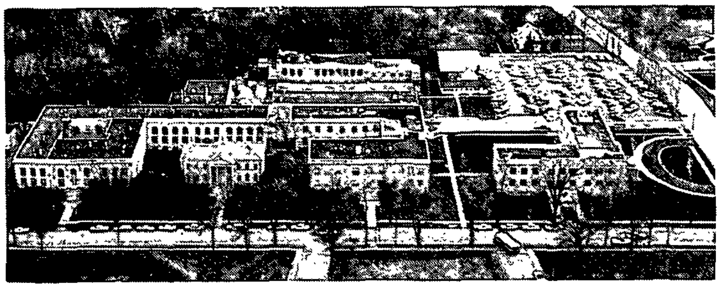

THE INSTITUTE OF PAPER CHEMISTRY, APPLETON, WISCONSIN

IPC TECHNICAL PAPER SERIES
NUMBER 107

THE EFFECT ON SHEET PROPERTIES OF RESIDENCE TIME AND IMPACT DISTRIBUTION IN FLOW—THROUGH REFINERS

HARRY T. CULLINAN, JR.

FEBRUARY, 1981

The effect on sheet properties of residence time and impact distribution in flow-through refiners

Harry T. Cullinan, Jr.
The Institute of Paper Chemistry, P.O. Box 1039, Appleton, Wis. 54912

ABSTRACT

In continuous refining operations, individual particles spend varying lengths of time in the refining zone and experience differing numbers of impacts (or discrete refining events). It is shown that, rather generally, the distributions of residence time and impacts have only a small effect on the development of sheet properties.

Introduction

It is now generally accepted that the fibers emerging from a flow-through refiner are not uniformly beaten as a result of the spread of residence times and discrete refining events (impacts) in the active refining zone. Also, the mean fiber residence time does not necessarily correspond to the hydraulic residence time. When these two mean times differ, material balance dictates a difference between consistency within the refiner and that of the feed (or exit) stream. The magnitude of this difference gives clues concerning the physics of flow and mechanical action within the refiner.

Yet to appear, however, is a quantitative relationship which connects refining action, residence time distribution and impact variation to final sheet properties. This paper presents such a development and an analysis of the expected magnitude of the effects of residence time and impact distributions. The data of Ryti and Arjas are shown to be consistent with the results presented here.

Analysis

With respect to any given final sheet property, a refined pulp is assumed to consist of a distribution of particle fractions which each contribute differently to the final value of the property in question. Fiber treatment in flow-through refiners can confidently be expected to be inhomogeneous at any instant of time because of such factors as variable plate patterns, nonuniform clearances and inhomogeneous stapling configurations. For present purposes, however, the refiner is considered to exert a spatially averaged set of actions on the particles. It is assumed that this set is invariant for a given steady-state operation. To the extent that the distributions of particle fractions are caused by a distribution of particle residence times in the refining zone, the first part of the investigation seeks the influence of this effect on the development of sheet properties.

To arrive at the connection between a final sheet property and the distribution of residence times, a series of thought-experiments can be conducted and subjected to statistical analysis.

Consider the ideal case of plug flow in a flow-through refiner. Imagine that a large number of runs are made in the plug flow mode at various residence times. For a given property, each run at a specific residence time, t, results in a sheet with a certain value for the property, P(t). We construct a population of property values, [P(t)], such that the frequency of occurrence of a given value of P(t) in the population is equal to the probability, f(t), that particles in the actual refiner will reside in the refining zone for time, t. For the population, the mean value is

$$\overline{P} = \sum_{t} f(t)\, P(t) \qquad (1)$$

and this value is the expected value of the property for an actual refining run with a residence time distribution given by the probability function, $f(t)$.

In an actual flow-through refiner the distribution of residence time is continuous, rather than discrete. In this case, the probability function, $f(t)$, is a differential quantity, and it is related to the particle residence time distribution function (RTD), $F(t)$, according to

$$df = F(t)dt \tag{2}$$

In terms of the RTD of particles the sheet property is then expressed as

$$\overline{P} = \int_0^\infty P(t)F(t)dt \tag{3}$$

The meaning of $P(t)$ is confirmed by considering the hypothetical case in which the particle RTD corresponds to plug flow, i.e.:

$$F(t) = \delta(t-\tau) \tag{4}$$

where $\delta\ (t-\tau)$ is the Dirac delta function, representing a spike at $t=\tau$ of unit area. In this case,

$$\overline{P} = \int_0^\infty P(t)\ (t-\tau)dt = P(\tau) \tag{5}$$

Therefore, $P(t)$ represents the property development function which could be measured by plug flow continuous refining.

