</td>
<td>21</td>
</tr>
<tr>
<td>7.</td>
<td>Photograph of the Spacing Control.</td>
<td>22</td>
</tr>
<tr>
<td>8.</td>
<td>Photograph of the Position Control.</td>
<td>23</td>
</tr>
<tr>
<td>9.</td>
<td>Photograph of the Phasing Unit.</td>
<td>24</td>
</tr>
<tr>
<td>10.</td>
<td>Photograph of a Single Antenna Calculator Unit.</td>
<td>25</td>
</tr>
<tr>
<td>11.</td>
<td>Photograph of the Rear View of the Electromechanical Antenna Calculator.</td>
<td>27</td>
</tr>
<tr>
<td>12.</td>
<td>Photograph of the Front View of the Electromechanical Antenna Calculator.</td>
<td>28</td>
</tr>
<tr>
<td>13.</td>
<td>A Two-Element Polar Pattern Using Units #1 and #4 of the Antenna Calculator.</td>
<td>35</td>
</tr>
<tr>
<td>14.</td>
<td>A Two-Element Polar Pattern Using Units #1 and #2 of the Antenna Calculator.</td>
<td>37</td>
</tr>
<tr>
<td>15.</td>
<td>A Two-Element Field Pattern Using Units #1 and #2 of the Antenna Calculator.</td>
<td>38</td>
</tr>
<tr>
<td>16.</td>
<td>A Three-Element Polar Pattern Using Units #1, #2, and #3 of the Antenna Calculator</td>
<td>39</td>
</tr>
</tbody>
</table>
FIGURE PAGE
17. A Four-Element Polar Pattern Using Units #1, #2, #3, and #4 of the Antenna Calculator ............... 40
18. A Five-Element Polar Pattern Using All Five Units of the Antenna Calculator ......................... 41
### TABLE OF SYMBOLS

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>DEFINITION</th>
<th>PAGE FIRST USED</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Antenna...</td>
<td>7</td>
</tr>
<tr>
<td>a</td>
<td>Initial starting angle of cosine generator</td>
<td>14</td>
</tr>
<tr>
<td>$B_k$</td>
<td>Phase angle of the current in kth antennas</td>
<td>9</td>
</tr>
<tr>
<td>d</td>
<td>Amplitude of swing of cosine generator</td>
<td>14</td>
</tr>
<tr>
<td>E</td>
<td>Total relative field intensity vector produced by the antenna array and measured in the horizontal plane at point P with respect to the reference antenna.</td>
<td>10</td>
</tr>
<tr>
<td>$E_k$</td>
<td>The magnitude of the field intensity produced by the kth antenna.</td>
<td>9</td>
</tr>
<tr>
<td>e</td>
<td>$2.718...$ the base of natural logarithms.</td>
<td>9</td>
</tr>
<tr>
<td>F</td>
<td>Total field intensity vector produced by the antenna array and measured in the horizontal plane at point P with respect to a reference axis.</td>
<td>9</td>
</tr>
<tr>
<td>$I_k$</td>
<td>Current in kth antenna.</td>
<td>9</td>
</tr>
<tr>
<td>j</td>
<td>$\sqrt{-1}$ imaginary j operator</td>
<td>9</td>
</tr>
<tr>
<td>k</td>
<td>kth antenna in system</td>
<td>7</td>
</tr>
<tr>
<td>$M_k$</td>
<td>Current ratio of kth antenna to reference antenna</td>
<td>10</td>
</tr>
<tr>
<td>N</td>
<td>Number of independent parameters.</td>
<td>10</td>
</tr>
<tr>
<td>n</td>
<td>Number of antennas in system.</td>
<td>9</td>
</tr>
<tr>
<td>P</td>
<td>Point of observation of field intensity</td>
<td>7</td>
</tr>
<tr>
<td>SYMBOL</td>
<td>DEFINITION</td>
<td>PAGE FIRST USED</td>
</tr>
<tr>
<td>--------</td>
<td>---------------------------------------------------------------------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>$S_k$</td>
<td>Electrical length of spacing of the kth antenna from reference antenna</td>
<td>7</td>
</tr>
<tr>
<td>$x$</td>
<td>Angle of drive</td>
<td>14</td>
</tr>
<tr>
<td>$\phi$</td>
<td>True horizontal azimuth angle of the direction to the observation point P</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>(measured clockwise from true north).</td>
<td></td>
</tr>
<tr>
<td>$\phi_k$</td>
<td>True horizontal azimuth angle orienting the kth antenna with respect to</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>the reference axis</td>
<td></td>
</tr>
<tr>
<td>$\theta_k$</td>
<td>Total phase relation between the voltage (or current) of the kth antenna and the reference antenna.</td>
<td>9</td>
</tr>
</tbody>
</table>
AN ELECTROMECHANICAL CALCULATOR FOR
HORIZONTAL PATTERNS OF MULTI-ELEMENT ANTENNAS

