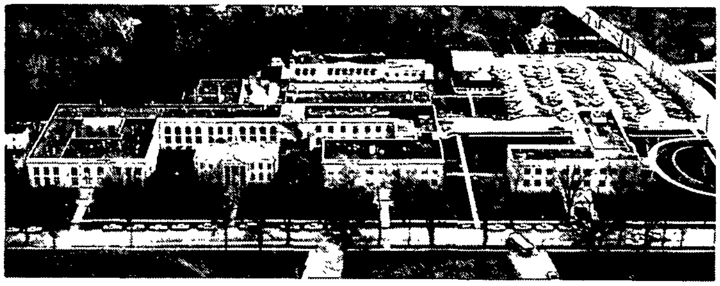


THE INSTITUTE OF PAPER CHEMISTRY, APPLETON, WISCONSIN

MAGAFILE SERIES

IPC TECHNICAL PAPER SERIES
NUMBER 72

ELASTIC WAVE PROPAGATION IN PAPER

R. W. MANN, G. A. BAUM, AND C. C. HABEGER

JANUARY, 1979

Elastic wave propagation in paper

R. W. Mann, G. A. Baum, and C. C. Habeger
The Institute of Paper Chemistry, Appleton, WI 54912

ABSTRACT

This paper describes part of a theoretical and experimental investigation of wave propagation in paper. Treating paper as a three-dimensional orthotropic material, techniques are described for determining seven of the nine independent orthotropic stiffness constants. These constants are used in conjunction with orthotropic plate wave theory to predict plate wave dispersion curves. These curves are then compared with experimental plate wave velocities, determined using a plate wave resonance technique. The good agreement between theory and experiment indicates that orthotropic wave theories, developed for homogeneous continuous media, apply to paper as well.

Introduction

The use of a sonic pulse technique in the nondestructive measurement of paper elasticity was first reported by Craver and Taylor (1,2). In this early work, paper was considered to be a planar material (two-dimensional). Craver and Taylor demonstrated that wave theory developed for homogeneous orthotropic planar materials could describe the propagation of sound waves through paper at low frequencies. The fact that paper is orthotropic in the plane was later verified by Jones (3) who determined in-plane elastic constants using a combination of mechanical techniques.

The existence of dispersive modes at high frequencies was discovered by Luukkala, *et al.* (4). The orthotropic planar model does not predict dispersion. Dispersion can be explained in a homogeneous sheet if it is

This paper has been submitted for publication in Tappi.

treated as a plate of finite thickness. This was done by Luukkala, et al., who assumed an isotropic plate model. The experimental dispersion, however, began at much lower frequencies than predicted by the theory.

In the present work, the validity of treating paper as a three-dimensional orthotropic material and of applying existing wave theories, developed for homogeneous continuous materials, will be established. The general approach taken is to compare experimental and theoretically predicted plate wave dispersion curves. Based on orthotropic bulk wave theory, ultrasonic techniques for measuring the bulk orthotropic stiffness constants were developed. These constants, including the Z-direction constants, are then used in conjunction with orthotropic plate wave theory to predict plate wave dispersion curves. Finally, experimental dispersion curves are obtained using a plate wave resonance technique, and the experimental and theoretical results compared.

Orthotropic wave theories

The theory of wave propagation in an infinite (or bulk) orthotropic medium is well-established. Theoretical developments (5,6) yield expressions for the phase velocities of the normal modes in terms of the orthotropic stiffness constants. These velocities are functions of the direction of wave propagation. For wave propagation in the x-y plane, the results are as follows.

$$\rho V_L^2 = 1/2(c^2C_{11}+s^2C_{22}+C_{66})+1/2((c^2(C_{11}-C_{66})+s^2(C_{66}-C_{22}))^2 +4c^2s^2(C_{12}+C_{66})^2)^{1/2} \quad (1)$$

$$\rho V_{S(in)}^2 = 1/2(c^2C_{11}+s^2C_{22}+C_{66})-1/2((c^2(C_{11}-C_{66})+s^2(C_{66}-C_{22}))^2 +4c^2s^2(C_{12}+C_{66})^2)^{1/2} \quad (2)$$

$$\rho V^2_{S(out)} = c^2 C_{55} + s^2 C_{44} \qquad (3)$$

where ρ = material density

V_L = velocity of longitudinal wave

$V_{S(in)}$ = velocity of shear wave having particle displacements in the x-y plane

$V_{S(out)}$ = velocity of shear wave having particle displacements out of the x-y plane (i.e., in the Z-direction)

c = cos θ

s = sin θ

θ = angle between propagation direction and X-direction

C_{ij}'s = stiffness coefficients

Though paper cannot be considered an infinite medium, Equations (1)-(3) and similar expressions for propagation in the other two principal planes, can be used under certain circumstances to relate measured ultrasonic velocities to the orthotropic stiffness coefficients.

