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HEAD-INJURY STUDIES

A Research Report
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Head-Injury Studies

(Response of a Prototype Material in a Rigid Cylindrical Container Subject to a Rotational-Acceleration Transient)

Abstract

With the objective of studying the response of brain tissue in a transient rotational acceleration type loading of the head, the problem of a cylindrical container containing a prototype material of silicone gel and subjected to a rotational acceleration around the axis of the cylinder, is analysed. The prototype material is considered to be homogeneous and isotropic, and is modeled alternatively as a linear elastic or a linear viscoelastic solid. This linearity holds between an appropriate strain measure and an appropriate stress measure, when the deformations are finite. Finite deformations and finite strains of the prototype material are accounted for.

The computational model for the present problem consists of a 3-dimensional isoparametric finite element model, wherein large deformations and large strains are treated through an updated Lagrangian approach. The data computed at node points in the computational finite element grid is extrapolated/interpolated through a smooth-function interpolation technique, onto a grid that was used in an independent experimental study on silicone gel. Thus a comparison of the results of the present 3-dimensional computations, with the attendant assumptions of on material data is made with the results of an independent experimental study. These comparisons of the deformation profiles and strains are quite encouraging. The phenomenon of large shear strains near the periphery of the cylindrical case, in the prototype material, under intense rotational acceleration is captured well in the computations. A computer visualization of the transient response of the prototype material is carried out, through modern computer animation techniques.

1 Introduction

A number of experimental, analytical, and numerical studies have been carried out on the subject of brain damage under a transient dynamic loading of the head.

Holbourn (1943) postulated that a rotation of the head caused by impact resulted in a rotatory distortion of brain, with high shear. It was pointed out by Strich (1969), that the high shear strain may damage tissues in the cortical and subcortical regions of the brain tissue. The translation component of the impact was considered to be non-injurious. Experiments have been conducted by Ommaya in 3 sub-human primate species, and he was able to confirm that the rotational excitation of a short duration may produce cerebral
concussion. At the University of Pennsylvania (1987), physical models of the brain-skull subject to inertial loading have been studied experimentally.

A. E. Engine (1972) studied analytically the steady state response of a solid sphere of an elastic material, under a radial harmonic excitation, and discussed the concept of a complex dynamic shear modulus. The case of a viscoelastic material was also treated, and transformed to the case of an elastic one, to solve the problem by the formulation suitable for an elastic material. H. C. Wang (1972) also studied analytically the response of brain which is modeled as a linear isotropic viscoelastic solid constrained by a rigid skull. C. Lung (1975) investigated analytically the motion of a viscoelastic material enclosed in a rigid shell which is rotated about its axis, for three types of a shell; a semi-infinite cylindrical shell is one of them. Excitational displacement was assumed to be a ramp function, with a Kelvin-Voigt model for the material.

H. C. Merchant (1974) modeled the head as a fluid-filled spherical shell, or alternatively as a prolate ellipsoid of revolution. A 3-D model with one axis of symmetry described in a Lagrangian coordinate system was used. The partial differential equations of equilibrium, continuity, and conservation of energy were solved by an explicit finite difference technique. C. C. Ward (1974) wrote a thesis about a dynamic finite element model of the human brain, and wrote a paper with the title, "Biodynamic finite element models used in brain injury research". A three dimensional analysis has been performed in this thesis. S. N. Atluri (1975) dealt with the finite-element analysis of the state of stress and strain in the vicinity of a blunt indenter applied to the exposed surface of the pia-arach-noid of an anesthetized rhesus monkey. M. Akkas (1975) modeled the head as a fluid-filled 3-layered spherical sandwich shell. The governing linear equations of the transient axisymmetric response of the system are derived using Hamilton principle and solved by a finite difference method. J. C. Misra (1984) considered the brain as a viscoelastic material contained in a shell. The problem was formulated in terms of prolate spheroidal coordinates. The solution of the boundary value problem is sought in the Laplace transform space, by employing a finite difference technique. Use of the alternating direction implicit metod together with the Thomas algorithm was made for obtaining a solution for angular acceleration. The Laplace inversion is also carried out with the help of a numerical procedure (Gauss quadrature formula).

