MANAGING WORKSTATION CAPACITY VARIATION THROUGH THE DESIGN OF A PROCESS PARAMETER PLATFORM

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1. OFFERING VARIETY THROUGH PLATFORM DESIGN

Offering product variety affordably is the crux of mass customization. Unfortunately, this is the foremost difficulty that enterprises face in making the transition to this paradigm. Anderson (1997) addresses the problem of offering affordable variety through the identification of the cost of variety. The cost of variety is the sum of all the costs of attempting to offer customers variety with inflexible products that are produced in inflexible factories and sold through inflexible channels. This cost includes the cost of customizing or configuring products, the cost of excessive variety, the cost of excessive procedures, and the cost of excessive processes and operations, among others. The key to mass customization, therefore, is the development of a product and a production process that minimize the components of this cost.

It is neither feasible nor effective to cope with customers’ demands for product variety through a simple increase in inventory, a reaction commonly found in mass production. Manufacturing enterprises are recognizing that product design presents the best control over offering such a variety (Anderson, 1997). The core issue of transitioning to mass customization now becomes how to design a product and its manufacturing process for affordable customization.

While previous chapters of this text focus on the design of the product, the focus in this chapter is the design of an aspect of the manufacturing process; specifically, the determination of process parameters for a single workstation. In order to provide some context, a brief overview of current manufacturing philosophies is provided in Section 1.1. In Section 1.2, the focus is narrowed to address the problem of workstation process parameter design in the presence of changing capacity requirements. In Sections 1.3 and 1.4, the authors present the chapter’s focus: a methodology for the design of a process parameter platform.

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1.1. Agile Manufacturing of Customized Products

As a result of the shift to mass customization, enterprises are forced to manufacture more complex products (multiple features, multiple variants) with reduced product life cycles, reduced time-to-market, and volatile demand (McFarlane and Bussmann, 2000). As such, the complexity of the production process design problem is dramatically increased. Today’s manufacturing approach must enable the quick launch of new product models, rapid adjustment of the manufacturing system capacity to market demands, and rapid integration of new process technologies into existing systems (Mehrabi et al., 2000). Realizing that improving the flexibility and productivity of a manufacturing system is the “crucial challenge of modern industrial management” (Ferrari et al., 2003), many system-level production process design approaches have been developed to enable manufacturing enterprises to affordably produce customized products.

Using the fundamental concepts of Group Technology - grouping parts with similar production processes together (Rolstadas, 2001) - cellular manufacturing involves processing a collection of similar parts (part families) on a dedicated cluster (or cell) of machines or manufacturing processes (McAuley, 1972). This strategy has the potential to reduce setup times, reduce in-process inventory, improve part quality, shorten lead time, reduce tool requirements, and improve productivity. Cellular manufacturing systems have a high level of flexibility that allow organizations to “quickly respond to changes in the market demand or product structure with minimum disruption to their prior manufacturing commitments” (Malakooti et al., 2004).

Flexible Manufacturing Systems (FMS) is a manufacturing technology and system-level philosophy that focuses on designing a production system that is capable of producing several families of parts, with shortened changeover time, and without major retooling. FMS is a programmable machining system configuration that incorporates software to handle changes in work orders, production schedules, part programs, and tooling for several families of parts (Hopp and Spearman, 2001).

Reconfigurable Manufacturing Systems (RMS) take this concept further by striving to create production processes that are capable of not only adapting to producing a variety of parts, but also changing the system itself easily (Mehrabi et al., 2000). Modular machines and open architecture controllers are the key enabling technologies for RMS, and have the ability to integrate/remove new software/hardware modules in response to changing market demands or technologies without affecting the rest of the system. “This offers RMS the ability to be converted quickly to the production of new models, to be adjusted to exact capacity requirements quickly as market grows and product changes, and to be able to integrate new technology” (Mehrabi et al., 2002). The objective of a RMS is to provide exactly the functionality and capacity that is needed, precisely when it is needed.

These system-level philosophies and their related implementation technologies provide general strategic direction for the planning of the production process. These ideologies have generated research towards the planning and design of various aspects of the production process - from sequencing and synchronization of multiple machining and assembly operations, to line balancing and capacity planning. In this chapter, however, the authors focus on improving the agility of individual workstations.
1.2. The Need for the Agile Definition of Process Parameters

Consider a manufacturer of customized widgets. Due to the volatile demand of the different widget variants, and the manufacture-to-order nature of the process, the capacity requirement of each of the workstations in the production line changes daily. In the context of a single workstation, this change forces the manufacturing engineer to reconfigure the process parameters of the workstation (e.g., turning speed, tool size, laser power, temperature, etc.) in order to maintain the best compromise between three conflicting objectives: minimization of cost, maximization of throughput, and maximization of quality. This reconfiguration not only requires a new evaluation of the process parameters, but also entails a costly and lengthy setup of the workstation. The engineer is in need of a means of making the setup of this specific workstation more efficient and effective at adapting to changing capacity requirements.

This scenario describes a problem of process parameter design. While there are parameter design techniques in the literature, these do not address defining process parameters so as to improve the agility of the workstation in the face of varying capacity requirements. Robust parameter design, inspired by Taguchi’s robust design principles, is focused on choosing the values of controllable parameters so as to improve a defined quality characteristic, while minimizing the variation imposed on the process via uncontrollable (noise) parameters/factors (Robinson et al., 2004). While typically used to maintain quality in the presence of uncertainty, robust parameter design has also been used to make decisions regarding capacity planning and machine investment (Paraskevopoulos et al., 1991).

In order to direct the reader’s focus towards the process parameter design problem in the context of mass customization, the authors pose the following question, How can a designer determine a workstation’s process parameters so as to efficiently and effectively handle fluctuating capacity requirements? In order to maximize a workstation’s efficiency in the presence of different production capacity requirements, one must identify a means to make efficient transitions between process parameters (i.e., minimize the cost and time of the workstation setups). The authors look to the development of product platforms as inspiration of achieving this goal.

1.3. Product Platform Development as Inspiration

Although they are two separate domains, product design and process parameter design share similarities in the context of producing customized goods. In the realm of product design, a designer must find an affordable manner in which to offer variety in product specification and/or product function. In the realm of process parameter design for individual workstations, a manufacturer must find an efficient means to offer variety in production capacity requirements.

In the context of designing customized products, variety is efficiently offered through the development of product platforms – a set of common components, modules or parts from which a stream of derivative products can be created (Lehnerd, 1987). The design of product platforms for customized products enables the manufacturer to maintain the economic benefits of having common parts and processes while still being able to offer product variety to customers.
It is the authors’ assertion that the core concept of platform design – offering variety efficiently through commonality and/or modularity – can be applied to the design of the process parameters for a workstation involved in the manufacture of customized goods. As such, the concept of a *process parameter platform* is introduced:

A *process parameter platform* is defined as a set of common process parameters from which a stream of derivative process parameters can generate a customized product efficiently despite changes in required capacity.

The concept of a process parameter platform is very similar to a product platform. Product platforms are a set of design parameters that are commonalized across various intervals in the design space in order to offer product variety. Process parameter platforms comprise a set of process parameters that are commonalized across various intervals in order to offer variety in the workstation’s production capacity requirement. Just as the commonality of design parameters in a product platform lowers the cost of offering product variety, the commonality of parameters in a process parameter platform lowers the cost of the setups encountered with the reconfiguration of a production workstation for different capacity requirements.

The concept of grouping similar system-level manufacturing/assembly processes into a family is the crux of agile manufacturing. The authors introduce the concept of the process parameter platform in order to extend this philosophy to the lower-end of the production hierarchy – the design of a family of process parameters for individual workstations.

### 1.4. Context

The goal in creating a platform of process parameters is to reduce the cost and time of workstation setups and thus create an efficient manner of offering variety in production capacity. The authors address two main issues in this chapter: how one should design a process parameter platform and, more importantly, whether or not the development of a process parameter platform is an advantageous venture. In order to answer these questions, the authors look to different product platform design techniques as potential foundations for the design of this new type of platform.

