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FINAL REPORT

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NATIONAL SCIENCE FOUNDATION

for

GRANT GK 191.

STUDIES OF THE DEFORMATION OF METALS
AND ALLOYS AT ELEVATED TEMPERATURES

by

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SUMMARY

A torsion testing machine is described for producing shear stress-shear strain data with the absence of axial stresses over a wide range of constant true strain rate and temperature. It is proposed that such data would be of use in metal working studies.

An analysis of the development of anisotropy during the torsional straining of hollow cylinders is given and expressions derived relating the anisotropic parameters and the associated yield stresses along the anisotropic axes (in terms of the through thickness yield stress) to the geometry of deformation as determined by the change in axial strain with shear strain.

The results from an extensive series of tests on the free torsional straining of hollow cylinders of aluminum and two commercial aluminum alloys (2017 and 6061) over a wide range of temperature and strain rate are presented. It is shown that all materials undergo a change in length when deformed: the direction and magnitude depending on the strain rate and temperature. No expression could be found relating shear stress to shear strain except in the case of aluminum at ambient temperature when a power law can be used to interpret the data. Some of the shear stress-shear strain curves contain stress maxima at elevated temperatures. The shape of these curves is discussed in terms of the relative effects of work hardening and recrystallization.

It is shown that the effect of strain rate on the flow stress of the materials at constant strain and temperature can be described by a power law: the strain rate sensitivity increasing with increasing temperature and strain. No relationship could be found to describe adequately the effect of temperature on the flow stress of the materials at constant
strain and strain rate over the whole range of variables considered. However, it is shown that in the case of aluminum at temperatures above 200°C the combined effect of strain rate and temperature on the flow stress is described accurately by the empirical expression.

\[ \tau |_{\gamma} = A_1 (\dot{\gamma})^S_1 \ln \frac{1}{T} + A_2 (\dot{\gamma})^S_2 \]

at constant strain where \( A_1, A_2, S_1 \) and \( S_2 \) are constants.
I. TITLE. Studies of the Deformation of Metals and Alloys at Elevated Temperatures.

II. SCOPE.

The present report is a description of work carried out under National Science Foundation Grant GK 191.

III. INTRODUCTION.

The work under National Science Foundation Grant GK 191 was carried out in the School of Mechanical Engineering at the Georgia Institute of Technology. The grant was for the sum of $21,264.00 and provided partial support for the principal investigator and one graduate student for a period of two years. The work has now been completed and the graduate student awarded a Ph.D.

The work will be presented in summary, but will include all pertinent information. Further details, including the review of literature on which the original proposal was based can be found in a recent dissertation given in Appendix I.

It is anticipated that much of the work will be published. A list of proposed publications is given in Appendix I. Copies of publications will be forwarded to the National Science Foundation when available.

IV. FUNDAMENTAL CONSIDERATIONS.

The rapid development of the defense and space programs has led to demands for materials of superior mechanical and physical properties to those currently in use. Considerable progress has been made recently in the development of these new materials but many difficulties have been encountered in their subsequent processing. These difficulties usually arise because of a lack of knowledge concerning the properties of materials
under the conditions encountered in the processing operation and clearly every effort must be made to overcome them.

For many metal working processes theoretical analyses have been made in order to derive formulae relating the variables of the process. These formulae usually contain terms involving both the geometry of the operation and the yield or flow stress of the material and enable loads, torques and power requirements to be calculated before new equipment is designed or new material processed. If the analyses are to give meaningful results then the appropriate value of the flow stress must be used, which is, of course, determined by the conditions of strain, strain rate and temperature employed in the operation.

There is a large amount of information available concerning the plastic properties of metals deformed at high temperatures and low strain rates (creep). However, in those ranges of temperature and strain rate employed in metal working processes, available information is limited and until quite recently such data have almost been nonexistent. There is an urgent need for more work in this area.

In cold working processes the determination of the appropriate flow stress is reasonably straightforward because changes in strain rate have little influence on the stress-strain characteristics of the materials. In hot working processes the situation is more complicated because at elevated temperatures the plastic properties of a metal depend very much on the rate of deformation and the effects of work hardening, recovery, polygonization and recrystallization must be considered. The experimental determination of the flow stress is difficult. In view of the vast tonnages of metal fabricated any advance in the understanding of the effects of strain rate and temperature on the plastic properties of metals and of the fundamental mechanisms involved in the deformation process would be of significant value.
Many methods have been used for determining the plastic properties of materials including tension, plane strain and axisymmetric compression and torsion using solid and hollow bars. However, most of these methods have certain limitations which restrict their usefulness.

In tension uniform strains are limited because of necking and non-homogeneous deformation. The difficulties associated with necking are accentuated at high testing speeds because adiabatic heating becomes localized in the neck region leading to softening and decreased flow. This latter effect is important since it limits the usefulness of the data still further, particularly with respect to the effect of strain rate on strain hardening and ductile fracture.

In axisymmetric compression the total strain that can be applied is greater than in tension but it is still rather limited because the increase in cross sectional area of the specimen with reduction results in the need of a greater load to cause further deformation. This adds to the mechanical difficulties of the test. The effect of friction between the dies and workpiece, which can cause barreling, further increases the load required for deformation.

