Total Variational Based Optical Flow for Cardiac Wall Motion Tracking

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ABSTRACT

In this note, we apply an L^1 based approach to optical flow to measure heart wall motions. Our method captures discontinuities and suddens changes in the flow field much better than conventional quadratic gradient approaches.

Keywords: Optical flow, tracking, cardiac wall motion, gradient direction, multiscale image processing

1. INTRODUCTION

In this note, we apply the L^1-based optical flow methods from[5] to the problem of tracking wall motion from cardiac imagery. The computation of optical flow has proved to be an important tool for problems arising in active vision. The optical flow field is the velocity vector field of apparent motion of brightness patterns in a sequence of images.[4] One assumes that the motion of the brightness patterns is the result of relative motion, large enough to register a change in the spatial distribution of intensities on the images. Thus, relative motion between an object and a camera can give rise to optical flow. Similarly, relative motion among objects in a scene being imaged by a static camera can give rise to optical flow.

In our computation of the optical flow we use work on generalized *viscosity* solutions to Hamilton-Jacobi type equations. Indeed, these techniques seem ideally suited for the variational Euler-Lagrange approaches to this problem (see also[2,4,9] and the references therein). Utilizing such generalized solutions, we have been able to handle the singularities and regularity problems for several distinct variational formulations of the optical flow that occur in this area.

2. L^1 BASED OPTICAL FLOW

In our work,[5] we consider a spatiotemporal differentiation method for optical flow. Even though in such an approach, the optical flow typically estimates only the isobrightness contours, it has been observed that if the motion gives rise to sufficiently large intensity gradients in the images, then the optical flow field can be used as an approximation to the real velocity field and the computed optical flow can be used reliably in the solutions of a large number of problems; see[3] and the references therein. Thus, optical flow computations have been used quite successfully in problems of three-dimensional object reconstruction, and in three-dimensional scene analysis for computing information such as depth and surface orientation. In object tracking and robot navigation, optical flow has been used to track targets of interest. Discontinuities in optical flow have proved an important tool in approaching the problem of image segmentation.

The problem of computing optical flow is ill-posed in the sense of Hadamard. Well-posedness has to be imposed by assuming suitable *a priori* knowledge. In,[5] we employ a variational formulation for imposing such *a priori* knowledge.

One constraint which has often been used in the literature is the "optical flow constraint"(OFC). The OFC is a result of the simplifying assumption of constancy of the intensity, $E = E(x, y, t)$, at any point in the image.[4] It can be expressed as the following linear equation in the unknown variables u and v

$$E_x u + E_y v + E_t = 0, \tag{1}$$

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where E_x, E_y and E_t are the intensity gradients in the x, y, and the temporal directions respectively, and u and v are the x and y velocity components of the apparent motion of brightness patterns in the images, respectively. It has been shown that the OFC holds provided the scene has Lambertian surfaces and is illuminated by either a uniform or an isotropic light source, the 3-D motion is translational, the optical system is calibrated and the patterns in the scene are locally rigid.

It is not difficult to see from equation (1) that computation of optical flow is unique only up to computation of the flow along the intensity gradient $\nabla E = (E_x, E_y)^T$ at a point in the image.[4] (The superscript T denotes "transpose.") This is the celebrated *aperture problem*. One way of treating the aperture problem is through the use of regularization in computation of optical flow, and consequently the choice of an appropriate constraint. A natural choice for such a constraint is the imposition of some measure of consistency on the flow vectors situated close to one another on the image.

In their pioneering work, Horn and Schunk[4] use a quadratic smoothness constraint. The immediate difficulty with this method is that at the object boundaries, where it is natural to expect discontinuities in the flow, such a smoothness constraint will have difficulty capturing the optical flow. For instance, in the case of a quadratic constraint in the form of the square of the norm of the gradient of the optical flow field,[4] the Euler-Lagrange (partial) differential equations for the velocity components turn out to be *linear* elliptic. The corresponding parabolic equations therefore have a linear diffusive nature, and tend to blur the edges of a given image. In the past, work has been done to try to suppress such a constraint in directions orthogonal to the occluding boundaries in an effort to capture discontinuities in image intensities that arise on the edges; see[6] and the references therein.