CHAPTER I

THE PROBLEM AND DEFINITIONS OF TERMS USED

Since the design engineer of a directional antenna system is confronted with designing the antenna system from the radiation pattern, a machine that would calculate the system from the pattern would be very useful. As it is not feasible to trace a pattern in two dimensions and come out with a large number of parameters to give an antenna system, the problem will be tackled in the reverse.

I. THE PROBLEM

Statement of the problem. It is the purpose of this study to: (1) design and develop an electromechanical calculator which will compute horizontal radiation patterns of multi-element antennas; (2) incorporate into it a system of readily changeable and continuously variable parameters, controllable from a panel, and; (3) present the pattern in a manner suitable for instantaneous observation so that the inverse function can be performed by continuously varying the parameters until the desired pattern is obtained and noting the setting of the parameters.

Importance of the study. It cannot be denied that the need for directional antenna systems has increased considerably in the past few years. Applications to the Federal Communications Commission in the
United States for licenses for broadcasting stations were at an all time high in 1946. The large number of stations and the trend toward higher power in broadcasting stations have necessitated the use of directional antenna systems to minimize the interference due to frequency allocation difficulties. The design of these directional antenna systems is straightforward but a very laborious process for the engineer. Several mechanical, electromechanical and electronic instruments to render short-cuts to the solution of the problem have been described in literature. A brief review of some of these devices is presented. The author feels that the calculator herein described will have several advantages over similar devices previously described in literature.

II. DEFINITIONS OF TERMS USED

Antenna. Antenna for the purpose of this paper is defined to mean that part of a radio transmitting station from which radio waves radiate into surrounding space.

Element. Element is interpreted to mean a single radiating antenna tower or fundamental unit of which antenna systems are built up.

Array. An array is defined to mean a system of antennas composed of one or more elements.

Differential Gear. A differential gear is an assembly of gears having the property of a third shaft rotating an angle which is a fixed


ratio times the difference of the angles of rotation of two other shafts.3

Radiation Pattern. A radiation pattern in this paper will be confined to mean the pattern in the horizontal plane formed by plotting the intensity of field strength of the radiation from an antenna system as the point of observation is moved around the antenna system at a constant radius.

III. ORGANIZATION OF REMAINDER OF THE THESIS

A summary of the pertinent literature is presented in Chapter II. In Chapter III a brief review of directional antenna theory is presented. The next two chapters deal with the theory of operation and the construction of the calculator. Chapter VI describes the operation and discusses the accuracy of the calculator described in this paper. A discussion of the results is given in Chapter VII. The conclusions and recommendations in Chapter VIII are followed by the Bibliography. Several patterns which appear in Chapter VI are calculated in the Appendix.

CHAPTER II
REVIEW OF THE LITERATURE

To aid the design engineer of a directional array antenna for broadcast station use, several mechanical, electromechanical, and electronic devices have been described in literature. This chapter is devoted to a brief review of some of this literature.

Literature on mechanical antenna calculators. Everest and Pritchett\(^1\) describe a mechanical machine for tracing out horizontal polar diagrams of two and three-element arrays.

Hutton and Pierce\(^2\) give us a mechanical calculator for calculating both horizontal and vertical polar diagrams of two and three-element arrays using the point by point method of plotting.

The operation of both of these devices depends on one vector being fixed and used as a reference with the vectors of the other two antennas revolving about its ends in a specific manner. Extending these calculators to handle more than three antennas would become mechanically involved because of the fact that the additional vectors must revolve about the ends of free-ended vectors. Therefore, it seems that these mechanical devices are limited to three towers.\(^3\)


Literature on electromechanical antenna calculator. Williams\textsuperscript{4} came out with an electromechanical machine which was an improvement over the mechanical devices previously described. This machine uses simple mechanical generating mechanisms, similar to the Everest-Pritchett, Hutton-Pierce devices, in conjunction with selsyn motors used as phase-shifting transformers, having three-phase stators as primaries and single-phase rotors as secondaries. The output of these selsyn-transformers represent electrical vectors, any number of which may be added in series to give a resultant vector. Thus, the principle of Williams' machine is not limited in the number of antennas it can be extended to handle. Williams obtains his plot by a point-by-point method.

