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ROBUST STABILIZATION OF SYSTEMS WITH UNCERTAIN PARAMETERS

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ABSTRACT

This paper considers, from a complex function theoretic point of view, certain robust synthesis problems. We consider both real and complex parameter variations. It is shown that several apparently different problems can be treated in a unified general framework. A new result on the gain margin problem for multivariable plants in also given.

NOTATION

$C = \{\text{complex numbers}\}$.
$R = \{\text{real numbers}\}$.
$P^1 = C \ \{\infty\}$.
H = open right half plane = $\{s \in C: \text{Re } s > 0\}$.

$\tilde{H}$ = closed right half plane = $\{s \in C: \text{Re } s > 0\}$.
$\overline{H} = H \ \{\infty\}$.
D = open unit disc = $\{s \in C: |s| < 1\}$.
$\overline{D}$ = closed unit disc = $\{s \in C: |s| < 1\}$.
T = unit circle = $\{s \in C: |s| = 1\}$.
H and D are well-known to be conformally equivalent.

0. INTRODUCTION

This paper is devoted to solving certain kinds of robust stabilization problems using techniques from complex analysis, and, in particular, interpolation theory. Particular cases of these problems have been considered by TANNENBAUM [1980, 1981, 1982]. In this paper, we continue the investigation of these robust design problems.

In general terms, the problems may be formulated as follows: Let $P_k(s)$ be a parametrized family of (linear, continuous-time, finite-dimensional, time-invariant, strictly proper) plants, where the parameter vector k takes values in some compact set K. Then we want to design a controller C(s) such that for each k in K, the following closed loop system is (internally) asymptotically stable:

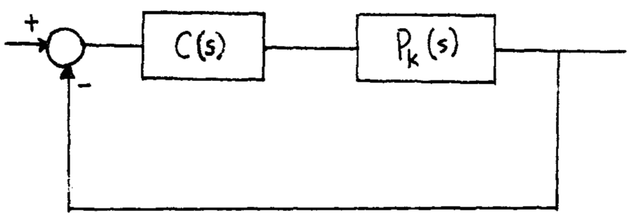

Fig. 1

The problem state above, in its complete generality, is very hard and no general solution is known. However, for certain special cases of importance in practical design, one can give a complete algorithmic solution. For example, consider the following family of SISO plants:

$$P_k(s) = kP_0(s), \qquad (0.1)$$

where $P_0(s)$ is the (fixed) nominal plant and k is the variable parameter taking values in [a, b], b > 1 > a > 0. Then the above problem becomes one of finding (if possible) a proper compensator C(s) which stabilizes the closed loop system for all k in [a, b]. It turns out that given the nominal model $P_0(s)$, one can compute a number β such that this problem is solvable if and only if

$$\frac{b}{a} < \beta. \qquad (0.1)$$

Indeed, it is easy to see that 20 log β is the maximual attainable gain margin for the nominal plant $P_0(s)$ by suitable design of C(s). Thus, this special problem may be viewed as the problem of maximization of gain margin by feedback. It will be seeen that this new invariant β depends only on the zeros and poles of $P_0(s)$ in the open right half plane. Given a, b such that (0.2) holds, we give an explicit parametrization of all controllers that solve this design problem. The above problem (which was considered in HOROWITZ and GERA [1979] and HOROWITZ and SIDI [1978], and solved by TANNENBAUM [1980]) is a very special case of the whole class of design problems for which our techniques work. In point of fact, we will argue that most of the standard robustness and H^∞-sensitivity minimization problems can be embedded in a unified framework and solved using essentially the same techniques. Explicitly, our techniques will be shown to be applicable to certain problems posed in ZAMES [1981], ZAMES and FRANCIS [1983], FRANCIS and ZAMES [1984], DOYLE, WALL, and STEIN [1982], and KIMURA [1983].

It is important to note that while the techniques of the above authors are essentially functional analytic, our techniques are complex analytic going back to some of the ideas of Nevanlinna and Pick (NEVANLINNA [1953] and AHLFORS [1973]). In particular, we make strong use of Pick's formulation of the Schwarz lemma in terms of a certain noneuclidean (hyperbolic) metric. This approach enables us to treat real as well as complex variations in the same framework. We feel that this is an important contribution of this paper. Full details of the results of this paper including the proofs can be found in KHARGONEKAR and TANNENBAUM [1984].

