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A PROCESS FOR EVALUATION AND RESOURCE ALLOCATION
IN DOMESTIC PUBLIC SPENDING PROGRAMS

A THESIS
Presented to
The Faculty of the Graduate Division
by
Melvin Elwood Case

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in the School of Industrial and Systems Engineering

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A PROCESS FOR EVALUATION AND RESOURCE ALLOCATION

IN

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Approved: 

[Signature]

Chairman

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SUMMARY

A process for evaluating the comparative progress that Office of Economic Opportunity (OEO) Community Action Agencies (CAAs) are making toward reducing poverty is developed and applied to seven CAAs in the State of Georgia. The process consists of four basic progress models, each for an identified parameter of poverty. The results from each progress model are then combined to determine CAA total progress and statistical methods are used to determine significant progress differences between CAAs and areas without CAAs.

The results from the progress models are combined with administrative performance results for input into a resource allocation model. The allocation model then optimizes progress and performance to determine the allocation of Federal funds to each CAA.

Even though the process is developed principally for OEO, it has application to social action agencies in other Federal Departments.
CHAPTER I

INTRODUCTION

This administration believes that every American should have the opportunity to participate in our Nation's economic life to the full extent of his abilities. The Office of Economic Opportunity will make this objective its highest priority... OEO is to be the cutting edge by means of which the government moves into unexplored areas.¹

To achieve this objective, the Office of Economic Opportunity (OEO) is organized into ten regions with each regional headquarters responsible for the Community Action Agencies (CAAs) in its region. The Southeast Regional Office, Atlanta, Georgia, has 196 CAAs under its control throughout the states of North Carolina, South Carolina, Kentucky, Tennessee Georgia, Mississippi, Alabama, and Florida, as shown in Figure 1. Each CAA is responsible at the local level for carrying out the purpose of Community Action which is

to stimulate a better focusing of all available local, State, private and Federal resources upon the goal of enabling low-income families, and low-income individuals of all ages...to attain the skills, knowledge and motivations, and secure the opportunities needed for them to become fully self-sufficient.²

Pursuant to the accomplishment of this purpose, the CAA seeks to mobilize resources such as funds, facilities and equipment from public

Figure 1. Office of Economic Opportunity Organization
and private resources, and talents and energies directed toward the elimination of poverty. The local CAAs are assisted in these efforts by the National and Regional Offices which are responsible for providing "financial assistance, basic policy direction, information and guidance, and training and technical assistance."³

To provide better direction for CAA assistance, the Office of Plans, Budget and Evaluation (PB&E) seeks annually to evaluate the efficiency and effectiveness of all CAAs in its region by commissioning an evaluation team composed of personnel from the Regional Office and the State Office of the CAA being evaluated. The impetus for an evaluation is normally provided by the evaluation schedule or by problems existing within a CAA, as shown in Figure 2. The team conducts an on-site evaluation of the agency with the emphasis being on the administration of the agency's programs and administrative compliance with OEO regulations. The evaluation team then prepares a narrative description of their findings for the agency and the OEO headquarters concerned. In most cases, the personnel sent to evaluate a given agency and the evaluation criteria used change from year to year making it difficult to detect any trend or change in the agency's performance.

In addition to this annual evaluation, each agency annually submits reports (CAP 5 and CAP 81) in which they assess their own performance during the reporting period. The CAP 81 is a narrative description of what the agency has tried to accomplish through the administration of

Figure 2. Evaluation Process
its programs. Attempts at a quantitative assessment are the exception rather than the rule. The CAP 5 has quantitative potential, but the agencies in most instances do not have the resources to gather the statistical data required on the report; hence, they use census figures which range from one to twelve years old, estimates from local surveys, and information such as family income and education attainment level, obtained from people seeking the agency's assistance.

These evaluation procedures raise the question of where to put evaluation emphasis. Joseph Wholey, et al. [53] state that the impact of activities that cost the public millions of dollars has not been measured, and that it is difficult to point with confidence to the difference, if any, that most social programs cause in the lives of Americans. They further emphasize that evaluation should examine policies and programs from the broadest National level down to specific operations of projects at the local level, including their impact on individuals.

In addition to issuing a call for more extensive evaluation, the authors make it clear that the emphasis should not be on an evaluation system for its own sake, but that its purpose should be to provide objective information to program managers and policy makers on the costs and effects of National programs and local projects. This information would assist in effective management and efficient allocation of limited resources.

Edward A. Suchman [51] also points out the need for more extensive evaluation.
All social institutions or sub systems, whether medical, educational, religious, economic, or political are required to provide "proof" of their legitimacy and effectiveness in order to justify society's continued support.... The current desire to judge the worthwhileness of community programs is but one aspect of modern society's belief that many of its social problems can be met most effectively through planned action based upon existing knowledge.4

However, Suchman and Wholey, et al. differ on their definitions of evaluation. Suchman states that evaluation "implies a logical or rational basis for making judgements but does not require any systematic procedures for marshalling and presenting objective evidence to support the judgement."5 Wholey, et al. state that evaluation "assesses the effectiveness of an on-going program in achieving its objectives, relies on the principles of research design to distinguish a program's effects from those of other forces working in a situation, and aims at program improvement through modification of current operations."6

In a more recent call for evaluation of social programs, Gold [20] states that "the programs comprising the War on Poverty can become much more effective if evaluation schemes are designed and executed to assess not only benefits, but the actual and potential impact of the programs on the poor."7

The need for improved evaluation has been recognized by management in the Plans, Budget, and Evaluation (PB&E) Division, Southeastern Regional

5. Ibid., p. 77.
Office, Office of Economic Opportunity. Efforts have been made to standardize and improve their evaluation system. These efforts have resulted in the development of a "Qualitative Factors Assessment Sheet (QFAS)" which uses a scoring model to rank on-going agencies based on planning and program policy development, personnel policy and management, and project-activity. The model is used by the Regional Office, State Offices and individual agencies in an attempt to gain the best judgement as a basis for scoring the performance of the agency. For on-going agencies, QFAS is now used in the annual agency evaluation by the same previously mentioned evaluation teams. Even though QFAS is a step in the direction of a more quantitative appraisal of an agency's performance, it fails to address the problem of evaluating the impact or progress that CAA's are making toward eliminating or reducing poverty.

As stated earlier, one of the benefits of a thorough evaluation system is the increased information available on which budget decisions can be made [53]. The current budgetary process used by PB&E is administrative and compliance oriented. If a CAA received $10,000 of Federal funds in the past program year, and has met its administrative requirements and complied with OEO regulations, and if the Federal allocation to the Region is constant, that CAA will be allocated $10,000 again in the current program year. Even though this strategy works, planners in PB&E recognize that it may not be the best strategy and they are in the process of developing additional strategies based on QFAS, and a Poverty Index which is determined by decennial income data from the Bureau of the Census, Department of Commerce. However, these additional strategies will still be inadequate since QFAS is administrative and compliance oriented, and
the Poverty Index is determined from census data that ranges from one to twelve years old.

Research Problem

The research problem is the development of a model which uses identified poverty parameters to measure the progress that a Community Action Agency is making toward eliminating or reducing poverty. The results from the progress model and agency performance will be used as input into a fund allocation algorithm that will determine the optimum Federal funding level for each on-going CAA in the Southeast Region.

Research Objectives

There are three objectives to be accomplished within this research:

1. To identify the parameters of poverty on which to base a description of Community Action Agency impact and progress toward reducing poverty.

2. To develop a model which uses the identified parameters of poverty to measure the comparative progress that each CAA in the Southeast Region is making toward reducing poverty in its area of responsibility.

3. To provide OEO management a method for optimizing the progress and performance of CAAs by allocating Federal funds subject to legal limitations and regional policy.
CHAPTER II

LITERATURE SURVEY

The literature survey was conducted to determine the significant elements affecting this research and what work had been done in the areas related to the specified research problem. As such, the survey was conducted in four general phases. The purpose of Phase I (Evaluation of Social Programs) was to determine the factors affecting the evaluation of social programs and what evaluative efforts have been made. Since progress can be represented by evaluation over time, Phase I was deemed essential and proved to be worthwhile. The purpose of Phase II (Value Methodologies) was to determine the methodologies available to determine the importance of several related factors, such as the parameters of poverty, for use in measuring social program progress. The purpose of Phase III (Resource Allocation Methodology) was to determine the existing methodologies applicable to the resource allocation problem of OEO. The purpose of Phase IV (Smoothing Methodologies) was to compare data "smoothing" techniques.

Evaluation of Social Programs

The literature on social programs, their evaluation, criteria for evaluation studies is voluminous. An attempt was made to restrict the survey to publications no more than five years old; however, exceptions were made where the source had potential importance.
In 1965, Burton A. Weisbrod [52] focused on two principle questions concerning poverty: why does poverty exist in the United States? and what can be done to minimize or eliminate poverty without impinging on other social and economic goals? Even though this research is not principally concerned with the whys of poverty and the hows to eliminate it, Weisbrod offers some important philosophies on relevant aspects of poverty.

On measuring and defining poverty, he states that

...it does make a difference how "poverty" is defined. The number of the poor differs according to the definition used. Yet the proponents of prompt action are also right: whichever definition is used, there is much poverty to be found.

More important, perhaps, than the total number of poor is the composition of the group. This characteristic is quite sensitive to the measure of poverty that is used. A measure that adjusts for family size and that accounts for assets and non-money income will tend to exclude many of the aged families, and particularly those living on farms; it will increase the absolute and relative number of children who are classified in poor families, and will increase the absolute and relative number of Negroes classified as poor. The poor families will be, predominately, the younger and larger families, and children will constitute a larger fraction of the total group. On the other hand, any measure of poverty that includes medical needs and income prospects will cause many of the aged to be classified as poor. Thus, effective allocation of a limited anti-poverty budget will vary--perhaps drastically--with the poverty measure applied.8

Thus, as he implies, there is a need for a measure that does not exclude any impoverished characteristic group. Concerning the allocation of resources, Weisbrod further states that

The wisdom of devoting a given amount of money to any antipoverty program, and the most effective way to spend that money, ought to depend not merely on "whether" the problem exists, but also on "how many" poor people there are, "why" they are poor, and "where" they are located.9

In 1967, the United States Senate Subcommittee on Intergovernmental Relations [50] discussed the problems involved in establishing criteria to measure the effectiveness of government programs, the qualifications criteria should have to meet, and gave an example of illustrative criteria for evaluating groups of programs. Pertinent criteria qualifications are:

1. Criteria must relate to governmental objectives.
2. More than one criterion will frequently be needed for individual problems.
3. Interactions occur among program areas and among criteria.
4. It is necessary to distinguish target groups.
5. Criteria need to be thoroughly defined.
6. Criteria can be expressed in different forms.
7. Monetary criteria can be very complex.
8. Criteria frequently will be difficult to measure.
9. Intangibles will always be with us.10

However, the Subcommittee did not suggest methods for developing criteria or measuring criteria already developed.

Fitzpatrick [16] also addressed the problem of selecting measures for evaluating social programs. He suggests two principle methods for the selection of measures:

9. Ibid., p. 5.
1. Method of Rationales -- a written version of the thought process by which the evaluator should have arrived at his decision concerning each measure.

2. Sampling of Measures -- when there are many measures from which to select, the best that can be done is a purposive selection to achieve a balanced representation.

In most of the literature surveyed, the authors pointed out philosophical considerations as opposed to operational procedures for evaluation. Howard E. Freeman and Clarence C. Sherwood [18] discussed the need for research to develop impact evaluation instead of process evaluation that is oriented toward the quality of the procedures used to administer social programs. The authors gave broad concepts for evaluation as follows:

1. Efficacy -- power to produce intended results.

2. Accountability -- a target population that can be dealt with by means of a program.

3. Efficiency -- yields the greatest per unit change in the classic sense, output/input.

They also stated that there are two lessons of relevance to the evaluation of anti-poverty programs as follows:

1. "Random allocation to treatment and non-treatment groups is not likely to be possible frequently.

2. Broad scale anti-poverty programs are not likely to be well-off with regard to knowledge of the representativeness of the population treated."11

Walter Williams discussed the methodological and institutional problems faced by a social action agency in trying to make evaluation an important input to its policy process in which major decisions are formulated and implemented. He stated two factors that indicate that the path towards evaluation becoming a major element in the policy process may be long and involved. The two factors are:

1. "Present methodological tools are inadequate.
2. It will be time consuming work to overcome these weaknesses."12

Efforts have also been made in modeling the evaluation of social welfare programs. Perry Levinson used a "Goal-Model Approach" and a "System-Model Approach" as a comparison for evaluation in very general terms. The Goal Model Approach facilitates the measurement of inputs, outputs, and outcomes in relation to formal agency goals and in terms of effectiveness and efficiency criteria. The System-Model includes Goal-Model evaluations and uses them for a basis of comparison of programs. Abraham Levine described several research studies designed to lay the ground work for a systematic effort directed toward evaluating both the effectiveness and efficiency of public welfare programs. Included in this description was a procedure for developing a program evaluation model as follows:

1. A review of the various project plans and typing these projects into a few broad classes on the basis of similarity.

2. Selections of representative projects from each of the major types for on-site study.

3. Development of a preliminary model which incorporates all of the significant dimensions and spells out the outcome variables in terms of program objectives.

4. Test model on one or more local projects and revise as necessary.

A more precise evaluation model was presented by Herman D. Stein, et al. [49] in which they proposed the examination of the agency's flow of service beginning with contact between potential clients and the agency, proceeding through intake and culminating in the provision of on-going services. Their objective was to relate agency operations as a process of achievement or a failure of the agency's output goals leading to the development of a methodology for identifying the agency's goals. They defined agency goals as

1. Outcome goals, e.g. reduce poverty.
2. Output goals, e.g. number of adoptions.
3. Input goals--in which is specified resources needed to achieve the output goals.
4. System-maintenance goals, e.g. personnel management policy.

Even though the proposed success of treatment approach measures output and goal achievement, it offers no way to determine success.

An experimental approach to social reform was advocated by Donald T. Campbell [7] in which he considered the political setting of program evaluation. He suggested four designs for evaluation:

1. Interrupted time-series design.
2. Control series design.
3. Regression discontinuity design.
4. True experiments.

He also suggested that decision-makers are not at the stage of continuing or discontinuing social programs based on assessed effectiveness, even though many people think they are.

Another more precise evaluation method was presented by Earl D. Maine [29] in which a Nationwide Evaluation of Manpower Development and Training Act (MDTA) trainees were compared against non-trainees. The purpose of the evaluation was to learn what effects MDTA job training programs had on income and employment for at least a year after courses ended. Results were obtained by interviewing both trainees and non-trainees retrospectively, and experimental designs were used to control outside variables.

A simple rating system for projects within Work Experience and Training Programs, which attempt to increase the earning power of the poor by providing basic education and training, was presented by Bateman [3]. In his article, Bateman categorized projects by population similarity and used historical data for comparison by ranking each project within a category based on specified criteria. In "Another Look at the Poverty Profile," Mollie Orshansky [42] studied income and food consumption. The results led to the assumption that a farm family would need 40 percent less cash than a non-farm family of the same size and composition since farm families generally can count not only some of their food but most of their housing as part of the farm operation. She also
suggested that the allowance for geographic variables of community size and region would improve the poverty index. Later, in another poverty study article, Orshansky [43] adjusted the needed cash for a farm family from 40 percent to 30 percent less than a non-farm family.