A. S. Hu (1980) investigated the load curve on the head. He concluded that integration of load curve is a kind of low-pass filter, and accumulates a bias-error proportional to time.

The purpose of this report is to study the response of the brain-tissue by using a cylindrical container type model of the brain-skull system, which is subjected to impact loading.

2 Preliminaries

The current state of the science of computational mechanics renders numerical simulation of a head injury to be an alternative to an experiment. When mechanical loads are applied to the head, the deformation, strains or stress within the brain can exceed the tolerance limit, resulting in a head injury. Therefore, it is necessary to better understand the biomechanical aspects of head injury under certain specific kinematic conditions and loading.
It is quite natural to expect that most biological materials would display both elastic or viscoelastic properties which include nonlinearity.

The data from the head injury experiments conducted at the University of Pennsylvania (Model C1, first generation model, B200) is used to define geometry, density, and mass of the model for the present numerical analyses. The boundary conditions used by the University of Pennsylvania are also employed in this analysis. From the acceleration load curve of the experiment, a displacement load curve is generated.

The experimental apparatus used by the University of Pennsylvania is shown in Figure 2.1.

3 Finite Element Formulation

Equilibrium Equations:

\[
\frac{\partial t_{ij}}{\partial x_i} + b_j = \rho \ddot{x}_j, \quad F_{ij}t_{jk} = F_{kj}t_{ji}
\]

Traction Boundary Conditions:

\[
\bar{T}_j = t_{ij}n_i \quad \text{at } S_t
\]

Displacement Boundary Condition:

\[
u_j = \bar{u}_j \quad \text{at } S_u
\]

\[
F_{ij} : \text{Deformation gradient}
\]

\[
t_{ij} : 1-st \text{ Piola Kirchhoff stress}
\]

\[
\sigma_{ij} : \text{Cauchy stress}
\]

\[
u_j : \text{displacement}
\]

\[
b_j : \text{body force}
\]

Large strains, large displacements, and large rotations that occur in the brain damage problem give rise to geometrical nonlinearities. We consider here an updated Langrangian, displacement formulation.

We first divide the loading part of the solid body into \( \Omega^1 \cdots \Omega^N, \Omega^{N+1} \cdots \Omega^J \) where the previous state \( \Omega^N \) is the reference state.

For a given load increment, the incremental equations are solved to obtain the next equilibrated state. With this equilibrated state as a current state, a new load increment is applied and the same procedure is repeated until the total load reaches the desired value.

Weak form: We write the weak forms of the linear momentum balance laws and the traction boundary conditions. It is well known [Atluri (1980)] that angular momentum balance laws for the first Piola stresses are embedded in the incremental potential defined for the Jaumann increment of the Kirchhoff stress.

\[
\int_{\Omega^N} \omega_j \left( \frac{\partial t_{ij}}{\partial x_i} + b_j \right) dV + \int_{\Omega^N} \omega_j \left( \frac{\partial \Delta t_{ij}}{\partial x_i} + \Delta b_j \right) dV - \int_{\Omega^N} \omega_j \rho \ddot{x}_j^{N+1} dV
\]

\[
+ \int_{S_e} \bar{t}_{ij}n_i + \Delta t_{ij}n_i - \bar{T}_j - \Delta \bar{T}_j dS = 0
\]

(1)
Take $\omega_j = -\omega_j$ on $S_r$, $w_j = 0$ on $S_u$. Then

$$
\int_{\Omega^N} \omega_j \left( \frac{\partial t_{ij}}{\partial x_i^N} + b_i \right) dV + \int_{\Omega^N} \left[ \frac{\partial}{\partial x_i^N} (\omega_j \Delta t_{ij}) - \Delta t_{ij} \frac{\partial \omega_j}{\partial x_i^N} \right] dV
$$

$$
+ \int_{\Omega^N} \omega_j (\Delta b_j - \rho \dddot{x}^{N+1}) dV - \int_{S_r + S_u} \omega_j (t_{ij} n_i + \Delta t_{ij} n_i - \overline{T}_j - \Delta \overline{T}_j) dS = 0.
$$