1. INTERPOLATION THEORY

Interpolation theory plays a major role in certain feedback design problems. In this section, we will describe those aspects of the classical Nevanlinna-Pick interpolation theory which are relevant to the design problems treated in the subsequent sections. See HELTON [1982] and the references cited there for an indepth discussion of interpolation theory and related

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topics.

Let $a_i \in D$, $\tilde{b}_i \in D$, $i = 1, 2, \ldots, q$ with $a_i \neq a_j$, $i \neq j$. The classical Nevanlinna-Pick interpolation problem is to find (if one exists) an analytic function $f: D \to D$ such that $f(a_i) = b_i$, $i = 1, 2, \ldots, q$. As is well known (PICK [1916], NEVANLINNA [1919]) an interpolating function f exists if and only if the following Nevanlinna-Pick matrix

$$N: = \left[\frac{1 - \tilde{b}_i\bar{\tilde{b}}_j}{1 - a_i\bar{a}_j}\right]_{i, j=1,2,\ldots,q}$$

is positive semi-definite.

Our work depends on the following slight variation of the above problem. Let $a_i \in D$, $b_i \in C$, $i = 1, 2, \ldots, q$ with the a_i distinct as above. Let $\alpha > 0$ be in R. Then we are interested in finding an analytic f_α: $D \to D$ such that $f_\alpha(a_i) = \alpha b_i$, $i = 1, 2, \ldots, q$. Clearly, for $\alpha = 0$, one can find such a function, namely $f_\alpha \equiv 0$. Therefore, by continuity, one can do this for α sufficiently small. Indeed, it is an easy exercise to compute the maximal α, $\hat{\alpha}_{max}$, such that for each $\alpha < \hat{\alpha}_{max}$, f_α exists. Explicitly, α_{max} can be computed as follows: Define

$$A: = \left[\frac{1}{1 - a_i\bar{a}_j}\right]_{i, j=1,2,\ldots,q}$$

$$B: = \left[\frac{b_i b_j}{1 - a_i\bar{a}_j}\right]_{i, j=1,2,\ldots,q}$$

Clearly, in order that the above problem be solvable we must require that $A - \alpha^2 B > 0$. If $b_i = 0$, $i = 1, 2, \ldots, q$ then $A - \alpha^2 B > 0$ for all α in R. In this case, we set $\hat{\alpha}_{max}: = \infty$. On the other hand if at least one of the $b_i \neq 0$, then

$$\hat{\alpha}_{max} = 1/\sqrt{\lambda max} \quad ,$$

where λ_{max} is the largest eigenvalue of $A^{-1}B$. (It is not difficult to see that $\lambda_{max} > 0$ if $B \neq 0$.) Note that $\alpha_{max}: = \hat{\alpha}_{max}(a_i, b_i)$ only depends on the interpolation data a_i, b_i, $i = 1, 2, \ldots, q$. We will see in Section 2 that $\hat{\alpha}_{max}$ plays a central role in robust stabilization problems.

It will be seen that we need to consider certain kinds of interpolation problems with some of the points lying on the boundary **T** of the unit disc **D**. Here we extend our notion of α_{max} to cover boundary interpolation. Let $a_j \in D$ $j = 1, \ldots, \ell$, $a_{\ell+r} \in T$ $(r = 1, \ldots, q - \ell)$, and $b_i \in C$ $i = 1, \ldots, q$. Given a real number $\alpha > 0$, we are require to find an analytic function f_α: $D \to D$ such that $f_\alpha(a_i) = \alpha b_i$ for $i = 1, \ldots, q$. Let α_1 be the $\hat{\alpha}_{max}$ for the "interior" interpolation data a_j, b_j, $j = 1, 2, \ldots, \ell$. Define

$$\alpha_{max}(a_j, b_i) := \min(\alpha_1, \frac{1}{|b_{\ell+1}|}, \frac{1}{|b_{\ell+2}|}, \ldots, \frac{1}{|b_q|}) \qquad (1.1)$$

for

$$j = 1, \ldots, \ell \text{ and } i = 1, \ldots, q.$$

We can now state the general

(1.2) THEOREM. Let a_i in $\bar{D}$ and b_i in C, $i = 1, 2, \ldots, q$ be as above. Then there exists an analytic function f_α: $D \to D$ such that $f_\alpha(a_i) = \alpha b_i$ if and only if $\alpha < \alpha_{max}(a_j, b_i)$.