In plane strain compression testing the cross sectional area of the specimen beneath the dies remains constant with reduction and allows very high strains to be applied. However, the effects of friction between the dies and workpiece present serious problems and very laborious, subsidiary experiments and corrections must be carried out in order to obtain values of stress relating only to the deformation properties of the material.

Of significance in tension and compression testing is the manner by which deformation is produced. In general, tests may be carried out either at a constant velocity of deformation or at a constant strain rate. Testing
in compression at constant velocity results in an increase in the instantaneous strain rate during deformation while testing in tension results in a decrease prior to necking then an increase. For a clearer understanding of the effects of strain rate on the flow stress of materials at elevated temperatures, data should be presented at constant strain rate: otherwise incorrect interpretation may result.

The testing of materials in tension and compression at a constant velocity of deformation is a fairly simple matter, since commercial equipment is readily available. However, testing at a constant strain rate is considerably more complex and usually special equipment must be constructed involving serious mechanical difficulties.

Torsion as a method of testing has proved useful for obtaining data for the analysis or laying down of hot working schedules. In torsion data can be produced at high constant strain rates, high temperatures and strains comparable with those encountered in metal working operations and is free of the problems associated with friction found in compression tests. A further advantage of the torsion test is that a measure of the ductility of the materials may be obtained from a knowledge of the number of revolutions to failure.

A major difficulty with torsion tests on solid bars arises from the variation of strain and strain rate across the section of the bar which can limit quantitative interpretation of the data. This variation may be reduced to an acceptably narrow range if hollow test pieces are chosen with carefully controlled dimensions. A further difficulty with torsion testing arises from the fact that specimens tend to elongate or contract axially during twisting; the magnitude and direction of the change in length, depending largely on the work hardening characteristics of the material. The change in length can lead to the development of axial
stresses in fixed torsion which may effect the shear stress of the material and lead to difficulties in interpreting the data. These stresses can be relieved if the specimen is allowed to undergo its natural change in length.

The tendency for specimens to change in length during torsional deformation is apparently due to the development of anisotropy. Metals are generally isotropic when their structure consists of randomly oriented grains of small size. However, when they are deformed grain movement takes place with the subsequent development of preferred orientation and anisotropy. The most obvious manifestation of anisotropy appears in the form of "ears" produced in the deep drawing of some cold rolled metal sheets.

Theoretical work in the past has shown that equations can be derived expressing the current state of anisotropy of a material during deformation in terms of six "so called" parameters of anisotropy. It is possible to calculate the values of these parameters from a knowledge of the tensile yield stress along the principal directions of anisotropy. In the case of rolling the yield stresses can be determined easily as the axes of anisotropy remain fixed during deformation. Such determination is not possible in the case of torsion of a hollow cylinder as the axes of anisotropy rotate during deformation.

Determination of the parameters of anisotropy is important as their variation indicates quantitatively the development of anisotropy and can lead to a better understanding of the deformation process. This is particularly important in the case of the torsion test if data is to be used in metal working studies.

Of the various testing procedures discussed above it was felt that torsion using hollow test specimens of suitable dimensions where the specimen is free to extend or contract axially was most suitable for producing data for use in metal working studies. Accordingly a research program based on this test was developed and executed.
The research program was composed of several phases including, a review of pertinent literature, design, construction and evaluation of a high speed torsion testing machine, experimental and analytical study of anisotropy in torsion and the determination of the effects of strain rate and temperature on the properties of commercially pure aluminum (1100) and two aluminum alloys (2027 and 6061). This work will now be described.

V. EXPERIMENTAL EQUIPMENT.

The first phase of the investigation was the design, construction and evaluation of a torsion testing machine for use in metal working studies. A total of twelve feasible drive units were designed and analysed with particular reference to their behavior under dynamic conditions. The unit finally selected is shown schematically in Figure 1. It is composed of two main sections, the drive assembly consisting of the power source and speed reduction units and the test assembly consisting of the clutch, specimen, furnace and torque sensor.

The power source is an electric motor capable of delivering 100 in-lbs. of torque at 1800 RPM and 450 RPM. The output shaft of the motor is connected through an intermediate system of gears to either of two high efficiency planetary reducers with reductions of 50:1 and 1238:1, respectively. The intermediate system of gears can be adjusted to provide reduction ratios of 1:1, 2:1, 3:1, and 5:1 by replacement of gear pairs. The planetary reducers may be operated separately or in series, thus providing a speed range of 0.0015 RPM to 35 RPM. For speeds greater than 35 RPM the power requirements exceed the rated horsepower of the motor and a flywheel must be used. In this case the motor is connected directly to the flywheel, which is mounted on the clutch input shaft, by means of a timing belt drive. A total of 31 speeds can be obtained over six orders of magnitude.
Figure 1  Schematic Diagram of Test Apparatus
The test assembly of the machine is illustrated in Figure 2. Torque is transmitted from the drive section to the forward end of the test specimen through a clutch. The specimen is held on the reaction side by a unique linear bearing designed to allow axial freedom while providing torsional constraint. The linear bearing is connected to a strain gage reaction torque sensor which is fastened rigidly to the main frame of the machine. The specimen is enclosed in a split (to allow easy access) resistance furnace with controller capable of maintaining temperatures up to 2400°F ± 5°. The shafts on either side of the specimen are water cooled to protect the torque sensor and clutch from thermal damage.

