THE BETA TRON AND ITS APPLICATIONS

A THESIS

Presented to

the Faculty of the Division of Graduate Studies

Georgia Institute of Technology

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Physics

by

Walter Wilkinson Atkins

June 1951
Original Page Numbering Retained.
THE BETATRON AND ITS APPLICATIONS

Approved:

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Date Approved by Chairman

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Apr. 30, 1951
ACKNOWLEDGMENTS

I should like to express my sincere appreciation to Dr. J. E. Boyd, my advisor, for his aid and guidance during the course of this work. Also, I should like to thank Dr. V. D. Crawford for his assistance in the preparation of specific sections, and to Dr. L. D. Wyly and Dr. W. M. Spicer for their constructive suggestions.

I should like to thank the staff of the library of the Georgia Institute of Technology, especially Miss Harris and Mrs. Jackson, for their cooperation and help in the location of reference material. Without the assistance of Miss Mildred Jordan of the Emory University Medical Library, sections of this thesis could not have been written.
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THE BETATRON AND ITS APPLICATIONS

CHAPTER I

INTRODUCTION

In the past thirty years many attempts have been made to accelerate electrons to high energies. Many of the methods employed were based on the electrodynamic equation for magnetic induction which, in simple terms, states that an electric field is produced by a varying magnetic field. If an electron is placed in a time-varying magnetic field, it will travel in a curved path and, at the same time, be accelerated by the electric field. The betatron was the first instrument produced which achieved, to any marked degree, success in the acceleration of electrons to high energies. With the betatron it is possible either to have the electrons strike a target and produce penetrating X-rays or to utilize the electron beam directly.

Since all attempts to produce an electron induction accelerator prior to 1940 failed, the various investigators were reluctant to publish their work (1)*. However, a few papers were published and several patents were requested. One of the first to propose methods for electron acceleration was Slepian (2) in 1922. In order to increase the energy of the electrons in an X-ray tube without increasing the potential, Slepian's arrangement was to use the electrostatic force to give electrons an

*See bibliography at end of thesis.
acceleration, and then to give them an additional acceleration by employing a varying magnetic field parallel to the electric field. In this manner, all the electrons making an angle with the electric field would be accelerated also by the magnetic field. Since very little energy could be gained by this spiralling action, Slepian suggested a second method for electron acceleration. This method utilized an evacuated circular glass tube around the central leg of what appeared to be a transformer. It was proposed that permanent magnets be used to bend the electrons into a circular path. Then, if the orbit could be linked by a varying magnetic field, the electrons should be accelerated by the resultant electric field. When the peak flux was reached, the electrons would no longer be accelerated and in some manner would then be utilized. This "transformer method" provides a method of obtaining a very high flux linkage. To make the electrons remain in a circular path, the permanent magnets were shaped in such a way that a strong field was produced at large radii and a weak field at small radii. Thus, as the electrons gained energy and spiralled outward they would encounter the strong magnetic field and be forced back to smaller radii. This shape of field, however, was later found to be exactly opposite to the requirements for electron focusing. Slepian did not pursue his ideas, but they have since been applied by many investigators.

Later, during the same decade, Breit and Tuve (3), at the Department of Terrestrial Magnetism in Washington, D. C., intensively studied various methods for producing high-energy particles. Magnetic induction was one of the methods studied. In their arrangement the acceleration chamber was placed between two large air core solenoids and electrons were
injected into this region of rapidly increasing magnetic field. The varying field was produced by discharging a high voltage condenser through the solenoids. The injected electrons were then to spiral inwardly while the field rose and to strike a target near the center at a small radius from the axis of the solenoids. Before Breit and Tuve could perfect their induction accelerator, a Van de Graaff electrostatic generator was installed, and the work on the induction accelerator was discontinued.

The next step in the development of a magnetic induction acceleration was made in Germany by R. Wideröe (3), who showed how to arrange Slepian's iron core system with a slowly varying magnetic field so as to reach high flux-linkage and high energies. Wideröe's method was to link the electron orbits with the central leg of an iron core transformer which contained an air gap in the plane of the orbit. Without the central flux thus obtained, the orbits would spiral inwardly as in the Breit-Tuve system. Wideröe developed the basic flux relations for a stable orbit but failed to realize the necessity for axial and radial focusing forces.

About the same time that Wideröe was carrying out his experiments, E. T. S. Walton (4), in England, added much to the understanding of electron stability requirements. His idea was to produce a rapidly varying magnetic field within a solenoid by means of an induction coil, condenser, and inductance. The solenoid was wrapped around a vacuum tube, and within this tube the electrons were to be accelerated. The electron source was simply a wire filament. His experimental arrangement failed, but his theoretical calculations solved the problem of obtaining a stable orbit - already solved by Wideröe. Walton also determined the requirements for
radial and axial stability.

Thus, by 1929, the necessary flux conditions for an equilibrium orbit and the requirements for the existence of focusing forces had been determined. There were other problems, however, that were not solved until 1939-1940.

The experimental equipments developed by Breit and Tuve, and by Walton, are quite similar to the betatron made by the Philips Laboratory at Eindhoven, Netherlands, about 1948. This betatron (5) is one without an iron yoke and has several advantages over the conventional machines. These advantages will be discussed in Chapter III.

In 1937, Steenbeck (6) applied for a patent for an induction accelerator based on the ideas brought out by Wideröe. He employed Slepian's transformer arrangement with an air gap in the central leg, and also shaped the pole pieces in such a manner that the magnetic field fulfilled the necessary conditions for radial stability for a fixed equilibrium orbit. The source of electrons was a heated filament hung in the vacuum chamber. In another source arrangement the electrons were injected from the outside. Steenbeck's methods of electron injection were the primary reason for his failure. His patent application made no reference to actual experiments with this machine, but in 1942 he stated that, in 1935, he had measured a small intensity of radiation (1a).

In 1940, G. W. Penny, using Wideröe's methods, attempted to build an induction accelerator, as did F. R. Abbott, at the University of Washington, and J. L. Tuck, at the Clarendon Laboratory, in England. All of these men failed because of minor faults or because their work was interrupted by the war.
The first successful induction accelerator (7) was built by Donald W. Kerst at the University of Illinois. This machine was tested on July 15, 1940. The energy of the X-rays produced was 2.3 million electron volts. Kerst utilized the required magnetic field conditions established by Wideröe for a stable electron orbit in the acceleration chamber, and the requirements for electron stability outlined by Walton. Before building the accelerator, however, Kerst and Serber developed the complete mathematical relations for electron acceleration. These relations and the other important considerations for acceleration will be discussed in Chapter II.

Since Röntgen's discovery of "a new kind of rays" (8), on November 8, 1895, numerous attempts have been made to increase the energy of X-rays. By heating a filament so that electrons are emitted and then applying a difference of potential between this filament and another plate, termed the target, electrons can be accelerated to such an energy that when they impinge on the target X-rays are produced. By increasing the difference of potential between the filament and target, the electrons will acquire greater energy and hence produce higher-energy X-rays. With this type of process, the maximum accelerating voltage is limited by the breakdown voltage of the insulation on the transformer and also by the danger of sparking across the tube elements. By the use of special insulation, it is possible to produce X-rays of 2 to 3 million electron volts by conventional means, but with the magnetic induction accelerator a means is available for the production of X-rays of many times this energy.

Shortly after the first accelerator was proven a success, Kerst
took a position with the General Electric Company in order to supervise the building of a 20 Mev machine (9). After this larger instrument was built, one having a maximum energy of 100 Mev was undertaken by General Electric (10). The 100 Mev machine was finished in 1945, and since that time accelerators having a maximum energy of 4, 10, 24, 40, 50, and 80 Mev have been made. The 24 Mev betatron is now being produced commercially by Allis-Chalmers Manufacturing Company. The latest and largest machine (11), maximum energy of 315 Mev, has recently been completed at the University of Illinois. The British have produced a 150 Mev machine, and the Germans have made several having maximum energies of 6 Mev and 15 Mev (12). The Germans have been studying and making plans to produce a 200 Mev betatron in the near future (13).

The electron accelerator which Kerst produced has been given several names. The one which is used most widely, however, is the name "betatron." This is derived from the Greek letter "beta" which is commonly utilized to represent a stream of electrons, and "tron" which means "an agency for producing" (14). The betatron has sometimes been referred to as a "rheotron," where "rheo" is a Greek word meaning flow or flux (15). In the early stages of the development of the betatron, it was referred to as a magnetic induction accelerator.

The applications of the betatron are numerous and of varied extent. In general, we can classify its applications into three groups: physical, industrial, and medical. The specific applications of the betatron may be further influenced by the maximum energy of the instrument.

As a research tool, in the field of physics, the instrument is extremely useful because it is not only possible to produce high energy
radiation, but also to extract and utilize the beam of high energy electrons. One disadvantage of this instrument is that the radiation produced is not monoenergetic but has a continuous spectrum. Since nuclear reaction measurements require a knowledge of the energy distribution of the incident radiation, many attempts have been made to determine the spectrum. The betatron is very useful in photo-disintegration studies to determine the threshold energies for gamma ray induced reactions. Other applications include studies of neutron yields, the Compton effect, pair production, and the scattering and absorption of both \( \gamma \)-rays and electrons by various materials. Since the new 300 Mev betatron has been completed, it is now possible to produce mesons artificially, aiding greatly in the study of these particles. The physical applications of the betatron will be considered in more detail in Chapter V.

The betatron is also a great asset to industrial radiography. From the study of absorption curves for iron and steel, it has been found that the maximum transmission of X-rays exists for an energy of approximately 7 or 8 Mev. Since a large intensity of 7 or 8 Mev photons is available from a 22 Mev betatron, an instrument of this type is ideal for industrial purposes. In addition to this minimum absorption characteristic, the secondary electrons and radiations produced in the specimen being radiographed are scattered primarily in the forward direction. This advantage of high-energy radiography substantially reduces fog on the photographic plate and eliminates the necessity for blocking of the specimen. Under all conditions, the time of exposure is greatly reduced as compared to conventional X-ray radiography or to the application of radium. It is also possible, using 22 Mev radiation, to detect flaws of 1/32 of
an inch thickness, and some experts claim that they can "see" a thickness variation of 1/64 of an inch by using high contrast film. Details and curves applying to radiographs are discussed more fully in Chapter VI.

In the field of medicine, the betatron has practically unlimited possibilities. For approximately the past fifty years, radiologists have employed X-rays in the treatment of malignant tissue. X-rays having energies up to 200 or 400 kev are very effective for certain types of malignancies when the growth is located on or very near the skin. But for tumors well within the body this radiation is of little avail. This ineffectiveness is primarily due to the absorption and scattering of the rays before they reach the region where the radiation is needed. In order to overcome this difficulty, the exposure of the skin and tissue in front of the tumor would reach a prohibitively high value. If, however, the X-ray energy is increased, the maximum dose does not occur on the skin but below the surface. For example, with X-rays having an energy of 24 Mev, the maximum dosage is at a depth of approximately four centimeters beneath the skin. By applying special techniques, or increasing the energy of the radiation, this maximum depth dose distance may be still further increased. Another possibility for the utilization of the betatron for therapy is the direct application of the electron beam.

In the past, and also at the present time, the use of the betatron in medical therapy for the treatment of malignant diseases, such as cancer, has been restricted. Experimentation using the betatron has been in progress for several years with encouraging results. But before this
instrument can be used widely, further experimentation must be performed. Chapter VII gives a much more detailed description of the medical applications of the betatron.
CHAPTER II

ELECTRON ACCELERATION

As has been previously stated, an electric field is induced by a time-varying magnetic field. In the betatron, the magnetic field provides the force required to keep electrons in a circular orbit while the electric field produces tangential acceleration of the particles. By placing a coil around each pole piece and applying an alternating current, the magnetic field is made to increase in the air gap between the poles. By correct shaping of the pole pieces, the field exerts forces on the electrons and causes them to take up stable orbits as they are injected into the vacuum chamber. Just after the magnetic field passes through its zero value, electrons are injected, and as the magnetic field reaches a maximum, the equilibrium orbit is disturbed and the electrons strike a target, thereby producing X-rays. Fig. 1 (16a) shows the basic arrangement for acceleration of electrons in the betatron.

For the successful operation of a betatron, there are many factors which must be considered. The following paragraphs discuss, with varying degrees of detail, the important points of theory for electron acceleration in the betatron.

Dynamics of Electrons

The equation of motion of the electrons is

\[ \frac{d(mv)}{dt} = e(E \times B). \]

The symbols used have their accepted meaning, and a wavy line under a symbol denotes that it is a vector quantity. The minus sign shows that
Figure 1. Diagram illustrating the principle of operation of a betatron. Electrons are injected at time A in the magnetic cycle and directed at time C against either an inside target or an outside target.
the electrons are negatively charged, and therefore \( e = 1.6 \times 10^{-19} \) coulomb. The number combination /1/ = (17) refers to equation /1/ and reference (17) in the bibliography. The relativistic mass, \( m \), is given by \( m = m_0/(1 - v^2/c^2)^{1/2} \), and \( e/m_0 = 1.76 \times 10^{11} \) coulomb per kilogram. Thus, the equations employ the meter-kilogram-second (mks) system of units.

By the use of cylindrical coordinates and equation /1/, the equations of motion of the electrons in the radial, tangential, and axial directions are

\[
\begin{align*}
\frac{d(mr^2)}{dt} &= -e(E_r + r\dot{B}_z - \dot{z}B_r), \quad (18a) \\
\frac{d(m\dot{r}^2)}{dt} &= -e(r\dot{E}_r + \dot{z}B_r - \dot{r}B_z), \quad (18b) \\
\frac{d(mz)}{dt} &= -e(E_z + \dot{r}B_r - r\dot{B}_r). \quad (18c)
\end{align*}
\]

In the ideal betatron the electric field is assumed to be produced solely by induction from the changing magnetic field. This assumption eliminates any sources such as charges or polarized materials. In addition, the magnetic field is symmetric about an axis of revolution and a plane of reflection (midway between the poles), as illustrated in Fig. 1. By placing the origin of the cylindrical coordinates at the axis of symmetry and making \( z = 0 \) at the plane of reflection, it can be assumed that the magnetic component \( B_\theta \) and the rate of change of \( B_z \) with respect to \( \theta \) are everywhere zero. Also by applying Maxwell's equations

\[ \nabla \times E = -\frac{\partial B}{\partial t} \quad \text{and} \quad \nabla \cdot B = 0 \]

and the assumptions just given, we find that the only components differing from zero are \( B_z \), \( \dot{E}_r \), and \( E_\theta \). Vector expansions in cylindrical
coordinates are given in Appendix I. Now by using the integral form of
Faraday's Law, which states
\[ \mathcal{E} \cdot dl = - \int \frac{\partial \mathbf{B}}{\partial t} \cdot ds, \quad /3/ - (19) \]
a relation between \( E_a \) and \( B_z \) may be determined. Since the path length
is \( 2\pi r \) and the area is \( \pi r^2 \) for \( E_a \) and \( B_z \), respectively, we have
\[ rE_a = - \int_0^r r \frac{\partial B_z}{\partial t} dr. \quad /4A/ \]

By expanding \( \nabla \cdot \mathbf{B} = 0 \), in cylindrical coordinates and applying the
assumption previously given, a relation between \( B_r \) and \( B_z \) may be found.
\[ rB_r = - \int_0^r r \frac{\partial B_z}{\partial z} dr. \quad /4B/ \]
Therefore, equations \( /4A/ \) and \( /4B/ \) express the relationship between \( B_r \),
\( B_z \), and \( E_a \). For a more detailed derivation of equation \( /4B/ \) see Appendix
II.

Employing the Maxwell equation
\[ \nabla \times \mathbf{H} = i_v \times \frac{\partial \mathbf{D}}{\partial t}, \quad /5/ - (20) \]
another requirement of \( B_z \) can be determined. In a vacuum, the current
density, \( i_v \), equals zero; also \( D \) equals \( \varepsilon E \), and \( B \) equals \( \mu H \). This gives
\[ \nabla \times \mathbf{B} = \mu \varepsilon \frac{\partial \mathbf{E}}{\partial t} \quad /6/ \]
By expanding \( \nabla \times \mathbf{B} \) and substituting equations \( /4A/ \) and \( /4B/ \) into equation
\( /6/ \), the result is
\[ - \frac{1}{r} \int_0^r \frac{r^2}{\partial z^2} B_z dr - \frac{\partial B_z}{\partial r} \frac{1}{e^2} \int_0^r \frac{r^2}{\partial t^2} B_z dr = 0. \quad /7/ \]
The derivation of equation /7/ is discussed in Appendix III.

By assuming complete symmetry of the fields as previously given, the equations of motion /2A-B-C/ may be simplified. If equations /4A/ and /4B/ are also used, the resulting equations of motion are

\[
\frac{d(m\dot{r})}{dt} = m\ddot{r} - e\dot{\phi}B_z, \quad /8A/ - (18b)
\]

\[
\frac{d(m\dot{r}^2\dot{\phi})}{dt} = e\dot{\phi}\int_0^r rB_zdr, \quad /8B/ - (18b)
\]

and

\[
\frac{d(m\ddot{z})}{dt} = -e\dot{\phi}\int_0^r r\frac{dE_z}{dz}dr. \quad /8C/ - (18b)
\]

For a derivation of equation /8B/, see Appendix IV. Upon integration of equation /8B/, a constant, \(K\), is required, which is

\[
K = m_o r_o^2 \ddot{r}_o/e - \int_0^r rB_z(r,z_0,t_0)dr. \quad /8D/ - (18b)
\]

The subscripts indicate initial conditions. Therefore, the equations of motion are

\[
\dot{r} = \frac{e}{m} \left[ \frac{\int_0^r rB_zdr}{r} \right] - \frac{K}{r}, \quad /9B/ - (18b)
\]

and equations /8A/ and /8C/ become, if we let the factor within the bracket equal \(S\),

\[
\frac{d(m\dot{r})}{dt} = -\frac{e^2}{2m} \frac{\partial(S^2)}{\partial r}, \quad /9A/ - (18b)
\]

and

\[
\frac{d(m\ddot{z})}{dt} = -\frac{e^2}{2m} \frac{\partial(S^2)}{\partial z}. \quad /9C/ - (18b)
\]

Since the total energy of the particle can change only by its motion along the existing component of the electric field, \(E_z\), the energy variation may be found by equations /4A/ and /9B/. It is
\[
\frac{d(mc^2)}{dt} = -e \dot{E}_{0} = -\frac{e^2}{2m} \frac{\partial E}{\partial t}.
\]

Now, \(m\) is given directly by substituting equation /9B/ into the equation for mass; therefore,

\[
m^2c^2 = \frac{e^2S^2 - m_0^2c^2}{1 - (r^2 - z^2)/c^2}.
\]

See Appendix V, for derivation of equation /11/. The kinetic energy of the particle is

\[
T = (m - m_0)c^2 = m_0c^2 \left\{ \frac{eS^2}{m_0} \sqrt{\frac{m_0 c^2 [1 - (r^2 - z^2)/c^2]}{e}} - 1 \right\} - 1.
\]

Equation /12/ is an exact expression by which the energy of the electron may be determined. \(B_z\) is a function of \(r, z,\) and \(t\); \(K\) depends on the injection characteristics; \(r^2\) and \(z^2\) take into account any possible radial or axial oscillations.

If the value of the kinetic energy is desired just before the electrons strike the target, equation /12/ may be reduced to a much simpler form. \(K\) becomes less and less important relative to the time increasing \(\int_0^r rB_z dr\); therefore, \(K\) can be assumed zero. For a perfectly stable orbit \(\dot{r}\) and \(\dot{z}\) are both zero. Under these conditions, equation /12/ becomes

\[
T = \left[ (m_0c^2)^2 - c^2(mv)^2 \right]^{\frac{1}{2}} - m_0c^2.
\]

The momentum \(mv\) has the value \(e(\Phi - \Phi_0)/2 \pi r\), where \((\Phi - \Phi_0)\) is the change in flux while the electrons are being accelerated. For details of the simplification of equation /12/, see Appendix VI. Fig. 2 (5a)
Figure 2. Graph illustrating the relation between the velocity, $v$, the mass, $m$, and the kinetic energy, $T$, of an electron in motion. The quotient $v/c$ is plotted as a function of $T$. Classically the dotted curve would apply, but at energies of the order of 1 Mev the velocity of the electron approaches the velocity of light, so that the relativistic deviation from the classical relation is then very noticeable. The factor $m/m_0$, indicating the relativistic increase of the mass at high velocities, is shown along the curve. (The scale for protons is also shown in the diagram.)
shows the relation between the velocity, the mass, and the kinetic energy of an electron.

During the early acceleration of the electron after injection, the energy gained is very small compared to the rest mass of 0.512 Mev of the particle. For this reason, for a time shortly after injection, the mass can be considered as constant. This assumption eliminates any damping force caused by the mass variation, but strong damping is produced by the changing magnetic field. At relativistic velocities, however, the mass variation is the predominant factor in damping the oscillations.

