A NEW GENERALIZED DETAILED BALANCE FORMULATION TO CALCULATE SOLAR CELL EFFICIENCY LIMITS

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ABSTRACT: Solar cells efficiency limits can be calculated either by thermodynamic or detailed balance approaches. For a single energy (i.e., single junction) solar cell, detailed balance equations are identical to the thermodynamic equations. However, standard detailed balance techniques cannot be directly used to analyze new approaches for higher efficiency solar cells if mechanisms other than radiative generation or recombination are included. A generalized form of detailed balance is presented, which is consistent with thermodynamic approaches but retains the benefits of detailed balance approaches. The generalized detailed balance is used to analyze recombination and thermalization in quantum well solar cells. This analysis demonstrates that an advantage of new approaches to high efficiency solar cells is that energy can be recovered from non-ideal or non-radiative processes and that if the solar cell system is constrained in some way, the inclusion of these non-ideal mechanisms can be beneficial.

Keywords: Fundamentals – 1: Thermodynamics – 2: High Efficiency – 3.

1. INTRODUCTION

The performance limits of solar cells are a fundamental parameter, for both practical and theoretical reasons. The efficiency limits can be calculated either by thermodynamic or by detailed balance [1] approaches. In this paper we show that these two approaches are equivalent. Detailed balance approaches, however, are simpler as they do not require the explicit calculation of entropies and are also easier to relate to the physical mechanisms in a solar cell. However, “standard” detailed balance calculations include two key assumptions, the existence of a single, constant quasi-Fermi level separation and the generation of exactly one electro/hole pair from each photon, that preclude their use on approaches designed to increase efficiency without the addition of extra junctions or band gaps. A generalized set of the detailed balance equations can be formulated and is also presented in this paper.

2. THERMODYNAMIC CALCULATIONS

Thermodynamics is commonly used to calculate the efficiency of energy conversion processes. In thermodynamic approaches, the efficiency is calculated by applying the First and Second Law of Thermodynamics. The model of the converter is shown in Fig 1, in which it is assumed that part of the converter is in perfect thermal contact with the ambient. In a thermal system, T1 is the actual temperature at which the converter is operating and will depend on the operating point of the converter. In a quantum converter such as a solar cell, T1 is an effective temperature. A summary of thermodynamic conversion is given in [2,3,4]

The First Law of Thermodynamics states that energy must be conserved, or equivalently that the work is the difference between the net energy input to the device and the energy out of the device. In the notation of Fig 1, $W = Q_{1} - Q_{2}$. Assuming that energy is not lost to the surroundings as it is transported to the converter, $Q_{1} = Q_{3}$. $Q_{2}$ is the energy that is lost to the surroundings (usually as heat) from the converter itself. $Q_{1}$ is the NET heat flux to the converter, and is the difference between the heat flow from the source to the converter, $Q_{source→con}$, and the heat flow from the converter to the source, $Q_{con→source}$, so that $Q_{1} = Q_{source→con} - Q_{con→source}$.

![Figure 1: Schematic of a thermal converter.](image)

The Second Law of Thermodynamics states that the entropy of a given system, S, must be greater than or equal to zero, or $\Sigma S \geq 0$. In all of the efficiency limit calculations performed, it is assumed that the converter does not create any entropy beyond that inherent in a particular energy flux (denoted by the equation $S = Q/T$ in Fig 1). Under this condition, the solar cell is called reversible and $\Sigma S = 0$ for the solar cell or $S_{2} = S_{3}$. However, the total entropy in the system (which includes the heat source or sun) is under most circumstances larger than zero and is given by $S_{tot} = S_{2} - S_{1}$.

The combination of the equations for W and S gives rise to an expression of work as $W=S_{2} - S_{1}$. The efficiency of the system is defined as $\eta = W/Q_{1}$, then the efficiency
can be found as:
\[ \eta = 1 - \frac{T_3}{T_1} \]

This efficiency is called the Carnot efficiency. The Carnot efficiency reaches its maximum value when no work is extracted from the system and the converter temperature is the same as the source temperature or \( T_3 = T_1 \). This occurs because as the converter temperature \( T_3 \) increases, the energy radiated back to the sun increases and the net energy input to the system approaches zero. Therefore, although the efficiency is increasing, the energy input to the converter and hence the work performed is dropping. This type of efficiency is useful when the goal is to conserve a heat source or fuel. In this case, heat returned to the source is not lost, and less fuel must be expended to maintain the source at a temperature \( T_1 \).

