RESEARCH PROJECT INITIATION

Project Title: Study of Non-Linear Rarefied Gas Flows by the Discrete Ordinate Method

Project No.: E-208 (E-208)

Project Director: Dr. A. H. Huang

Sponsor: National Aeronautics and Space Administration, Washington, D.C.

Agreement Period: From 1 February 1968 to 31 January 1969

Type Agreement: Grant NGR 18-002-062

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Grant Administrator:

Headquarters, Contracts Division
Code DH-4
National Aeronautics and Space Administration
Washington, D.C. 20546

Reports Required:

Annual Status - 1 July 1968, ten (10) copies
Final - Upon completion of project, ten (10) copies

Assigned to: School of Aerospace Engineering

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Mr. R. A. Martin, EE8
GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF RESEARCH ADMINISTRATION
RESEARCH PROJECT TERMINATION

Date: April 9, 1973

Project Title: Study of Non-Linear Rarefied Gas Flows by the Discrete Ordinate Method

Project No.: E-16-604 (old E-208)

Principal Investigator: Dr. A. B. Hwang

Sponsor: NASA; Washington, D. C.

Effective Termination Date: March 30, 1973 (Final Report Delivered)

Clearance of Accounting Charges: by Jan. 31, 1973 (Grant Expiration)

Grant/Contract Closeout Actions Remaining:
- Obtain record copies of Final Tech. Report from Sponsor.
- Final Quarterly Financial Report (Form 1031)
- Final Report of Inventions
- Final Equipment Inventory Report (if applicable)

Assigned to: School of Aerospace Engineering

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RA-4 (6-71)
STUDY OF NON-LINEAR RAREFIED GAS FLOWS BY
THE DISCRETE ORDINATE METHOD

Prepared under NASA Grant NGR 11-002-062

Semiannual Report
Covering the period from February 1, 1968 through July 30, 1968

by

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July 31, 1968
SUMMARY

A brief report of the present status of the research program supported by NASA under the Research Grant NGR 11-002-062 is included. In this program several non-linear re-entry problems in gasdynamics have been solved from the molecular approach of kinetic theory using the Boltzmann equation as the fundamental governing relation and using the discrete ordinate method as a tool. The method as applied to the solution of both steady and unsteady non-linear B-G-K Boltzmann equation has been established on a rigorous, fundamental, and systematic basis by the author. The results obtained in this study represent, to the best of the author's knowledge, the first meaningful solution to the nonlinear and two-dimensional B-G-K Boltzmann equation. It is thus concluded that the proposed method is a powerful tool for solution of some practical rarefied gasdynamic problems. A list of publications during the period of February 1, 1968 through July 31, 1968 is also included.

In the extended program, the proposed method is being applied to the problem of hypersonic rarefied gas flow with the effects of collisional ionization, non-equilibrium radiation, and chemical dissociation.
I. PRESENT STATUS

In 1966, the author studied the discrete ordinate method in a rigorous analysis. The method was applied to the linearized (steady and unsteady) Boltzmann-Bhatnagar-Gross-Krook equation in an effort to establish a new technique for the solution of this equation over as wide a range of Knudsen numbers as possible. A new and very useful Gaussian quadrature was developed particularly for the type of problems encountered in kinetic theory, namely, those problems in which the distribution function possesses a streamwise character. The development of this new quadrature is one of the primary contributions of the author's research since it is this device which allows the discrete ordinate method to give accurate solutions for quite large values of knudsen number, namely, this new quadrature yields very fast convergence for numerical quadratures in the near free molecular flow regime. The method with the new quadrature has been applied to the steady and unsteady linearized Couette flow, to the linearized channel flow, to the slip velocity problem, and to the linearized Rayleigh problem. It has been concluded that this method gives accurate solutions over a wider range of Knudsen numbers for a given amount of computational effort than any other existing approximate method.