To carry out tests at constant strain rate it is essential that the engagement time of the clutch be as small as possible. After considering a wide variety of clutches it was decided to use an air actuated disc clutch. The particular model selected (Force Control Model SC) can reach full torque capacity of 4600 in-lbs. in 20 milliseconds when actuated by an air pressure of 80 psi. At lower torques the reaction times are considerably less.

The machine is completely instrumented. Torque is measured by applying the output from the torque sensor to one beam of a multibeam oscilloscope or pen recorder depending on the testing speed. Angular displacements are measured by gearing the clutch output shaft to an energized helical potentiometer and applying the signal to the oscilloscope or pen recorder. Linear displacements are measured by monitoring the output from a small differential transformer connected to the linear bearing.

The operation of the torsion testing machine was found to be entirely satisfactory. The machine was free from any dynamic vibration effects
Figure 2  Test Assembly
over the whole of the speed range and gave a constant rate of twisting. The engagement time of the clutch was about 3 milliseconds (neglecting the actuation time of the solenoid valve) for all tests carried out.

VI. EFFECT OF STRAIN RATE AND TEMPERATURE ON THE MECHANICAL PROPERTIES OF SELECTED MATERIALS.

1. Experimental Procedure

The high speed torsion testing machine described in the previous section was used to determine the effects of strain rate and temperature on the properties of selected materials over the normal range of these variables encountered in metal working processes. In all tests the specimens were allowed to undergo a natural change in length and the data corrected after testing.

Investigations were carried out on commercially pure aluminum (1100) and two aluminum alloys (2017 and 6061). The analyses are given in Table I. All materials were received in rod form from which hollow test specimens with reduced gage section were machined and heat treated to the O-temper. The heat treatment procedures are given in Table II. The dimensions of the specimens are given in Figure 3.

2. Results and Discussion

Shear stress-shear strain curves were obtained for all the materials over a strain rate range 0.015 to 150 per second and at a variety of test temperatures. The results for commercial pure aluminum showed that the stress required to produce a given deformation increased with increasing strain rate and decreasing temperature and that the effect of strain rate became more pronounced at high temperatures. At temperatures above 300°C some of the curves showed maximum values of stress at shear strains of \(\sim 1.1\), further deformation occurring at a lower steady stress. The strain rate below which such maxima occurred increased with increasing temperature.
Figure 3 Dimensions of Test Specimen (not to scale)
The curves for the aluminum alloys (2017 and 6061) showed pronounced peaks under all experimental conditions employed in the present work. The peaks for all materials generally broadened with increasing strain rate and decreasing temperature and are attributed to the momentary predominence of thermal softening over work hardening. At higher strains a dynamic balance is formed. Further work is required to investigate the cause of such peaks.

Attempts to find empirical formulae relating flow stress to strain for the various materials were generally unsuccessful except for aluminum at room temperature. In this instance the effect of strain ($\gamma$) on the flow stress ($\tau$) could be expressed by a power law of the form:

$$\tau \mid_{\gamma T} = C(\gamma)^n$$  \hspace{1cm} (1)

where $\tau$ is the shear flow stress at constant strain rate and temperature, $C$ is a constant, $\gamma$ the shear strain and $n$ the work hardening exponent. The value of $n$ for aluminum was found to be 0.15. No relationship could be found to express correctly the shape of any stress strain curves that contained stress maxima.

For all the materials tested the effect of strain rate ($\dot{\gamma}$) on the flow stress ($\tau$) at a given strain ($\gamma$) and temperature ($T$) could be expressed by the relationship:

$$\tau \mid_{\gamma T} = \tau_o (\dot{\gamma})^m$$  \hspace{1cm} (2)

where $m$ and $\tau_o$ are constants. The exponent $m$ is known as the strain rate sensitivity. Values of $m$ for a variety of shear strains for the three materials are given in Tables III, IV and V. In the case of the commercial pure aluminum $\tau_o$ decreased uniformly with increasing temperature and
generally increased with strain; \( m \) generally increased with strain and

temperature. For the aluminum alloys neither \( \tau_o \) nor \( m \) varied systematically

with strain but \( \tau_o \) decreased and \( m \) increased with increasing temperature. An interesting feature in the behavior of the aluminum alloys is that two

regions of strain rate sensitivity were observed, the material being more

sensitive to strain rate at the lower strain rates. The values of \( m \) given

in Tables IV and V correspond to the region of higher strain rate sensitivity. The strain rate dividing the sensitive from the insensitive regions was about 15.0 per second.

