Exploring Frequency Space: Applying the 2D FFT

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EXPLORING FREQUENCY SPACE: APPLYING THE 2D FFT

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ABSTRACT

The shape and arrangement of features across a paper or paperboard sheet may be characterized using 2D FFT analysis. The present study explains the relationship between spatial features in images and their corresponding incarnations in the frequency plane of the 2D FFT. Topographic and textural data vary with the contrast mechanism used to obtain the images. Forming fabrics and papers formed upon them are used as examples. A software tool has been developed to facilitate the analysis of spatial data in paper images via the 2D FFT, especially those containing patterns.

INTRODUCTION

Irregularities at or near the surface of paper and board decrease product quality and marketability. The texture or roughness may impact only the appearance or feel. However, surface features can lead to difficulties in coating and printing. They may also be indicative of potential problems in strength, runnability, and convertibility.

A variety of methods are available for identifying and quantifying physical aspects of paper and board (Fig. 1). 2D mappings* from scanning triangulating laser sensors and Moiré interferometric surface images are useful for studying intermediate to large-scale surface topographical features like cockling and warp. At higher magnifications, confocal, scanning electron, and atomic force microscopies generate topographic representations for fiber-scale studies. Low-angle reflected light imaging can be useful for the study of surface texture, roughness, and orientation, but it requires a shadow-angle transform in order to infer surface height and is subject to large noise and biases due to the lighting itself. Incident ultraviolet light in conjunction with fluorescent dyed fiber mixtures of varying densities allows the study of surface fiber orientation and distribution. Multiple volumes of differently colored dyed fibers and corresponding color filters may be used to study different fiber ‘families’, each of which might have, for example, been injected into different points into a headbox for the study of turbulent flow patterns and patterns of deposition onto a forming fabric. Through-sample transmitted light images (with and without a logarithmic transform from transmitted light intensity to mass density) and beta radiography are useful for studying formation and other forms of mass distribution. Transmitted light through thin papers containing a small portion, by volume, of dyed fibers or very low fiber density specimens (e.g., tape pulls) provides a means of studying fiber orientation and density distributions and, at low enough densities, individual fiber characteristics such as fiber orientation, kinking, length, and curl.

PATTERNS IN PAPER AND BOARD

Surface and internal markings have a number of possible sources, including forming wire or other fabrics used in pressing and drying. Jet-wire speed and formation irregularities can contribute to marking. Also, spatial variations in web stresses can cause periodic changes in the fiber structure. The markings may be periodic or appear randomly. They may be different on each side of the paper. They may be more obvious within the formation than at the surface, or vice versa.

* In this paper, when the number of dimensions of a dataset is referred to (e.g., 2D), the number will be a reference to the spatial dimensions upon which an additional amplitude/intensity dimension is encoded. Multidimensional intensities (i.e., color) are outside the scope of this paper.
Pattern detection research has been driven mainly by competitive pressures and potential liability for markings on paper and board. The objective of these studies is to detect and extract surface or internal patterns. This will help to isolate individual contributors to the markings, better understand the processes that form them, and, ultimately, to minimize the patterns. Pattern characterization can also aid in product development efforts of machine clothing manufacturers.

Many patterns seen in paper and board have periodic components. As such, they have characteristic frequencies and orientations that are detectable using two-dimensional Fast Fourier Transform (FFT) analysis (1-5). Image analysis with FFT has become more common as more powerful computers have become available at lower cost. Andersson (6) used Fourier descriptors to characterize textures and formation variation in paper. I’Anson (3,4) showed that machine fabrics have unique FFT spectra.

CONTRAST MECHANISMS AND FREQUENCY SPACE SIGNATURES

One of the most universally applicable analysis tools available for examining and quantifying two-dimensional data from many sources is the 2D FFT. The transform of the spatial image data is represented as an energy intensity dimension encoded upon a rectangular subset of a Cartesian frequency plane. The 2D Fast Fourier Transform is computationally efficient and excellent for identifying periodic components of features in 2D images. However, it is useful for working with nonperiodic features, too, since even seemingly nonperiodic features are ultimately periodic, because the FFT assumes that the boundaries of the window are joined seamlessly and that the entire image is a periodic component with a wavelength/spatial frequency that is perfectly aligned with the dimensions of the window. Being 'nonperiodic', they are guaranteed to be complicated, involving energy distributions through regions of the frequency plane and possibly the entire rectangular window into the plane. For example, a nonperiodic image consisting of a single pixel whose intensity (amplitude) is markedly different from an otherwise homogeneous background of pixels fills the frequency plane with an essentially homogeneous distribution. More complex objects, like line segments and ellipsoids (Figs. 2,5), also fill the frequency plane with energy distributions that are condensed into agglomerations of energy and which have a degree of directionality associated with them in proportion to the 'degree of directionality' associated with the source image. The orientation of a line segment is identifiable and measurable in the frequency plane even though a lone line segment would not normally be considered a periodic feature.