Very recently, the method has been generalized for a much wider class of non-linear rarefied gas dynamic problems by the author. It has been demonstrated by the author that the generalized method can be applied to the non-linear Boltzmann equation with hard sphere model as well as with the B-G-K model. It has also been shown by the author that a previous technique which was successfully applied to the linearized problems is a particular case of this generalized method. An important ingredient which allows the method to be extended to non-linear cases is the development of the equally spaced quadratures. It has been discussed in great detail that the main difficulty in applying the discrete ordinate method to the non-linear collision integral can be overcome by utilizing the equally spaced quadratures. After developing the discrete ordinate method for the non-linear Boltzmann
equation, the technique has been applied to several flow problems in order to establish the accuracy and practicality of the method. First, the case of non-linear Couette flow with heat transfer between two infinite parallel plates was considered. Since numerically exact solutions of this problem are available, it is thus possible to calibrate the proposed method against the known standard solutions. The results calculated using \( n=8 \) (\( n \) is the order of discrete velocity quadrature used) are in excellent agreement with Anderson's numerically exact results (the agreement is good up to the third decimal point). In addition, there are three points which are worthy of notice. First, the proposed method is capable of obtaining accurate solutions from large Knudsen numbers (the free molecular regime) all the way to small Knudsen numbers (the continuum flow regime). It should be noted that the numerical scheme used by Anderson can not be applied to calculate the small Knudsen numbers cases (the smallest Knudsen number which Anderson calculated is 0.1). Second, the proposed method is also capable of obtaining good solutions from small Mach numbers (subsonic flow) to large Mach numbers (supersonic or hypersonic flow). The numerical scheme used by Anderson can not yield good solutions for high Mach number cases. Third, in contrast to the usual numerical scheme (for instance, the Anderson's approach) and the moment scheme (for instance, the Macomber's approach), the proposed method always starts with the calculation of the distribution functions for the chosen discrete velocity points. This information is very useful and important for those who are working with the molecular beam experiments, because the most direct comparison between theory and experimental measurements is to compare the local distribution function and not to compare the macroscopic moments.

The problem of the translational relaxation process (pseudo shock relaxation) of a rigid sphere gas was also considered. The solutions obtained using \( n=4 \) and a Burroughs B-5500 digital computer are in excellent agreement with those obtained by Nordsieck and Hicks utilizing a Monte-Carlo scheme. It is very important to note that the computer time needed in the above calculations for each time step is approximately 30 seconds (very much shorter than that of Nordsieck and Hicks' scheme).
In order to examine the method for time dependent problem, the non-linear Rayleigh flow problem was next considered.\textsuperscript{14} The results obtained agree in the small Mach number limit with linearized kinetic theory.\textsuperscript{6} In the high Mach number cases, the solutions on the shear velocity and density profiles agree well with those of Chu\textsuperscript{15} utilizing a modified numerical method of finite difference.

One of the most important applications of the proposed method is that of the sharp leading edge problem.\textsuperscript{16} The flow field near the leading edge of a sharp flat plate which is aligned with a high speed gas stream is a classical problem in the mechanics of viscous fields. Under rarefied conditions at high speeds, a sharp flat plate generates a wide spectrum of flow regimes. These range from a near free molecular region near the leading edge to the continuum picture of a classical boundary layer far from the plate. Several attempts to use the continuum equations and strong interaction theories have failed to adequately predict the behavior of the flow near the leading edge. In Reference 16, the leading edge problem is formulated and studied within the framework of the kinetic theory of gases. Numerical solutions, capable of describing the flow field from the leading point of the plate to the downstream end where the strong-interaction theory (or the continuum boundary layer theory) is valid, are given. The solutions are compared with the existing experimental data. The local distribution functions for the entire flow field have been calculated, and thus the complete flow field has been generated for the cases of $M_\infty = 1.5$ and 3, and plate temperatures $T_v = 2T_\infty$, $T_v = T_\infty$. The formation of the shock wave and boundary layer are completely specified. The shock wave thickness exhibited is quite broad, in contrast to several earlier theoretical assumptions,\textsuperscript{17} yet agreeing with experimental results.\textsuperscript{18} The existence of the "pressure plateau" which was experimentally observed\textsuperscript{19,20} in the near-free molecular region is predicted by the present theory. It is also interesting that the present theory predicts a small "slip velocity plateau" in the transition flow regime. Some experimental data\textsuperscript{19} also to indicate the existence of this "slip velocity plateau" in the transition
flow regime. All macroscopic property profiles are calculated throughout the flow field and agree qualitatively with continuum results in the downstream regions. Variation of heat transfer coefficients and skin friction coefficients are computed and compared to various theories. These are used also to verify adiabatic wall conditions. To the best of the author's knowledge, this is the first solution to the non-linear and two-dimensional Boltzmann equation with the B-G-K model for this leading edge problem. The solutions which are based on the framework of kinetic theory are capable of describing the entire flow field from near-free molecular flow at the leading edge to continuum conditions.