Conditions for Stable Orbit

To obtain the required field configuration for electron capture, the mass variation can be considered zero. Then, the equations of motion for the r and z directions are of the form describing two-dimensional potential motion, and may be written as

\[ m_0 \ddot{r} = -e \frac{\partial V_m}{\partial r} \quad \text{and} \quad m_0 \ddot{z} = -e \frac{\partial V_m}{\partial z} \]

where \( V_m = \frac{e m_0 S^2}{2m^2} \).

Equations /14A-B/ may also be written \( F_r = -e \frac{\partial V_m}{\partial r} \) and \( F_z = -e \frac{\partial V_m}{\partial z} \) which shows that the total force is \( F = -e V_m \). Therefore, we may refer to \( V_m \) as a scalar potential.

By the approximate equivalence of the radial and axial electron motion to two-dimensional classical motion in a potential field slowly varying with time, the necessary conditions for an instantaneous equilibrium orbit, having a radius of \( r_i \), can be determined. The force in the radial direction, for an orbit to exist at \( r_i \), must be a minimum;
therefore, the potential surface at the radius $r_1$ must conform to the conditions that

$$\frac{\partial V_m}{\partial r} = 0 \quad \text{and} \quad \frac{\partial^2 V_m}{\partial r^2} > 0. \quad /15A/ - (21)$$

For the $z$ direction, a minimum is obtained if

$$\frac{\partial V_m}{\partial z} = 0 \quad \text{and} \quad \frac{\partial^2 V_m}{\partial z^2} > 0. \quad /15B/ - (21)$$

The final condition for a minimum to exist between two variables is that

$$\left( \frac{\partial^2 V_m}{\partial r^2} \right) \left( \frac{\partial^2 V_m}{\partial z^2} \right) - \left( \frac{\partial^2 V_m}{\partial r \partial z} \right)^2 > 0. \quad /15C/ - (21)$$

Because of symmetry of the electric and magnetic fields with respect to the plane $z = 0$, the condition that $\frac{\partial V_m}{\partial z} = 0$ is satisfied everywhere on this plane. Therefore, if the conditions that $\frac{\partial^2 V_m}{\partial r^2} > 0$ and $\frac{\partial^2 V_m}{\partial z^2} > 0$ are satisfied, equation $/15C/$ is also satisfied. Hence, the only conditions which are of importance are those of equation $/15A/$ and the latter equation of $/15B/$, since the electrons are usually injected in the plane $z = 0$. These equations specify the requirements for radial equilibrium, radial focusing, and axial focusing, respectively.

Considering the radial equilibrium condition applied to $V_m$, we have

$$S = \int_0^r r B_z \, dr \neq \frac{\mathbf{r}}{K} = 0 \quad /16/$$

and

$$B_z - S/r = 0. \quad /17/$$

By referring to equation $/9B/$, it is seen that equation $/16/$ is for an electron at rest, and $/17/$ is the condition for equilibrium in the radial
direction. If by using the latter expression in the equations, when
the radial and axial focusing conditions are applied to $v_m$, the two
expressions

$$\frac{r}{B_z} \frac{\partial B_z}{\partial r} > -1 \quad \text{(18a)}$$

and

$$B_z \int_0^r r \frac{\partial^2 B_z}{\partial z^2} \, dr > 0 \quad \text{(18b)}$$

are obtained. Now, by taking into consideration the quasistatic nature
of the magnetic field, the time derivative term in equation (7) may be
neglected. If the results from equation (7) are substituted into equa-
tion (18b), this gives

$$1 > \left[ -\frac{r}{B_z} \frac{\partial B_z}{\partial r} \right] > 0. \quad \text{(19)}$$

This means that the lines of the magnetic field must be convex outward
and that the field intensity diminishes with increasing radius, while
the corresponding diminishing in centripetal magnetic force must occur
more slowly than the decrease in centrifugal force at constant electron
velocity. During the acceleration, the constant $K$ has less and less im-
portance relative to the time increasing $\int_0^r r B_z \, dr$. Therefore, the lim-
it- ing value of equation (17) is

$$B_z = \frac{\int_0^r r B_z \, dr}{r^2}. \quad \text{(20)}$$

Hence, the magnetic induction at the equilibrium orbit is simply one-
half of the average magnetic induction within the orbit. Therefore,
equations (19) and (20) express the necessary conditions for the estab-
ishment of an equilibrium orbit.
Another method of determining the necessary conditions for an equilibrium orbit was developed by Kerst and Serber (22). Since $\nabla \cdot \mathbf{B} = 0$, we can write that $\mathbf{B} = \nabla \times \mathbf{A}$, where $\mathbf{A}$ is the magnetic vector potential. Considering only the $z$ component of the magnetic field, the expression

$$B_z = \frac{A}{r} \frac{\partial A}{\partial r}$$  \hspace{1cm} (22a)  

can be obtained. At the equilibrium orbit equation /21/ becomes

$$B_{ze} = \frac{A_e}{r_e}.$$  \hspace{1cm} (22)  

By using the basic condition for magnetic lines of flux through the area enclosed by the orbit, the function $A$, and Stokes theorem, we have

$$\oint \mathbf{B} \cdot d\mathbf{r} = \oint \nabla \times \mathbf{A} \cdot d\mathbf{r} = \oint \mathbf{A} \cdot d\mathbf{r} = 2\pi r_e A_e.$$  \hspace{1cm} (23)  

This reduces to

$$\oint \mathbf{B} \cdot d\mathbf{r} = 2\pi r_e B_{ze}.$$  \hspace{1cm} (24)  

Therefore, the magnetic induction at the orbit is one-half the average value of $B_z$ within the orbit.

The condition that $B_z$ vary proportionally as $r^{-n}$ was assumed as an additional requirement to provide radial and axial focusing of the electrons and that $n$ should have a value greater than zero and less than one. Symbolically, this states that

$$0 < n < 1.$$  \hspace{1cm} (25)  

Since $\partial B_z/\partial r$ from equation /19/ is inherently negative, the two expressions /19/ and /25/ have identical meanings. This condition for the shape
of the magnetic field for electron focusing was first derived by Walton (4). Equations /20/ and /24/ have identical meanings and express the flux requirement for an equilibrium orbit. This necessary flux condition, known as the 1:2 relation, was first derived by Wideröe (3).

Induction accelerators are usually built so that condition /18B/ is satisfied for all positions within the vacuum chamber. Since the equilibrium plane, at $z = 0$, is symmetrical, the axial sections of $V_m$ produce surfaces of revolution which are paraboloidal in shape increasing outwardly on either side of $z = 0$. These surfaces form potential bowls, and the shape of the surface at the equilibrium orbit determines whether the electrons will remain in the bowl, if the sides are steep enough, or collide with the walls of the vacuum chamber. The shape of the bowl depends on the negative value of $(r/B_z) \cdot B_z/\partial r$. The radial sections of the potential surfaces are of no greater importance than the axial ones; but because of the constant $K$ in the equation for $V_m$, the radial surfaces are critically dependent on the initial injection conditions. This is caused by the fact that the function $B_z$ in the expression for $K$, equation /8D/, varies with $r$ only, since $z_0$ and $t_0$ are for initial conditions. Fig. 3 (18d) shows the instantaneous magnetic field distribution and the $V_m$ surface for a normal electron. The outline of the pole pieces generating this field is also shown. The term "normal electron" refers to an electron that has a $K$ value equal to zero. Fig. 4 (18e) shows the profile of the $V_m$ function with respect to the radial distance from the axis for the $z = 0$ plane and varying values of $K$. The curves have the same general shape for different values of $z$. 
Figure 3. Cross section of a small betatron showing the variations of the magnetic field intensity and the potential function as the radius varies. By graphical integration, it can be shown that the field intensity at the orbit is \(\frac{1}{2}\) the average intensity enclosed by the orbit.
Figure 4. Radial profiles of the potential function $V_m$. (The C in the figure is equivalent to the K used in the text.)
From the curves of Fig. 4, a large variety of injection phenomena can be understood. A complete discussion and interpretation of these curves may be found in reference (18).

Periods of Oscillation

For small amplitudes of oscillation and an instantaneous radius of \( r = r_1 \), the periods or rotational, radial and axial oscillation are

\[
T_o = 2\pi (eB_z/m)^{-1}, \quad /26A/- (18f) \\
T_r = 2\pi (eB_z/m)^{-1}(1 - n)^{-\frac{1}{2}}, \quad /26B/- (18f) \\
\text{and} \quad T_z = 2\pi (eB_z/m)^{-1}(n)^{-\frac{1}{2}}. \quad /26C/- (18f)
\]

For large amplitudes, all the equations hold with the exception of \( T_r \).

Since the particle approaches the rounded crests of the potential barrier, the period becomes greater than that given by equation /26B/. It is also noted that the periods of \( T_r \) and \( T_z \), because of equation /19/, are always greater than \( T_o \). Since the amplitude of the oscillation is proportional to the period (22a), the ratio between the radial and axial amplitudes can be found from equations /26B/ and /26C/. This ratio is

\[
\Delta r/z = n^{\frac{1}{2}}/(1-n)^{\frac{1}{2}}. \quad /27/
\]

If, for example, \( n = -(r/B_z) \partial B_z/\partial r = 2/3 \), then the amplitude in the \( z \) direction is 0.707 times the amplitude in the radial direction.

The size of the beam also would have this same ratio for its dimensions.

Because of the time variation of the magnetic field, the magnitude and shape of the potential surfaces, given by \( V_m \) as a function of \( r \), change. The energy of the electron increases at first with the
square of $rB_z$ and at relativistic velocities directly with $rB_z$ while the radial and axial oscillations vary with changes in the potential surface. If the radial and axial potential bowls are assumed to be paraboloidal, and if the electron is executing harmonic motion, the fractional decrease in the amplitude of the radial or axial oscillation is given by

$$\frac{\Delta a}{a} = -(1/4) \frac{\Delta V_m}{V_m} = -(1/2) \frac{\Delta B}{B},$$

where $a$ is the amplitude of either the radial or axial oscillations.

Equation /28/ is valid when $K = 0$ and includes any variation in mass; $\Delta B/B$ is the fractional increase in the magnetic induction. A complete derivation of the expression is given in references (18) and (22). For non-relativistic energies, equation /28/ reduces to that derived by Kerst and Serber, which is

$$\frac{\Delta a}{a} = -(1/4) \frac{\Delta V}{V},$$

where $\Delta V$ is the voltage gain of the electrons per revolution and $V$ is the injection voltage.

Equation /29/ should give the maximum injection voltage that can be employed so that the electrons will miss the injector. However, it has been found experimentally that the injection voltage can be many times that given by the equation. Consider a 22 Mev betatron with $a = 1$ cm, $V = 60,000$ volts, and $\Delta V = 90$ volts per revolution. Under these conditions $\Delta a = 0.0004$ cm which is obviously insufficient damping to allow the electrons to miss even the smallest injector. Kerst (23) attempted to solve this difficulty in the theory by
considering the action of the electron beam on itself. As the electron beam circulates between the poles of the betatron, it radiates an electromagnetic energy which is obtained from the kinetic energy. This reduces the kinetic energy of the electrons and causes them to move into a smaller radius. The energy in the electromagnetic field is proportional to the square of the circulating current; hence, the loss of energy per electron will be proportional to the circulating current. To determine the orbital shift, the expression

\[
\frac{\Delta r_i}{r_i} = \frac{\Delta V}{2V(1 - n)}
\]

is used. In this equation \( r_i \) is the instantaneous orbital radius, and \( \Delta V \) is the loss of energy in electron volts. This loss can be found by \( \Delta V = I d_i/dt \). For a 22 Mev betatron the inductance, found by placing a wire at the orbit, was \( 4 \times 10^{-6} \) henries. If the initial circulating current is 1 ampere and the time to establish this current is 1 revolution, which is \( 10^{-8} \) seconds, then the energy lost by each electron is 400 ev. The electrostatic energy associated with an ampere of circulating current is approximately 150 ev. Therefore, the total energy loss is 550 ev per electron. If the initial energy given the electron is 60,000 ev, \( r_i = 20 \) cm, and \( n = 3/4 \), then the shift in the orbit is 3.66 mm. By beam current measurements, which will be discussed in the next chapter, only 0.1 ampere is held in the equilibrium orbit, but it is quite possible that the one ampere of beam current circulates a few times around the tube before the electrons strike the walls or injector.

The ideal position of the injector is somewhere in the plane of \( z = 0 \). This position is best because of the fact that the radial
oscillations are decreasing in amplitude, and the instantaneous orbit is shifting rapidly away from the injector. The theory pertaining to injector position and orbit shift will not be discussed here because at best it represents only partially the mechanism by which the electrons miss the injector.

**Energy Limitation**

When the betatron was first developed, it was thought that the energy obtainable was dependent only on the dimensions of the magnet and the maximum field at the orbit. But, since that time, many investigators have considered the effects due to radiation from the accelerating electrons and have found that this radiation imposes a limiting value of the energy obtainable from the accelerator. Schiff (24) has derived an expression for the fraction of the total energy, $E$, radiated by the acceleration of randomly-spaced electrons. This equation is

$$\frac{\Delta E}{E} = \frac{\pi}{8} \left( \frac{e^2}{m_0 c^2 R} \right) \left( \frac{\omega}{\omega_0} \right)^2 \left( \frac{E}{m_0 c^2} \right)^3.$$  \hspace{1cm} (24a)

In this equation, $\omega$ is the angular velocity of the electrons and approximately equal to $c/R$; $\omega_0$ equals $2\pi$ times the frequency of the sinusoidally varying magnetic field. The radius of the equilibrium orbit equals $R$. If the magnetic field varies linearly instead of sinusoidally the factor $\pi/8$ is reduced to $2/15$. This reduces the radiation to 34 per cent of that for the previous case. If $E = 300$ Mev, $\omega_0/2\pi = 60$ cycles per sec, $R = 2$ m, and $B_z$ max. $= 0.5$ weber/m$^2$ at the orbit, the energy lost by radiation would be 4.7 or 1.6 per cent for sinusoidal or linear variations, respectively, depending on the wave form of the excitation.
Because of this loss in energy, the equilibrium orbit shrinks to a smaller radius. Blewett (25) derived the necessary equations for the electron radiation and orbital shrinkage. By using a 100 Mev machine, it was found that the beam displacement at maximum energy was approximately 7 cm.

Schwinger (26) and McMillan (27) contributed much to the understanding of the radiation problem. The radiation spectrum consists of harmonics of the rotational angular frequency and has a range of more than $10^7$. Schwinger found that the radiation is from two sources: the single electron and the electron group. If $E \gg m_0c^2$, the energy from the individual electrons, or incoherent radiation, $\Delta E$, is mostly in the very high harmonics and is dependent upon the total energy of the electron. Also if $E \gg m_0c^2$, the radiation is partially coherent, in the low harmonics, and is dependent on the number of electrons, $N$, covering uniformly an arc with an angular extent which is $1/u$ of a circle. (If the electron group extends over $1/5$ of the circumference, then $u$ equals 5.) In this case the coherent radiation, $\Delta E'$, is not a function of $E$. For the single electron, the incoherent radiation in electron volts per revolution is

$$\Delta E = \frac{4\pi}{3} \frac{e^2}{R} \left( \frac{E}{m_0c^2} \right)^4.$$  \hspace{1cm} /32/ - (26)

For the electron group, the coherent radiation in electron volts per electron per revolution is

$$\Delta E' = \frac{4\pi}{3} \frac{e^2}{R} 2.4(u - 1)N.$$  \hspace{1cm} /33/ - (27)

There are three methods by which this radiation loss may be
reduced. First, it is possible to greatly reduce the coherent radiation loss by metallic shielding. The second method is to have a large radius of curvature of the electron path. The third arrangement is to operate the betatron at a high frequency. By employing a frequency of 600 cycles per second instead of 180 cycles per second, the voltage gain per turn will be higher; therefore, for the same final energy, the electrons will be required to make fewer revolutions.
Having discussed the general characteristics of the betatron and the specific requirements for electron acceleration, the following chapter presents a detailed outline of the instrument. The general specifications of different energy betatrons are similar, but the specific design of each machine varies. For this reason, the major part of the chapter will be confined to the betatron having a maximum energy of approximately 24 Mev. Topics such as weight, size, vacuum chamber, electron injector, primary electrical circuits, auxiliary circuits, methods of increasing the electron energy, and pulsed operation will be discussed.

Kerst's Original Betatron

The betatron is inherently a small instrument as compared to other particle accelerators having the same maximum energy. Kerst's first machine (28) measured approximately 19 in. long, 10 in. high, and 8 in. wide. Fig. 5 (28a) is a diagram of this machine. M represents the magnetic yoke, P the pole pieces, C the excitation coils, V the vacuum chamber, G the injector, and T the target. The magnet and pole pieces were constructed of 0.003 in. silicon steel laminations, and were baked in an adhesive.

The pole pieces were of such a shape that \(- (r/B_z) \partial B_z / \partial r\), which is equivalent to n, equaled 2/3 (29). A compressed iron dust cap was placed on the central section of each pole piece. By varying the
Figure 5. Diagram illustrating Kerst's original 2 Mev betatron. The magnet measures 19 in. long, 10 in. high, and 8 in. wide. The X-ray output is equivalent to 1 gm of radium.

Figure 6. Cross section of the acceleration chamber in Fig. 5. The equilibrium orbit is at $r_o$, T is the tungsten target, A is the injector, and B is a top view of the injector. A beam of electrons from the filament F is shot out through slots in the positive plates P. G are negative focusing electrodes.
air gap between the pole caps, the radius of the equilibrium orbit could be changed. As the magnetic field approached its peak value of $0.16 \text{weber/m}^2$, the pole caps became saturated while the field at the orbit still increased. This disturbed the ratio between the flux enclosed by the orbit and the flux at the orbit; therefore, the electrons spiralled in to a smaller radius and struck a tungsten target.

The excitation coils for the magnet were made of two hundred strands of No. 20 enameled wire having ten turns each. Connected to these coils were eight 5 microfarad Pyranol condensers rated at 660 volts a-c. To supply power to this resonant circuit, one turn of wire was placed around each pole piece and connected to a 4 kilowatt, 600 cycle per second generator which was driven by a variable speed, direct-current motor (30a). The root-mean-square voltage in the primary could be above 100 volts, but 60 to 80 volts were used in operation since this was sufficient to saturate the pole caps.

The acceleration chamber, hereafter referred to as the donut, had inner and outer diameters of 7 cm and 20 cm, respectively. The injector located inside the tube fired electrons into an equilibrium orbit having a diameter of approximately 15 cm. To eliminate the charge collected on the inner surface, a silver coating was deposited on this surface and grounded. Fig. 6 (30b) gives the details of the vacuum chamber and injector. The donut was evacuated to a pressure of at least $10^{-5}$ mm of mercury and then sealed.

The energy of the electrons was estimated to be approximately 2.2 Mev. This estimate was substantiated by measurement of the absorption of the output X-rays in lead. The intensity of the radiation from the
The Allis-Chalmers 22 Mev Betatron

Immediately after the initial success of the first accelerator, work began on an instrument to produce 20 Mev X-rays. Since that time the betatron has been redesigned constantly until the present-day instruments shown in Fig. 7 (16b) and Fig. 18 (31) were obtained. The latter device will be discussed in a later section of this chapter. Although both instruments are being produced by the Allis-Chalmers Manufacturing Company, the one illustrated in Fig. 7 was developed under the auspices of the Office of Scientific Research and Development and will be discussed in the following paragraphs (32).

The magnet assembly consists of the magnetic yoke, the poles, and the central air gap which contains a wafer. The yoke, consisting of two symmetrical halves, is stamped from 0.014 in. thick silicon steel sheets. These laminations are varnished and stacked into 1 in. thick bundles which are separated by 0.25 in. spaces. These spaces allow air from a blower to circulate through the yoke to facilitate its cooling. Each half-yoke is clamped, and one is placed on the other to form the complete yoke. The overall dimensions of the yoke are 62 in. long, 39 in. high, and 22 in. wide. The rectangular hole extending through the yoke measures 36 in. by 13 in., and in this space are placed the poles, wafer, coils, and donut. The entire assembly is placed on a base so that the X-ray beam will be at a convenient height. A motor-operated jack arrangement is incorporated in the design to provide a means of easily raising
Figure 7. Illustration of the 22 Mev betatron. The donut and centerpiece are not shown. Accelerated current is approximately 0.15 microamperes, and the duration of the X-ray pulse is approximately 0.2 microseconds.
or lowering the top structure which weighs approximately two tons.

The laminated poles are formed from 0.014 in. thick silicon steel sheets. Because the construction and shape of these poles greatly affect the operation of the betatron, extreme care must be used in their manufacture. Various methods of pole production were tried. The most satisfactory arrangement, however, is to die-cut the steel and spot-weld the sections together. The laminations, of varying length, are stacked into groups of eight having the same length, and the longest ones are welded to a central tube. The bundles of lesser length are then fitted into the wedge-shaped space and spot-welded only at one end. Since each lamination is insulated from its neighbor by varnish and welded in one spot, the only eddy currents produced are in each thin piece of material. An exception to this is the currents which circulate in the central tube, but these are of little importance. Because of this method of radial stacking, small wedge-shaped holes are left in the poles, thereby permitting passage of air. By this method, a very rugged, precisely constructed pole can be produced. Each pole is fastened to the yoke by a centrally located non-magnetic steel alloy bolt. Circular spacers are placed between the yoke and pole so that air can circulate between these units.