By selecting the imaging method and contrast mechanism, the researcher has a degree of control over the frequency space signature of the features to be identified, filtered, and/or measured. Every microscopist knows that a contrast mechanism is needed to differentiate the features of interest from the background. Likewise, to obtain an optimum image for analysis, the researcher may be obliged to use dyes, color filtration, polarization, lighting angle, or whatever possible means to induce a gray-scale differential between the features of interest and the background prior to imaging and application to the FFT. On other occasions, the researcher may seek to maximize the differential between the perimeters of the features of interest and the background. The signatures of the two cases within the frequency space of the FFT are somewhat different, and, depending on the analytical objectives and imaging conditions, this should be taken into account (Fig. 3).

Consider a cell that is being imaged with a light microscope and is roughly the shape of an ellipse. The cell may be imaged as a surface or as an interior cross section. The cell could have been any contiguous object, such as a shive on the surface of paper. In any case, the interior and the cell boundary may or may not appear very different in terms of average brightness from the surrounding medium. The FFT will present a distribution of energy that can be considered to be the bisection of the cell at all angles of rotation (Fig. 2). Each bisection can be considered to be a 1D array, and 1D FFT analogies are applicable (Fig. 4). In the case of the ellipsoid cell, if the interior brightness is essentially the same as that of the surrounding medium, but the cell wall is of a substantially different brightness, then the FFT will see at all angles two discrete impulses, separated by a distance that is the width of the ellipsoid along the angle of bisection.

If the area (surface or interior cross section) of the cell is either relatively dark or relatively bright as compared to the surrounding medium, then the FFT will see at all angles a very different situation, essentially a step function.
Impulses tend to distribute energy more or less evenly throughout the frequency plane. Step functions, on the other hand, carry more energy than a pair of discrete impulses and tend to concentrate energy about the origin of the frequency plane. Both impulses and step functions produce distributions of harmonics that radiate from the origin and have a shape that is a function of the diameters (wavelengths) of the bisection of the step (Fig. 4). In both cases, the harmonics account for the sharp transition from one brightness to another for both the step and impulse functions.

**LINE SEGMENTS AND LOW-DENSITY FIBER NETWORKS**

A line segment, when viewed along a direction perpendicular to its long axis, is interpreted by the FFT as an impulse (Fig. 5). The length of the straight line segment effectively multiplies the contribution of the signature of the impulse in the frequency plane. When viewed along an orientation parallel to the line segment, the FFT sees a step function. This produces a relatively large agglomeration of energy at the zero-frequency origin with a series of harmonics radiating away from the origin and decaying with distance from it. Thus, the line segment's length concentrates energy, while the thin width distributes it, resulting in a series of parallel lines (highly elliptical distributions) that are oriented in a direction that is perpendicular to the length of the fiber. The dominant energy distribution (or fundamental frequency) passes through the origin, with decreasingly energetic harmonics on either side. The distribution is centered at the origin, because the FFT interprets the lone line segment to be an image-wide and image-high pseudo-boundary (of largest wavelength), regardless of the position of the line segment in the source image. The pixels immediately adjacent to the zero-frequency origin carry the longest wavelength information present in the image, so the fundamental frequency is centered at the origin.

With decreasing length, the line segment looks increasingly like a point source (Fig. 5). The FFT signature of the line segment does not change appreciably in a direction perpendicular to the segment length. However, along the direction of the length, the origin-centered fundamental wavelength and its harmonics grow wider and the harmonics decrease in frequency. With decreasing length, the line segment shrinks to an impulse (a single image pixel) and the fundamental widens until it spans the entire FFT, just as in a 1D impulse analogy (Figs. 4, 5).