2. ROBUST STABILIZATION AND OTHER PROBLEMS

In this section, we will consider certain types of robust stabilization and related problems which were alluded to in the Introduction. To motivate our approach, let us begin by reviewing precisely how the problem of internal stabilization feedback amounts to an interpolation problem. Let $P_o(s)$ be a fixed SISO nominal plant with closed right half plane zeros $z_1, z_2, \ldots, z_m$, and closed right half plane poles $p_1, p_2, \ldots, p_n$. (Note that some of the z_i's will be ∞ since we are dealing with a strictly proper plant.) For a given compensator C(s) define the sensitivity function

$$S(s) = (1 + P_o(s)C(s))^{-1}. \qquad (2.1)$$

As is well known, in order for the closed loop system to be internally asymptotically stable, it is necessary and sufficient that S(s) have the following properties:

- (i) S(s) is real rational and analytic in H;
- (ii) the zeros of S(s) contain $\{p_1, p_2, \ldots, p_n\}$ multiplicities included; and
- (iii) the zeros of S(s) - 1 contain $\{z_1, z_2, \ldots, z_m\}$ multiplicities included.

Given any such S(s), one can find the corresponding (proper) compensator C(s) using (2.1).

Let us begin by considering the problem of internal stabilization for plants with parameter uncertainty as discussed in the Introduction. Consider the family of SISO plants $P_k(s) = kP_o(s)$ as given by (0.1) where $P_o(s)$ is the nominal model and k belongs to the interval [a,b], b>1>a>0. Let C(s) be a proper compensator. We can now state the following:

(2.3) LEMMA. The feedback system (of Fig. 1) is internally asymptotically stable for all k in [a,b] if and only if the sensititivy function S(s) satisfies (2.2-i,ii,iii) and

$$S(s) \notin (-\infty, \frac{a}{a-1}] \cup [\frac{b}{b-1}, \infty)$$

for all s in $\tilde{\mathbf{H}}$.

(2.4) GAIN MARGIN PROBLEM. Lemma (2.3) shows that the gain margin problem of the Introduction is equivalent to the following interpolation problem: For given $P_o(s)$ and interval [a,b], $0 < a < 1 < b$, find a real rational function S(s) such that

(i) $S(s)$: $\tilde{H} \to C \setminus \{(-\infty, \frac{a}{a-1}] \cup [\frac{b}{b-1}, \infty)\}$

(ii) S(s) satisfies (2.2-i,ii,iii).

Next let us consider the problem of sensitivity minimization of ZAMES [1981], ZAMES and FRANCIS [1983], and FRANCIS and ZAMES [1984]. First we will consider the unweighted sensitivity function and then, a bit later, consider the weighted sensitivity function. Let $P_o(s)$ be the fixed SISO plant Then we are required to find

$$\inf \{\sup_{s \in \tilde{H}} |S(s)| : C(s) \text{ internally stabilizes } P_o(s)\}.$$

We can reformulate this problem in the following way:

(2.5) MINIMAL SENSITIVITY PROBLEM. Let $r > 0$ be a real number such that there exists

$$S(s): \tilde{H} \to D_r := \{s \text{ in } C: |s| < r\},$$

satisfying (2.2-i,ii,iii). Clearly, the Francis-Zames problem stated above is to find the infimum, r_o, of all such real numbers r.

Next we would like to consider a kind of parameter variation which is motivated by the work of DOYLE, WALL, and STEIN [1982] and LEHTOMAKI [1981]. These authors consider various types of uncertainties in modelling dynamics. Their work shows that in several cases these uncertainties are equivalent to complex uncertainties in the multiplicative factor. We will therefore consider the following family of plants. Let $r > 0$ be given.

Define

$$K_r := \{k: k = (1 + a)^{-1} \text{ where } s \in C \text{ and } |s| < r\}. \quad (2.6)$$

Now consider the family of plants

$$P_k(s) = kP_0(s)$$

where k belongs to K_r, and $P_0(s)$ is the nominal plant. (DOYLE, WALL, and STEIN [1982] consider other types of modelling uncertainties as well. Each of these cases can also be translated into interpolation problems with different data and interpolating functions.) For this family of plants we consider the corresponding robust stabilization problem. Using the same method as in (2.3), it is easy to see that this problem can be formulated as follows:

(2.7) COMPLEX PARAMETER VARIATIONS.