Simpson provides a thorough review of 32 existing optimization-based product platform design approaches wherein their different characteristics are compared and contrasted (Simpson, 2003). The following limitations are identified in Simpson’s review:

1. Two-thirds of the techniques require *a priori* specification of the platform to optimization;
2. Half of those techniques surveyed assume that maximizing product performance maximizes demand, maximizing commonality minimizes production costs, and that resolving the tradeoff between the two yields the most profitable product family;
3. Only half of the methods integrate manufacturing costs directly;
4. Less than one-third incorporate market demand or sales into the problem - those that do assume that demand is uniform, and use single objective optimization (with the goal of either minimizing cost or maximizing profit);
5. Only two methods are capable of handling multiple methods of managing variety (modularity and product scaling).

These limitations are significant, as the design of a process parameter platform requires a methodology that enables a designer to handle multiple design objectives, synthesize multiple manners in which to offer variety, model the manufacturing process and the non-uniform demand of the market effectively, and handle the inherent tradeoffs between commonality and platform performance. Of those product platform techniques surveyed by Simpson, only one technique is capable of satisfying all of the above listed requirements - the Product Platform Constructal Theory Method.

The Product Platform Constructal Theory Method (PPCTM) is a top-down product platform design approach for developing product platforms that facilitates the realization of a stream of customized product variants, and which accommodates the issue of multiple levels of commonality and multiple customizable specifications (Hernandez et al., 2003; Williams et al., 2004). The result of the use of the PPCTM is a hierarchical organization of multiple approaches for achieving commonality, as well as the specification of their range of application across the product platform.

In this chapter, the authors present a methodology for the development of a process parameter platform, using the PPCTM as a theoretical basis. In Section 2 the PPCTM, and the abstraction of its principles for its application to process parameter design, is described in detail. The methodology is presented in Section 3 and is explained and validated through its application to an example problem. Results are presented in Section 4 and closing remarks are offered in Section 5.

2. HIERARCHICAL PROCESS PARAMETER PLATFORM DESIGN AS A PROBLEM ACCESS IN GEOMETRIC SPACE

The Product Platform Constructal Theory Method (PPCTM) serves as the theoretical foundation for the methodology of designing process parameter platforms. The fundamental problem addressed in the PPCTM is how to determine and organize different methods of offering variety systematically in order to create an efficient platform (Hernandez et al., 2002; Hernandez et al., 2003).

2.1. The Product Platform Constructal Theory Method

In the Product Platform Constructal Theory Method, the approach for organizing common components for a very large number of product variants is anchored in the thesis of Herbert Simon (Simon, 1996), who observed that complex structures adapt and evolve more efficiently when they are organized hierarchically. Considering this, Hernandez and coauthors (2003) propose to determine and organize commonality of product parameters in a hierarchic manner. With the formulation of the PPCTM, platform design is represented as a problem of optimization of access in a geometric space.
An optimal access problem is characterized by the need to determine the optimal “bouquet of paths” that link all points, \( P_{x,y} \), of a geometric space, \( S \), with a common destination, \( O \) (Figure 1). Adrian Bejan (1996) initiated constructal theory as a result of studying problems of optimal access. Bejan’s constructal theory embodies the notion that the hierarchic organization we observe in Nature is the result of a sequential process of optimization that works towards improving the “access” of elementary geometric space elements, which are then assembled into larger space elements until the entire relevant space is connected (Bejan, 1997). Constructal theory has been applied to several different types of engineering problems ranging from thermodynamics, fluid analysis, heat transfer, and the design of product platforms. Fundamentally, the crux of constructal theory is that access in a geometric space can be made most efficient through (i) a hierarchic organization of the several means of achieving access, and (ii) the use of a multistage decision process to determine the range of application of each technique (Bejan, 2000).

In order to abstract constructal theory and problems of optimal access to product platform development, Hernandez and coauthors introduce the concept of space of customization as the geometric space set of all feasible combinations of values of product specifications that a manufacturing enterprise is willing to satisfy (i.e., space \( S \) in Figure 1) (Hernandez et al., 2002).

Mathematically, let \( N \) be the number of quantitative parameters that define the requirements of a product. Let \( r_i \) represent these parameters, where \( i = 1, \ldots, N \). Then the space of customization, \( M^N \), is the set:

\[
M^N \equiv \{ (r_1, r_2, \ldots, r_N) \}
\]  

(1)

It should be noted that a space of customization is not limited to continuous variables; it can be formed by continuous, discrete or mixed-valued requirements.

Based on this mathematical definition of space of customization, a product \( i \) can be represented by a unique specification of product requirements in an \( N \)-dimensional space of customization, i.e., a vector \( r_i(r_1, \ldots, r_N) \):

\[
r_i = r_{i1}\hat{e}_1 + r_{i2}\hat{e}_2 + \ldots + r_{iN}\hat{e}_N
\]  

(2)
where \( \hat{e}_k \) is the unit vector in each direction \( k \) of the space of customization. Using this representation of a product, the derivation of a new product, \( r_p \), based on an existing product, \( r_i \), is referred to as a "product customization," represented by a vector in the space of customization:

\[
\Delta r_{ji} = \sum_{k=1}^{N} (r_{jk} - r_{ik}) \hat{e}_k = \sum_{k=1}^{N} \Delta r_{jik} \hat{e}_k
\]

(3)

This representation of product customization is illustrated in Figure 1. Generic approaches to "access" points in the space of customization, i.e., to achieve product customizations from a baseline design, are referred to as modes for managing product variety (as shown in Figure 1 as \( \Delta r \)).

With the introduction of these definitions, the problem of designing a platform for customizable products becomes an effort to define a baseline set of components (the product platform) from which all the points of a space of customization can be accessed through the systematic use of a series of modes for managing product customization, and improving a (set of) given objective(s) (e.g., cost, profit, product performance, etc). The fundamental problem addressed in the application of the PPCTM to product platform design is how to organize and determine the extent of application of modes for managing product variety systematically in order to create a product platform for customized products. Through the application of the tenets of constructal theory, this optimal access problem is formulated as a multi-stage decision wherein the ranges of application of each mode for managing product variety are the decision variables. The goal of each decision is to improve the objective functions in order to provide the most efficient manner of offering product variety. More detailed information on the abstraction of constructal theory to product platform design can be found in (Hernandez et al., 2003).

2.2. Abstracting the PPCTM to Process Parameter Platform Design

In order to abstract constructal theory and problems of optimal access to the design of a process parameter platform, one must first define a geometric space that captures the essence of the problem. Since the production capacity requirement of the workstation is the only specification being varied, it serves as the lone dimension of the resulting geometric space. This geometric space is defined as the space of capacity:

A space of capacity is the range of workstation production capacity that a manufacturing enterprise is willing to satisfy with a process parameter platform.

The bounds of this single dimension are determined by the manufacturing enterprise as the amount of production capacity that their manufacturing system should satisfy. Each point in this geometric space represents a unique required level of production capacity (Figure 2).
Production Capacity
(parts / day)

**Figure 2.** Visualization of the Space of Capacity

With this geometric space defined, the crux of the application of the PPCTM to process parameter platform design is the synthesis of multiple methods of offering variety in order to provide any variant within the geometric space. These methods are called modes for managing capacity variety, and are defined as:

*A mode of managing capacity variety* is any generic approach in the design of the process parameters of a workstation for achieving a change in the required manufacturing capacity.

Examples of modes of managing capacity variety include, but are not limited to, process parameter standardization, batch size commonalization, machine type commonalization, and the modular combination of machine capacity (i.e., adding a new machine to the manufacturing system). These modes serve as the linking mechanism between the individual production capacity requirements that compose a family of process parameters.

With a means for linking the geometric space established, the problem of designing a platform for customizable workstation setups becomes an effort to define a set of parameters from which one can access all the points of a space of capacity through the systematic use of a series of modes for managing capacity variety, and improving some given objective(s) (e.g., cost, throughput, quality, etc). The fundamental problem addressed in the application of the PPCTM in this realm is how to organize and determine the extent of application of the modes for managing capacity variety systematically in order to create a process parameter platform that will enable the efficient reconfiguration of a workstation.