Straight line segments with identical orientation (Fig. 6) result in similar FFT spectra. Since the wavelength of the harmonics is a function of the line segment length and the harmonic distributions occupy the same regions in the frequency plane, it is easy to see why the harmonics of a collection of line segments interfere in complicated ways. Other than the interferences, which cause the FFT to appear rough and grainy, no other differences are obvious. As such, it is difficult to isolate individual line segments with similar orientations using the FFT, regardless of the relative positions of those line segments within the source image. This difficulty will apply to more complicated shapes of similar orientation, since the FFT signatures of the shapes will occupy similar positions within the FFT. For images of multiple line segments with varying orientations, the situation is quite different (Fig. 7). For images containing relatively few line segments, the cumulative contributions within the FFT of each of the line segments is obvious. Individual line segments may be partially resolved from each other with good angular exclusion using highly elliptical filters (Fig. 8).

Images of multiple line segments of varying orientation make a reasonably good model of images of actual low-density fiber networks (Fig. 9). If a highly elliptical filter, as described above, is rotated to a certain angle and applied to the FFT of a low-density fiber network, the mean intensity of the pixels enclosed by the filter may be calculated. If the filter is rotated through an angular range and repeatedly applied, then a plot of the mean value of the intensities of the pixels that fall within the filter at each orientation may be calculated. If the fiber network is not perfectly random or has a dominant fiber orientation associated with it, then the polar coordinate plot should reflect this with local extrema corresponding to variations in fiber orientation distribution. It may be advisable to exclude the lowest frequencies from the FFT with the addition of a high-pass filter (Fig. 9). This filter can improve the angular resolution of the polar plot by excluding pixels that would otherwise fall within the bounds of the elliptical filter at all orientations. Pourdeyhimi et al. (9) discussed rotated filter techniques but did not elaborate on the shape of filters used. In fact, the size and shape of the rotated filter makes a great deal of difference as to the resulting polar coordinate plot. The size and shape of the filter could be chosen to suit a particular application. For example, the filter dimensions could be chosen for optimal retention of a particular range of fiber widths with maximum exclusion of other wavelengths. In another example, the rotatable filter could be chosen to include only a portion of
a particular harmonic for a detection application involving simple elliptical shapes of varying orientation and constant size (Fig. 19). Brodeur et al. (7) have used high-pass filters in combination with rotated line filters to measure the fiber orientation distributions of binarized images of dyed fibers in non-dyed fiber matrices. These distributions were compared with stiffness orientation distributions measured ultrasonically. That study demonstrated that stiffness orientation is sensitive to strained/restrained drying, while fiber orientation is not (Fig. 10).

LOW-ANGLE INCIDENT LIGHT IMAGING

Low-angle incident light can cause misleading shape and shadow discontinuities in images of surfaces. Shape distortions occur, because the portions of the surface that are either closer to the light source or have surface orientations close to optimal for reflecting incident light into the lens of the imaging system generally appear brighter than other portions of the image. Therefore, light intensity is related to the surface topography, but only indirectly. Shadow discontinuities occur as the topographically higher portions of the surface cast shadows upon lower features, and even relatively smooth areas can cast stark shadow boundaries. Recall that sharp discontinuities of all kinds fill the frequency plane with energetic harmonic distributions, which invariably complicate FFT analyses. For example, the complexity of the FFTs of actual surface topographic representations of typical forming fabrics are generally less complicated than the FFTs of their reflected light representations.

These distortions and biases can often make accurate amplitude measurements difficult or impossible, and feature orientations can show angular favoritism due to the angle of lighting (Fig. 11). In addition to shadow boundaries, rough and fibrous surfaces often show evidence of glare, which tends to produce features that approximate sharp impulse and line sources that can inject distributed harmonic energy throughout the FFT. Also, large-scale specimen gradations like warp or curl can dramatically influence the interaction of reflected light with rough surfaces (Fig. 11). In the absence of functions to transform shadow patterns into topographical amplitudes, the energy distribution within the FFT can be overwhelmed by energy dominated by the lighting itself.