In the space between the pole faces is placed the wafer or center-piece. This wafer consists of two steel disks, radially stacked, in a manner similar to the poles, and separated by a machined stone disk. The assembly is clamped with a plastic ring, varnished, and baked. By separating the wafer from the two poles by thin pieces of fish paper and fastening the wafer to the lower pole with a non-magnetic bolt, a strong
unit having a minimum of vibration is obtained. Since the 1:2 flux relation is dependent on the ratio of magnetic reluctances, it is of utmost importance that the thickness of the components comprising the wafer be properly proportioned with respect to the main gap in which the donut is located. A method by which the gap may be eliminated will be considered on page 60.

Closely fitted around the wafer is the donut. The details and dimensions of the tube are given in Fig. 8 (32a). With the cooperation of the Ceramics Department of the University of Illinois, a porcelain tube was developed that could be evacuated to a pressure less than $10^{-6}$ mm of mercury and permanently sealed. The lower extension on the tube contains the electron gun. If the tube is to be continuously pumped, the upper extension is provided for connection of the pump. Two barium getters are located in the upper extension. One of these is flashed just before the tube is sealed, and the other may be used later to prolong the life of a gassy tube.

The inner surface of the donut is coated with a layer of palladium having a thickness of a few millionths of an inch. This coating provides a path to ground for the electrons which strike the tube, thus preventing accumulation of charge and destruction of the electron beam. The resistance of this coating measured between two point probes one inch apart must be between 20 and 80 ohms. If the resistance is much lower, eddy currents which disturb the magnetic field will be set up; and if much higher, the spray of electrons striking the walls will not be conducted to ground rapidly enough to prevent the coating from becoming negatively charged.
Figure 8. Details of the porcelain donut used in the production of 22 MeV X-rays.
To insure the correct position of the equilibrium orbit, \( r_e \), at approximately 19 cm, the tube must be mounted as rigidly as possible and still be free of vibrations. The tube rests on sponge rubber pads and has additional pads between it and the wafer. A detailed discussion of tube mounting is given by Westendorp and Charlton in reference (10a).

**Electron Injectors**

Perhaps the most stringent requirements for the successful operation of a betatron are placed on the electron injector more than on any other part of the instrument. Fig. 9 (33a) is a drawing of one of the latest developed injectors. The requirements are such that the injector must withstand a high-voltage pulse (100kv), prevent undue sparking between electrodes, and dissipate the heat developed. In addition, the injector must produce a copious supply of electrons and have a reasonably long life. Because of these specifications, and since the same gun can be used in the synchrotron, research has been continuous in attempts to improve the unit.

The early injectors were of the same general shape and size as the present guns except that the filament was of tungsten wound around a mandrel. Because of the evaporation and embrittlement of the tungsten, the life of the injector was approximately 125 hours.

The new injector (33), Fig. 9, is 3.175 cm long with a helically wound filament of molybdenum wire. The filament has a square cross section of 0.368 cm on an edge and a length of 0.687 cm. The electron-emitting surface is concave inward and has a radium of curvature of 0.757 cm. This recessed portion aids in obtaining correct electron optics for the beam. The distance from the center of the filament to the edge of
Figure 9. Betatron injector utilizing a barium aluminate cathode.
the target is 0.642 cm. Within the coil is a core of barium aluminate which is the active electron-emitting material. Because of the large supply of active material, the injector has a long life. A gun of this type has been used over 1200 hours in a 100 Mev betatron. The power required for cathode heating is approximately 50 watts.

Modifications (33b) of this injector have been made in an attempt to trap more electrons in the equilibrium orbit and thus to increase the yield of the betatron. One method utilizes a third element pulsed positively in an attempt to improve the focusing qualities of the gun. This scheme has been found quite successful. In order that the electrons may miss the injector as they spiral out to an extraction device, a gun which is located slightly above the $z = 0$ plane has been developed.

A new type of injector (34) has been designed at the Naval Research Laboratory. The electrons are emitted from a circular washer-shaped cathode which is centered on the equilibrium orbit. As the electrons leave the injector, they are collimated by a series of electron lenses such that the axis of the beam coincides with the stable orbit. The part of the injector behind the cathode can be designed so that electrons passing through the injector on subsequent transits can be refocused as they pass through the low potential region near the center of the cathode ring. Since the electron velocity during the injection pulse falls as the electron enters the injector, reaches a minimum as it passes through the cathode ring, and rises to its initial value as the electron leaves the gun, it is necessary to supply a compensating magnetic field in the neighborhood of the injector during injection.
Electrical Circuits

The electrical circuits required for the operation of a betatron have constantly been revised and increased in their complexity. Consequently, detailed circuit diagrams will not, in general, be included. However, block diagrams will be discussed, and references will be given to specific circuits. Fig. 10 (35) is a block diagram of the general power circuit and includes the majority of the auxiliary circuits employed. Fig. 11 (16c) and Fig. 12 (16d) show in more detail these two main circuit divisions.

The line voltage is 440 volts, 3 phase, 60 cycles per second, and supplies about 85 kva to the frequency tripling transformer. Since the power factor is about 34 per cent, approximately 29 kilowatts of power are required. The primary of the tripling transformer is connected in open wye, and the secondary is open delta connected. The theory of the open delta shows that only the third, ninth, fifteenth, ... harmonics are additive, the others cancelling identically. The third harmonic is large compared with the higher ones, and since the betatron is tuned to this frequency, the higher harmonics have a negligible effect as a power source. The potential of 4170 volts at 180 cycles per second is across the secondary which is connected to the resonant tank circuit of the instrument as shown in Fig. 11.

Manual and automatic amplitude controls are provided as well as a stabilizer circuit (32b). The function of this section of the power circuit is to help maintain the peak strength of the magnet at a constant value in spite of line voltage or frequency fluctuations. Since fluctuations might occur which the stabilizer would be unable to correct, an
Figure 10. Block diagram of the essential electrical elements of an automatically controlled betatron. The integrating monitor circuit (lower right) automatically stops the betatron when a predetermined X-ray exposure is reached.
Figure 11. Main power connections and controls for a 22 Mev betatron.
Figure 12. Diagram of the auxiliary circuits for a 22 Mev betatron.
overload cut-out system is incorporated in the load circuit to shut off the machine.

The main excitation coils (32c) around each pole are wound of stranded cable having sixty turns in each coil box. The bottom of this box contains a slot through which air is directed to the layers of the coil and on throughout the betatron. Since the voltage developed across each coil is approximately 6600 volts rms, the combination of coils being grounded at the electrical center, each cable is insulated with two layers of glass tape. Excitation is accomplished by connecting a tripler circuit across one of the four condenser banks which is part of the betatron tank circuit.

Electron Injection and Capture

The injector circuit, shown in a block diagram in Fig. 12 and in detail in reference (32d), supplies a high voltage negative pulse to the injector filament and shield. Since the amplitudes of oscillations in the r and z directions for electrons vary proportionally as the period, the amplitudes decrease with $1/B_z$. Hence, it is important to inject the electrons when the field is small and increasing. A biased-peaking transformer strip of steel permalloy triggers the injection circuit. Since the strip saturates very readily, there is no flux change through it except when the field changes direction. An RLC series circuit with a coil around the leg of the betatron causes the peaker flux to decrease just after the magnetic field has begun to build up again. By varying the resistance $R$, the time lag may be altered. A negative pulse of approximately $\approx 70$ kv and of 2 to 8 microseconds duration is applied to the injector filament from the secondary of an r-f transformer.
The filament potential, from the secondary of the filament transformer, is controlled by a variac connected to a 110 volt, 60 cycles per second source. A current transformer is incorporated on the core of the filament transformer so that the filament current may be measured without subjecting the ammeter to the high voltage pulse. These three units: the r-f transformer, the filament transformer, and the current transformer, are all enclosed in the same oil-filled case. Under normal operating conditions, the filament current should be approximately 6.1 amperes.

A relatively new circuit, in comparison to the circuits of the early betatrons, has been developed to aid in the capture of electrons during injection (36), (37a). This is an orbit contractor circuit consisting of one turn of wire on each pole face. Each turn has a radius equal to that of the equilibrium orbit. As the electrons are injected, a large, rapidly rising current is started in these coils in such a direction as to retard the rate of rise of flux produced by the main coils within the orbit. Because of this increasing current, the ratio of magnetic field to enclosed flux undergoes a large change outside the orbit, which causes the contraction of orbits having radii larger than that of the coil. The contraction is small for radii smaller than that of the coil since this ratio does not appreciably change. Increasing the current in the coil also decreases the radial variation of the magnetic field strength which causes a damping of the radial oscillations of the injected electrons. Both of these effects, contracting and field shaping, help prevent the electrons from colliding with the injector. If a separate time control is used to energize the contractor circuit when the injection voltage is a maximum, it is possible to increase the X-ray
yield by 100 per cent (36).

The explanation of orbital shrinkage during injection given by Kerst (23), and discussed in Chapter II, is similar to the contraction coil action. In Kerst's method, the beam current, instead of a contraction coil, acts to change the field shape and reduce the orbit.

Another method of orbit contraction, by application of a negative bias to the inner surface of the donut, has been described by Heymann (38).

The different theories and practical applications pertaining to electron injection and capture have been discussed in detail, since the beam current controls the X-ray yield of the betatron. By obtaining a greater circulating charge, the X-radiation yield can be increased because of the increased number of electrons which impinge on the target.

**Electron Deflection to Target**

Two methods are available by which the electrons may be shifted from the equilibrium orbit to the target. One method is to allow the central flux to saturate; the electrons then spiral inward to a target. This was the process used in the first betatron. However, it has been found that a greater X-ray yield is obtained if the beam radius is increased so that a target at a radius greater than $r_e$ is bombarded.

The process usually employed to expand the beam is to send a high current through a coil placed at the equilibrium orbit radius on each pole face. When the current in the main coils reaches its maximum value, a thyratron is triggered, and a condenser is discharged through a 4:1 pulse transformer connected to the expander coils. The current is sent through these coils in such a direction as to decrease the flux within
the orbit. Because of the widening air gap, the flux is decreased more within the orbit than beyond the orbit. For this reason, the beam moves outward in an attempt to re-establish the required equilibrium condition. A thin target, usually made of platinum, located on the edge of the injector, is bombarded by the particles.

**Maximum Energy Measurements**

The applications of the betatron to physical and medical research require variation of the maximum energy of the X-rays. The energy must also be known precisely. The Allis-Chalmers betatron shown in Fig. 7 is not equipped to comply with these requirements. However, so-called integrator-expander circuits have been developed to control the energy and determine its maximum value (39), (40), (41).

By substituting \( V_e = e \oint B \cdot dl \) into equation \(/3/\) and solving, the instantaneous energy of the electrons is obtained. This is per revolution

\[
\text{Energy}_{\text{inst.}} = 2\pi e \int_0^r r \frac{\partial B_z}{\partial t} \, dr. /34/\]

Since the rate of change of voltage across the excitation coils is proportional to \( \frac{\partial B_z}{\partial t} \), and since the target is at a fixed radius, \( r \), the instantaneous energy of the electrons depends upon the voltage across the coils. Therefore, by connecting a high resistance in series with a condenser across one of the coils and biasing the condenser, a variable energy control can be produced. When the predetermined voltage, set by the bias control, is developed across the condenser, a signal is transmitted to the expander circuit connected to the expander coils. A block diagram of the required circuitry is given in Fig. 13 (41a). Detailed
Figure 13. Block diagram of the integrator-expander circuit for accurate control of the maximum X-ray energy.
diagrams of each circuit unit are given in (41).

Calibration of this integrator-expander circuit may be done in two ways: (1) by assuming a linear energy scale and correcting for errors caused by time delays in the various circuits and (2) by electromagnetic calibration and application of necessary corrections for time delays.

Circuits utilizing the first method were developed for the 22 Mev betatron at the University of Saskatchewan (41b). Time delays in the amplifier, multivibrator, and expander totaled 2.1 microseconds or an energy equivalent to 0.05 Mev. Nonlinearities in the expander pulse and delays in circuits produced an additional error of 0.05 Mev. Therefore, a total correction of 0.1 Mev was necessary. To obtain an energy versus bias voltage curve, photo-disintegrations, Chapter V, of substances having known threshold energies were used. Three reactions with accurately
known thresholds are $^{12}\text{C}(\gamma,n)^{11}\text{C}$ at 18.7 Mev, $^{63}\text{Cu}(\gamma,n)^{62}\text{Cu}$ at 10.9 Mev, and $^{9}\text{Be}(\gamma,n)^{8}\text{Be}$ at 1.67 Mev (40). For the particular circuit used a potential difference of 86 volts was developed across the condenser at 23.1 Mev.

Electromagnetic calibration requires measurement of the voltage produced by the flux through a search coil placed in the plane of the orbit. Since the energy depends on the flux, the integrator circuit may be calibrated. The probable error in this calibration is about 1.5 per cent. However, because of variable time delays, the error should be higher but not greater than 2 per cent.

Because of the large impedance ratio of the integrator components, the 180 cycle magnet voltage is phase shifted almost 90 degrees, lacking only about 31 minutes of phase angle. If the time constants of the coils
and the integrator stack were equal, the signal voltage on the integrating condenser would be exactly in phase with the magnet current. Since the magnet coils have an effective resistance, a phase shift of 23 minutes results. These phase differences require the integrator voltage to be 8 minutes of phase angle ahead of the magnet current, which corresponds to a time difference of approximately 2.1 microseconds. This phase advance is advantageous because it aids in compensating for the time lags produced in the other components of the circuit. Since the maximum energy can be varied, the time delay in the expander also varies between 3 and 6 microseconds.

**Exposure Control**

The last auxiliary circuit to be discussed is the exposure control system shown in Fig. 10. The purpose of this circuit is to cut off the power to the excitation coils automatically when the specimen has received a predetermined amount of radiation. This is accomplished by using an ionization chamber in conjunction with an integrator circuit. Although the ionization chamber is placed in the center of the beam close to the donut, the circuit is calibrated to record the total irradiation of an object usually at a distance of one meter from the target. When the object has received the required amount of radiation, a switch is opened which stops the machine.

**X-Ray Yield**

Measurements of X-radiation are usually made in terms of a unit called the roentgen. The roentgen, r, defined at the Radiological Congress at Chicago in 1937, (42a), is "that quantity of X- or gamma-radiation
such that the associated corpuscular emission per 0.001293 gm of air produces, in air, ions carrying 1 electrostatic unit of quantity of electricity of either sign." The mass of air referred to is that contained in 1 cm$^3$ of dry air at a temperature of 0°C and a pressure of 760 mm of Hg. From the definition it can be calculated that 1 r produces $2.083 \times 10^9$ ion pairs per cm$^3$ of air or $1.6 \times 10^{12}$ ion pairs per gm of air.

Since the average ionization potential for air is 32.5 ev, then 1 r corresponds to the absorption of $5.24 \times 10^{13}$ ev or to 83.8 ergs of energy per gram of air. Physically, the roentgen refers to the quantity of ionization produced in air by the secondary electrons formed in gamma-ray interaction with air molecules and does not depend on the time. Hence, the roentgen is a unit of energy dissipation.

Two additional terms used in connection with X-ray measurements are ionization intensity and gamma-ray intensity. Since the unit r is for the quantity of ionization produced, the term r/sec expresses what is known as the ionization intensity in dry air under standard conditions. Gamma-ray intensity is the rate at which photon energy flows past any point. This intensity can be expressed only in absolute units such as Mev per cm$^2$-sec. The ionizing effects depend not only on the true intensity but also on the number and energy of the photons included.

To illustrate the difference between these two units, let us consider the case when 1000 photons per cm$^2$-sec, each with an energy of 1 Mev, traverse an area of 1 cm$^2$; the gamma-ray intensity is 1000 Mev/cm$^2$-sec. The same gamma-ray intensity could be obtained by using 500 photons/cm$^2$-sec with each having an energy of 2 Mev. In this energy range, the energy dissipated is caused practically entirely by the
Compton effect which has a coefficient of absorption, of $3.60 \times 10^{-5}$/cm and $3.00 \times 10^{-5}$/cm for 1 Mev and 2 Mev photons, respectively, for standard air. Therefore, the energy absorbed per cm$^3$ of air will be:

$1000 \times 1 \times 3.6 \times 10^{-5} = 0.036 \text{ Mev/cm}^3$-sec for the 1 Mev photons and $500 \times 2 \times 3.0 \times 10^{-5} = 0.03 \text{ Mev/cm}^3$-sec for the 2 Mev photons. Also, since $1 \text{r} = 6.77 \times 10^4 \text{ Mev/cm}^3$ for standard air, we have for the ionization intensity

$$\frac{0.036}{6.77 \times 10^4} = 5.3 \times 10^{-7} \text{ r/sec}$$

and

$$\frac{0.03}{6.77 \times 10^4} = 4.4 \times 10^{-7} \text{ r/sec}$$

respectively. Thus, the ionization intensity varies with photon energy even though the gamma-ray intensity remains constant. To determine the energy absorbed from the X-ray beam, ionization intensity measurements are made usually in units of roentgens per minute at a standard distance of 1 meter from the target. This measurement is referred to as the X-ray yield.

There are several other methods by which the betatron X-ray yield may be increased. The first of these methods is to increase the voltage at which the electrons are injected into the donut. This usually causes more electrons to be captured in an equilibrium orbit and hence increases the photon output. Westendorp and Elder (43) conducted extensive experiments with betatrons having maximum energies of 100, 50, and 10 Mev to determine the curves of X-ray output versus electron gun emission for different injection voltages from 20 to 50 kv. For all three machines, the maximum X-ray output, measured in r/min at 1 m, varied very nearly as the 1.25 power of the gun voltage.
By obtaining two X-ray beams, it is possible to increase the total yield from the betatron. In Kerst's original machine (30c), the electrons were accelerated in one direction as the magnetic field increased positively from zero and in the other direction as the field increased negatively from zero. This action alternately produced X-ray beams in opposite directions. A method by which the two beams were made to cross was developed by Kerst (44) in 1949. His arrangement was to place two injectors back to back in the donut. When the magnetic field began to increase, the electron gun injected the electrons. Those electrons injected from one gun were trapped in the orbit and accelerated while the electrons going in the other direction were curved into the wall of the vacuum chamber. At ejection time, current passed through the expander coil and the captured beam moved out to the target. As the field went negative, electrons were again injected, and the process was opposite to the conditions set up a half cycle earlier. When the peak negative field was reached, current passed through the expander coil in the same direction as in the previous case. Since the direction of the flux had reversed, the beam contracted to a target located on the inner surface of the wall. The two targets were on opposite sides of the donut, so that the beam patterns would intersect. Wang (45), in 1945, developed a similar system to produce two intersecting X-ray beams.

Another factor which can radically affect the X-ray yield is the nonuniformity of the magnetic flux, or the azimuthal variation, during the time of electron injection. This variation can greatly reduce the number of electrons captured in stable orbits or make the betatron completely useless. The magnetic field distribution, near zero time
(instant of field reversal) is strongly influenced by slight phase differences between different sections of the magnetic poles. These phase differences are caused by differing power factors of sections of the magnetic circuit. The power factor is dependent upon the amount of iron in the magnetic circuit, the hysteresis and eddy current losses, and accidental short circuits between laminations. Also different bundles of laminations may have slightly varying power factors. A local increase in the power factor causes a proportional lag in the magnetic field.

Kerst and Scag (46) investigated the azimuthal variation of the field of four 22 Mev betatrons; Fig. 14 (46) shows the field variations of each instrument near zero time.

From the figures, it is evident that the bumps are from 3 to 4 oersteds high; but since the injection voltage is approximately 65 kv, the field at the time of injection is about 45 oersteds. Therefore, the bumps are less than 10 per cent of the field amplitude. The width and height of these bumps as well as the number and distribution of them determine the scattering impulse given the electrons as they travel in their orbits. The worst type of bump distribution is illustrated by betatron number 50. In this machine, there is one positive and one negative bump. This field variation can be reduced by placing a small correction coil beneath the orbit at 90° with a current in it to advance the phase slightly. When this coil is used, the X-ray yield increases by 20 to 50 per cent.

Betatron No. 51 has four main cycles of field variation around its pole, but the use of correction coils has no effect on the yield. No. 52 has many small negative and positive bumps in its field, but
Figure 14. Azimuthal field plot near zero time of four 22 Mev betatrons.
experiment has shown that no extra coils are necessary. With No. 53, however, the yield was increased from 50 r/min to 80 r/min at one meter by the use of correction coils. In many betatrons, it is necessary to use these correction coils. Their usefulness is determined experimentally after the yoke and pole pieces have been built. Additional information concerning this subject is found in reference (47); methods of field measurement are given in references (37), (46), and (48).

Proposed methods of increasing the betatron yield are given by Wideröe in a review article by H. F. Kaiser (13). By the use of electrostatic or electromagnetic lenses, the X-ray yield should be increased. In addition to lenses for the collimation of the electrons, it was proposed that many injectors be installed around the donut — 28 cathodes for a 100 Mev unit. The idea of using guide fields, discussed on page was also considered in the design of this proposed 100 Mev machine.

