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A CONTROL MODEL FOR WASTE PERFORMANCE IN A COTTON SPINNING MILL

A THESIS
Presented to the Faculty of the Graduate Division by Lynwood A. Johnson

In Partial Fulfillment of the Requirements for the Degree Master of Science in Industrial Engineering

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A CONTROL MODEL FOR WASTE PERFORMANCE
IN A COTTON SPINNING MILL

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PREFACE

Acknowledgements.--The study reported in this thesis was conducted under the Callaway Fellowship, a research grant sponsored by the Callaway Mills Company. The personnel of that organization were most generous with their time and effort in providing practical information, past records, and operating experience concerning the problem under study. In particular, Mr. A. W. Gunn, Mr. James C. Teaver, and Mr. Kenneth Combs were most cooperative and informative.

Dr. David C. Ekey, of the School of Industrial Engineering, was the faculty advisor for this study and gave much assistance, advice, and encouragement, thereby enabling this project to be successfully concluded. Dr. Joseph J. Noder, School of Industrial Engineering, and Professor Ralph L. Hill, School of Textile Engineering, served as members of the reading committee, and their advice and suggestions have been incorporated in this paper.

Type of study.--This research concerned itself with an investigation of the economics involved in the problem of controlling unnecessary material waste in textile manufacturing and the methods by which an integrated control system could be established to realize the greatest gain to the enterprise. A problem of this magnitude requires a system which fully coordinates
the efforts of all segments of the textile organization. Scope of the system studied.--While waste is an important economic consideration in all phases of textile manufacture, it was desirable that the scope of this study be limited to a particular phase or process, as the multitude of raw material-process combinations make it impossible to adequately treat every possible case.

The process of preparing cotton yarn was selected as the system for study because of the following reasons:

1. Cotton is a common textile raw material.
2. Yarn preparation is the initial manufacturing process for most textile products.
3. Yarn is the finished product for some mills.
4. The processing steps are well defined.
5. Conclusions drawn from a study of this process can easily be extended to other textile manufacturing processes.
6. One of the purposes of the cotton yarn preparation process is the removal of undesirable matter from the raw cotton; that is, waste is an inherent part of the process.
7. Undoubtedly, more economic gain can be realized through sound regulation of waste removal in this process than through similar control of other textile processes.

The system studied consisted of all process steps necessary to convert raw cotton into cotton yarn. It was recognized that several process routes are possible within this
system, depending upon the desired quality characteristics of the yarn. Therefore a general theory was developed to include all operational possibilities within the defined boundaries.
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SUMMARY

The purpose of this research was to investigate the evaluation of waste performance in a cotton spinning mill, with the objective of developing appropriate models for controlling the economic losses attributable to waste.

A symbolic model representing the dollar loss incurred by a mill because of waste was developed. Examination of this model revealed that the percentage of waste, commonly used as a measure of the waste performance of a mill, does not give an exact indication of the magnitude of the dollar loss involved. The loss may vary, even though the percentage of waste remains constant. Since the goal of the textile mill is primarily economic, it was concluded that the dollar loss caused by waste should be the measure of effectiveness used to evaluate waste performance.

It is submitted that the goal of any program installed to control waste losses would be to maximize the difference between the waste losses averted by the program and the cost of the program. Thus the effectiveness of different control programs could be compared by this criterion.

The effect of the volume of raw material input (in pounds) on the percent waste was found to be significant for the mill studied. The percent waste decreased as the level of production input increased. The possibility of using the
regression of percent waste on production input was advanced as an aid in planning.

The problem of designing a waste control program was studied and a model, consisting of the functional elements of measurement, evaluation, correction, and communications, was developed. Two levels of control were recognized: local control involves control within a process step; system-wide control refers to the control of the mill as a whole.

It was found that a need exists for experimentation on and analysis of the problem of waste. The exact nature of the parameters and variables affecting waste losses in a particular mill must be determined if maximum return from the control effort is to be realized. Many mills have large quantities of historical data on waste performance, but the usefulness of this data for analytical purposes is limited by its incompleteness and unknown accuracy.
CHAPTER I

INTRODUCTION TO THE PROBLEM OF COTTON WASTE

To provide a sound foundation for this study, certain facts are briefly reviewed. First, the raw material, cotton, must be discussed in light of the types of undesirable matter that it contains. Secondly, the nature of the yarn preparation processes should be examined, but only to such an extent as is necessary to supplement later developments in this paper. Thirdly, the meaning of "waste" as used in this study needs clarifying.

Raw material.—Cotton is usually received at a spinning mill in bales. A bale is a specified weight of compressed and matted cotton fibers, containing a small percentage of undesirable foreign material (e.g., stems, bolls, leaves, and dirt), covered with jute fiber bagging, and bound together with metal straps called ties. The bagging and ties are removed from the bale as the initial step in the opening process and are sold as waste.

In addition to the foreign matter, a bale of cotton will contain some undesirable cotton fibers. These may be short fibers, immature cotton, damaged and broken fibers, or neps (fibers that have become tangled and rolled into a small bunch). If allowed to remain in the process, all of the above
will tend to produce inferior yarn by adversely affecting its strength and appearance. Therefore the early stages of the yarn preparation process are devoted to the removal of this material. The amount of such material that must be removed depends upon the grade of cotton used and the desired quality characteristics of the yarn.

Objectives of the yarn preparation process.—There are four primary goals of the yarn preparation process (1):

1. To take the proper mixture of raw cotton and spread out the compressed fibers into a looser state, at the same time removing the heavier impurities. This is done primarily in the opening and picking process steps.

2. To perform a finer cleaning of the cotton by extracting the short undesirable fibers and any foreign material which may have escaped the opener and picker cleaning.

3. To make the individual cotton fibers parallel and to attenuate and even the strands into which the fibers are arranged. This is accomplished to some extent in each of these process steps -- carding, combing, drawing, fly-frame processes, and final spinning.

4. To strengthen and complete the manufacture of the yarn by adding the desired amount of twist. The spinning operation performs this function.

The exact number and combination of process steps and types of machines used in the yarn preparation process will
vary from mill to mill, depending upon the desired quality attributes of the yarn being manufactured and the equipment limitations and other characteristics of a particular mill.

Spinning Mill Waste.—The term "waste" as used in this paper refers to all material removed from the yarn preparation process other than finished yarn of acceptable quality. (Note that the amount of waste per time period will not necessarily equal the difference between the measured input in baled cotton and the measured output in yarn, since such factors as moisture, added material, and weighing and accounting errors must also be considered.)

Waste may be classified in several ways. Initially, all waste may be placed in two broad categories, "avoidable" and "unavoidable."

Unavoidable waste refers primarily to the foreign matter and undesirable cotton fibers which are purposely removed from the stock in order that the final product will be of acceptable quality. Some good cotton is also unavoidably made waste through the inherent limitations of operators and machines. However, the amount of unavoidable waste is largely a function of the grade of raw cotton used and the desired quality of the final product.

Avoidable waste is good spinnable cotton which is removed by machine action with the unavoidable waste or through operator carelessness or poor operating procedures.
Waste may also be classified as "reworkable" or "non-reworkable."

Reworkable waste consists of good cotton fibers which come out of the various process steps as waste and are suitable for reinsertion into the opening process step to be reworked. The literature (2) mentions that what is reworkable waste in one mill may not be reworkable waste in another, because of differences in product, equipment, operating policy, etc.

Non-reworkable waste is waste that is not suitable for return to the process. Most types of non-reworkable waste may be sold; however, the remaining portion is of no value and must be disposed of.

The term "invisible waste" is frequently seen in articles on cotton waste, and often it is given different meanings. In this paper, invisible waste is defined as the unaccounted for material waste from the process. It does not include moisture gain or loss, materials added (e.g., oil), or weighing and accounting errors. This type of waste could also be call "unaccounted for non-reworkable waste" and the remaining non-reworkable waste (actually collected) could be referred to as "accounted for non-reworkable waste."

To avoid this cumbersome nomenclature, the convention of using "non-reworkable waste" to refer only to the accounted for portion of waste that cannot be returned to the process and "invisible waste" to denote the unaccounted for portion
will be adopted.

Need for waste control.—The extraction of waste, both intentionally and unintentionally, from textile processes serves to increase mill costs in several ways.

1. Raw material cost is increased, since more material than the desired output quantity must be purchased to cover waste losses within the process. This would also result in increased raw material inventories with associated carrying and handling costs.

2. The cost of labor, machine time, and associated factors involved in processing the stock to the point where it becomes waste is chargeable to waste extraction.

3. There are costs involved in handling and disposing of waste.

4. Reprocessing of reworkable waste may adversely affect the spinning characteristics of the yarn and result in excessive downtime in the spinning room.

5. Inadequate removal of waste may cause yarn made from such stock to be rejected for poor quality. It could also cause poor operating performance in the fly-frame and spinning processes.

Waste can be sold for various prices, depending upon the type of waste, but the income derived will only partially offset the above mentioned cost. Some mills process the lower grades of waste so that a higher sale price can be obtained.
The cost of this processing must be charged against income from waste sold.