FABRIC AND PAPER/BOARD CHARACTERISTICS

The topography of forming fabrics can produce an intricate distribution of energetic and localized wave modes in frequency space. A low-angle reflected light image of the fabric may generate an FFT that is dense with localized and highly energetic wave modes (Fig. 16). Much of the energy is due to the presence of numerous harmonics that are representative of sharp apparent boundaries that are accentuated by the reflected light. Shadows and accentuated boundaries represent a distortion of the actual surface topography and hamper the usefulness of low-angle reflected light. An actual surface topography generator like a confocal or Moiré system does not suffer this limitation. However, even a perfect representation of the fabric surface would show a considerable number of real and highly energetic and condensed wave modes that contribute to the lattice structure of the forming fabric in the frequency plane of the FFT. The contributions of some of them are subtle and not immediately apparent from the original pre-FFT image.

A paperboard handsheet that was pressed (while wet) against the forming fabric described above has a distinctive frequency space signature (Fig. 16). Every pixel of the finite distribution of dominant energies evident in the FFT (all non-white pixels, as represented in the lower right frequency diagram in Fig. 16), has a substantially higher intensity than all of the other surrounding `background' pixels. This is quite dramatic and somewhat surprising. It suggests that each of those bright pixels is essential for a complete characterization of the dominant features in the source image. Although these are reflected light representations of the surface and are, therefore, severely distorted in terms of relative amplitude, one can be reasonably confident that at least some of the wave modes evident in the image of the paper would also be present and in roughly the same locations as the FFT of the fabric (allowing for dimensional changes of the paper during drying). When the FFTs of the fabric and paper are compared more closely, this appears to be the case. One of the predominant periodicities observed in the paper is a series of waves whose crests roughly define a series of lines projecting diagonally downward with a slope of about sixty degrees below the horizontal (Fig. 17). This seems to represent a twill pattern. The corresponding frequency space incarnation is evidently dominated by the rough line of pixels that passes through the origin with a slope of about thirty degrees above the horizontal.
Looking at the FFT of the fabric, a correspondence with the paperboard FFT is not immediately apparent. The FFT signatures appear quite different and the fabric is a good deal more complicated than that of the paperboard that was pressed against it. Clearly, there should be some relationship between the two, and it is possible to quickly decipher at least a part of that relationship in frequency space. A relatively modest contributor to the FFT of the fabric is similar in orientation angle and wavelength to the roughly linear component distribution in the FFT of the paper mentioned above. The two brightest cross shapes on either side of the horizontal axis encode most of the energy in the series of thick, vertically oriented threads (Fig. 16). The four small, less intense agglomerations just above and below the two cross shapes on either side of the vertical axis are modest contributors that show a correspondence. These are partly responsible for constructing the bright spots in the image of the wire of the horizontally oriented cross threads, which go alternately above and below the vertical threads in the weave and which shine brightly under the low-angle reflected light as they cross from below to above the vertical threads. Notice that those bright spots could be said to form a line with a slope of about sixty degrees below the horizontal or sixty degrees above the horizontal, depending on how those bright spots are viewed. The FFT accounts for both possibilities with those four agglomerations (and undoubtedly other energies not yet included in the subset). This correspondence suggests that at least some of the patterns prominent in the paper could have as much to do with drainage patterns as with expected wire indentations of the paper surface. The researcher could continue to decipher the relationship between wire and paper in this way.

TOOLS NEEDED TO MAKE THE FULLEST USE OF THE 2D FFT

Many software tools can generate the 2D FFT from digital gray-scale images, but none have seemed to include all of the supporting functionality necessary to make the FFT the efficient, precise, insightful, and, above all, easy-to-use analysis tool that it should be. A software program, the 2D FFT Explorer, was developed in order to address these needs (8). The features implemented in that program include the following:

1) All of the basics of supporting the FFT are included:
   - Managing the pixel calibrations needed for determining physical dimensions from pixel dimensions. Fast and easy zooming is available for close inspections.
   - Managing the padding (if needed) of source images to power-of-two and square dimensions. It is useful to keep the image dimensions and pixel aspect ratio spatially 'true' from the source image to the FFT to the inverse FFT so that distances and angles observed in one correspond precisely to distances and angles in the other two. Thus, it is possible to automatically mean value pad the source image to square and power-of-two dimensions and automatically square the subset window into the inverse FFT image.
   - Supporting the rapid selection of subset windows of source images for application to the FFT allows the quick comparison of the characteristics of different regions of the source images. Also included is the ability to directly enter the exact size and position of the visible subset window into the nonlinear (triangular) coordinate system of the two-quadrant-redundant FFT.
   - Support for rapid real-time control of the gray-scale magnitude representation of the FFT (a necessity when searching for subtle patterns and features).
   - Importing and exporting image files in popular graphic file formats and raw matrices of numbers stored in ASCII files (common in the lab).