Because paper is a plate material, having one dimension small relative to the other two, it is necessary to consider plate wave theory. Plate wave theory predicts the existence of an infinite number of dispersive plate wave modes (7-10). The frequency dependency (dispersion) of the various plate wave velocities results from the zero-stress boundary conditions that exist at the plate surfaces.

A theoretical treatment of plate waves in an orthotropic medium has recently been developed in this laboratory (7). The treatment is restricted to plate wave propagation in the two principal directions. A dispersion equation which relates the phase velocity to the frequency is developed in terms of the sheet density, caliper, and stiffness coefficients.

For plate waves propagating in the X-direction the dispersion equations required C_{11}, C_{33}, C_{13}, and C_{55}. The coefficients for Y-direction propagation are C_{22}, C_{33}, C_{23}, and C_{44}. Details are included in Reference (7). Therefore, by evaluating seven of the nine orthotropic stiffness constants, the theoretical dispersion curves can be predicted for plate wave propagation in the X and Y-directions.

Determination of elastic constants

Three of the needed seven stiffness constants, C_{33}, C_{44}, and C_{55}, were determined by propagating longitudinal and shear bulk waves in the Z-direction. The following relationships, from Equations (1) and (2), are applicable:

$$C_{33} = \rho V_{Lz}^2 \tag{4}$$

$$C_{44} = \rho V_{Sy-z}^2 \tag{5}$$

$$C_{55} = \rho V_{Sx-z}^2 \tag{6}$$

where V_{Lz} = bulk longitudinal velocity in the Z-direction

V_{Sy-z} = bulk shear velocity in the Z-direction, with polarization in the Y-direction

V_{Sx-z} = bulk shear velocity in the Z-direction, with polarization in the X-direction.

For these measurements, 1 MHz piezoelectric contact transducers were used. The 1/2" diameter transducers are placed on opposite sides of the paper specimen (see Fig. 1a). High frequency (200-800 kHz) bursts of sine waves are fed to the sending transducer, which responds mechanically to the voltage signal. The faces of the longitudinal transducers displace parallel to the axis of the cylindrical transducers, normal to the paper sheet. The shear wave transducers displace parallel to the plane of the sheet.

[Fig. 1 here]

The receiving transducer responds to this disturbance, reproducing the electrical signal. The transit time of the pulse through the specimen is taken as the difference in the measured delay time with and without the specimen between the transducers. Delay times were measured to the nearest nanosecond using a time interval counter (Hewlett Packard 5300B/5308A) in conjunction with an oscilloscope (Hewlett Packard 1740A).

It was necessary to use a small amount of coupling material between the transducers and the specimens. Vacuum grease was used with the longitudinal transducers, while honey was used with the shear transducers. The coupling material was applied to both transducers, which were then affixed to the paper specimens.

The estimated measurement error for each of these velocities is ± 5%, for samples having a thickness of about 25 mils. The three main sources of error are as follows: (1) a slight penetration of coupling material into the specimen, (2) uncertainty in the thickness measurement and (3) extraneous delay times resulting from acoustical impedance mismatches at the transducer-specimen interfaces. These factors constitute greater error for thinner samples.

Constants C_{11} and C_{22} were determined by propagating longitudinal bulk waves in the $\underline{X}$ and $\underline{Y}$-directions, respectively. The following relationships, from Equation (1), are applicable in this case:

$$C_{11} = \rho V_{Lx}^2 \tag{7}$$

$$C_{22} = \rho V_{Ly}^2 \tag{8}$$

where $\underline{V}_{\underline{Lx}}$ = bulk longitudinal velocity in the $\underline{X}$-direction

$\underline{V}_{\underline{Ly}}$ = bulk longitudinal velocity in the $\underline{Y}$-direction

Since paper is not a bulk material with respect to wave propagation in the $\underline{X}$-$\underline{Y}$ plane, it was necessary to construct three-dimensional paper stacks. One method for doing this is to glue individual sheets together with a rubber cement. The glue reduces the measured velocity slightly, but this effect is small and can be accounted for.

$\underline{V}_{\underline{Lx}}$ and $\underline{V}_{\underline{Ly}}$ were determined by measuring the pulse transit time through two stacks of different widths, using 1 MHz longitudinal transducers. The stacks, typically 1 1/2" high, 3" long, and 1/2" to 2" wide, should be large relative to the transducer size, as depicted in Fig. 1b, as well as large compared with the wavelength. Measurement error for both velocities has been estimated to be about $\pm$ 1% using this method.