We change the surface integral to the volume integral by using Gauss theorem

$$
\int_{S_r + S_u} \omega_j (t_{ij} n_i + \Delta t_{ij} n_i) dV = \int_{\Omega^N} \left( \frac{\partial \omega_j t_{ij}}{\partial x_i^N} + \frac{\partial \omega_j \Delta t_{ij}}{\partial x_i^N} \right) dV.
$$

Substituting (3) into (2) gives

$$
- \int_{\Omega^N} t_{ij} \frac{\partial \omega_j}{\partial x_i^N} dV + \int_{\Omega^N} \omega_j (b_j + \Delta b_j - \rho \dddot{x}^{N+1}) dV
$$

$$
- \int_{\Omega^N} \Delta t_{ij} \frac{\partial \omega_j}{\partial x_i^N} dV + \int_{S_u} \omega_j (\overline{T}_j + \Delta \overline{T}_j) dS = 0.
$$

Rewriting the above equation, we can obtain

$$
\int_{\Omega^N} \Delta t_{ij} \frac{\partial \omega_j}{\partial x_i^N} dV = \int_{\Omega^N} \omega_j \Delta b_j dV + \int_{S_u} \omega_j \Delta \overline{T}_j dS
$$

$$
- \left[ \int_{\Omega^N} t_{ij} \frac{\partial \omega_j}{\partial x_i^N} dV - \int_{S_u} \overline{T}_j \omega_j dS - \int_{\Omega^N} (b_j - \rho \dddot{x}^{N+1}) \omega_j dV \right].
$$

We introduce Jaumann stress increment $\Delta \tau'$ for objectivity. In the updated Lagrangian formulation, reference state is $\Omega^N$.

$$
\Delta \tau' = \Delta \tau + \Delta \varepsilon \tau + \tau \Delta \varepsilon - \tau L^T \Delta \tau
$$

where

$$
\Delta \varepsilon = \frac{1}{2} \left[ \frac{\partial u_i}{\partial X_j} + \frac{\partial u_j}{\partial X_i} \right]
$$

$J$ : determinant of deformation gradient $F$

t : First Piola-Kirchhoff stress

$S$ : Second Piola-Kirchhoff stress

t : Kirchhoff stress

Q : rotation tensor

L : velocity gradient.

At $t = t^N$, $t = \tau$. At $t = t^{N+1}$

$$
S(t^{N+1}) = S(t^N)
$$

$$
t(t^{N+1}) = t(t^N)Q^T
$$
\[ \Delta t = t(t^{N+1}) - t(t^N) = t(t^N)Q^T - t(t^N). \]

We calculate the left-hand side of equation (4).

\[ \int_{\Omega^N} \Delta t_{ij} \frac{\partial \omega_j}{\partial x_i^N} dV = \int_{\Omega^N} \left( \Delta \tau_{ij} - \Delta \varepsilon_{im} \tau_{mj} - \tau_{im} \Delta \varepsilon_{mj} + \frac{\tau_{im}}{\partial x_m \partial x_i} \frac{\partial \Delta u_j}{\partial x_i^N} dV \right) \]

Taking \( \omega_j = \Delta \delta u_j \)

\[ \int_{\Omega^N} \Delta t_{ij} \frac{\partial \omega_j}{\partial x_i^N} dV = \delta \int_{\Omega^N} \left[ \Delta \tau_{ij} - \Delta \varepsilon_{ij} - \Delta \varepsilon_{im} \tau_{mj} + \frac{1}{2} \frac{\partial \Delta u_j}{\partial x_m} \frac{\partial \Delta u_j}{\partial x_i} \tau_{im} \right] dV. \] (5)

We introduce a linear relation between the Jaumann stress increment and incremental strain in the updated Lagrangian frame.

\[ \Delta \tau_{ij} = C_{ijkl} \Delta \varepsilon_{kl} \]

Displacement increment \( \Delta u_i \) at an arbitrary point on the element can be interpolated by the shape function \( N_{ik} \).

\[ \Delta u_i = N_{ik} \Delta \psi_k, \quad \omega_j = \delta \Delta u_j = N_{jk} \delta \Delta \psi_k \]

\( \psi_k \): nodal displacement in an element.