The end result of the application of the PPCTM to the design of process parameters is the synthesis, hierarchic organization, and determination of the ranges of multiple modes of managing capacity variety in order to construct a process parameter platform. With these fundamental theoretical constructs presented, the Process Parameter Platform Constructal Theory Method is presented in detail in Section 3. An example problem is presented alongside each of the six steps of this augmented version of the PPCTM to serve as a tutorial for the reader.

### 3. THE PROCESS PARAMETER PLATFORM CONSTRUCTAL THEORY METHOD

Through the application of the tenets of constructal theory, the development of a process parameter platform for a workstation is formulated as an optimal access problem and solved as multi-stage decision wherein the ranges of application of each mode for managing capacity variety are the decision variables. The goal of each decision is to improve the given objective function(s) in order to provide the most efficient manner of...
offering production capacity variety. The six steps of the methodology are shown in Figure 3.

The first step of the PPCTM involves abstracting the development of a process parameter platform as a problem of access in a geometric space by identifying the space of capacity. In the second step, the objective functions are defined. Typical objective functions include production performance metrics such as the minimization of average cost, or the maximization of throughput and/or product quality. The modes for managing variety are identified in the third step and are hierarchically organized in Step 4. The determination of the range of application of each mode for managing variety is done through the formulation and solution of a multi-stage utility-based compromise Decision Support Problem. With the extent of application of each mode known, a designer is capable of fully defining the process parameter family that offers the best compromise to the objective functions. An example problem, the design of a process parameter platform for the manufacture of customizable hearing aid shells, is presented as a means to illustrate the method.

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3.1. Example Problem: Process Parameter Platform Design for the Rapid Manufacture of Customized Hearing Aid Shells

Consider a manufacturer of hearing aids that seeks a competitive advantage by offering personalized hearing aids (Figure 4a). The manufacturer wishes to create these personalized hearing through the use of an additive fabrication technology (i.e., rapid prototyping). Specifically, the manufacturer can create a personalized hearing aid shell from a 3D CAD model of a patient’s ear canal (obtained by laser scanning a clay impression of the ear), using Stratasys’ Fused Deposition Modeling (FDM). As shown in Figure 4b, FDM is a rapid prototyping technology that creates objects by extruding a heated filament through a nozzle that lays down the part’s cross-section one layer at a time (Stratasys, 2003).

The manufacturer believes that the use of the FDM rapid prototyping technology will provide the manufacturing enterprise the agility and flexibility to offer a personalized hearing aid to each customer at a competitive price. Due to the nature of the technology, a rapid prototyping machine is capable of creating multiple, different geometries in a single build without having to change the primary machine tool, and is thus capable of creating thousands of unique hearing aids more efficiently than traditional hard tooling alternatives. This particular example is based on an actual current product line that is the result of a collaboration between Siemens and Phonak (Masters, 2002).

While the manner in which product variety will be offered has been determined, the specific manner in which the manufacturing process will be configured has not. The manufacturing engineer faces a dilemma, as the selection of process parameters for Fused Deposition Modeling is a very difficult and complex problem, especially when confronted with the need to satisfy three conflicting objectives: the minimization of production cost, the minimization of production time, and the maximization of the quality of each part. Specifically, the manufacturing engineer must determine the appropriate batch size, process parameters specific to FDM, as well as the type and number of machines to be used for different levels of production capacity.

The problem is further obfuscated by a production constraint: the parts must be completed no more than one week from the date they were ordered. Furthermore, as is typical with most mass-customized parts, the demand for this product is highly non-uniform – the demand for customized hearing aid shells range from an arrival rate of 120 parts per day to 1000 parts per day. As a result, each time the capacity requirement...
changes, the manufacturer is required to re-evaluate all of the process parameters in order to maintain maximum production efficiency, as well as pay the costly setup penalty of changing the process parameters.

In this example problem, the manufacturing engineer will benefit from the use of the PPCTM to generate a process parameter platform for the range of capacity requirements presented by this problem. Through the commonalization of process parameters, the setup and reconfiguration of the FDM machine will be more efficient when faced with changes in capacity requirement.

There are two key assumptions in the formulation of this example problem:

1. This manufacturing enterprise seeks to offer variety only through hearing aid shell geometry. Offering variety through a change in product material, product color, or in functionality is not considered in this example problem.

2. All relevant information and models needed to apply the augmented PPCTM are available, complete, and certain. The role of uncertainty and risk is not taken into account in the formulation of the problem.

The motivation behind the inclusion of this example problem is to highlight the authors’ core focus: the ability of a designer to use the PPCTM to design process parameter platforms. As such, the explanation of the modeling of the example problem is kept to a minimum in order to focus the reader’s attention to the method itself. The reader is directed towards (Williams et al., 2003) for an in-depth analysis of the modeling effort of this complex problem.

3.2. Step 1: Define the Geometric Space

The first step of the PPCTM is the abstraction of a geometric space from the problem. For the development of a process parameter platform, this geometric space is the space of capacity (defined in Section 2.2). The definition of an appropriate space of capacity involves the identification of the range of production capacity that the manufacturing enterprise wishes to offer. The resulting space of capacity is a one-dimensional space that is bounded by the range of required production capacity.

As stated in the problem definition the production capacity will fluctuate from 120 parts per day to 1000 parts per day. The resulting space of capacity is provided in Figure 5. Each point along this space of capacity represents a different level of production capacity.
3.3. Step 2: Define the Objective Functions

As stated in the problem description in Section 3.1, the manufacturing engineer wishes to find the best compromise between three conflicting objectives: to minimize the cost of the production process, to minimize the amount of time to build a batch of parts, and to maximize the quality of each part. The focus in this particular step of the PPCTM is to define the necessary objective functions.

The calculation of average time for the entire process family is based on the amount of time to build one batch, $t_{\text{batch}}$, of a particular capacity requirement from Equation 4. The average build time of the process family is calculated as:

$$
t_{\text{avg}} = \left[ \frac{\sum_{i=1}^{D_{\text{max}}} t_{\text{batch},i} N_{\text{batch},i}}{D_{\text{max}} - D_{\text{min}}} \right] + \frac{\sum_{j=1}^{N_{\text{setup}}}}{D_{\text{max}} - D_{\text{min}}}$$  \hspace{1cm} (4)

where $D_{\text{max}}$ and $D_{\text{min}}$ represent the upper and lower bounds of the capacity space respectively. It is important to note that this build time metric includes a setup time penalty, $t_{\text{setup}}$, of 30 minutes that is accrued for each different arrangement of process parameters, $N_{\text{setup}}$, across the production family. The time to build a single batch, $t_{\text{batch}}$, is calculated as:

$$
t_{\text{batch}} = t_{\text{warm}} + (t_{l} N_{l} + t_{\text{base}}) N_{pb}$$  \hspace{1cm} (5)

where:

- $t_{\text{warm}}$ = setup time; warming the machine for a build ($\cong$ 0.5 hr)
- $t_{l}$ = time to build each layer
- $N_{l}$ = number of layers
- $t_{\text{base}}$ = time to build the base of the part
- $N_{pb}$ = number of parts per batch

The time to build each layer is directly dependent on the road width parameter; as road width increases, the amount of material deposited in a single pass is increased; thus the amount of time spent depositing a layer decreases. The number of layers ($N_{l}$) needed to complete a part is the quotient of the height ($h$) of the part and the layer thickness ($t_{\text{layer}}$).

$$
N_{l} = \frac{h}{t_{\text{layer}}}$$  \hspace{1cm} (6)

While it is naïve to assume that the time to build each layer will be the same for each layer of each hearing aid shell, it is understood that this will not significantly influence the overall results or interfere with the validation of the augmented PPCTM.