To illustrate the amount of money which avoidable waste can cost in raw material dollars alone, consider the following example. Raw cotton of the grade used in a hypothetical mill costs $0.30 per pound. Production, in pounds of raw cotton processed, may vary from 100,000 to 300,000 pounds per week. The graph shown in Figure 1 illustrates the increase in weekly costs if total mill waste is in excess of the unavoidable waste percentage.

For instance, if waste is one percent above the unavoidable level and the mill is processing 200,000 pounds of cotton per week, the elimination of this excess percent would result in a savings of $600 per week in raw material costs—more than equivalent to a reduction of five operators on the plant's labor rolls.

The economic considerations of waste performance are discussed more completely in Chapter II. It is sufficient at this point to state that avoidable waste can result in unnecessary cost to a textile mill and, through the preceding simple example, to note that this cost can be of large proportions.

The possible economies associated with waste control have not gone unnoticed by textile manufacturers. To eliminate unnecessary cost, many mills have introduced formal waste control programs; however, this practice is not universal and
Figure 1. Weekly Raw Material Cost Caused by Various Percentages of Unnecessary Waste.
(Raw Cotton Cost = $0.30 Per Pound.)
many other mills have little or no planned activities regarding the reduction of avoidable waste. Authorities in the field are unanimous in recommending waste control for cost reduction in mill operations. Reflection upon the previously listed savings that might accrue from such control would seem to substantiate the popular opinion. More specifically, in 1948, United States mills produced 642 million pounds of cotton waste with a value at the mill of 71 million dollars. (3)

The problem now resolves itself into determining how to go about controlling unnecessary waste.

**Present Control Methods.**—Existing methods of waste control have been patterned around one or more primary factors in determining the amount of waste produced. These factors are the following:

1. Cotton grade (i.e., amount of impurities and fiber properties).
2. Desired quality characteristics of the product.
3. Machine characteristics (e.g., settings, limitations).
4. Human element, both supervisors and operators.

This would include operating procedures pertaining to humans.

There are a great variety of waste control methods used in current practice. Among the formal methods, the following are listed and described briefly:
1. Proper raw material mix. Use of an optimum blend of cotton grades should insure that the most economical balance between waste cost, raw material cost, and product quality has been obtained.

2. Proper settings on machinery. Machine settings are basically determined by the desired quality of the final product and the characteristics of the cotton being processed. Research at North Carolina State (4) showed that waste savings could be realized through using more optimum card settings than were being used by all mills under study. Careful selection and maintenance of machine settings are ways of controlling waste.

3. Supervisory and operator training and motivation. Realizing that the human element is very much a factor in waste performance, some mills have undertaken programs to train both supervisors and operators in proper methods of machine operation, material handling, and waste classification. Campaigns have been conducted to reduce unnecessary waste, using competition between departments or shifts as a motivating factor. Usually these campaigns are sporadic and are more in the form of "fire-fighting" than continuing control.

In this regard, a group of British textile people, in this country to study our textile industry, reported as follows on a visit to a southern textile consulting firm.
WASTE REPORT

Mill:_________________________ Period Covered:_________________________

Pounds Opened by Grade:__________________________________________

<table>
<thead>
<tr>
<th>Process &amp; Waste Type</th>
<th>Actual Waste</th>
<th>Standard Waste</th>
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<tr>
<td></td>
<td>Pounds</td>
<td>Percent</td>
</tr>
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</table>

Opening and Picking:
- Bagging and Ties
- Damaged Cotton
- Motes and Fly
- Sweeps
- Invisible
- Total

Carding:
- Lap Waste
- Strips
- Clearer Waste
- Motes and Fly
- Sweeps
- Invisible
- Total

Drawing:
- Sliver Waste
- Clearer Waste
- Sweeps
- Invisible
- Total

Roving:
- Sliver Waste
- Roving Waste
- Clearer Waste
- Sweeps
- Invisible
- Total

Spinning:
- Roving Waste
- Thread Waste
- Clearer Waste
- Sweeps
- Invisible
- Total

Total Mill Waste:

Figure 2. A Typical Waste Reporting Form
It was found that the firm operated entirely as consultants on waste reduction and control. They were not so much concerned with the controllable waste or dirt extracted by the machines as with the waste caused by inadequate managerial arrangements or supervision, badly maintained machinery or carelessness on the part of the operatives (including the results of carelessness at a preceding process).

In this same section of its report, the British team gives numerous examples of how the human element can increase waste costs (5).

4. Waste records and reports. A typical waste reporting form is shown in figure 2. Some mills use this type of report to summarize waste performance for a certain time period, usually a week. Note the columns headed "Standard Waste, Percent" and "Standard Waste, Pounds." The use of standards will be discussed below.

Waste tickets, tabulations of waste, sent to the waste house, and records of reworkable waste, production data, and unusual incidents pertinent to waste performance are types of records that may be maintained.

5. Standards for waste performance. Since the aim of waste reduction and control is to minimize the amount of spin-nable fiber lost as waste, progressive mills have attempted to determine the minimum waste percentage that they can maintain and still attain the desired product quality.

These endeavors have resulted in waste "standards," against which actual performance is evaluated. These standards are usually expressed as a percentage of input, either to the
mill or to the applicable step of the process.

Three primary methods of developing these standards are found in the literature:

a. Calculation. These calculations use such data as yarn count, sliver weight, machine speeds, percent ends-down, and production rates (6)(7).

b. Past history. Past records are studied to obtain a standard waste percentage. Usually the experience of supervision is a factor in this type of determination.

c. Through waste tests. Under controlled conditions, a certain amount of raw cotton is processed and each category of waste is carefully collected and weighed. The percentage of waste is calculated from this data. Tests are run periodically to insure up-to-date standards.

The collection of actual waste data and the computation of standards are rather costly items and will be discussed in greater detail later.

6. Waste tests. One use of waste tests was mentioned above, that is, the determination of waste standards. Another use, usually where involved reporting and standards are not used, is periodic sampling of performance under controlled conditions.

7. Special devices on machines. Use of such devices as Pneumafil, continuous stripping, card fly separators, and automatic knockoff are claimed to aid in reduction of waste costs (8).
3. Addition of ingredients to cotton mix. Oil and static reducing chemicals are sometimes added to cotton to control certain kinds of waste losses (9).

9. Humidity control. Certain fluctuations in waste percentages are attributed to the humidity. One Georgia mill reported that at 70-75% relative humidity there was a noticeable reduction in waste over 50-55% humidity (in opening and picking). However, no precise amounts were given. Another mill stated that undesirable foreign matter is not removed in such high humidity (10).

10. Proper collection and segregation of waste. Care in collecting and handling waste through use of proper equipment and procedures will prevent high grade waste from becoming mixed with lower grade waste. Economies are to be realized through efficient handling methods. Proper segregation of waste will help insure that maximum income will be received from the sale of the waste.

There are less formal control methods, such as the experience of operators and supervisors which enables them to judge when waste is excessive. In mills which consider all waste as an unavoidable expense, this may be the only form of regulation.

**Objective of study.**—George Dockray, Barkley Meadows, and Leonard Smith (11) say in their excellent article on cotton waste:
Since present cotton manufacturing processes and normal cotton grades yield a certain amount of waste, the aim of waste reduction and control is to minimize the amount of spinnable fiber which is removed with the leaves, dust, notes, and other foreign matter. To accomplish this requires continuing effort by all mill personnel working according to a carefully planned program.

The last sentence of the preceding quotation spells out the need for a well-designed and implemented control system to minimize unnecessary waste costs on a continuing basis. A search of the literature revealed that no detailed attempt to present such an integrated system has been made.

This study had as its objective the design of a system to control, evaluate, and predict waste performance in a spinning mill or multi-mill organization.
CHAPTER II

SYMBOLIC DESCRIPTION OF WASTE PROBLEM

Mathematical Model of Material Flow

It was felt desirable to construct an analytical model of the spinning mill's material flow to facilitate a better understanding of the process. The word "material," as used here, refers to stock being processed through the mill. An attempt was made to include all factors affecting the weight flows of this material.

Figure 3 is a flow diagram of a typical carded yarn spinning mill and illustrates the movement of the cotton which eventually becomes yarn. Reworkable waste is shown because it eventually becomes product. The input is in baled cotton, the output in yarn. Output pounds, of course, do not equal input pounds. The reasons for this discrepancy are listed below:

1. Non-reworkable waste.
2. Unaccounted for (invisible) waste.
3. Moisture change.
4. Added material.
5. Errors in weighing, collecting, and otherwise accounting for the material movements.

Figure 4 shows the addition of the flow of non-rework-
able waste to Figure 3. A portion of the waste is sold and the remainder is disposed of.

To formulate a model of the weight flows encompassing the factors listed above, consider a spinning mill with a variable number (n) of distinct process steps. This is done to generalize the model, since the actual number of process steps may vary from mill to mill.

Let \( x_j \) be the number of pounds of good cotton leaving the \( j \)th process step to become the input to the \((j+1)\)st process step. Then,

\[
x_{j-1} - x_j = \text{net loss in weight during the } j \text{th step, or}
\]

\[
x_{j-1} - x_j = r_j + w_j + m_j + e_j - a_j,
\]

where \( r_j \) = reworkable waste produced in the \( j \)th process step in pounds,

\( w_j \) = accounted for non-reworkable waste produced in the \( j \)th process step in pounds,

\( u_j \) = unaccounted for (invisible) waste produced in the \( j \)th process step in pounds,

\( m_j \) = moisture change in pounds in \( j \)th process step,

\( a_j \) = materials added in pounds in \( j \)th process step, and

\( e_j \) = weighing, collecting, and accounting errors in the \( j \)th process step in pounds.