In solar energy conversion systems, we are not concerned about the fuel consumption over the life of the converter system, but instead in extracting as much work as possible from a particular source (the sun) that uses the same amount of fuel weather we radiate energy back to it or not. In this case we need to find the efficiency where the work extracted \( W = Q_1 - S_1 T_2 \) is a maximum. In addition, for solar energy conversion, \( Q_1 \), the net energy to the converter, should be replaced by the energy generated by the sun. Therefore, instead of using \( Q_{\text{input}} = Q_1 = \sigma (T_1^4 - T_3^4) \), we use \( Q_{\text{input}} = \sigma T_1^4 \).

As a result of the different possible definitions of efficiency, we can define four possible efficiencies for a converter of radiative energy. Two definitions arise from the definition of input energy as the net flux or the solar flux and two more arise depending on if the efficiency is defined as the maximum work condition. The four definitions of efficiency are shown in Table 1 and Figure 2. The Carnot efficiency is the standard thermodynamic efficiency. The maximum Carnot efficiency occurs where the converter and sun temperature are equal. This is the maximum condition since the entropy generated by the source depends on the net energy to the converter, and when \( T_1 = T_3 \), then this entropy term drops to zero. The solar analogue of the Carnot efficiency is the Landsberg efficiency \([5]\), which recognizes that the upper limit on conversion efficiency is achieved when the entropy in the converter is zero, leaving just the entropy generated due to the radiation emitted by the source and the converter. The maximum corresponds to short circuit conditions. The efficiency most relevant to solar energy conversion is the efficiency assuming the solar flux is the input to the converter and the efficiency is defined at the maximum work point. This value is 85.4%.

**Table 1: Different possible definitions of efficiency.**

<table>
<thead>
<tr>
<th></th>
<th>Maximum work?</th>
<th>Input</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carnot</td>
<td>No</td>
<td>Net flux</td>
<td>95.5%</td>
</tr>
<tr>
<td>Landsberg</td>
<td>No</td>
<td>Solar flux</td>
<td>93.3%</td>
</tr>
<tr>
<td>Müser Engine</td>
<td>Yes</td>
<td>Net flux</td>
<td>88.2%</td>
</tr>
<tr>
<td>Solar</td>
<td>Yes</td>
<td>Solar flux</td>
<td>85.4%</td>
</tr>
</tbody>
</table>

**Figure 2: Three possible definitions of efficiency as a function of converter temperature.**

3. **FACTORS AFFECTING EFFICIENCY**

The efficiencies calculated in the previous section can be changed by several factors. Firstly, the efficiency can be improved by splitting the incident spectrum and shining this light into separate converters, with each converter optimally matched to a narrow portion of the spectrum. In this case, the energy lost as waste heat \( Q_2 \) is reduced, and the efficiency increases. The efficiency rises from 85.4% to a value of 86.8%.

A second factor that affects the efficiency of the converter is the geometry or optical concentration of the converter. In the previous calculations, it is assumed that the converter is surrounded by the source at a uniform temperature and equal to the temperature of the sun. However, because we are some distance from the sun, the sun only occupies a small fraction of the sky and the remainder is ambient illumination. The net temperature of illumination of the converter is then the average of the sun and the ambient. This value can be increased by geometrical manipulation of the region surrounding the cell or by concentrating the sunlight. Finally, the temperature of the sun and the earth and the spectrum of the sun will also affect the efficiency. The previous values are calculated with the sun’s temperature = 6000 K and the earth's temperature = 300 K.

4. **QUANTUM CONVERTERS**

A quantum or particle converter (e.g., a solar cell) is a converter that converts a flux of quanta such as photons into work. It differs from the thermal converter described in the previous section by the addition of one addition rule (the sum of all the particle flows must be zero), by the fact that a particle source may be described by a chemical potential and a temperature rather than just a temperature and by a different form of the entropy equation. Most importantly however, a quantum converter does not absorb the entire energy or particle flux from the source (unless the band gap is zero). Therefore, the fluxes of the sun and converter cannot simply be written as \( \sigma T_1^4 \) and \( \sigma T_3^4 \), but instead need to integrated as:
The efficiencies of a quantum converter, including the effects of spectral splitting concentration and sun/earth temperatures can be calculated in an identical fashion to the thermal converter and are given in Table 2.