Value Methodologies

This phase of the literature survey is an examination of the methodologies for determining the relative importance of several related factors. In an "Overview of Value Methods," Norman R. Baker [2] presented a descriptive narrative with examples of many of the methods for determining relative importance. Three of the principle methods discussed are comparative methods, scoring models, and value contribution models.

Comparative methods are used to determine relative project preference by comparing one project against another project or group of projects. Baker's discussion included four comparative methods as follows:

1. **Q-Sort Methodology** determines qualitative differences in the value of a series of items and puts the items into categories.

2. **Paired Comparisons** assign items a quantitative rating by pairwise rating with the scale anchored at the lowest value or rating.

3. **Successive Ratings** assign numerical value to each item with the scale anchored at both ends.

4. **Successive Comparisons** compare each item against combinations of all others using the procedure developed in [11].

The second principle method to determine relative importance discussed by Baker was scoring models. Scoring models are designed so that
the respondent determines the merit of a project based on established criteria which can yield either an absolute or relative measure, depending on the criteria. To properly design a scoring model, Baker [2] suggests the following three steps:

1. "Construct a concise, exhaustive list of criteria.
2. Develop project measurement distributions and scales.
3. Weight the criteria according to relative importance."

Baker's discussion on value methods also includes the use of value contribution models. Value contribution models force the respondent to tie projects directly or indirectly into program objectives. The result gives an absolute measure of contribution of the project to the stated objectives. This method requires more value judgements, but each is restricted to a smaller segment of the total problem. Mathematical techniques can then be used to assimilate the judgements into an overall evaluation of the project.

John R. Moore and Norman R. Baker [34] discuss the use of scoring models for research and development project selection. The scoring models are used to "compute an overall project score based on ratings assigned to each project for each relevant decision criterion."

One of the important advantages of the scoring model is that it operates with subjective input data.

Sigford and Parvin [58] developed a different method for determining relative importance. Their method, PATTERN, reduces decision-making

judgement errors by establishing a relevance tree which reliably repre-
sents the combined value judgements of the participants. By relevance
voting, the PATTERN methodology establishes relative values for any
number of related factors. A methodology similar in purpose to PATTERN
is the DELPHI METHOD developed by Olaf Helmer [24, 10]. The DELPHI
METHOD also seeks to reduce the influence of intuitive judgement on the
outcome of analysis by successive questioning of individual experts,
without face-to-face confrontation, interspersed with controlled feedback
of the group's opinions and reasons offered in support of such opinions.

John C. Flanagan [17] developed a technique for measuring typical
performance, principally for evaluation in the field of psychology. Even
though the technique does not establish relative importance among the
systematically generated critical incidents, it does provide a thorough
statement of the critical requirements and a checklist for evaluating
performance.

Resource Allocation Methodologies

In the field of mathematical programming for resource allocation,
James E. Bruno [6] presented a linear programming model for optimally
allocating state and local funds to schools. The author's development
started from the shortcomings of actual school financing, where basic
state aid was given to every school district, regardless of its own
financing capacity. He proposed several alternative objective functions,
primarily minimization of the percentage spread in total district expendi-
tures per average daily attendance, and a constraint set with percentage
relationships between amounts of funds from various sources. A paper by
Edward N. Dodson [13] described the cost-effectiveness portion of a study where various systems in urban transportation were investigated. The objective function used is the improvement of the quality of urban living. An attempt was made toward the classification of quantitative measures of cost-effectiveness.

In order to assist corporate management in achieving a balanced allocation of research and development funds, Marvin J. Getron [9] described the elements of a quantitative resource allocation technique for exploratory development funds. The technique uses collective judgements and policy decisions from corporate planning, marketing-engineering teams and specialists in relevant technologies as inputs to obtain a utility measure. The technique then produces an optimal allocation of the development budget consistent with the stated measures. In a paper by Donald Gross and Richard M. Soland [21], an algorithm is presented for allocation problems in which constraint coefficients depend upon decision variables. The authors state that problems of this form cannot be solved by linear programming. Their algorithm is based on piecewise linear functions and takes the general form

$$\text{Minimize } \tilde{c}'\tilde{x}$$

Subject to

$$\tilde{A}x \geq \tilde{b}$$

$$x \geq 0$$

where

- $\tilde{x}$ is the allocation or decision vector
- $\tilde{c}'$ is the constant cost vector transposed
- $\tilde{b}$ is the constant vector of requirements or availabilities
- $\tilde{A}$ is the constraint matrix whose element $a_{i,j}$ is productivity or a coefficient of effectiveness.
The paper assumes that some of these productivities are not constant, but depend on the allocation variables. The authors approximate the dependence of productivity by piecewise linear functions, and develop a branch and bound algorithm to solve the resulting sequence of linear programming problems.

A sequential procedure to evaluate the value of interacting alternatives and to maximize their combined benefits was developed by J. J. Moder and J. J. Nickl [32]. Their procedure, as do most of the procedures discussed, assumed that a fixed sum of money, insufficient to fully finance all alternative activities, was given. The interactions they considered were:

1. Mutually exclusive alternatives.
2. Alternatives contingent upon one another
3. Interdependence of alternatives due to duplication of efforts.

Mathematically stated, the problem is

$$\text{Maximize } R = \sum_{j=1}^{k} f(x_j) + h(x)$$

Subject to

$$g_i(x) \geq b_i \quad i = (1, 2, \ldots, m) \quad (1)$$

$$\sum_{i=1}^{m} x_{i} = A \quad (2)$$

where

- $f(x_j)$ is the return from $x_j$ allocated to alternative $j$
- $h(x)$ is the interaction function expressing interactions between the activities.
Eq. (1) is the constraint on each project and Eq. (2) is the budget constraint.

A finite iterative method "Subopt" for solving interval programming problems was presented by Philip D. Robers and Adi Ben-Israel [45, 46]. The advantages of their method are:

1. In some cases, it is possible to express the solution of an interval programming problem explicitly and in closed form.

2. Problems arising in the interval programming form may be solved more efficiently with an interval programming method such as Subopt, which uses the special structure of the two-sided constraint, than ordinary linear programming techniques.

To use this method, the problem should be of the form

Maximize $\tilde{c}'\tilde{x}$

Subject to $\tilde{b}^- \leq \tilde{A}\tilde{x} \leq \tilde{b}^+$

where

$\tilde{c}'$ is the objective function coefficient vector transposed

$\tilde{x}$ is the activity vector

$\tilde{b}^-$ is the constraint lower bound vector

$\tilde{b}^+$ is the constraint upper bound vector

$\tilde{A}$ is the constraint coefficient matrix.

This method is ideally suited for allocation problems having specified upper and lower limits for each activity.

Another resource allocation model, developed by Elizabeth E. Bailey and John C. Malone [1], uses Lagrangian analysis to solve the
constrained systems in relation to various management objectives so far as their interactions on resource allocation are concerned. The model takes the general form

$$\text{Maximize } \Gamma(G,K)$$

$$\text{Subject to } \pi = f[F(G,K)]$$

where

(G,K) represent categories of profit (π), rate of return on investment, and volume of output

F(G,K) is some measurable characteristic of the firm and "f" is the fair return on F(G,K).

Ambrose Ben Nutt [38] developed the complex Research and Development Effectiveness (RDE) program used as an aid to laboratory management for allocating the annual laboratory research budget for the U. S. Air Force Flight Dynamics Laboratory. Nutt used the Churchman, Arnoff, Ackoff [11] method of determining relative value of importance of laboratory systems and goals, and linear programming to maximize the effectiveness of the total laboratory effort for the optimal budget. The linear programming model uses the measure of effectiveness of each resource level of each task, dollar and manpower resource levels of each task, and total dollars and manpower available. In a later article, Nutt [39] discussed an Air Force experiment in testing the resource allocation model which attempted to provide a balanced allocation of resources within the laboratory. Nutt's model was principally "designed to supplement the intuition of managers at all levels by combining expert subjective
judgements in a structured fashion to serve as an aid in the decision-making process."

Smoothing Methodologies

One of the essential characteristics of the United States' economic system is that it is continually changing. Since reducing poverty is also a function of time, the need exists to use the data available for determining progress toward reducing poverty in a dynamic manner consistent with our economic system. Several methods of using historical data, developed principally for predicting demand in inventory and production control, are applicable to the research problem.

Hadley and Whitin [22] discussed two methods for using historical data to predict demand for dynamic inventory systems with no strong seasonal pattern. They suggest using either least squares or exponential smoothing. One of the advantages of exponential smoothing over least squares is that exponential smoothing uses any time sequence of data as opposed to requiring data for a number of back periods.

R. G. Brown [5] pointed out advantages and disadvantages of three smoothing techniques. Moving average has the desirable characteristics for smoothing out fluctuations in variable history and has a stable response to change which can be controlled by time interval selection. However, the method requires keeping track of all past variable data, makes changing the rate of response difficult and does not correct for errors in computations. Exponential smoothing cuts down on data requirements

and has a stable response to change. The rate of response can be readily adjusted, computational errors are gradually eliminated, and trends or changes in trends can be calculated. Second-order systems can be used to track a variable that combines variable average and trend, but the method has a tendency to oscillate when a sudden change occurs in the variable.

In another discussion of smoothing techniques, J. F. Muth [36] suggested that "the main 'a priori' justification of exponential smoothing is that it leads to correction of persistent errors without responding very much to random disturbances."^{17}

As a result of investigating these smoothing techniques, the exponential smoothing technique will be applied to the historical data in Chapters III and IV to develop CAA progress results for each parameter of poverty.

CHAPTER III

THE DEVELOPMENT OF THE PROGRESS EVALUATION AND RESOURCE ALLOCATION PROCESS

Introduction

As discussed in Chapter I, the evaluation and budgetary process currently used by the Regional Office, OEO, is based on administrative performance only. As such, Federal funds are allocated without regard to the effectiveness of the CAAs. Thus, decisions are being made without considering all the relevant data.

In developing a process to assist the decision maker to optimally allocate scarce financial resources among competing CAAs, on an annual basis, a rationale other than administrative performance had to be developed with a quantitative foundation. The underlaying rationale is that Federal funds should be allocated to the CAAs based on the comparative progress that each CAA is making toward reducing poverty and the comparative performance of each CAA. To achieve this objective, the progress model is developed to provide pertinent information, in addition to the information provided by the existing performance model, for input into the fund allocation model.

To be functional, the progress model is developed heuristically with respect to the parameters of poverty and based on all relevant data available for each parameter. Consequently, the output from each model or phase of the total process is used as input for the succeeding model. The overall process can be represented by Figure 3.
Figure 3. Progress Evaluation and Resource Allocation Process
The output from the allocation model provides the optimum funding level for each CAA in the Southeast Region based on the comparative progress and performance of each CAA.

Parameters of Poverty

Before the evaluation process could be developed, the staff in PB&E had to identify those parameters which could be used to measure or describe poverty. Prior to using the method of successive comparisons [2,11], the parameters of poverty were identified in their order of importance to be:

\[ P_1 = \text{Income} \quad P_2 = \text{Education} \quad P_3 = \text{Housing} \quad P_4 = \text{Health} \]

In accordance with the method of successive comparisons, an arbitrary tentative value of 1.00 was assigned to income. The remaining parameters were then assigned a tentative and relative value with the result as follows:

\[ P_1 = 1.00 \quad P_3 = 0.61 \]
\[ P_2 = 0.75 \quad P_4 = 0.34 \]

The following relative importance comparisons were made:

1. \( P_1 \) vs \( P_2, P_3, P_4 \)
2. \( P_2 \) vs \( P_3, P_4 \)
3. \( P_3 \) vs \( P_4 \)

18. The parameters and their relative importance were determined during a meeting with the staff in PB&E, August 3, 1971.
On querying the decision makers about the comparisons, it was found that the initial values were satisfactory.

Hence, the final values are

\[ P_1 = 1.00 \quad P_3 = 0.61 \]
\[ P_2 = 0.75 \quad P_4 = 0.34 \]

The final values are then normalized by

\[ R_1 = \frac{P_1}{P_1 + P_2 + P_3 + P_4} = 0.370 \]
\[ R_3 = \frac{P_3}{P_1 + P_2 + P_3 + P_4} = 0.226 \]
\[ R_2 = \frac{P_2}{P_1 + P_2 + P_3 + P_4} = 0.278 \]
\[ R_4 = \frac{P_4}{P_1 + P_2 + P_3 + P_4} = 0.126 \]

**Development of the Progress Model**

To determine the comparative progress that CAAs are making toward reducing poverty implies the necessity to compare what each agency is doing to what it has done in the past. To make a quantitative comparison requires the use of historical data to form a base from which the comparisons can be made. The techniques of using historical data discussed in Chapter II are used for forecasting demand primarily in the fields of inventory and production control. However, the rationale for using these techniques for forecasting in inventory and production control is the same as the rationale for using them in the progress model developed herein, even though forecasting is not the principal aim. The rationale for using the techniques is to consider all historical data available, consider trends of progress in reducing poverty, and adjust for random variations.
in the data. The resulting similarity of rationale and application justify the use of these smoothing techniques in the development of the progress model.

**Income Model**

**Earned Income.** For the parameter of income to be used to measure progress toward reducing poverty, the model must consider data from previous reporting periods so that a quantitative analysis of that progress can be made. Since the data are in the form of numbers of people, and total income within specified income ranges by area (CAA), the model should incorporate as many meaningful measures that can be developed to describe the changes in the relative income situation.

Let

\[ P_{i,t} \]  
\[ p_{i,t} \]  
\[ M_{i,t} \]  
\[ m_{i,t} \]

where:

- \( P_{i,t} \) population of area \( i \) during period \( t \)
- \( p_{i,t} \) number of people in area \( i \) during period \( t \) under $3000
- \( M_{i,t} \) mean income of area \( i \) during period \( t \)
- \( m_{i,t} \) mean income of those people under $3000 in area \( i \) during period \( t \).

Then for a given period:

\[ \frac{P_i}{p_i} = \text{fraction of people in area } i \text{ under } $3000, \]

\[ \frac{M_i}{m_i} = \text{factor by which the mean income of area } i \text{ is greater than the mean income of those under } $3000, \]

and the Relative Earned Income Quotient (E) for area \( i \) is defined to be
\[ E_i = \frac{P_i}{P_i} \cdot \frac{M_i}{m_i} \]  

(1)

\[ = \frac{m_i/P_i}{m_i/P_i} \]  

(2)

\( E_i \) is computed using Eq. (1) and measures the extent to which the mean income of the population, adjusted for population size, is greater than the mean income of those under $3000, adjusted for the number of people under $3000. To reduce poverty with respect to earned income, \( E_i \) must decrease over time.

To determine the progress that a CAA has made toward reducing poverty with respect to earned income in its area of responsibility during period \( t \),

\[ \Delta E_{i,t} = \tilde{E}_{i,t-1} - E_{i,t} \]

where

\( \tilde{E}_{i,t-1} \) is the exponentially smoothed value of \( E_{i,j} \) for \( j = 1, \ldots, t-1 \)

\( E_{i,t} \) is computed for \( j = t \).