Consider a mill with \( n \) process steps as shown in Figure 5. Here \( x_0 \) denotes the pounds of baled cotton put into the
Figure 3. Flow of Cotton Which Eventually Becomes Yarn for a Typical Carded Yarn Mill.
Figure 4. Non-reworkable Waste Flow Added to the Flow of Usable Cotton in Figure 3.
process during the time period under consideration and $x_n$, the output in pounds of yarn. The total amount of non-reworkable waste would be

$$W = \sum_{j=1}^{n} w_j.$$ 

Similarly, the unaccounted for waste would be

$$U = \sum_{j=1}^{n} u_j;$$

the net moisture effect,

$$M = \sum_{j=1}^{n} m_j;$$

the materials added,

$$A = \sum_{j=1}^{n} a_j;$$

and the net effect of errors,

$$E = \sum_{j=1}^{n} e_j.$$ 

Therefore, when considering only the inputs and outputs of the whole process, the model of the system is

$$x_0 - x_n = W + U + M - A + E,$$  \hspace{1cm} (2a)
M = \sum m_j \quad A = \sum a_j

W = \sum w_j \quad U = \sum u_j \quad E = \sum e_j

Figure 5. Weight Inputs and Outputs to Spinning Process.

\begin{align*}
X_{i-1} & \quad \vdots \quad X_i \\
\text{Process Step} & \quad j=i
\end{align*}

m_i \quad a_i

\begin{align*}
r_i & \quad w_i & \quad u_i & \quad e_i
\end{align*}

Figure 6. Weight Inputs and Outputs to a Step in the Spinning Process.

R = \sum r_j

\begin{align*}
X_0 & \quad \vdots \quad X_1 \\
\text{Opening} & \quad (Step \ 1)
\end{align*}

m_1 \quad a_1

\begin{align*}
r_1 & \quad w_1 & \quad u_1 & \quad e_1
\end{align*}

Figure 7. Weight Inputs and Outputs to Opening Step Illustrating Reworkable Waste Input.
or

\[ x_0 - x_n = \sum w_j + \sum u_j \pm \sum m_j - \sum a_j \pm \sum e_j. \quad (2b) \]

The term corresponding to reworkable waste was not included since it is contained wholly within the process.

Note that the inputs to the spinning process are the pounds of baled cotton, the pounds of moisture gained, and the pounds of material added. The outputs are the pounds of finished yarn of acceptable quality from the nth process step; the pounds of non-reworkable waste and invisible waste made at the various steps in the system; and the pounds of moisture loss. The errors in accounting for the weight flows at each process step are shown as either an output or an input. This is merely a sign convention. Any error which would tend to make \((x_0 - x_n)\) larger would be positive and conversely, any error that would make \((x_0 - x_n)\) smaller would be given a negative sign.

To examine more closely the internal flows of the spinning process, consider the ith process step (i.e., \(j=i\)). It is shown diagrammatically in Figure 6.

The symbolic model for this process step's weight flow would be

\[ x_{i-1} - x_i = r_i + w_i + u_i \pm m_i - a_i \pm e_i. \quad (3) \]

Reworkable waste must be included when considering the individual process steps, since it is returned to the opening
process step for reworking. This calls for a slight modification to the above model for the opening process, because reworkable waste from steps \( j = 1, \ldots, n \) is an input as well as the raw cotton. See Figure 7.

The symbolic model representing the opening process step would be

\[
(x_0 + \sum_{j=1}^{n} r_j) - x_1 = r_1 + w_1 + u_1 \pm m_1 - a_1 \pm e_1. \tag{1}
\]

When all process steps are placed in order of occurrence and the weight flows are illustrated, Figure 8 is the result.

As a check on the correctness of the preceding formulation, the summing of the weight flows for each process step should result in Equation (2), the model for the whole process. The individual process step models are:

Step 1: \[(x_o + \sum r_j) - x_1 = r_1 + w_1 + u_1 \pm m_1 - a_1 \pm e_1,\]

Step 2: \[x_1 - x_2 = r_2 + w_2 + u_2 \pm m_2 - a_2 \pm e_2,\]

\[\vdots \]

Step n: \[x_{n-1} - x_n = r_n + w_n + u_n \pm m_n - a_n \pm e_n.\]

The sum of the above n equations is

\[
(x_0 + \sum r_j) - x_n = \sum r_j + \sum (w_j + u_j \pm m_j - a_j \pm e_j).
\]
Figure 8. Complete Diagram of Weight Flow Within a Textile Spinning Mill.
or

\[ x_0 - x_n = \sum v_j + \sum u_j + \sum n_j - \sum a_j + \sum e_j. \]

This equation is identical with Equation (2).

The above flow models illustrate where changes in process material weights occur, where waste is made, and where it goes. The equations account for the variation between measured input and measured output of a process step or of the entire process. The benefits to be gained from this formulation accrue from a better understanding of where certain parameters influence real and apparent waste losses. Furthermore, it provides a foundation upon which the economic analysis of the following section is based.

Economic Model of Waste Problem

Because the goals of a textile firm are primarily economic, the economics associated with the waste problem were studied. Initially, the various incomes and costs which affect or are affected by the amount of waste made were determined through analysis of the process, interviews with persons closely associated with the problem, and review of the literature. The economic parameters considered to be significant are:

1. Cost of raw material.
2. Cost of direct labor.
3. Distributed costs.
4. Cost of waste collecting, handling, sorting, and
selling which is proportional to the amount of waste made.

5. Cost associated with inadequate removal of foreign matter or excessive amounts of waste reworked; e.g., downtime caused by excessive ends-down in spinning, finished yarn rejected for quality reasons, customer goodwill.

6. Income derived from the sale of waste.

The possibility of reducing selling price and thereby perhaps increasing net income, if waste losses are substantially reduced, is not considered. To take this into consideration would severely complicate the model, since market conditions, described by appropriate variables and parameters, would have to be incorporated into any description of waste economies. It was felt that the ability to predict the effect of price changes on net income is not compatible with the ability to determine the values of the six parameters previously mentioned.

The costs associated with waste collecting, handling, sorting, storing, and selling are considered to consist of a fixed portion, virtually independent of the amount of waste made, and a variable portion, which varies directly with the waste poundage. It is this variable part that is referred to in the fourth parameter. The fixed part is placed in the category of distributed costs.

Distributed costs, often referred to as burden or overhead, consist of all costs of manufacture not specifically included in the other parameters. These costs may be dis-
tributed to various process steps through several methods, such as percent floor space, direct labor charges capital investment, etc. How they are allocated is not relevant to the problem, so long as the distribution is reasonable.

Poor quality costs of two types are considered. One type is caused by failure to remove enough of the foreign matter and short fibers from the stock. The second type accrues from reworking waste. In practice these costs may be difficult to evaluate.

Income derived from the sale of non-reworkable waste tends to offset the waste costs mentioned above and therefore should be taken into account.

By choosing appropriate symbols for these economic parameters and relating them to the model of material flow presented in the initial section of this chapter, an economic model of the waste losses of an n-process spinning mill is developed. The following symbolism is introduced in addition to that previously mentioned in this chapter.

Let $m =$ cost of raw cotton (in dollars per pound),

$L_j =$ direct labor cost at jth process step (in dollars),

$D_j =$ distributed cost to jth process step (in dollars),

$h =$ variable cost of waste collecting, handling, sorting, and selling (in dollars per pound),

$q_r =$ poor quality cost associated with reworking
waste (in dollars per pound of reworkable waste),

\( q_f \) = poor quality cost associated with failure to remove enough foreign matter and short fibers (in dollars per pound of finished product),

\( I \) = income from waste sold (in dollars),

\( V_j \) = total value of material in process at completion of \( j \)th process step (in dollars), and

\( v_j \) = value of material in process at completion of \( j \)th process step (in dollars per pound).

The following cost relations were derived for any arbitrarily chosen time period.

**Cost of Material Input to Opening Process.**

\[
\text{Material Input Cost (MIC)} = m x_0 + \sum v_{j-1} r_j + h \sum r_j
\]

This relation states that the cost of material placed in the openers is equal to the raw cotton cost, plus the value of the waste from the various process stages that is reworked, plus the cost of handling this reworked waste. The value of reworkable waste is defined here as the cost of processing the cotton to the process step preceding the one where it becomes waste. This is not exactly correct, but it is a satisfactory approximation. Also the possibility of some cotton being reworked twice is not considered, since in a mill making six percent reworkable waste only about 0.36 percent
would be reworked for the second time.

**Value of Material in Process at Completion of *ith* Process Step.**

\[ V_i = MIC + \sum_{j=1}^{i} (L_j + D_j) \]

\[ = m_x + \sum_{j=1}^{n} v_{j-1} \cdot r_j + h \sum_{j=1}^{n} r_j + \sum_{j=1}^{i} (L_j + D_j), \]

and

\[ v_i = \frac{V_i}{x_i}. \]

This relation states that the total value of material in process at the completion of the *ith* process step equals the material input cost plus the sum of the direct labor and distributed costs through the *ith* process. The value per pound is the total value divided by the number of pounds of output from the *ith* process step.