The stress required to produce a given strain (\( \gamma \)) at a given strain rate (\( \dot{\gamma} \)) generally varied with temperature (\( T \)) in a complex manner and could not be expressed by the relationship:

\[
\tau |_{\dot{\gamma}, \gamma} = C \, e^{\frac{Q}{RT}}
\]  

(3)

where \( R \) is the gas constant, \( C \) is a constant and \( Q \) is the activation energy for plastic flow, or

\[
\tau |_{\gamma} = f(Z)
\]  

(4)

where (\( Z \)) is the so called Zener-Hollomon parameter and is given by

\[
Z = \dot{\gamma} \exp \frac{\Delta H}{RT}
\]  

(5)

where \( \Delta H \) is the activation energy for plastic flow and the remaining

symbols have their usual significance. However, it was found that in the case of aluminum at strains greater than 0.5, the effect of temperature (\( T \)) on the shear flow stress (\( \tau \)) could be expressed by the relationship:

\[
\tau |_{\gamma} = C_1 \ln \frac{1}{T} + C_2
\]  

(6)
where $C_1$ and $C_2$ are constants. Further investigation showed that $C_1$ and $C_2$ were functions of strain rate ($\dot{\gamma}$) and could be expressed by the relationship:

$$C_1 |_{\dot{\gamma}} = A_1 (\dot{\gamma})^{S_1}$$

(7)

and

$$C_2 |_{\dot{\gamma}} = A_2 (\dot{\gamma})^{S_2}$$

(8)

at constant strain ($\gamma$) where $A_1$, $A_2$, $S_1$ and $S_2$ are constants. Equations 6, 7 and 8 may be combined to give

$$\tau |_{\gamma} = A_1 (\dot{\gamma})^{S_1} \ln \frac{1}{T} + A_2 (\dot{\gamma})^{S_2}$$

(9)

Equation (9) shows the combined effect of strain rate ($\dot{\gamma}$) and temperature ($T$) on the yield shear stress ($\tau$) of a material at a given strain ($\gamma$).

Equation (9) could be of value in metal deformation calculations, as the resistance to deformation of the material could be determined at known strain rates and temperatures for any desired strain. If the constants were known for various strains then the information could be stored in a computer and calculation made of working loads under any desired set of conditions.

In torsion tests the number of revolutions to failure can often be used as a measure of ductility providing the effects of strain rate, temperature and specimen geometry are considered. In the case of all materials no sharp drop in the torque was observed and the strain to fracture could not be determined. It appeared that cracks were initiated at the specimen surface and propagated through the section over several revolutions. However, generally ductility decreased with increasing speed and decreasing temperature.
### Table I

**Composition of Alloys wt%**

<table>
<thead>
<tr>
<th>Alloy</th>
<th>Si</th>
<th>Fe</th>
<th>Cu</th>
<th>Mn</th>
<th>Mg</th>
<th>Cr</th>
<th>Zn</th>
<th>Ti</th>
<th>Al</th>
</tr>
</thead>
<tbody>
<tr>
<td>1100</td>
<td>0.5</td>
<td>0.5</td>
<td>0.2</td>
<td>0.05</td>
<td>-</td>
<td>-</td>
<td>0.10</td>
<td>-</td>
<td>Rem</td>
</tr>
<tr>
<td>2017</td>
<td>0.8</td>
<td>1.0</td>
<td>3.5</td>
<td>0.40</td>
<td>0.20</td>
<td>0.16</td>
<td>0.25</td>
<td>-</td>
<td>Rem</td>
</tr>
<tr>
<td>6061</td>
<td>0.4</td>
<td>0.7</td>
<td>0.15</td>
<td>0.15</td>
<td>0.8</td>
<td>0.15</td>
<td>0.25</td>
<td>0.15</td>
<td>Rem</td>
</tr>
</tbody>
</table>

### Table II

**Heat Treating Procedures**

<table>
<thead>
<tr>
<th>Alloy</th>
<th>Temperature</th>
<th>Time</th>
<th>Cooling</th>
</tr>
</thead>
<tbody>
<tr>
<td>1100</td>
<td>650°F</td>
<td>1 hr.</td>
<td>Air cool.</td>
</tr>
<tr>
<td>2017</td>
<td>775°F</td>
<td>2 hrs.</td>
<td>50°F/hr to 500°F then Air Cool.</td>
</tr>
<tr>
<td>6061</td>
<td>775°F</td>
<td>2 hrs.</td>
<td>50°F/hr to 500°F then Air Cool.</td>
</tr>
</tbody>
</table>
### Table III

**Variation of Strain Rate Sensitivity Coefficient (m) for Commercially Pure Aluminum (1100)**

<table>
<thead>
<tr>
<th>Temperature</th>
<th>Shear Strain (γ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>°C</td>
<td>Homologous</td>
</tr>
<tr>
<td>22</td>
<td>0.32</td>
</tr>
<tr>
<td>200</td>
<td>0.52</td>
</tr>
<tr>
<td>300</td>
<td>0.63</td>
</tr>
<tr>
<td>400</td>
<td>0.74</td>
</tr>
<tr>
<td>500</td>
<td>0.84</td>
</tr>
<tr>
<td>550</td>
<td>0.90</td>
</tr>
</tbody>
</table>