To compute \( \tilde{E}_{i,t-1} \),

\[ \tilde{E}_{i,t-1} = \alpha E_{i,t-1} + \alpha(1-\alpha) E_{i,t-2} + \alpha(1-\alpha)^2 E_{i,t-3} + \]

\[ + \ldots + \alpha(1-\alpha)^{t-2} E_{i,1} \quad 0 < \alpha < 1 \]
or

$$
\hat{E}_{i,t-1} = \alpha \sum_{j=1}^{t-1} (1 - \alpha)^{t-j-1} E_{i,j} \\
0 < \alpha < 1
$$

This reduces to the simpler form

$$
\hat{E}_{i,t-1} = \alpha E_{i,t-1} + (1 - \alpha) \hat{E}_{i,t-2} \\
0 < \alpha < 1 \quad (3)
$$

When using Eq. (3), the stored data required to update $\hat{E}_{i,t}$ is simply $\hat{E}_{i,t-1}$ [22].

The selection of the value of $\alpha$ is not made arbitrarily. The best value of $\alpha$ is selected by minimizing the variability of the observed $E_{i,t}$ from the expected $E_{i,t}$ ($\hat{E}_{i,t-1}$) over all $i$. This is done by successively choosing $\alpha$, $0 < \alpha < 1$, and computing the variance of the observed $E_{i,t}$ with respect to the expected $E_{i,t}$ over all $i$ and selecting $\alpha^*$ to be that $\alpha$ which minimizes

$$
\sigma^2 = \frac{\sum_{i=1}^{z} (\Delta E_{i,t} - \hat{\Delta} E_{i,t})^2}{z - 1}
$$

where $z$ is the number of CAAs evaluated, and

$$
\Delta E_{i,t} = \frac{\sum_{i=1}^{z} (E_{i,t} - \hat{E}_{i,t-1})}{z}
$$

Selection of $\alpha$ close to 1 emphasizes the importance of recent data and conversely, selection of $\alpha$ close to 0 emphasizes older data. To improve the accuracy of measuring each parameter, a separate $\alpha$ must be selected for each parameter.
When computed for all i, $\Delta E_{i,t}$ positive indicates progress by an amount $\Delta E_{i,t}$ and provides a basis for quantitatively comparing each CAA against the other CAA's to determine the comparative progress made toward reducing poverty with respect to earned income.

It should be noted, however, that the possibility exists that the measure of $E_i$ may be influenced in part by a factor of economic activity. This may be especially true in highly industrial areas. To measure economic activity, Moore, et al. [33] present the most precise model to date in which business cycle indicators are used with an explicit scoring system to produce an index of economic activity. The model is used principally for the United States as a whole but could be applied to county areas if the data were available in county form. This, however, is not the case.

At such a time when the data are available to support the use of the model developed by Moore, the change in the level of economic activity ($\Delta X_{i,t}$) would be computed using the same procedure used for $\Delta E_{i,t}$. To determine the degree of correlation between $E_i$ and $X_i$, and to correct $E_i$ for the correlation, see Appendix I.

Social Security Income. Social Security payments are dispersed to retired workers, disabled workers, dependents of retired and disabled workers, survivors and special age 72 beneficiaries.\(^{19}\) Since all age groups are considered in the Progress Model, all categories of Social Security payments must be considered.\(^{20}\)

---

19. Payments are made up to $45 per month to persons over age 72 if they are not eligible for regular Social Security and are receiving no public assistance or retirement program income.

20. Social Security data are not reported with income data.
Let

\[ P_{i,t} = \text{population of area } i \text{ during period } t \]
\[ q_{i,t} = \text{number of people receiving Social Security payments in area } i \text{ during period } t \]
\[ s_{i,t} = \text{mean Social Security payment in area } i \text{ during period } t. \]

Then for a given period

\[ \frac{q_i}{P_i} = \text{fraction of the population of area } i \text{ receiving Social Security payments,} \]
\[ \frac{M_i}{s_i} = \text{factor by which the mean income of area } i \text{ is greater than the mean Social Security payment in area } i, \]

and the Relative Social Security Quotient (S) for area i is

\[ S_i = \frac{q_i}{P_i} \cdot \frac{M_i}{s_i} \]

To determine progress toward reducing poverty in area i, period t with respect to Social Security for n periods

\[ \Delta S_{i,t} = S_{i,t-1} - S_{i,t} \]

where \( \Delta S_{i,t} \) is determined by the same procedure used to determine \( \Delta E_{i,t} \).

**Welfare Income.** Public welfare payments are considered in the same general form as Social Security payments since welfare statistics are not considered in the income statistics. Public welfare payments are disbursed in the categories of old age assistance, aid to the blind, aid to families with dependent children, and aid to the disabled. Again the model considers all categories in developing progress information for welfare.
Let

\[ P_{i,t} = \text{population of area } i \text{ during period } t \]

\[ r_{i,t} = \text{number of people receiving welfare payments in area } i \text{ during period } t \]

\[ w_{i,t} = \text{mean welfare payment in area } i \text{ during period } t. \]

Then for a given period

\[ \frac{r_{i,t}}{P_{i,t}} = \text{fraction of the population of area } i \text{ receiving welfare payments}, \]

\[ \frac{M_i}{w_i} = \text{factor by which the mean income of area } i \text{ is greater than the mean welfare payment in area } i, \]

and the Relative Welfare Quotient (W) for area i is

\[ W_i = \frac{r_{i,t}}{P_{i,t}} \cdot \frac{M_i}{w_i} \]

and

\[ \Delta W_{i,t} = W_{i,t} - W_{i,t-1} \]

where \( \Delta W_{i,t} \) is determined using the previously described procedure.

**Total Income.** To determine progress with respect to total income, the individual terms \( \Delta E \), \( \Delta S \), and \( \Delta W \) are summed so that the aggregate change for the Income Model for area i, period t is

\[ \Delta I_{i,t} = \Delta E_{i,t} + \Delta S_{i,t} + \Delta W_{i,t} \]

**Health Model**

To determine progress toward improving the general health of an area for which a CAA is responsible, mortality statistics are used since levels of physical health may be adequately measured by mortality rates. Even though mortality data of the poor have not been collected separately,
it appears that mortality rates, particularly during infancy, childhood, and even the younger adult years are higher in areas of poverty than for the rest of the population \([26,44]\). Since the mortality rates of the rest of the population will remain relatively stable in the absence of epidemics, a reduction in mortality rates would indicate an improvement in the health of the poor. These mortality statistics are available in the form of fetal deaths, infant deaths, and maternal deaths.

Let

\[
\begin{align*}
P_{i,t} &= \text{population of area } i \text{ during period } t \\
f_{i,t} &= \text{number of fetal deaths in area } i \text{ during period } t \\
i_{i,t} &= \text{number of infant deaths in area } i \text{ during period } t \\
d_{i,t} &= \text{number of maternal deaths in area } i \text{ during period } t.
\end{align*}
\]

Then for a given period

\[
\begin{align*}
\frac{f_{i}}{P_{i}} &= \text{number of fetal deaths per person in area } i, \\
\frac{i_{i}}{P_{i}} &= \text{number of infant deaths per person in area } i, \\
\frac{d_{i}}{P_{i}} &= \text{number of maternal deaths per person in area } i,
\end{align*}
\]

and the Relative Health Quotient \((A)\) for area \(i\) is

\[
A_i = \frac{f_i}{P_i} + \frac{i_i}{P_i} + \frac{d_i}{P_i}.
\]
Hence a reduction in death rates will lower $A_i$ indicating progress toward improving the health status of an area. This progress is measured by

$$\Delta A_{i,t} = \bar{A}_{i,t-1} - A_{i,t}$$

where $\Delta A_{i,t}$ is determined by the same procedure used for $\Delta E_{i,t}$.

**Education Model**

Since education of the poor may be a way out of their dilemma, a model is needed to describe the progress that education is making in reducing poverty. Even though participation in the educational process will improve the plight of the poor, it is principally the completion of a given level of schooling that yields the greatest return rather than any of the years leading up to graduation [31]. In addition to considering the students enrolled in high school, the model also considers Adult Vocational Education and Adult Basic Education Programs, since these inputs effect the education of a people.

Unfortunately, education data on the poor as a group are not available except in the form of isolated case studies. However, case studies reveal that a student's socio-economic background relates to his educational achievement levels as well as to his growth in achievement [41,4,19]. Even though the educational attainment of the low socio-economic class lags behind the attainment of the middle and high socio-economic classes, and the differential rates of growth of attainment differ, educational attainment levels for each socio-economic group increases as the level of attainment of all groups increase [40,41]. Therefore, the Education Model measures, with an associated lag for all CAAs, the increase in educational attainment of low-income persons.
Let

\[ P_{i,t} = \text{the population of area } i \text{ during period } t \]
\[ e_{i,t} = \text{the high school (grades 9-12) enrollment} \]
\[ \text{in area } i \text{ during period } t \]
\[ a_{i,t} = \text{the enrollment in adult basic education} \]
\[ \text{programs in area } i \text{ during period } t \]
\[ v_{i,t} = \text{the enrollment in adult vocational education} \]
\[ \text{in area } i \text{ during period } t. \]

Then for a given period,

\[ \frac{e_{i}}{P_{i}} = \text{the fraction of the population enrolled in} \]
\[ \text{high school in area } i, \]
\[ \frac{a_{i}}{P_{i}} = \text{the fraction of the population enrolled in} \]
\[ \text{adult basic education in area } i, \]
\[ \frac{v_{i}}{P_{i}} = \text{the fraction of the population enrolled in} \]
\[ \text{adult vocational education in area } i. \]

When comparing the number of students in each category from one area to another, the measure must be adjusted by the population of the respective area.

Hence, the Relative Education Quotient \((D)\) for area \(i\) is

\[ D_{i} = \frac{e_{i}}{P_{i}} + \frac{a_{i}}{P_{i}} + \frac{v_{i}}{P_{i}} \]

and the progress that is being made toward increasing the level of education in area \(i\) during period \(t\) is determined by

\[ \Delta D_{i,t} = D_{i,t} - \bar{D}_{i,t-1} \]

where \(\Delta D_{i,t}\) is determined by the same procedure used for \(\Delta E_{i,t}\).
ΔD, positive indicates an increase in the level of education and provides a means for comparing the progress in one area with the progress in another area.

Housing Model

To reduce poverty with respect to housing, sub-standard housing would have to be reduced with standard housing being built to house those who once lived in the sub-standard dwellings. The model to describe progress toward improving housing includes inputs from both aspects of the housing problem and is adjusted by total housing units per area.

Let

- \( H_i \) = total all-year-round housing units in area i from the most recent census
- \( h_i \) = number of sub-standard units in area i from the most recent census
- \( \alpha_i \) = number of low income units completed since the last census
- \( L_{i,t} \) = number of low income units under construction in area i during period t
- \( \%_{i,t} \) = percentage of completion of housing units under construction in area i during period t
- \( Y_i \) = number of sub-standard units razed since last census
- \( \beta_i \) = number of sub-standard units renovated since last census

Then, for a given period

\[
\frac{h_i - Y_i - \beta_i}{H_i - Y_i} = \text{fraction of all-year-round units that are sub-standard,}
\]

\[
\frac{\alpha_i}{H_i + \alpha_i - Y_i} = \text{fraction of low income all-year-round units that have been completed since last census,}
\]
units that are in process, and the Relative Housing Quotient (N) for area i during period t is

\[
N_{i,t} = \frac{L_i + \psi_{i,t}(L_{i,t})}{H_i + L_i - Y_i + \psi_{i,t}(L_{i,t})} - \frac{h_i - \gamma_i - \beta_i}{H_i - \gamma_i}.
\]

To determine the progress made toward reducing poverty with respect to housing for area i during period t over the past n periods

\[
\Delta N_{i,t} = N_{i,t} - \bar{N}_{i,t-1}
\]

where \(\Delta N_{i,t}\) is determined by the same procedure used for \(\Delta E_{i,t}\), \(\Delta N_{i,t}\) positive indicates progress by the amount \(\Delta N_{i,t}\).

**Total Progress Model**

To determine the total progress that any agency has made toward reducing poverty in its area of responsibility during a specified period of time, all four parameters (income, education, health, and housing) and the relative importance of each is considered. As such, the total progress measure provides a basis for comparing the impact that each agency has had on its population during the most recent time period considered.

The Total Progress (T) for area (agency) i during period t is

\[
T_{i,t} = (R_1 \cdot \Delta I_{i,t}) + (R_2 \cdot \Delta D_{i,t}) + (R_3 \cdot \Delta N_{i,t}) + (R_4 \cdot \Delta A_{i,t}). \tag{4}
\]
The linear relationship given by Eq. (4) appears to be the most meaningful method to aggregate the four parameters of poverty. A multiplicative relationship would provide erroneous total progress information if the progress with respect to one or more parameters were either zero or negative. For the same reason, second order or higher order equations are eliminated.

Despite a concerted effort to develop the model so that the progress measures of the parameters of poverty would be mutually independent the possibility of income interacting with each of the other parameters exists. To determine if progress with respect to income interacts with progress with respect to either health, housing or education, a correlation test must be made.

Let

\[ z = \text{the number of CAAs considered} \]
\[ \rho = \text{the coefficient of correlation} \quad -1 \leq \rho \leq 1 \]
\[ r = \text{the estimate of } \rho. \quad -1 \leq r \leq 1 \]

To test for an income-housing correlation during period \( t \), compute

\[
r_{I,N} = \frac{\sum_{i=1}^{z} (\Delta I_{i,t} - \Delta \overline{I},t)(\Delta N_{i,t} - \Delta \overline{N},t)}{\sqrt{\sum_{i=1}^{z} (\Delta I_{i,t} - \overline{\Delta I},t)^2 \sum_{i=1}^{z} (\Delta N_{i,t} - \overline{\Delta N},t)^2}}
\]

where

\[
\Delta \overline{I},t = \frac{\sum_{i=1}^{z} \Delta I_{i,t}}{z}
\]
Using the "t" statistic to test the hypothesis that income progress and housing progress are uncorrelated \((\rho = 0)\), compute

\[
|t| = \frac{|\rho|}{\sqrt{1 - \rho^2}} \sqrt{\frac{N - 2}{\chi^2_{(z-2)}}}
\]

and if \(|t| \geq t_{\alpha/2, z-2}\), where \(\alpha\) is the probability of rejecting a true hypothesis (level of significance, \(\alpha = 0.05\)), reject the hypothesis. If \(|t| < t_{\alpha/2, z-2}\), then the progress measures of the parameters are uncorrelated.

If the measures are uncorrelated, \(\rho = 0\), then total progress can be determined as in Eq. (4). If \(\rho \neq 1\), then the measures are perfectly correlated and the housing measure should be eliminated for the current period giving preference to the income measure. In this case, the relative importance factors \((R_1, R_2, R_3)\) would have to be normalized again. However, if \(\rho \neq 0\) and \(\rho \neq 1\), a subjective judgement will be necessary to determine if the housing measure should be eliminated for the current program period. Preferably all data should be used, but if the data are hard to obtain, and the parameters are highly correlated where \(\rho\) is close to \(1\), then the housing measure should be eliminated; otherwise, both measures should be used.
To test for correlations between the other parameters, income-education, income-health, health-education, health-housing, and housing-education, use the procedure defined by Eqs. (5), (6), (7), and (8) making the appropriate parameter substitutions.