**Incremental Value Added by the *ith* Process Step.**

\[ V_{i-1} = MIC + \sum_{j=1}^{i-1} (L_j + D_j). \]

\[ V_i = MIC + \sum_{j=1}^{i} (L_j + D_j). \]

Value Added = \[ V_i - V_{i-1} = L_i + D_i. \]

Value Added Per Pound = \[ \frac{L_i + D_i}{x_i}. \]
As would be expected, the incremental value added by a process step to material processed during an arbitrarily chosen time period is the sum of the direct labor and distributed costs charged to that time period. Value added per pound would be calculated on the basis of incremental value per unit output.

Cost of Poor Quality.---

\[ \text{Total Cost of Poor Quality} = q_t x_n + q_r \sum_{j=1}^{n} r_j. \]

Of all the costs considered, this is the most difficult to obtain. The cost of failure to remove undesirable matter is assumed to vary linearly with the pounds of finished yarn produced. This assumption is based on the premise that ends-down in spinning, rejected product, and customer dissatisfaction vary linearly with the final output, \( x_n \). Similarly, it is assumed that the poor quality costs caused by reworking waste are proportional to the amount of waste reworked.

Net Return from Waste Sold.---

\[ \text{Net Return} = I - \sum v_{j-1} w_j + h \sum w_j. \]

Net return from the sale of waste is here defined as the total income from selling waste, less the value of the non-reworkable waste and the variable costs associated with collecting, handling, sorting, and selling all non-reworkable waste. The return may be negative.
Economic Loss Attributed to Waste.

\[
\text{Loss} = \sum v_{j-1}(r_j + w_j + u_j) + h \sum (r_j + w_j) + q_f x_n + q_r \sum r_j - I.
\]

This loss function states that the economic loss attributable to waste equals the cost of processing the cotton to the point where it becomes waste, plus the cost of collecting, handling, sorting, and selling the waste, plus the loss attributed to poor quality, less the income derived from the sale of waste.

The minimization of the above function is a desirable objective for any mill interested in optimizing its operations. However, a necessary step in using the above relations is the determination of the values of the parameters involved. Methods of accomplishing this objective are discussed in Chapter IV.
CHAPTER III

EFFECT OF VOLUME OF PRODUCTION ON WASTE

Because of widespread recognition by authorities in the textile industry, the effect of such factors as raw material blend, machinery, and the human element on waste were accepted in this investigation without study. However, during the course of this research, the question arose as to the effect of production volume on the amount of waste made. Naturally, as more cotton is processed, more waste is produced; but what is the nature of the relationship? If it were linear, as most waste standards assume, the ratio of the total waste to total production input for a time period would be a constant, regardless of the level of input. In other words, the same percent total waste (as a percentage of input) would be expected at a high level of throughput as at a lower level.

To investigate this effect, data were obtained from an actual spinning mill. These data consisted of thirteen weekly reports of production volume and waste made. During the weeks selected, the mill was processing a blend of approximately 83 percent Type "A" and 17 percent Type "B" cotton.* These weeks, when the mill was processing almost identical

* "A" and "B" have no significance other than to indicate two different grades of cotton.
blends of cotton, were selected in an attempt to control the
effect of the raw material grade on the amount of waste.
Production varied from 162,217 to 273,402 pounds per week.
Waste percentages ranged from 14.854 to 18.587 percent. The
data are displayed in Table 1.

Initially, a scatter diagram of waste in pounds versus
pounds of production input was made. See Figure 9. Superim-
posed on the diagram is a line of constant percent waste, with
the slope of the line equaling the total percent waste for the
thirteen week period. Inspection of this plot reveals that
five of the six points with production greater than 225,000
pounds per week fall below the line of constant percentage.
Conversely, for production less than 225,000 pounds per week,
six of the seven points fall above the line. This suggests
that for higher production levels the waste percentage is
smaller than at lower production levels.

Correlation Coefficient.—The next step was to determine the
degree of relationship between production level and waste per-
centages. This was done by computing the correlation coeffi-
cient between waste percentage and production input in thou-
sands of pounds. The formula,

\[ r = \frac{n \Sigma xy - \Sigma x \Sigma y}{\sqrt{n \Sigma x^2 - (\Sigma x)^2} \sqrt{n \Sigma y^2 - (\Sigma y)^2}} \]

where \( n \) = the number of weeks,
Table 1. Production and Waste Data from Spinning Mill Processing an Average Blend of 83.4 Percent Type "A" and 16.6 Percent Type "B" Cotton

<table>
<thead>
<tr>
<th>Week</th>
<th>Production Input (Pounds)</th>
<th>Total Waste (Pounds)</th>
<th>Total Waste (Percent of Input)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>162,217</td>
<td>30,152</td>
<td>18.687</td>
</tr>
<tr>
<td>2</td>
<td>273,402</td>
<td>43,648</td>
<td>15.978</td>
</tr>
<tr>
<td>3</td>
<td>238,123</td>
<td>41,424</td>
<td>17.397</td>
</tr>
<tr>
<td>4</td>
<td>269,247</td>
<td>41,728</td>
<td>15.318</td>
</tr>
<tr>
<td>5</td>
<td>220,667</td>
<td>39,669</td>
<td>17.977</td>
</tr>
<tr>
<td>6</td>
<td>210,543</td>
<td>35,618</td>
<td>16.917</td>
</tr>
<tr>
<td>7</td>
<td>232,551</td>
<td>34,524</td>
<td>14.854</td>
</tr>
<tr>
<td>8</td>
<td>246,148</td>
<td>38,837</td>
<td>15.752</td>
</tr>
<tr>
<td>9</td>
<td>217,374</td>
<td>38,383</td>
<td>17.658</td>
</tr>
<tr>
<td>10</td>
<td>205,335</td>
<td>35,645</td>
<td>17.360</td>
</tr>
<tr>
<td>11</td>
<td>206,357</td>
<td>36,082</td>
<td>17.405</td>
</tr>
<tr>
<td>12</td>
<td>213,583</td>
<td>32,011</td>
<td>14.988</td>
</tr>
<tr>
<td>13</td>
<td>245,393</td>
<td>33,047</td>
<td>13.468</td>
</tr>
<tr>
<td>Total</td>
<td>2,941,243</td>
<td>480,876</td>
<td>16.349</td>
</tr>
</tbody>
</table>
Figure 9. Waste Versus Production Input. (Data from Table 1.)
\( x_i \) = the weekly production level for the \( i \)th week,
\( y_i \) = the weekly waste made in percent of \( x_i \) for the
\( i \)th week,

and summation is understood to be over \( i = 1, \ldots, n \), gives
a correlation coefficient of \( r = -0.6009 \). (Calculations are
in the appendix.)

To test the significance of this correlation coefficient, certain normality conditions must be satisfied; i.e.,
the distribution of \( y \) for any \( x \) must be a normal distribution. While data are not available to test this requirement, it is
felt that the error in assuming the distribution to be normal
would not be large. Therefore, after establishing the null
hypothesis that no relationship between \( x \) and \( y \) exists (i.e.,
\( H_0: \rho = 0 \)), the alternate hypothesis (\( H_1: \rho \neq 0 \)), and the
desired level of significance, \( \alpha = 0.05 \), a table of percentiles
of the distribution of the correlation coefficient was
consulted (12). This showed the calculated \( r \) to be barely
significant at the five percent level. Although the null hy-
pothesis is rejected on this basis, it would be desirable to
have a larger sample size to insure that this is a correct
decision.

Regression Equation.--An investigation was made into the na-
ture of the relationship between \( x \) and \( y \) through the use of
regression analysis.

Assuming a linear relation, the regression model of \( y \)
on $x$ is

$$
\mu_{y|x} = A + B (x - \bar{x}) ,
$$

where $\mu_{y|x}$ is the mean of the distribution of $y$'s for any given $x$, and $A$ and $B$ are parameters defining the relationship. The problem is one of parametric estimation.

Under the assumption that $\sigma^2_{y|x}$, the variance of the distribution of $y$'s for a given $x$, is constant for all $x$'s, the maximum-likelihood estimators for $A$ and $B$ are defined as

$$
\hat{A} = a = \bar{y} \quad \text{and} \quad \hat{B} = b = \frac{\Sigma xy - n\bar{xy}}{\Sigma x^2 - n\bar{x}^2} .
$$

Let $y'_x$ denote the estimate of $y$ when $x$ is given (i.e., $\mu_{y|x}$). Then

$$
y'_x = \bar{y} + b(x - \bar{x}) .
$$

The following regression equation was calculated using the data in Table 1. (See Appendix for calculations.)

$$
y'_x = 23.397 - 0.0302 x .
$$

Obviously this relation is defined only for $x > 0$, because negative production is impossible and with $x = 0$, $y = 0$ also. However, when $x = 0$, $y'_x = 23.397$, which cannot be true; therefore, the function ceases to be linear for some small value of $x$ below the range of values investigated.
The unbiased estimator of $\sigma^2_{y \cdot x}$ is $s^2_{y \cdot x}$, and it is defined by the following equation:

$$s^2_{y \cdot x} = \frac{n - 1}{n - 2} (s^2_y - b^2 s^2_x)$$

where $s^2_x = \text{variance of the observed } x$, and $s^2_y = \text{variance of the observed } y$.