### Table 2: Efficiency of thermal and quantum converters under several conditions.

<table>
<thead>
<tr>
<th></th>
<th>Thermal Converter</th>
<th>Quantum Converter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum conc., multiple devices</td>
<td>86.8%</td>
<td>86.8%</td>
</tr>
<tr>
<td>Maximum conc., one device</td>
<td>85.4%</td>
<td>40.8%</td>
</tr>
<tr>
<td>One sun, one device</td>
<td>12.9%</td>
<td>30.8%</td>
</tr>
</tbody>
</table>

### 5. Detailed Balance Calculations

The detailed balance approach to calculate solar cell efficiency limits was first used by Shockley and Queisser [1] to calculate the efficiency limits for a single junction solar cell. In detailed balance calculations, the current from a solar cell is calculated based on the continuity equation. The current out of the device is the difference between the light absorbed and the light emitted via radiative recombination. Both the absorbed and emitted light have equations of the same form, given by:

\[
\phi(E_1, E_2, T, \mu) = \frac{2\pi}{h^3 c^2} \int_{E_1}^{E_2} \frac{E^2}{\exp((E-\mu)/kT)-1} dE
\]

where \( \phi \) is the particle flux, \( E_1 \) and \( E_2 \) are the range of energies over which particles are absorbed or emitted, \( T \) is a temperature and \( \mu \) is the chemical potential or quasi-Fermi level of the system. For black body light emitters, the generation fluxes have \( \mu = 0 \) and \( T \) is the temperature of the black body emitter. For recombination fluxes, the temperature is that of ambient and \( \mu \) is the quasi-Fermi level separation with \( \mu = qV \), where \( V \) is the voltage across the solar cell. In both absorption and recombination fluxes, for a single band gap \( E_1 = E_g \) and \( E_2 = \infty \). The current is then:

\[
I = q(\phi(E_g, \infty, T_{sun}, 0) - \phi(E_g, \infty, T_{amb}, \mu))
\]

The work from the solar cell is calculated by \( W = I \times V \).

The equation of work above is equivalent to the thermodynamic equation of a quantum conversion or a thermal converter with a band gap if the radiation emitted at the chemical potential \( \mu \) is mathematically transformed into an equivalent blackbody radiation. Such a transformation is possible since for any arbitrary radiation with a chemical potential, we can find an equivalent radiation emitted at some temperature \( T_3 \) such that:

\[
\frac{E^3}{\exp((E-\mu)/kT_{amb})} = \frac{E^3}{\exp(E/kT_3)}
\]

The relation that links \( \mu \) and \( T_3 \) is:

\[
\frac{E - \mu}{kT_{amb}} = \frac{E}{kT_3} \quad \text{or} \quad \mu = E \left(1 - \frac{T_2}{T_3}\right).
\]

For a single photon energy (i.e., the above equations are not in their integral form) the work from detailed balance then becomes:

\[
W = \sigma E \left(1 - \frac{T_2}{T_3}\right) \frac{E^2}{\exp(E/kT_1)} - \frac{E^2}{\exp(E/kT_1)}
\]

\[
= \sigma \left(1 - \frac{T_2}{T_3}\right) \frac{E^3}{\exp(E/kT_1)} - \frac{E^3}{\exp(E/kT_1)}
\]

where \( \sigma \) is Stefan-Boltzmann’s constant.

In a thermal converter, the work is given by:

\[
W = \sigma \left(T_1^4 - T_3^4\right) \left(1 - \frac{T_2}{T_3}\right)
\]

In the thermal equations, the absorbed and emitted radiation is integrated over all wavelengths to obtain \( \sigma T_1^4 \) and \( \sigma T_3^4 \) respectively. In the quantum converter, only the spectrum above \( E_g \) is important and therefore the equation of work for a particular energy level (i.e., leaving out the integral over \( E_{1g} \) to infinity) becomes:

\[
W = \sigma \left(1 - \frac{T_2}{T_3}\right) \frac{E^3}{\exp(E/kT_1)} - \frac{E^3}{\exp(E/kT_1)}
\]

This is exactly the same equation as the work for a single photon energy as from detailed balance. The equivalence of the two approaches can be understood by examining the energies represented by the equations. In detailed balance, the energy input into the solar cell is the number of particles above the band gap, multiplied by chemical potential \( qV \). This value is identical to the calculation of the total photon energy, and then subtracting off the energy that is passed as heat to the ambient. Thus in the work equation for thermodynamics, \( W = U_1 - Q_2 - U_{sun} - U_{converter} - Q_2 \) while detailed balance only needs to calculate \( qV N_{phot} \) which equals \( U_{sun} - Q_2 \). The energy of the converter to the sun \( (U_{converter}) \) is the same in both detailed balance and thermodynamic equations. Essentially in detailed balance approaches the energy above the band gap is not considered an input (i.e., it never reaches the solar cell) and can be considered that it is “lost” before it ever gets to the solar cell. Consequently, entropy arising from an attempt to extract work from this energy does not need to be considered, making detailed balance a simpler, elegant way to perform efficiency limit calculations for solar converters.