**Analysis of Progress Data**

In addition to determining the progress that each CAA has made toward reducing poverty, the need exists to determine if that progress is the result of the efforts of the CAA and if there are significant differences among CAAs. This is determined by using analysis of variance on a randomized complete block design for the progress data for each parameter of poverty over all past periods considered, and can be represented by a mathematical model which can be applied for each parameter. Using the parameter of health as an example the model is

\[ \Delta A_{i,t} = \mu + B_t + \epsilon_{i,t} \]

where

- \( \Delta A_{i,t} \) is the progress in area (CAA) \( i \) during period \( t \)
- \( \mu \) is the fixed effect
- \( B_t \) is the time effect
- \( \epsilon_{i,t} \) is the random error.

Since \( \mu \) is composed of effects common to all areas and effects peculiar to only the \( i \)th area, the model can be further subdivided into

\[ \Delta A_{i,t} = \mu + \varphi_i + B_t + \epsilon_{i,t} \]
where

- $\mu$ is the component of the fixed effect common to all areas
- $\varphi_i$ is the component of the fixed effect peculiar to the $i$th CAA.

Hence, if all $\varphi_i$ are equal, the progress in each area with respect to the parameter tested is equal.

The hypothesis to be tested is that the progress in each area is equal, and can be written as

$$H_0: \varphi_0 = \varphi_1 = \varphi_2 = \ldots = \varphi_i = \ldots = \varphi_z$$

where

- $i = 0$ represents the areas for which there is no CAA.

The hypothesis is tested using two-way analysis of variance to compute the mean square for the between-areas ($MS_3$), within-areas ($MS_2$), and error ($MS_1$) sources, and using an "F" test so that

$$F = \frac{MS_3}{MS_1}$$

Thus, the hypothesis of equality of the $\varphi$'s is rejected if

$$F = \frac{MS_3}{MS_1} \geq F_{1-\alpha, \nu_1, \nu_2}$$

where

- $\alpha$ is the level of significant ($\alpha = 0.05$)
- $\nu_1$ is the number of areas minus one, $(z-1)$ degrees of freedom
- $\nu_2$ is the number of areas minus one, multiplied by the number of progress periods minus one, $(z-1)(n-1)$ degrees of freedom.

If the hypothesis of equality is rejected, there are several techniques available to determine which areas differ significantly [14,23,25,30,47]. One of these techniques, the Duncan Multiple Range Test [14,25],
permits the comparison of each CAA with other CAA to determine which areas have made progress significantly different from the others.

Confidence limits should also be set on progress with respect to each parameter for each area. These limits are used to provide a range for true progress with respect to each parameter of poverty. Using confidence limits, the probability is \( 1 - \alpha \) that the actual progress is within those limits. The confidence limits (CL) are computed by using the "t" statistic for \( \alpha = 0.05 \)

\[
CL_i = \Delta \hat{A}_{i,.} \pm t_{(1-\alpha), \nu_2} \cdot \sqrt{\frac{MS_{error}}{n}}
\]

where

\( \Delta \hat{A}_{i,.} \) is the mean progress with respect to health of area \( i \) over \( n \) periods

\( MS_{error} \) is the mean square of the error source

\( \nu_2 \) is the degrees of freedom of \( MS_{error} \)

\( n \) is the number of progress measurements.

Performance Model

As discussed in Chapter I, CAA performance is annually evaluated using the Qualitative Factors Assessment Sheet (QFAS). The output from the performance model is to be used as input for the fund allocation model.

However, total progress and performance are distinct measures on distinct scales. Consequently, the two measures must be converted to a common scale before they can be used as inputs into the fund allocation model. To convert CAA performance \( Q \) to the scale of CAA total progress \( T \) recall that
Let 

\[ T_{i,t} = \text{the total progress of CAA } i \text{ during period } t. \]

Then the mean total progress and mean performance during period \( t \) are respectively

\[ \bar{T}_{.,t} = \frac{\sum_{i=1}^{z} T_{i,t}}{z} \]

and

\[ \bar{Q}_{.,t} = \frac{\sum_{i=1}^{z} Q_{i,t}}{z} \]

To equate mean performance (\( \bar{Q}_{.,t} \)) to mean total progress (\( \bar{T}_{.,t} \)), find the scale factor (\( \tau \)) such that

\[ \bar{T}_{.,t} = |\tau| \cdot \bar{Q}_{.,t} \]

The procedure defined above puts progress and performance on a common scale for use as input to the fund allocation model.

**Development of the Allocation Model**

**Introduction**

Title II of the Economic Opportunity Act of 1964, as amended [15], "The Act," provides general guidance to the Regional Directors, OEO, for allocating Federal funds to CAAs. Even though the Act does not delineate specific allocation strategies, it provides limitations on financial assistance. Of the funds provided to each region by the Federal Office, the Regional Director must allocate at least 80 per cent of that amount to the States within the region. The Regional Director determines the
amount of funds distributed to each State in his region in accordance with the latest available data on the basis of

1. the relative number of public assistance recipients in each State as compared to all States,

2. the average number of unemployed persons in each State as compared to all States, and

3. the relative number of related children living with families with income of less than $1000 in each State as compared to all States.²¹

Even though the amount of funds to be allotted to each State is regulated by law, the Regional Office must determine the amount of funds to be allocated to each on-going CAA in each State. The Act also puts limitations on CAA funding levels. As a minimum, the Regional Office can allocate to each CAA 80 per cent of the CAA's previous year's fund allocation without having a legal hearing to justify cutting the CAA's allocation. As a general rule, the Regional Office will not cut a CAA's funding by more than 20 per cent unless the Region Office is going to de-fund that CAA for reasons of non-compliance with OEO regulations, in which case, the de-funded CAA is not considered in the budgetary process.

If, however, the Regional Office wants to increase the funding level for a CAA, it may do so providing the conditions of the statutory State funding level and the minimum CAA funding levels are not violated. In other words, an increase in funding for any CAA in a given State must come from that State's allocated resources. Thus, the funding for each on-going CAA takes the form of being bounded by a minimum and maximum amount for a given period.

Allocation Model

From the above discussion, it is obvious that the problem of allocating funds to the CAAs in a given State takes the form of a mathematical programming problem. As discussed in Chapter II, Robers and Ben-Israel [45,46] used Interval Programming (IP) to solve Chemical Engineering Problems. The OEO funding problem takes the same form as the problems discussed by Robers and Ben-Israel.

The Constraints. As stated earlier, the funds allocated to the CAAs in a given State must not exceed the State's statutory total, and each CAA must be allocated an amount within the established legal limits. Let

\[ x_{i,t} \] = the funds to be allocated to the \( i^{th} \) CAA for the current budget period \( t \)

\[ M_{i,t} \] = the statutory limit of funds for a given State

\[ y_{i,t}^- \] = the lower bound for funding CAA \( i \) for period \( t \)

\[ y_{i,t}^+ \] = the upper bound for funding CAA \( i \) for period \( t \)

\( z \) = the number of CAAs in a given State.

Then, the budget constraint for all the CAAs in a given State for period \( t \) can be written as

\[
M_{i,t} \leq \sum_{i=1}^{z} x_{i,t} \leq M_{i,t}
\]

and the constraint for each CAA is

\[
y_{i,t}^- \leq x_{i,t} \leq y_{i,t}^+\]

\( i = (1, \ldots, z) \)
where

\[ y_{1,t}^- = 0.80 \times_{i,t-1} \]

\[ y_{1,t}^+ = x_{i,t-1} + 0.20 \sum_{j=1}^{p} x_{j,t-1} \]

\[ i = (1, \ldots, 2) \]

Hence the constraint set for a given State can be written as

\[ M_0, t \leq x_{1,t} + x_{2,t} + x_{3,t} + \ldots + x_{2,t} \leq M_0, t \]

\[ y_{1,t}^- \leq x_{1,t} \leq y_{1,t}^+ \]

\[ y_{2,t}^- \leq x_{2,t} \leq y_{2,t}^+ \]

\[ y_{3,t}^- \leq x_{3,t} \leq y_{3,t}^+ \]

\[ \vdots \]

\[ y_{2,t}^- \leq x_{2,t} \leq y_{2,t}^+ \]

The Objective Function. The objective function of the allocation of Federal funds problem takes the form of maximizing the return for the funds allocated based on the total progress (T) that each CAA has made toward reducing poverty and the CAA's performance (Q) as reflected in the annual Qualitative Factors Assessment Sheet (QFAS) evaluation. This is represented by

\[ \text{Maximize } \vec{v}_{i,t} \times_{i,t} \]

where

\[ \vec{v} \] is the progress and performance coefficient vector transposed.

For CAA \( i \), period \( t \), \( \vec{v}_{i,t} \) is found by
\[
\tilde{v}_{i,t} = (R_5 \cdot T_{i,t}) + (R_6 \cdot \tau \cdot Q_{i,t}) \quad i = (1, 2, \ldots, 2)
\]

where

- \(R_5\) is the relative importance value for using total progress for funding
- \(R_6\) is the relative importance value for using performance for funding
- \(T_{i,t}\) is the total progress of CAA \(i\) during period \(t\)
- \(Q_{i,t}\) is the QFAS performance score of CAA \(i\) during period \(t\)
- \(\tau\) is the scale factor to equate progress and performance.

Therefore, the allocation of funds for a given State for period \(t\) can be written as the Interval Program, noted as IP,

\[
\text{Maximize} \quad \tilde{v}_{1,t} x_{1,t} + \tilde{v}_{2,t} x_{2,t} + \tilde{v}_{3,t} x_{3,t} + \cdots + \tilde{v}_{2,t} x_{2,t}
\]

Subject to

\[
\begin{align*}
M_{i,t} & \leq x_{1,t} + x_{2,t} + x_{3,t} + \cdots + x_{2,t} & \leq M_{i,t} \\
y_{1,t}^- & \leq x_{1,t} & \leq y_{1,t}^+ \\
y_{2,t}^- & \leq x_{2,t} & \leq y_{2,t}^+ \\
y_{3,t}^- & \leq x_{3,t} & \leq y_{3,t}^+ \\
\vdots & \vdots & \vdots \\
y_{2,t}^- & \leq x_{2,t} & \leq y_{2,t}^+
\end{align*}
\]

Since the Southeast Region is composed of eight States, it may be convenient to represent the fund allocation problem in one IP instead of eight separate ones. The Region IP is represented in a graphic form since a mathematical programming formulation quickly becomes notationally

22. The relative importance values, \(R_5, R_6\), are developed in Appendix II.
cumbersome. The Regional IP is

$$\text{Maximize } \sum_{i=1}^{n} v_i x_{1,i} + \sum_{i=2}^{n} v_i x_{2,i} + \sum_{i=3}^{n} v_i x_{3,i} + \cdots + \sum_{i=2}^{n} v_i x_{2,i}$$

Subject to

$$B_1 \quad B_2 \quad \ldots \quad B_j \quad \ldots \quad B_u$$

where

- $B_j$ is the constraint set for the $j^{th}$ State
- $j$ ranges from 1 to $u$ States.

Considerations Before Using the Allocation Model

After the coefficients of the objective function, and the upper and lower bounds of the constraint set have been determined, feasibility and boundedness must be checked. This insures that the model will produce an optimal solution [45].

**Feasibility.** Feasible solutions are obvious, e.g., fund each CAA at last year's level. If the resources available to the Region have been reduced, then a straight percentage reduction for each CAA will provide a feasible solution. If the IP has feasible solutions, then the IP is feasible.

**Boundedness.** If the IP is feasible, boundedness is then checked. The form of the constraint set for the OEO funding problem guarantees that the IP is bounded since each variable (CAA) must be funded within the specified interval.
Summary

Using the parameters of poverty, a model has been developed to measure the comparative progress that CAAs are making toward reducing poverty in their respective areas of responsibility. The output from the progress model has been combined with output from the existing performance model to provide input for the fund allocation model. The fund allocation model has been developed to optimize both progress and performance and provides an optimal allocation of Federal funds to CAAs. Chapter IV is an example of the progress evaluation and resource allocation process using relevant data.
CHAPTER IV

EXAMPLE OF THE PROGRESS EVALUATION AND RESOURCE ALLOCATION PROCESS

The example in this chapter uses the models developed in Chapter III and the data for each parameter of poverty to determine the comparative progress that CAAs are making toward reducing poverty. Progress and performance outputs are then used as inputs for the resource allocation model to optimally allocate Federal funds.

To provide a comparative analysis, seven CAAs and four counties without CAAs in Georgia are used in the example. Data sources are identified in the computational section for each parameter.

The following CAAs and counties served are used in the example:

<table>
<thead>
<tr>
<th>CAA Number</th>
<th>CAA Name</th>
<th>Counties Served</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Central Ogeechee Community Action Authority, Inc.</td>
<td>Emanuel, Jefferson</td>
</tr>
<tr>
<td>2</td>
<td>Central Savannah River Area</td>
<td>Burke, Richmond</td>
</tr>
<tr>
<td>3</td>
<td>Clayton County Economic Opportunity Authority</td>
<td>Clayton</td>
</tr>
<tr>
<td>4</td>
<td>DeKalb County Economic Opportunity Authority</td>
<td>DeKalb</td>
</tr>
<tr>
<td>5</td>
<td>Economic Opportunity for Savannah-Chatham County Area, Inc.</td>
<td>Chatham</td>
</tr>
<tr>
<td>6</td>
<td>Gainesville-Hall County Economic Opportunity Organization, Inc.</td>
<td>Hall, Jackson</td>
</tr>
<tr>
<td>7</td>
<td>Macon-Bibb County Economic Opportunity Council</td>
<td>Bibb</td>
</tr>
</tbody>
</table>
The following counties without CAAs are used in the example:

Calhoun, Cobb, Jones, and Warren.

The population for the respective areas are shown in Table 1. 23

Table 1. CAA and County Population

<table>
<thead>
<tr>
<th></th>
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<th></th>
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<th></th>
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</tr>
</thead>
<tbody>
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<td>2</td>
<td>170,849</td>
<td>173,343</td>
<td>175,793</td>
<td>178,243</td>
<td>180,692</td>
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<tr>
<td>3</td>
<td>77,372</td>
<td>82,539</td>
<td>87,707</td>
<td>92,875</td>
<td>98,043</td>
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<tr>
<td>4</td>
<td>351,945</td>
<td>367,805</td>
<td>383,666</td>
<td>399,526</td>
<td>415,387</td>
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<tr>
<td>5</td>
<td>187,980</td>
<td>187,927</td>
<td>187,873</td>
<td>187,820</td>
<td>187,767</td>
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<td>6</td>
<td>75,594</td>
<td>76,820</td>
<td>78,046</td>
<td>79,272</td>
<td>80,498</td>
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<tr>
<td>7</td>
<td>142,550</td>
<td>142,767</td>
<td>142,984</td>
<td>143,201</td>
<td>143,418</td>
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<tr>
<td>Calhoun</td>
<td>6,900</td>
<td>6,826</td>
<td>6,753</td>
<td>6,679</td>
<td>6,606</td>
</tr>
<tr>
<td>Cobb</td>
<td>163,745</td>
<td>172,007</td>
<td>180,269</td>
<td>188,531</td>
<td>196,793</td>
</tr>
<tr>
<td>Jones</td>
<td>10,718</td>
<td>11,093</td>
<td>11,468</td>
<td>11,843</td>
<td>12,218</td>
</tr>
<tr>
<td>Warren</td>
<td>6,945</td>
<td>6,876</td>
<td>6,807</td>
<td>6,738</td>
<td>6,669</td>
</tr>
</tbody>
</table>

23. Population data were obtained from the Bureau of the Census, Department of Commerce, Atlanta, Georgia.
Progress Model Results

Health Model

The data used in the Health Model are shown in Table 2.24

Table 2. Health Data

<table>
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<td></td>
</tr>
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<td></td>
</tr>
<tr>
<td>i</td>
<td></td>
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<td></td>
</tr>
<tr>
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<td></td>
</tr>
<tr>
<td>d</td>
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<td></td>
</tr>
<tr>
<td>i</td>
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<tr>
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</tr>
<tr>
<td>d</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Using the Health Model from Chapter III,

\[ A_1 = \frac{f_i}{p_i} + \frac{i_i}{p_i} + \frac{d_i}{p_i} \]

For 1966 and CAA 1,

\[ A_1 = \frac{23 + 35 + 1}{35,331} = 0.00167 \]

24. Health data were obtained from the Biostatistics Division, Georgia Department of Public Health, Atlanta, Georgia.
For 1967 and CAA 1,

\[ A_1 = \frac{25 + 31 + 1}{35,339} \approx 0.00161. \]

For the years 1966-1970, Table 3 shows \( A_{i,t} \) for each CAA and county.