2) Some researchers consider windowing to be essential for excluding edge discontinuities. Several window types (e.g., Hanning, Hamming, Blackman, Exact Blackman, and Blackman-Harris) are available for easy application to the source image. However, windowing should be considered a necessary evil, because it has drawbacks of its own. Windowing reduces the area of the source image that effectively contributes to the calculation of the FFT, attenuating the edges of the image to zero amplitude with decreasing attenuation as the center of the image is approached (Fig. 15). Windowing consequently injects distortions of its own into the FFT, making interpretation of energies in the FFT and amplitudes in the real space of the inverse FFT more ambiguous. As an alternative to windowing, it is often possible to collect the energies in the FFT responsible for the edge discontinuities. In some cases, this is the best option, since this can avoid both windowing distortions and distortions arising from the exclusion of the energies responsible for the edge discontinuities. Even when edge
discontinuity energies are excluded, the centers of corresponding inverse FFT images generally show a minimum of distortion. For detection applications in which the measurement of amplitudes/intensities is not important, windowing can be an especially useful tool (Fig. 13).

3) Because a linear representation of the FFT is a frequency plane, which operators invariably wish to transform into wavelengths in order to interpret in terms of spatial dimensions, associated with the FFT display should be a point tool for rapid, live display of positions in the frequency plane as both frequencies and as wavelengths. The display of these quantities should include the horizontal and vertical complex components of their distance from the zero frequency (infinite wavelength) origin and the vector magnitude and angle of orientation associated with the origin-referenced vector which points to that location. An FFT feature at a certain angle of orientation is directly associated with the angle of its real space incarnation. Knowing the expected angle of the distribution of features in real space, which may not be immediately apparent from the source image, is often a first step in associating their real space and frequency space equivalents.

4) A point tool is often sufficient for browsing the FFT and for situations in which maximum precision is not a necessity. However, even an isolated feature present in the FFT will generally not occupy a single pixel, but will be an agglomeration of pixels that are close in frequency (Fig. 15). Only if a feature were singular in frequency/wavelength and precisely coincident with the finite resolution of the FFT could it possibly occupy only a single pixel in the frequency plane. So, it should be possible to isolate a contiguous region of the frequency plane, which encloses all pixels constituting a feature of interest, and then to calculate a centroid position corresponding only to those enclosed pixels. From this centroid position, X, Y and magnitude frequency plane location components and an origin-referenced angle of orientation calculation may be made, as well as the average intensity of the enclosed pixels. Several useful shapes (point, ellipse, rectangle and line) of arbitrary size, eccentricity, and angle of orientation should be available for rapidly selecting regions throughout the frequency plane. Additionally, a calculation of the primary axis of orientation of the pixels enclosed should be available in order to determine the orientation of the feature itself (Fig. 20).

5) More generally, features of interest in the source image may be constituted by a number of noncontiguous energies that may be distributed throughout the frequency plane. It should be easy to design potentially detailed filters that may include a number of separate energy clusters. The filters may be tailored to suit the study of the individual characteristics of any of a number of different types of specimens by excluding noise and other complicating energies. For example, several dominant wave modes will generally be apparent in the FFTs of images of visible light transmitted through lower basis weight papers, which have evidence of wire mark (Fig. 15). Although their individual contributions may not always be apparent to the eye, together these distinct wave modes constitute the lattice structure inherent in mass distribution patterns induced during formation. Each constitutes some distinct ‘aspect’ of the patterns present.

6) An FFT filter should be constructable by intensity/amplitude thresholding alone, or in combination with region selection, as described above. For example, when a researcher wishes to isolate/enhance anomalies within an otherwise periodic structure, then the thresholding tool can be particularly helpful. In specimens that show evidence of periodicities, the largest amplitude contributors to the FFT are generally associated with those periodicities and are generally distributed in finite portions of the frequency plane. As such, it is often possible to remove those largest contributors through thresholding. Anomalies/deviations from the periodicities are generally distributed through different regions of the frequency plane than those occupied by the periodicities. As such, they are often resolvable from the threshold-excluded periodicities. Figs. 12 and 13 illustrate this resolvability. The artificial anomaly could as easily have been a chink or a cockle in a fluted surface or a missing thread in a forming fabric. Both region selection methods (shape and threshold) should be repeatedly applicable alone or in combination for the study and accentuation of arbitrarily complex FFT distributions that may be either filtered out or retained.