Since detailed balance and thermodynamic approaches are mathematically equivalent, it is no surprise that they yield the same answers. In Table 2, the efficiencies of the quantum converter system are also those of detailed balance calculations.
6. MULTIPLE LEVEL DETAILED BALANCE

A central assumption in detailed balance calculations is that the quasi-Fermi level separation across the region of interest remains constant and has only a single value. Consequently, using this assumption, the only possible way to realize efficiency increases above a single junction solar cell is to add additional band gaps and hence additional pn junctions. This assumption did not allow detailed balance modelling to be applied to several types of devices, including hot carrier solar cells [6], solar cells based on Auger generation [7] and quantum well solar cells [8].

Ketterman and Guillmoines [9] were the first to recognize that the assumption of constant quasi-Fermi levels could be violated by some devices and hence may allow higher efficiencies without the necessity of adding additional layers. Although the concept of a quasi-Fermi level may not be valid for some such advanced devices, for simplicity in this paper we will use quasi-Fermi levels to describe the average energy of an arbitrary meta-stable carrier population. Luque [10] performed detailed balance calculations for an impurity band solar cell and Brenner [11] performed detailed balance calculations for quantum well solar cells. In all of these calculations, the central feature is that more than a single quasi-Fermi level can exist, and radiative transitions (both generation and recombination) can occur between the multiple quasi-Fermi levels. Similar techniques can be used to calculate efficiency limits for Auger generation [12,13]. Green [14] has summarized multiple energy level calculations for a variety of approaches to achieving high efficiency solar cells. A key feature of these results is that

7. GENERALIZED DETAIL BALANCE

Multiple energy level detailed balance removes the key assumption in detailed balance for single n junctions that the quasi-Fermi level must be constant across the device. However, it still does not take into account the full range of physical mechanisms in solar cells. In particular, it neglects a generalized approach to the inclusion of other recombination and generation terms. Such terms have been previously included by several authors beginning with Shockley and Queisser [1]. However, these approaches disturb the mathematical equality that exists between detailed balance and thermodynamic approaches. Consequently, as demonstrated by Luque [15] for Auger generation, these approaches are not automatically thermodynamically consistent. Luque [15] checks the thermodynamic consistency of the detailed balance model for Auger recombination. Here we provide a generalized set of conditions that the fluxes must meet in order to be thermodynamically consistent.

The total entropy in the system is defined as the difference between the entropy to the sink, \( S_2 \), in the notation of Fig 1. and the entropy generated by the source, \( S_1 \). Luque [15] calls this entropy, \( S_{\text{irr}} = S_2 - S_1 \). We shall also call this entropy \( S_{\text{irr}} \), but as with the other notation in this paper, take the remainder of the notation from De Vos [2]. The entropy for a quantum converter is

\[
S = \frac{U - \mu N}{T}
\]

where \( U \) is the energy, \( \mu \) is the chemical potential of quasi-Fermi level separation and \( N \) is the particle flow. \( S_2 \) is the entropy associated with an energy flow to the sink \( T_2 \). For the earth at \( T_2 \), the chemical potential is 0, and the energy flow to \( T_2 \) is \( U - W \), where \( U \) is the NET input energy to the solar cell and \( W \) is the work extracted from the system. This makes the entropy

\[
S_2 = \frac{U - W}{T_2} \quad \text{and} \quad S_1 = \frac{U}{T_1}
\]

Substitution of these two equations for entropy gives:

\[
T_2 S_{\text{irr}} = U \left( 1 - \frac{T_2}{T_1} \right) - W
\]

where \( W \) is the work extracted from the system and is given by:

\[
W = \mu N
\]

and where \( N \) is the difference between the particles from the sun to the solar cell and from the solar cell to the sun. It consists of two parts, \( N = N_{\text{rad}} + N_{\text{g}} \), where \( N_{\text{rad}} \) is the net radiative flux and \( N_{\text{g}} \) is the net flux associated with other processes. \( qN \) is equal to the current from the device.