Smith and Gove\textsuperscript{5} have constructed a calculator using the same principle as the Williams apparatus but with the refinements of: (1) being motor driven; (2) having a recording mechanism that gives a direct plot of the pattern calculated; and (3) having a planimeter that gives the rms value of the pattern. In addition, this machine will also automatically trace out the field intensity contours at various elevations.

Literature on electronic antenna calculator. In all the previously described calculators there still remained inconvenient mechanical adjustments of the parameters. Brown and Morrison\textsuperscript{6} have developed


\textsuperscript{5} Smith and Gove, op. cit. pp.78-83.

the "R. C. A. Antennalyzer" which is an example of solving antenna design problems by electronic means. This instrument has the advantage in that the parameters can be varied simply by turning dials while instantaneous changes can be observed on the face of a cathode ray oscillograph on which the plot of the pattern appears.

The inverse function, i. e. the function of analysis, can be performed with this instrument by observing the changing pattern while varying the parameters until the desired pattern is obtained and then noting the settings of the parameters which produced the pattern.
CHAPTER III

REVIEW OF DIRECTIONAL ANTENNA THEORY

Derivation of radiation pattern equation. As a brief review of the theory of directional arrays, the radiation pattern equation is derived.

At any point $P$ on a horizontal plane passing through the base of a vertical tower antenna, the field strength is directly proportional to the current in the antenna. With an array of identical towers, the field strength is the vector sum of the contribution from each of the individual elements.\(^1\)

Assume a point $P$ to be such a distance from the array compared to the element dimensions that the signals from each of the towers to the point $P$ will follow essentially parallel paths. Only the directive characteristics of the array are considered. All propagation effects are neglected.\(^2\)

Referring to Figure 1, it will be seen that, compared with a radiation starting at the reference antenna $A_1$, the phase of the signal from antenna $A_2$ is advanced by $S_2 \cos (\phi - \phi_2)$ degrees while that from antenna $A_k$ is advanced by $S_k \cos (\phi - \phi_k)$ degrees. Where $\phi$ is the true horizontal azimuth angle of the direction to the observation point $P$ measured clockwise from true north; $\phi_k$ is the true horizontal azimuth angle orienting the $k$th antenna with respect to the reference axis;

\(^1\) Everest and Pritchett, op. cit., p. 222.
\(^2\) Ibid, p. 228.
FIGURE 1

PLAN VIEW OF AN ANTENNA CONFIGURATION HAVING K ANTENNAS
and $S_k$ is the electrical length of spacing of the kth antenna from the reference point.

Let the current $I_k$ in the kth antenna lead the current $I_1$ in the reference antenna $A_1$ by the phase angle $B_k$. Then the total phase relation between the voltage (or current) of the kth antenna and the reference antenna is,

$$\theta_k = B_k + S_k \cos (\phi - \phi_k) \quad (1)$$

The field at a distant point in the horizontal plane in the general case is given by,

$$F = \sum_{k=1}^{k=n} E_k e^{j\theta_k} \quad (2)$$

Where,

- $E_k$ is the magnitude of the field intensity produced by the kth antenna,
- $e^{j\theta_k}$ is the unit operator determining the phase of the voltage vector of the kth antenna,
- $e$ is $2.718...$ the base of natural logarithms,
- $j$ is the square root of $(-1)$, imaginary $j$ operator.

Since $A_1$ is the reference antenna, $\theta_1$ is zero and $e^{j\theta_1} = 1$, equation (2) becomes

$$F = E_1 + \sum_{k=2}^{k=n} E_k e^{j\theta_k} \quad (3)$$
Now if we let \( \frac{E_k}{E_i} = M_k \) and \( \frac{F}{E_i} = E \),

The general equation for calculating the horizontal radiation pattern becomes,

\[
E = 1 + \sum_{k=2}^{n} M_k e^{ij\phi_k}.
\]  

(4)

Equation for number of parameters involved. From this we see that for \( n \) antennas we have one dependent parameter \( E \) and a number of independent parameters equal to

\[
N = 4(n-1) + 1
\]

(5)

The independent parameters being,

\( \phi, \phi_k, S_k, B_k, \) and \( M_k \).

