Creating software to compute the optimal number of newspapers to deliver to each sales outlet

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Finalist Presentation
April 29, 2009

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Overview

Current Model:
- Maximizing Circulation

Optimization Model:
- Historical Sales Data
- Optimization Model
- Optimal Draw
- Maximized Profit

Additional profit per year: $1.8 million
Optimization Model Inputs

- Marginal Revenue
- Marginal Cost
- Salvage Revenue
- Demand Distributions
- Theft
- Shrinkage

Optimization Model
Outlet Types

Theft Outlet

Shrinkage Outlet

Non-Theft/Non-Shrinkage Outlet
Case without shrinkage or theft:

\[
E_D[g(x, D)] = \sum_{d=0}^{x} P[D = d] (rd + s(x - d)) + \left(1 - \sum_{d=0}^{x} P[D = d]\right)(rx) - cx
\]

\[
x = \text{Decision variable, draw} \quad r = \text{Marginal revenue}
\]

\[
D = \text{Demand} \quad c = \text{Marginal cost}
\]

\[
s = \text{Salvage revenue}
\]
Profit Function

Maximum Profit: $5.81

Optimal Draw: 7
Optimization Model Inputs

- Marginal Revenue
- Marginal Cost
- Salvage Revenue
- Demand Distributions
- Theft
- Shrinkage
Demand Distributions

Outlet 11386 on Friday Histogram

Frequency

Demand, d

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Demand Distributions

- Censored data
- Shifting sales trends


Histograms

CDFs
## Covariates

### Covariates tested:

<table>
<thead>
<tr>
<th>Weather</th>
<th>4\textsuperscript{th} of July</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unemployment Rate</td>
<td>Tax Free Holiday</td>
</tr>
<tr>
<td>Seasonality</td>
<td>Labor Day</td>
</tr>
<tr>
<td>Price</td>
<td>Wednesday before Thanksgiving</td>
</tr>
<tr>
<td>Time</td>
<td>Thanksgiving</td>
</tr>
<tr>
<td>New Year’s Day</td>
<td>Thanksgiving Weekend</td>
</tr>
<tr>
<td>Martin Luther King Day</td>
<td>Christmas Eve</td>
</tr>
<tr>
<td>Memorial Day</td>
<td>Christmas Day</td>
</tr>
</tbody>
</table>
Covariates

Covariates with greatest impact:

- Weather
- Unemployment Rate
- Seasonality
- Price
- Time
- New Year’s Day
  - Martin Luther King Day
  - Memorial Day
- 4th of July
- Tax Free Holiday
- Labor Day
- Wednesday before Thanksgiving
- Thanksgiving
- Thanksgiving weekend
- Christmas Eve
- Christmas Day

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Demand Distributions

Demand distributions considered:
- Exponential
- Gaussian
- Log-Logistic
- Log-Normal
- Logistic
- Poisson
- Weibull

Maximum Likelihood Estimation (MLE)
Maximum Likelihood Estimation

MLE example for Poisson distribution without theft:

\[ LL(\beta) = \sum_{j=1}^{n} \left[ (1 - z_j) \log \left( \frac{e^{-\lambda_j} (\lambda_j)^{k_j}}{k_j!} \right) + z_j \log \left( 1 - \sum_{l=0}^{k_j-1} \frac{e^{-\lambda_j} (\lambda_j)^{l}}{l!} \right) \right] \]

- \( \lambda_j = \beta_0 + \beta_1 x_1 + \ldots + \beta_m x_m \)
- \( \beta_0 - \beta_m \) = Parameters to be estimated
- \( x_1 - x_m \) = Covariate values
- \( k_j \) = Sold quantity of data point \( j \)
- \( z_j \) = 1 if data point \( j \) is censored; 0 otherwise

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MLE example for Poisson distribution with theft:

\[ LL(\beta) = \sum_{j=1}^{n} \left( 1 - z_j \right) \log \left( \frac{e^{-\lambda_j} \left( \lambda_j \right)^{k_j} \left( 1 - p \right)^{k_j}}{k_j!} \right) \]

\[ + z_j \log \left( 1 - \sum_{l=0}^{k_j-1} \frac{e^{-\lambda_j} \left( \lambda_j \right)^{l} \left( 1 - p \right)^{k_j-1}}{l!} \right) + \sum_{l=1}^{k_j} \frac{e^{-\lambda_j} \left( \lambda_j \right)^{l} \left( 1 - \left( 1 - p \right)^l \right)}{l!} \]

\[ \lambda_j = \beta_0 + \beta_1 x_1 + ... + \beta_m x_m \]

\[ \beta_0 - \beta_m = \text{Parameters to be estimated} \]

\[ x_1 - x_m = \text{Covariate values} \]

\[ k_j = \text{Sold quantity of data point } j \]

\[ z_j = 1 \text{ if data point } j \text{ is censored; } 0 \text{ otherwise} \]

\[ p = \text{Probability of theft} \]
Probability of Theft

- Pick initial value, \( p_i \), of theft probability \( p \)
- Calculate \( \beta \) parameters with fixed \( p_i \) using MLE
- Use \( \beta \) parameters to compute calculated theft % as a function of \( p \)
- Find value, \( p_f \), of \( p \) that makes calculated theft equal to 14.4% of sales
- Adjust initial value, \( p_i \)

\[ \text{Does } p_i = p_f ? \]

- Yes
  - \( p_i \) is probability of theft
- No

- Remember:
  - We compute separate \( \beta \) parameters for every outlet and every day of the week, but we only have one aggregate percentage of theft: 14.4%
Demand Distributions

- Demand distributions tested:
  - Exponential
  - Gaussian
  - Log-Logistic
  - Log-Normal
  - Logistic
  - Poisson
  - Weibull