A mechanical device by which the X-ray target can be shifted horizontally or vertically while the betatron is operating was developed at the Naval Research Laboratory (49). By slight motion of the donut, the position of the target with respect to the beam can be varied. By this means, the best position of the target for maximum X-ray yield can be determined quickly. Also by slight variations of the target position, it is possible to produce small changes in the maximum energy of the photons.

Field Biasing

In 1945, Westendorp (50) and Kerst (51) published papers discussing methods by which the a-c power required for a betatron could be
greatly reduced. Also methods by which the maximum energy of the electrons could be increased were considered.

The first major modification (50) reduced the a-c power requirement to approximately 30 per cent of its former value. This reduction was accomplished by superposition of a direct current upon the a-c current, and by maintaining constant maximum energy in either case. This energy can be constant only if the wafer does not, because of the two currents used, become saturated sooner than when the a-c is used alone. From experiments it was found that the iron losses were approximately proportional to the square of the total flux density and that the d-c component of the flux does not produce an increase in the hysteresis losses for a given a-c excitation. Also, the hysteresis losses formed the major portion of the losses in the magnetic circuit. Therefore, by application of a d-c bias, the same energy of electrons can be obtained by reducing the a-c power. By application of the d-c bias, the total power necessary was still less than 50 per cent of the original value.

None of the fundamental requirements for electron acceleration, the 1:2 relation or the $1 > n > 0$ relation, is changed by the addition of the direct current. The 1:2 relation is still determined by the wafer and air gap between the pole pieces. However, it is important that the value of the direct current never exceed the peak a-c value. This is an obvious condition since the magnetic induction at the orbit must pass through zero just before electron injection. Since it is important to have a strong electrical field gradient present during injection, a direct current value, $I_{d-c}$, of $0.866 I_{a-c}$ peak would give a gradient during injection of one-half the peak gradient during acceleration.
Under these conditions, the injection point would be located $30^\circ$ beyond the negative peak value of the a-c wave. Fig. 15a (50a) illustrates the wave pattern while Fig. 15b (50a) shows the circuit required.

If the d-c flux passing through the core could be eliminated and the d-c component at the orbit be retained, higher flux densities at the orbit could be used before causing saturation of the center. An arrangement of this type would increase the maximum energy of the electrons, and the machine would be known as a field-biased betatron.

To suppress the central d-c flux, backwound bucking coils are placed in grooves around the pole faces. These coils are referred to as groove coils and are illustrated in Fig. 15c (50a). These coils carry slightly fewer direct-current ampere turns than the main coils since they block out only the d-c field in the core. Fig. 15d (50b) is the circuit diagram for this system.

Because of the separated groove coils and centerpiece, a strong horizontal component of the magnetic field is produced. Since this radial field would interfere with electron capture, the centerpiece must be redesigned. By making one large central section with air gaps at the top and bottom, the horizontal component of the field can be eliminated. The 1:2 flux condition, however, is still determined by the ratio of these air gaps to the main air gap. By employing field biasing, the energy of the electrons can be increased and the weight of steel required for the instrument can be reduced.

Another circuit, Fig. 15e (50b), is designed so that all of the power condensers can be placed in one bank. To prevent any alternating current from flowing in the groove coils, these coils have been given a
(a) Magnetizing current as a function of time.  

(b) Circuit diagram for the application of direct current to a betatron.

(c) Machine with grooved pole pieces, extra coils, and large centerpiece.

(d) Circuit diagram for machine of Fig. 15(c).

(e) Circuit diagram for biasing a betatron having a closed central core.

Figure 15. Circuits for biasing a betatron.
larger number of turns than the main coils. In this way the induced emf equals the voltage across the main coils. Since the groove coils now have more turns than formerly, the direct current through these coils must be lower to keep the d–c ampere turns at the correct value. This is accomplished by a control resistance and a by–pass reactor. The conductor loops shown in series with the by–pass condenser across the control resistance and the generator represent turns coupled with the reactor to compensate for the capacitive voltage drop.

By using this circuit, Fig. 15e, the 1:2 flux condition is no longer determined by the ratio of the magnetic reluctances. Now, the relation is determined by the turns ratio of the groove to main coils, their voltages being identical and their magnetic flux being inversely proportional to their numbers of turns. The air gaps can therefore be eliminated and the betatron made with a closed central core. An auto–transformer, not shown in Fig. 15e, connected between the main coils and groove coils, adjusts the flux ratio slightly so that fine adjustment of the electron orbit diameter can be made. Since the reluctance of the closed core requires very few ampere turns for magnetization, the groove coils will have to carry both d–c and a–c bucking ampere turns; therefore, an alternating current will flow which counteracts the effects of the main coils.

There are two additional advantages of the closed central core betatron. For the betatron with the wafer and air gaps, the energy stored in the gap is large and requires a larger condenser bank. But by elimination of the gap by a closed core, the energy stored in the field can be reduced by 50 per cent or more, depending on the main air
gap which contains the donut. This reduction results in an appreciable decrease in the size of the required capacitor bank. The second advantage is in the maximum energy control of X-rays when orbit contraction by means of saturation of the core is used. A small reduction of the direct current in the groove coil, by means of the control resistor, will cause saturation of the central core and contraction of the beam. In this manner a wide range of X-ray energies can be obtained.

A field biased, closed central core betatron was tested by Westendorp (52) and found to be very satisfactory. With the instrument operating at 11 MeV, the energy could be increased to 20.5 MeV simply by applying the d-c bias and rephasing the electron-injection pulse. With oil-cooled coils, the machine will produce X-rays having an energy of 50 MeV.

The method of increasing the energy of the betatron proposed by Kerst (51) is flux biasing. This scheme requires a direct current biased winding around the closed central core. Since the direct current negatively biases the central core, the instrument cannot be run on alternating current without seriously saturating the core. Consequently, excitation must be accomplished by unidirectional pulses of current. The new 300 MeV betatron is flux biased.

Field and flux biasing are briefly discussed in reference (37b), and a mathematical discussion of biasing is given in reference (53). Frequently a biased betatron is referred to as a two-field instrument. The fields are a guide field at the orbit, and an accelerating field in the closed central core. The 16 MeV betatron at the Clarendon Laboratory is described as a two-field instrument in the complete article of
Pulsed Operation

The unidirectional pulse operation of a betatron has two distinct advantages. First, for a flux biased betatron it is essential that the magnet be pulsed since the power dissipation for continuous operation is prohibitive because of the saturation of the iron during the negative swing of the flux. Second, this method of operation is more efficient when the betatron is used in conjunction with a cloud chamber. Experiments employing the betatron and cloud chamber will be discussed in Chapter V.

In betatron-cloud chamber experiments, the ideal arrangement is to synchronize the instruments so that when the chamber becomes sensitive the betatron will produce a burst of X-rays. Two methods for accomplishing this were developed by Baldwin, Klaiber, and Hartzler (55). The first method was to excite the magnet continuously and to inject, accelerate, and to expand the orbit to produce X-rays only during one or two cycles, intermittently. The second arrangement was to produce a damped oscillation of the energy in the condenser bank through the magnet coils so that operation takes place only for the first cycle. The disadvantage of the first system is in the unnecessary expenditure of power, and the weakness of the latter system is caused by slight variations in the magnet current.

To overcome these difficulties, a new circuit for a 22 Mev betatron was developed by Koch and Robinson (56). The magnet is operated by a unidirectional pulse. A relay, operated by a signal from the cloud chamber, allows two ignitrons in series to conduct and energize the
betatron tank circuit. Because of time delays in the circuit, approximately 3.1 sec, and the poor regulation of the half wave rectifier which recharges the condensers, the total time between pulses is approximately 30 seconds.

In addition to being an instrument for producing high energy X-rays, the betatron is also a device for obtaining a beam of highly energetic electrons. However, it was several years after the first instrument was built before the problem of electron extraction was solved. By placing the injector slightly above the equilibrium orbit plane, the radius of the electron beam can be increased out to the wall of the donut by the orbit expander circuit. As the electrons reach a region near the edge of the magnetic field, they start moving in spirals of increasing pitch and are scattered out of the vacuum chamber in all directions. To counteract this spiraling action, a magnetic shunt (57) is placed in such a position that as the electrons begin spiraling they enter a groove in the shunt. This shunt, made of laminated steel sheets, effectively shields the slot from the magnetic field so that the electrons entering the groove will travel approximately in a straight line for about 10 cm. The electrons emerge into such a weak field that they receive only slight magnetic deflection before they pass through the window of the vacuum chamber and escape entirely from the influence of the field. Fig. 16a (57a) and Fig. 16b (57a) show the position and dimensions of the magnetic shunt used in a 22 Mev accelerator.

For the electron beam to emerge from the donut, it must pass through a thin window of aluminum 0.001 in. thick. At the window, the cross-sectional area of the beam is approximately $2 \times 5 \text{ mm}^2$, but this area
(a) Diagram of vacuum chamber showing the equilibrium orbit, spiralling region, and magnetic shunt.

(b) Diagram of magnetic shunt used to extract electrons from donut.

Figure 16. Magnetic shunt for extracting electron beam from betatron.

(a) Section of donut showing scattering foil and deflector. (b) Variations in electron output as a function of foil position.

Figure 17. Electrostatic deflector for extracting electrons from the betatron.
quickly increases because of electron scattering in air. At 10 cm from
the window, the beam is about 6 mm high and 13 mm wide; at one meter,
the mean diameter at the position of one-half maximum intensity is about
12 cm. The electron current of a beam having an energy of 17 Mev is of
the order of 0.01 microampere (58), and at a distance of 45 cm from the
window the ionization measured with a Victoreen thimble chamber is
equivalent to 18,000 r per minute of X-rays. The duration of the elec-
tron pulse is less than 0.5 microsecond.

Another method of electron extraction, superior to the magnetic
shunt, is by electrostatic deflection plates. This method was thoroughly
studied mathematically by Courant and Bethe (59) and also by Widerøe
(60). In this process, the beam is expanded from the equilibrium orbit
as before and enters the space between two deflection plates. The beam
is deflected outward more sharply by the electric field and emerges into
the weak region of the magnetic field and on out of the donut. The re-
sults from reference (59) state that the angular divergence of the beam
is approximately proportional to \((H/E)^{3/4}\), where \(H\) is the magnetic field
at the orbit and \(E\) the electric deflecting field. If \(H\) equals 10 kilo-
gauss and \(E\) equals 60 kv/cm, the deflected beam will be collimated into a
cone having an angular divergence of approximately 10 degrees.

A 6 Mev betatron (12), built in Germany, utilizes the electrostatic
method of removing electrons. Fig. 17a (12a) shows the deflection plates
and scattering foil; while Fig. 17b (12a) illustrates the effect of foil
position on electron yield. Slightly inside the limiting circle, shown in
dashes, an adjustable deflecting capacitor of field strength up to 150
kv/cm is arranged. Approximately 20 degrees azimuthally in front of it
is a thin aluminum strip, 2 to 3 microns thick, with its face perpendicular to the electron beam. The inner edge of the strip is about 0.1 mm nearer the center than the inner deflection plate. This strip aids in the collimation of the beam and in the trapping of the electrons by the deflector. The highest beam current produced, after the electrons passed through a thin copper foil window, was approximately 0.1 microampere. It was estimated that about 70 per cent of the electron beam was extracted from the betatron. A method of measuring the electron current of the beam in the donut by the principle of induced current in a coil placed around the donut is given in reference (61).

As has been previously stated, the principal applications of the betatron are in the field of medicine, physics, and industrial X-ray. To aid the radiologist, Allis-Chalmers Manufacturing Company has produced a new 24 Mev machine which has been especially designed for medical therapy. There is no reason, however, why this instrument should not be used for other applications. Fig. 18 (31) shows this new betatron. The primary features of this machine are the following:

1. The energy is variable from 5 to 25 Mev.

2. It may be continuously operated for an 8-hour day at 22 Mev.

3. At three feet from the target the X-ray output is at least 100 r/min. (This is measured with a standard Victoreen meter surrounded by a cylinder of plastic having a diameter of approximately 4 inches).

4. A motor-operated jack raises the top yoke to facilitate changing the donut.

5. The magnet assembly is mounted so that the X-ray beam can be rotated from 40 degrees above the horizontal, through the horizontal, to 40 degrees beyond the vertical, making a total angular variation of 170 degrees.
Figure 18. ALLIS-CHALMERS 24 MILLION VOLT BETATRON
6. Rate of rotation of the magnet assembly is 0.5 revolution/min.

7. The angle of beam setting is accurate to within 0.05 degree.

8. The unit contains a total exposure meter and circuit to automatically shut off betatron when predetermined amount of radiation has been reached.

In addition to these features, the instrument operates at a frequency of 180 cycles per second with a peak voltage of 18 kv across the magnet coils. The injection pulse can be a maximum of -60 kv and is approximately 4 microseconds wide. To expand the orbit 800 ampere turns are necessary. The weight of the instrument is approximately 8 tons. It has a length of about 104 in., a maximum height of 109 in., and a maximum width of 36 in.

The cost of the betatron, June, 1950, including the condenser bank, operator's desk, and tripler transformers is about $90,000. The information and photograph of this machine are from reference (62).

For the physicist, betatrons having an X-ray energy up to 100 Mev and 300 Mev are available. The 100 Mev machine (10) operates at a frequency of 60 cycles per second and at full load requires 200 kilowatts of power. With a Victoreen ionization chamber shielded by 0.25 in. of lead, the output is 2600 r/min at one meter from the target. The instrument weighs about 130 tons and is approximately 9 feet high, 6 feet wide, and 15 feet long. The 300 Mev betatron (63) is pulse operated at the rate of 6 pulses per second. The output of this new machine, tested with an ionization chamber behind 0.125 in. of lead, was about 1000 r/min at a distance of one meter when the machine was
operated at 315 Mev.

A betatron without an iron core (5), mentioned in Chapter I, has several advantages over the conventional instrument. One primary advantage is its great reduction in weight. A 5 Mev iron-core machine weighs about 600 pounds, while a 9 Mev air-core instrument weighs only 110 pounds. By eliminating the poles, it becomes much easier, and less expensive, to make this type of betatron than it does to make an instrument of the conventional type. Small iron bars are inserted in the center of the coils and form an air gap. To vary the output energy, it is necessary to simply remove this core and replace it by one having a different size. Since the instrument operates at a frequency of 2500 cycles per second, strong focusing forces are exerted on the electrons and fewer revolutions are required for them to reach a final energy than is required in the conventional machine. X-ray production occurs for a few cycles at intervals of approximately one second. A disadvantage of the instrument is the high current required. This current can be obtained easily, however, by discharging through the coils a condenser charged to a high voltage.

A small 4 Mev betatron designed principally for radiographic applications will be discussed in Chapter VI.
CHAPTER IV

BETATRON INSTALLATION

Since the installation requirements vary for different energy betatrons, the installation of 24 Mev machines only will be discussed in the present chapter. The problems pertaining to the location of the various units, the protection of personnel from electrical and radiation hazards, and special features of existing betatron laboratories will be discussed.

Auxiliary Betatron Units

Several auxiliary units are necessary for operation of the betatron, Fig. 18. The principal units are the control panel or console, the capacitor rack, and the three tripler transformers. If variations in the input voltage are greater than four per cent, two feeder voltage regulators are required. Additional units are a peaker transformer, magnetic contractor transformer, and other miscellaneous pieces of equipment. The console is designed as a large desk having the back portion raised to contain panels of instruments and switches. The desk is approximately 66 in. long, 34 in. wide, working surface 30 in. from floor, and top 43 in. above the floor. Fig. 19 (62) shows a picture of this unit.

The capacitor rack, 116 in. long, 54 in. wide, and 97 in. high, contains amplitude control resistors, relays, and 64 varied-size capacitors. The unit is built for indoor installation and cooled by an air circulation system which removes 4000 cubic feet per minute of air
Figure 19. A scale model of a typical betatron installation for medical therapy.
from the housing. The weight of this unit is approximately 5 tons.

The tripling transformers, each rated at 23 kva and 60 cycles per second, are provided with intermediate taps so that the voltage may be varied from 254 volts to 2457 volts. Each transformer is approximately 26 in. wide, 64 in. high, 26 in. thick, and weighs about 1400 pounds. These units are suitable for either indoor or outdoor mounting.

Two feeder voltage regulators, rated at 9.6 kva, 200 ampere, single phase, and 60 cycles per second, control the input line variations of ±10 per cent in 32 steps. Each unit is approximately 21 in. wide, 67 in. high, 34 in. thick, and can be mounted either indoors or outdoors. The gross weight of one regulator is approximately 1900 pounds. Cost of all the equipment, except the voltage regulators, is included in the initial price of $90,000 for the betatron assembly. The regulators, if required, cost $1,375 each. These prices, quoted from Allis-Chalmers, were of June, 1950, (62).

The accelerator is mounted on a base which extends between the magnet supports. The top of the base is flush with the floor so that the height of the X-ray beam, when the magnet assembly is horizontal, is 66 in. above the floor. In positioning the instrument, at least four feet of space should be left between the back of the magnet assembly and the back wall and four feet of clearance between the instrument and each side wall.

Forced air cooling of the magnet unit is accomplished by one or two air blowers located on the magnet assembly. Air is drawn through the poles and coils and exhausted into the radiation room. Provision must
be made to remove 4000 cfm of air from this room as well as from the capacitor bank. In addition to air cooling, a water system having a capacity of 0.25 gal/min must be provided at the magnet for cooling an ignitron tube.

One X-ray emission sealed-off vacuum tube, or donut, is supplied with the betatron. The tube is guaranteed on a pro-rata basis for 150 hours of actual use and has an unlimited shelf-life guarantee. By using a current between 5.7 and 6.0 amperes to heat the injector filament, instead of the maximum rating of 6.1 rms amperes, the tube life can be greatly increased. A new tube costs $3,500, and the old tube has a salvage value of approximately $300. An electron-emission tube may be purchased from the Picker X-ray Corporation for $5000. These tubes have the same guarantee and salvage value as the X-ray emission donut (64).

**Personnel Protection**

The primary hazards of betatron operation are electrical and radiological. To minimize the electrical hazards, the principal cables going from the tripler transformer to the capacitor bank and to the magnet assembly are in large conduits and may be buried underground. Almost all of the high voltage connections are enclosed except for those around the frequency tripler and magnetic contractor transformers. The auxiliary circuits are located either in the pedestal (support for the magnet assembly) or in the condenser housing. Safety-interlock-switch circuits are incorporated to block the operation of the betatron when pedestal or capacitor doors are open and could also be used at other critical positions, such as at the transformer room door.
To protect the betatron from inexperienced operators or from excessive power variations, additional safety circuits and units are required. The first precaution taken is to install fuses in all supply lines. Also, the power circuit and auxiliary circuits are interlocked and employ time-delay switches. Thus, the circuits are closed in a certain sequence, and a definite time is required for the tubes and circuits to be ready to operate before the magnet can be energized. If the cooling system stops, the accelerator is automatically shut off.

Because of the high-energy radiation emitted by the betatron, it is imperative that the instrument be properly shielded. The most common material for this purpose is concrete. Although lead is a good shield against X-rays, it is not so widely used as concrete because of the supports required by its weight and because of the cost involved. Poured natural sand concrete, which may contain gravel or crushed stone, or solid concrete building blocks can be used for shielding.

For a 24 Mev betatron installation, using poured concrete, the recommended height of all protective walls is nine feet. The wall behind the instrument should be at least four feet thick, and the side walls should have a minimum thickness of three feet. The thickness of the wall facing the X-ray beam varies as the distance from the betatron to the wall is changed. Table I gives the minimum requirements of wall thickness as a function of the magnet to wall distance. If standard solid concrete building blocks are used, the wall dimensions, from Table I, should be increased 1.8 times. For a lead-shielded installation, the dimensions should be multiplied by 0.12.

To this point, the source of information for this chapter, with
the exception of quoted tube prices, was from private communications (62).

TABLE I

Thickness Requirements of a Poured Concrete Shielding Wall

<table>
<thead>
<tr>
<th>Distance from front of magnet to wall - (feet)</th>
<th>Thickness of concrete wall - (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>72</td>
</tr>
<tr>
<td>30</td>
<td>65</td>
</tr>
<tr>
<td>40</td>
<td>62</td>
</tr>
<tr>
<td>50</td>
<td>58</td>
</tr>
<tr>
<td>60</td>
<td>55</td>
</tr>
</tbody>
</table>

The thickness requirements given in the previous paragraph should reduce the radiation to at least the tolerance dose level. This so-called safe dose has been 0.1 roentgen/8 hr day, but at the last meeting of the International Commission on Radiological Protection, July, 1950, the tolerance dosage was reduced to 0.3r/week (65a). The shielding values given are probably sufficient to reduce the quantity of radiation to this new safe level, but if operating personnel are to be exposed for extended periods, the possibility of additional shielding being required should not be overlooked.