### Table 3. Progress with Respect to Health

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( A_{1,1} )</td>
<td>( A_{1,2} )</td>
<td>( A_{1,3} )</td>
<td>( A_{1,4} )</td>
<td>( A_{1,5} )</td>
<td>( \Delta A_{1,5} )</td>
</tr>
<tr>
<td>1</td>
<td>.00167</td>
<td>.00161</td>
<td>.00139</td>
<td>.00133</td>
<td>.00153</td>
<td>-.000192</td>
</tr>
<tr>
<td>2</td>
<td>.00169</td>
<td>.00155</td>
<td>.00168</td>
<td>.00159</td>
<td>.00164</td>
<td>-.0000423</td>
</tr>
<tr>
<td>3</td>
<td>.00119</td>
<td>.00101</td>
<td>.00113</td>
<td>.00144</td>
<td>.00173</td>
<td>-.000322</td>
</tr>
<tr>
<td>4</td>
<td>.00070</td>
<td>.00068</td>
<td>.00091</td>
<td>.00110</td>
<td>.00115</td>
<td>-.0000714</td>
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<tr>
<td>5</td>
<td>.00150</td>
<td>.00161</td>
<td>.00152</td>
<td>.00189</td>
<td>.00205</td>
<td>-.000196</td>
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<tr>
<td>6</td>
<td>.00091</td>
<td>.00052</td>
<td>.00064</td>
<td>.00050</td>
<td>.000459</td>
<td>+.0000542</td>
</tr>
<tr>
<td>7</td>
<td>.00110</td>
<td>.00085</td>
<td>.000911</td>
<td>.00087</td>
<td>.00111</td>
<td>-.000236</td>
</tr>
<tr>
<td>Calhoun</td>
<td>.00058</td>
<td>.00059</td>
<td>.00103</td>
<td>.00149</td>
<td>.00121</td>
<td>+.0002295</td>
</tr>
<tr>
<td>Cobb</td>
<td>.00087</td>
<td>.00070</td>
<td>.00083</td>
<td>.00088</td>
<td>.00112</td>
<td>-.0002462</td>
</tr>
<tr>
<td>Jones</td>
<td>.00177</td>
<td>.00063</td>
<td>.00070</td>
<td>.00068</td>
<td>.00090</td>
<td>-.0002177</td>
</tr>
<tr>
<td>Warren</td>
<td>.00100</td>
<td>.00102</td>
<td>.00162</td>
<td>.00074</td>
<td>.00135</td>
<td>-.000528</td>
</tr>
</tbody>
</table>

To determine progress with respect to health, recall that

\[ \Delta A_{i,t} = \bar{A}_{i,t-1} - A_{i,t}, \]

where

\[ \bar{A}_{i,t-1} = \alpha \sum_{t=1}^{n-1} (1 - \alpha)^{n-t-1} A_{i,t}, \quad 0 \leq \alpha \leq 1. \]
Recall also that $\alpha$ is selected by minimizing

$$\sigma^2 = \frac{\sum_{i=1}^{z} (\Delta A_{i,t} - \bar{\Delta A}_{.,t})^2}{z - 1}.$$

Exponential smoothing constants ($\alpha$) to one decimal place are used in practical application in the fields of inventory and production control. Consequently, only one decimal place $\alpha$'s are used in this example.

Let $\alpha = 0.5$

$$\Delta A_{1,5} = \bar{A}_{1,4} - A_{1,5}$$

$$= 0.5 \left[ .00133 + 0.5(.00139) + 0.25(.00161) + 0.125(.00167) \right]$$

$$- 0.00153$$

$$= -0.000212$$

$$\Delta A_{2,5} = -0.000527$$

$$\Delta A_{3,5} = -0.000244$$

$$\Delta A_{4,5} = -0.000430$$

$$\Delta A_{5,5} = +0.00073$$

$$\Delta A_{6,5} = -0.000272$$

$$\Delta A_{7,5} = -0.000126$$

$$\Delta A_{8,5} = -0.000521$$

and

$$\sigma^2_{0.5} = \frac{\sum_{i=1}^{7} (\Delta A_{1,5} - \bar{\Delta A}_{.,5})^2}{6}$$

$$= \frac{\left[ (-0.001599)^2 + (-0.000006)^2 + \ldots + (+0.000394)^2 \right]}{6}$$

$$= 0.0000005353.$$
Let $\alpha = 0.9$

\[ \Delta A_{1,5} = 0.9 \left[ 0.00133 + 0.1(0.00139) + 0.01(0.00161) + 0.001(0.00167) \right] \]

\[ - 0.00153 \]

\[ = -0.000192 \]

\[ \Delta A_{2,5} = -0.000322 \]

\[ \Delta A_{3,5} = -0.0000714 \]

\[ \Delta A_{4,5} = -0.000196 \]

\[ \Delta A_{5,5} = +0.0000542 \]

\[ \Delta A_{6,5} = -0.000236 \]

\[ \Delta A_{7,5} = -0.0000423 \]

\[ \Delta A_{8,5} = -0.000143 \]

\[ \sigma^2_{0.9} = \frac{[(-0.000048)^2 + (-0.000178)^2 + \ldots + (0.000101)^2]}{6} \]

\[ = 0.0000000414 \]

Values for $\Delta A_{i,5}$ were also computed for $\alpha = 0.1, 0.2, 0.3, 0.4, 0.6, 0.7, 0.8, \text{ and } 1.0$, but $\alpha = 0.9$ is the value of $\alpha$ that minimizes $\sigma^2_{\alpha}$.

Hence, $\Delta A_{i,5}$ for $\alpha = 0.9$ are the values of progress with respect to health that are to be used in the Total Progress Model. These values are recorded under $\Delta A_{i,5}$ in Table 3.

As shown in Table 3, CAA 6 and Calhoun County are the only areas that exhibit positive progress with respect to health.

Education Model

The data used in the Education Model are shown in Table 4.\(^{25}\)

\(^{25}\) Education data were obtained from the Division of Statistics and Research, Adult Education Branch, Vocational Education Branch, Georgia Department of Education, Atlanta, Georgia.
Table 4. Education Data

<table>
<thead>
<tr>
<th>CAA/County</th>
<th>Year</th>
<th>$e_i$</th>
<th>$v_i$</th>
<th>$a_i$</th>
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<td>2599</td>
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<td>131</td>
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<td>1970</td>
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<td>( v_1 )</td>
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<td></td>
<td>1970</td>
<td>483</td>
<td>407</td>
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</tbody>
</table>
Using the Education Model from Chapter III,

\[ D_1 = \frac{a_i}{P_i} + \frac{a_i}{P_i} + \frac{v_i}{P_i} \]

for 1966, and CAA 1,

\[ D_1 = \frac{1}{35,331} \left( 2599 + 0 + 2014 \right) = 0.13057 \]

For 1968, and CAA 1,

\[ D_1 = \frac{1}{35,347} \left( 2614 + 148 + 2419 \right) = 0.14657 \]

For the years 1966-1970, Table 5 shows \( D_1 \) for each CAA and county.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
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<tbody>
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<td>( D ), .4</td>
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<tr>
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<td>.15519</td>
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<td>Cobb</td>
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<td>.06707</td>
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<td>.11120</td>
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<td>-0.01013</td>
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<td>.12619</td>
<td>.11888</td>
<td>.15010</td>
<td>0.03073</td>
</tr>
</tbody>
</table>
To determine progress with respect to education, compute

$$\Delta D_{i,t} = D_{i,t} - \bar{D}_{i,t-1}$$

and select $\alpha$ by the same procedure used for the Health Model results.

Let $\alpha = 0.5$

$$\Delta D_{1,5} = 0.17648 - 0.5 \left[ 0.15605 + 0.5(0.14657) + 0.25(0.15866) + 0.125(0.13047) \right]$$

$$= 0.03382$$

$$\Delta D_{2,5} = 0.03899$$

$$\Delta D_{3,5} = -0.00581$$

$$\Delta D_{4,5} = 0.03238$$

$$\Delta D_{5,5} = 0.01456$$

$$\Delta D_{6,5} = 0.00779$$

$$\Delta D_{7,5} = 0.04790$$

$$\Delta \bar{D}_{7,5} = 0.02423$$

and

$$\sigma_{0.5}^2 = \frac{\sum_{i=1}^{7} (\Delta D_{i,5} - \Delta \bar{D}_{7,5})^2}{6}$$

$$= \frac{[(0.00958)^2 + (0.01476)^2 + \ldots + (0.02367)^2]}{6}$$

$$= 0.0003922$$

Let $\alpha = 0.9$

$$\Delta D_{1,5} = 0.17648 - 0.9 \left[ 0.15605 + 0.1(0.14657) + 0.01(0.15866) + 0.001(0.13047) \right]$$

$$= 0.02129$$
\[ \Delta D_{2,5} = 0.02997 \]
\[ \Delta D_{3,5} = -0.01036 \]
\[ \Delta D_{4,5} = 0.02687 \]
\[ \Delta D_{5,5} = 0.00436 \]
\[ \Delta D_{6,5} = 0.00390 \]
\[ \Delta D_{7,5} = 0.03574 \]
\[ \Delta D_{.5} = 0.01597 \]

and

\[ \sigma_{0.9}^2 = \frac{[(0.00532)^2 + (0.01400)^2 + \ldots + (0.01977)^2]}{6} \]

\[ = 0.0002845 \]

\( \alpha = 0.9 \) is the value of \( \alpha \) that minimizes \( \sigma_{\alpha}^2 \); hence, \( \Delta D_{1,5} \) for \( \alpha = 0.9 \) are the values of progress with respect to education that are to be used in the Total Progress Model. These values are recorded under \( \Delta D_{.5} \) in Table 5.

As shown in Table 5, all CAAs, except CAA 3, have positive progress with respect to education. Of the counties considered, only Cobb and Warren Counties have positive progress.

Housing Model

The data used in the Housing Model are shown in Table 6.\(^{26}\) Since data are not available for the number of sub-standard units razed since the last census \( (Y_i) \) and the number of sub-standard units renovated since the last census \( (\beta_i) \), \( Y_i \) and \( \beta_i \) are neither shown in Table 6 nor used in

---

26. Data for total all-year-round housing units and sub-standard units are provided by the Bureau of the Census, Department of Commerce. Data for low income units completed, under construction, and percentage of completion are provided by the Program Coordination and Services Office, Region IV, Department of Housing and Urban Development, Atlanta, Georgia.
the Housing Model example. However, when the data for $\gamma_i$ and $\beta_i$ become available, their values should be used in the Housing Model as developed in Chapter III.

Table 6. Housing Data

<table>
<thead>
<tr>
<th>CAA/County</th>
<th>1969</th>
<th></th>
<th>1970</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$H$</td>
<td>$h$</td>
<td>$N_i$</td>
<td>$L_i$</td>
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<tr>
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<td>10898</td>
<td>2665</td>
<td>168</td>
<td>0</td>
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<td>2</td>
<td>44095</td>
<td>8683</td>
<td>898</td>
<td>0</td>
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<td>3</td>
<td>12864</td>
<td>1307</td>
<td>35</td>
<td>0</td>
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<td>76875</td>
<td>5382</td>
<td>235</td>
<td>0</td>
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<td>10703</td>
<td>400</td>
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<td>4523</td>
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<td>0</td>
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<td>742</td>
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<td>0</td>
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<td>33135</td>
<td>2963</td>
<td>100</td>
<td>0</td>
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<td></td>
<td>2282</td>
<td>532</td>
<td>0</td>
<td>0</td>
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<td></td>
<td>2142</td>
<td>798</td>
<td>0</td>
<td>0</td>
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Using the Housing Model without the variables $\gamma_i$ and $\beta_i$,

$$N_i,t = \frac{\mathcal{L}_i}{H_i + \mathcal{L}_i} - \frac{\psi_i,t(L_i,t)}{H_i + \mathcal{L}_i + \psi_i,t(L_i,t)} - \frac{h_i}{H_i} .$$

For 1969 and CAA 4,

$$N_4 = \frac{235}{76875} - \frac{5382}{76875} = -0.066962 .$$

For 1970 and CAA 4,

$$N_4 = \frac{0}{129656} + \frac{.97(108)}{129656 + .97(108)} - \frac{1941}{129656} = -0.014163 .$$
For 1969 and 1970, Table 7 shows \( N_i \) for each CAA and county.

Table 7. Progress with Respect to Housing

<table>
<thead>
<tr>
<th>CAA/County</th>
<th>1969 ( N_{i,1} )</th>
<th>1970 ( N_{i,2} )</th>
<th>Progress ( \Delta N_{i,2} )</th>
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<tr>
<td>1</td>
<td>- .229359</td>
<td>- .349787</td>
<td>- .1662998</td>
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<tr>
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<td>- .176957</td>
<td>- .090092</td>
<td>+ .0514736</td>
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<tr>
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<td>- .098888</td>
<td>- .029964</td>
<td>+ .0491464</td>
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<td>4</td>
<td>- .066962</td>
<td>- .014163</td>
<td>+ .0394066</td>
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<td>5</td>
<td>- .173716</td>
<td>- .071099</td>
<td>+ .0678735</td>
</tr>
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<td>6</td>
<td>- .192366</td>
<td>- .147990</td>
<td>+ .0059420</td>
</tr>
<tr>
<td>7</td>
<td>- .099694</td>
<td>- .079389</td>
<td>+ .0003662</td>
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<td>Calhoun</td>
<td>- .320380</td>
<td>- .465728</td>
<td>- .2094224</td>
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<td>Cobb</td>
<td>- .086413</td>
<td>- .028310</td>
<td>+ .0408204</td>
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<tr>
<td>Jones</td>
<td>- .233129</td>
<td>- .257974</td>
<td>- .0714708</td>
</tr>
<tr>
<td>Warren</td>
<td>- .372549</td>
<td>- .415018</td>
<td>- .1169788</td>
</tr>
</tbody>
</table>

Progress with respect to housing is determined by

\[
\Delta N_{i,t} = N_{i,t} - \bar{N}_{i,t-1}
\]

For CAA 1, and \( \alpha = 0.8 \)

\[
\Delta N_{1,2} = - .349787 - 0.8(-.229359) = - 0.1662998
\]

The value of \( \alpha \) that minimizes

\[
\sigma^2 = \frac{1}{6} \sum_{t=1}^{7} (\Delta N_{1,2} - \bar{\Delta N}_{1,2})^2
\]
is determined using the same procedure previously described. The value of \( \alpha \) that minimizes \( \sigma_\alpha^2 \) is \( \alpha = 0.8 \). For \( \alpha = 0.8 \), progress with respect to housing is recorded under \( \Delta N \), of Table 7.