- Poisson distribution
Optimization Model Inputs

Marginal Revenue
Marginal Cost
Salvage Revenue
Demand Distributions
Theft
Shrinkage

Optimization Model
Software Snapshot

Start

In Progress

Complete

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Update Parameters

ONE outlet on ONE day of the week

Covariates

Historical Sales Data

News Rack: MLE Using Probability of Theft

OR

Non-News Rack: MLE

Store Estimated Parameters, $\beta_i$
Calculate Draw

- Covariates $x_i$
- Estimated Parameters $\beta_i$
- Profit Function Parameters $r, c, s$
- Distribution Parameter $\lambda$
- Profit Function Inputs

Maximize Profit

- News Rack $E_D[g(x,D,p)]$
- Kroger or Wal-Mart $E_D[g(x,D,q)]$
- Other $E_D[g(x,D)]$

Store Optimal Draw
Validation

- Optimization model data:
  - July 2007 – November 2008

- Validation:
  - December 2008

- Assumptions:
  - Calculate AJC estimated profit for comparison
Validation

Sample of results:

GT profit versus AJC profit summary:

<table>
<thead>
<tr>
<th>Compare GT &amp; AJC</th>
<th>GT Profit</th>
<th>AJC Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Per outlet per day:</td>
<td>$6.04</td>
<td>$5.28</td>
</tr>
<tr>
<td>All outlets per day:</td>
<td>$45,500</td>
<td>$39,700</td>
</tr>
<tr>
<td>All outlets per year:</td>
<td>$14.2 million</td>
<td>$12.4 million</td>
</tr>
</tbody>
</table>
Sensitivity Analysis

What if we used the Poisson distribution to calculate the draws, but the demand is actually Gaussian distributed?

<table>
<thead>
<tr>
<th>Comparison</th>
<th>Expected Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>AJC Profit</td>
<td>$12.5 million</td>
</tr>
<tr>
<td>GT Profit using Poisson draws</td>
<td>$14.45 million</td>
</tr>
<tr>
<td>GT Profit using Gaussian draws</td>
<td>$14.47 million</td>
</tr>
</tbody>
</table>
Sensitivity Analysis

Include 100% of advertising:

<table>
<thead>
<tr>
<th>Comparison</th>
<th>Expected Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>AJC Profit</td>
<td>$12.4 million</td>
</tr>
<tr>
<td>GT Profit</td>
<td>$14.2 million</td>
</tr>
</tbody>
</table>

Include 50% of advertising:

<table>
<thead>
<tr>
<th>Comparison</th>
<th>Expected Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>AJC Profit</td>
<td>$7.8 million</td>
</tr>
<tr>
<td>GT Profit</td>
<td>$8.7 million</td>
</tr>
</tbody>
</table>

Include 0% of advertising:

<table>
<thead>
<tr>
<th>Comparison</th>
<th>Expected Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>AJC Profit</td>
<td>$3.1 million</td>
</tr>
<tr>
<td>GT Profit</td>
<td>$3.55 million</td>
</tr>
</tbody>
</table>

Additional Profit

- $1.8 million
- $900,000
- $450,000
Thank You

Client Advisor: Mike Burlingame
Senior Director, Consumer Sales & Retention
The Atlanta Journal-Constitution

Faculty Advisor: Dr. Anton Kleywegt
Appendices

- Appendix I: Objective Profit Function (Shrinkage)
- Appendix II: Objective Profit Function (Theft)
- Appendix III: Profit Function (Theft) Comparisons
- Appendix IV: Fixed Point Calculation
Objective Function

Case with shrinkage:

\[ E_D[g(x, D, q)] = \sum_{d=0}^{x} P[D = d](r_{adv}d + (1 - q)r_{sales}d + s(x - d)) + \left(1 - \sum_{d=0}^{x} P[D = d]\right) (r_{adv}x + (1 - q)r_{sales}x) - cx \]

- \( x \) = Decision variable, draw
- \( D \) = Demand
- \( q \) = Probability of shrinkage
- \( r_{adv} \) = Marginal advertising revenue
- \( r_{sales} \) = Marginal sales revenue
- \( c \) = Marginal cost
- \( s \) = Salvage revenue
Objective Function

Case with theft:

\[ E_D[g(x, D, p)] = \sum_{d=1}^{x} P(D = d) \left( \sum_{j=1}^{d} (1 - p)^{j-1} p(r_{sales} j + r_{adv} x) + (1 - p)^{d} (r_{sales} d + r_{adv} d + s(x - d)) \right) \]
\[ + \left( 1 - \sum_{d=0}^{x} P(D = d) \right) \left( \sum_{j=1}^{x} (1 - p)^{j-1} p(r_{sales} j + r_{adv} x) + (1 - p)^{x} (r_{sales} x + r_{adv} x) \right) \]
\[ + \left[ P(D = 0)(sx) \right] - cx \]

- \( x \) = Decision variable, draw
- \( D \) = Demand
- \( p \) = Probability of theft
- \( r_{adv} \) = Marginal advertising revenue
- \( r_{sales} \) = Marginal sales revenue
- \( c \) = Marginal cost
- \( s \) = Salvage revenue

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Example 1: Marginal advertising revenue is low

Example 2: Marginal advertising revenue is high
Probability of Theft

\[ \sum \left[ \sum_{d=1}^{x_i} P_i(D = d) \sum_{j=1}^{d} (1 - p)^{j-1} p(x_i - j) + \left( 1 - \sum_{d=0}^{x_i} P_i(D = d) \right) \sum_{j=1}^{x_i} (1 - p)^{j-1} p(x_i - j) \right] = 14.4\% \text{ of Total Sales} \]

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