We have proposed in[5] a novel method for computing optical flow based on the theory of the evolution of curves and surfaces. The approach employs an L^1 type minimization of the norm of the gradient of the optical flow vector rather than quadratic minimization as has been undertaken in most previous regularization approaches. The equations that arise are nonlinear degenerate parabolic equations. The equations diffuse in a direction orthogonal to the intensity gradients, i.e., in a direction along the edges. This results in the edges being preserved. The equations can be solved by following a methodology very similar to the evolution of curves based on the level set ideas of Osher and Sethian.[7] Proper numerical implementation of the equations leads to solutions which incorporate the nature of the discontinuities in image intensities into the optical flow.

We can summarize our procedure as follows:

1. Let $E = E(x, y, t)$ be the intensity of the given moving image. Assume constancy of intensity at any point in the image, i.e.,
$$E_x u + E_y v + E_t = 0,$$
where
$$u = \frac{dx}{dt}, \quad v = \frac{dy}{dt},$$
are the components of the apparent motion of brightness patterns in the image which we want to estimate.

2. Consider the regularization of optical flow using the L^1 cost functional
$$\min_{(u,v)} \int\int \sqrt{u_x^2 + u_y^2} + \sqrt{v_x^2 + v_y^2} + \alpha^2 (E_x u + E_y v + E_t)^2 dxdy,$$
where α is the smoothness parameter.

3. The corresponding Euler-Lagrange equations may be computed to be
$$\begin{aligned} \kappa_u - \alpha^2 E_x (E_x u + E_y v + E_t) &= 0, \\ \kappa_v - \alpha^2 E_x (E_x u + E_y v + E_t) &= 0, \end{aligned}$$
where the curvature
$$\kappa_u := \mathrm{div}(\frac{\nabla u}{\|\nabla u\|}),$$
and similarly for κ_v.

4. These equations are solved via "gradient descent" by introducing the system of nonlinear parabolic equations

$$\begin{aligned} \hat{u}_{t'} &= \kappa_{\hat{u}} - \alpha^2 E_x(E_x\hat{u} + E_y\hat{v} + E_t), \\ \hat{v}_{t'} &= \kappa_{\hat{v}} - \alpha^2 E_x(E_x\hat{u} + E_y\hat{v} + E_t), \end{aligned}$$

for $\hat{u} = \hat{u}(x, y, t')$, and similarly for $\hat{v}$.

The above equations have a significant advantage over the classical Horn-Schunck quadratic optimization method since they *do not blur edges.* Indeed, the diffusion equation

$$\begin{aligned} \Phi_t &= \Delta\Phi - \frac{1}{\|\nabla\Phi\|^2}\langle\nabla^2\Phi(\nabla\Phi), \nabla\Phi\rangle, \\ &= \kappa_\Phi\|\nabla\Phi\| \end{aligned}$$

does not diffuse in the direction of the gradient $\nabla\Phi$. Our optical flow equations are perturbations of the following type of equation:

$$\Phi_t = \frac{\kappa_\Phi}{\|\nabla\Phi\|}\|\nabla\Phi\|.$$

Since $\|\nabla\Phi\|$ is maximal at an edge, our optical flow equations do indeed preserve the edges. Thus the L^1–norm optimization procedure allows us to retain edges in the computation of the optical flow.

This approach to the estimation of motion will be one of the tools which we will employ in our tracking algorithms. The algorithm has already proven to be very reliable for various type of imagery.[5] We are also interested in applying a similar technique to the problem of stereo disparity. We also want to apply some of the *multigrid* techniques of Vogel[10] in order to significantly enhance the performance of both our optical flow and stereo disparity algorithms.

3. EXPERIMENTAL RESULTS

We applied the L^1 based optical flow to several cardiac cines provided to us by the University of Minnesota Medical School, Department of radiology. The cines each consisted of 20 frames of 256 by 256 16 bit gray scale data which was subsequently scaled to have values between 0 and 255. The algorithm was run on a standard Sparc Ultra 10 machine.