To calculate the required thickness of a protective barrier, the thickness necessary to reduce the radiation to 1/2 of its incident value is useful. This thickness is known as the half-value layer (hvl), (66). By interposing a series of barriers, each having a thickness such that the radiation transmitted is 1/2 of the incident radiation, a relation
can be determined for the number of hvl required to reduce the radiation to a safe value.

If 1 r/min at 1 m is to be reduced to 0.3 r/wk (based on an 8 hr day, 5 days/wk) at the same distance, the reduction factor is 8000. This reduction would require \( 2^n = 8000 \) or approximately 13 half-value layers of an absorber, where \( n \) equals the number of hvl required. Since the decrease in radiation follows the inverse square law, the number of hvl to reduce the intensity to 0.3 r/wk at any distance, \( x \), from the target may be determined. The equation is

\[
2^n = \frac{8000 R_1}{x^2} ,
\]

where \( R_1 \) is the intensity of the primary beam 1 m from the target of the betatron. Therefore, equation /35/ becomes

\[
n = 13 + 3.32 \log R_1 - 6.56 \log x .
\]

If \( R_1 \) equals 100 r/min at 1 m and a barrier is to be placed 8 m from the target, the number of half-value layers required to reduce the radiation to 0.3 r/wk is 13.64 or 14 hvl.

**Betatron Installations**

A typical betatron installation, designed primarily for medical therapy, is shown in Fig. 19 (62). Above the operator's console is an observation window. A mirror inside the betatron room, rotated by the operator, enables the operator or radiologist to view the patient.

Illustrations of complete betatron installations are shown in Fig. 20a (67) and Fig. 20b (68). The University of Saskatchewan's
(a). Installation of a 22 Mev betatron at the University of Saskatchewan.

(b). Basement installation of a 20 Mev betatron at the Naval Research Laboratory.

Figure 20. Betatron installations.
installation (67) of a 22 Mev betatron, Fig. 7, has many points of in-
terest. It is arranged to utilize all three applications of the
instrument. For medical therapy the magnet room has a variable position
mirror and a window for viewing the patient. Wide corridors and absence
of heavy doors facilitate the movement of stretchers. The walls and
ceiling of the magnet room are covered with sound-absorbing tile to
reduce the noise level to approximately 92 decibels. In the control
room the sound is approximately 52 decibels - a common level in offices.
For radiographic uses, the room contains an overhead I beam with a chain
block for lifting heavy equipment and moving it along a line normal to
the face of the machine in line with the X-ray beam. Beneath the I
beam, two steel rails and a scale are embedded in the floor so that
heavy pieces of metal may be placed on a truck and quickly rolled to a
certain distance from the betatron. For physical research, no special
features are required.

The inside dimensions of the room are approximately 27 ft by 15
ft. The concrete wall directly in line with the X-ray beam is 7 ft
thick. The thickness of the far side wall, from the operator, and the
back wall is about 4 ft, while the other side wall is 3 ft thick.
Since the installation is partially underground, the thickness of the
front wall below ground level is much less than 7 ft.

A radiation survey of the installation was made while the beta-
tron operated at 24 Mev and produced an output of 100 r/min at 3 ft
from the target. The results of this survey are given in Table II
(67a). Since the instrument is rarely operated for eight hours a day,
the actual radiation received by any person is considerably less than
that shown in the table.

In addition to the usual interlock switches, time delays, and other safety devices, another safety circuit is employed. This circuit utilizes a warning horn in the betatron room which sounds five seconds before the magnet is energized. Three blocking switches which prevent the operation of the instrument are located in the room so that anyone has time either to get out or to open one of the switches.

TABLE II

Survey of the Stray Radiation
for a 24 Mev Betatron Installation
(Based on an 8 hr-day)

<table>
<thead>
<tr>
<th>Location</th>
<th>Radiation (r)</th>
</tr>
</thead>
<tbody>
<tr>
<td>In line with the X-ray beam at ground level</td>
<td>0.02</td>
</tr>
<tr>
<td>outside the building</td>
<td></td>
</tr>
<tr>
<td>Maximum observed on the roof of the building</td>
<td>0.40</td>
</tr>
<tr>
<td>and above the axis of the X-ray beam</td>
<td></td>
</tr>
<tr>
<td>Near the door between the control room and</td>
<td></td>
</tr>
<tr>
<td>the corridor leading to the betatron room</td>
<td></td>
</tr>
<tr>
<td>Control room side</td>
<td>0.06</td>
</tr>
<tr>
<td>Corridor side</td>
<td>0.25</td>
</tr>
<tr>
<td>In the corridor leading to the betatron room</td>
<td></td>
</tr>
<tr>
<td>near the entrance to the betatron room proper</td>
<td>1.0</td>
</tr>
<tr>
<td>At the control panel in the control room</td>
<td>0.05</td>
</tr>
<tr>
<td>In the small laboratories</td>
<td>0.02</td>
</tr>
<tr>
<td>Directly behind the betatron in the hallway</td>
<td>0.10</td>
</tr>
<tr>
<td>In the dark room</td>
<td>0.05</td>
</tr>
</tbody>
</table>

The installation of a 20 Mev betatron (68), Fig. 20b, at the Naval Research Laboratory employs concrete blocks as the shielding
material. Since the machine is located in one corner of the basement and because of the thick ceiling already existing, additional shielding was required only on two sides of the room. The solid blocks were made from a 4:1 cement mixture. A heavy door, electrically operated, may be used either to close the room completely or to act as a baffle in front of the opening.

A 10 Mev betatron located at the Naval Ordnance Laboratory, has a rather novel mounting (69). The machine is suspended from the ceiling in such a manner that the vertical distance above the floor may be varied. Parallel I beams on the ceiling provide a method by which the betatron may be moved in a horizontal direction the length of the building. Finally, the instrument can be rotated 360 degrees about a vertical axis. With this type of mounting, the machine may be moved to almost any position to radiograph large structures.
CHAPTER V

PHYSICAL APPLICATIONS

The previous chapters have dealt primarily with the electron accelerator. Early developments of the betatron, mathematical conditions for electron acceleration and stability, characteristics of the instrument, and installation of the accelerator have been the principal topics. Except for the brief summary of applications, ionization and gamma intensities, and the definition of the roentgen, discussion of the X-ray and electron beams has been minimized. The remaining chapters, however, present a detailed discussion of the applications of the betatron.

Cyclotrons and electrostatic generators were operating several years before the betatron, but these accelerators were of relatively low energy. Because of the relativistic mass change, the cyclotron was limited to the acceleration of heavy particles. The maximum energy for deuterons was 22 Mev. However, by frequency modulation it became possible to produce much higher energies with the cyclotron. Although the electrostatic generator could produce an electron, X-ray, or heavy ion beam, its usefulness, initially, was limited to an energy region of one or two Mev. This limitation was primarily caused by breakdown of the electrical insulation, but modifications have made it possible to increase the energy to about 10 Mev. While the betatron is limited to either X-ray or electron beam production, the instrument can produce energies to approximately 500 Mev.

With the betatron, physicists were able to attack new problems and to attempt verification of theoretical relations. The production of
Bremsstrahlung, the Bremsstrahlung spectrum, the thresholds of many gamma-induced reactions, the nuclear cross sections for these reactions, photofission, and many other problems have been investigated. The theoretical and experimental aspects of these problems are discussed in this chapter.

Concentration of X-Ray Beam

The narrow beam of photons produced by the betatron is caused by the high energy and therefore momentum of the impinging electrons and is a characteristic of high-energy X-rays. The angular divergence of the beam is determined by the target thickness and the energy of the impinging electrons. Schiff (70) has calculated this angular width, and Table III gives the angle measured from the beam center to the position where the intensity is reduced to one-half of its maximum value. These values are in good agreement with those found experimentally by Keret. The half angle intensity for the 22 Mev betatron is about 4.5 degrees. Hence, the target thickness must be approximately 0.05 cm thick.

TABLE III

Half-Angle Width of the Betatron Beam

<table>
<thead>
<tr>
<th>Energy (Mev)</th>
<th>0.050 cm</th>
<th>0.015 cm</th>
<th>0.005 cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>4.8°</td>
<td>3.7°</td>
<td>3°</td>
</tr>
<tr>
<td>50</td>
<td>2.0°</td>
<td>1.5°</td>
<td>1.2°</td>
</tr>
<tr>
<td>100</td>
<td>1.0°</td>
<td>0.73°</td>
<td>0.60°</td>
</tr>
<tr>
<td>200</td>
<td>0.5°</td>
<td>0.36°</td>
<td>0.30°</td>
</tr>
</tbody>
</table>
The electromagnetic radiation emitted from the betatron target has been referred to as either X-rays or γ-rays. However, according to Heitler (71) the radiation is actually Bremsstrahlung. When an electron with incident energy \( E_0 \) passes through the field of a nucleus, or atom, it generally is deflected. Since this deflection always produces an acceleration, the electron, according to classical theory, emits a radiation known as Bremsstrahlung. In the quantum theory, there is a certain probability that a photon of momentum \( k \) and energy \( k \) is emitted. In the field of the nucleus, the electron makes a transition to another state with energy \( E \), so that

\[
E/k = E_0. \tag{71}
\]

Intensity distribution curves of the Bremsstrahlung are shown in Fig. 21 (71a). The probability that a photon of energy \( k \) will be emitted per atom of the target for an incident electron of energy \( E_0 \) is \( \sigma_k \), which is also called the cross section for the process. In Fig. 21, \( k\sigma_k \) is plotted in units of \( E_0/\mathcal{J} \) for various values of \( k/(E_0 - \mu) \), where

\[
\mathcal{J} = 2^2 r_0^2/137 = 2^2 r_e^2/\hbar c.
\]

Here

\[
\mu = m_0 c^2,
\]

\[
\hbar = h/2\pi,
\]

and

\( r_0 \) = classical radius of electron.

Therefore, the values on the abscissa vary from zero to one. The numbers affixed to the curves refer to the primary kinetic energy \( E_0 - m_0 c^2 \). The dotted portions of the curves, except for the non-relativistic curve,
Figure 21. Intensity distribution of the Bremsstrahlung as a function of $\hbar/(E_0 - m_0c^2)$. The numbers affixed to the curves refer to the primary kinetic energy $E_0 - \mu$ in units of $\mu = m_0c^2$.

Figure 22. Absorption coefficient for Bremsstrahlung in Pb, Sn, Cu, and Al. The dotted curves show the three components of $\tau$ for lead.
are calculated for the case where screening has been neglected. In the high-energy quanta region, the full dotted curves coincide. Therefore, after these curves merge, the screening is not important, and the curves are once again valid for all values of Z. The non-relativistic case is of no interest to our problem and will not be discussed.

X-Ray Absorption Processes

Three experimental methods of determining the radiation spectrum from a betatron are by the Compton effect, the pair production process, or by analysis of the range-energy relation for protons from the photo-disintegration of deuterium. Since these processes are directly concerned with absorption and scattering of the radiation, the different absorption and scattering methods will now be discussed. Later, their application in the determination of the Bremsstrahlung spectrum will be considered.

The intensity of a beam of X or \( \gamma \)-rays decreases owing to absorption or scattering. If we neglect all types of selective absorption, such as processes in which atoms or nuclei are excited to a discrete level, there remain three processes which give rise to continuous absorption:

1. Photoelectric effect,
2. Compton scattering,
3. Creation of pairs.

Absorption by the photoelectric effect diminishes as the energy of the photon increases. For the non-relativistic case, the cross section for removing the k level electrons is proportional to \( (\mu/k)^{7/2} \), (71c), while for high energies, \( k \gg \mu \), the cross section varies as \( \mu/k \).
This simply states that the absorption coefficient decreases more rapidly in the non-relativistic than in the relativistic region. The energy region where the photoelectric effect is of most importance is where \( k < 0.5 \text{ Mev} \) for lead and \( k < 0.02 \text{ Mev} \) for aluminum. In Fig. 22 (71d) a graph is shown of the variations in absorption caused by the photoelectric effect, as a function of \( k/\mu \) for a lead scatterer.

The absorption coefficients of lead for Compton scattering are also shown in Fig. 22. Since this process has a much greater effect on the absorption of the primary radiation than does the photoelectric effect in the region from 1 to 10 Mev, Compton scattering will be more fully discussed.

Briefly, the Compton scattering process occurs when an incoming quantum of initial momentum \( k_o \), energy \( k_o \), is deflected in a collision with a free electron having an initial momentum \( p_o \) equal to zero, and energy \( E_o \) equal to \( \mu \). The particle is deflected at some angle, and in the final state has a momentum \( p \) and energy \( E \). After collision, the photon, reduced in energy to \( k \), is scattered at an angle \( \theta \) formed between \( k_o \) and \( k \). The photon energy after scattering is

\[
\frac{k}{k_o} = \frac{k_o \mu}{\mu + k_o (1 - \cos \theta)}.
\]

The linear absorption coefficient of a material owing to the Compton effect is \( \tau_c = N Z \sigma \). \( N \) equals the number of atoms per \( \text{cm}^3 \), \( Z \) represents the number of electrons per atom, and \( \sigma \) is the cross section per electron of the scattering process. Klein and Nishina (72) have derived the cross section expression for this type of scattering. Their result for the relativistic energy case using unpolarized incident radiation is
Thus, it is seen that the cross section, and hence absorption, decreases with increasing energy of the incident radiation. It is also evident that the absorption increases as heavier elements are used. A method for experimentally determining the intensity spectrum of a 2.8 Mev betatron utilizing the Compton effect is discussed on page 89.

The third type of absorption is caused by the conversion of energy into mass. If a photon having a minimum energy of $2 m c^2$ enters the coulomb field of a nucleus, an interaction may occur so that the photon vanishes and a positron and electron appear. The energy of the photon in excess of twice the rest mass of the electron is given the two particles as kinetic energy.

To determine the absorption coefficient, $\tau_p$, due to pair production, the cross section of the reaction must be known. Limiting values of this cross section exist when either no screening or complete screening is assumed. If $2 E_+ E_- / k \mu \ll 137Z^{-1/3}$, where $E_+$ and $E_-$ are the energies of the positron and electron, respectively, screening effects may be neglected. In this case

$$\sigma_{\text{pair}} = \overline{\phi} \left( \frac{28 \log 2k}{9} \frac{218}{27} \right),$$

where

$$\overline{\phi} = 2^2 r^2 / 137.$$

On the other hand, for $2E_+ E_- / k \mu \gg 137Z^{-1/3}$, complete screening results,
and we have

\[ \mu_{\text{pair}} = \bar{\beta} \left( \frac{28}{9} \log 183Z^{-1/3} - \frac{2}{27} \right). \]  

/40B/ = (71e)

For lead, the deviation from equation /40A/ begins at approximately 25 Mev.

A comparison of the Compton effect to pair production has two points of interest: (1) Scattering in the former case is proportional to Z of the material and in the latter case to \( Z^2 \). (2) The cross section for Compton scattering decreases with increasing incident energy, while the pair production cross section increases with energy. Lead, used as the absorbing material, has approximately equal absorption coefficients for the two processes at about 5 Mev.

In addition to these three processes for the absorption and scattering of Bremsstrahlung, other reactions complicate the determination of energy dissipation. Pairs may be produced by fast electrons entering the Coulomb field, by collision of two electrons, by two photons having a total energy greater than \( 2m_0c^2 \), by X-rays emitted by a nucleus, and by heavy particles if they have the required energy. Pair annihilation in which one or two quanta are emitted is another complicating factor. Compton-scattered electrons can produce ionization in the material as well as give rise to additional radiation.

**Determination of Radiation Spectrum**

For the betatron to be an ideal research tool, the absolute intensity spectrum of the radiation must be known. However, this is practically an experimental impossibility. Using a machine of low energy, 2 to 3 Mev, the spectrum may be approximated by measuring the
number, energy, and angle of the Compton scattered electrons from a thin target. Assuming the measuring equipment to be one hundred per cent efficient and that measurements are taken for all angle and energy intervals, the information obtained can be used to determine the incident intensity if the cross section for Compton scattering is known. These cross sections are not known and cannot be found unless the intensity of the incident beam is known. This difficulty can be surmounted if the theoretical cross section derived by Klein and Nishina is used. Now, since the number of electrons scattered having an energy between E and E + dE is known, and the cross section at this energy is available, the number of incident photons and, hence, the intensity may be found. Since the photoelectric effect depends on $Z^5$, carbon is utilized as the scatterer to minimize the error caused by the ejection of electrons by this effect which is of minor importance even for energies of 0.5 Mev. The cross section for this reaction is approximately $10^{-31}$ cm$^2$. For a constant Z, the cross section varies inversely with k for the high energy case. Pair production causes an increasingly large error in determining the radiation spectrum by the Compton effect for increasing energies. At approximately 3 Mev the cross section for the two effects, using carbon, are approximately equal.

The intensity distribution spectrum (73) of a 2.8 Mev betatron was determined using the Compton effect. A thin carbon target was mounted in a cloud chamber, placed in the X-ray beam, and a magnetic field was applied to the region where stereoscopic photographs were taken. By measuring the path curvature and initial direction of the ejected electrons, the energies for a specific angular distribution could be determined. Using the Klein-Nishina formula and the intensity distribution,
a histogram of the intensity versus energy was computed. The results showed a maximum intensity at about 1 Mev. The effect of pair formation was not considered. The spectral shape did not follow that expected. From the theoretical curves for monoenergetic electrons shown in Fig. 21, it appears that this discrepancy may be caused by electrons of various energies striking the tube target. It is possible, under certain magnetic field conditions, to have more than one stable equilibrium orbit. Hence, the electrons may be accelerated to different final energies.

Koch and Carter (74), (75) determined the intensity spectrum of X-rays, produced by bombarding a platinum target, 0.005 in. thick, with 19.5 Mev electrons. Their calculations were based on the energy distribution of pairs produced in an air-filled cloud chamber. Their primary purpose was to obtain a set of experimental data which could be compared with the predictions of the Bethe-Heitler Bremsstrahlung theory. To make a valid comparison between theory and experiment, the following conditions were important:

(1) Use of a thin X-ray target was necessary.

(2) Precise collimation of the beam was required so that only quanta originating in the target and traveling in the forward direction could pass through the cloud chamber.

(3) A low Z material in which electron-positron pairs were produced was used so that the theoretical cross section for pair production would permit an accurate conversion of the pair energy into an X-ray intensity distribution.

(4) The pair particle characteristics were examined in detail in order to eliminate instrument discrimination against certain energy groups.
NUMBER OF QUANTA SPECTRUM IN THE FORWARD DIRECTION FOR ELECTRONS FROM THE TARGET.

DOTTED LINE - THEORETICAL CURVE AS CALCULATED BY SCHIFF NORMALIZED WITH 3% MORE EXPERIMENTAL QUANTA THAN PREDICTED THEORETICALLY FROM 13 TO 20 MEV.

(a). Schematic diagram of the experimental arrangement.

(b). Electron pair energy spectrum.

(c). Number of quanta as a function of energy.

Figure 23. Equipment and data for the experimental determination of the Bremsstrahlung spectrum having a maximum energy of 19.5 Mev.
Equipment for this experiment may be separated into two divisions: (1) apparatus necessary to collimate the beam, and (2) devices to detect, photograph, and analyze the pairs created. Fig. 23a (75a) shows the experimental arrangement of beam collimation. The extensive collimator was necessary so that only the radiation emitted from the thin target could enter the chamber. The intensity of the uncollimated beam was one-half of its maximum value three degrees from the beam axis. (This intensity decrease agrees well with the value in Table III). As the beam entered the cloud chamber, its total angular width was approximately $0.24^\circ$. In addition to restricting the X-ray beam, the collimator reduced the number of secondary electrons which reached the chamber.

Integral X-ray intensity measurements were made for each X-ray burst by means of a high pressure argon-filled ionization chamber and a charge integrating circuit. The integrator readings were recorded on an Esterline-Angus recorder which permitted monitoring of the X-ray yield during the periods of the cloud-chamber exposures. At the chamber position, the yield was approximately $1.9 \times 10^{-6}$ r/pulse measured with a thick-walled ionization chamber. The betatron was operated at two pulses per minute; see page 62 for pulsed operation.

Using a stereoscopic camera and 35 mm film, 40,000 pictures were taken. Of this number, 10,300 pictures were analyzed and measurements were obtained on 1122 pairs. First, the direct image on the film was projected on a plane normal to the cloud-chamber magnetic field. The $x$ and $y$ coordinates of the pair origin, the average radius, and chord length of each particle's arc were measured. Second, to describe the electrons and positrons in space, the related angles had to be measured. This was done by reprojecting the film through a three-dimensional projector (76).
Fig. 23b (75b) shows the histogram of the number of pairs in each specific energy region. The solid line represents the corrected values obtained by applying the weighting factors. Now, by taking Heitler's theoretical cross sections for pair production in air and dividing by the histogram of Fig. 23b, the spectrum of the number of quanta as a function of the energy may be obtained. Fig. 23c (75b) shows this spectral distribution. The relative intensity distribution of Bremsstrahlung is found by multiplying the number of quanta at a particular energy, by that energy, and plotting the result as a function of the energy. Fig. 24 (75b) shows the calculated and theoretical intensity spectra of Bremsstrahlung from a betatron operating at 19.5 Mev. The dashed curve was calculated by Schiff from the Bethe-Heitler theory and normalized with 8 per cent more experimental quanta between 13 and 20 Mev than was theoretically predicted.