Since precise plug flow of particles in a flow-through refiner is not attainable under the usual operating conditions, some information on $P(t)$ might be obtained from batch refining. In batch refining, all particles "reside" in the refiner for the specified period of operation. However, the beating experienced by a given particle during batch refining of a specified duration can be different from that experienced by the same particle during plug flow refining with the same residence time. Only if the time average of the beating rate in batch refining is equal to the space average of the beating rate in plug flow refining, can the beating results be expected to be exactly the same.

Equation (3) is the basic result which connects sheet properties to refining action and particle flow characteristics. The form of P(t) will vary from property to property and will also depend on refining geometry and operating variables.

To investigate the effect of particle RTD consider a refiner operating at a mean particle residence time of τ. We expand P(t) as a Taylor's series around τ and evaluate the integral in Eq. (3).

$$P = \int_0^\infty \sum_{n=0}^\infty \frac{1}{n!} \left. \frac{d^n P}{dt^n} \right|_{t=\tau} (t-\tau)^n F(t)dt \tag{6}$$

Interchanging summation and integration and noting that

$$\int_0^\infty F(t)dt = 1 \tag{7}$$

and

$$\int_0^\infty tF(t)dt = \tau \tag{8}$$

the result is

$$\overline{P} = P(\tau) + \sum_{n=2}^\infty \frac{1}{n!} \left. \frac{d^n P}{dt^n} \right|_{t=\tau} \int_0^\infty (t-\tau)^n F(t)dt \tag{9}$$

The integrals in Eq. (9) are the higher moments of the RTD. Neglecting moments beyond the skewness of the distribution the result is

$$\overline{P} = P(\tau) + \frac{1}{2}\left.\frac{d^2P}{dt^2}\right|_{t=\tau}\sigma^2 + \frac{1}{6}\left.\frac{d^3P}{dt^3}\right|_{t=\tau}S^3 \tag{10}$$

where σ^2 is the variance of F about τ and S^3 is the skewness.

Equation (10) expresses the sheet property in terms of characteristics of the property development function, P(t), and the particle residence time distribution, F(t).

Implications

The first obvious result to be noted from Eq. (10) is that if P(t) is a linear function in the region around τ, then the shape of the particle RTD has no bearing on final sheet property. For reasonably smooth behavior of P(t), the result also shows that the variance and skewness of the particle RTD are second- and third-order effects, respectively.

With the exception of pathological operating conditions, the property development function, P(t), is expected to be fairly smooth. Under these conditions we expect that

$$\tau \left.\frac{d^3P}{dt^3}\right|_{t=\tau} \ll \left.\frac{d^2P}{dt^2}\right|_{t=\tau} \tag{11}$$

so that a good approximation [valid except for region in which wide variations in P(t) occur] is

$$\overline{P} \doteq P(\tau) + \frac{1}{2}\left.\frac{d^2P}{dt^2}\right|_{t=\tau}\sigma^2 \tag{12}$$

If the property development function, P(t), is smooth and concave upward ($\frac{d^2P}{dt^2} > 0$) then a wide distribution of particle residence times (as expressed by σ^2) actually improves the sheet property over that obtainable in a batch or plug flow refiner operating at the same mean fiber retention time. This is interesting; but, as noted above, it is a second-order effect.

If P(t) is concave downward ($\frac{d^2P}{dt^2} < 0$), a wide distribution of particle residence times has a negative effect on final sheet property.

Comparison with experiment

In a series of papers (1,2,3), Ryti and Arjas developed the theory of residence time distribution in a flow-through refiner and presented experimental results to demonstrate the effects on sheet properties.

This careful work shows the property development functions for freeness and porosity to be increasing and concave upward. The functions of tear factor and scattering coefficient are decreasing and concave upward. For each of the properties which show concave upward shape in development, the corresponding continuous refining property is slightly higher than the batch result. This is consistent with the present analysis, provided that the batch refining results are assumed to correspond to the ideal plug flow limit. For tensile strength, the property development function is increasing and concave downward. In this case, the continuous refining property is slightly lower than the batch result. Again, this is consistent with the behavior, predicted by Eq. (12).