The calculation of the average cost of the process family is performed in a similar fashion. The cost per batch of a single capacity requirement, $C_{\text{batch}}$, is calculated as:
\[ C_{\text{batch}} = (V_{\text{material}} \cdot C_{\text{material}}) \cdot N_{\text{pb}} + (C_{\text{labor}} + C_{\text{operation}}) \cdot t_{\text{batch}} + \left[ (C_{\text{maint}} + C_{\text{machine}}) \cdot N_{\text{machine}} / N_{\text{ppy}} \right] \cdot N_{\text{pb}} \]  

(7)

where:

- \( V_{\text{material}} \) = volume of material used to build one part (~229 mm\(^3\))
- \( C_{\text{material}} \) = cost of material; includes build and support materials (0.26 $/cm\(^3\) and 0.23 $/cm\(^3\), respectively)
- \( C_{\text{labor}} \) = cost of labor ($25 / hour)
- \( C_{\text{operation}} \) = hourly cost of machine operation ($80 / hour)
- \( C_{\text{maint}} \) = annual machine maintenance cost ($50,000 / year)
- \( C_{\text{machine}} \) = cost of purchasing machine, to be paid through one year of production
- \( N_{\text{ppy}} \) = number of products produced in one year

The hourly and annual fees for labor, operation, and maintenance are estimates of actual costs. The cost of purchasing the machine is presented in Table 1 along with other machine specifications. Their values do not change the fundamental validation strategy of applying the augmented PPCTM to this example problem.

The average cost is calculated as:

\[
C_{\text{avg}} = \left( \sum_{i=D_{\text{min}}}^{D_{\text{max}}} C_{\text{batch},i} \cdot N_{\text{batch},i} \right) + \left( \sum_{j=1}^{N_{\text{setup}}} C_{\text{setup},j} \right) \left/ D_{\text{max}} - D_{\text{min}} \right.
\]

(8)

A penalty for the setup of a different arrangement of process parameters is added to this metric with \( C_{\text{setup}} \), which is equal to $50.

For many products, quality is a metric with a subjective nature. When manufacturing with rapid prototyping, quality can be quantified by modeling the difference between the desired geometry and the actual, produced geometry. This error occurs because of the nature of this layer-based, additive manufacturing technology (known as the “stair stepping” effect, Figure 6). The quality of the part is directly related to two process parameters: the road width (\( w_{\text{road}} \)) and the layer thickness (\( t_{\text{layer}} \)) of the deposition.

As can be observed in Figure 6, the best quality is achieved with a minimal layer thickness and minimal road width. The average quality of the process family is calculated as:

\[
Q_{\text{avg}} = \left( \sum_{i=D_{\text{min}}}^{D_{\text{max}}} Q_i \right) \left/ (D_{\text{max}} - D_{\text{min}}) \right.
\]

(9)

Table 1. Machine Characteristics (Stratasys, 2003)

<table>
<thead>
<tr>
<th>Machine</th>
<th>Max. Parts</th>
<th>Scan Speed</th>
<th>Layer Thickness</th>
<th>Road Width</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prodigy Plus</td>
<td>234</td>
<td>64 mm/s</td>
<td>0.178 – 0.33 mm</td>
<td>0.19 – 0.21 mm</td>
<td>$70,000</td>
</tr>
<tr>
<td>Titan</td>
<td>818</td>
<td>127 mm/s</td>
<td>0.24 – 0.26 mm</td>
<td>0.19 – 0.21 mm</td>
<td>$210,000</td>
</tr>
<tr>
<td>Maxum</td>
<td>1703</td>
<td>254 mm/s</td>
<td>0.127 – 0.25 mm</td>
<td>0.193 – 0.965 mm</td>
<td>$260,000</td>
</tr>
</tbody>
</table>
where $Q_i$ is the quality of a single part, estimated as the sum of the road width and the layer thickness.

Unfortunately, these objectives are inherently contradictory. For example, to maximize the quality, one would set the layer thickness and road width parameters to a minimum. This however, would drastically increase material costs as well as production time.

3.4. Step 3: Identify the Modes for Managing Variety

For this step, a designer identifies appropriate methods for providing variety in production capacity for the manufacturing system. Modes of managing variety are the linking mechanism between the different production capacity requirements. For this third step in the augmented PPCTM, the designer identifies those modes that will be used to offer variety in the production capacity of the manufacturing process. Since the space for capacity is one-dimensional, each mode will be directed towards achieving variety in capacity only. For this example problem, five different modes for managing process customization are implemented.

3.4.1. Mode D1: Customization of the Batch Size

One mode of managing capacity variety for this example problem is the standardization of the batch size (i.e., number of parts per build). With the use of this mode, changes in capacity are achieved by choosing a different batch size. The determination of batch size is a very important parameter decision. Larger batch sizes are preferred because of the associated decrease in costs. The batch size cannot be too large as the process is constrained both by the physical capacity limitation of each machine type (see Table 1), and by the weeklong production-time constraint.

3.4.2. Modes D2 and D3: Standardization of the Process Parameters

For these two modes, changes in capacity are achieved by standardizing the process parameters of the manufacturing process: layer thickness (the height of each individual layer deposited by the process) and road width (the width of the material deposited by the extrusion nozzle). The determination of these parameters is very important to the overall process, as each of the parameters plays a large role in the calculation of each of the three
objective functions. In comparison to modifying the road width of the process (Mode D3), production capacity changes of higher fidelity can be achieved by slightly changing the layer thickness (Mode D2).

3.4.3. Mode D4: Commonalization of Machine Type

Changes in capacity are also achieved by changing the type of machine used in the production process. Changing the machine type changes the maximum batch size constraint, and also affects the speed at which the layers of the parts can be drawn. Of course, different machines cost different amounts of money, so the objective of minimizing cost is severely altered with each different machine type. Similar to all forms of technology, larger, faster machines cost more money and typically offer better production quality (see Table 1). As such, the selection of machine type is crucial. The use of this mode for managing variety commonalizes the scan speed and maximum parts allowed per batch over a range of capacity needs.

3.4.4. Mode D5: Modular Combination of Machines

This fifth and final mode of managing capacity variety for this example problem is the selection of the number of machines used for the production of the shells. This mode involves the notion of modular combination to achieve variety. For this example, “modular combination” refers to the addition of machines to compensate for large demands of capacity. Since it is assumed that only similar machines can be added to the production process, the addition of a machine simply doubles the capacity of the manufacturing process. This doubling comes at a large cost however.

These five modes of managing process customization are the approaches considered for accessing all the points of the space of capacity generated in Step 1 (Section 3.2) of this example. Next, in Steps 4 through 6 (Sections 3.5 – 3.7), a hierarchic organization of these modes is synthesized in order to offer a variety of production capacity in the space of customization.

3.5. Step 4: Identify the Number of Hierarchy Levels and Allocate the Modes for Managing Variety to the Levels

In the fourth step of the PPCTM, it is established how and when each mode of managing variety is used. Modes that are capable of achieving the smallest variations in production capacity are typically used at the lower levels of the hierarchy (i.e., before modes that can only achieve large variations in capacity). Economical and technological considerations place an important role in establishing the hierarchic use of the modes for managing variety.

Following the tenets of constructal theory, each level of the hierarchy represents a geometric “sub-space” of the entire space of customization. The sizes of each sub-space represent the extent of application of each mode for managing variety, and are the decision variables of this multi-stage design problem.
3.5.1. The First Stage and the First Space Element

For this example problem, of all the process parameters to be varied, altering the batch size provides the best “control” over changes in the required production capacity. Customizing the batch size provides the simplest (changing the batch size does not require additional operator input) and most cost effective manner (there are no setups costs associated with changing the batch size) for a designer to offer a specific production capacity requirement. For these reasons, this mode is placed at the bottom of the mode hierarchy. The mode D1, “Customization of the Batch Size,” is used to define a common batch size for a range of capacities that are bounded in the first space element, $\Delta D_1$, as shown in Figure 7.

3.5.2. The Second Stage and the Second Space Element

The mode D2, “Standardization of Layer Thickness,” is chosen as the mode for managing variety for the second stage in the hierarchy. Of the modes remaining for selection, altering this process parameter provides a designer with the ability to make smaller adjustments in the level of production capacity. Mode D2 is used to define a common layer thickness for a range of capacities that are bounded by the second space element, $\Delta D_2$ (Figure 7). Each second space element is composed of a number of first space elements.