In order for the entropy to always be positive, a sufficient condition is that the work extracted from the device never exceeds \( \eta_c U_1 \), where \( \eta_c \) is the Carnot efficiency and \( U_1 \) is the NET energy flux to the solar cell. In order to demonstrate this condition, we would need to check at all points along the IV curve of a solar cell. A simpler but sufficient condition is to show that the entropy is zero at the minimum entropy point. The minimum entropy point can be found by calculating \( \frac{dS_{\text{irr}}}{d\mu} = 0 \). Differentiating the equation for \( S_{\text{irr}} \) gives:

\[
T_2 \frac{dS_{\text{irr}}}{d\mu} = -N = 0
\]

At the minimum entropy condition, the total particle flux \( qN \) must go to zero, which corresponds to the open circuit condition. At open circuit, the entropy must be at least zero, giving:

\[
U_1 \left( 1 - \frac{T_2}{T_1} \right) = \mu \left( N_{\text{rad}} - N_{\text{g}} \right)
\]

where \( N_{\text{g}} \) is the additional particle flux due to other recombination or generation processes. At open circuit, \( U_1 \) is zero (there is no NET energy to the solar cell) and in addition, we know that the particle flow in detailed balance is zero at open circuit. Consequently, the particle flux due to other processes must be zero at open circuit condition.

The entropy calculated by the derivative is either a minimum or a maximum condition. To show that it is a minimum, it is sufficient to demonstrate that the entropy is larger than zero at any other point. The simplest point to calculate the work is at short circuit. At short circuit it is sufficient to show that the work extracted is zero, giving:
\[ T_2 S_{pr} = U \left( 1 - \frac{T_2}{T_1} \right) > 0 \]

Therefore, the two conditions necessary to insure thermodynamic consistency are that the sum of any added flux terms is zero at open circuit and that no work is extracted at short circuit.

These two conditions hold if both generation and recombination fluxes are included in the model. If only recombination processes are included in the generalized detail balance approaches, then the calculations are inherently thermodynamically consistent since the work decreases, hence increasing the entropy.

8. INCLUSION OF HEAT LOSS MECHANISMS

In addition to the inclusion of other recombination/generation mechanisms, the calculation of heating effects should be allowed in detailed balance approaches. Heat loss mechanisms are inherently excluded in standard detailed calculations since any energy above the band gap is not considered as an input into the solar cell. However, these heating effects can be calculated by determining the energy above the band gap. This energy is used to heat the light generated electrons to some temperature above the lattice temperature. Assuming that these electrons interact with each other such that they can be described by a Maxwell-Boltzmann distribution but that they do not interact with the lattice, the electron temperature can then be described by an average temperature.

The average temperature of the electrons is calculated by finding the energy of the electrons above the band gap and the number of light generated carriers. The total number of electrons should be the light generated carriers plus the intrinsic carrier calculations. However, the intrinsic carrier concentration is several orders of magnitude below the number of light generated carriers and hence makes no difference to the numbers for the cases considered. Its inclusion would require assumptions about the density of states in the conduction and valence band, \( N_C \) and \( N_V \), and hence is theoretically material dependant. Consequently, it is neglected in the calculations.

The total energy of the electrons above the band gap divided by the number of electrons will be equal to the average energy of the electrons. Since they are described by a Maxwell Boltzmann distribution, the average energy will equal 3/2 kT, where T is the effective electron temperature. The equations are shown below.

\[ \frac{3kT}{2} = \frac{E_{\text{photon}} - E_G}{N_e} \]

where the energy of the photon \( E_{\text{photon}} \) is given by:

\[ E_{\text{photon}} = K \int_{E_G}^{\infty} \exp\left( \frac{E - \mu}{kT_{\text{amb}}} \right) - 1 \, dE \]

where \( K \) is a constant, \( T_{\text{amb}} \) is the ambient temperature and \( \mu \) is the quasi-Fermi level separation. The number of light generated electrons is given by:

\[ N_e = K \int_{E_G}^{\infty} \frac{E^3}{\exp((E - \mu)/kT_{\text{amb}}) - 1} \, dE \]

Calculations show that for an average energy above the band gap of 0.77 eV (the value that results for relatively optimum values of band gaps), the temperature that describes the electron distribution is approximately 6000K.