Two disadvantages of this experiment were that the statistics were poor and that the theoretical cross section for pair production was required in the calculations. By this method of pair detection, it is possible to improve the statistics, but it would be a very laborious task to completely analyze a hundred thousand pictures. A method for overcoming this difficulty is described in the next section. It is almost impossible to eliminate the theoretical cross sections from the calculations. As was previously stated, the intensity spectrum and the cross section are interrelated since both are functions of the energy; therefore, one theoretical value must be used when experimentally determining the other quantity. Another limitation on the results is that the intensity spectrum is critically dependent upon the geometry of the
Figure 24. Experimental and theoretical curves of the spectrum for 19.5 Mev radiation.

Figure 25. Experimental photo-fission threshold curve of U238. The fission activities in terms of fission counts per r of X-ray intensity are plotted as a function of the peak betatron energy. The background count level is shown by the dotted line.
experimental arrangement. By altering the physical arrangement of apparatus, the spectrum obtained may be materially changed.

By using a spectrum analyzer in conjunction with the betatron, work required to determine the Bremsstrahlung spectrum is greatly reduced. An analyzer at the Research Laboratory of General Electric has been briefly described by Lawson (77). Basically, the device furnishes the same information as is obtained from photographs using a cloud chamber. Pairs produced by the irradiation of a thin target are curved by a magnetic field and detected by Geiger-Müller counters. The counters are arranged so that the electrons or positrons entering a specific tube must be within a small energy region. In one experiment with 88 Mev gamma-rays, 76 G-M tubes were grouped so that each group covered an energy region of 15 Mev. The circuitry and recorders for the instrument and magnet are extensive. It is possible to record single events of either sign, pair production, and multiple events. The "single" is when only one counter of either sign is discharged. Although the equipment requires more than 500 electronic tubes, it is relatively free from electrical troubles. The analyzer is large and expensive but data from hundreds of thousands of events can be obtained with relative ease.

Baldwin and Klaiber (78) have pointed out that an X-ray spectrum can be determined without observing both members of each pair. The energy of one particle of a pair does not determine the energy of the photon which produced that pair, but it is possible to calculate statistically the energy distribution of quanta by observing the distribution of positrons. Results, based on this method, however, have not yet been reported.
Wang and Wiener (79) have determined the intensity spectrum of 10 Mev radiation by analyzing the range-energy of photo-protons in a deuterium loaded nuclear emulsion. A nuclear plate, Ilford C-2, 100 microns thick, was soaked in D₂O while a similar control plate was immersed in water. After soaking, the two plates and a non-soaked control plate were irradiated with 0.3 roentgens. About 2000 tracks were observed on the D₂O plate while only 30 were found on the water plate and simply background on the non-treated control plate. Five hundred tracks were measured and those having a slope of 45° or less (312 tracks) were used. On an equal area of the water plate, only seven tracks were found. Since the emulsion changed in thickness from 100 microns to 380 microns and then to 40 microns, the depth component, d, of the measured track was multiplied by a factor of 9/5 to determine the slope of the track in the wet emulsion.

The range-energy relation for protons was used to determine the variation of stopping power with energy for the wet plate. Although the betatron was designed to produce 10 Mev radiation, the longest path measured indicated a value of 11 Mev as the maximum energy. The number of proton tracks having a range between R and R + ΔR is related to the intensity I, for a specified energy interval by

\[ N_p = I \sigma(2E_p \neq 2.2)N_D \frac{dE_p}{dR} \Delta R \tau \]

where

- \( N_p \) = number of proton tracks having lengths between R and R + ΔR
- I = intensity of photon
- \( \sigma \) = cross section as function of energy and scattering angle, 45° to 135°
\[ E_p = \text{energy of proton in Mev} \]
\[ N_D = \text{number of deuterons per cm}^2 \text{ of emulsion} \]
\[ N = 2 \times 6.0 \times 10^{23} \times (\text{gm of D}_2\text{O/cm}^2 \times 1/20) \]
\[ dE_p = \text{stopping power of emulsion (Mev/micron)} \]
\[ dR \]
\[ A = \text{area of emulsion examined} \]
\[ t = \text{time of irradiation (in experiment } t = 12 \text{ seconds).} \]

Since the binding energy of the deuteron is approximately 2.2 Mev and the masses of the proton and neutron are about equal, the term \((2E_p - 2.2)\) gives the approximate energy of the incident photon.

The theoretical histogram, \(N_p\) versus photon energy, presented by Adams (80) and derived by Schiff, fits the experimental data from the 10 Mev betatron in all but the low energy region. This discrepancy could be caused by the failure to measure all of the short range proton tracks. By applying corrections for the low energy region, the intensity can be determined by equation /41/, thereby obtaining the intensity distribution spectrum.

To measure gamma radiation, it is necessary to measure the ionization produced by secondary electrons in the absorption of the original quanta. The absorption varies both energy and absorbing material. Variation of the ionization intensity as a function of photon energy was discussed on page 51. Also, since the ionization varies with absorber, the intensity of a betatron beam is critically dependent upon the measuring device. The measured value of a beam of radiation may be 100 r/min at 1 meter using one instrument and 185 r/min at the same distance with another measuring device. This discrepancy is caused by
changing the absorbing material which surrounds the ionization chamber.

For intensity measurements to have a meaning, the method of measurement must be given, or a standard instrument, placed at a specific distance from the target, must be used. There are indications that the Victoreen r-thimble meter is beginning to be used as a standard. This ionization chamber may be shielded by different materials of calibrated thicknesses so that maximum ionization intensities can be measured. Another type of monitor, often used in specific experiments, utilizes the activity from a gamma-induced radioactive substance. This method will be discussed in a later section of this chapter.

Photo-Fission Measurements

To obtain cross sections of gamma-induced photo-fission processes a yield curve is plotted against the maximum X-ray energy as the energy is changed. The ordinate of the curve is the ratio of the measured fission yield to the yield of the X-ray monitor. To interpret such data, the spectrum of the radiation and the response of the monitor to this spectrum at different X-ray energies must be known. Baldwin and Klaiber (81), (82) have roughly determined the response of a Victoreen r-meter surrounded by 1/8 in. of lead for incident radiation from 10 to 100 Mev.

To make an exact calculation of the meter response requires consideration of the detailed energy and angular distribution of the secondaries, the specific ionization as a function of electron energy, the scattering of the electrons, the trinary processes, the geometry, and the exact spectrum of the radiation. The first assumption of Baldwin and Klaiber was that the Bremsstrahlung intensity was constant for all energies. (Photo-fission in heavy elements was studied in this particular
experiment. Since the photo-fission yield decreases rapidly for $E > 20$ Mev, this indicates a reduction in cross section; hence, little error will be introduced by squaring off the spectrum.) For other types of experiments, this assumption would not be valid. Now, the number of quanta per square centimeter with energy between $k$ and $k + dk$ can be written

$$n(k,E)dk = cdk/k, \text{ for } k < E,$$  \hspace{1cm} (82) \hspace{1cm} /42/ \\

where $E$ is the maximum energy for a particular setting. The number of quanta per square centimeter in an absorber of thickness $t$ at a distance $x$ is

$$n(k,E,x)dk = cdk/k, \text{ for } k < E.$$  \hspace{1cm} /43/ \\

The number of electrons which reach the counter is

$$\mathcal{N}(E) = \int_0^E \left[ 2(T_f)_{\text{phot.}} + (T_f)_{\text{C}} + (T_f)_{\text{pair}} \right] \frac{dk}{k}.$$  \hspace{1cm} /44/ \\

The factors $f$ take account of absorption of quanta in the wall together with the fact that an electron of low energy can emerge from the wall only if its point of production lies within its range of the second surface. The range of an electron was taken to be proportional to its energy. For electron energies above 300 kev, the specific ionization for air is approximately independent of energy. An electron produces about 76 ion pairs per centimeter at 300 kev, 53 ion pairs at 2 Mev, and slowly increases to 70 ion pairs at 100 Mev. Low energy electrons which produce higher ionization are also highly scattered so that their effect is not serious. Since the meter is surrounded by lead, the effect of multiple
scattering and initial angular distribution of produced secondaries is reduced. Because of these variations, it is assumed that the electrons which enter the chamber produce an average ionization of 60 ion pairs per centimeter.

Baldwin and Klaiber integrated equation /44/ graphically. For a maximum energy, \( E \), of 10 Mev, the number of secondaries produced by all three absorption processes was calculated. To this was added the recoils and pairs resulting from absorption of photons above 10 Mev for a sequence of energies up to 100 Mev. The rectangular spectrum was used, and \( C \) was adjusted to unity. The reading of the r-meter can then be written

\[
R(E) = e\bar{s}v(E)C = 2.9 \times 10^{-8} (E), /45/ - (82)
\]

in esu, where \( \bar{s} \) = average ionization of 60 ion pairs per cm. Therefore to produce a reading of 1 roentgen in the meter surrounded by 1/8 in. lead requires approximately \( 2.5 \times 10^7 \) quanta of energy \( k \) per unit energy interval per square centimeter.

The ordinate of the yield curve is

\[
\Phi(E) = \frac{F(E)}{R(E)}, \text{ in fissions/roentgen} /46/ - (82)
\]

which may be termed the "X-ray fission yield."

The number of fission processes is \( F(E) \). These occur in a target of area \( A \) containing \( N \) atoms/cm\(^2\) when irradiated normally with X-rays of maximum energy \( E \) and intensity such that the quanta per cm\(^2\) between \( k \) and \( k \neq dk \) are incident on the target. If \( \sigma_k \) is the cross section for
photo-fission at the energy $k$, then

$$F(E) = \frac{N_A}{47} \int_0^E \sigma_h(k, E) dk.$$  \hspace{1cm} (82)

Therefore, by substituting equations (45) and (47) into (46) and solving for the cross section, we have

$$\int \frac{E}{2.9 \times 10^{-8} E \frac{d(\Phi \nu)}{dE}} \frac{dE}{N_A}.$$  \hspace{1cm} (48)

Now, by plotting $\nu(E)$, equation (45), versus $E$ and multiplying by the X-ray fission yield, which is determined experimentally, a graph of $\Phi \nu$ versus $E$ is obtained. Differentiation of the curve gives $d(\Phi \nu)/dE$ and hence the cross section $\sigma$.

Because of the approximations and uncertainties of the experiment, the authors state that the data have only qualitative significance.

For uranium $\sigma$ has a resonance shape of curve having a maximum of about $6 \times 10^{-26}$ cm at 17 Mev and 4 Mev width at one-half maximum. Thorium oxide was irradiated and found to have a maximum $\sigma$ of about $3 \times 10^{-26}$ cm. Tungsten, lead oxide, lead, bismuth, gold, thallium, and samarium were irradiated but no fission was observed. Sugarman (83) has reported photo-fission yields from bismuth, however, in 1950.

Measurements of threshold energies for photo-fission were first attempted by Baldwin and Koch (39) and later by Koch, McElhinney, and Gasteiger (84). In the later work, fission thresholds were determined using a 22 Mev betatron, for $^{233}U$, $^{235}U$, $^{238}U$, $^{239}Pu$, and $^{232}Th$. Basically, the arrangement was to irradiate a specimen which coated one of two plates of an ionization chamber and record the voltage pulses across the plates produced by the fission fragments. A thick-walled
ionization chamber was utilized to measure the beam intensity. By vary-
ing the maximum X-ray energy, a plot of the number of counts per
roentgen as a function of maximum photon energy determined the threshold
for the process. Fig. 25 (84a), is a graph used to determine the
threshold energy for U$^{238}$.

For the highly alpha-active samples of U$^{233}$ and Pu$^{239}$, the count-
ing chamber was modified. A third plate, a central grid through the
middle plate, and baffles over the sample were added. The potential of
the sample plate was $-800$ volts, while the top plate had a potential of
$800$ volts. Since alpha particles from the sample were directed, by the
baffles, through both chambers, a self-cancelling voltage pulse was pro-
duced. The shorter-range fission fragments produced a pulse across only
one chamber. Table IV (84a) gives the results of these experiments.

### TABLE IV

Photo-Fission Threshold Measurements
Utilizing a 22 Mev Betatron

<table>
<thead>
<tr>
<th>Sample</th>
<th>Photo-fission threshold (Mev)</th>
<th>Theoretical thresholds (Mev)</th>
</tr>
</thead>
<tbody>
<tr>
<td>U$^{238}$</td>
<td>5.08 ($\pm$ 0.15)</td>
<td>7.0</td>
</tr>
<tr>
<td>U$^{235}$</td>
<td>5.31 ($\pm$ 0.25)</td>
<td>6.1</td>
</tr>
<tr>
<td>Pu$^{239}$</td>
<td>5.31 ($\pm$ 0.27)</td>
<td>4.9</td>
</tr>
<tr>
<td>U$^{233}$</td>
<td>5.18 ($\pm$ 0.27)</td>
<td>5.7</td>
</tr>
<tr>
<td>Th$^{232}$</td>
<td>5.40 ($\pm$ 0.22)</td>
<td>-</td>
</tr>
</tbody>
</table>
Gamma—Induced Reactions

Extensive experiments of threshold reactions for twenty-three elements have been conducted by McElhinney et al. (85). The 22 Mev betatron utilized an integrator—bias control to determine the peak X-ray energy and was calibrated by measuring thresholds of several known \((\gamma, n)\) reactions. The energy calibration points were the thresholds for beryllium and carbon at the low end and copper and nitrogen for the higher energies. This gave approximately a linear scale.

Three experimental arrangements were used for threshold determination of the various samples. The first arrangement was to measure short period activities in Li, Mg, Al, Si, S, and Ca. Cylindrical samples were placed in the beam just beyond the coil box of the betatron and irradiated at different energies for a short period. After each exposure, the sample was dropped over a Geiger counter and the activity was measured. The second method was for the measurement of Mg, Cu and Ta which involved irradiations of about an hour. These samples were usually taped to the donut to receive the maximum amount of X-ray flux. Measurements of their activity were then made by a counter. Since bismuth gave no measurable radioactivity, the third system was to use a rhodium neutron detector to determine the bismuth threshold. After these experiments were conducted, the energy scale was checked against the thresholds of light elements and additional information was obtained on P, K, Fe, Br, Sb, and I.

Table V (85a) shows the reaction, observed activity, observed threshold in Mev, and the calculated threshold.
TABLE V

Observed Activities and Thresholds
for \((\gamma,n)\) and \((\gamma,p)\) Reactions

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Observed Activity</th>
<th>Observed thresholds (Mev)</th>
<th>Calculated thresholds from masses (Mev)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^1\text{H}(\gamma,n)^1\text{H})</td>
<td>Neutrons</td>
<td>2.20 ± 0.05</td>
<td>2.19 ± 0.03</td>
</tr>
<tr>
<td>(^9\text{Be}(\gamma,n)^9\text{Be})</td>
<td>Neutrons</td>
<td>Calibration</td>
<td>1.63 ± 0.03</td>
</tr>
<tr>
<td>(^6\text{Li}(\gamma,p)^6\text{He})</td>
<td>0.85 sec (\beta^-)</td>
<td>9.8 ± 0.5 (^a)</td>
<td>10.1 ± 0.5</td>
</tr>
<tr>
<td>(^{11}\text{C}(\gamma,n)^{11}\text{C})</td>
<td>20.5 min (\beta^+)</td>
<td>Calibration</td>
<td>18.7 ± 0.1</td>
</tr>
<tr>
<td>(^{13}\text{N}(\gamma,n)^{13}\text{N})</td>
<td>10 min (\beta^+)</td>
<td>10.65 ± 0.2</td>
<td>10.51 ± 0.1</td>
</tr>
<tr>
<td>(^{24}\text{Mg}(\gamma,n)^{24}\text{Mg})</td>
<td>11.6 sec (\beta^+)</td>
<td>16.2 ± 0.3</td>
<td>15.5 ± 1.0</td>
</tr>
<tr>
<td>(^{25}\text{Mg}(\gamma,p)^{25}\text{Na})</td>
<td>14.8 hr (\beta^-)</td>
<td>11.5 ± 1.0 (^a)</td>
<td>10.9 ± 1.0</td>
</tr>
<tr>
<td>(^{26}\text{Mg}(\gamma,p)^{26}\text{Na})</td>
<td>62 sec (\beta^-)</td>
<td>14.0 ± 1.0 (^a)</td>
<td>15.0 ± 1.6</td>
</tr>
<tr>
<td>(^{27}\text{Al}(\gamma,n)^{27}\text{Al})</td>
<td>7 sec (\beta^+)</td>
<td>14.0 ± 0.4</td>
<td>11.1 ± 1.0</td>
</tr>
<tr>
<td>(^{31}\text{S}(\gamma,n)^{31}\text{S})</td>
<td>3.2 sec (\beta^+)</td>
<td>14.8 ± 0.4</td>
<td>16.7 ± 0.5</td>
</tr>
<tr>
<td>(^{39}\text{K}(\gamma,n)^{39}\text{K})</td>
<td>7.5 min (\beta^+)</td>
<td>13.2 ± 0.2</td>
<td>12.7 ± 1.5</td>
</tr>
<tr>
<td>(^{40}\text{Ca}(\gamma,n)^{40}\text{Ca})</td>
<td>1 sec (\beta^+)</td>
<td>15.9 ± 0.4</td>
<td>13.7 ± 1.5</td>
</tr>
<tr>
<td>(^{62}\text{Cu}(\gamma,n)^{62}\text{Cu})</td>
<td>10 min (\beta^+)</td>
<td>10.9 ± 0.2</td>
<td></td>
</tr>
<tr>
<td>(^{78}\text{Br}(\gamma,n)^{78}\text{Br})</td>
<td>6.4 min (\beta^+)</td>
<td>10.7 ± 0.2</td>
<td></td>
</tr>
<tr>
<td>(^{126}\text{I}(\gamma,n)^{126}\text{I})</td>
<td>Neutrons</td>
<td>9.3 ± 0.2</td>
<td></td>
</tr>
<tr>
<td>(^{180}\text{Ta}(\gamma,n)^{180}\text{Ta})</td>
<td>8.2 hr (\beta^-)</td>
<td>7.7 ± 0.2</td>
<td></td>
</tr>
<tr>
<td>(^{208}\text{Bi}(\gamma,n)^{208}\text{Bi})</td>
<td>Neutrons</td>
<td>7.45 ± 0.2</td>
<td></td>
</tr>
</tbody>
</table>

\(^a\) These values are reduced from experimentally-observed thresholds by approximately one-half the height of the barrier for proton emission.
Masses of elements having a mass number greater than 40 are in most cases so poorly known that thresholds, i.e., neutron binding energies, cannot be calculated directly with any accuracy. Bohr and Wheeler (86) have derived a relation for the neutron binding energy from the packing fraction curve, and Mayer (87) has derived an empirical relation to express the masses as functions of the atomic and mass numbers, \( M(A, Z) \). The binding energy of the neutron is

\[
E_b(A, Z) \equiv M(A - 1, Z) - M_n - M(A, Z),
\]

which with Mayer's equation becomes

\[
E_b(A, Z) \equiv 83(Z^2/A^2) \left[ 1 - 0.0025A^{2/3} \right] - 9.33A^{-1/3} - 5.72
\]

\[
\neq 36A^{-3/4} \text{ millimass units.} \quad /49/ - (88b)
\]

The coefficient, 36, in the last term is for even-even or odd-odd nuclei. For even A and odd Z it becomes -36, and zero for odd A and even Z. Feenberg (88) has attempted to find a more detailed empirical relation which takes into account the fact that Z, for nuclei along the mass valley, is not a smooth function of the mass. Calculated and observed binding energy data are presented in Table VI (85a).

Many photo-disintegrations induced by 100 Mev X-rays have been reported by Baldwin and Klaiber (89). Fifteen hundred photographs of reactions in an air-filled cloud chamber yielded 105 single proton tracks of energies up to 9 Mev, 7 tracks of a proton or alpha particle and a heavy recoil nucleus, and 3 four-particle stars could be identified. Residual nuclei have been identified from measurements of the induced radioactivities in exposed samples. Many multiple-disintegration
reactions have been observed. Strong activities were observed from \((\gamma, p)\) and \((\gamma, 2n)\) reactions; moderately strong activities from \((\gamma, pn)\), \((\gamma, 2p)\) and \((\gamma, 2pn)\) were also observed. In addition to these reactions, weak activities were observed which could possibly be attributed to \((\gamma, p2n)\), \((\gamma, 3pn)\), and \((\gamma, \alpha n)\) or \((\gamma, 2p3n)\).