Then

\[ \Delta \varepsilon_{kl} = \frac{1}{2} \left( \frac{\partial \Delta u_k}{\partial x_i} + \frac{\partial \Delta u_i}{\partial x_k} \right) = \frac{1}{2} \left( N_{km,l} + N_{lm,k} \right) \Delta \psi_m \]

\[ = B_{km} \Delta \psi_m \] (6)

\[ \delta \Delta \varepsilon_{kl} = B_{km} \delta \Delta \psi_m, \quad \frac{\partial \Delta u_j}{\partial x_m} \frac{\partial \Delta u_j}{\partial x_i} = \frac{1}{2} N_{jk,m} N_{jn,i} \Delta \psi_{n,m} \delta \Delta \psi_{l}. \] (7)

Substituting (6) and (7) into (5) gives

\[ \int_{\Omega^N} \Delta t_{ij} \frac{\partial \omega_j}{\partial x_i^N} dV = \int_{\Omega^N} \delta \Delta \psi_n \left[ C_{ijkl} B_{kjm} B_{lji} \right] \Delta \psi_m dV \]

\[ - \int_{\Omega^N} \left\{ \delta \Delta \psi_n \left| B_{kjm} \tau_{ji} B_{lji} \right| \Delta \psi_m + \delta \Delta \psi_m \left| B_{lji} \tau_{ij} B_{kjm} \right| \Delta \psi_n \right\} dV \]

\[ + \frac{1}{2} \int_{\Omega^N} \left\{ \delta \Delta \psi_n \left| N_{jm,k} N_{jn,i} \tau_{ik} \right| \Delta \psi_m + \delta \Delta \psi_m \left| N_{jm,k} N_{jn,i} \tau_{ik} \right| \right\} dV. \]

The second and third integrals are symmetric in the indices \( m, n \).

We have

\[ \int_{\Omega^N} \Delta t_{ij} \frac{\partial \omega_j}{\partial x_i^N} dV = \delta \Delta \psi_n \int_{\Omega^N} C_{ijkl} B_{kjm} B_{lji} dV - \int_{\Omega^N} 2B_{kjm} \tau_{ji} B_{lji} dV \]

\[ = \delta \psi_n K_{mn} \Delta \psi_m \] (8)

where \( K_{mn} \): stiffness matrix.
We calculate the right-hand side of equation (4).
\[
\int_{\Omega} \omega_j \Delta b_j dV + \int_{\partial \Omega} \omega_j \Delta T_j \tau_j dS = \int_{\Omega} N_{jn} \Delta b_j dV \delta \psi_n + \int_{\partial \Omega} N_{jn} \Delta T_j \tau_j dS \delta \psi_n = \delta \Delta \psi_n \int_{\Omega} N_{jn} \Delta b_j dV + \int_{\partial \Omega} N_{jn} \Delta T_j \tau_j dS = \delta \Delta \psi_n \Delta P_{n+1}^N
\]  
\[
(9)
\]
\[
\int_{\Omega} \tau_{ij} \frac{\partial \omega_j}{\partial x_i^N} dV = \int_{\Omega} \tau_{ij} \frac{\partial \delta \Delta \psi_j}{\partial x_i^N} dV = \int_{\Omega} \tau_{ij} \delta \Delta \epsilon_{ij} dV = \int_{\Omega} \tau_{ij} \beta_{ijk} \Delta \psi_k dV = F_k(x^N) \delta \Delta \psi_k
\]  
\[
(10)
\]
\[
\int_{\Omega} \rho \ddot{\psi}_j^N \omega_j dV = \int_{\Omega} \rho \ddot{\psi}_m^N \omega_m dV = \int_{\Omega} \rho N_{jm} \ddot{\psi}_m^N \delta \Delta \psi_k = M_{km} \ddot{\psi}_m^N \delta \Delta \psi_k
\]  
\[
(12)
\]
Substituting Eqs. (8), (9), (10), (11), and (12) into (4) yields
\[
M_{ij} \ddot{\psi}_j^{N+1} + K_{ij} \Delta \psi_j^{N-1} = \Delta P_{i}^{N+1} + P_{i}^{N} - F_i(x^N)
\]  
\[
(13)
\]
Using the Newmark integration method, we obtain
\[
\ddot{K}_{ij} \Delta \psi_j^{N+1} = \Delta P_{i}^{N+1} + P_{i}^{N} - \ddot{F}_i(x^N)
\]  
\[
(14)
\]
where \(\Delta \psi_j = \psi_j^{N+1} - \psi_j^N\)
\(\psi_j^{N+1} = \psi_j^N + [(1 - \delta) \ddot{\psi}_j^N + \delta \dot{\psi}_j^{N+1}] \Delta t\)
\(\Delta \psi_j = \psi_j^N \Delta t + [(\frac{1}{2} - \alpha) \dot{\psi}_j^N + \alpha \psi_j^{N+1}] \Delta t^2\)
\(\ddot{K}_{ij} = K_{ij} + \frac{1}{\Delta t^2} M_{ij}\)
\(\ddot{F}_i(x^N) = F_i(x^N) - M_{ij} \left[ \frac{\ddot{\psi}_j^N}{\alpha \Delta t} + (\frac{9 \Delta t}{2 \alpha} - 1) \dddot{\psi}_j^N \right] + \left(\frac{\Delta t - 2 \alpha}{2 \alpha}\right) \Delta t \dddot{\psi}_j^N\)