### Table IV

**Variation of Strain Rate Sensitivity Coefficient (m) for Aluminum Alloy 2017**

<table>
<thead>
<tr>
<th>Temperature</th>
<th>Shear Strain (γ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>°C</td>
<td>Homologous</td>
</tr>
<tr>
<td>350</td>
<td>0.73</td>
</tr>
<tr>
<td>400</td>
<td>0.78</td>
</tr>
<tr>
<td>450</td>
<td>0.85</td>
</tr>
<tr>
<td>500</td>
<td>0.90</td>
</tr>
<tr>
<td>Temperature</td>
<td>Homologous</td>
</tr>
<tr>
<td>-------------</td>
<td>------------</td>
</tr>
<tr>
<td>°C</td>
<td></td>
</tr>
<tr>
<td>350</td>
<td>0.73</td>
</tr>
<tr>
<td>400</td>
<td>0.78</td>
</tr>
<tr>
<td>450</td>
<td>0.85</td>
</tr>
<tr>
<td>500</td>
<td>0.90</td>
</tr>
</tbody>
</table>
VII. ANISOTROPY IN TORSION.

1. Analytical Investigation

The object of the present analytical study was to investigate the development of anisotropy during torsional deformation.

It is possible to state the von Mises criteria for yielding of an anisotropic material in the form

\[
2f (\sigma_{ij}) = F(\sigma_{yz} - \sigma_{xy})^2 + G(\sigma_{zx} - \sigma_{yx})^2 + H(\sigma_{xy} - \sigma_{yz})^2 + 2L \tau_{yz}^2 + 2M \tau_{zx}^2 + 2N \tau_{xy}^2 = 1
\]  

(10)

where \( F, G, H, L, M \) and \( N \) are six parameters indicating the current state of anisotropy and the axes of anisotropy are mutually orthogonal. If \( X, Y \) and \( Z \) are the tensile yield stresses in the principal directions of anisotropy, then

\[
\frac{1}{X^2} = G + H, \quad 2F = \frac{1}{Y^2} + \frac{1}{Z^2} - \frac{1}{X^2}
\]

\[
\frac{1}{Y^2} = H + F, \quad 2G = \frac{1}{Z^2} + \frac{1}{X^2} - \frac{1}{Y^2}
\]

\[
\frac{1}{Z^2} = F + G, \quad 2H = \frac{1}{X^2} + \frac{1}{Y^2} - \frac{1}{Z^2}
\]

(11)

If \( R, S, \) and \( T \) are the yield stresses in shear with respect to the principal axes of anisotropy then

\[
2L = \frac{1}{R^2}
\]

\[
2M = \frac{1}{S^2}
\]

\[
2N = \frac{1}{T^2}
\]

(12)
Hence to determine the anisotropic parameters the six yield stresses must be known.

By analogy with the Lévy-Misses equations for isotropic materials, it is supposed that $f(\sigma_{ij})$ in equation (10) is the plastic potential. The strain increment relations referred to the principal axes of anisotropy, are then,

$$
d\varepsilon_x = d\lambda [H(\sigma_x - \sigma_y) + G(\sigma_x - \sigma_z)]
$$

$$
d\gamma_{yz} = d\lambda L_{yz}
$$

$$
d\varepsilon_y = d\lambda [F(\sigma_y - \sigma_z) + H(\sigma_y - \sigma_x)]
$$

$$
d\gamma_{zx} = d\lambda M_{zx}
$$

$$
d\varepsilon_z = d\lambda [G(\sigma_z - \sigma_x) + F(\sigma_z - \sigma_y)]
$$

$$
d\gamma_{xy} = d\lambda N_{xy}
$$

(13)

The above equations may now be applied directly to the development of anisotropy in a thin wall cylinder subject to torsional deformation.

Consider the extended surface of a thin wall cylinder shown in Figure 4. The initial direction of greatest compression for any shear strain $\gamma$ can be shown to lie on OC which bisects angle AON, where the lines AO and BO respectively represent a line which is first compressed and then extended an equal amount due to the shear strain. After applying any shear strain $\gamma$, OC is compressed to the position OC' and this can be shown to represent the maximum compressive strain. The initial and final directions of the greatest extensions are found to lie in the directions OE and OE' respectively. The directions OC' and OE' are defined as the principal axes of anisotropy and are perpendicular to each other. If the cylinder is initially isotropic, the angle $\theta$ will increase from $\pi/4$ to $\pi/2$ and the anisotropic axes will rotate through an angle $(\theta - \pi/4)$. The rotation is confined to the x - y plane about the z - axis which is directed outwards in the radial direction. If $\tau$ is the principal shear stress then
\[ \sigma_x = -\sigma_y \]
\[ = -\tau \sin 2\theta \]

and
\[ \tau_{xy} = \tau \cos 2\theta \]

From equation (10)
\[ \tau = [2N + (F + G + 4H - 2N) \sin^2 2\theta] \cdot \frac{1}{2} \]