**TABLE VI**

<table>
<thead>
<tr>
<th>Element</th>
<th>A</th>
<th>Z</th>
<th>Theoretical Binding Energy</th>
<th>Observed Binding Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fe</td>
<td>54</td>
<td>26</td>
<td>12.6</td>
<td>13.8 ± 0.2</td>
</tr>
<tr>
<td>Cu</td>
<td>65</td>
<td>29</td>
<td>10.0</td>
<td>10.2 ± 0.2</td>
</tr>
<tr>
<td>Se</td>
<td>82</td>
<td>34</td>
<td>8.0</td>
<td>9.8 ± 0.5</td>
</tr>
<tr>
<td>Br</td>
<td>81</td>
<td>35</td>
<td>9.0</td>
<td>10.2 ± 0.2</td>
</tr>
<tr>
<td>Ay</td>
<td>107</td>
<td>47</td>
<td>9.6</td>
<td>9.5 ± 0.5</td>
</tr>
<tr>
<td>In</td>
<td>115</td>
<td>49</td>
<td>8.7</td>
<td>9.5 ± 0.5</td>
</tr>
<tr>
<td>Sb</td>
<td>121</td>
<td>51</td>
<td>8.4</td>
<td>9.2 ± 0.2</td>
</tr>
<tr>
<td>Ta</td>
<td>181</td>
<td>73</td>
<td>7.7</td>
<td>7.7 ± 0.2</td>
</tr>
<tr>
<td>Bi</td>
<td>209</td>
<td>83</td>
<td>7.1</td>
<td>7.4 ± 0.2</td>
</tr>
</tbody>
</table>

Perlman and Friedlander (90) conducted a series of experiments to determine the yields of \((\gamma, n)\), \((\gamma, p)\), \((\gamma, 2n)\), and \((\gamma, 2p)\) reactions at energies of 50 and 100 Mev. The yields were relative to the yield from N \(^{14}\) \((\gamma, n)\) N \(^{13}\) reaction taken as 1.00. An abrupt transition
at \( Z > 29 \) took place for the \((\gamma, n)\) process in which the yield increased by about one order of magnitude over the yields obtained for \( Z < 29 \). The \((\gamma, p)\) reactions studied gave a yield approximately equal to the \((\gamma, n)\) yields below the transition and decreased slowly as \( Z \) became larger than 29. Yields from \((\gamma, 2p)\) reactions were about one-twentieth of the \((\gamma, p)\) yields, and the \((\gamma, 2n)\) yields were in the order of one-hundredth the \((\gamma, n)\) yields.

In the experimental arrangement of Perlman and Friedlander, irradiation of the target was monitored by measuring the \( ^{11}C \) or \( ^{18}F \) activity induced in polystyrene or lithium fluoride. The use of a monitor whose half-life approximates that of the product, reduces the errors caused by X-ray intensity fluctuations. Calibration of the monitors at the two X-ray energies was accomplished by measuring the activity produced in them when they were bombarded together for ten minutes at nearly constant intensity. Monitor-decay curves at each energy were made.

The decay curves of most irradiated samples showed more than one decay scheme, but the activity of the component sought was determined by extrapolation. This value was used to calculate the activity at the end of a saturation exposure if there were no absorption or back scattering and if the product from one milligram-atom of the parent isotope was present. Corrections for absorption and back scattering caused by the thickness and mounting of the specimen were calculated. The ratio of the saturation product activity to the saturation activity of the monitor was then taken as a measure of the yield of the reaction.

Using a 22 Mev betatron and photographic nuclear emulsions, Diven and Almy (91) studied the energy and angular distribution of protons
emitted by irradiation of silver and aluminum. In the experimental arrange-
ment, a thin foil, nearly parallel to the X-ray beam, was placed in 
an evacuated chamber and exposed. To one side and perpendicular to the 
foil were two Ilford C-2 photographic plates placed parallel to each 
other in the chamber. Water vapor, to a pressure of 18 mm Hg, was emitted 
into the chamber to avoid dissociation of the nuclear emulsion. The lead 
collimated beam passed through the chamber and into a Victoreen 100-r 
thimble surrounded by 4 cm of aluminum. The response of the monitor was 
calculated using the methods of Fowler, Lauritsen, and Lauritsen (92). 
The beam also passed through a tantalum foil inducing, by a (r,n) reac-
tion, an 8.2 hr activity which served as an additional monitor.

Using silver as the specimen, 676 proton tracks with energies 
greater than 3 Mev were selected from a total area of 291 mm$^2$. To be ac-
cepted, a track was required to start at the surface of the emulsion and 
to have a direction compatible with an origin in the irradiated part of 
the foil. Angles to the beam from 20° to 160° were accepted. The foil 
was exposed to photons having a maximum energy of 20.8 Mev. Graphs were 
made of the number of protons per unit solid angle for various angles be-
tween the X-ray beam and emitted proton for three energy regions. Protons 
of energies between 4 and 7 Mev and between 7 and 10 Mev appeared to have 
a spherically-symmetrical distribution. For energies between 10 and 14 
Mev, however, a pronounced asymmetric distribution was observed with the 
most probable emission at 90° to the beam.

The total yield was 1.29 protons/r/mm of foil length. The total 
yield of neutrons during the same exposure was 54.6 neutrons/r/mm length 
of foil.
With an experimental arrangement similar to that for silver, an aluminum foil (4.74 mg/cm²) was irradiated by photons of 13.9, 17.1, and 20.8 Mev energies. The proton distribution was spherically symmetric at all energies, and the number of protons/atom/r x 10¹⁹ for an X-ray energy of 20.8 Mev was 2.8.

The (γ,p) cross section in aluminum gave a maximum cross section of 6 millibarns at an energy of 18 Mev. For (γ,n) reactions the maximum cross section had not been attained at 22 Mev. For silver the (γ,n) cross section reached a maximum of 0.32 barns at 16.5 Mev.

The applications discussed have been in using the betatron to produce various radioactive substances by (γ,n), (γ,p), and other processes. The activities of these substances have been used to determine threshold energies, yields, and cross sections for the various reactions. In most cases, a Victoreen r-meter has been used as the monitor, but other monitors may be employed.

Neutron Yields from Gamma-Induced Reactions

Another method for obtaining cross section measurements of (γ,n) reactions and for determining the absolute neutron yields from these reactions is to measure the activity produced by captured neutrons in a foil. Price and Kerst (93) have obtained absolute neutron yields as a function of Z, for 53 elements, using 18 and 22 Mev radiation. A schematic diagram of their experimental arrangement is shown in Fig. 26a (93a). Fig. 26b (93b) is a detailed diagram of the detector.

The detector was a rhodium foil in a large paraffin block placed in a double-walled lucite box. Fast neutrons from the irradiated sample passed through a layer of boron carbide, placed between the walls of the
(a). Schematic diagram of apparatus.

(b). Diagram of the fast neutron detector. The front plate which is made of lucite and boron carbide has been removed to show location of the rhodium detector.

Figure 26. Equipment for measurement of the photo-neutron yield of various samples.
lucite box, and were moderated by the paraffin. As slow neutrons, they were captured by the rhodium, and the 44-second beta activity which resulted was measured by a Geiger counter. Large paraffin blocks in conjunction with the boron carbide shielded the foil from stray neutrons. The concrete wall shielded the detector from X-rays and helped to moderate the fast neutrons from the betatron. The detector was calibrated by a 25 milligram standard radium-beryllium neutron source placed at the sample position.

To eliminate the effect of fluctuations in the X-ray beam intensity, a Rh foil was used as monitor. Since the number of neutrons generated in the betatron depends on the X-ray intensity, the activity of the Rh monitor foil produced a measure of the X-ray intensity. By placing a thick-walled Victoreen ionization chamber at the normal sample position, the Rh monitor was calibrated.

Utilizing the small, portable detector, Fig. 26a, measurements were made to determine the angular distribution of neutrons. Samples tested were iron, lead, beryllium, and deuterium. Beryllium and deuterium were found to have the previously observed asymmetric distribution with a maximum intensity of 90° from the X-ray beam. Iron and lead were found to be spherically symmetric.

The counting scheme was to irradiate the sample at a fixed maximum energy for three minutes, wait 30 seconds, and measure the Rh 44-second beta activity with a Geiger counter for three minutes. From the detector calibration curve, the number of neutrons impinging on the Rh foil could be determined. Then, by using the monitor calibration curve, the number of photo-neutrons emitted by the sample per mole per roentgen could be
determined. Fig. 27a (93c) shows the neutron yield for elements up to 
Z equal 92. The curves are for maximum X-ray energies of 18 and 22 Mev. 
Below Z equal 20, the curves are drawn to indicate general trends only. 
Neutron yields from thorium and uranium do not fit the curves because of 
the additional neutrons produced by photo-fission. Curve "A" is the yield 
predicted by Goldhaber and Teller (94) for 22 Mev radiation and normalized 
at Z equals 90.

An experimental arrangement similar to that used by Price and Kerst 
was utilized by Baldwin and Elder (95) to determine absolute neutron 
yields. They measured the neutrons per mole per roentgen from elements 
ranging in Z from 5 to 92 when irradiated with 50 Mev X-rays. Fig. 27b 
(95) shows the neutron yield as a function of Z.

Recently, Kerst and Price (96) determined the neutron yield ob-
tained by bombarding samples with 320 Mev radiation. Except for the 
determination of the X-ray intensity by the temperature increase in an ir-
radiated lead block, the equipment was the same as that previously used. 
With a Victoreen 100-r ionization chamber shielded by 1/8 in. of lead, 
the intensity was $7.8 \times 10^{-4}$ joules/cm$^2$/r. The yield of neutrons per 
mole/erg/cm$^2$ as a function of Z is given in Fig. 27c (96).

Becker, Kirk, and Buck (97) developed a special probe with which 
the betatron may be used to produce highly radioactive samples of Cu$^{62}$. 
A thin, folded copper foil, spring-clipped to one side of a small probe, 
is inserted in the donut. Since Skaggs et al. (98) found that the cross 
section for photo-disintegration is about 400 times greater than that for 
electro-disintegration, a thick lead block is placed on the other side of 
the probe. When the electron beam is expanded, X-rays are produced at
(a). Yields of photo-neutron sources. Curve A is the theoretical yield for 22 Mev radiation and normalized at Z = 90.

(b). Neutron yields as a function of Z using 22 and 50 Mev X-rays.

(c). Neutron yields for 320 Mev X-rays. The 22 Mev yield curve is adjusted to coincide at copper. Numerical ordinates refer only to the 320 Mev curve.

Figure 27. Curves of photo-neutron yield as function of Z for X-ray energies of 18, 22, 50, and 320 Mev.
the probe and irradiate the copper foil. This technique produces activities more than a thousand times greater than those realized when the betatron is used in the conventional manner. The probe is removed through an air-lock in the donut. By this method of irradiation, specific activities in excess of 5 millicuries of Cu\textsuperscript{62} positron activity per gram have been obtained. It should also be possible to increase activity in other materials by this same method.

**Electro-Disintegration**

Experimental studies of the electro-disintegration of Cu\textsuperscript{63}, Ag\textsuperscript{107}, and Ag\textsuperscript{109} have been made by Skaggs et al. (98) employing the electron beam extracted from a 22 Mev betatron by a magnetic shunt. The beam current was in the order of 0.01 microampere and concentrated in an area 5 cm wide and 2 cm high at a distance of 45 cm from the donut window. Different specimens, placed in the center of the beam were irradiated, and the sample activity was measured by a thin window Geiger counter. Another sample of the same material, placed just beneath the electron beam, provided a means for measuring the photo-activity caused by stray X-rays. The cross section for the electro-disintegration of Cu\textsuperscript{63} was 1.4, 16, and 32, with each value multiplied by 10\textsuperscript{-29} cm\textsuperscript{2} for electron energies of 13, 16, and 17.5 Mev, respectively. Cross sections for electro-disintegrations of Ag\textsuperscript{107} and Ag\textsuperscript{109} were 54 and 79 \times 10\textsuperscript{-29} cm\textsuperscript{2} respectively when irradiated with 16 Mev electrons.

**Electron Scattering**

The elastic scattering of 16.5 Mev electrons has been studied by Lyman, Nanson, and Scott (99) to determine the scattering cross sections
of different materials. The electron beam, focused to a 2 mm spot 4 meters from a 22 Mev betatron by a magnetic lens, passed through a low pressure ion chamber and impinged upon a thin target at the center of an evacuated scattering chamber. Scattered electrons were selected in angle by a lucite aperture, analyzed and refocused by a 75° Nier type magnetic analyzer, and detected by coincidence Geiger counters. Scattering cross sections in barns per unit solid angle are given in Table VII (99).

TABLE VII

Cross Sections for the Elastic Scattering of Electrons

<table>
<thead>
<tr>
<th>Element</th>
<th>Cross Sections (in barns)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(30°)</td>
</tr>
<tr>
<td>C</td>
<td>0.13</td>
</tr>
<tr>
<td>Al</td>
<td>0.65</td>
</tr>
<tr>
<td>Cu</td>
<td>3.1</td>
</tr>
<tr>
<td>Ag</td>
<td>9.6</td>
</tr>
<tr>
<td>Au</td>
<td>30</td>
</tr>
</tbody>
</table>
CHAPTER VI

RADIOGRAPHY WITH THE BETATRON

The application of the betatron to thick-section radiography has been investigated. For a device to be of radiographic usefulness for industrial purposes, it must produce on a photographic plate a clear picture showing as many of the details as possible. This means that the focal spot must be small, the absorption of the radiation must be a minimum, and there must be a minimum of X-ray scattering in the sample. These required conditions are fulfilled by a betatron.

Characteristics of High-Energy Radiography

Measurement of the focal spot was accomplished by Almy and Adams (100) with a pin hole camera. Midway between the betatron target and film was placed a lead plate having holes ranging from 0.0135 in. to 0.040 inches. From the spot size on the film the target focal size could be determined. A wedge-shaped platinum target having a height of 0.01 in. was used, and the X-ray origin was measured to be 0.01 in. high and less than 0.005 in. wide.

Adams (80) determined experimentally the absorption coefficients for lead, iron, and aluminum and found that these coefficients are essentially the same as predicted by theory. The minimum absorption for these materials occurs at an energy of about 4 Mev, 7 Mev, and 15 Mev, respectively. High X-ray intensities for each of these energies are produced with a betatron operating at 22 Mev. For a twenty-inch-thick section of steel, the efficiency of penetration is 3 for a 10 Mev betatron, relative
to a maximum efficiency of 100 for a 25 Mev betatron. Then the efficiency decreases slowly to 40 for a 90 Mev betatron. For peak energies from 90 Mev to 105 Mev, the penetration is approximately constant.

The radiographic figure of merit, F, is the ratio of the total exposure on the film, caused by the primary rays plus the secondary rays originating in the specimen, to the film exposure caused by the primary X-rays alone. Ideally, F is unity. From measurements, F is 1.2 for a 20 Mev betatron and increases to 1.7 for a 10 Mev betatron when a specimen of steel 4 inches thick is radiographed. At one or two Mev, F is of the order of 10; therefore, at these low energies about 80 to 90 percent of the film density is produced by secondaries. Hence, it is found that F increases as the X-ray energy decreases and also increases with specimen thickness.

As a result of the low intensity and diversions of the secondary radiation, it is not necessary for the radiographed specimen to be of a single thickness. Thus, the laborious job of blocking the specimen, often required in low energy X-ray work, is eliminated.

Because of the X-ray beam characteristics, focal spot size, and low intensity of scattered radiation, Almy and Adams (101) found that it is possible to obtain magnification of the radiographic image using the betatron. An example of this was the radiograph taken of the jaws of a caliper opened 0.001 inch. The caliper, between 1 in. thick sections of iron, was placed one meter from the target while the film was 3.3 m from the target. The developed film showed clearly the slit and all of the mechanical details of the caliper enlarged 3.3 times.

Since the X-ray beam is primarily in the forward direction,
intensity being \(\frac{1}{2}\) of the maximum 45° from the beam center, the correct
distance between the target and film is important for good film coverage.
A general working rule is that this distance should be at least five times
the largest dimension of the film.

X-Ray Films and Techniques

There are several types of industrial X-ray films and many different
radiographic techniques (100a). To detect 1/32 in. flaws in steel
sections up to 8 in. thick, Eastman Industrial, Type A film is recommended,
and for sections up to 14 in. thick, Eastman No-Screen film should be
used. For detection of 1/16 in. flaws in steel or to inspect assemblies
with great variation in thickness, Eastman Industrial, Type F film with a
0.040 in. front lead screen for sections up to 12 in. thick or 0.030 in.
lead and "W" screens for sections up to 20 in. thick give the clearest
radiographs. In addition to "W" screens, which are fluorescent, Patterson
Hi-speed screens are used.

Fig. 28a (101a) shows the exposure time required to radiograph
various thicknesses of iron using different film and screen techniques.
The exposure time is that required to produce a density of 2 on the film
when it is developed 10 minutes in D-19B developer. X-ray intensity, as
measured by a thick-walled Victoreen r-meter, is 50 r/min at 1 m; the
target to film distance is 72 inches.

Fig. 28b (100b) is a graph of the quantity of radiation, in
roentgens, three feet from the betatron target required to produce a
density of 2 on the film for different thicknesses of steel. Industrial,
Type A film with a 0.040 in. front lead screen is used. The different
curves show the effect of changing the target to film distance. If
(a). Exposure time as a function of specimen thickness using various films.

(b). Effect of varying the target-specimen distance.

Figure 28. Exposure curves for thick section radiography.
Eastman No-Screen film is used, the exposure given in Fig. 28b will be 2\frac{1}{2} times too much. Exposure time may be calculated by determining the X-ray intensity in r/min at 3 ft.

Fig. 29 (102) is a graph of exposure time versus plate thickness to radiograph steel plates using various peak X-ray energies and screens. The target to film distance is 48 inches.

The 4 Mev Betatron

Film-developing procedure is outlined in reference (100), and a detailed discussion of films and screens is presented by Sproull (103).

A 4 Mev portable betatron has been developed for radiographic purposes. The intensity output is approximately 0.04 r/min at 1 meter. Since the \( \gamma \)-radiation from 1 kg of radium produces approximately 13.8 r/min at 1 m (105a), this betatron has an intensity equivalent to about 2.9 gm of radium. Since the focal spot is less than 0.010 in. along its greatest dimension, the source of radiation is much smaller than that for an equivalent radium source. The definition in detail is not so good as with the 20 Mev machine since much more of the image is caused by secondary radiation. However, clear radiographs can be taken without blocking the specimen. Using Industrial, Type F film and 0.02 in. lead screens at a distance of 30 in., an exposure time of about one hour is required to radiograph a 4 in. section of steel.

The intensity of radiation from this betatron was measured using a Victoreen r-meter; the beam struck a masonry wall 4 ft. from the target. The quantity of radiation received during an eight-hour day ten feet from the target is small. In 45° increments, positive angles 10 ft from the direct beam the intensity measured 0.30, 0.01, 0.05, 0.075, 0.05, 0.00
Figure 29. Exposure time curves for steel utilizing various X-ray energies.
and 0.42 r/8 hr day. Therefore, any position ten feet away directly to the side or behind the betatron should be a safe location for personnel. Since the details of the method of measurement were not stated, and since the scattered radiation may increase when large metal specimens are radiographed, this radiation survey is not too reliable.

Several 4 Mev units have been built at the University of Illinois, at an estimated cost, in 1945, of $5,000 per unit. This price included two sealed-off vacuum tubes which cost approximately $400 each (101).
CHAPTER VII

MEDICAL APPLICATIONS

The aim of radiation therapy is to destroy completely malignant growth while causing only a tolerable injury to the skin and surrounding tissue. All radiation therapy developments since Röntgen's discovery, more than fifty years ago, are directed toward this objective. The use of penetrating gamma-rays in beam and interstitial radiotherapy, higher voltage equipment in X-ray therapy, neutron beam therapy, high energy electrons therapy, and finally the treatment of localized tumors by the selective absorption of artificially produced radioactive substances seek primarily to improve the effectiveness with which a tumor is subjected to high energy radiation.

High Voltage Radiology

Applications of external radiation sources such as X-rays, usually referred to as roentgen rays in medical applications, are the most customary approach to the deep therapy problem. A beam of the required aperture and intensity may be directed toward any desired region. Such therapy, however, has suffered in the past from the limitation that the beam must enter through the radiosensitive skin and undergo considerable absorption and scattering before it reaches the region of a deep tumor. Many of these difficulties are reduced by an ingenious choice of treatment distance, radiation field size, and the number of beam portals through which the radiation enters the body. More often than not, however, in the treatment of deep-seated malignancies with roentgen rays of
200 or 400 kev, the total dose and daily dose are established not by the optimum amounts needed to destroy the tumor, but rather by the limitations of surrounding normal tissue or the exposure of the skin through which the radiation passes.

Approximately thirteen years ago at the Collis P. Huntington Memorial Hospital, Boston, use of high voltage roentgen rays, having an energy of 1 Mev, was initiated in the treatment of deep-seated malignancies (104). Since then, roentgen rays of energies to 4 Mev have been utilized. This increase in energy was a definite step forward in the more effective treatment of tumors well within the body. With 3 Mev energy rays it became possible to deliver upwards of 5,000 r utilizing a single port of entry, in daily doses of from 300 to 400 r, without skin irritation or epilation of hair. The maximum ionization or dose received is not at the skin, and the electrons within the body are scattered more in the forward direction than when lower energy rays are used. The conditions are a consequence of high energy radiation.

By utilizing the betatron, a beam of high energy roentgen rays becomes available for possible use in medical therapy. The instrument is also a source of approximately monoenergetic electrons which may be useful to the radiologist. Electron therapy will be discussed in the latter part of this chapter.