We consider several methods to solve the following finite element equation.
\[K \Delta U = P^{N+1} - F^N\]  
\[
(15)
\]
where \(P^{N+1}\) : load vector at time \(N+1\)
\(F^N\) : internal force vector at time \(N\)
\(\Delta U\) : corresponds to \(\Delta \psi\) in equation (11)

Modified Newton method:
\[\tau K \Delta U_i = P^{N+1} - F_{i-1}\]  
\[
(16)
\]
where \( \mathbf{T} \mathbf{K} \) : tangent stiffness matrix.
\( \mathbf{F} \) : internal force at \((i - 1)\text{th}\) iteration
\( \mathbf{r} \) : one of the accepted equilibrium configuration at times \(0, \Delta t, 2\Delta t, \cdots t\).

Newton's Method:

\[
K_{i-1} \Delta \mathbf{U}_i = P^{N+1} - F_{i-1}
\]

where \( K_{i-1} \): the tangent stiff matrix in iteration \(i - 1\).

Quasi-Newton Method:

\[
\delta_i = U_i - U_{i-1}.
\]
\[
\gamma_i = \Delta \mathbf{R}_{i-1} - \Delta \mathbf{R}_i.
\]
\[
K_i \delta_i = \gamma_i
\]

where \( \Delta \mathbf{P}_i = P^N - F_{i-1} \).

BFGS Method

1. Evaluate a displacement vector increment

\[
\Delta \mathbf{U} = (K^{-1})_{i-1}(P^{N+1} - F_{i-1}).
\]

This displacement vector defines a direction for the actual displacement increment.

2. Perform a line search in the direction \( \Delta \mathbf{U} \) to satisfy equilibrium in this direction.

\[
U_i = U_{i-1} + \beta \Delta \mathbf{U}
\]

\( \beta \) is varied until the following equation is satisfied.

\[
\Delta \mathbf{U}^T (P^{N+1} - F_i) \leq \varepsilon \Delta \mathbf{U}^T (P^{N+1} - F_{i-1})
\]

where \( \beta \) : scalar multiplier
\( \varepsilon \) : convergence tolerance.

3. Evaluate the correction to the coefficient matrix.

\[
(K^{-1})_i = A_i^T (K^{-1})_{i-1} A_i
\]
\[
A_i = I + V_i W_i
\]
\[
V_i = - \left[ \frac{\delta_i^T \gamma_i}{\delta_i^T K_{i-1} \delta_i} \right]^{1/2} K_{i-1} \delta_i - \gamma_i
\]
\[
W_i = \frac{\delta_i}{\delta_i^T \gamma_i}
\]

When we solve equation (15) the new deformed position vector is obtained at the \( i \)-th iteration point corresponding to \( i \)-th iteration solution of \( \Delta \mathbf{U}_i \). We also obtain the strain increment corresponding to the displacement and the associated stress increment from the constitutive relation. Thus we can compute the internal force \( F(x^N - \Delta x_i) \). This enables us to perform the equilibrium iteration.