3.5.3. The Third Stage and the Second Space Element

This third stage follows the same formulation as found in the previous stages. Mode D3, “Standardization of Road Width,” is used to define a common road width for a range of capacities that are bounded by the third space element, $\Delta D_3$ (Figure 7). Mode D3 is placed at a higher level of the hierarchy than mode D2 (changing the layer thickness) because changing the value of road width does not provide as such high fidelity changes in production capacity. The build time metric is not as sensitive to changes in road width as it is with changes in layer thickness. Similar to layer thickness, increasing the road width of a part decreases the build time and cost, as well as lowers the average part quality.
3.5.3. The Fourth Stage and the Second Space Element

The fourth and final space element is composed of a number of assemblies of the third space element as shown in Figure 7. In this final space the remaining two modes of managing variety, Mode D4 ("Commonalization of Machine Type") and Mode D5 ("Altering the Number of Machines") are used.

The application of these modes is different than the application of the other modes in the previous stages. Mode D4 is based on the concept that certain capacity requirements are suited for different types of machines. Whether limited by maximum build size or by a slow scan speed, lower-end machines simply cannot produce parts at a sufficient rate to meet larger demands. This applies to the other end of the capacity space spectrum as well; higher-end machines may be too expensive to justify their use for lower capacity needs.

Mode D5, the combination of similar machines, is investigated in this space as an opportunity to offer more capacity. Increasing the number of machines produces an interesting tradeoff: it not only increases the amount of capacity of the production process by decreasing the build time, but it also increases the costs associated with purchasing and maintaining the machines. Since these two modes are only able to offer expensive, large, discrete changes in production capacity, they are used at the highest level of the hierarchy.

The focus in this decision stage is the assignment of different machine types and quantities to specific ranges of the capacity space. This is achieved by identifying six cutoff points along the space (each cutoff representing each of the discrete upgrade of production capacity). As can be seen in Figure 8, each cutoff point represents a different combination of machine type and quantity. This specific ordering of each cutoff point is based on the maximum number of parts per build of each machine type/quantity combination (see Table 1).

With all of the modes for managing variety successfully organized (Figure 9), the formulation of the multi-stage utility-based compromise Decision Support Problem begins.
3.6. Step 5: Formulate a Multi-Stage Utility-Based Compromise Decision Support Problem

Following the tenets of constructal theory, the determination of the range of application for each mode for managing variety that composes a level of the hierarchy (or sub-space) represents one stage in a multi-stage decision. With the order of the use of the modes established, a designer proceeds by formulating a proper multi-stage decision problem.

In this work, each decision stage is formulated with a utility-based compromise Decision Support Problem. The utility-based compromise DSP (u-cDSP) is a decision support construct that is based on utility theory and permits mathematically rigorous modeling of designer preferences such that decisions can be guided by expected utility in the context of risk or uncertainty associated with the outcome of a decision. While any appropriate decision formulation technique is serviceable, the authors prefer to use the u-cDSP because its use “provides structure and support for including human judgment in engineering decisions involving multiple attributes, while simultaneously providing an axiomatic basis for accurately reflecting the preferences of a designer with regard to feasible tradeoffs among these attributes under conditions of uncertainty” (Fernández et al., 2001). Furthermore, the u-cDSP has proven useful in previous product platform techniques as it provides a decision construct in which a designer can model multiple, conflicting objectives (Seepersad et al., 2002). The formulation of each utility-based compromise Decision Support Problem follows the four steps presented in (Seepersad et al., 2002) (Figure 10).

First a utility function for each of the objectives is formulated by qualitatively and quantitatively assessing the preferences of the designers (all designers’ preferences are modeled as risk averse in this work). These individual utility functions are then combined into a multi-attribute utility function as a weighted average of the individual utilities. Finally, goal and deviation functions are developed for each stage. The deviation function of the u-cDSP is formulated to minimize deviation from the target expected utility (i.e., 1, the most preferable value), which is mathematically equivalent to
maximizing expected utility. The goal and deviation functions formulated for each u-cDSP inherently consider the compromise of the tradeoffs between the each objective function. With the goal of minimizing the deviation of the expected utility from the ideal value, parameters that provide the best values for this overall objective are chosen while maintaining consistency with the designer’s preferences. With the presence of the u-cDSP, designers are given the ability to model multiple objectives in each decision stage of the PPCTM.

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In this example, the multi-attribute utility function for each stage is based on the individual utility functions of cost, time, and quality. The values to be used in the utility assessment are calculated using the average values of the metrics calculated in each decision stage. Each utility function is presented in Table 2.

The $k$-values presented in the table above are calculated by solving a system of equations wherein a designer establishes quantitative preferences for each scaling constant. The $k$-values are used in concert with the individual utility functions for the determination of the overall expected utility, $E[U(D_o)]$. 

Figure 10. The Formulation of the Utility-Based Compromise Decision Support Problem
Table 2. Utility Functions for Cost, Time, and Quality

<table>
<thead>
<tr>
<th>Utility Value</th>
<th>Cost ($), $k_c = 0.25$</th>
<th>Time (hrs), $k_t = 0.25$</th>
<th>Quality (mm), $k_q = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>500</td>
<td>13</td>
<td>0.35</td>
</tr>
<tr>
<td>0.75</td>
<td>8500</td>
<td>63</td>
<td>0.65</td>
</tr>
<tr>
<td>0.5</td>
<td>15250</td>
<td>105</td>
<td>0.9</td>
</tr>
<tr>
<td>0.25</td>
<td>21000</td>
<td>139</td>
<td>1.12</td>
</tr>
<tr>
<td>0</td>
<td>25000</td>
<td>168</td>
<td>1.3</td>
</tr>
</tbody>
</table>

- Utility Function:

\[ u(C) = 2 - e^{\frac{0.685(C - C_{min})}{(C_{max} - C_{min})}} \]
\[ u(t) = 2 - e^{\frac{0.685(t - t_{min})}{(t_{max} - t_{min})}} \]
\[ u(q) = 2 - e^{\frac{0.6851(q - q_{min})}{(q_{max} - q_{min})}} \]

*the min and max subscripts refer to the values of each parameter with utilities of 0 and 1 respectively.

\[ U(D_j) = k_c u(C) + k_t u(t) + k_q u(Q) \] (10)

The goal of each objective is to maximize the value of each individual utility function (i.e., for the value to reach the ideal value, 1).

\[ E[u(C)] + d_+^C + d_-^C = 1 \] (11)
\[ E[u(t)] + d_+^t + d_-^t = 1 \] (12)
\[ E[u(q)] + d_+^q + d_-^q = 1 \] (13)

The deviation function for the multi-attribute utility function of each decision stage, therefore, is to minimize the deviation of the expected utility function from the ideal value:

\[ Z_i = 1 - E[U_i(D_j)] = \sum_j k_j (d_+^j + d_-^j) \] (14)

In order to evaluate the values of the three objective functions for each subspace, the values of parameters to be commonalized across the space must first be determined. In order to satisfy the property of near-decomposability of hierarchic systems, the choice of these design variables for each space element must be independent of the choice for the other space elements. The decision of appropriate values to be commonalized is based on the largest values of capacity requirement of the subspace, or geometrically the rightmost point of each space element, since its solution will be sufficient for all variants within that space.

Due to the multiple objectives involved with this example, one cannot identify a specific value of the process parameter that satisfies production constraints and also minimizes cost and time, and maximizes quality without a detailed analysis. In order to determine the appropriate values of the parameters to be commonalized for each subspace...
a designer must employ a utility-based compromise DSP. This initial decision is presented in Figure 11 as “Decision 0.” The max and min subscripts in Figure 11 refer to the imposed bounds of the machine for each process parameter (see Table 1). The individual utility functions of Decision 0 (E[u(C)], E[u(t)], E[u(q)]) are evaluated using Eqs. (4), (8), and (9) respectively. Decision 0 will be used inherently in each of the decision stages in order to determine the value of the parameter that will be commonalized across each sub-space.

3.6.1. The First Stage and the First Space Element

The first space element is defined by the range of application of Mode D1: “Customization of the Batch Size”. \( \Delta D \) determines the extent of the commonality of the batch size, and is therefore the decision variable for this first stage. The focus in this decision is the determination of \( \Delta D \) that provides the best value for the overall objective – the maximization of the expected utility of all the objectives.

In this first stage, the calculation of cost, time, and quality at each node begins with the determination of the value of the batch size that should be commonalized. As stated above, this value is determined at the maximum capacity value of the current sub-space.