As calculated in the Appendix, $s^2_{y \cdot x} = 0.1555$, and $s_{y \cdot x} = 0.395$.

Under the assumption that the distribution of $y$ for any given $x$ is a normal one, it is possible to indicate by confidence intervals estimates of $B$ and $\sigma^2_{y \cdot x}$.

The 95 percent confidence interval for $B$ is given by

$$b + t_{.025} \frac{s_{y \cdot x}}{s_x \sqrt{n - 1}} < B < b + t_{.975} \frac{s_{y \cdot x}}{s_x \sqrt{n - 1}}$$

which computed is $0.0277 < B < 0.0327$.

The 95 percent confidence interval for $\sigma_{y \cdot x}$ is given by

$$\sqrt{\chi^2_{.025}} \sqrt{(n - 2) s^2_{y \cdot x}} > \sigma_{y \cdot x} > \sqrt{\chi^2_{.975}} \sqrt{(n - 2) s^2_{y \cdot x}}$$

which computed is $0.670 > \sigma_{y \cdot x} > 0.230$.
Test for Independence of Production Level and Waste Percent.—
One criterion for the independence of \( x \) and \( y \) is that the
mean \( y \) be the same for each \( x \), or in other words that the
average percent waste be the same for each level of produc-
tion. With reference to the linear regression model, this
means that \( B = 0 \). The test for the hypothesis \( B = 0 \) is given
below (13).

1. \( H_0: B = 0 \); \( H_1: B \neq 0 \).
2. Choose \( \alpha = 0.05 \).
3. As a test statistic, use
   \[
   t = \frac{(b - 0)}{s_{y.x} \sqrt{n - 1}} = 23.560
   \]
4. If the distribution of \( y \) for each \( x \) is normal
   with the same mean \( (\mu_{y.x} = \mu) \), then the sampling
distribution of this statistic is a \( t \) distribution with \( n - 2 = 11 \) degrees of freedom.
5. The critical region is \( t < -2.20 \) or \( t > 2.20 \), (14).
6. Here \( t = 23.560 \), which is larger than 2.20, and
   so there is sufficient reason to say that, at the
   5.0 percent level, \( y \) is dependent on \( x \).

Testing Reliability of the Regression Model.—The data from
which the model,

\[
y'_x = 23.397 - 0.0302 x
\]
was derived are plotted on a scatter diagram in Figure 10.
The regression line is drawn through the points.

To test the reliability of this regression equation, eight weeks additional data were obtained from the same mill. The production input in pounds and the total waste made in pounds for each week, plus total waste as a percent of the input, are recorded in Table 2. A scatter diagram of input versus waste percentage is shown in Figure 11, with the previously calculated regression line drawn to indicate the reliability with which the curve predicts actual performance.

The actual error incurred through use of the regression estimate was calculated and the results displayed in Table 3. The average weekly input for the eight week period was 233,178 pounds and the average weekly waste was 37,376 pounds. The regression estimate of the average weekly waste, based on the stated input, was calculated to be 38,136 pounds, or 760 pounds above the actual average for the period. The percent error of the regression estimate from the actual is 2.03 percent.

It should be pointed out that the two sets of data, upon which the preceding analysis was based, were obtained from periods where the mill was processing different blends of cotton. For the thirteen week period covered by the first set of data (Table 1), the mill processed a blend of two grades of cotton, say "A" and "B", consisting of approximately 83 percent Type "A" and 17 percent Type "B". However, other available data were for different blends, requir-
Figure 10. Percent Waste Versus Weekly Production Input. (Data from Table 1.)
Table 2. Production and Waste Data from Spinning Mill Processing an Average Blend of 61.4 Percent Type "A" and 38.6 Percent Type "C" Cotton

<table>
<thead>
<tr>
<th>Week</th>
<th>Production Input (Pounds)</th>
<th>Total Waste (Pounds)</th>
<th>Total Waste (Percent of Input)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>240,289</td>
<td>38,525</td>
<td>16.033</td>
</tr>
<tr>
<td>2</td>
<td>243,065</td>
<td>37,735</td>
<td>15.524</td>
</tr>
<tr>
<td>3</td>
<td>197,800</td>
<td>35,146</td>
<td>17.920</td>
</tr>
<tr>
<td>4</td>
<td>238,657</td>
<td>36,474</td>
<td>15.283</td>
</tr>
<tr>
<td>5</td>
<td>211,949</td>
<td>32,763</td>
<td>15.453</td>
</tr>
<tr>
<td>6</td>
<td>199,089</td>
<td>36,253</td>
<td>18.210</td>
</tr>
<tr>
<td>7</td>
<td>258,549</td>
<td>41,021</td>
<td>15.366</td>
</tr>
<tr>
<td>8</td>
<td>276,022</td>
<td>40,793</td>
<td>14.779</td>
</tr>
<tr>
<td>Total</td>
<td>1,065,420</td>
<td>299,010</td>
<td>16.029</td>
</tr>
</tbody>
</table>
Figure H. Percent Waste Versus Weekly Production Input. (Data from Table 2.)

\[ y = 23.44 - 0.03x \]
Table 3. Error Incurred Through Use of the Regression Equation to Estimate Waste Pounds from a Given Production Volume

<table>
<thead>
<tr>
<th>Week</th>
<th>Production* (Pounds)</th>
<th>Actual Waste* (Pounds)</th>
<th>Theoretical Waste** (Pounds)</th>
<th>Error Pounds</th>
<th>Error Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>240,289</td>
<td>38,525</td>
<td>38,785</td>
<td>260</td>
<td>0.67</td>
</tr>
<tr>
<td>2.</td>
<td>243,065</td>
<td>37,735</td>
<td>39,027</td>
<td>1292</td>
<td>3.42</td>
</tr>
<tr>
<td>3.</td>
<td>197,800</td>
<td>35,166</td>
<td>34,463</td>
<td>-983</td>
<td>-2.77</td>
</tr>
<tr>
<td>4.</td>
<td>238,657</td>
<td>36,474</td>
<td>38,639</td>
<td>2165</td>
<td>5.96</td>
</tr>
<tr>
<td>5.</td>
<td>211,949</td>
<td>32,763</td>
<td>36,023</td>
<td>3260</td>
<td>9.95</td>
</tr>
<tr>
<td>6.</td>
<td>199,039</td>
<td>36,253</td>
<td>34,613</td>
<td>-1640</td>
<td>-4.52</td>
</tr>
<tr>
<td>7.</td>
<td>258,549</td>
<td>41,021</td>
<td>40,291</td>
<td>-730</td>
<td>-1.78</td>
</tr>
<tr>
<td>8.</td>
<td>276,022</td>
<td>40,793</td>
<td>41,572</td>
<td>779</td>
<td>1.91</td>
</tr>
<tr>
<td>Ave.</td>
<td>233,178</td>
<td>37,376</td>
<td>38,136</td>
<td>760</td>
<td>2.03</td>
</tr>
</tbody>
</table>

* From Table 2.

**Waste Pounds (from regression equation) =

\[
10 \cdot y'_{x} = 10 \left(23.397 x - 0.0302 x^2\right),
\]

where \( x \) = production input in thousands of pounds, and \( y'_{x} \) = regression estimate of waste percentage for the given \( x \).
ing selection of a period when the mill was running a similar, but not identical, blend. Therefore, the second group of data (Table 2) was selected from a period when the mill was processing approximately 61 percent Type "A" and 39 percent Type "C" cotton.

A test was made on the two sets of data to ascertain if both came from populations having the same mean waste percentage. See the Appendix for details of the test. The results of the test can be stated in terms of the following 99 percent confidence interval for the difference in population means:

$$0.00232 < p_1 - p_2 < 0.00419,$$

where

- $p_1$ = percent waste of population from which the first group of data was drawn, and
- $p_2$ = percent waste of population from which the second group of data was drawn.

Since the interval does not include zero, the null hypothesis, $p_1 = p_2$, would be rejected.

**Interpretation and Conclusions.**—While the above analysis is not as complete as could be desired, it is sufficient in so far as the objective of the study is concerned. Two inferences may be drawn from the above discussion:

1. The expected percentage of total waste to mill input is not constant for all levels of input, and
2. There is a significant difference in expected waste percentage because of variations in the blend of raw stock.

The correlation coefficient between input and waste percent was calculated to be \(-0.601\). Although barely significant because only thirteen weeks of data were used, this coefficient indicates that a relationship does exist between input volume and waste percent. Furthermore, the negative sign attached to the coefficient shows that this relationship is negative; that is, as mill input increases, percent waste decreases. Assuming the relationship to be linear, the equation,

\[
y' = 23.397 - 0.0302x,
\]

was calculated using the principle of least squares. This means that an increase of 1000 pounds per week in the mill's input would result in a decrease in percent waste of 0.0302 percent. It should be pointed out that this relationship is valid only in the region of production encompassed by the data.