The final two constants, C_{13} and C_{23}, were determined by measuring the low frequency longitudinal plate wave velocities, and using relationships obtained from orthotropic plate wave theory in the limit as frequency goes to zero [Reference (7)].

$$C_{13} = \pm [(C_{11} - \rho V^2_{SOx}) C_{33}]^{1/2} \quad (9)$$

$$C_{23} = \pm [(C_{22} - \rho V^2_{SOy}) C_{33}]^{1/2} \quad (10)$$

V_{SOx} and V_{SOy} are the low frequency velocities of the zeroth order symmetric plate modes (SO) for propagation in the X and Y-directions, respectively. These velocities could be measured using the Morgan Dynamic Modulus Tester as done by Craver and Taylor (1, 2). Alternatively, contact transducers can be used in the transducer-specimen configuration shown in Fig. 1c. In the present work, the latter technique was used with longitudinal contact transducers at a frequency of 50 kHz. The error in the SO velocities is estimated to less than ± 0.5%.

The coefficients, C_{13} and C_{23}, are calculated using the positive roots of Equations (9) and (10). This assumes positive out-of-plane Poisson ratios. The sign of these coefficients has no effect on the dispersion equation and cannot be determined by techniques employing only elastic wave velocity measurements.

Plate wave resonance technique

To our knowledge, the only successful measurements of dispersive waves in paper have been achieved using a plate wave resonance technique (4). An adaptation of this method has been used in this work.

In the plate wave resonance technique, a sheet of paper is mounted between an ultrasonic transmitter and receiver (see Fig. 2). The paper can be rotated relative to the stationary transducers. At a fixed frequency, as the angle of incidence is changed, the wavelength of the air disturbance projected along the sheet changes. If at some combination of angle and frequency the wavelength along the sheet corresponds to that of a plate wave mode, an optimum transfer of energy occurs, resulting in a maximum in the signal detected by the receiver. The velocities of plate wave modes can thus be measured by recording the peaks in the received signal vs. angle curves, at different frequencies.

[Fig. 2 here]

The frequency range over which measurements can be made using the plate wave resonance technique is limited by a number of factors. Attenuation of ultrasonic waves in air limits the measurements to below about 400 kHz. Since the wavelength along the sheet is always greater than that for the high frequency sound in air, paper velocities below that of sound in air cannot be measured. One result of these limitations is that a sheet must be relatively thick (about 0.4 mm) in order to excite waves by this method.

Results and discussion

Measurements were made on a bleached kraft milk carton stock and two 90-lb linerboard samples. The samples were conditioned at 22% RH for 48 hours before being placed in a controlled environment (73°F, 50% RH) for subsequent testing. The milk carton stock results are discussed below.

The average thickness, determined using a Schopper micrometer calibrated according to TAPPI Standard 411, was 26.7 mils (0.679 mm). The average basis weight was 107.8 lb/1000 ft^2 and the apparent density was 0.775 g/cm^3.

The results of the velocity measurements are given in Table I, along with the estimated experimental uncertainties. Seven of the nine independent orthotropic stiffness constants can be calculated from these velocities, using Equations (4)-(10). The calculated stiffness constants are given in Table II.

[Tables I and II here]

These stiffness constants were used to construct the predicted dispersion curves as outlined in Reference (7). Dispersion curves for X and Y-direction plate wave propagation are the solid and dotted curves presented in Fig. 3 and 4, respectively.

[Fig. 3 and 4 here]

The theoretical dispersion curves are designated S or A, and by a subscript. All plate wave modes have particle displacements which are either symmetric (S) or antisymmetric (A) upon reflection about the midplane of the plate. The subscript gives the mode order. These characteristics of plate wave modes are illustrated in Fig. 5, where the deformation patterns of several X-direction plate waves, as indicated in Fig. 3, are shown.

[Fig. 5 here]

Several features of the theoretical dispersion curves in Fig. 3 and 4 result primarily from the low Z-direction stiffness characteristic of the samples tested (e.g., compare C_{33}/ρ with C_{11}/ρ and C_{22}/ρ in Table II). Compared to an isotropic plate, with stiffness equal to one of the measured in-plane values, the dispersion curves for the orthotropic board are shifted to the left, to much lower frequencies. At high frequencies all modes approach a common behavior. This corresponds to the nondispersive Rayleigh wave for which the motion is concentrated at the surface. The orthotropic treatment predicts a significantly lower Rayleigh velocity than the comparable isotropic model. Also, there are wide nondispersive regions on the symmetric mode curves. For the S1 and higher modes, these occur at approximately the bulk longitudinal velocities. Wave A in Fig. 5 corresponds to the S1 mode at the bulk longitudinal velocity. Interestingly, there is no particle displacement at the surfaces. In the comparable case of bulk longitudinal wave propagation, there is no Z-direction particle displacement at all. The low frequency SO plateau occurs at a slightly lower velocity.