Calculation of a simple two-element antenna pattern. For a two-antenna array, equation (4) becomes,

\[
E = 1 + M \cos [B_2 + S_2 \cos(\phi - \theta)] + j M_2 \sin [B_2 + S_2 \cos(\phi - \theta)].
\]

(6)

for \( M_2 = 1 \)  \( B_2 = 0 \)  \( \phi_2 = 0 \)  \( S_2 = 180^\circ \)

\[
E = 1 + \cos (180 \cos \phi) + j \sin (180 \cos \phi).
\]

(7)

Calculations using equation (7) are shown in Table III on Page 50 in the Appendix. The resulting pattern is plotted and shown in Figure 14 on Page 37.
CHAPTER IV

THE THEORY OF OPERATION

Having briefly reviewed the mathematical theory of directional antenna arrays in Chapter III, it is the purpose of this chapter to show how this machine solves the radiation pattern equation.

The author decided to use the electromechanical form of calculator since the purely mechanical forms are limited to three antennas as discussed in Chapter II, and material was more readily available for the construction of the electromechanical form than for the electronic form as described by Brown and Morrison.1 This machine in general is patterned after Williams' machine2, but incorporates some of the flexibility of Brown and Morrison's instrument.3

General Layout. In Figure 2 is shown a block diagram of the electromechanical five-antenna pattern calculator. A block diagram of a unit representing a single antenna element is shown in Figure 3. A motor and a hand crank are coupled together through a differential gear to drive a mechanical cosine generator. The cosine generator is coupled by a rack and gear to the rotor of a phase-shifting transformer. The output of the transformer is connected to a potentiometer.

The drive mechanism. A differential gear is incorporated in the drive mechanism to isolate the motor from the hand crank so that either

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1 Brown and Morrison, op. cit., pp. 994-997.
3 Brown and Morrison, op. cit., p. 995.
FIGURE 2

BLOCK DIAGRAM OF THE ELECTROMECHANICAL ANTENNA PATTERN CALCULATOR
FIGURE 3

BLOCK DIAGRAM OF A UNIT ANTENNA CALCULATOR
form of drive may be used independently. This feature permits the flexi-
bility of using either the point by point system of plotting when using
the hand crank or the instantaneous presentation of the whole pattern
when using the motor. The hand crank is locked in position when using
the motor.

**Antenna Position Control.** Panel dial control of the antenna
position is accomplished by having the cosine generator driven through
a differential gear. This permits the starting angle of the cosine
generator to be changed by turning a dial on the panel without dis-
connecting the generator from the drive.

**The Cosine Generator.** The cosine generator converts rotary
motion to simple-harmonic motion by the usual means of a wheel with a
pivot off center and scottish yoke to transmit the translatory motion.
However, in this generator the wheel is replaced by two wheels, one
stacked upon the other in such a manner that when one of the wheels is
rotated with respect to the other the pivot, which drives the scottish
yoke, is caused to change position radially. Thus, by setting the posi-
tion of one wheel with respect to the other, the amplitude of swing of
the cosine generator is controlled. The mechanical output of the cosine
generator is proportional to

\[ d \cos(x - a) \]

where \( d \) is the amplitude of swing, \( x \) is angle of drive, \( a \) is the initial
starting angle.

This mechanical output is rack-geared on to the rotor of the phase-
shifting transformer.
**Antenna Spacing Control.** Antenna spacing is accomplished by varying the amplitude of swing made by the cosine generator. Control of the antenna spacing is brought out to a dial on the panel by coupling together the two wheels of the cosine generator, described in the above paragraph, through a third differential gear. Thus, the amplitude of swing of the cosine generator is varied from a dial on the panel without having to disengage the generator from the drive or stop the drive mechanism.

**Phase-Shifting Transformer and Current Phase Control.** A selsyn motor is used as a phase-shifting transformer by exciting its three-phase wound-stator from a three-phase source. The phase of the voltage induced in the single-phase rotor-winding is exactly proportional to the angular position of the rotor. Since the rotor is rack-geared to the cosine generator, this phase is proportional to

\[ d \cos (x-a) \] degrees.\(^5\)

The selsyn motor is mounted in a bearing and the stator is geared to a dial on the panel in order that the phase may be shifted by an additional angle \(b\), where \(b\) is an angle proportional to the rotation of the phase shift dial on the panel. The output from the selsyn transformer is now a 60-cycle a-c voltage whose phase is represented by the term

\[ [b + d \cos (x-a)] \] degrees.

\(^4\) Williams, op. cit., p. 106.

\(^5\) See paragraph on cosine generator for meanings.
Antenna Current Ratio Control. The magnitude of this a-c voltage is controlled on the panel by means of a potentiometer connected in shunt with the output of the selsyn-transformer.