From equation (13) the components of the increment of strain become

\[ d\varepsilon_x = -\tau \lambda (G + 2H) \sin 2\theta \]
\[ d\varepsilon_y = \tau \lambda (F + 2H) \sin 2\theta \]
\[ d\varepsilon_z = \tau \lambda (G - F) \sin 2\theta \]
\[ d\varepsilon_{xy} = \frac{d\gamma_{xy}}{2} \]

\[ = \tau \lambda N \cos 2\theta \]

The axial strain increment \( d\varepsilon_a \) and the shear strain increment \( d\gamma \) are given by

\[ d\varepsilon_a = d\varepsilon_x \sin^2 \theta + d\varepsilon_y \cos^2 \theta - d\gamma_{xy} \sin 2\theta \]
\[ d\gamma = (d\varepsilon_y - d\varepsilon_x) \sin 2\theta + 2d\gamma_{xy} \cos 2\theta \]

Hence
\[ \frac{d\varepsilon_a}{d\gamma} = \frac{[(N-G-2H) \sin^2 \theta - (N-F-2H) \cos^2 \theta] \sin 2\theta}{2N + (F+G+4H-2N) \sin^2 2\theta} \]
Equation (15) gives the change in axial strain with shear strain. The incremental state of strain of an element on the surface of the hollow cylinder is shown in Figure 4. Figure 4a represents an element oriented along the major axes of the cylinder while Figure 4b represents an element oriented along the principal axes of anisotropy which have rotated through some angle \( \phi - \pi/4 \) due to the shear strain \( \gamma \). This situation can be represented more conveniently by the Mohr strain circle shown in Figure 5.

To proceed with the analysis it is necessary to know the relationship between the axial strain and shear strain. For convenience the relationship may be expressed in the form:

\[
\varepsilon = f(\gamma) \tag{16}
\]

By differentiation:

\[
d\varepsilon = d f(\gamma) = f'(\gamma) d\gamma \tag{17}
\]

Now the angle \( \beta \) in the Mohr circle, Figure 6 is given by

\[
\beta = \cot^{-1} \left( \frac{3}{2} \frac{d\varepsilon}{d\gamma} \right) = \tan^{-1} \left( \frac{2}{3 f'(\gamma)} \right) \tag{18}
\]

and the incremental strains by

\[
d\varepsilon_x = -R \cos\alpha + \frac{df(\gamma)}{4}
\]

\[
d\varepsilon_y = R \cos\alpha + \frac{df(\gamma)}{4}
\]

\[
d\varepsilon_{xy} = R \sin\alpha. \tag{19}
\]
Figure 4  Element of Cylinder Surface

Figure 5  Strain on Element of Cylinder Surface
where \( R \) is the radius of the Mohr circle and is given by

\[
R = \frac{d\gamma}{2 \sin \beta}.
\]  \hspace{1cm} (20)

Substituting for \( R \) in equations (19) gives

\[
de_x = -\frac{\cos \alpha}{2 \sin \beta} \ d\gamma + \frac{f'(\gamma)d\gamma}{4}
\]

\[
de_y = \frac{\cos \alpha}{2 \sin \beta} \ d\gamma + \frac{f'(\gamma)d\gamma}{4}
\]  \hspace{1cm} (21)

and

\[
de_{xy} = \frac{\sin \alpha}{2 \sin \beta} \ d\gamma
\]

where

\[
\alpha = 2\phi - \beta
\]

The constant of proportionality \((d\lambda)\) may be eliminated from equation (14) using the relationship:

\[
d\lambda = t d\gamma
\]

to give

\[
de_x = -t^2 (G + 2H) \sin 2\phi \ d\gamma
\]

\[
de_y = t^2 (F + 2H) \sin 2\phi \ d\gamma
\]

\[
de_{xy} = \frac{d\gamma_{xy}}{2}
\]  \hspace{1cm} (22)

\[
= t^2 N \cos 2\phi \ d\gamma
\]
Figure 6 Mohr Strain Circle

Figure 7 Comparison of Empirical Equation and Experimental Data
Equations (21) and (22) may now be combined to give the following expressions for the anisotropic parameters:

\[
G + 2H = \frac{1}{2\tau^2} \left[ 1 + \cot 2\phi \cot \beta - \frac{f'(\gamma)}{2 \sin 2\phi} \right] \\
F + 2H = \frac{1}{2\tau^2} \left[ 1 + \cot 2\phi \cot \beta + \frac{f'(\gamma)}{2 \sin 2\phi} \right] \\
N = \frac{1}{2\tau^2} \left[ 1 - \tan 2\phi \cot \beta \right]
\]  

(23)

It is now necessary to find a suitable function \( f(\gamma) \) to evaluate \((G + 2H), (F + 2H)\) and \(N\). This function cannot be determined analytically but may be obtained from experimental data. By inspection of equation (15) it can be seen that

\[
\frac{d\varepsilon_a}{d\gamma} \bigg|_{\theta = 45^\circ, 90^\circ} = 0
\]

A function which fulfills this requirement is

\[
f(\gamma) = k \cos^3 2\theta
\]  