7) Once a filter has been constructed, it should be easy to save the filter to a file and thereafter recall the filter for further refinement or for application to new data. This is helpful for speeding the analysis of the numerous specimens involved in comparative studies and for maintaining consistency between them.
8) Of the four quadrants of the window into the frequency plane of the FFT, two of them are redundant with respect to their diagonal opposites, and it would be reasonable to exclude them. However, some researchers have an intuitive preference for the redundancy of the four-quadrant representation. In support of constructing filters within this redundant representation, a facility for automatically reflecting changes made within a given quadrant into the diagonal opposite of that quadrant is a helpful addition.

9) The researcher should have the choice of whether or not to maintain the two-quadrant redundancy of the FFTs of real images, either automatically or by manually repeating changes in one quadrant into identical changes in the opposite quadrant. Breaking the symmetry of the FFT can be useful in order to determine the centroid position of complex distributions of energy in the FFT as a whole, before or after the FFT has been filtered. Some FFT energy distributions cannot be enclosed (with good exclusion) by a single shape as described above, so, in many cases, it is better to filter out portions of the FFT as needed and then to apply a global centroid calculation to the filtered FFT as a whole rather than the collection of pixels enclosed by a single shape. From the centroid position in the frequency plane, an angle of orientation relative to the origin may be determined along with a mean intensity of the entire filtered FFT. If the symmetry of the FFT is not broken, then the calculated centroid position will always be equal to zero.

10) The 2D FFT may be represented with the lowest frequencies at the center of the window into the frequency plane with the highest frequencies near the edges of the window. Alternatively, the FFT may be represented with the high frequencies at the center and the low frequencies near the edges. In most analyses involving paper and paper-related specimens, the low-frequency centered representation is much more useful, because the features evident in most papers tend to be relatively low in frequency. In this representation, the zero-frequency (infinite wavelength) image mean pixel is at the center of the FFT. When constructing zero-frequency centered filters, it can be difficult to position a cursor over a specific pixel in large (high-resolution) images. In support of quickly finding the frequency plane origin for the application of the filtration shapes mentioned above, it should be easy to make the origin "sticky", holding the cursor at the origin, if a mouse pointer is moved relatively close to it.

11) Fig. 18 illustrates the use of band-pass filtration for size discrimination of simple filled circular objects. It is easy to turn circular objects with a particular diameter “on and off” by capturing the fundamental wave mode using an appropriate band-pass filter. However, if the circular objects are replaced with ellipsoids, which have a directionality associated with them, then their signatures in frequency space will also have a corresponding directionality associated with them. The researcher could design a wide circular filter in an attempt to capture the fundamental frequency (at every orientation) of the frequency space signature of the shape. However, this filter would have a poor exclusion of competing energies, especially those of shapes of similar size and orientation. A better solution may be to design a filter with optimal exclusion for a single arbitrary shape orientation. If this filter can then be rotated through an angular range and repeatedly applied, then the angular distribution of the mean energies (or other arbitrary measure) of the resulting series of filtered images may be used to either make angular comparisons for angular distribution studies or to accumulate all angles to accommodate all possible shape rotations.

12) There are a number of situations in which the automated application and iteration of the tools described above in various combinations can further facilitate complex analyses of specimens and save more of the researcher’s time. This is also true of situations in which there are many specimens to analyze (i.e., comparative evaluations), there are many area subsets within a given image to analyze, or there are a number of different FFT features that need to be individually examined. Automatable functions should include the positions of cursors, filter construction and rotation, exportation and importation of component images, etc. A complete system should have an expanding list of available high-level functions, like the rotated filter analyses described above, and other custom analyses.

13) Attempting to mentally translate between energy distributions seen in frequency space and their corresponding spatial incarnations for images of only modest complexity is difficult. And many researchers prefer to measure heights, reflectivities, orientations, etc. from amplitudes and planar dimensions in real space rather than from the energy distributions observed in frequency space. So, a quick, integrated method to facilitate the isolation
of features in the FFT and then to translate the filtered FFT back into real space via the inverse FFT is needed. In fact, the ability to observe the real space results of progressive modifications to the FFT makes working in frequency space more intuitive and the FFT a better teaching tool, helping one (mainly through simple trial and error) to learn to mentally project from real space into frequency space and vice versa.