The Reference Antenna. Since, for the antenna used as a reference, the spacing, position and phase shift are all zero, all that is required for the reference antenna is a potentiometer to vary the magnitude of the reference voltage. Although a selsyn-transformer was not required, one was used so that the reference voltage would be isolated from the three-phase source and also so that the loads on the three-phase source would all be balanced. The rotor of the reference selsyn-transformer is locked in the reference position.

An Analogy. An analogy showing the correlation of the antenna system to the parameters of the calculator is shown in Table I below.

**TABLE I**

<table>
<thead>
<tr>
<th>Antenna System</th>
<th>Parameters of Calculator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Antenna Current Ratio $M_k$</td>
<td>Ratio of voltages $M_k$</td>
</tr>
<tr>
<td>Phase Angle between Currents $B_k$</td>
<td>Phase Angle $b$</td>
</tr>
<tr>
<td>Tower Spacing $S_k$</td>
<td>Amplitude of Swing $d$</td>
</tr>
<tr>
<td>Azimuth Angle of Tower $\phi_k$</td>
<td>Initial Starting Angle $a$</td>
</tr>
<tr>
<td>Azimuth Angle of Measuring Point $\phi$</td>
<td>Angle of drive $x$</td>
</tr>
</tbody>
</table>

From here on the symbols of the antenna system will be adopted as the calculator parameters.

Details of the Electromechanical Calculator. An electrical circuit diagram of the electromechanical calculator is shown in Figure 4. The power source is a 220 volt, three-phase, 60 cycle, a-c supply.
3 phase fises
220 volt
60 cycle 2 amp.

Phase Sequence
Reversing
On-Off
Line Switch

Transformer
Step Down
Delta to Wye
220v to 10v

Phase Sequence
Indicator

Line Voltage
Pilot Lights

10 volt 60 cycle 3 phase line

Ant. 1
Ant. 2
Ant. 3
Ant. 4
Ant. 5
Sync Selsyn

Selsyn
Phase
Shifting
Transformers

Antenna
Off-On Switch
Calculator
Pilot Lights

Voltage Ratio
Potentiometers
Synchronizer
Selector Switch

Meter Output
Selector Switch

Saw-Tooth
Potentiometer

Volteimeter

Calculator Output

FIGURE 4

ELECTRICAL CIRCUIT DIAGRAM OF THE ELECTROMECHANICAL ANTENNA CALCULATOR
stepped down to five volts by three single-phase transformers. However, by connecting the transformers delta to wye, a 8.6 volt three-phase excitation is supplied to the selsyn-transformers. A phase-sequence indicator composed of two pilot lights and an inductance is incorporated to indicate the proper phase sequence of the supply voltage. The completed calculator is designed to handle five antennas, although it could be expanded to handle any number. The outputs from the five units are added in series and terminated on the panel. A continuous travel potentiometer geared to the mechanical drive forms the basic part of a saw-tooth sweep-generator for presentation of the pattern on an oscilloscope in rectilinear coordinates. An additional selsyn phase-shifting transformer geared to the mechanical drive forms a basic part of polar pattern sweep synchronizer. The outputs from the saw-tooth potentiometer and sweep selsyn are terminated on panel through a selector switch. The machine is designed to run the cosine generators at approximately 30 revolutions per minute. A speed faster than 30 rpm, the author felt, would impose unnecessary wear on the gears and also increase the effect of inertia on the reciprocating mechanisms.

CHAPTER V

CONSTRUCTION OF THE CALCULATOR

I. PHASES OF CONSTRUCTION

Sources of material. Material for the construction of this calculator is available from war surplus equipment. The single-parallax gunsight computers, manufactured by General Electric Company and used in the B-29 bomber, contain about all the equipment used in the construction of this calculator. The selsyn-transformers, the differential gears, the cosine generator, and an assortment of gears are found in these single-parallax computers. A photograph of one of these computers is shown in Figure 5.

The Cosine Generator. The cosine generator is found mounted in a ball-bearing race which is contained in a bell housing. It is necessary to remove the generator from the housing and ball-bearing race. A photograph of the cosine generator is shown in Figure 6.

Antenna Spacing Control. The antenna spacing control was found to exist in the computer almost usable as is. The only modification necessary was to lengthen one shaft and remove a couple of unnecessary gears. A photograph of the spacing control is shown in Figure 7.

Antenna Position Control. The antenna position control unit had to be fabricated. It consists mainly of a differential and a few other gears as shown in the photographs of Figures 8 and 10. These gears were all found in the computer.