Once the incremental displacements are computed, stress updates are carried out using the incrementally objective algorithms discussed in detail in Rubinstein and Atluri (1983) and Reed and Atluri (1985).
4 Constitutive Law and Finite Strain Measure

4.1 Constitutive Law

We consider a linear, homogeneous, isotropic solid for both the viscoelastic and the elastic cases.

**Viscoelastic**

The internal damping of the brain material is described by means of a standard linear viscoelastic model in this study. This model does not thoroughly describe the characteristics of the real brain material, but serves as a simple model.

The schematic of the standard linear model is shown in Figure 4.1

The shear relaxation function and differential equation are expressed as follows:

$$R(t) = G_2 + (G_1 - G_2)e^{-at}$$  \hspace{1cm} (16)

where $R(t)$ = shear relaxation function

$G_2 = K$

$G_1 = K_1 + K$

$\alpha = K_1/C$

$$S + \frac{C}{K_1} \frac{ds}{dt} = KE + C(1 + \frac{K}{K_1}) \frac{dE}{dt}$$

If we write the above differential equation in terms of $G_1$, $G_2$, and $\alpha$

$$S + \frac{1}{\alpha} \frac{dS}{dt} = G_2 E - \frac{G_1 dE}{\alpha}$$  \hspace{1cm} (17)

where $S$ : deviatoric stress

$E$ : deviatoric strain x 2.

From equation (17) we can obtain the shear modulus $G$, which corresponds to $\frac{\dot{s}}{\dot{E}}$, in terms of sinusoidal input frequency as follows:

$$G(i\omega) = \frac{\alpha^2 G_2 + \omega^2 G_1}{\alpha^2 + \omega^2} + \frac{(G_1 - G_2) \alpha \omega}{\alpha^2 + \omega^2}$$

A relationship between the mean stress and mean strain for a viscoelastic medium can be expressed as:

$$P_1(D)\sigma_m = P_2(D)\varepsilon_m$$

where $P_1(D)$ and $P_2(D)$ are linear differential operators in relation to time $t$.

Assuming that during the extensive distention or compression, the viscoelastic medium behaves in the same way as an elastic body,

$$P_1(D) = 1, \quad P_2(D) = 3K.$$  \hspace{1cm} $K$ : compressibility modulus

**Elastic**

$$S = 2GE$$

$$\sigma_m = 3K\varepsilon_m.$$
4.2 Finite Strain and Deformation

The Deformation gradient referred to the undeformed configuration is denoted by $F$ and is defined as a tensor whose components are $\frac{\partial x_i}{\partial X_j}$ in Cartesian coordinates.

$$d\xi = \frac{\partial x_i}{\partial X_j} dX_j = FdX$$

Green-Lagrangian strain tensor $E$ and Almansi strain tensor $e$ are defined as follows:

**Green-Lagrangian strain:**

$$ds^2 - dS^2 = 2dXEdX$$

**Almansi strain:**

$$ds^2 - dS^2 = 2d\xi ed\xi$$

where $X, \xi$ : position vectors in the undeformed and deformed body respectively

d$S, ds$ : length of infinitesimal element in the undeformed and deformed body respectively

We define deformation terms $C$ and $B^{-1}$ as follows:

$$ds^2 = dXCX$$

$$dS^2 = d\xi B^{-1}d\xi$$

Comparison with the defining equations for each tensor shows that

$$C = F^TF$$

$$B^{-1} = (F^{-1})^TF^{-1}$$

$$E = \frac{1}{2}|F^TF - I| = \frac{1}{2}|C - I|$$

$$e = \frac{1}{2}|I - (F^{-1})^TF^{-1}| = \frac{1}{2}|I - B^{-1}|.$$
Number of nodal points 1325
Number of elements 984
Height 7.02 cm
Radius 3.97 cm

Newmark constants of 0.45 and 0.75 are used for $\alpha$ and $\delta$. These constants satisfy unconditional stability criteria. In addition, $\delta$ was chosen to be a value greater than 0.5 so that a numerical damping can be introduced.