Once a temperature for the electrons is determined, then a thermodynamic analysis of the system can be calculated. In this system, the heat fluxes are dependant on conduction processes rather than radiative processes, and hence \( Q_e \) is given by

\[ Q_e = g(T_e - T_1) \]

where \( g \) is a constant that describes the conduction process. For the conduction process described here, \( g \) = \( \frac{3}{2} kN_e \). This definition assumes that the number of electrons in the well is the same as that in the barrier material. However, this does not incur any loss of generality since \( T_1 \) is an adjustable parameter and is never explicitly calculated. If the temperature of the electrons in the well is to be calculated, the \( T_1 \) must be adjusted by \( N_C/N_V \) where \( N_C \) is the number of light generated carriers in the well.

Once the form of \( Q_e \) is known, the work and efficiency of this process can be calculated following the calculation of a Curzon-Ahlborn heat engine as in DeVos [2]. In this calculation, the efficiency is most appropriately defined as the net heat flux towards the well (which is the difference of the heat flow from the well to the barrier minus the heat flux from the barrier to well) rather than the flux of heat from the well only. Hence it is a “thermodynamic” efficiency rather than a “solar” efficiency. Based on these calculations, the equation for work can be found:

\[ W = \frac{3}{2} kN \left( \sqrt{T_1} - \sqrt{T_2} \right)^2 \]

where \( N \) is the number of light generated particles in the barrier region. This is the work that can be thermodynamically extracted from a system that has a source temperature of \( T_w \), in which heat flow by conduction and in which the conduction parameter \( g \) is \( 3kN_c/2 \). In our system, we will use this extracted work to raise carriers from the bottom of the well to the bottom of the conduction band in the barrier region. The flux due to the extraction of this work, is given by the equation

\[ \phi = \frac{W}{\Delta E_c} \]

This flux terms does not need a corresponding downward flux to compensate since it is the net result of two heat flow mechanisms and the entropy generation (which is larger than zero in the general case) has already been included.

Inclusion of this flux into a detailed balance calculation of a quantum well efficiency allows the determination of the
efficiency of extracting carriers from the well region by thermal means. Any heat generated in quantum well by the carriers with energy between that of the barrier material and the well is considered lost in this calculation.

Including the extra flux in detailed balance calculations results in a set of equations dependant on the band gap of the barrier material on the well material. Raising the band gap of the barrier material reduces the temperature of the electrons, hence reducing the source temperature for the thermal energy, the efficiency and the amount of work that can be extracted from the heat engine. However, if the barrier band gap is too low, then there is more than enough energy to extract all the carriers from the well region. Any work in excess of $\Delta G \times N_w$ is lost, and hence an optimum exists for a particular barrier and well band gap.

The calculation of efficiency based on the equations above represents an upper bound on efficiency using the extra energy in the barrier material, since we have assumed essentially that carriers can interact with as many phonons as needed and further that they only extract exactly the amount of energy needed to reach the conduction band of the barrier material. The results of the one sun calculations are shown in Table 3 and in Figure 3.

<table>
<thead>
<tr>
<th>$E_b$ barrier</th>
<th>$E_c$ well</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.34 eV</td>
<td>0.82 eV</td>
<td>48.5%</td>
</tr>
<tr>
<td>1.18 eV</td>
<td>0.64 eV</td>
<td>65.7%</td>
</tr>
</tbody>
</table>

The optimum for one sun conditions is 48.5%. This is below the efficiency of quantum well in which radiative escape is used as the means of carrier escape from the well. However, the efficiency under maximum concentration is 65.7%, which is higher than the corresponding fully radiative quantum well case. The reason for this variation is that thermal converters have a stronger dependency on concentration than quantum converters do. Consequently, under one-sun conditions, the thermal converter has a relatively low efficiency, which increases as the efficiency is increased under maximum concentration.

9. CONCLUSION

New approaches to solar cell efficiency allow efficiencies close the theoretical maximum efficiency for solar energy conversion of 86.8%. These approaches have several advantages over an infinite stack of tandems, which given an efficiency of 86.8%. One key advantage is that the new approaches can give a high efficiency without requiring an infinite number of band gaps. Another advantage, which is demonstrated by the development and application of generalized detailed balance calculations, is that solar cells based on new approaches can extract energy solar cell loss mechanisms such as thermalization of carriers. When applied to quantum well solar cells, the calculations show that thermal escape of carriers from the well can give an increase in efficiency above a homojunction solar cell by extracting work from hot carriers in the barrier region.

Figure 3: Efficiency as a function of $\Delta E_c$ ($E_{G,barrier} - E_{G,well}$) and $E_{G,well}$.