---

**Figure 11. Decision Formulation for “Decision 0”**

| Given: | Capacity Requirement; \( D_d \) (parts/day) |
| Machine Type; Prodigy Plus, FDM Titan, or FDM Maxum |
| Machine Number; \( N_{mach} \) |
| Find: | Batch Size; \( N_{pb} \) |
| Layer Thickness; \( t_{layer} \) |
| Road Width; \( w_{road} \) |
| Deviation variables; \( d_i \) and \( d_i^* \) |
| Satisfy: | Bounds: \( 0 \leq N_{pb} \leq N_{pb,max} \) |
| \( t_{layer,min} \leq t_{layer} \leq t_{layer,max} \) |
| \( w_{road,min} \leq w_{road} \leq w_{road,max} \) |
| Constraints: | \( t_{cycletime} \leq 1 \) week |
| \( d_i, d_i^* \geq 0 \) |
| \( d_i \cdot d_i^* = 0 \) |
| Goals: | \( E[u(C)] + d_c \cdot d_c^* = 1 \) |
| \( E[u(t)] + d_t \cdot d_t^* = 1 \) |
| \( E[u(Q)] + d_q \cdot d_q^* = 1 \) |
| Minimize: | \( Z = 1 - E[U(D)] = 1 - \left[ k_c (d_c - d_c^*) + k_t (d_t - d_t^*) + k_q (d_q - d_q^*) \right] \) |
Given the layer thickness and road width, the batch size that provides the largest expected utility within the space is solved for using the u-cDSP outline in “Decision 0” (Figure 11).

This common batch size is then applied to each capacity requirement of the first space. The resulting time, cost, and quality metric values are then averaged across the first space. This is done by first evaluating the number of first space elements in the space of capacity:

$$N_i = \frac{(D_{\max} - D_{\min})}{(D_{\max,1} - D_{\min,1})}$$  \hspace{1cm} (15)

The values of $D_{\max}$ and $D_{\min}$ are total range of capacity offered; $D_{\max,1}$ and $D_{\min,1}$ are based on the corresponding value of $\Delta D_i$:

$$D_{\max,1} = D_{\min,1} + \Delta D_i$$  \hspace{1cm} (16)

The average time of all of the first space elements is calculated as:

$$t_{\text{avg}} = \frac{\sum_{i=1}^{N} t_{\text{avg},i,i} + \sum_{j=1}^{n} t_{\text{setup},j}}{(D_{\max} - D_{\min})}$$  \hspace{1cm} (17)

where $t_{\text{avg},i,j}$ is the average time of a single first space element. Similarly the average cost of the first space elements is calculated with:

$$C_{\text{avg}} = \frac{1}{D_{\max} - D_{\min}} \left[ \left( \sum_{i=1}^{N} C_{\text{avg},i,i} \right) + \left( \sum_{j=1}^{n} C_{\text{setup},j} \right) \right]$$  \hspace{1cm} (18)

The third objective, the maximization of average quality, is averaged across the space of capacity via:

$$Q_{\text{avg}} = \frac{1}{D_{\max} - D_{\min}} \left( \sum_{i=1}^{N} Q_{\text{avg},i,i} \right)$$  \hspace{1cm} (19)

These averaged values are then used to calculate an expected utility of the first space. The individual utility functions ($u(C)$, $u(t)$, and $u(Q)$), and the resulting expected utility function ($E[U(D)]$), are calculated as presented in Table 2. The resulting decision formulation for the first stage is shown in Figure 12.

Inherent in this first stage decision is the value of layer thickness and road width process parameters, as well as the type and number of machines used in the production process. The individual utility functions cannot be evaluated without these values. These crucial details are determined in the following decisions.
Given: The one-dimensional capacity space
Mode D1: Customization of the Batch Size

Find: The value of the decision variable $\Delta D_1$
Deviation variables, $d_C^-, d_C^+, d_t^-, d_t^+, d_Q^-, d_Q^+$

Satisfy: Bounds: $0 \leq \Delta D_1 \leq 880$

Constraints: $t_{cycle} \leq 1$ week
$d_i^-, d_i^+ \geq 0$
$d_i^+ \cdot d_i^- = 0$

Goals: $E[u(C)] + d_C^- - d_C^+ = 1$
$E[u(t)] + d_t^- - d_t^+ = 1$
$E[u(Q)] + d_Q^- - d_Q^+ = 1$

Minimize: $Z_1 = 1 - E[U(D_d)] = 1 - \left[ k_1(d_C^- - d_C^+) + k_2(d_t^- - d_t^+) + k_3(d_Q^- - d_Q^+) \right]$

Figure 12. Decision Formulation for the First Space Element

3.6.2. The Second Stage and the Second Space Element

The range of application of Mode D2, “standardization of the layer thickness,” defines the size of the second space element. Each second space element is composed of a number of first space elements. The number of first space elements that compose a second space element, $N_{1,2}$, is calculated as:

$$N_{1,2} = \Delta D_2 / \Delta D_1 = \left( D_{max,2} - D_{min,2} \right) / \left( D_{max,1} - D_{min,1} \right)$$

(20)

The range of commonality of the layer thickness process parameter is defined by the decision variable $\Delta D_2$. Similar to the previous decision stage, “Decision 0” is used to identify the layer thickness that provides the best compromise between the three conflicting objectives for the largest capacity requirement of each second sub-space. This value is then commonalized over a series of capacity requirements as dictated by the value of $\Delta D_2$.

Similar to the first stage, the focus of this second decision stage is the selection of a value of $\Delta D_2$ that maximizes the expected utility of the space of customization defined in Step 1. The calculation of the average time, cost, and quality of each second space element is dependent on the number of first space elements that compose the second space element:

$$t_{avg,2} = \frac{\sum_{i=1}^{N_{1,2}} t_{avg,1,i}}{N_{1,2}}$$

(21)
The average time, cost, and quality of all of the second space elements is calculated in a similar fashion as is done for the first space elements (Eqs. 17-19). Following the format of Stage 1, the averaged values are then used to calculate the individual utility functions (Table 2). Finally, these individual utility functions are combined into an overall expected utility function. The focus in the decision of this second stage (Figure 13) is the minimization of the deviation of this expected utility from 1.

3.6.3. The Third Stage and the Third Space Element

For the third space Mode D3, standardization of road width, is used to offer variety in the production capacity of the manufacturing process. The range of commonality of road width, \( \Delta D_3 \), determines the size of each third space element. Each third space element is composed of a number of second space elements.

Given: The one-dimensional capacity space
Mode D2: Standardization of Layer Thickness
The value of \( \Delta D_1 \)

Find: The value of the decision variable \( \Delta D_2 \)
Deviation variables, \( d_C^- \), \( d_C^+ \), \( d_t^- \), \( d_t^+ \), \( d_Q^- \), \( d_Q^+ \)

Satisfy: Bounds: \( 0 \leq \Delta D_2 \leq 880 \)
Constraints:
\( \Delta D_1 \leq \Delta D_2 \leq 880 \)
\( t_{cycle} \leq 1 \) week
\( d_i, d_i^+ \geq 0 \)
\( d_i^- \cdot d_i^+ = 0 \)

Goals:
\( E[u(C)] + d_C^- - d_C^+ = 1 \)
\( E[u(t)] + d_t^- - d_t^+ = 1 \)
\( E[u(Q)] + d_Q^- - d_Q^+ = 1 \)

Minimize: \( Z_2 = 1 - E[U(D)] = 1 - \left[ k_c (d_c^- - d_c^+) + k_t (d_t^- - d_t^+) + k_Q (d_Q^- - d_Q^+) \right] \)

Figure 13. Decision Formulation for the Second Space Element
The number of second space elements that compose a third space element, \( N_{2,3} \), is calculated as:

\[
N_{2,3} = \frac{\Delta D_3}{\Delta D_2} = \left( \frac{D_{\text{max},3} - D_{\text{min},3}}{D_{\text{max},2} - D_{\text{min},2}} \right) \tag{24}
\]

The formulation of the decision third for the third stage is identical to that of the previous two. There are two decision variables for this third stage, the value of road width to be commonalized, and its range of commonality, \( \Delta D_3 \). “Decision 0” is used to identify the road width that provides the best compromise between the three conflicting objectives for the largest capacity requirement of each second sub-space. This value is then commonalized over a series of capacity requirements as dictated by the value of \( \Delta D_3 \).