As shown in Table 7, all CAA's, except CAA 1, have made positive progress toward reducing poverty with respect to housing. Of the four counties considered in the example, only Cobb County exhibits positive progress with respect to housing.

**Income Model**

**Earned Income.** The data used in the Earned Income Model are shown in Table 8.

<table>
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<th>CAA/County</th>
<th>( P_1 )</th>
<th>( P_i )</th>
<th>( M_1 )</th>
<th>( m_i )</th>
<th>( P_1 )</th>
<th>( M_1 )</th>
<th>( m_i )</th>
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<td>806</td>
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<td>8721</td>
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<td>624</td>
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<td>347</td>
<td>34098</td>
<td>1757</td>
<td>856</td>
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<td>1538</td>
<td>403</td>
<td>8798</td>
<td>1684</td>
<td>645</td>
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<td>32166</td>
<td>2072</td>
<td>435</td>
<td>38196</td>
<td>2369</td>
<td>763</td>
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<td>405</td>
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<td>1827</td>
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<td>406</td>
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<td>1899</td>
<td>706</td>
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<td>493</td>
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<td>1708</td>
<td>437</td>
<td>22639</td>
<td>2161</td>
<td>761</td>
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<td>1523</td>
<td>1474</td>
<td>620</td>
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<td>288</td>
<td>1413</td>
<td>1351</td>
<td>658</td>
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</table>

27. The data for 1960 income were provided by the Bureau of the Census, Department of Commerce, Atlanta, Georgia. The data for 1966 were consolidated from personal income tax returns by the Internal Revenue Service and distributed by the National Technical Information Service, Department of Commerce, Springfield, Virginia. 1968 personal income
E_i and \( \Delta E_{i,t} \) are computed using the Earned Income Model from Chapter III,

\[
E_{i,t} = \frac{P_{i,t}}{P_{i,t}} \cdot \frac{M_{i,t}}{m_{i,t}}
\]

and

\[
\Delta E_{i,t} = \bar{E}_{i,t-1} - E_{i,t}
\]

\( \alpha = 0.3 \) is the value of \( \alpha \) that minimizes \( \sigma^2 \) for earned income. Using the computational procedure previously described, the values of \( E_{i,t} \) for 1960 and 1966, and \( \Delta E_{i,t} \) (\( \alpha = 0.3 \)) for each CAA and county are shown in Table 9.

Table 9. Progress with Respect to Earned Income

<table>
<thead>
<tr>
<th>CAA/County</th>
<th>1960 E_{i,t}^1</th>
<th>1966 E_{i,t}^2</th>
<th>Progress ( \Delta E_{i,t} )</th>
</tr>
</thead>
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<td>0.25431</td>
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<tr>
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<td>-0.18602</td>
</tr>
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<td>-0.20264</td>
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<tr>
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<td>1.1477</td>
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<tr>
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<td>0.02107</td>
</tr>
</tbody>
</table>

data will be available in early 1972, and annual data will be available in 1973 or 1974.

28. The mean incomes are mean incomes per person as opposed to mean incomes per family, and are adjusted to the 1957-1959 base in accordance with the Consumer Price Index for Urban Wage Earners and Clerical Workers, U. S. Department of Commerce.
As shown in Table 9, CAA 2 is the only CAA that exhibits positive progress with respect to earned income. Of the counties used in the example, only Jones and Warren Counties exhibit positive progress.

**Social Security Income.** The data used in the Social Security Model are shown in Table 10.29

**Table 10. Social Security Data**30

<table>
<thead>
<tr>
<th>CAA/County</th>
<th>1967 q_i s_i</th>
<th>1968 q_i s_i</th>
<th>1969 q_i s_i</th>
<th>1970 q_i s_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4686 665</td>
<td>4560 616</td>
<td>4794 676</td>
<td>4988 736</td>
</tr>
<tr>
<td>2</td>
<td>17049 741</td>
<td>17547 744</td>
<td>18338 812</td>
<td>19092 890</td>
</tr>
<tr>
<td>3</td>
<td>4001 875</td>
<td>4344 821</td>
<td>4775 897</td>
<td>5316 989</td>
</tr>
<tr>
<td>4</td>
<td>16995 890</td>
<td>21267 881</td>
<td>27583 977</td>
<td>31258 1050</td>
</tr>
<tr>
<td>5</td>
<td>20023 799</td>
<td>20711 806</td>
<td>21354 886</td>
<td>22042 940</td>
</tr>
<tr>
<td>6</td>
<td>9098 711</td>
<td>9500 719</td>
<td>9886 787</td>
<td>10359 869</td>
</tr>
<tr>
<td>7</td>
<td>16702 755</td>
<td>17314 759</td>
<td>18019 829</td>
<td>18662 909</td>
</tr>
<tr>
<td>Calhoun</td>
<td>1217 584</td>
<td>1266 587</td>
<td>1247 578</td>
<td>1313 719</td>
</tr>
<tr>
<td>Cobb</td>
<td>11578 836</td>
<td>12339 843</td>
<td>13171 902</td>
<td>14244 1024</td>
</tr>
<tr>
<td>Jones</td>
<td>953 613</td>
<td>974 621</td>
<td>1018 667</td>
<td>1059 721</td>
</tr>
<tr>
<td>Warren</td>
<td>689 616</td>
<td>721 618</td>
<td>719 605</td>
<td>792 737</td>
</tr>
</tbody>
</table>

\[ S_{i,t} \] and \( \Delta S_{i,t} \) are computed using the Social Security Model

\[ S_{i,t} = \frac{q_{i,t}}{p_{i,t}} \cdot \frac{M_{i,t}}{s_{i,t}}, \]

29. The data for Social Security were provided by the Social Security Administration Office, Atlanta, Georgia.

30. The mean Social Security payments are adjusted to the 1957-1959 base as previously discussed.
and

\[ \Delta S_{i,t} = S_{i,t-1} - S_{i,t} \]

\( \alpha = 0.6 \) is the value of \( \alpha \) that minimizes \( \frac{\sigma^2}{\alpha} \) for Social Security. The computational procedure described for \( E_{i,t} \) and \( \Delta E_{i,t} \) is used to compute the values of \( S_{i,t} \) and \( \Delta S_{i,t} \) shown in Table 11.

A decrease in \( S_{i,t} \) from period to period generally represents positive progress which should take the form of a stable ratio \( q_i/P_i \), while decreasing the ratio \( M_i/S_i \). Four of the seven CAAs and three of the four separate counties exhibit positive progress with respect to Social Security.

Table 11. Progress with Respect to Social Security

<table>
<thead>
<tr>
<th>CAA/County</th>
<th>1967</th>
<th>1968</th>
<th>1969</th>
<th>1970</th>
<th>Progress</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( S_{.,1} )</td>
<td>( S_{.,2} )</td>
<td>( S_{.,3} )</td>
<td>( S_{.,4} )</td>
<td>( \Delta S_{.,4} )</td>
</tr>
<tr>
<td>1</td>
<td>.27949</td>
<td>.29337</td>
<td>.28094</td>
<td>.26871</td>
<td>-.00291</td>
</tr>
<tr>
<td>2</td>
<td>.23321</td>
<td>.23583</td>
<td>.22244</td>
<td>.20848</td>
<td>.00397</td>
</tr>
<tr>
<td>3</td>
<td>.09327</td>
<td>.10167</td>
<td>.09649</td>
<td>.09231</td>
<td>-.00106</td>
</tr>
<tr>
<td>4</td>
<td>.12297</td>
<td>.14915</td>
<td>.16744</td>
<td>.16976</td>
<td>-.02169</td>
</tr>
<tr>
<td>5</td>
<td>.24350</td>
<td>.24973</td>
<td>.23442</td>
<td>.22189</td>
<td>.00207</td>
</tr>
<tr>
<td>6</td>
<td>.28076</td>
<td>.28537</td>
<td>.26708</td>
<td>.24952</td>
<td>.00617</td>
</tr>
<tr>
<td>7</td>
<td>.29425</td>
<td>.30315</td>
<td>.28815</td>
<td>.27189</td>
<td>.00200</td>
</tr>
<tr>
<td>Calhoun</td>
<td>.41898</td>
<td>.43322</td>
<td>.44278</td>
<td>.37918</td>
<td>.03068</td>
</tr>
<tr>
<td>Cobb</td>
<td>.17408</td>
<td>.17554</td>
<td>.16739</td>
<td>.15276</td>
<td>.00652</td>
</tr>
<tr>
<td>Jones</td>
<td>.14871</td>
<td>.20168</td>
<td>.19004</td>
<td>.17707</td>
<td>-.00037</td>
</tr>
<tr>
<td>Warren</td>
<td>.21967</td>
<td>.23154</td>
<td>.23838</td>
<td>.21784</td>
<td>.00185</td>
</tr>
</tbody>
</table>
Welfare Income. The data used in the Welfare Model are shown in Table 12.31

Table 12. Welfare Data32

<table>
<thead>
<tr>
<th>CAA/County</th>
<th>1968</th>
<th>1969</th>
<th>1970</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( r_i )</td>
<td>( w_i )</td>
<td>( r_i )</td>
</tr>
<tr>
<td>1</td>
<td>3555</td>
<td>435</td>
<td>4622</td>
</tr>
<tr>
<td>2</td>
<td>8447</td>
<td>408</td>
<td>10920</td>
</tr>
<tr>
<td>3</td>
<td>1030</td>
<td>454</td>
<td>1212</td>
</tr>
<tr>
<td>4</td>
<td>5319</td>
<td>439</td>
<td>6796</td>
</tr>
<tr>
<td>5</td>
<td>11613</td>
<td>408</td>
<td>13606</td>
</tr>
<tr>
<td>6</td>
<td>3294</td>
<td>470</td>
<td>3749</td>
</tr>
<tr>
<td>7</td>
<td>8091</td>
<td>396</td>
<td>10215</td>
</tr>
<tr>
<td>Calhoun</td>
<td>516</td>
<td>502</td>
<td>631</td>
</tr>
<tr>
<td>Cobb</td>
<td>2214</td>
<td>452</td>
<td>3024</td>
</tr>
<tr>
<td>Jones</td>
<td>995</td>
<td>339</td>
<td>1107</td>
</tr>
<tr>
<td>Warren</td>
<td>860</td>
<td>428</td>
<td>1098</td>
</tr>
</tbody>
</table>

\( \alpha = 0.9 \) is the value of \( \alpha \) that minimizes \( \sigma^2 \) for welfare. \( W_{i,t} \) and \( \Delta W_{i,t} \) are computed using the procedure previously described and the equations

\[
W_{i,t} = \frac{r_{i,t}}{p_{i,t}} \cdot \frac{M_{i,t}}{w_{i,t}},
\]

and

\[
\Delta W_{i,t} = W_{i,t-1} - W_{i,t}.
\]

31. The data for Welfare were provided by the State Department of Family and Children Services, Atlanta, Georgia.

32. The mean welfare payments are adjusted as previously discussed.
The values for $W_{i,t}$ and $\Delta W_{i,t}$ are shown in Table 13.

### Table 13. Progress with Respect to Welfare

<table>
<thead>
<tr>
<th>CAA/County</th>
<th>1968 $W_{i,1}$</th>
<th>1969 $W_{i,2}$</th>
<th>1970 $W_{i,3}$</th>
<th>Progress $\Delta W_{i,3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.32400</td>
<td>0.46149</td>
<td>0.52601</td>
<td>-0.08151</td>
</tr>
<tr>
<td>2</td>
<td>0.20711</td>
<td>0.29053</td>
<td>0.39438</td>
<td>-0.11426</td>
</tr>
<tr>
<td>3</td>
<td>0.04352</td>
<td>0.05189</td>
<td>0.07279</td>
<td>-0.02217</td>
</tr>
<tr>
<td>4</td>
<td>0.07476</td>
<td>0.10198</td>
<td>0.12834</td>
<td>-0.02983</td>
</tr>
<tr>
<td>5</td>
<td>0.27702</td>
<td>0.34248</td>
<td>0.42615</td>
<td>-0.09299</td>
</tr>
<tr>
<td>6</td>
<td>0.15121</td>
<td>0.18100</td>
<td>0.20430</td>
<td>-0.02779</td>
</tr>
<tr>
<td>7</td>
<td>0.27131</td>
<td>0.36744</td>
<td>0.48829</td>
<td>-0.13318</td>
</tr>
<tr>
<td>Calhoun</td>
<td>0.20860</td>
<td>0.29306</td>
<td>0.40023</td>
<td>-0.11770</td>
</tr>
<tr>
<td>Cobb</td>
<td>0.05878</td>
<td>0.08430</td>
<td>0.10049</td>
<td>-0.01933</td>
</tr>
<tr>
<td>Jones</td>
<td>0.37763</td>
<td>0.42227</td>
<td>0.40663</td>
<td>0.00739</td>
</tr>
<tr>
<td>Warren</td>
<td>0.39886</td>
<td>0.57505</td>
<td>0.64478</td>
<td>-0.09134</td>
</tr>
</tbody>
</table>

Positive progress with respect to welfare is achieved in the same manner as positive progress with respect to Social Security. However, unlike Social Security, the number of recipients of welfare payments in each area increased proportionally more from period to period than did the population, and the ratio $M_i/w_i$ also generally increased. Consequently, all CAAs and three of the four counties exhibit negative progress.

**Total Income.** Progress with respect to total income is computed by

$$\Delta I_{i,t} = \Delta F_{i,t} + \Delta S_{i,t} + \Delta W_{i,t}$$

$\Delta I_{i,t}$ for each CAA and county is shown in Table 14.
Table 14. Progress with Respect to Total Income

<table>
<thead>
<tr>
<th>CAA/County</th>
<th>$\Delta E_{i,t}$</th>
<th>$\Delta S_{i,t}$</th>
<th>$\Delta W_{i,t}$</th>
<th>$\Delta I_{i,t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-.10825</td>
<td>-.00291</td>
<td>-.08151</td>
<td>-.19257</td>
</tr>
<tr>
<td>2</td>
<td>.25431</td>
<td>.00397</td>
<td>.11426</td>
<td>.14402</td>
</tr>
<tr>
<td>3</td>
<td>-.13934</td>
<td>-.00106</td>
<td>-.02217</td>
<td>-.16257</td>
</tr>
<tr>
<td>4</td>
<td>-.15825</td>
<td>-.02169</td>
<td>-.02983</td>
<td>-.20977</td>
</tr>
<tr>
<td>5</td>
<td>-.18602</td>
<td>.00207</td>
<td>-.09299</td>
<td>-.28108</td>
</tr>
<tr>
<td>6</td>
<td>-.24530</td>
<td>.00617</td>
<td>-.02779</td>
<td>-.26692</td>
</tr>
<tr>
<td>7</td>
<td>-.22150</td>
<td>.00200</td>
<td>-.13318</td>
<td>-.35268</td>
</tr>
<tr>
<td>Calhoun</td>
<td>-.20264</td>
<td>.03068</td>
<td>-.11770</td>
<td>-.28966</td>
</tr>
<tr>
<td>Cobb</td>
<td>-.20767</td>
<td>.00652</td>
<td>-.01933</td>
<td>-.22048</td>
</tr>
<tr>
<td>Jones</td>
<td>.00639</td>
<td>-.00037</td>
<td>.00739</td>
<td>.01341</td>
</tr>
<tr>
<td>Warren</td>
<td>.02107</td>
<td>.00185</td>
<td>-.09134</td>
<td>-.06842</td>
</tr>
</tbody>
</table>

As shown in Table 14, CAA 2 and Jones County are the only areas that have positive progress with respect to total income.