Antenna Phasing Control. The antenna phasing control is part of the unit containing the phase-shifting selsyn-transformer which is the heart of the calculator. This unit was constructed. It uses a selsyn-
FIGURE 6
PHOTOGRAPH OF THE COSINE GENERATOR
FIGURE 7
PHOTOGRAPH OF THE SPACING CONTROL
FIGURE 8
PHOTOGRAPH OF THE POSITION CONTROL
FIGURE 10
PHOTOGRAPH OF A SINGLE ANTENNA CALCULATOR UNIT
transformer, a rack and gear assembly, a set of miter gears, a ball bearing race previously used to mount the cosine generator, and a set of 2\(\frac{1}{2}\) inch spur gears. All this was obtained from the parallax computer. The phasing unit is shown in the photograph of Figure 9.

One Antenna Calculator Unit. A single antenna unit is shown in the photograph of Figure 10. This shows the spacing control, the cosine generator, the positioning control, the connecting rod coupling the cosine generator to the phasing unit, the phasing unit and output potentiometer.

II. THE FIVE-ANTENNA CALCULATOR

The five-antenna calculator is mounted on a six foot standard 19 inch relay rack.

Rear View. The rear view of the calculator is shown in the photograph of Figure 11. This shows the four antenna units coupled together mechanically using shafts, miter gears and couplings. The power supply is shown at the top with the polar sweep selsyn and saw-tooth potentiometer just below it. The bottom panel contains the reference antenna voltage ratio potentiometer, the reference selsyn-transformer, the hand drive, and the motor. The hand drive and the motor are not shown in the photograph.

Front View. A front view of the five-antenna calculator is shown in the photograph of Figure 12. The antenna spacing dials are located at the right. The antenna positioning dials are located next to the spacing dials. The voltage ratio dials are on the left with the phasing
FIGURE 11
PHOTOGRAPH OF THE REAR VIEW OF THE ELECTROMECHANICAL ANTENNA CALCULATOR
FIGURE 12
PHOTOGRAPH OF THE FRONT VIEW OF THE ELECTROMECHANICAL ANTENNA CALCULATOR
dials next to the ratio dials. Switches for switching the individual
antenna calculating units on and off are located just below the voltage
ratio dials. Red pilot lights for indicating which antenna calculating
units are in use are located just below the antenna spacing dials. Dial
locks are shown on all the dials except the voltage ratio dials. These
locks are necessary to hold the parameter settings while the machine is
in operation. The bottom panel contains the reference antenna ratio
dial on the left and the hand-drive dial on the right. At the top is
the power panel.

The Power Panel. At the top of the power panel are three green
pilot lights indicating when the three-phase supply voltage is on. Be-
tween each pair of green lights is a pilot light: one is white; the other
is red. These two pilot lights make up the phase-sequence indicator
mentioned in Chapter IV. At the upper right are located the output ter-
minals for the polar sweep selsyn or the saw-tooth potentiometer. The
selector switch for selecting either of the two is located just below
output terminals. Output terminals for either the combined antenna
calculator output or any of the unit antenna calculator outputs are
located at the upper left. A selector switch for selecting any of the
calculator outputs is located just below the terminals. At the lower
center of the panel an on-off-on selector switch is located, its use
being to turn on the supply and select the proper phase sequence as in-
dicated by the phase-sequence indicator. Just above this switch are
located three two-ampere fuses, one in each of the three-phase lines
for protection.
CHAPTER VI

THE CALCULATOR IN OPERATION

I. CALIBRATION

The calibration of the antenna calculator is fairly simple. The dials are calibrated according to their function in the following order: current ratio, current phase, antenna position, and antenna spacing.

The Current Ratio Dials. The current ratio dials are calibrated by balancing one antenna against another. Antenna number one alone is switched on and its current ratio dial is set to a position which gives a voltage less than the maximum obtainable from the settings of each of the remaining four antennas. A vacuum tube voltmeter is used to read these output voltages. This arbitrarily selected point is the unit ratio point used as a reference for the remaining calibrations.

Number two antenna is now switched on together with antenna number one. The phasing and ratio dials are adjusted for a minimum output voltage. An Oscilloscope is useful in obtaining the balance. This determines the unity point on the ratio dial of antenna number two. By a similar process of comparing magnitudes, this unity point is located on ratio dials of the remaining antennas.

Now again using antenna number one alone, with the aid of the oscilloscope, the magnitude of its output voltage is reduced to one half. This determines the one half point on the reference ratio dial. By the process previously used, this point is transferred to the other antenna ratio dials. By continuing the process outlined above, the calibration
of all the ratio dials is obtained.