For given $P_0(s)$ and $r > 0$, find

(i) $S(s): \tilde{H} \to D_{1/r}$, and
(ii) $S(s)$ satisfies (2.2-i,ii,iii).

We will now solve problems (2.4), (2.5), (2.7) (and their weighted analogs) in a unified way. Let us first note that the conditions (2.4-i), (2.5-i), and (2.7-i) require the sensitivity function $S(s)$ to have range in a domain which is simply connected and not all of **C**. But by the Riemann mapping theorem (RUDIN [1966]) these domains are all conformally equivalent to the unit disc D. In point of fact, in all these cases it is trivial to write explicit conformal equivalences between these domains and D which we will do shortly. But first, let us abstract the problem.

(2.8) GENERAL PROBLEM. *Let G C be a given simply connected domain containing 0, 1. Find (if possible) a rational analytic function*

$$S(s): \tilde{H} \to G$$

satisfying (2.2-i,ii,iii).

We will now give a simple procedure to solve this general problem which will lead to explicit solutions of problems (2.4), (2.5), and (2.7) (and their weighted analogs). In order to do this, we will have to describe, briefly, a certain notion from complex function theory, namely the *hyperbolic* or *Poincaré metric*. For complete details, see the classical work of NEVANLINNA [1953]. We should note that in HELTON [1982] noneuclidean metrics and their relations to problems in system theory have also been discussed.

Let **D** denote the open unit disc. Then the *hyperbolic distance* between two points z_1, z_2 in **D** is given by

$$d_D(z_1, z_2) = \log \frac{1 + \left|\dfrac{z_1 - z_2}{1 - z_1 z_2}\right|}{1 - \left|\dfrac{z_1 - z_2}{1 - z_1 z_2}\right|}. \quad (2.9)$$

In particular, for $r > 0$,

$$d_D(0, r) = \log \frac{1 + r}{1 - r}.$$

Next let $G \subseteq C \cup \{\infty\}$ be a simply connected domain with at least two boundary points. Then by the Riemann mapping theorem there exists $\lambda: G \to D$ a conformal equivalance. We define the *hyperbolic distance* on G by

$$d_G(z_1, z_2) = d_D(\lambda(z_1), \lambda(z_2)). \quad (2.10)$$

It is a fact that this definition is independent of the choice of conformal equivalence λ.

The key fact which we need is the following version of the Schwarz lemma. See AHLFORS [1973] for a proof.

(2.11) THEOREM. *Let G_1, $G_2 \subseteq C \cup \{\infty\}$ be simply connected domains with at least two boundary points. Let $f: G_1 \to G_2$ be an analytic map. Then for all z_1, z_2 in* G_1,

$$d_{G_1}(z_1, z_2) \geq d_{G_2}(f(z_1), f(z_2)).$$

Moreover, one has equality if and only if f is a conformal equivalence.

This result will be the key in our treatment of robust stabilization. Before stating our solutions to the general problem (2.8), we need to set up some notation. Let $P_0(s)$ be the nominal plant as

above with z_i in $\tilde{H}$ the zeros and p_j in $\tilde{H}$ the poles.

Let $\phi: \tilde{H} \to \bar{D}$ be a fixed conformal equivalence. Let $\xi_i := \phi(z_i)$ and $\psi_j := \phi(p_j)$.

In the notation of Section 1, define the interpolation data:

$$a_i = \xi_i, \; i = 1, 2, \ldots, m$$

$$a_{j+m} = \psi_j, \; j = 1,2, \ldots, n$$

$$b_i = 1, \; i = 1,2, \ldots, m$$

$$b_{j+m} = 0, \; j = 1,2, \ldots, n.$$

As in (1.1) consider now the α_{max} defined relative to this interpolation data. We can now state the following key result:

(2.12) THEOREM. *The general problem (2.8) is solvable if and only if*

$$d_G(0, 1) < d_D(0, \alpha_{max}) = \log \frac{1 + \alpha_{max}}{1 - \alpha_{max}}.$$

REMARK. Theorem (2.12) essentially solves problems (2.4), (2.5), and (2.7). Indeed, we see that solving these problems can be divided into two parts. the first part requires computation of α_{max} which depends only on the zeros and poles of the nominal plant in the open right half plane when the plant has at least one open right half plane zero, and is 1 otherwise.

The second part of the solution of these problems is the computation of $d_G(0,1)$. Certainly, this depends on the choice of G which in turn depends on the kind of uncertainty in the given problem. Given the domain G, $d_G(0, 1)$ can be computed as explained in (2.9).