**Total Progress**

Using the relative importance values developed in Chapter III for each parameter of poverty, and progress made toward reducing poverty with respect to each parameter, the total progress for each CAA and county can be computed. Total progress is computed by using Eq. (4), Chapter III:

$$T_{i,t} = (R_1 \cdot \Delta I_{i,t}) + (R_2 \cdot \Delta D_{i,t}) + (R_3 \cdot \Delta N_{i,t}) + (R_4 \cdot \Delta A_{i,t})$$

CAA and county total progress is shown in Table 15.
Table 15. Total Progress

<table>
<thead>
<tr>
<th>CAA/County</th>
<th>$T_{.,t}$</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.10298</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>0.07325</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>-0.05228</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>-0.06135</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>-0.08747</td>
<td>7</td>
</tr>
<tr>
<td>6</td>
<td>-0.09633</td>
<td>8</td>
</tr>
<tr>
<td>7</td>
<td>-0.12050</td>
<td>10</td>
</tr>
<tr>
<td>Calhoun</td>
<td>-0.15932</td>
<td>11</td>
</tr>
<tr>
<td>Cobb</td>
<td>-0.02567</td>
<td>3</td>
</tr>
<tr>
<td>Jones</td>
<td>-0.01403</td>
<td>2</td>
</tr>
<tr>
<td>Warren</td>
<td>-0.04328</td>
<td>4</td>
</tr>
</tbody>
</table>

Even though the ranking in Table 15 is not used for the purpose of analysis, it is interesting to note that three of the four counties used in this example rank in the top four in total progress.

**Progress Measure Correlation Tests**

As discussed in Chapter III, it is necessary to determine if interactions exist between progress with respect to each parameter. To test for an interaction, Eqs. (5), (6), (7), and (8) are used. Using these equations and the progress values from Tables 3 and 14, the test for an income-health correlation is shown below.

$$
\Delta T_{.,t} \approx \sum_{i=1}^{7} \frac{\Delta I_{i, t}}{7} = \frac{-1.3217}{7} = -0.1881
$$
The format shown in Table 16 is helpful for computing correlation tests.

Table 16. Income-Health Correlation Computational Data

<table>
<thead>
<tr>
<th>CAA</th>
<th>((\Delta I_i,t - \Delta I_i,t))</th>
<th>((\Delta I_i,t - \Delta I_i,t)^2)</th>
<th>((\Delta A_{i,t} - \Delta A_{i,t}))</th>
<th>((\Delta A_{i,t} - \Delta A_{i,t})^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-.00386</td>
<td>.000015</td>
<td>-.000480</td>
<td>.0000002304</td>
</tr>
<tr>
<td>2</td>
<td>.33283</td>
<td>.110775</td>
<td>-.000100</td>
<td>.0000000100</td>
</tr>
<tr>
<td>3</td>
<td>.02683</td>
<td>.000689</td>
<td>-.000179</td>
<td>.0000000320</td>
</tr>
<tr>
<td>4</td>
<td>-.02096</td>
<td>.000460</td>
<td>.000072</td>
<td>.0000000052</td>
</tr>
<tr>
<td>5</td>
<td>-.09227</td>
<td>.008510</td>
<td>-.000052</td>
<td>.0000000027</td>
</tr>
<tr>
<td>6</td>
<td>-.07811</td>
<td>.006100</td>
<td>.000198</td>
<td>.0000000391</td>
</tr>
<tr>
<td>7</td>
<td>-.16387</td>
<td>.026850</td>
<td>-.000092</td>
<td>.0000000085</td>
</tr>
</tbody>
</table>

\[ r_{I,A} = \frac{(-.00386)(-.00048) + \ldots + (-.16387)(-.000092)}{\sqrt{(.153389)(.0000003281)}} = -0.148087 \]

Using the "t" statistic to test the hypothesis that income progress and health progress are uncorrelated \((\rho = 0)\), compute

\[ |t| = \left| \frac{r}{\sqrt{1 - r^2}} \right| \sqrt{2z - 2} \]

If \(|t| \geq t_{\frac{z}{2}}\), reject the hypothesis that \(\rho = 0\).

\[ |t| = \left| \frac{.148087}{.98897} \right| \sqrt{2(7) - 2} = 0.5181 \]
\[ t_{.025,12} = 2.179 \quad (\alpha = 0.05) \]

Hence, \( |t| < t_{.025,12} \). Therefore, progress with respect to income is uncorrelated with progress with respect to health.

Next, check for an income-education correlation. Since the values for income have been computed for the income-health correlation, only those values for education need to be computed. The values are shown in Table 17.

\[
\Delta_{\text{D},t} = \frac{\sum_{i=1}^{7} \Delta D_{i,t}}{7} = 0.11177 = 0.01597
\]

**Table 17. Income-Education Correlation Computational Data**

<table>
<thead>
<tr>
<th>CAA</th>
<th>((\Delta D_{i,t} - \Delta_{\text{D},t}))</th>
<th>((\Delta D_{i,t} - \Delta_{\text{D},t})^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.00532</td>
<td>0.0000283</td>
</tr>
<tr>
<td>2</td>
<td>0.01400</td>
<td>0.0001960</td>
</tr>
<tr>
<td>3</td>
<td>-0.02633</td>
<td>0.0006933</td>
</tr>
<tr>
<td>4</td>
<td>0.01090</td>
<td>0.0001188</td>
</tr>
<tr>
<td>5</td>
<td>-0.01161</td>
<td>0.0001348</td>
</tr>
<tr>
<td>6</td>
<td>-0.01207</td>
<td>0.0001457</td>
</tr>
<tr>
<td>7</td>
<td>0.01977</td>
<td>0.0003908</td>
</tr>
</tbody>
</table>

\[
r_{I,D} = \frac{(-.00386)(.00532) + \ldots + (-.16387)(.01977)}{\sqrt{(1.53387)(.0017077)}} \\
= 0.3333
\]
\[ |t| = \left| \frac{r}{\sqrt{1 - r^2}} \right| \sqrt{2z - 2} = 1.225 \]

\[ |t| < t_{0.025, 12}; \text{ therefore, progress with respect to income is uncorrelated with progress with respect to education.} \]

The last correlation test is for an income-housing correlation. The values for this correlation test are shown in Table 18.

\[ \Delta_{i,t} = \frac{\sum_{i=1}^{7} \Delta N_{i,t}}{7} = \frac{0.047908}{7} = 0.006844 \]

Table 18. Income-Housing Correlation Computational Data

<table>
<thead>
<tr>
<th>CAA</th>
<th>((\Delta N_{i,t} - \Delta N_{*,t}))</th>
<th>((\Delta N_{i,t} - \Delta N_{*,t})^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.1731439</td>
<td>0.0299790</td>
</tr>
<tr>
<td>2</td>
<td>0.0446295</td>
<td>0.0019918</td>
</tr>
<tr>
<td>3</td>
<td>0.0423023</td>
<td>0.0017895</td>
</tr>
<tr>
<td>4</td>
<td>0.0325630</td>
<td>0.0010603</td>
</tr>
<tr>
<td>5</td>
<td>0.0610294</td>
<td>0.0037246</td>
</tr>
<tr>
<td>6</td>
<td>-0.0009021</td>
<td>0.0000008</td>
</tr>
<tr>
<td>7</td>
<td>-0.0064779</td>
<td>0.0000419</td>
</tr>
</tbody>
</table>

\[ r_{I,N} = \frac{(-0.00386)(-0.1731439) + \ldots + (-0.16387)(-0.0064779)}{\sqrt{(0.153387)(0.0385879)}} \]

\[ = 0.148837 \]
\[ |t| = \left| \frac{r}{\sqrt{1 - r^2}} \sqrt{2z - 2} \right| = 0.52078 \]

\[ |t| < t_{0.025, 12} \]; hence, progress with respect to income is uncorrelated with progress with respect to housing.

This correlation test procedure can also be used to test for possible interactions between health-education, health-housing, and housing-education. However, only the correlation tests for possible interactions between income and the other parameters are shown in this example.

**Analysis of Progress Results**

To determine if progress with respect to each parameter is significantly different among CAAs and the areas without CAAs, a two-way analysis of variance is used as discussed in Chapter III. The hypothesis being tested is that the comparative CAA progress values, with respect to the parameter tested, are equal. The hypothesis can be written as

\[ H_0: \varphi_0 = \varphi_1 = \varphi_2 = \ldots = \varphi_7 \]

where

\[ \varphi_0 \] represents the areas for which there is no CAA.

Since the parameters of health and education have four progress periods, and income and housing have only one, the analysis of variance will be computed for health and education only. For the parameter of health, compute the four progress values \((\Delta A_{i,2}, \Delta A_{i,3}, \Delta A_{i,4}, \Delta A_{i,5})\) for each CAA using the progress computational procedure previously described.
To determine the progress values for the areas without CAAs (ΔA₀,ₜ), use the mean Aᵢ,ₜ for the four counties.

The results of computing ΔAᵢ,ₜ for each progress period are shown in Table 19.

### Table 19. CAA Health Progress Comparisons

<table>
<thead>
<tr>
<th>Year</th>
<th>ΔA₀,ₜ</th>
<th>ΔA₁,ₜ</th>
<th>ΔA₂,ₜ</th>
<th>ΔA₃,ₜ</th>
<th>ΔA₄,ₜ</th>
<th>ΔA₅,ₜ</th>
<th>ΔA₆,ₜ</th>
<th>ΔA₇,ₜ</th>
<th>δₜ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1967</td>
<td>2.15</td>
<td>-1.07</td>
<td>-.29</td>
<td>.61</td>
<td>-.50</td>
<td>-2.60</td>
<td>2.99</td>
<td>1.40</td>
<td>2.69</td>
</tr>
<tr>
<td>1968</td>
<td>-2.89</td>
<td>2.58</td>
<td>-1.33</td>
<td>-1.14</td>
<td>-2.35</td>
<td>.64</td>
<td>-.90</td>
<td>-.47</td>
<td>-5.86</td>
</tr>
<tr>
<td>1969</td>
<td>.69</td>
<td>3.30</td>
<td>.77</td>
<td>- .31</td>
<td>-2.14</td>
<td>-3.64</td>
<td>1.32</td>
<td>.29</td>
<td>.28</td>
</tr>
<tr>
<td>1970</td>
<td>-1.91</td>
<td>-1.92</td>
<td>-.42</td>
<td>-3.22</td>
<td>-.71</td>
<td>1.96</td>
<td>.54</td>
<td>-2.36</td>
<td>-8.04</td>
</tr>
<tr>
<td>Totals</td>
<td>-1.96</td>
<td>2.89</td>
<td>-1.27</td>
<td>-4.06</td>
<td>-5.70</td>
<td>-3.64</td>
<td>3.94</td>
<td>-1.14</td>
<td></td>
</tr>
<tr>
<td>ΔAᵢ,ₜ</td>
<td>-.49</td>
<td>.72</td>
<td>-.32</td>
<td>-1.02</td>
<td>-1.43</td>
<td>-.91</td>
<td>.99</td>
<td>-.29</td>
<td></td>
</tr>
<tr>
<td>∑ΔAᵢ,ₜ²</td>
<td>17.10</td>
<td>22.38</td>
<td>2.62</td>
<td>12.14</td>
<td>10.99</td>
<td>24.26</td>
<td>11.80</td>
<td>7.82</td>
<td></td>
</tr>
</tbody>
</table>

\[ \varphi_\ldots = \sum_{i=0}^{7} \sum_{t=2}^{5} \Delta A_{i,t} = 2.15 + \ldots - 2.36 = -10.93 \]

\[ \sum_{i=0}^{7} \sum_{t=2}^{5} \Delta A_{i,t}^2 = 17.10 + 22.38 + \ldots + 7.82 = 109.11 \]

33. Each entry in Table 19 is multiplied by 10,000.
\[
SS_{\text{total}} = \sum_{i=0}^{7} \sum_{t=2}^{5} \frac{\Delta A_{i,t}^2}{N} = 109.11 - \frac{(-10.93)^2}{32} = 105.38
\]

\[
SS_{\text{time}} = \sum_{t=2}^{5} \frac{Z_{i,t}^2}{N} = \frac{(2.69)^2}{8} + \ldots + \frac{(-8.04)^2}{8} - \frac{(10.93)^2}{32} = 4.35
\]

\[
SS_{\text{area}} = \sum_{i=0}^{7} \frac{\Delta A_{i,i}^2}{n} = \frac{(-1.96)^2}{4} + \ldots + \frac{(-1.16)^2}{4} - \frac{(10.93)^2}{32} = 19.47
\]

\[
SS_{\text{error}} = SS_{\text{total}} - SS_{\text{area}} - SS_{\text{time}} = 105.38 - 19.47 - 4.35 = 81.56
\]

The ANOVA for health progress is

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>area</td>
<td>19.47</td>
<td>7</td>
<td>2.78</td>
<td>0.72</td>
</tr>
<tr>
<td>time</td>
<td>4.35</td>
<td>3</td>
<td>1.45</td>
<td></td>
</tr>
<tr>
<td>error</td>
<td>81.56</td>
<td>21</td>
<td>3.88</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>105.38</td>
<td>31</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Since $F = 0.72 < F_{0.95,7,21} = 2.48$, do not reject the hypothesis of equality. Hence, there is no significant CAA/area effect on progress with respect to health.

To set confidence limits (CL) on progress with respect to health, compute

$$CL_i = A_{\hat{A}_i} \pm t(1-\alpha), \sqrt{MS_{\text{error}}} \sqrt{\frac{n}{2}}$$

$CL_1 = 0.72 \pm 2.08 \sqrt{\frac{3.88}{4}} = 0.72 \pm 1.94$

$CL_2 = -0.32 \pm 1.94$

$CL_3 = -1.02 \pm 1.94$

$CL_4 = -1.43 \pm 1.94$

$CL_5 = -0.91 \pm 1.94$

$CL_6 = 0.99 \pm 1.94$

$CL_7 = -0.29 \pm 1.94$

These confidence limits establish a range for mean progress with respect to health for each CAA. Using $\alpha = 0.05$, the probability that the true progress with respect to health for each CAA is within its established limits is 0.95.

The procedure to determine significant differences for progress with respect to health is used to test for significant differences for progress with respect to education. The application of the analysis of variance for progress with respect to education yields the following ANOVA:
<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>area</td>
<td>14.85</td>
<td>7</td>
<td>2.12</td>
<td>1.72</td>
</tr>
<tr>
<td>time</td>
<td>17.63</td>
<td>3</td>
<td>5.88</td>
<td></td>
</tr>
<tr>
<td>error</td>
<td>25.81</td>
<td>21</td>
<td>1.23</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>58.29</td>
<td>31</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Since $F = 1.72 < F_{0.05,7,21} = 2.48$, do not reject the hypothesis of equality. Hence, there is no significant CAA/area effect on progress with respect to education.