As a time step, $\Delta t = 2.5 \times 10^{-4}$ sec is used. The effect of time step on the accuracy of the transient response is small within its range of $6.0 \times 10^{-5} \sim 4.0 \times 10^{-4}$ sec.

5.2 Models

Four models are classified according to their material properties.

\[
\begin{align*}
\text{Mass:} & \quad 105 \text{ gm} \\
\text{Density:} & \quad 0.95 \text{ gm/cm}^3
\end{align*}
\]

(Model 1)

Elastic

\[
\begin{align*}
\text{Young's Modulus } \sigma & : \quad 8.0 \times 10^4 \text{ pa} \\
\text{Poisson's ratio } \nu & : \quad 0.49
\end{align*}
\]

(Model 2)

Viscoelastic

\[
\begin{align*}
G1 & : \quad 2.69 \times 10^4 \text{ pa} \\
G2 & : \quad 7.54 \times 10^3 \text{ pa} \\
\text{Bulk modulus } K & : \quad 1.25 \times 10^6 \text{ pa} \\
\text{Decay constant } \alpha & : \quad 60 \text{ 1/sec}
\end{align*}
\]

\[
\begin{align*}
\text{equivalent } \sigma
\text{at the frequency } & : \quad 8.0 \times 10^4 \text{ pa} \\
\text{of 419 rad/sec}
\end{align*}
\]

\[
\begin{align*}
\text{equivalent } \nu
\text{at the frequency } & : \quad 0.49 \\
\text{of 419 rad/sec}
\end{align*}
\]

(Model 3)

Elastic

\[
\begin{align*}
\text{Young's Modulus } \sigma & : \quad 1.39 \times 10^5 \text{ pa} \\
\text{Poisson's Ratio } & : \quad 0.49
\end{align*}
\]
Viscoelastic

\[ G_1 : 4.84 \times 10^4 \text{ pa} \]
\[ G_2 : 7.54 \times 10^3 \text{ pa} \]

Bulk modulus \( K \) : 2.25 \times 10^6 \text{ pa}

Decay constant \( \alpha \) : 20 \text{ 1/sec}

Equivalent \( E \)

at the frequency : 1.39 \times 10^5 \text{ pa}

of 419 rad/sec

Equivalent \( \nu \)

at the frequency : 0.49

of 419 rad/sec

where \( G_1 \) : maximum value of shear modulus in the relaxation function

\( G_2 \) : minimum value of shear modulus in the relaxation function

Relaxation functions and frequency characteristics for viscoelastic models are shown in Figure 5.4 and Figure 5.5.

Static shear modulus \( G_2 \) is determined from the experimental data (\( E = 2.246 \times 10^4 \text{ pa}, C_1, B200 \)) obtained from the University of Pennsylvania, and by using the relation \( G = \frac{E}{2(1+\nu)} \). The dynamic maximum shear modulus \( G_1 \) was selected arbitrarily. The reason is that, if we assume the brain model as a standard linear solid, the static value of shear modulus \( K_1 \), and viscosity constant \( C \) are insufficient to determine the shear modulus \( G \). An additional constant \( K \) would be needed to determine \( G \) [Figure 4.1]. But \( K \) and hence \( G_1 \) are selected so that the dynamic shear modulus \( G \) is within the range of \( 8.274 \times 10^3 \sim 6.137 \times 10^4 \text{ pa} \), which is the dynamic shear modulus surmised from the experimental data.

5.3 Load curve

Twenty-eight points are sampled from the angular acceleration load curve of the experiment. This sampled data is multiplied by one half. Chebyshev polynomials of the order of twenty-seven is used to interpolate the data. The resulting coefficients \( C_i \) are listed in Table 5.1. Acceleration is expressed by \( \sum_{i=1}^{28} C_i t^{i-1} \text{ (rad/sec}^2) \), \( t \) being in sec. Application of this load in the finite element program is carried out by the prescribed displacement along \( \Omega_a \) which consists of the side and bottom face of the cylinder.

We need to have the displacement curve corresponding to the acceleration curve through exact integration. The acceleration and displacement curve are plotted and listed in Figure 5.6 and Table 5.2
5.4 Visualization

Computer visualization (animation) was done for the deformed shapes, and cross-sectional component of shear strain at the mid-height cross-section, by using the Alliant FX-8 mini supercomputer at the National Center for Supercomputing Application, University of Illinois.