When tested by using data for eight additional weeks, the regression equation predicted the total waste pounds with an average error of 2.03 percent from the actual, even though the mill was processing a different blend of cotton when the test data were obtained. Statistical analysis showed this blend to result in a slightly lower waste percent than the original blend, which partially accounts for the average estimated waste being higher than the average actual waste.
CHAPTER IV

ELEMENTS OF A WASTE CONTROL SYSTEM

Control systems.--The word "control" is defined as the exercise of restraint or direction over, or the holding in check or curbing, and "system" is defined as any formulated, regular or special method of procedure (15). Thus a control system can be defined as any formulated, regular, or special method or plan of procedure designed to exercise restraint or direction over some activity.

While a wide variety of control systems have been designed (e.g., radar-controlled guns, inventory control systems, cost controls, automatic pilots, and gyroscopes), they all have three common functions--measurement, evaluation, and correction (see Figure 12).

Measurement concerns the monitoring of the activity being regulated and its associated sources to collect data pertinent to the evaluation function.

Evaluation is the analysis of the collected data to determine if the activity is being performed satisfactorily, and, if not, what corrective action should be taken.

Correction involves the implementation of actions selected by the evaluation function.

As an illustration of the above functions, consider
Figure 12. Functional Block Diagram of a Control System.

Figure 13. Automobile Speed Control System.
the activity of regulation of the speed of an automobile. Here the speedometer performs the function of measuring the speed of the automobile. The driver, thus informed, evaluates his present speed through consideration of such factors as speed limits, road conditions, traffic congestion, stop signals, his mental and physical alertness, and any other pertinent factors. If he is satisfied with his present speed, he takes no action; if not, he has several corrective measures at his disposal--altering the rate of gas flow to the engine or putting on brakes. The correction function would be the increase or decrease in pressure on the accelerator or the depression of the brakes. This simple control system is illustrated in Figure 13.

The preceding discussion can be extended to the problem of controlling cotton waste in a spinning mill. However, before doing so, the concept of the "measure of effectiveness" must be introduced.

Measure of effectiveness.--Essentially, the measure of effectiveness is the criterion by which the performance of the activity under control is to be judged. Goode and Mandel (16), writing on the subject of measures of effectiveness in system design, point out many desirable characteristics of such measures. Several of these characteristics, slightly modified to fit the control system problem, are listed here:

1. It should measure the true effectiveness of the
Evaluation of data and action decisions.

Implementation of corrective action.

Collection of appropriate data.

TEXTILE PROCESS WASTE

Figure 11. Simple Functional Diagram of Textile Waste Control System.
activity being controlled and thereby be in harmony with the long-range goals of the activity.

2. It should be quantitative.

3. It should be obtainable with reasonable accuracy and without excessive cost or delay. Note that if the measure were statistically efficient (i.e., small variance), it would tend to have this quality.

4. It should be complete, telling the whole story.

5. It should be simple, where this is compatible with completeness.

6. It should have physical meaning, if possible.

These authors further state (17) that it is not necessary for the same measure to be applied to the overall system and to each of its subsystems. Thus, both system-wide and local measures of effectiveness may be developed. However, the writers inject a word of caution against exclusive use of local measures; "...the overall system must not be ignored ...because a local optimum may not yield the best performance systemwise."

In the example of controlling the speed of an automobile, the truest measure of effectiveness might be the avoidance of any accident. However, this measure is not quantitatively determined in a short period of time. This is the reason that speed limits are established and speedometers are placed in automobiles. Conformance to speed limits (tempered by weather, road, traffic, and driver conditions) is used as a
measure of effectiveness of the driver-automobile combination. It is obvious that the measure (or measures) of effectiveness must be chosen before the control system (or subsystems) can be efficiently designed.

The assumption that the long-range goal of a textile organization is to maximize the return on the owners' investment would probably be a reasonable one. In such a case all activities of this organization should work toward that goal and the effectiveness of these activities should be measured in terms of their contribution toward achieving it. This definitely applies to any waste control system that might be established. To say that the waste control program can best contribute to the enterprise by minimizing cotton waste and to measure the effectiveness of the system in that light is not correct. It has been pointed out that failure to remove enough foreign material and short fibers will adversely affect the quality characteristics of the yarn. This could result in increased downtime because of breaks in spinning ("ends-down"), excessive rejection of product, loss of customer goodwill, etc. These results would tend to offset the savings resulting from any excessive reduction in waste made. Obviously some sort of balance between amount of waste and quality of product is desirable. This balance must be economic, since the goal of the enterprise is an economic goal.

The most common measure used in present waste control systems is percent waste. Percent waste may be calculated
as pounds of waste per pound of mill input or as pounds of waste within a process step per pound of input to that process step. Waste standards, previously mentioned in Chapter I, are maintained by some mills for comparison with the actual waste percentages to obtain an indication of the effectiveness of the system. However, the use of an overall mill waste percentage as a measure of effectiveness of that mill's waste performance has shortcomings. Examination of the economic loss function developed in Chapter II reveals that even though the overall mill waste percentage remains constant for two consecutive periods of time, the economic loss incurred may vary. This is partially explained by the fact that waste made at the initial process steps has less value than that made at later process stages. Therefore, a shift in the location of waste made from the earlier to the later process steps could cause the economic loss to increase and yet the same amount of waste would be obtained. The same shift could have an effect on the income from waste sold, handling costs, and quality losses. On the other hand, the evaluation of waste performance of an individual process step through comparison of an actual percent waste with a standard percent, where waste is calculated as a percentage of input to that process step, is logically and economically sound.

The difficulty involved in establishing a reliable and accurate quantity with which to measure the effectiveness of waste performance may be resolved through use of both a sys-
The system-wide measure would be the dollar loss attributed to waste (as defined in Chapter II). It would consider the mill as a whole and would be used to evaluate the waste performance of that mill.

The local measure would apply to each process step. It would be the total waste for the process step divided by the total input to that step, which would give the waste made as a percent of material processed by that particular step. Dollar loss could be used in this case as well as for the system-wide measure, but percent waste would be easier and cheaper to obtain.

The reasons behind the above proposals are logical. It was previously submitted that the goal of the enterprise is economic, that the measure of effectiveness of any control system should be in harmony with the objectives of the enterprise, that percent waste for the system does not give a true picture of the economic losses caused by improper amounts of waste, and that an approximate function could be established to represent this economic loss. Therefore, considering the mill as an economic entity and evaluating its waste performance on a dollar-loss basis would be a more meaningful measure than use of waste percentages. The proposed measure is quantitative and complete in the sense that it takes into account the pertinent parameters of performance. Questionable characteristics are the cost, accuracy, and delay involved
with the collection of data to evaluate the effectiveness function.

When considering the effectiveness of an individual process step, one encounters a different situation. Here the value of waste made is constant and variations in value added per pound would be caused primarily by fluctuations in the level of input to the process step. In addition, the optimizing of waste losses within each process step might sub-optimize the overall process economics, because of the inter-relations between the process steps. Hence, the use of percent waste as a criterion for evaluating the effectiveness of the individual process steps does not have the disadvantages that it does when applied to the whole mill.

Note that a distinction is made between "system-wide" and "local" control. Thus the control of waste from the overall mill viewpoint would be different from the control of a process step.

An extension of the above discussion to include multi-mill operations is relatively simple. The evaluation would be accomplished through an analysis of the sum of the dollar waste losses of the various mills.

Once the measures of effectiveness are decided, there are four phases of the control system remaining to be solved. These phases are measurement, evaluation, correction, and communication.
Measurement.—Measurement involves the collection of data pertinent to the waste control problem. The purposes of data gathering are to measure waste performance in terms of the chosen measure(s) of effectiveness and to allow detailed analysis of the system to determine methods of optimizing waste losses. The measurement function provides information required in the evaluation function. Since indiscriminate data collection is wasteful, the types of data desired and methods of collection and processing should be carefully designed. In present practice, information requirements are, in general, determined by the type of evaluation used. However, evaluation methods may change periodically or special analyses may need to be conducted which require data not needed on a routine basis. Therefore, to insure that proper historical information is available, there are certain data which should always be maintained. The literature search and discussions with persons in the textile industry revealed that types of data that would be of considerable value in analyzing certain facets of the waste problem are not being collected and processed. In most cases, the additional effort involved in obtaining this data would be negligible.

Information pertinent to the evaluation function may be classified into the categories of production data, waste data and supplementary data.

Production data for any time period under consideration would consist of such factors as the following:
1. The pounds of raw cotton opened by grade or type.
2. The pounds of input to and output from each process step.
3. The in-process inventories at the end of the time period (and therefore at the start of a new time period).
4. The amount and location of any material, other than cotton, added to the process.
5. Direct labor charges by process step.
6. Distributed costs to each process step.
7. Amount and lost value of final product rejected either by final inspection or by customers.

Waste data would consist of such factors as the following:

1. Pounds of waste made at each process step, classified by the type of waste; e.g., card room, flat strips.
2. Pounds of waste reworked, identifying it by the process step from which it came.
3. Variable costs of handling and selling waste.
4. Income from waste sold.

Supplementary data would contain such items as these:

1. Time period covered by the data.
2. Identification of production and waste data by date, shift and mill.
3. Results of periodic checks of scales, meters,
counters, etc., from which production and waste data are obtained. This is to eliminate systematic errors caused by inaccurate instruments.

4. Results of special experiments, such as double or triple reweighing and reaccounting for production and waste poundages in order that the errors involved may be statistically estimated.