The experimental plate wave data for the milk carton stock are also presented in Fig. 3 and 4. The plate wave resonance technique was capable of detecting only three modes because of the limitations noted earlier. The agreement between theory (solid and dotted curves) and experimental data is very good. The bars shown in these figures define the limit of error which could be attributed to the uncertainties given in Table I. All of the data falls within these limits. The linerboard samples display the behavior predicted by the orthotropic theory although the deviations are greater than for the milk carton stock. This possibly results from the two-sidedness which characterizes linerboard samples made on double headbox fourdriniers.

Conclusions

The prediction of plate wave propagation using orthotropic theories has been demonstrated for milk carton stock. Experimental plate wave data for the linerboard samples were found to deviate slightly from theoretical predictions. Nevertheless, all of the results clearly demonstrate the validity of using these theories.

It is concluded that, based on the experimental data, machine made paper can be considered a three-dimensional, homogeneous, orthotropic material. In addition, stiffness constants calculated from the measured velocities can be used to determine all of the Young's moduli, shear moduli, and Poisson ratios for paper. The good agreement between theory and experiment when treating paper as an orthotropic plate has suggested other, improved, means of measuring certain of the stiffness coefficients. These improved methods and the determination of all nine orthotropic engineering constants will be reported in a subsequent paper.

Literature cited

1. Craver, J. K. and Taylor, D. L., Tappi 48(3):142(1965).

2. Taylor, D. L. and Craver, J. K. In Bolam's Consolidation of the paper web. Vol. 2:p. 852-72. London, Technical Section of the British Paper and Board Makers' Association, 1966.

3. Jones, A. R., Tappi 51(5):203(1968).

4. Luukkala, M., Heikkila, P. and Surakka, J., Ultrasonics 9(3):201 (1971).

5. Borngis, F. E., Phys. Rev. 98(4):1000(1955).

6. Musgrave, M. J. P., Proc. Royal Soc. A226(1166):339(1954).

7. Habeger, C. C., Mann, R. W., and Baum, G. A., Ultrasonics, January, 1979.

8. Graff, K. P. Wave motion in elastic solids. Ohio State University Press, 1975.

9. Viktorov, I. A. Rayleigh and Lamb waves. New York, Plenum Press, 1967.

10. Solie, L. P. and Auld, B. A., J. Acoust. Soc. Am. 54:50(1973).

ACKNOWLEDGMENTS

This work was supported by The Institute of Paper Chemistry. Portions of the work were used by one of the authors (R. W. M.) as partial fulfillment of the Ph.D. degree requirements. The authors are grateful to Dr. Peter Donofrio of Champion Papers who generously supplied the milk carton stock used in the investigation.

Table I. Measured velocities for the milk carton stock

Designation	Velocity, mm/μsec	Uncertainty
V_{Lx}	3.279	± 1%
V_{Ly}	2.307	± 1%
V_{S0x}	3.188	± 1%
V_{S0y}	2.199	± 1%
V_{Lz}	0.231	± 5%
V_{Sx-z}	0.418	± 5%
V_{Sy-z}	0.356	± 5%

Table II. Seven of the nine orthotropic stiffness constants for the milk carton stock ($mm^2/\mu sec^2$)

C_{11}/ρ =	10.75
C_{22}/ρ =	5.32
C_{33}/ρ =	0.0534
C_{44}/ρ =	0.127
C_{55}/ρ =	0.175
C_{13}/ρ =	0.177
C_{23}/ρ =	0.161