The Current Phase Dials. Before the current phase dials can be calibrated, the spacing dials must be zeroed. This is done as follows. The antenna to be zeroed (Nos. 2, 3, 4, or 5) is balanced against the reference antenna (antenna #1) by adjusting the phase and ratio dials for a minimum output voltage. Having obtained this balance, the spacing dial is set to such a point that by either turning the drive mechanism or the positioning dial, the balance is not affected.¹

After the spacing dials have been set to zero, the balance establishes the 180° point on the phasing dial. Since the gearing of the phasing dials to the selsyn stators are in a one to one ratio, 360° protractors are used for calibrated dials. This makes the phasing dial calibrations very simple. After aligning the dial with the 180° point, no further calibration is necessary.

The Antenna Position Dials. Before calibrating the position dials, an angular reference of the drive mechanism is arbitrarily selected by setting the plotting azimuth angle indicator to zero. With the spacing dials still in their zero position, two antennas are balanced in the manner described above. Now the position dial is set on a point so that when the spacing dial is rotated, the balance is not affected. This locates either the 90° or 270° point on the position dial. Since the gearing of this dial is also in a one to one ratio with the cosine

¹ In performing this and all remaining calibrations, the dials must be locked after each setting.
The Antenna Spacing Dials. The calibrating of the spacing dials proved to be as simple as that of the ratio dials. With the position dials set to 0° or 180° and two antennas (one the reference antenna) balanced against each other, the phasing dial should read 180° for zero setting of the spacing dial.

Now since the spacing is in electrical degrees, a convenient unit of calibration is tenths of a wavelength or 36° intervals. The balance is upset by rotating the phasing dial an amount in degrees equal to the interval of spacing desired on the spacing dial. Then the balance is restored by rotating the spacing dial giving the desired point.

This process is continued until all the desired points on the dial are obtained.

II. CALCULATION OF FIELD PATTERNS

The operation of the calculator is almost obvious from its description, construction and calibration. To check the calibration and accuracy of the calculator, several antenna configurations giving known patterns were set up.

Two-Element Array Patterns. First, to eliminate any ambiguity in the calibration in the positioning dial, a two-element configuration giving a non-symmetrical pattern was set up using each of the four antenna calculators separately with the reference antenna. The configuration,
being two antennas spaced three quarters of a wavelength apart, oriented east and west or $90^\circ$ to the reference line, and phased $90^\circ$ with equal magnitudes, is set up as follows: The spacing dial, located at the right, is set to .75; the position dial located next to the spacing dial is set on $90^\circ$; the phasing dial is also set on $90^\circ$; both the ratio dials, located at the left, are set on unity ratio. Both antennas are switched on and the pattern is obtained by the point by point method, using a vacuum tube voltmeter and hand crank.

The results in all four cases were comparable to the pattern calculated mathematically. Figure 13, giving the pattern of #4 antenna, is representative of the results.

Table II compares the results of all four antenna calculators to the computed result for a two-element broadside array, using one half wavelength spacing. The computed results are given in Column #1. Columns #2, #3, #4, and #5 are the results of respective calculators. One of these patterns is plotted against the computed pattern in Figures 14 and 15. These two-element patterns give a check on the calibration and accuracy of the calculators. The accuracy is within an allowable five percent.

**Multi-Element Array Patterns.** Patterns were calculated both mathematically and from the calculator for one half wavelength spaced broadside arrays using 3, 4, and 5 element arrays. These are shown in Figures 16, 17 and 18. Figure 16 demonstrates that the accuracy of the three-element patterns is comparable to that of the two-element pattern. Figures 17 and 18 show that the error increases greatly with the added fourth and fifth elements. The data for the mathematical computation
of these broadside arrays is shown in Table III on Page 50 in the Appendix.
FIGURE 13
A TWO-ELEMENT POLAR PATTERN USING UNITS θ1 AND θ2 OF THE ANTENNA CALCULATOR
## COMPARISON OF RESULTS

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Full line - Computed curve
Dotted line - Curves given by calculator

A two-element polar pattern showing the effects of $\theta_1$ and $\theta_2$ of the antenna calculator
A two-element planar pattern using delays $\phi_1$ and $\phi_2$ of the antenna elements.
Full line = Computed curve
Dotted line = Curve given by calculation