5. Any unusual occurrence or situation concerning the waste problem which would aid in evaluation of performance.

6. If standards are used, information about product types, routings through the mill, etc., will be needed.

7. Downtime or other production losses attributable to improper waste removal or excess reworked waste.

The above lists are not necessarily complete, but they do point out the variety of information available for collection which might be profitably put to use. The prime shortcomings observed in the data now being collected by mills practicing waste control are the lack of appropriate identifying information and the questionable accuracy of the data collected. Both of these factors severely restrict the application of statistical techniques to analyze the data.

Since the cost of a waste control program is charged against any savings resulting from such control, it is desir-
able to practice economies in all phases of the control system. The methods of measuring and recording needed data should be designed with this thought in mind. Mills which have access to punched card tabulating equipment might consider the use of this equipment, employing such devices as prepunched cards and mark-sensed cards for data collection purposes. Flow charting and forms design should be employed in all cases. This aspect will be discussed at the end of this chapter under the subject of communications.

**Evaluation.**—The evaluation function is most critical in the successful operation of the control loop. The design of measurement procedures and the proper choice of corrective actions depend largely upon the methods of evaluation used. Evaluation essentially involves comparing actual performance with expected or standard performance, where performance is measured in terms of the chosen measure of effectiveness.

The initial problem in establishing an evaluation function is the determination of standard performance. In Chapter I, three methods of establishing waste standards were mentioned. They were the analysis of past data, waste tests, and calculation.

In articles on waste standards by calculation, formulas for such calculations are presented which contain terms that are to be arbitrarily estimated. In addition, the influences of raw material blend, production volume, and ma-
chinery variations are not taken into consideration. This method therefore leaves several things to be desired.

Waste tests, on the other hand, are conducted in such a manner that raw material blend and the human elements are controlled. Furthermore, the error involved in accounting for the waste made is minimized, and, since the test is conducted on a specific set of machines to establish standards for those machines, the variation from machine set to machine set is not relevant. Disadvantages of waste tests are the relatively small sample (input volume) upon which the standards are established, the cost of conducting the tests, and the fact that the test covers a relatively small period of time when the mill may not be operating at normal conditions.

The analysis of past production data and waste data to determine standards of performance has been seriously handicapped by the lack of identifying information such as production shifts, dates, raw material blend, and inputs to individual process steps. Thus large quantities of data routinely collected are virtually useless for want of a small amount of additional facts. To determine standards from historical data, it is necessary that the amount of waste variation caused by the human element and the expected weighing and accounting errors be estimated statistically. Thus, the reason for identifying data by shifts is to allow statistical analysis of past data to isolate the effect of different shifts on waste. The error involved in accounting for production and
waste volume could be statistically estimated if the weighing and accounting in a particular time period were replicated. Since any variation in waste (for a particular mill) attributed to the operators or to error is unnecessary, this variation may be eliminated from the total waste to give an estimate of the standard waste for any product mix and raw material blend. The effect of the level of production of the mill on waste should be more completely analyzed than was done in Chapter III and included in establishing the waste standards.

Assuming that waste standards for a particular combination of raw material blend, product mix, and production level are available, the problem of the level of evaluation arises. Two levels are considered here: local evaluation and system-wide evaluation.

Local evaluation concerns the analysis of waste performance within a process step. In essence, the effectiveness of the process step with regard to waste is evaluated on the basis of waste made as a percent of input to that process step, i.e. the local measure of effectiveness previously proposed in this chapter. The actual waste percent at a process step is compared with a standard waste percent based upon the level of input, the type of stock being processed, and the desired quality characteristics of the final product.

The purpose of this type of evaluation is to provide
local control within a process step, or department, of a mill. Then the operators and their supervision have some immediate indication when the waste level goes out of control, allowing them to initiate corrective action immediately.

Obviously, the evaluation process on the local level must be simple, quick, and cheap, since this function is performed by the operating personnel of the process step. Therefore, full advantage should be taken of tables, charts, nomographs, etc., when providing each process step with a set of standards against which actual performance will be compared. Control chart techniques may be used to plot actual waste versus standard waste (both in percent) and control limits may be established to provide a decision rule concerning out-of-control points. Runs should be watched for since such a system is likely to creep out of control. Checks on performance should be conducted frequently, say each day.

Assuming that the economic loss function developed in Chapter II is used as the measure of effectiveness of the mill, an initial step must be the determination of the values of parameters in the equation. Having estimated these parameters and utilizing waste standards, mill supervision can calculate a standard waste loss for any time period. Actual waste losses could then be obtained and compared with the standard to evaluate performance in terms of the chosen measure of effectiveness.

If performance is judged to be satisfactory, no action
will be taken. However, if it is not satisfactory, corrective action will be required. As an aid in selecting the proper course of action, the available data should be analyzed to attempt to determine the cause of the substandard performance.

While not in the field of routine control, there are other uses for the data available in the evaluation function. By using the techniques of industrial experimentation, special analyses may be conducted to determine, for example, the relationship between money spent on preventive maintenance and changes in waste losses, the effect of different shifts on waste made, how waste affects the quality, or the relationship of money spent on waste control measures to the resulting economic gain. Similarly, past data may be used to develop a model for predicting waste losses as an aid to future planning or cost estimation. The simplest prediction tool would be a regression equation relating waste to production input. Such an equation was developed in Chapter III and, although based on a relatively small time period of thirteen weeks, it predicted waste pounds for eight more weeks with an average error of two percent. This model takes into account factors such as machines, human elements, and production level. By proper selection of development data, it can be made to account for raw material differences. That is, a model could be developed over a time period when only a particular raw material blend was being processed.
Correction.—Once the effectiveness of the systems' performance has been evaluated and found to be substandard, corrective action is required. For this purpose, several courses of action are available to mill management. For example, they may elect to take one or more of the following steps:

1. Inspect or adjust machine settings.
2. Increase preventive maintenance on equipment.
3. Change raw material blend.
4. Consider replacement of present equipment with machines having more economical operating characteristics.
5. Re-emphasize to supervision the importance of proper control of waste.
6. Review or modify existing operating procedures.
7. Instigate a program to train employees in proper operating procedures.
8. Improve waste handling procedures and methods.
9. Transfer or otherwise replace operating personnel.
10. Modify the amount of waste reworked.
11. Initiate a campaign to reduce waste similar to a safety campaign.

The proper selection of the corrective action(s) must be based upon the results of the evaluation function and tempered by the experience of management. Once selected, corrective action should be properly implemented under continuous supervision of responsible persons. The effects of various
actions should be observed and recorded for future reference.

The above discussion is not intended to minimize the importance of the continuing type of correction, such as a continuing waste campaign involving competition between departments, shifts, or mills, or a method whereby waste performance is included as a factor in an incentive plan. These preventive actions are desirable, as evidenced in the field of safety, where preventive safety programs have been shown to be more effective than sporadic after-the-fact efforts.

The process of corrective action is understandably less well defined than the functions of measurement and evaluation. Modern textile mills spend considerable money and effort to collect information on waste made by establishing waste standards and by continuously reporting actual waste percentages for comparison with these standards. The procedure to this point in the control loop is fairly well defined; however, once waste performance has been reported, the situation clouds considerably. Corrective action may be non-existent or stringent corrective measures may be initiated in the case of abnormally high waste figures. The degree of action to be taken in any given situation is a problem which remains amenable only to heuristic methods.

Abnormally low waste may be extremely satisfying to all concerned, but rarely does it stimulate an investigation into the reasons for the sudden change. As pointed out many times before, failure to remove the necessary amount of waste may
affect the quality characteristics of the final product; therefore, it is essential that excessively low waste be subject to corrective action as well as excessively high waste. If the economic loss function is used as the criterion of performance, waste, too high or too low, will show up as an excessive waste loss for the period. Investigation of low waste losses may uncover methods of maintaining this desirable performance.

Communication.—The three activities of making measurements on the system, evaluating the data, and correcting the system, must be linked by an efficient communications system if the control program is to function properly. Essentially, a complete communications system consists of a source, transmitter, channel, receiver, and destination. The system is acted upon by disturbing elements which are referred to as noise.

The source of any communication refers to the originator of the message. The destination is the person or place for which the message is intended. The transmitter, channel, and receiver are devoted to transporting the message from source to destination. The device or equipment which enables the source to put its message into the communications channel is called the transmitter, and the device or equipment which allows the destination to obtain the message from the channel is known as the receiver. Noise, acting on the transmitter,
channel, and receiver, tends to distort the message being transmitted so that the destination does not correctly obtain all of the information sent by the source. From this macroscopic viewpoint, any communications system may be represented by the diagram in Figure 15.

The design of the communications portion of a waste control system involves the determination of the composition of each of the segments shown in Figure 15. The type and quantity of information to be transmitted are established by determining the information requirements at the various functions of the control loop and at the various levels of the organization. In effect, this also determines the destinations involved. The next step comprises the identification of points in the organization where the required information is to be originated or obtained. These are the sources. The methods by which the sources pass the data on to the destinations are decided upon to complete the communications design. In this last step it is important to minimize the distortion of the message while in transmission. This can be done through proper forms design, avoidance of transmitting too much information at one time, segregation of important from unimportant information, proper identification of the data being transmitted, and minimization of errors.