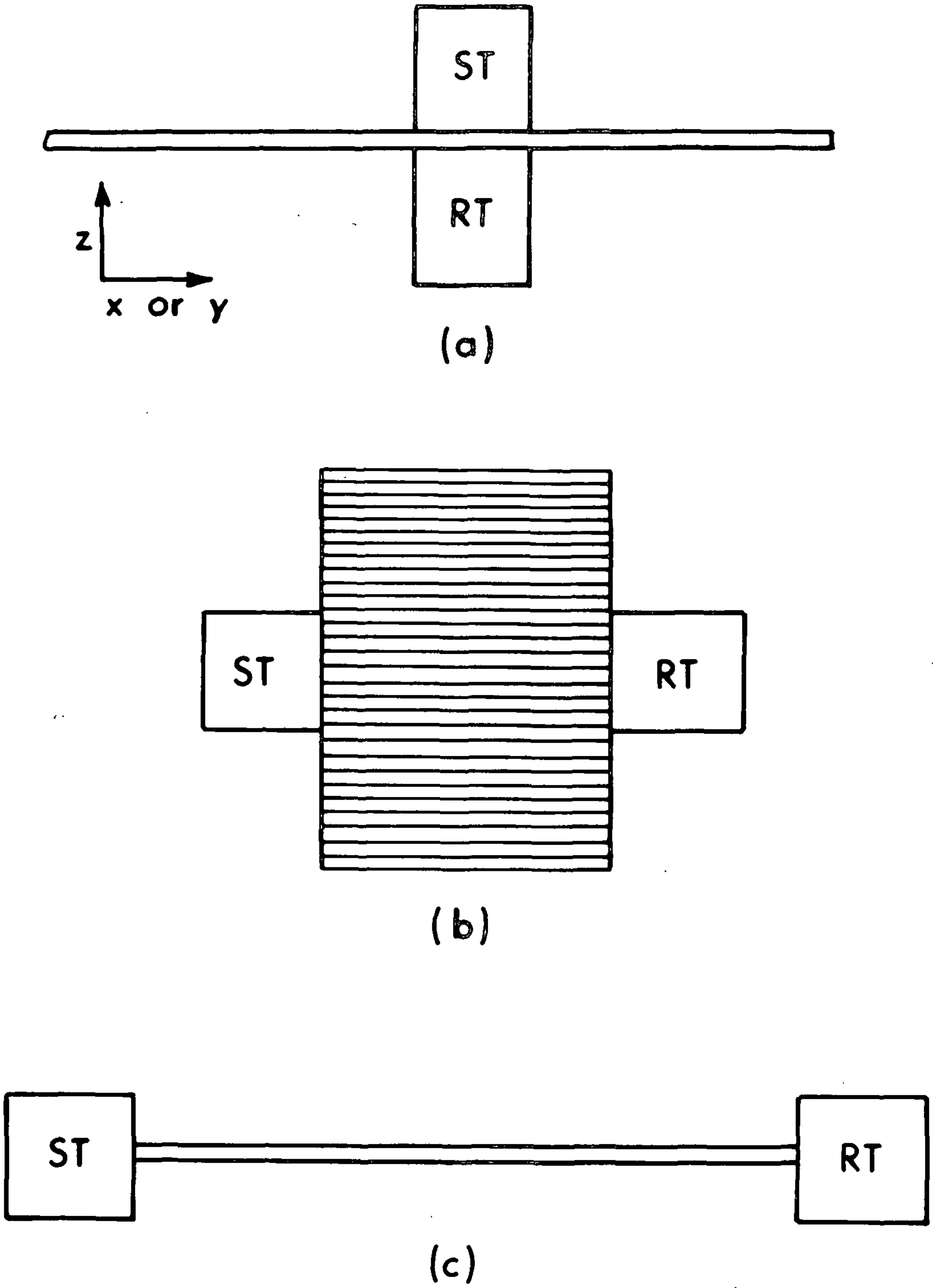


Fig. 1. Transducer-specimen Configurations for Wave Propagation (a) In the Z-direction, (b) Through Stacks, (c) Through Single Thickness Specimens. ST and RT are the Sending and Receiving Transducers, Respectively