Since the formulation of the objective functions and the decision of a third space element is extremely similar to that of a second space element (Eqs. 21-23; Figure 13), their explicit forms are not presented here for the sake of brevity.

### 3.6.4 The Fourth Stage and the Fourth Space Element

The size of the fourth and final space element is determined by the range of commonality of each machine type / quantity combination. The range of application of Modes D4 and D5 are defined by the placement of the “cut-off” points – the capacity requirements that require a different machine type and/or quantity. As with the previous decision stages, the focus of the implementation of Modes D4 and D5 in this fourth stage is the maximization of expected utility for the entire capacity space. Similar to the previous steps, average time, cost, and quality are all calculated as a sum of the spaces of the previous stage. The analysis begins with the calculation of the number of third space elements in each fourth stage cutoff point, \( 4,i \).

\[
N_{3,4} = \frac{\Delta D_{4,4,i}}{\Delta D_3} = \left( \frac{D_{\text{max},4,i} - D_{\text{min},4,i}}{D_{\text{max},3} - D_{\text{min},3}} \right) \tag{25}
\]

The average time, cost, and quality of a fourth space element is evaluated using Eqs. (29) – (31).

\[
t_{\text{avg},4} = \frac{\sum_{j=1}^{N_{3,4}} t_{\text{avg},3,j}}{N_{3,4}} \tag{26}
\]

\[
c_{\text{avg},4} = \frac{\sum_{j=1}^{N_{3,4}} c_{\text{avg},3,j}}{N_{3,4}} \tag{27}
\]

\[
q_{\text{avg},4} = \frac{\sum_{j=1}^{N_{3,4}} q_{\text{avg},3,j}}{N_{3,4}} \tag{28}
\]
The one-dimensional capacity space

Mode C4: Commonalization of Machine Type

Mode C5: Altering the Number of Machines

The value of $\Delta D_1$

The value of $\Delta D_2$

The value of $\Delta D_3$

Find:

The value of the decision variable $\Delta D_4$

The location of each cutoff point, $D_1', D_2', D_3', D_4', D_5'$

Deviation variables, $d_{C^-}, d_{C^+}, d_i^-, d_i^+, d_q^-, d_q^+$

Satisfy:

Bounds: $0 \leq \Delta D_4 \leq 880$

Constraints:

$\Delta D_1 \leq \Delta D_4 \leq 880$

$t_{cycle}\leq 1\ \text{week}$

$d_i^-, d_i^+ \geq 0$

$d_i^- \cdot d_i^+ = 0$

Goals:

$E[u(C)] + d_{C^-} \cdot d_{C^+} = 1$

$E[u(t)] + d_i^- \cdot d_i^+ = 1$

$E[u(Q)] + d_q^- \cdot d_q^+ = 1$

Minimize:

$Z_4 = 1 - E[U(D)] = 1 - \left[ k_C (d_{C^-} - d_{C^+}) + k_i (d_i^- - d_i^+) + k_q (d_q^- - d_q^+) \right]$

Figure 14. Decision Formulation for the Fourth Space Element

The average time, cost, and quality of all of the fourth space elements is calculated in a similar fashion as is done for the first space elements (Equations 17-19). Following the format of the previous stages, these averaged values are then used to calculate the individual utility functions as shown in Table 2. Finally, these individual utility functions are combined into an overall expected utility function (Equation 10). The focus in the decision of this fourth stage (Figure 14) is the minimization of the deviation of this expected utility from 1.

The result of the solution process is the determination of the ranges of the modes of managing process customization that will produce the best compromise between the three objectives (minimize cost, minimize build time, and maximize quality).

3.7. Step 6: Solve the Multi-Stage Utility-Based Compromise Decision Support Problem

The final step of the PPCTM is the formulation of an appropriate solution algorithm. Any appropriate solution technique can be used; the primary goal in the solution is the determination of the ranges of the modes that provide the largest expected utility (i.e., the best compromise between the conflicting objectives of minimizing cost, minimizing build time, and maximizing part quality). A graphical representation of the solution method for this problem is presented in Figure 15.
This solution method involves iterating through values of the modes of managing variety ($\Delta D_1$, $\Delta D_2$, $\Delta D_3$, and $\Delta D_4$), establishing the dimensions of the sub-spaces, commonalizing the design parameters ($N_{pb}$, $t_{layer}$, and $w_{road}$) across each sub-space, evaluating the objective functions, and comparing the resulting overall utility of each iteration.
4. RESULTS

The result of the application of the PPCTM to this example problem is the hierarchic organization of the five modes of managing production variety (Section 3.4), and the determination of their range of application across the process parameter platform that provides the best compromise among the three objectives. The extent of application of each mode and the resulting average cost, time, and quality for the platform is shown in Table 3.

Table 3. Range of Each Mode of Managing Capacity Variety for Hearing Aid Example

<table>
<thead>
<tr>
<th>Mode of Managing Customization</th>
<th>Parameter</th>
<th>Range of Commonalization (parts / day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>∆D₁ (customization of batch size)</td>
<td>Batch Size</td>
<td>5</td>
</tr>
<tr>
<td>∆D₂ (standardization of layer thickness)</td>
<td>Layer Thickness</td>
<td>21</td>
</tr>
<tr>
<td>∆D₃ (commonalization of road width)</td>
<td>Road Width</td>
<td>21</td>
</tr>
<tr>
<td>∆D₄ (commonalization of machine type: modular combination of machines)</td>
<td>One FDM Maxum</td>
<td>880</td>
</tr>
</tbody>
</table>

\[ t_{avg} = 47.42 \text{ hrs} \quad C_{avg} = $11,659.32 \quad Q_{avg} = 0.509 \text{ mm} \]

This table of results provides a manufacturing engineer with the range of commonalization for each mode of managing process customization. This is presented graphically in Figure 16. These results inform the manufacturing enterprise that, in order to achieve the best compromise between all three objectives, the best configuration of the modes of managing variety is to use one FDM Maxum, to commonalize the batch size for every interval of 5 parts per day, and to commonalize road width and layer thickness parameters for every interval of 21 parts per day. From these ranges of application for each mode of managing variety, the specific values of the design variables are derived by the use of the PPCTM (Table 4).

To illustrate the significance of this result, the following tutorial example is presented. Imagine that the manufacturing enterprise reports an arrival rate of demand of 234 parts per day (point A in Figure 16). Looking at Table 4, the manufacturing engineer, using one FDM Maxum, selects a batch size of 1299 parts/batch, a road width of 0.22 mm, and a layer thickness of 0.18 mm. If the manufacturing capacity requirement changes to 183 parts per day the following week (point B in Figure 16), the engineer is only required to change the batch size to 1047 parts/batch and the road width to 0.19 mm without the need to change the value of the layer thickness parameter.
Figure 16. Graphical Representation of Resultant Process Parameter Platform for Hearing Aid Example

Table 4. A Sample of the Mapping of the Process Parameters

<table>
<thead>
<tr>
<th>Capacity (parts/day)</th>
<th>Batch Size (parts)</th>
<th>Road Width (mm)</th>
<th>Layer Thickness (mm)</th>
<th>Machine Type</th>
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A thorough investigation of these results provides interesting lessons about the methodology and the example problem itself. For instance, it can be questioned why only one machine type and quantity was chosen to be commonalized across the entire space of capacity. Specifically, how can the most expensive machine be the best choice for low capacity requirements? After careful observation it is concluded that while the less expensive machine does offer a lower total cost, the difference in cost is not enough to compete with the higher-end machine which offers a faster build time and a better part quality. As a result, the higher-end machine is the best choice for the entire space of capacity; it simply offers more machine for its price. This observation provides a great opportunity to witness the ability of a designer to consider multiple objectives in the design of a production platform with the PPCTM.