14) Associated with the filtered image should be point and line inspection tools to facilitate the display and exportation of line scans from the filtered image at arbitrary line orientations (Fig. 21).

CONCLUSIONS

2D FFT analysis is a powerful method for comparing physical characteristics of paper and board measured across the MD-CD plane. Methods of imaging and illumination have a significant influence on the signature of textural features presented for 2D analysis. Not only patterns but also individual objects and other nonperiodic features may be manipulated and measured using the FFT. Machine fabrics may display very complex FFT signatures, especially when imaged under incident lighting. Patterns in paper/board may have contributions from multiple fabrics (top and bottom), and the relationship between the fabric and the paper/board may be somewhat indirect, making comparisons between their respective FFT spectra challenging. The addition of several software tools for enhancing the usability of the FFT can open new avenues to the researcher in the analysis and decomposition of complex patterns and other features in the images of paper, board, and associated materials.

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REFERENCES


8 A demonstration version of 2D FFT Explorer is available now for download. For more information, contact the author at ted.jackson@ipst.edu.

Fig. 1 - Common Measurement Methods

Mass Distribution (formation, wire mark, etc.)
- White light (thin papers/tape pulls/etc.)
- Beta rays (thick paper/board/etc.)

Surface fiber and topology orientation and distribution
- Confocal microscopy
- Moiré interferometry
- Scanning triangulating laser sensors
- Reflected visible light
- Incident ultraviolet light & fluorescent dye
Fig. 2 - The FFT can be considered to be a function of the bisection of simple objects along any angle of orientation.
Fig. 3 - Depending on the contrast mechanism, an object may appear as an area and/or a perimeter with a corresponding FFT signature.
Fig. 4 - 1D/2D FFT Analogies and the Effects of Shape, Size and Contrast
Fig. 5 - Thin beam segment study: Decreasing beam segment length

1D impulse analogy perpendicular to fiber length

1D step function analogy parallel to fiber length
Fig. 6 - Thin beam segment study: Multiple straight beams with constant orientation

random location

random location & length
Fig. 7 - Thin beam segment study: Increasing densities of beam segments with random length, orientation and location.
Fig. 8 - Thin beam segment study: Using a highly elliptical filter to obtain high angular exclusion
Fig. 9 - Fiber orientation distribution plots may be generated from low-density fiber networks.

High pass filtration included for improved angular resolution and exclusion of long wavelength gradations.
Fig. 10 - Brodeur, Gerhardstein, et al. have made comparative evaluations between:
* Stiffness orientation (in-plane ultrasonics)
* Fiber Orientation (successive oriented line filtration of the FFTs of binarized dyed fiber images)

Fiber orientation is not sensitive to built-in stresses (strained/restrained drying)

Stiffness orientation IS sensitive
Fig. 11 - Incident light imaging fills the FFT with distributed energy, making surface evaluation difficult, even in cases with features that are apparent to the eye.
Fig. 12 - Threshold blocking can be effective for detecting anomalies within predominantly periodic features.
Fig. 13 - An anomaly detection application where windowing is helpful.
Fig. 14 - The detected line segment encodes the discontinuity from the periodic background. The circular section is the detected anomaly.
Fig. 15 - Several dominant wave modes in the FFT encode the pattern of wire mark. These may be removed or retained, depending on the application.
Fig. 16 - A forming fabric and a corresponding sheet pressed in the laboratory and imaged with low-angle incident light.
Fig. 17 - Mode decomposition of a paper surface. One of the modes corresponds with the weave pattern of the forming fabric.
Fig. 18 - Size discrimination of simple circular objects using ring (2D bandpass) filters
Fig. 19 - For better angular exclusion and/or precision and to avoid the generally 'busy' low-frequency origin, capturing all or part of one or more high-frequency harmonics is an option for detection applications.
Fig. 20 - FFT point (P-) and centroid (C-) statistics:
location (x,y,m), amplitude (Z) and orientation (ϕ, θ)

\[ P\lambda = \frac{1}{Pf} \]
\[ CZ = \frac{\sum P_i}{n}, \quad C\lambda = \frac{1}{Cf} \]
(n enclosed pixels)