Extension

In the analysis above, the effect on sheet properties of a distribution of residence times in flow-through refining was investigated without explicit consideration of the possible variation in impacts imparted to the fibers. Even if the spread of residence times can be eliminated completely to achieve plug flow refining, there still will exist a variation in the number of impacts (or discrete refining events) experienced by individual particles.

This section extends the previous analysis, to the general case in which individual particles are subject to simultaneous variation in both residence time and discrete refining events.

Now imagine that plug flow refining can be carried out in such a way that all particles receive exactly the same number of impacts (or discrete refining events). Consider a series of experiments in which ideal plug flow refining of this type is carried out, each at a residence time of τ, and let the number of impacts, n, vary from run to run. Each run results in a sheet with a certain value of the property, $P^*(n)$, which is determined solely by the number of refining events experienced by each particle. We construct a population of property values, $[P^*(n)]$, such that the frequency of occurrence of a given value of $P^*(n)$ in the population is equal to the probability, $g(n,\tau)$, that a particle in the <u>actual</u> refiner will receive n impacts if it resides for a time, τ. For this population, the mean value is

$$P(t) = \sum_n g(n,\tau)\; P^*(n) \tag{13}$$

and this value is the expected value of the property for a plug flow run with residence time of τ and impact probability function, $g(n,\tau)$.

Equation (13) can be substituted into Eq. (3) to give the general result:

$$\overline{P} = \int_o^\infty \sum_n g(n,t)\; P^*(n)\; F(t)dt \tag{14}$$

which connects sheet properties to discrete refining events and continuous flow distribution. The central feature is the fundamental property development function which for a given refiner is taken to depend only on the number of discrete refining events.

An extended version of Eq. (12) can be obtained by applying Eq. (13) to a plug flow refiner operating at a particle residence time of τ and a corresponding average number of impacts of $\overline{n}$. If $P^*(n)$ is expanded as a Taylor's series around $\overline{n}$ and the following relations:

$$\sum_n g(n,\tau) = 1 \tag{15}$$

$$\sum_n g(n,\tau)n = \overline{n} \tag{16}$$

are used, the result is

$$P(\tau) = P^*(\overline{n}) + \frac{1}{2}\left.\frac{d^2P^*}{dn^2}\right|_{\overline{n}} \sigma_g{}^2 \tag{17}$$

where $\sigma_g{}^2$ is the variance of $g(n,\tau)$ and terms involving higher moments of $g(n,\tau)$ have been neglected.

If either the spread in the impact distribution is small or the nonlinearity of the fundamental property development function, $P^*(n)$, is small, then the last term in Eq. (17) is not significant. In this case, $P(\tau)$ is determined predominantly by the average numbers of impacts.

If it is assumed that the variance of $g(n,\tau)$ is not affected by flow rate, then Eq. (17) applies equally to a batch refiner, operating for a time, τ. This observation provides the basis for drawing the correspondence between batch refining and the plug flow limit, as was assumed above in the comparison with experiments.

Thus Eq. (12) is a good general representation of property development in a flow-through refiner. The leading term on the right can be equated with the corresponding batch refiner property development function, and it contains, according to Eq. (17) and the discussion above, all of the effects of impact distribution.

Discussion

In addition to providing the quantitative interrelationship among property development, particle flow and impact distribution in continuous refiners, the present results provide a framework for the experimental determination of the effect of operating variables. Gap setting, plate loading, RPM and throughput influence the property development function, the particle residence time distribution and the impact distribution. For a given set of operating variables, the property development function can be estimated by batch refining, as shown by Ryti and Arjas. The detailed shapes of the fiber RTD and the impact distribution are not crucial to the development of sheet properties; only the means and the variances of the distributions are required, and the latter produce only second order effects.

The simplicity of the results presented here and the experimental accessibility of the important parameters appear to obviate the need for concern over reasonably narrow distributions. Development work on refiners should focus instead on the details of the refining process proper. The results also provide a sound basis for optimization of refining operations for various end uses.

Acknowledgments

The author is indebted to D. Wahren (IPC) for helpful discussion and to A. Nissan for valuable criticism.

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