The confidence limits for CAA progress with respect to education are computed using the same procedure described for health. The confidence limits for progress with respect to education are:

- $CL_1 = 1.58 \pm 1.15$
- $CL_2 = 1.62 \pm 1.15$
- $CL_3 = -0.17 \pm 1.15$
- $CL_4 = 1.16 \pm 1.15$
- $CL_5 = 1.02 \pm 1.15$
- $CL_6 = 0.39 \pm 1.15$
- $CL_7 = 2.11 \pm 1.15$

As previously mentioned, the analysis of variance and confidence limits cannot be computed for income and housing since each has only one progress period. However, the preceding analysis has determined two important results:

1. Progress with respect to income does not correlate with progress with respect to the other parameters.
2. CAA progress with respect to health and education is not significantly different than the progress made in the areas without CAAs.
Performance Model Results

To use both total progress and performance values for input into the fund allocation model, the performance scores ($Q_{i,t}$) must be scaled as discussed in Chapter III. CAA total progress and performance scores are shown in Table 20.

Table 20. Progress and Performance Results

<table>
<thead>
<tr>
<th>CAA</th>
<th>$T_{i,t}$</th>
<th>$Q_{i,t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.10298</td>
<td>63.60</td>
</tr>
<tr>
<td>2</td>
<td>0.07325</td>
<td>84.80</td>
</tr>
<tr>
<td>3</td>
<td>-0.05228</td>
<td>73.45</td>
</tr>
<tr>
<td>4</td>
<td>-0.06125</td>
<td>53.61</td>
</tr>
<tr>
<td>5</td>
<td>-0.08747</td>
<td>82.24</td>
</tr>
<tr>
<td>6</td>
<td>-0.09633</td>
<td>73.70</td>
</tr>
<tr>
<td>7</td>
<td>-0.12050</td>
<td>74.00</td>
</tr>
</tbody>
</table>

To scale performance scores compute

$$\overline{Q}_{i,t} = \frac{\sum_{i=1}^{7} Q_{i,t}}{7} = \frac{505.4}{7} = 72.2$$

The scale factor ($\tau$) is determined by

$$\tau = \frac{|T_{i,t}|}{|\overline{Q}_{i,t}|} = \frac{0.0639}{72.2} = 0.00088$$
Allocation Model Results

The results from the Total Progress Model and the Performance Model can now be used for input into the Allocation Model. The allocation of Federal funds to each CAA during the past program period, and the lower and upper bounds for the allocation of Federal funds for the next program period are shown in Table 21.

Table 21. Federal Funding Limits

<table>
<thead>
<tr>
<th>CAA</th>
<th>Previous Program Year's Funding</th>
<th>Current Period Funding</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$241,000</td>
<td>$192,800</td>
<td>$608,400</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>564,000</td>
<td>451,200</td>
<td>866,800</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>185,000</td>
<td>148,000</td>
<td>563,600</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>222,000</td>
<td>177,600</td>
<td>593,200</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>595,000</td>
<td>476,000</td>
<td>891,600</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>89,000</td>
<td>71,200</td>
<td>486,800</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>182,000</td>
<td>145,600</td>
<td>561,200</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>$2,078,000</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

To determine the lower bound for each CAA, compute

\[ y_{i,t}^- = .80 x_{i,t-1} \]

For CAA 1

\[ y_{1,t}^- = .80(241,000) = 192,800 \]

The upper bound for CAA 1 is determined by

\[ y_{1,t}^+ = x_{1,t-1} + .20 \sum_{i=2}^{7} x_{i,t-1} \]
\[ y_{i,t}^* = 241,000 + .20(564,000) + \ldots + .20(182,000) \]
\[ = 608,400 \]

The coefficient vector \( \tilde{v} \) of the objective function is determined by

\[ \tilde{v}_{i,t} = (R_5 \cdot T_{i,t}) + (R_6 \cdot \tau \cdot Q_{i,t}) \]

Using the values of \( R_5 \) and \( R_6 \) as developed in Appendix II, and the values of \( T_{i,t} \) and \( Q_{i,t} \), the objective function coefficient for CAA 1 is

\[ v_1 = \left[ .57(-.10298) + (.43)(.00088)(63.60) \right] \]
\[ = -0.03463 \]

The objective function coefficient vector is shown in Table 22.

<table>
<thead>
<tr>
<th>CAA</th>
<th>( v_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.03463</td>
</tr>
<tr>
<td>2</td>
<td>0.07384</td>
</tr>
<tr>
<td>3</td>
<td>-0.00201</td>
</tr>
<tr>
<td>4</td>
<td>-0.01463</td>
</tr>
<tr>
<td>5</td>
<td>-0.01874</td>
</tr>
<tr>
<td>6</td>
<td>-0.02702</td>
</tr>
<tr>
<td>7</td>
<td>-0.04068</td>
</tr>
</tbody>
</table>
Hence, the fund allocation problem becomes

Maximize \(-0.03463 \, x_1 + 0.07232 \, x_2 + \ldots - 0.04068 \, x_7\)

Subject to:

\[\begin{align*}
192,800 & \leq x_1 & \leq 608,400 \\
451,200 & \leq x_2 & \leq 866,800 \\
148,000 & \leq x_3 & \leq 563,600 \\
177,600 & \leq x_4 & \leq 593,200 \\
476,000 & \leq x_5 & \leq 891,600 \\
71,200 & \leq x_6 & \leq 486,800 \\
145,600 & \leq x_7 & \leq 561,200 \\
2,078,000 & \leq x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 & \leq 2,078,000
\end{align*}\]

Using the computer program in Appendix III, the optimal solution to this fund allocation problem is to allocate the following amounts of Federal funds:

\[\begin{align*}
x_1 &= 192,800 \\
x_2 &= 866,800 \\
x_3 &= 148,000 \\
x_4 &= 177,600 \\
x_5 &= 476,000 \\
x_6 &= 71,200 \\
x_7 &= 145,600
\end{align*}\]

CAA 2 is funded at the upper bound, while the other CAAs are funded at the lower bound.
It should be emphasized that the comparative progress measures used to determine the objective function coefficients have adjusted for the population size of the respective CAA. Hence, the allocation of Federal funds is not influenced by the population size or the number of poor in the respective CAA areas.

Summary

The objective of this chapter was to present the computational aspects and results of the progress evaluation and resource allocation process using actual data. Progress with respect to each parameter was determined and analyzed for each CAA and county. It is significant to note that for this example, the progress measure for income is uncorrelated with the progress measures for each of the other parameters.

CAA progress and performance were then combined to provide input into the resource allocation model. The allocation model optimized comparative progress and performance to allocate Federal funds.
CHAPTER V
CONCLUSIONS AND RECOMMENDATIONS

This study has developed a process for evaluating and allocating resources to public spending programs. The basic rationale of the process is that the evaluation of social action agencies should be impact oriented and supplemented by agency performance evaluation. The results of the impact and performance evaluations should then be used as the basis for allocating Federal funds.

The process developed in this thesis was restricted by the availability of supportive data. The data available for each parameter of poverty determined the characteristics of the parameter models and progress measures. Given these conditions, the following are the main results of this thesis.

Conclusions

1. The results of the analysis of progress indicate that the progress made toward reducing poverty among CAAs and counties used in the example without CAAs is not statistically significantly different. However, it should be emphasized that a lack of sufficient data for income and housing prevented a more thorough analysis. The anticipated release of 1968 income data from the Internal Revenue Service and the 1970 income data from the Bureau of the Census will permit a more thorough analysis with respect to income. The consolidation of future quarterly low-income housing reports from the Regional Office, Department
of Housing and Urban Development will permit a more thorough analysis with respect to housing.

2. The task of collection Georgia data that would be common with the other States in the Southeast Region was tedious at best. To gather the required data for each State in the Region, an efficient information retrieval system would be a necessity.

3. Computer aided analysis would be essential to use the developed evaluation process for the 196 CAAs in the eight Southeastern States.

4. Use of the progress evaluation model provides quantitative information that will assist the decision maker in determining which anti-poverty programs are successful, and where the programs are successful.

5. The developed evaluation model does not address the question of why given CAAs exhibit more or less comparative progress toward reducing poverty than other CAAs.

Recommendations

1. Research directed toward answering the question, why given CAAs exhibit more or less comparative progress, is a logical extension of this thesis. Research of this nature would be helpful for improving the effectiveness of the CAAs.

2. Research is needed to determine if the resource allocation process can be improved by using CAA population size and the number of poor as an input with comparative CAA progress and performance.

3. Even though this research was directed toward OEO, the rationale and process have application to other social action agencies in other Federal departments.
APPENDIX I

CORRECTION OF EARNED INCOME FOR ECONOMIC ACTIVITY

As discussed in Chapter III, the measure of $E_i$ may be influenced in part by a factor of economic activity ($X_i$) and a correlation test should be made to determine the existence of a possible interaction between $E_i$ and $X_i$.

To determine if earned income is influenced by economic activity, compute

$$r_{E,X} = \frac{\sum_{i=1}^{n} (\Delta E_i, t - \Delta \tilde{E}, t)(\Delta X_i, t - \Delta \tilde{X}, t)}{\sqrt{\sum_{i=1}^{n} (\Delta E_i, t - \Delta \tilde{E}, t)^2 \sum_{i=1}^{n} (\Delta X_i, t - \Delta \tilde{X}, t)^2}}$$

and use the "t" statistic as discussed in the Total Progress Model, Chapter III, to determine if $E_i$ and $X_i$ are uncorrelated ($\rho = 0$).

If the measures are uncorrelated, then proceed with the process as discussed in Chapter III. However, if the measures are correlated, $E_i$ must be corrected to eliminate the influence of $X$. This is done by computing the regression coefficient $[23,30]$ and correcting each $\Delta E_i$.

Let

$$\theta_{X,E} = \text{the regression coefficient (slope of the actual regression line)}$$

$$\sigma_X = \text{the standard deviation of } X$$

$$\sigma_E = \text{the standard deviation of } E.$$
Then

\[ \theta_{x,E} = \frac{\sigma_x}{\sigma_E} \]

where

\[ \sigma_x = \sqrt{\frac{\sum_{i=1}^{z} (\Delta x_i, t - \Delta \bar{x}, t)^2}{z - 1}} \]

\[ \sigma_E = \sqrt{\frac{\sum_{i=1}^{z} (\Delta E_i, t - \Delta \bar{E}, t)^2}{z - 1}} \]

The corrected \( \Delta E_i \) (\( \Delta E_i' \)) is then

\[ \Delta E_i' = \theta_{x,E} \cdot \Delta E_i \]

and the value of \( \Delta E_i' \) is used instead of \( \Delta E_i \) for the remainder of the process.
APPENDIX II

THE RELATIVE IMPORTANCE OF PROGRESS AND PERFORMANCE

This questionnaire is to determine the relative importance of using progress toward reducing poverty information and CAA performance (QFAS) for allocating Federal funds to CAAs.

If you had quantitative information on the comparative progress toward reducing poverty that was being made in the Counties for which there are CAAs and CAA performance scores, how would you score the relative importance of the each for allocating Federal funds?

Give a score between 0 and 10 to the most important measure (performance or progress) and a relative score between 0 and 10 to the other measure so that the sum of the scores equals 10.

<table>
<thead>
<tr>
<th>MEASURE</th>
<th>SCORE</th>
</tr>
</thead>
<tbody>
<tr>
<td>PROGRESS</td>
<td>5.7</td>
</tr>
<tr>
<td>PERFORMANCE</td>
<td>4.3</td>
</tr>
</tbody>
</table>

TOTAL = 10.0

34. This questionnaire was given to the staff in PB&E, August 12, 1971. The scores entered above are the mean values for each measure.
APPENDIX III

RESOURCE ALLOCATION PROGRAM

The computer program in this appendix has been adapted from [46] for use on the Univac 1108.
PROGRAMMING PRINCIPLE

This program will solve any interval linear programming problem where \( A \) is an \( m \times n \) matrix.

The program will convert the problem to an equivalent problem where the matrix \( A \) is always bounded.

Using the Simplex method where \( A \) is an \( m \times n \) matrix.

Subject to

\[ B = A x + \beta \]
SUBROUTINE POUND

COMPLEX A(M), Y(1:1N)
COMPLEX M(1:1N), L(1:1N)
COMPLEX S1(1:1N), S2(1:1N), S3(1:1N)
COMPLEX Y1(1:1N), Y2(1:1N), Y3(1:1N)
REAL C, NULL
LOGICAL UNHMD

IF(ABS(X) LE 0.0) GO TO 120

DO 10 J = 1, MR

SUMM = SUMM + A(J,J) * X(J)

IF(ABS(SUM) LE 0.0) GO TO 120

CONTINUE

IF(M-R,EQ.0) GO TO 140

W(1) = 1

FORMAT(I0.0) "THIS PROBLEM IS EITHER UNBOUNDED OR INFEASIBLE."

END
SUBROUTINE AUXPRB

COMMON /FXTRA (10,10) .P (10,10) /M3/BIM (10,10) /V2/PMP /XOP (10) /V2/XPV /A (10) /C(10) /XOP(10) /DEL (10)

REAL GRAPE, PN, SL, INFEAS/54/LEAVE/S9/ITER

COMMON /S3/P/SP/OPVAL/SA/INFEAS/54/LEAVE/S9/ITER

LOGICAL INF, S, AS

A(N) = 0.0

DO 10 I = 1, N

A(N) = A(N) + B(I) * X(J)

CONTINUE

END OF COMPILATION: NO DIAGNOSTICS.


```
00367  116  346  XOP(J)=XOP(J)+EL(K)*B1M(J,K)
00371  117  346  K=K(P)
00372  116  330  X=1*P(J)=XOP(J)+THETA*BIN,J,K)
00374  116  OPVME
00375  124  DO 350 I=1,n
00379  125  350 OPVALPVAL=XOP(I)+C(I)
00380  124  IF(P(I,L))GO TO 370
00384  123  DO 360 J=1,nPM1
00387  124  K<351
00400  125  360 B<351(X)=X0(Y)+PFL(K)
00401  127  370 B<351(X)=X0(Y)+PFL(K+1)
00412  127  RETURN
00414  129  END
00414  131  C
00414  141  END

END OF COMPILATION!  NO DIAGNOSTICS.
```

```
00101  1*  COMMON /M3/INVRS(10*10)
00101  2*  COMMON /M7/K(10*10)
00101  3*  COMMON /S3/I=10*10
00107  6*  REAL M
00110  9*  LOGICAL ILI5
00111  10*  UU 100 I=1,n
00114  11*  N(J,L.I)
00115  12*  DO 100 L=1,n
00120  15*  100 N(J,L)=N(J,L)*MVR(L.I)
00123  14*  IF(ABS(N(J)) .GT. 0.0001) GO TO 105
00125  13*  SIGE=TRUE
00126  14*  RETURN
00127  17*  105 SIGE=FALSE
00130  19*  DO 110 I=1,n
00130  20*  110 B<INVRS(I)*INVRS(I,K)*NUK)
00135  22*  DO 120 J=1,n
00140  25*  IF(J,L.J)GO TO 125
00142  22*  DO 130 J=1,n
00145  23*  B<INVRS(I,J)*INVRS(I,K)*NUK)
00146  24*  130 CONTINUE
00150  28*  CONTINUE
00151  29*  CONTINUE
00153  23*  RETURN
00154  24*  END

END OF COMPILATION!  NO DIAGNOSTICS.
```
BIBLIOGRAPHY


44. Poverty and Health in the United States, Medical and Health Research Association of New York City, New York, 1968.