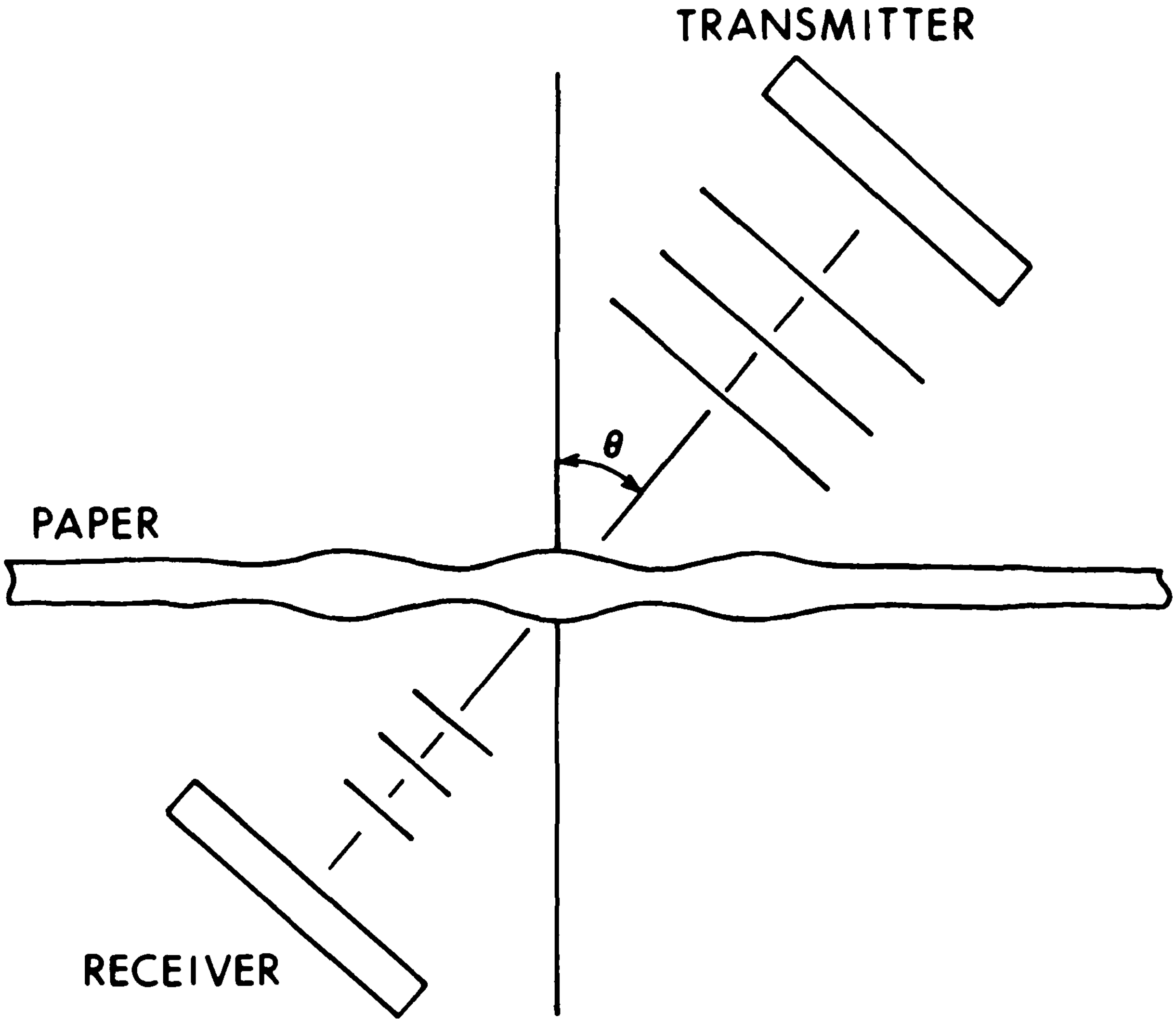


Fig. 2, Schematic of the Plate Wave Resonance Technique

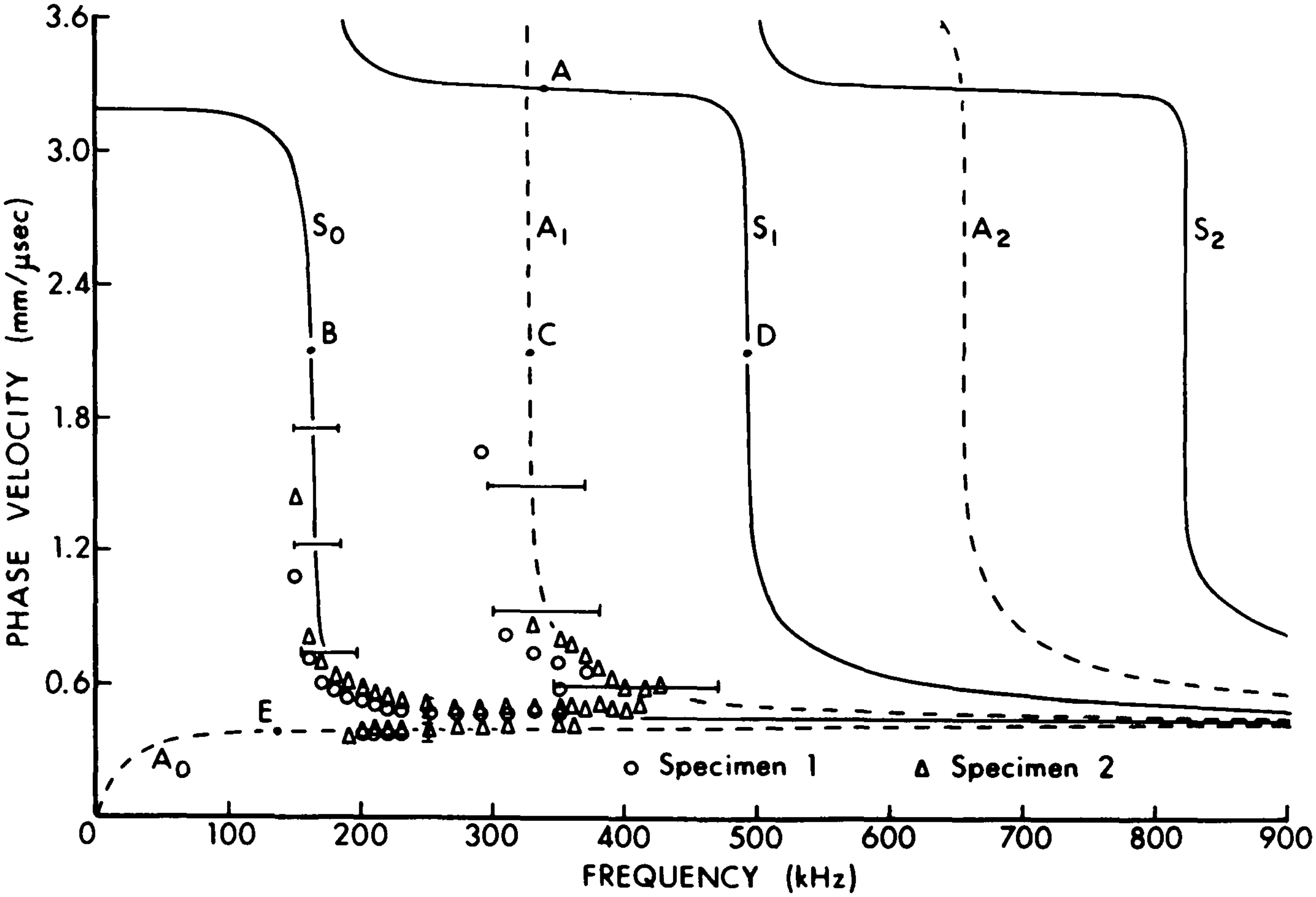


Fig. 3. Theoretical and Experimental Dispersion Curves for Milk Carton Stock in the X-direction. The Points A, B, C, D, and E Refer to the Waves Shown in Figure 5