Lines of responsibility and authority for the collection, transmission, and use of the information should be clearly established. This can be facilitated through the use
Figure 15. Schematic Representation of a Communications System.
of an information flow diagram, similar to an organization chart, on which the frequencies, origin, destination, and method of communication of all information are shown.

The importance of a well-designed and operated communications system should not be minimized, since the accurate and rapid transmission of information around the control loop is imperative to the success of the waste control program.
CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

Conclusions.—The results of this study are summarized in the following conclusions:

1. The potential economic gain to be realized from the control of cotton waste justifies some form of waste control program in every mill.

2. The appropriate measure of effectiveness for mill waste performance is the dollar loss caused by waste, rather than waste percent. Any function representing this loss must include such factors as the value of the waste made, the variable costs of collecting, handling, storing, sorting, and selling waste, the poor quality cost caused by failure to remove enough undesirable material from the stock, the poor quality cost attributed to reworking waste, and the income derived from the sale of waste.

3. The objective of a waste control program is to maximize the difference between the losses averted by the program and the cost of the program. Stated symbolically, the objective is

\[
\text{MAX} \left\{ \left( (W.L.)_o - (W.L.)_c \right) - C \right\},
\]

where \((W.L.)_o\) = the dollar loss attributed to waste without
a waste control program,

\[(W.L.)_c = \text{the dollar loss attributed to waste with a waste control program, and}
\]

\[C = \text{the cost of operating the waste control program.}\]

4. The problem of establishing a program to minimize losses caused by waste is amenable to standard methods of system design. In this case a control system is required which measures the waste performance of the mill, evaluates this performance in terms of the selected measure of effectiveness, and, if necessary, selects the proper corrective action. The elements of the control loop are linked by an effective communications network. The crux of the problem lies in the design of these functions of measurement, evaluation, correction, and communication.

5. In Chapter III, the effect of the level of input to the mill on the amount of waste made was studied. The results indicated that pounds of waste was not proportional to pounds of input to the mill. The expected percentage of waste was not constant at all levels of input, but decreased as mill input increased. Therefore, it is concluded that the level of production is a significant factor influencing variations in the amount of waste made. This conclusion, however, was based on historical data of unknown accuracy and for one particular mill. If the same result were obtained for a mill under study, the linear regression equation of
waste percentage on the production input would be a valuable tool for aiding in the prediction of waste losses for that particular mill.

6. In the mill studied, a small difference in the blend of raw stock processed had a significant effect on the amount of waste made.

7. There is a need for experimentation and analysis to determine the exact nature of the parameters and relationships affecting the waste problem.

8. The prime deterrents to the use of available historical data on waste for the above analyses are its incompleteness and unknown accuracy.

Recommendations for Future Study.—Very valuable contributions to the textile industry could be made through investigations into the exact nature of the relationships existing between the waste variables and parameters outlined in this study. For example, the effect of the amount of waste removed on roving and yarn breaks in spinning could be studied to determine the relationship present. The effect of the amount of waste reworked would also be included in the investigation. Many other similar studies exist.

Research could also be conducted on each of the functional stages of a waste control system. Optimum methods of obtaining information from the textile process could be developed for the measurement function. Methods of evaluating this data at minimum cost to obtain maximum knowledge of the
waste problem could be profitably studied. The effects of various types of corrective action might be analyzed. Information theory could be applied to the design of the communications network through which the waste information flows.

The problems associated with cotton waste are many and their scope wide. There is a great opportunity to understand more about methods of reducing the economic losses resulting from such waste, if only the techniques of scientific investigation will be further applied.
APPENDIX
APPENDIX I

CORRELATION AND REGRESSION CALCULATIONS

FOR ANALYSIS OF DATA IN TABLE 1

Symbols Used.--

\( x \) = weekly production input in thousands of pounds.
\( y \) = weekly total waste as a percentage of \( x \).
\( \mu_{y|x} \) = mean of distribution of \( y \)'s for a given \( x \).
\( \sigma^2_{y|x} \) = variance of distribution of \( y \)'s for a given \( x \).
\( \sigma^2_x \) = variance of distribution of \( x \)'s.
\( \sigma^2_y \) = variance of distribution of \( y \)'s.
\( s^2_{y|x} \) = unbiased estimator of \( \sigma^2_{y|x} \).
\( s^2_x \) = unbiased estimator of \( \sigma^2_x \).
\( s^2_y \) = unbiased estimator of \( \sigma^2_y \).
\( A, B \) = parameters of regression equation.
\( \bar{x}, \bar{y} \) = mean of observed values of \( x \) and \( y \), respectively.
\( y' \) = value of total waste percent estimated by regression equation.
\( a, b \) = maximum likelihood estimators of \( A \) and \( B \).
\( r \) = correlation coefficient.
\( n \) = number of weeks

Data from Table 1.--

\( n = 13 \)
\[ \sum x = 2941.246 \]
\[ (\sum x)^2 = 8,650,928.803 \]
\[ \sum x^2 = 675,933.725 \]
\[ \bar{x} = 226.249 \]
\[ \sum xy = 48,088.500 \]
\[ \sum y = 213.946 \]
\[ (\sum y)^2 = 45,772.391 \]
\[ \sum y^2 = 3547.509 \]
\[ \bar{y} = 16.457 \]

**Correlation Coefficient.**

\[
\begin{align*}
r &= \frac{n \sum xy - \sum x \sum y}{\sqrt{\left[ n \sum x^2 - (\sum x)^2 \right] \left[ n \sum y^2 - (\sum y)^2 \right]}} \\
&= \frac{625,150.5 - 629,267.8}{(136,210.1) (3144.726)} \\
&= -0.6009
\end{align*}
\]

**Regression Equation (of y on x).**

\[ y' = \bar{y} + b (x - \bar{x}) , \]

where \[ b = \frac{\sum xy - n \bar{y} \bar{x}}{\sum x^2 - n \bar{x}^2} = \frac{-316.72}{10,477.67} \]
\[ b = -0.0302 \]
\[ y' = 16.457 - 0.0302(x - 226.249) \]
\[ y' = 23.397 - 0.0302 \times \]

**Standard Error of Estimate.**

\[
s^2_{y|x} = \frac{n - 1}{n - 2} (s^2_y - b^2 s^2_x)
\]

where \[ s^2_x = \frac{\sum x^2 - n \bar{x}^2}{n - 1} = \frac{10.177.67}{12} = 873.139 \]

\[ s^2_y = \frac{\sum y^2 - n \bar{y}^2}{n - 1} = \frac{26.680}{12} = 2.223 \]

\[ s^2_{y|x} = \frac{12}{11} \times 2.223 - (0.0302)^2 (873.189) = 0.1555 \]

\[ s_x = 29.549 \]
\[ s_y = 1.491 \]
\[ s_{y|x} = 0.3943 \]
APPENDIX II

TEST FOR SIGNIFICANT DIFFERENCE
BETWEEN PERCENT WASTE IN TABLE 1 AND TABLE 2

<table>
<thead>
<tr>
<th></th>
<th>Table 1</th>
<th>Table 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Pounds Processed</td>
<td>2,941,246</td>
<td>1,865,120</td>
</tr>
<tr>
<td>Total Pounds of Waste</td>
<td>480,876</td>
<td>299,010</td>
</tr>
<tr>
<td>Percent Waste</td>
<td>16.3494</td>
<td>16.0291</td>
</tr>
</tbody>
</table>

Let

population waste proportion = \( p_1 \)
observed waste proportion = \( \bar{p}_1 \)
sample size = \( N_1 \)

Then \( \bar{p}_1 = 0.163494 \), \( N_1 = 2,941,246 \)
\( \bar{p}_2 = 0.160291 \), \( N_2 = 1,865,120 \)

(1) \( H_0: p_1 = p_2 \); i.e., \( p_1 - p_2 = 0 \); \( H_1: p_1 \neq p_2 \).

(2) Choose \( \alpha = 0.01 \).

(3) The difference between \( p_1 \) and \( p_2 \) can be estimated by the following approximate confidence interval (18):

\[
\bar{p}_1 - \bar{p}_2 - 2.576 \sqrt{\frac{\bar{p}_1(1 - \bar{p}_1)}{N_1} + \frac{\bar{p}_2(1 - \bar{p}_2)}{N_2}} < p_1 - p_2 < \\
\bar{p}_1 - \bar{p}_2 + 2.576 \sqrt{\frac{\bar{p}_1(1 - \bar{p}_1)}{N_1} + \frac{\bar{p}_2(1 - \bar{p}_2)}{N_2}}
\]
0.003203 - 0.000887 \leq p_1 - p_2 \leq 0.003203 + 0.000887

0.002316 \leq p_1 - p_2 \leq 0.004190

(4) \( H_0 \) is rejected at the 1.0 percent level, because the above confidence interval does not contain zero.
The problem was to calculate an estimate of total waste based upon a knowledge of production input and using the regression equation,

\[ y^1_x = 23.397 - 0.0302 x, \]

where the variables are the same as defined in Chapter III. Estimated waste equals estimated percent waste expressed as a fraction times production input. Since \( x \) is production input in thousands of pounds, multiplication by 1000 will give estimated waste in pounds.
BIBLIOGRAPHY

Literature Cited


13. Ibid., p. 160.


Other References