6 Results and Discussion

Before using the final mesh shape [Figure 5.3] for Model 1, Model 2, Model 3, and Model 4, several mesh shapes were tried [Figure 6.1]. First mesh shape \( M_1 \) seemed reasonable because we anticipated shear distortions in the central region to be minor. Therefore, mesh \( M_1 \)'s poor central region would introduce little error. However, the results showed that spurious response occurred as time exceeds 16 m sec. For this reason, the somewhat undesirably shaped mesh was pulled towards the center, resulting in improved response for mesh \( M_2 \). Meshes \( M_3, M_4, \) and \( M_5 \) are finer that \( M_2 \) in the radial direction. The results varied little between the three mesh shapes \( (M_3, M_4, M_5) \).

Before using the three-dimensional model, plate models were tried. Deformed shapes of plate model, and those at the mid-height cross-section of the three-dimensional model are shown in Figure 6.2. These were done for Model 1.

The effect of time step on the transient response was small within the range of \( 6.0 \times 10^{-5} - 4.0 \times 10^{-4} \) sec.

As pointed out by Stritch (1969), it is believed that shear strain is the leading contributor to the brain damage.

Deformed shapes, Green-Lagrangian strains, Almansi strains, and engineering strains at the mid-height cross-section for each model are plotted in Figure 6.3, Figure 6.4, Figure 6.5, and Figure 6.6. Deformed shapes obtained experimentally at the University of Pennsylvania are plotted along with the numerical results for comparison. The computed results for displacement profiles are similar to those of the experiments performed at the University of Pennsylvania. Similar to the experimental results, the numerical results showed that the brain model lags behind the input rotational displacement during the acceleration phase. Under the deceleration, the inner portion of the brain model continues to move, overshooting the outer input displacement up to a time of 10 m sec. Also, there is an undeformed core exhibiting rigid-body motion as in the experiment.

When memory effects are negligible in a viscoelastic material, the behaviour at short time is due essentially to elastic deformation. Viscoelastic models, Model 2 and Model 4, correspond to elastic models, Model 3 and Model 4, respectively.

Model 3 and Model 4 are 1.8 times stiffer than Model 1 and Model 2. The latter two models showed more similarity to the experiment in their predicted response. It may be pointed out that response largely depends on material properties chosen in the analysis.

7 References


Figure 21: Experimental apparatus (Univ. of Pennsylvania)
Figure 4.1 Model for the standard linear solid
Figure 5.1 Brain-skull model and B.C.

\[ \Omega_u \quad u_i = U_i(t) \]

\[ \Omega_\sigma \quad t_i = T_i(t) = 0. \]

Figure 5.2 3-D element
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Table 5.1 Coefficients of acceleration
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Table 5.2 Load curve data
Figure 5.3 3-D mesh discretization for cylindrical model
Figure 5.4 Relaxation function of viscoelastic models
Figure 5.5 Frequency characteristics
Figure 5.6 Load curve used
Figure 6.1 Mesh shapes
Time= 15.0 m sec
Linear elastic plate model

Time= 18.0 m sec
Linear elastic plate model

Time= 15.0 m sec
Linear elastic 3-D model

Time= 18.0 m sec
Linear elastic 3-D model

Figure 6.2 deformed shapes
Figure 6.3.1 Deformed shape and shear strains: Model 1
Figure 6.3.2.
Figure 6.3.3
Figure 6.3.4
Figure 6.3.5
Figure 6.3.6
Figure 6.4.1 Deformed shape and shear strains: Model 2
Figure 6.4.2.
Figure 6.4.3
Figure 6.4.4
Figure 6.4.5
Figure 6.4.6
Figure 6.5.1 Deformed shape and shear strains: Model 3
Figure 6.5.2.
Figure 6.5.4
Figure 6.5.6
Figure 6.6.1 Deformed shape and shear strains: Model 4
Figure 6.6.2.
Figure 6.6.3
Figure 6.6.4
Figure 6.6.5
Figure 6.6.6