In Figure 1, we show four frames from a given cardiac cycle. These preliminary results show that the algorithm works reasonably well for such data. Of course, before any conclusions can be made the algorithm will have to be much more extensively tested.

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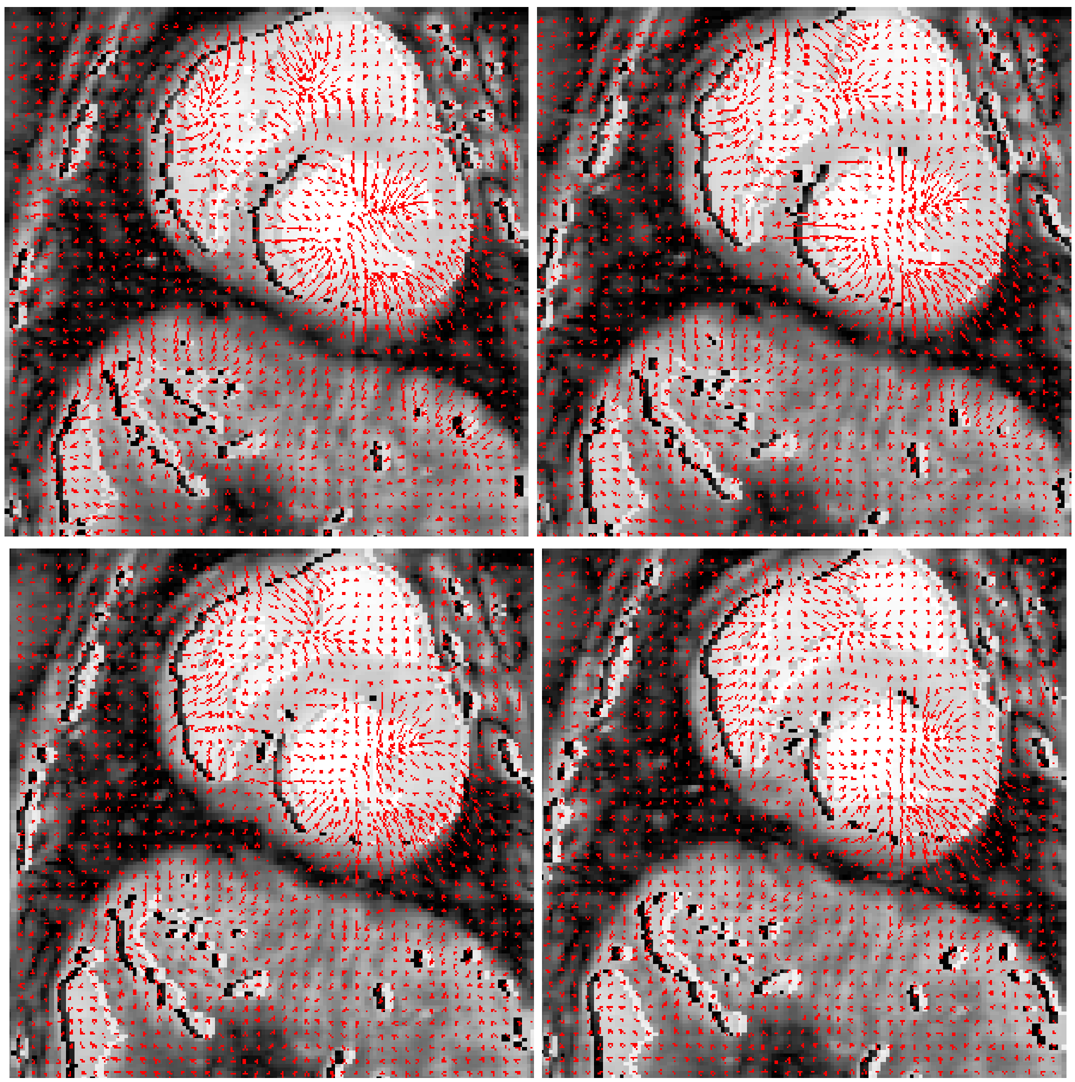

Figure 1. Four Frames of MRI Sequence