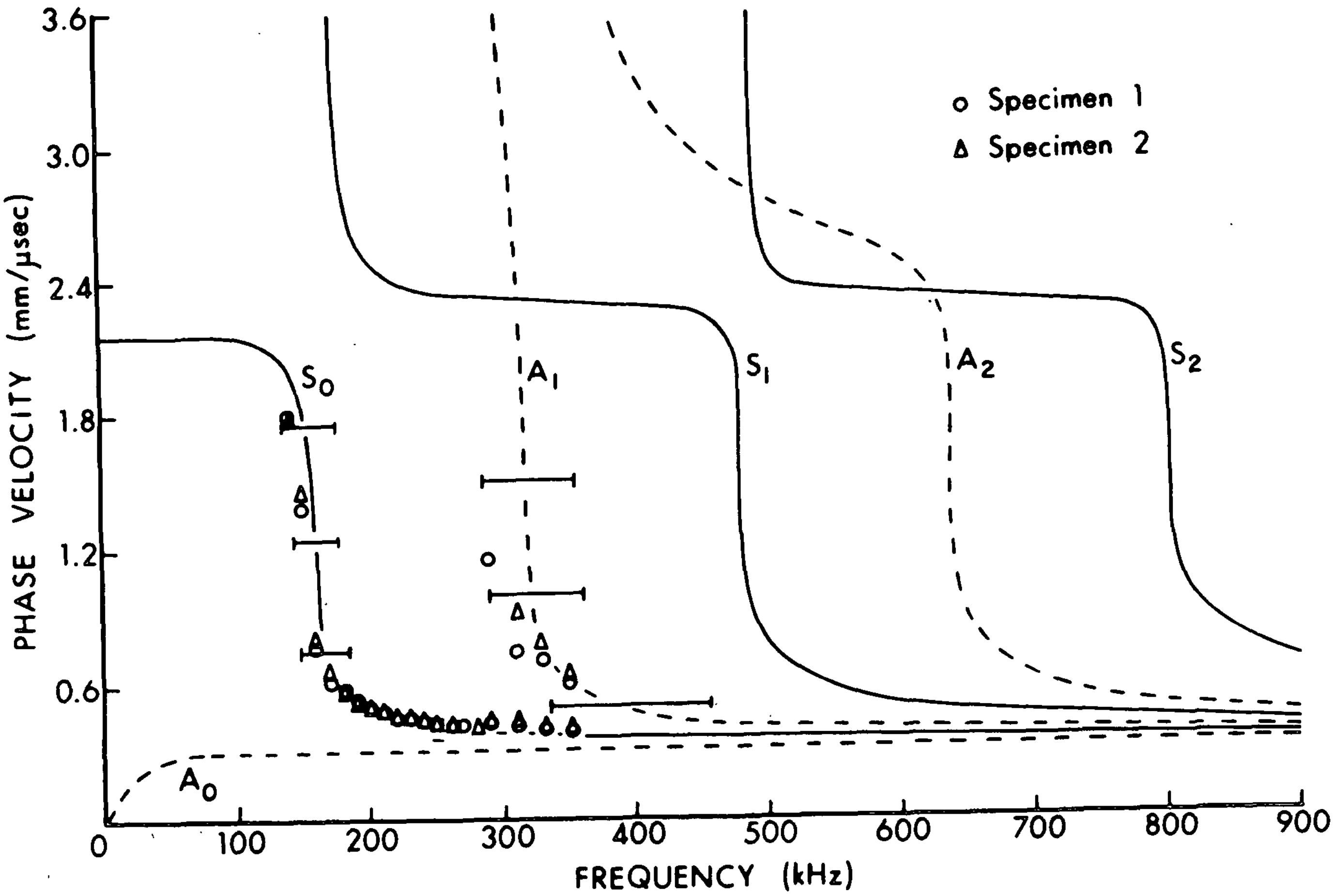


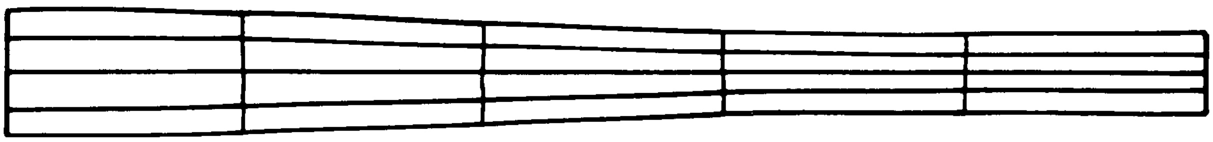
Fig. 4. Theoretical and Experimental Dispersive Curves for Milk Carton Stock in the Y-direction

A

S1 Mode: $V = V_{Lx}$

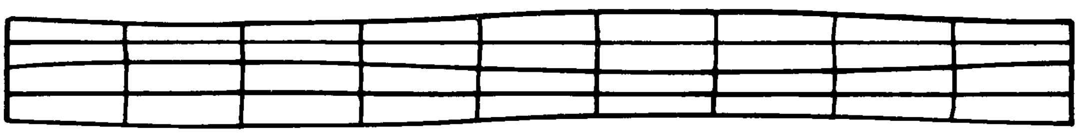
$= 3.279$ mm/μsec

B



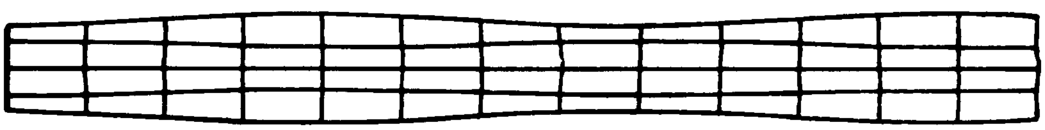
S0 Mode: $V = 2.1$ mm/μsec

C



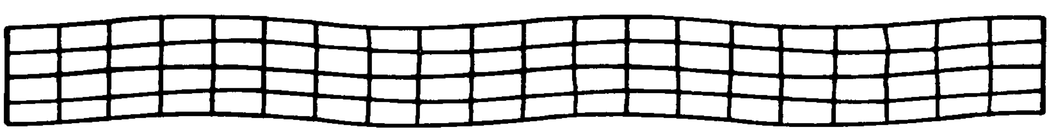
A1 Mode: $V = 2.1$ mm/μsec

D



S1 Mode: $V = 2.1$ mm/μsec

E



A0 Mode: $V = 0.38$ mm/μsec

Fig. 5. Various Cross-sectional Views of Plate Waves in Milk Carton Stock (not to scale). See Figure 3 for Identification of These Waves on the Dispersion Curves