(24)

where \(k\) is a constant. Differentiation of equation (24) gives

\[
f'(\gamma) = 6k \cos^2 2\theta \sin 2\theta \frac{d\theta}{d\gamma}
\]  

(25)

Substituting for \(f(\gamma)\) and \(f'(\gamma)\) in equations (23) and (18) gives
\[ G + 2H = \frac{1}{2\tau^2} \left[ 1 + \cot \theta \cot \beta + 3k \cos^2 \theta \frac{d\theta}{d\gamma} \right] \]

\[ F + 2H = \frac{1}{2\tau^2} \left[ 1 + \cot \theta \cot \beta - 3k \cos^2 \theta \frac{d\theta}{d\gamma} \right] \]

\[ N = \frac{1}{2\tau^2} \left[ 1 - \tan \theta \cot \beta \right] \]

\[ \beta = \tan^{-1} \left[ \frac{-1}{9k \cos^2 \theta \sin \theta \frac{d\theta}{d\gamma}} \right] \]

where \( \theta \) is a known function of \( \gamma \). The angle (\( \theta \)) and shear strain (\( \gamma \)) are related through the expressions:

\[ \theta = \sin^{-1} \left[ \frac{\sin \frac{\psi}{2}}{[1 + (2 \cot \psi \sin \frac{\psi}{2})^2 - 2 \cos \psi]^{\frac{1}{2}}} \right] \]

and

\[ \gamma = \frac{2}{\tan \psi}. \]

which are obtained from Figure 4. Differentiation of equations (27) and (28) gives

\[ \frac{d\theta}{d\gamma} = \left[ -\frac{2}{\gamma^2 + 4} \right] \left\{ \left[ \sin^2 \frac{\psi}{2} \right]^{\frac{1}{2}} \right\} \left[ 1 - \frac{\sin^2 \frac{\psi}{2}}{A} \right] \cdot \frac{\cos \frac{\psi}{2}}{2A^{\frac{1}{2}}} - \frac{\sin \frac{\psi}{2}}{2A^{\frac{1}{2}}} \]

\[ \left\{ \left( 4 \sin \frac{\psi}{2} \right) \left( \tan \psi \cos \frac{\psi}{2} - 2 \sin \frac{\psi}{2} \sec^2 \psi \right) \right\} \frac{\tan^2 \psi}{A}. \]

where \( A \) is \( \{1 + \left( \frac{2 \sin \frac{\psi}{2}}{\tan} \right)^2 - 2 \cos \psi \}. \)
For any given shear strain ($\gamma$), values for $\phi$ and $\frac{d\phi}{d\gamma}$ can be calculated from equations (27), (28) and (29). If these are then substituted into equation (26) the anisotropic parameters $(G + 2H)$, $(F + 2H)$ and $N$ can be expressed in terms of $1/2\tau^2$, the principal yield shear stress.

Of fundamental importance in the study of anisotropy are the yield stresses along anisotropic axes since these give a very good indication of the change and variation in mechanical properties produced by deformation.

In torsion the yield stresses $X$ and $Y$ along the $x$ and $y$ anisotropic axes on the surface of the cylinder are not likely to be determined directly by experimental means because of rotation of the axes. However, the yield stress $Z$ through the wall thickness along the radial $z$ axis could be determined quite readily: for example from a compression test on a sample taken from the wall of the cylinder.

Rearrangement of equation (11) gives

$$G + 2H = \frac{3}{2X^2} + \frac{1}{2Y^2} - \frac{1}{2Z^2}$$  \hspace{1cm} (30)

$$F + 2H = \frac{1}{2X^2} + \frac{3}{2Y^2} - \frac{1}{2Z^2}$$  \hspace{1cm} (31)

Solving equations (30) and (31) simultaneously gives

$$X = \frac{2}{\{3(F + 2H) - (G + 2H) + \frac{1}{Z^2}\}\frac{1}{2}}$$  \hspace{1cm} (32)

and

$$Y = \frac{2}{\{3(G + 2H) - (F + 2H) + \frac{1}{Z^2}\}\frac{1}{2}}$$  \hspace{1cm} (33)
Values for the anisotropic parameters \((F + 2H)\) and \((G + 2H)\), and an experimentally determined value of \(Z\) for various shear strains can now be substituted into equations (32) and (33) to give \(X\) and \(Y\), the yield stresses along the anisotropic axes. The values of \(X\), \(Y\) and \(Z\) can be substituted into equation (11) which can be solved simultaneously to give individual value for \(H\), \(G\) and \(F\). The remaining anisotropic parameter \(N\) and the associated yield shear stress can be determined from equations (14) and (15).

The above analysis has shown that it is possible to determine the yield stresses along the principal axes of anisotropy and the anisotropic parameters from a measure of the principal yield shear stress of the material and the yield stress of the material measured through the wall of the test specimen.

2. Experimental Procedure

Initial experiments on solid bars and hollow cylinders in fixed torsion showed that sufficient change in length occurred to cause buckling, indicating the existence of large axial stresses which could affect the yield shear stress of the test material. To eliminate the development of axial stresses tests must be carried out in free torsion where the specimen is allowed to undergo its natural change in length which is accompanied by a corresponding change in radius to maintain constant volume. If shear stress-shear strain curves are required experimental data must be corrected to account for these changes.