Another interesting observation is that, from watching the iteration history, the value of expected utility does not change drastically with different ranges of application of each mode for managing variety. From this observation one can witness a major limitation of the use of an expected utility function for this specific example problem. The formulation of the expected utility function (Table 2) is broad enough to capture the extremely different capabilities and properties of each machine. This results, however, in a function that is insensitive to small changes in decision parameters. It is therefore recommended that a designer’s preferences be clearly identified, and be as scoped as possible for the application of the PPCTM to this type of problem. It is noted that the use of the u-cDSP is not a limitation of the PPCTM; the expected utility function’s insensitivity to small parameter changes is a result of the model itself, as documented in (Williams et al., 2003).

Through the solution of this example problem, it is evident that the PPCTM is an effective means of designing process parameter platforms. The issue of the usefulness of production process itself has yet to be addressed, however. In order to answer this issue, the values of each objective are compared between the results from the development of a production platform (using the PPCTM), with the result of having no commonality in process parameters for each capacity requirement (i.e., parameters are reevaluated and reconfigured for each change), and with the result of making all process parameters common across the space of capacity (i.e., there is one parameter configuration for all capacity requirements). This comparison is provided in Table 5.

As can be observed from the table, the concept of developing a process parameter platform for this example problem provides an improvement to all three objective functions (and thus provides the largest expected utility) when compared to having “pure” and “no” commonality. Although there is only a small quantitative benefit for this specific problem, it is evident that the development of a process parameter platform improves the workstation’s ability to adapt efficiently to changes in its required capacity.

Table 5. Comparison of Results of a) PPCTM, b) No Commonality, and c) Strict Commonality of Process Parameters

<table>
<thead>
<tr>
<th></th>
<th>$t_{avg}$ (hrs)</th>
<th>$C_{avg}$ ($)</th>
<th>$Q_{avg}$ (mm)</th>
<th>$E[U(D)]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) PPCTM Result</td>
<td>47.42</td>
<td>11,659.32</td>
<td>0.509</td>
<td>0.8360</td>
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<tr>
<td>(b) No Commonality ($\Delta C_X = 1$)</td>
<td>48.16</td>
<td>11,734.33</td>
<td>0.507</td>
<td>0.835</td>
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<tr>
<td>(c) Pure Commonality ($\Delta C_X = 880$)</td>
<td>49.52</td>
<td>12,681.23</td>
<td>0.756</td>
<td>0.720</td>
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</table>
Although it is evident that the development of a process parameter platform is of some benefit for this example problem, it is important to note that this success is not necessarily universal. It is important to note that this methodology is most beneficial when the cost and time required for the changeover of a workstation is very costly. Furthermore, if the manufacturer expects the workstation to encounter wildly different required capacity from day to day (e.g., a change of over 200 parts/day for this example problem), the benefits of the process parameter will be lost. Since commonality between the process parameter variants is dependent upon the order of arrival, it is possible that no commonality between different setups will be encountered if the change in production capacity is continually larger than the range of application of each mode for managing variety. This limitation can be alleviated by accounting for the frequency of different capacity requirements with a probability distribution function.

While this specific example problem provides an appropriate means for validating the proposed methodology, it is important to note its unique characteristics. Rapid manufacturing is a special class of production process because it is capable of producing multiple products in one batch. It is also capable of producing an entire part with a single workstation without an assembly step, and thus, there is no differentiation between a “part” and the end product. Since it is possible to manufacture a product with a single rapid manufacturing machine, decisions regarding batch size, machine selection, and machine quantity are able to be made during the determination of process parameters. This is not a valid assumption when designing a production process with the more traditional and less flexible means of manufacturing. Despite the uniqueness of this example problem, the authors are confident in the adaptability of the methodology to a wide range of applications. The authors are aware that more traditional means of manufacturing involve complex decisions regarding sequencing and capacity planning. For this reason, this methodology is scoped to be applicable to the process parameter design of single workstations.

5. CLOSURE

In this chapter the authors present the concept of process parameter platforms. Process parameter platforms are a set of common process parameters from which a stream of derivate process parameters can generate a customized product efficiently despite changes in capacity requirement. The goal in creating a platform of process parameters is to create an efficient manner of offering variety in production capacity. Similar to product platforms, process parameter platforms achieve variety efficiently through commonality and/or modularity.

Along with this concept, the authors present a design methodology for realizing process parameter platforms. Leveraging from previous work with the Product Platform Constructal Theory Method, a powerful product platform design technique, the design of process parameter platforms is treated as a problem of access in a geometric space. The use of the PPCTM provides a designer the ability to accommodate the issues of:

- multiple design objectives: As shown in Section 3.3, the development of the platform required the compromise of three conflicting objectives: the
minimization of production cost, the maximization of quality, and the maximization of throughput.

- **multiple modes of offering variety**: As illustrated in Section 3.4, a designer must synthesize multiple modes of offering variety (standardizing process parameters, commonalizing batch size, standardizing machine type, and modularly combining machines) in order to provide a means of achieving all variants within the space of capacity. The use of the PPCTM synthesizes multiple modes of offering variety through hierarchic organization in order to offer variety efficiently.

- **volatile markets**: Through the definition of the space of capacity in Section 3.2, a designer is able to develop platforms in the presence of changing capacity requirements for workstations - a feature inherent in the manufacture of customized goods.

- **the inherent tradeoffs between platform extent and performance**: As described in Sections 3.6 and 3.7, the determination of the range of application of each mode for managing variety is achieved systematically through the rigorous formulation of a multi-stage utility-based compromise Decision Support Problem.

An example problem, the design of a process parameter platform for the manufacture of a line of customizable hearing aid shells, is presented as an example problem to aid in the description of the methodology. Through the application of the methodology to this example problem, it is shown that the design of a process parameter platform for a specific workstation minimizes successfully the necessary setup and changeover encountered with changes in capacity requirement; however, it is noted that this benefit is only seen when the changes in capacity are not widely distributed along the range of required capacity.

6. ACKNOWLEDGEMENTS

We gratefully acknowledge the support of NSF Grants DMI-0085136 and DMI-9900259. Christopher Williams is a Georgia Tech President’s Fellow and a NSF IGERT Research Fellow through the Georgia Tech T-GER program. The cost of computer time was underwritten by the Systems Realization Laboratory at the Georgia Institute of Technology.
## 7. NOMENCLATURE

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<th>Description</th>
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<td>Average cost of family of processes; $</td>
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<tr>
<td>( C_{\text{batch}} )</td>
<td>Cost of building a single batch of hearing aid shells; $</td>
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</tr>
<tr>
<td>( C_{\text{labor}} )</td>
<td>Cost of labor to operate FDM machines; $/hr</td>
<td>$/hr</td>
</tr>
<tr>
<td>( C_{\text{machine}} )</td>
<td>Cost of purchasing FDM machine; $</td>
<td>$</td>
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<tr>
<td>( C_{\text{maint}} )</td>
<td>Annual cost of maintaining FDM machine; $</td>
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<tr>
<td>( C_{\text{material}} )</td>
<td>Cost of FDM material; $/mm³</td>
<td>$/mm³</td>
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<td>( C_{\text{operation}} )</td>
<td>Annual cost of operating FDM machines; $</td>
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<td>( C_{\text{setup}} )</td>
<td>Cost of changing process parameters due to setup; $</td>
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<td>( D )</td>
<td>Production capacity; parts/day</td>
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</tr>
<tr>
<td>( d_i ), ( d_i^* )</td>
<td>Deviation variables</td>
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<td>( h )</td>
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<td>Number of layers</td>
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<td>Number of parts per batch</td>
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<td>Number of parts per year</td>
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<tr>
<td>( N_{\text{setup}} )</td>
<td>Number of production setups</td>
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<td>( Q_{\text{avg}} )</td>
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<td>Machine setup time; sec</td>
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<td>Machine warm-up time; sec</td>
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<td>mm³</td>
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<tr>
<td>( W_{\text{road}} )</td>
<td>Road width of FDM deposit; mm</td>
<td>mm</td>
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<tr>
<td>( Z )</td>
<td>Deviation function</td>
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<td>( \Delta D )</td>
<td>Range of capacity in hearing aid shell space of capacity</td>
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<td>The dimension of ( i^{th} ) space element in direction ( D ) of market space; a decision variable for a stage ( i ).</td>
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8. REFERENCES


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McFarlane, D. C. and Bussmann, S., 2000, Developments in holonic production planning and control, Production Planning and Control, 11:6, pp. 522-536.