Before a patient can be subjected to the intense ionization produced by a beam, 20 Mev or higher, energy X-rays, the early and late reactions caused by irradiation must be known as accurately as possible. It is not the purpose of this chapter to present a medical interpretation or evaluation of these reactions, but to present the physical properties
of the high energy radiation when it is absorbed in a tissue-like ma-
terial. The auxiliary equipment required for the medical application
of a betatron will also be discussed, and the physical characteristics of
200–400 kev, 2 Mev, and 22 Mev radiations will be compared.

Radiation from the betatron cannot be used effectively for medical
therapy before problems involving stray radiation, collimation of the
beam, production of radiation fields of a variety of different sizes,
depth-dose distribution, biological effectiveness of betatron radiation,
and construction of a betatron which is flexible with respect to its po-
sition and orientation of the beam are investigated. The problem in-
volving the fluxibility of the beam has been solved by Allis-Chalmers in
their new 24 Mev instrument, Fig. 18.

Auxiliary Equipment

Successful shielding of the betatron to reduce the stray radiation
to a safe level has been effected by Quastler, Adams et al., (105), and
later modified by Johns et al. (106). The stray radiation consists of
electrons from the betatron, unabsorbed primary roentgen rays, and second-
aries produced in the absorption and scattering of the primary rays by
any material placed in the beam. Fig. 30 a (107a) shows the arrangement
for shielding. Lead sheets, 1/32 in. thick, separated by fish paper, form
an inner shield placed between the coil boxes. On each side of this
shield and in the region near the X-ray target are placed blocks of lucite.
The electron-shielding material, lucite, was selected so that the X-rays
produced in the absorber would be minimized. Lead was the absorber for
the primary stray X-radiation. A second 3 in. thick lead shield covered
(a). Schematic diagram of shielding and auxiliary equipment.

(b). Apparatus for beam collimation.

Figure 30. Diagram of betatron equipped for medical therapy.

Figure 31. Depth dose graph: A, 15 cm circular field at 105 cm F.S.D.; B, 10 cm circular field at 70 F.S.D.
the entire front of the betatron to absorb any radiation which was not absorbed by the other shield. In addition to lucite being a shield for electrons, it also reduces the intensity of the stray neutrons produced. By employing the basic shielding arrangement shown in Fig. 30a with possible modifications for a specific instrument, a 22 Mev betatron can be adequately shielded for medical applications.

To make the high-energy X-ray beam useful, it is necessary to be able to select the size of the radiation field conveniently, and to be sure that the radiation outside this field is biologically insignificant. The apparatus required to meet these requirements is illustrated in Fig. 30a and Fig. 30b (107a). A master collimator A (Fig. 30b), consisting of a stainless-steel nonmagnetic rectangular case, filled with a mixture of lead pellets, lead oxide and glycerine, is inserted through the two shields so that its central axis is coincident with the X-ray beam axis. A square step-shaped hole passes through the collimator. Into this hole may be inserted a lead plug D (Fig. 30b), which determines the shape of the radiation field. To increase the accuracy in the determination of the exact size and position of the field as the radiation enters the body, a treatment cone, E (Fig. 30b) is connected to the face plate of the master collimator. The treatment cone acts only to indicate the beam position and has no collimating effects. A different lead plug and cone are required for each radiation field size. Because of the pronounced intensity of the X-rays in the forward direction, the intensity varies considerably across the beam. For a betatron operating at 22 Mev, a 10 cm diameter field 75 cm from the target has an intensity at the edge of only 1/2 that at the center (105). Ideally, the intensity should be
uniform over the entire field. This condition can be approximated by utilizing a conical filter. At conventional energies, 200 to 400 kev, filters of high atomic number are generally used to absorb the low energy components in the beam, but for energies of several Mev these filters are not satisfactory because of the absorption by pair formation. However, filters of carbon, aluminum, or copper can be utilized in the high energy region. Since the total absorption coefficient for copper is approximately constant from 5 Mev to 25 Mev, and since a copper filter is the easiest to make, this material is usually employed. Fig. 30a and Fig. 30b show the copper-compensating filter. The filter and a monitor are mounted in the master collimator.

**Depth-Dose Distribution**

The depth-dose distribution and the biological effectiveness of the radiation from the betatron are the two factors which primarily determine the therapeutic usefulness of high voltage roentgen rays. Koch, Kerst, and Morrison (108) measured the intensity distribution of the radiation as a function of depth in an absorber for 5, 10, 15, and 20 Mev X-rays. Charlton and Breed (109) have made depth-dose studies of roentgen rays of energies from 20 to 100 Mev, and Johns et al. (106) have investigated the depth-dose for radiation in the 22 Mev region. Also, Trump et al. (110) have investigated the characteristics of roentgen radiation from 1 to 4 Mev energy from an electrostatic generator. Quastler has studied the biological effectiveness of betatron radiation and has published several papers on this subject. One of the more recent articles is given in reference (111).

The intensity distribution of radiation in an absorber can be
determined by making approximately point measurements using a small ionization chamber. Since the density of water is nearly the same as that of tissue, water is often employed as the absorbing medium. A remote-control probe type of ionization chamber and a positioner (106) have been developed by the betatron group at the University of Saskatchewan for depth-dose measurements. (A similar probe and positioner were developed by Laughlin and Davies (112).) With this movable probe, a 22 Mev betatron, and the auxiliary equipment previously described, measurements were obtained of the dose distribution for a variety of sizes of the radiation field (106). The target to entrance surface distance, referred to as the focal skin distance (F.S.D.) was 70 cm and 105 cm for two sets of data. Curve A, Fig. 31 (106a) illustrates the per cent depth-dose for a 15 cm diameter circular field at 105 F. S. D.; curve B shows the conditions for a 10 cm diameter circular field at 70 cm F. S. D. By measuring the dose distribution throughout the volume, isodose curves may be plotted. Each curve represents a region of equal ionization and has a per cent value based on a maximum ionization of 100. Fig. 32a (106b) shows the isodose curves for 22 Mev radiation using a copper compensating filter, an F. S. D. of 105 cm, and a 6 cm x 15 cm rectangular-shaped field.

Because of the high energy of the incident radiation, the maximum depth-dose does not occur at the surface of the absorber. For a tissue-like substance, the radiation is absorbed principally by the Compton process. The Compton electrons are scattered primarily in the forward direction and produce, by collision, thousands of low energy secondary electrons. The maximum ionization is reached at that distance below the surface at which the first Compton electrons have on the average expended
(a). Distribution for short axis of a 6 cm x 15 cm field using 22 Mev radiation at F.S.D. of 105 cm with a copper filter.

(c). Distribution for short axis of 6 cm x 15 cm field using 400 kev radiation with hvl of 9 mm of Cu and F.S.D. of 80 cm.

Figure 32. Isodose distribution for 22 Mev, 2 Mev, and 400 kev X-rays.
their energy in ionization and stopped. Fig. 32a shows that this distance is about 4 cm. This maximum depth-dose distance is not appreciably affected by variations of 70, 105 (106), or 196 (109) cm in the F. S. D. for copper-filtered 22 Mev radiation. However, because of the effect of the inverse square law, there is a slight increase in this depth for unfiltered 50 Mev radiation as the F. S. D. is increased from 132 to 510 cm (109).

The action of the compensating filter not only produces a beam having a uniform cross-sectional intensity but also reduces the low energy components. Therefore, by utilization of a filter, the maximum depth-dose distance is increased from about 1.5 cm (109) to approximately 4 cm (106).

Exact measurements of the dose at the surface are critically dependent upon the wall thickness of the ionization chamber and the contamination of the beam by stray electrons. Therefore, the surface dose cannot be accurately determined, but this dose is estimated to be less than 30 per cent of the maximum.

Another interesting fact obtained by comparing the isodose curves for different sizes of radiation fields is that the ionization density for a specific depth is not a function of the field area for a constant target-skin distance. This can be illustrated by a comparison between a 9 cm circular field (106) and a 6 x 15 cm$^2$ rectangular field, Fig. 32a, when both have a F. S. D. of 105 cm. Both fields produce maximum ionization 4 cm within the absorber, 90% at 8 cm, 80% at 11 cm, 70% at 14 cm, 60% at 18 cm, and 50% at 23 cm.

Although the depth of the maximum dose does not vary with changes
in the field cross section or focal skin distance, the per cent dose in
the surrounding region does vary as the F. S. D. changes. This fact is
illustrated in Fig. 31 for F. S. D. of 105 cm and 70 cm. If a specimen
thickness of 16 cm is assumed, the exit doses are approximately 65 per
cent and 57 per cent of the maximum for the two distances when 22 Mev
radiation is used. If higher energy radiation is employed (109), the
maximum dose appears at a depth greater than 4 cm. The entrance dose is
decreased, but the exit dose is substantially increased. It is quite
possible that the exit dose will determine the maximum energy which may
be used, as well as the focal skin distance.

A method by which the per cent depth-dose may be calculated for
any depth within a specimen and any F. S. D. is given in reference (106).

22 Mev, 2 Mev, and 400 Kev Radiation Compared

For deep therapy the application of radiation having an energy in
the million volt region has several advantages over lower energy radia-
tion, 400 kv and below. One of the primary advantages is that the
maximum dose received does not occur on the skin but at a distance below
the surface, which increases as higher energy X-rays are used. Fig. 32
shows the depth-dose distribution for 22 Mev, 2 Mev and 400 Kev radia-
tion. This reduction in surface dose partially explains why the tolerance
of the skin to radiation increases as the X-ray energy is increased (113).

The reduction in erythema is another advantage of million volt ra-
diation in the treatment of deep-seated lesions.

In the application of 400 kv radiation to a region, the depth-dose
is critically dependent on the size of the radiation field. Hence,
for a region to receive a specified dose, it is quite possible that the
field must be so large that the surrounding tissue will receive a dose which prohibits the application of roentgen ray therapy. This disadvantage does not exist for 22 million volt radiation because now the intensity is independent of the radiation field size.

To apply low voltage X-rays in the treatment of deep-seated malignancies, it is often necessary to direct the beam toward the tumor from several directions. This technique, known as cross firing, enables the radiation to be distributed in the surrounding tissue while still being concentrated in the tumor region. Cross firing requires the movement of either the X-ray machine or the patient. Often six or even eight portals of entry are required to produce a given depth-dose.

By using radiation in the million volt region, however, cross firing becomes less important. For 22 Mev radiation, the maximum dose occurs 4 cm below the surface and reduces to only one-half of this maximum 22 cm within the specimen (Fig. 32a). Therefore, it is now possible to irradiate a deep-seated tumor utilizing only one portal. However, when a depth-dose greater than that obtainable from a single beam at a specified depth is required, cross firing using any number of beams may be employed. Johns et al. (114) have made an extensive study of the depth-dose distribution of 22 Mev radiation for cross firing utilizing 2, 3, 4, and 6 field portals. Studies were also made of the distribution in dose when cross firing was applied to 400 kva X-rays. Fig. 33a (114a) and Fig. 336 (114a) show the dose distribution when four fields are applied to a region.

Trump et al. (110) have obtained isodose curves for 2 Mev X-rays, and they have studied the ionization density within a specimen when single portal, three portal and continuous portal cross fire techniques
(a). Isodose distribution for two 15 cm x 10 cm fields and two 10 cm x 8 cm fields arranged to treat case of cancer of the cervix. 22 Mev radiation at F.S.D. of 105 cm with Cu filter was used.

(b). Isodose distribution for two 15 cm x 10 cm fields and two 10 cm x 8 cm fields for treatment of cancer of the cervix. 400 kev radiation with hvl of 4 mm Cu at 80 cm F.S.D. was used.

Figure 33. Depth dose distribution for cross firing with 22 Mev, 2 Mev, and 400 kev radiation.
(c) Dose distribution for 2 MeV radiation using a 5 cm x 5 cm field and F.S.D. of 100 cm. Relative intensities for single, triple, and continuous portal irradiation are shown.

Figure 34. Efficiencies of high and low energy beams compared by normalizing exposure at two different depths.

Figure 35. Depth dose of electrons in a water specimen.
were employed using 200 kev and 2 Mev radiations. From Fig. 32b (110a), it is evident that the dose distribution is approaching that for 22 Mev X-rays. However, in this low million volt region, the intensity distribution is partially controlled by the size of the radiation field. Fig. 33c (110a) shows the relative intensity of 2 Mev radiation necessary to produce an intensity of 100 at a depth of 10 cm using single beam and cross firing techniques.

Fig. 34 (105a) shows the relative intensities of 20 Mev and 200 kev radiation as a function of depth within an absorber. The curves have been normalized so that the same intensity is produced by the two beams at a depth of 7 cm and 12 cm in a tissue-like material.

**Electron Therapy**

Since the biological effects of roentgen rays and radium are caused by the secondary electrons which are produced when roentgen rays pass through tissue, many attempts have been made to utilize directly a beam of electrons. The primary reason why electron therapy has not been more successful in the past is that the incident electrons were of too low an energy. However, with the application of the betatron a source of penetrating electrons is available.

The principal advantages of electrons relative to the X-ray beam are shown in Fig. 35 (115). A peak depth-dose is obtained which increases with the incident electron energy. The depth-dose decreases rapidly beyond this peak so that there is usually no effect at the exit portal. In tissue, the maximum electron range, measured in centimeters, is approximately one-half the maximum energy, in Mev, minus 0.5 cm.

Electrons from a 6 Mev betatron have been used in Germany in the
treatment of a number of cancer patients (116). The results of these treatments were most encouraging.

Additional information concerning electron therapy is given in references (117) and (118).

From this review of the medical applications of the betatron, it was shown that the instrument produces an X-ray beam which has many of the fundamental characteristics required for the treatment of deep-seated malignancies. Also, there is justification for additional experimentation to further determine the properties and effects of the electron beam.
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APPENDIXES
APPENDIX I

VECTOR NOTATION IN CYLINDRICAL COORDINATES

Using mutually perpendicular unit vectors \( \mathbf{r} \), \( \mathbf{\theta} \), and \( \mathbf{z} \), we have
\( x = r \cos \theta \), \( y = r \sin \theta \), and \( z = z \). Also,

\[
\nabla = r_1 \frac{\partial}{\partial r} \mathbf{r} + \frac{1}{r} \frac{\partial}{\partial \theta} \mathbf{\theta} + \frac{\partial}{\partial z} \mathbf{z},
\]

\[
\nabla^2 A = \frac{1}{r} \frac{\partial}{\partial r}(r \frac{\partial A}{\partial r}) + \frac{1}{r^2} \frac{\partial^2 A}{\partial \theta^2} + \frac{\partial^2 A}{\partial z^2},
\]

\[
\nabla A = r_1 \frac{\partial A}{\partial r} + \frac{1}{r} \frac{\partial A}{\partial \theta} + \frac{\partial A}{\partial z},
\]

\[
\nabla \times A = \frac{1}{r} \frac{\partial}{\partial r}(r A_r) + \frac{1}{r} \frac{\partial A}{\partial \theta} + \frac{\partial A_z}{\partial z},
\]

\[
\nabla \cdot A = \begin{vmatrix}
\frac{r_1}{r} & \theta_1 & k_1 \\
\frac{1}{r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\
A_r & r A_\theta & A_z
\end{vmatrix}
\]

\[
= \frac{r_1}{r} \left( \frac{\partial A_z}{\partial \theta} - \frac{\partial (r A_\theta)}{\partial z} \right) + \frac{1}{r} \left( \frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \right) + \frac{k_1}{r} \left( \frac{\partial (r A_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta} \right).
\]
APPENDIX II

DERIVATION OF RELATION BETWEEN $B_r$ AND $B_z$

By the expansion of $\nabla \cdot \mathbf{B}$, we have,

$$\nabla \cdot \mathbf{B} = \frac{1}{r} \frac{\partial (r B_r)}{\partial r} + \frac{1}{r} \frac{\partial B_\theta}{\partial \theta} + \frac{\partial B_z}{\partial z} = 0.$$ 

Since $\frac{1}{r} \frac{\partial B_\theta}{\partial \theta} = 0$, $\mathbf{B}$ becomes

$$\frac{\partial (r B_r)}{\partial r} = - r \frac{d B_z}{d z}.$$ 

Upon integration this becomes

$$r B_r = - \int_0^r r \frac{d B_z}{d z} dr.$$
Beginning with Maxwell's equation

\[ \nabla \times \mathbf{B} = \mathbf{i}_v \times \frac{\partial \mathbf{D}}{\partial t}, \]

where

- \( \mathbf{i}_v \) is current density which is zero for a vacuum,
- \( \mathbf{D} = \epsilon \mathbf{E} \),
- \( \mathbf{B} = \mu \mathbf{H} \), and
- \( \mu \epsilon = 1/c^2 \).

By substitution we have,

\[ \nabla \times \mathbf{B} = \mu \epsilon \frac{\partial \mathbf{E}}{\partial t}. \]

By expanding \( \nabla \times \mathbf{B} \), the terms which are not zero give

\[ \nabla \times \mathbf{B} = \frac{\partial B_r}{\partial z} - \frac{\partial B_z}{\partial r} = \mu \epsilon \frac{\partial E_z}{\partial t}. \]

Since \( r \) is not a function of \( z \), by equation /4B/, we have,

\[ \frac{\partial B_r}{\partial z} = -\frac{1}{r} \int_0^r \! \frac{1}{r} \frac{\partial^2 B_z}{\partial z^2} \, dr \]

and by /4A/

\[ \frac{\partial E_z}{\partial t} = -\frac{1}{r} \int_0^r \! \frac{1}{r} \frac{\partial^2 B_z}{\partial t^2} \, dr. \]

Therefore, by substitution equation /7/ is obtained, which is

\[ -\frac{1}{r} \int_0^r \! r \frac{\partial^2 B_z}{\partial z^2} \, dr - \frac{\partial B_z}{\partial r} + \frac{1}{c^2} \frac{1}{r} \int_0^r \! r \frac{\partial^2 B_z}{\partial t^2} \, dr = 0. \]
APPENDIX IV

PROOF OF EQUATION /8B/

Substitution of equations /4A/ and /4B/ into equation /2B/ gives

\[ \frac{d(mr^2e)}{dt} = e \int_{0}^{l} r \frac{\partial B_z}{\partial t} \, dr + e^2 \int_{0}^{l} r \frac{\partial B_z}{\partial z} \, dr - erfB_z. \]

Now let \( Q = e \int_{0}^{l} rB_z \, dr \). Also \( \frac{dQ}{dt} = \frac{\partial Q}{\partial r} \frac{\partial r}{\partial r} + \frac{\partial Q}{\partial z} \frac{\partial z}{\partial t} + \frac{\partial Q}{\partial t} \frac{\partial t}{\partial t} \)

Since \( B_z = f(r,z,t) \), we have

\[ \frac{\partial Q}{\partial r} = erB_z, \quad \frac{\partial Q}{\partial z} = e \int_{0}^{l} r \frac{\partial B_z}{\partial z} \, dr, \quad \frac{\partial Q}{\partial t} = e \int_{0}^{l} r \frac{\partial B_z}{\partial t} \, dr \]

Therefore,

\[ \frac{dQ}{dt} = erB_z + 2e \int_{0}^{l} r \frac{\partial B_z}{\partial z} \, dr \int_{0}^{l} r \frac{\partial B_z}{\partial t} \, dr. \]

Hence,

\[ \frac{dQ}{dt} = \frac{d(mr^2e)}{dt} = \frac{e}{dt} \int_{0}^{l} rB_z \, dr \]

which is equation /8B/.
Determining the Relativistic Mass Equation

Using the expression /9B/ and the equation for the velocity of the particle, a mass equation may be derived from the general mass formula. Here

\[ (r \dot{\theta})^2 = \frac{e^2 S^2}{m^2} \]

and

\[ \dot{v}^2 = r^2 \dot{\phi}^2 + \dot{z}^2. \]

By substitution of these equations into \( m = m_0 / (1 - v^2 / c^2)^{1/2} \), we obtain

\[ m^2 = \frac{m_0^2 c^2}{c^2 - \left( r^2 \dot{\phi}^2 + \dot{z}^2 \right) / m^2}. \]

This expression reduces to equation /11/, which is

\[ m^2 c^2 = \frac{m_0^2 c^2 + e^2 S^2}{1 - \left( \dot{r}^2 + \dot{z}^2 \right) / c^2}. \]
APPENDIX VI

SIMPLIFICATION OF THE KINETIC ENERGY EQUATION

The kinetic energy equation /12/, which states that

\[ T = m_0 c^2 \left\{ \frac{e^2}{m_0} \left[ 1 - \left( \frac{\mathbf{r} \cdot \mathbf{v}}{c^2} \right)^2 \right] \right\} \] 

may be reduced to the more general form for the relativistic energy.

If we assume that \( \mathbf{r} \) and \( \mathbf{z} \) are zero and recall that

\[ S = \frac{m \mathbf{v}}{e} = \frac{mv}{e} , \]

then the expression reduces to

\[ T = \sqrt{(mv)^2 c^2 - (m_0 c^2)^2} - m_0 c^2 , \]

which is the general relativistic energy equation.