A comprehensive series of experiments on hollow cylinders of aluminum and two aluminum alloys were performed over a wide range of temperature and strain rate to obtain further knowledge concerning the development of anisotropy and the changes in specimen geometry with torsional straining.
3. Results and Discussion

The change in axial strain with shear strain for commercially pure aluminum and two aluminum alloys (2016 and 6061) were determined in the temperature range 22°C to 550°C and 300°C to 500°C respectively and in the strain rate range 0.015 to 150 per second. Tests on commercially pure aluminum indicated that an overall increase in length occurred over the entire range of temperature and testing speed at large strains. At temperatures above 400°C an initial decrease in length was noted which was followed by the usual increase in length. At the highest temperatures and at very large strains, the specimens stopped lengthening and began shortening again. This behavior is apparently related in a complex manner to the work hardening characteristics of the materials as influenced by strain rate and temperature.

The behavior of the two alloys was generally similar to that of the commercially pure aluminum. The alloys showed an initial shortening at temperatures of 400°C and above which increased with increasing temperature and decreasing strain rate. The total increase in length at a given shear strain generally decreased with increasing temperature and decreasing strain rate. Selected results showing the change in length with shear strain for the three materials at a variety of strains are given in Table VI.

Several experiments were carried out at ambient temperature and at a strain rate of 1.0 per second to compare shear stress-shear strain data for commercially pure aluminum specimens twisted with and without axial freedom. The mean ultimate shearing stress based on the original dimensions for the tests with axial freedom was found to be 14,800 psi while the specimens subjected to axial constraint had a mean ultimate shearing stress of 15,300 psi, a difference of approximately 3%. This
is a small amount which is opposite in sign to that which would be anticipated.

The change in shear stress in the above case is the result of two conflicting tendencies. During the test, high compressive stresses develop due to constraint in fixed torsion which tend to reduce the torque required for deformation. However, it was noted that the torque in the constrained condition was actually higher than in the free case. This abnormality can be explained by the fact that the compressive stresses are of sufficient magnitude to cause plastic deformation and an increase in the cross sectional area of the specimen which will increase with increasing strain. Apparently the increase in torque due to an increase in cross sectional area of the specimen is more than sufficient to overcome the drop caused by the compressive stress.

The analytical work described above indicates that a knowledge of the change in axial strain with shear strain is necessary so that the stresses X, Y and Z and the anisotropic parameters F, G, H, and N can be evaluated. In the analysis the expression:

\[ f(\gamma) = k \cos^3 2\theta \]  

was used. Figure 7 shows a comparison of experimental data for aluminum at room temperature with equation (34). The value of \(k\) was chosen to be -0.07. It can be seen that agreement is very good over the range of shear strains considered. It is anticipated that equation (34) would be applicable to most materials with changes in the value of \(k\).
Change in length is measured in thousandths of an inch.
VIII. CONCLUSIONS.

From the work presented above, the following conclusions can be drawn.

1. Torsion testing using hollow cylinders of carefully controlled dimensions provides an excellent means for obtaining data for use in metal working studies.

2. The change in length which occurs during the torsional plastic straining of hollow cylinders of commercially pure aluminum and two aluminum alloys (2017 and 6061) is due to the development of anisotropy which can be related qualitatively to the work hardening characteristics of the material.

3. Analytical expressions could be derived relating the anisotropic parameters or the associated yield stresses along the anisotropic axes (in terms of the through thickness yield stress) to the geometry of torsional deformation as determined by the change in axial strain with shear strain. These expressions describe completely the development of anisotropy during torsional straining.

4. The change in length with shear strain for commercially pure aluminum at room temperature could be represented by the expression:

\[ \varepsilon = f(\gamma) \]

where

\[ f(\gamma) = -0.07 \cos^3 2\phi \]

and where the angle \( \phi \) represents the rotation of the anisotropic axes.

5. No relationship could be found to express the shape of the shear stress-shear strain curves over the whole of the temperature and strain rate range considered.

6. The stress maxima which occur in some tests carried out at elevated temperatures could be explained by a qualitative consideration of the relative effects of work hardening and recrystallization.
7. The effect of strain rate on the flow stress of the three materials tested at a given strain and temperature could be expressed by a power law.

8. No relationship could be found to express the effect of temperature on the flow stress of the three materials at a given strain and strain rate over the whole of the temperature range considered. However in the case of commercial aluminum at temperatures above 200°C the combined effect of temperature and strain rate could be described by the equation:

\[ \tau|_\gamma = A_1(\dot{\gamma})^{S_1} \ln \frac{1}{T} + A_2(\dot{\gamma})^{S_2} \]

where \( A_1, A_2, S_1 \) and \( S_2 \) are constants.
APPENDIX I

The following dissertations and papers based on work carried out under grant GK 191 have been published or are in preparation.


Copies of all publications will be forwarded to the National Science Foundation as soon as reprints become available.