$\theta = \theta_1, \theta_2$
$\alpha = \alpha_1, \alpha_2$

A THREE-ELEMENT POLAR PATTERN OBTAINED IN A, $\alpha_1$, AND $\theta_1$ OF THE ANTENNA CALCULATOR
Full line - Computed curve
Dotted line - Curve given by calculator

\[ L = L_1 L_2 \]
\[ = L_1 e^{i \theta} \]
\[ = L_2 e^{i \phi} \]
\[ \theta = \phi = \phi_1 - \phi_2 = \phi_3 - \phi_4 = \phi_5 \]

Figure 17

A four-element polar pattern using units \( A_1, A_2, A_3, \) and \( A_4 \) in the antenna calculator
Figure 3: A five-element polar pattern using all five parts of the antenna calculator.
CHAPTER VII

DISCUSSION OF RESULTS

Inadequacy of the Cosine Generator. In the construction of the first antenna unit, the author found that the swing of the cosine generator was limited to approximately one half an inch in either direction. It was necessary to amplify this swing by a five to one ratio to obtain an antenna spacing of one wavelength. The results being found fairly good for a two-element antenna, the author decided to find the limit to which this could be extended without sacrificing reasonable accuracy.

To accomplish this end, the five-unit calculator was constructed with the swing of the cosine generators amplified to give a maximum of three wavelengths spacing in units #4 and #5 and only one wavelength spacing in units #2 and #3, #1 unit being the reference antenna.

From Figures 17 and 18 it is apparent that errors in the four and five-element antenna patterns exceed the tolerable limit. This can be accounted for in several ways.

Sources of Error. The largest source of error is due to gear slack in the cosine generator and differential gears. The effect is amplified from about five to about fifteen times in gear-train to increase the swing to the phase-shifter. The errors due to line-voltage fluctuations, unbalanced phase voltages and harmonics present in the wave form would also have a considerable effect as the number of antennas were increased. This is particularly true since a peak-reading vacuum tube voltmeter was used in reading the output voltage.
Presentation of the Pattern. Although the calculator was designed to be motor driven, the motor was not tried by the author. However, the author feels that no difficulty should be experienced in obtaining a presentation of the pattern by using a voltagraph recording instrument such as an Esterline-Angus meter with the motor. Also with a few circuit additions, the author feels that a pattern presentation can be put on the face of an oscilloscope. A synchronizing selsyn and a continuous travel potentiometer were built into the calculator for that purpose.
CHAPTER VIII

CONCLUSIONS AND RECOMMENDATIONS

I. CONCLUSION

It is concluded that an electromechanical antenna pattern calculator can be constructed to a practical degree of accuracy incorporating a panel presentation of readily changeable parameters and an instantaneous picture presentation of the pattern.

II. RECOMMENDATIONS

Improvement of Cosine Generator. It is recommended that the cosine generator be redesigned to have a larger crank wheel giving a swing large enough to drive the phase shifting unit directly or even through a step-down gear ratio, if practical, to decrease the error due to this source.

Improvement of Power Supply. It is desirable to have a well-regulated three-phase balanced supply voltage free from harmonic content. However, error due to this source is not as bad as that due to mechanical sources.

Use of a Linear Recording Voltograph. A voltograph recording linearly with voltage is desirable for presenting recorded polar diagrams on a turntable.

Use of an Oscilloscope. The author strongly feels a suitable oscilloscope circuit can be worked out to be used on this machine for the instantaneous presentation of polar and rectilinear field strength patterns.
BIBLIOGRAPHY
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Ebel, A. James, "Directional Radiation Patterns," Electronics, 9:29-30, April, 1936.


Lumped-Constant Systems.


APPENDIX

THE CALCULATION OF POLAR PATTERNS

The calculation of a polar pattern for a 2, 3, 4 and 5 element broadside arrays using one-half wavelength spacings is given by the equation,

\[ E = 1 + \sum_{k=2}^{k=n} M_k e^{j\theta_k} \]

This equation becomes

\[ E = 1 + \cos(180 \cos \phi) + \cos(360 \cos \phi) + \cos(540 \cos \phi) + \cos(720 \cos \phi) \]
\[ + j \left[ \sin(180 \cos \phi) + \sin(360 \cos \phi) + \sin(540 \cos \phi) \right. \]
\[ \left. + \sin(720 \cos \phi) \right] \]

for \( n = 5 \)

\[ M_k = 1 \]
\[ B_k = 0 \]
\[ \phi_k = 0 \]

\[ S_2 = 180 \]
\[ S_3 = 360 \]
\[ S_4 = 540 \]
\[ S_5 = 720 \]

The patterns obtained using the above equation and data are calculated in Table III.
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