It is very important to note that, based on the applications of the proposed method to the Boltzmann equation for the leading edge flow problem, the difficulty of singularity which is usually encountered in the conventional gasdynamics can be overcome. This will supply some justification for the application of the proposed method to the rarefied wake and sharp corner flow problems.

Very recently, the method used in Reference 16 has been modified and extended to the hypersonic sharp leading edge problem \( M_\infty = 10, 20 \text{ and higher} \). The results which are excellent in comparison with the experimental data are discussed in detail in a paper \(^1\) which is to be presented at the 19th International Astronautical Congress, New York, October 13-19, 1968.

The determination of the flow field around a sharp, mathematically thin, finite (length) flat plate parallel to a rarefied supersonic flow has been of practical interest in the field of rarefied gasdynamics, since it represents not only the problem of the sharp leading edge, but also the problem of the low density wake. This problem has also been solved using the proposed method. The results which give considerable insight into the understanding of the downstream effects on the upstream flow properties are discussed in detail in a paper \(^2\) which was presented at the 6th International Symposium on Rarefied Gas Dynamics, MIT, Boston, July 22-26, 1968. This solution represents the first solution to the non-linear and two-dimensional Boltzmann (B-G-K) equation based on the closed boundary value approach.
The proposed method has also been applied to solve the supersonic wedge-flow problem. The rarefied wakes after a wedge, and the rarefied wakes after a sharp corner. Meaningful results have been obtained and will be summed up for publication in the near future. It is seen that the proposed method can make the previously unapproachable two (space) dimension problem (with the difficulty of singularity) accessible to solution based on the Boltzmann equation.

II. REFERENCES


III. LIST OF PUBLICATIONS (February 1, 1968 - July 31, 1968)


IV. EXTENDED PROGRAM

The combination of high temperatures, high velocities, and low densities, such as are encountered in the real hypervelocity flight, has led to the study of such non-equilibrium gasdynamic phenomena as vibrational and dissociative relaxation, collisional ionization, and non-equilibrium radiation in flows. The determination of the complete flow field around a vehicle which travels with high velocities in a rarefied plasma has been of practical importance, since the information is essential in the determination of particle drag on vehicle, in obtaining the information about the properties of the upper atmosphere, and in understanding the effects of the boundary layer and plasma sheaths in connection with communication and detection aspects. Furthermore, the study is directly of importance in connection with the re-entry flight problem.
Theoretically, the problem of the three-component plasma (consisted of ions, electrons, and neutral particles) flow over a vehicle can be analyzed by treating the Poisson equation, which governs the electrical potential, in conjunction with the Boltzmann equation for ions, electrons, and neutral particles, which yield the local flow properties of ions, electrons, and neutral particles. In general, however, the Boltzmann equation is a very complicated non-linear integro-differential equation. To date there is no mathematically rigorous method of solving this equation analytically for typically non-linear problems of interest in high speed, high altitude aerodynamics. It can thus be seen that the problem becomes much more complicated when the effects of collisional ionization, nonequilibrium radiation, and chemical reaction are taken into account.

As discussed in the previous section, the method of discrete ordinates is a systematic method which has sufficient flexibility to be adapted to many practical problems in hypersonic rarefied gasdynamics. It is felt that the proposed method can be extended to solve the practical hypersonic rarefied gasdynamic problems with the effects of non-equilibrium radiation, collisional ionization, and chemical reaction. As a matter of fact, for the first test, the proposed method has been applied to the supersonic plasma flow past the sharp leading edge of a flat plate. Very favorable solutions have been obtained. This will be reported in the next semi-annual report.