We shall now give explicit solutions to the above three problems.

(2.4)' SOLUTION TO (2.4). We need to find

$$\theta: \mathbb{C}\setminus\{(-\infty, \frac{a}{a-1}] \cup [\frac{b}{b-1}, \infty)\} \to D,$$

a conformal equivalence, such that $\Theta(0) = 0$. Following standard procedures in conformal mapping theory, (see, e.g., TANNENBAUM [1980]), we find

$$\theta(s) = \frac{1 - \left[(1 - (\frac{b-1}{b})s)/(1 - (\frac{a-1}{a})s)\right]^{1/2}}{1 + \left[(1 - (\frac{b-1}{b})s)/(1 - (\frac{a-1}{a})s)\right]^{1/2}} .$$

It is easy to compute that

$$\theta(1) = \frac{1 - \sqrt{a/b}}{1 + \sqrt{a/b}} .$$

Theorem (2.12) implies that the gain margin problem is solvable if an only if

$$d_G(0, 1) = d_D(0, \theta(1)) < d_D(0, \alpha_{max})$$

or equivalently,

$$\frac{b}{a} < \left(\frac{1 + \alpha_{max}}{1 - \alpha_{max}}\right)^2 =: \beta_{max}. \tag{2.13}$$

From this expression, certain interesting control theoretic implications can be drawn. For example, as α_{max} approaches 1, the maximal attainable gain margin goes to ∞. If the nominal plant $P_0(s)$ has no zeros in the open right half plane, i.e., we have a minimum phase plant, then it is immediate that $\alpha_{max} = 1$. Thus, for such plants given $b > 1 > a > 0$, one can always solve (2.4). In Section 3, we shall prove a similar result for multivariable plants. On the other hand as α_{max} approaches zero, the maximal b/a approaches 1.

(2.5)' SOLUTION TO (2.5). In this case we need to find $\theta: D_r \to D$ such that $\theta(0) = 0$. Trivially $\theta(s) = s/r$, and

$$d_{D_r}(0, 1) = d_D(0, 1/r) = \log \frac{1 + 1/r}{1 - 1/r} .$$

Applying Theorem (2.12), (2.5) is solvable if and only if

$$\log \frac{1 + 1/r}{1 - 1/r} = d_{D_r}(0, 1) < d_D(0, \alpha_{max}) = \log \frac{1 + \alpha_{max}}{1 - \alpha_{max}}$$

That is,

$$r > 1/\alpha_{max}.$$

Therefore by definition, the minimal sensitivity

$$\inf_C \sup_{s\in\tilde{H}} |S(s)| = 1/\alpha_{max}, \tag{2.14}$$

where the infimum is taken over all internallly stabilizing compensators.

This result reveals a basic connection between the sensitivity minimization problem and the gain margin problem. From this new general viewpoint, it is clear that TANNENBAUM [1980] and ZAMES and FRANCIS [1983]

(2.7)' SOLUTION TO (2.7). In this case

$$G = D_{1/r} = \{s \in C: |s| < 1/r\}.$$

Thus this is precisely the ZAMES-FRANCIS [1983] problem and for each $r < \alpha_{max}$, the problem is solvable.

Finally it is not difficult to incorporate the question of weighted sensitivity minimization into our general framework. This is the general problem considered by ZAMES and FRANCIS [1983]. For details see KHARGONEKAR-TANNENBAUM [1984].

3. REMARKS ON THE MULTIVARIABLE CASE

In this section, we present a simple result on the multivariable version of the gain margin problem. Let us consider the family of p x m real rational proper transfer matrices

$$P(s) = kP_0(s), \; k \in [a, b], \; b > 1 > a > 0.$$

We want to find a real rational compensator transfer matrix C(s) such that the feedback system shown in Fig. 1 is internally asymptotically stable for all k in [a, b]. Let R denote the ring of stable proper rational functions. It is well know that R is a Euclidean domain (see Hung and Anderson [1979] and Morse [1976]). Let $P_0(s) = N(s)D^{-1}(s)$ be a comprime factorization of $P_0(s)$, where N(s), D(s) have their entries in R. (See VIDYASAGAR [1978]). Let $\alpha(s)$ be the g.c.d. (over R) of all entries of N(s). Then the zeros of $\alpha(s)$ in $\tilde{H}$ are the blocking zeros of $P_0(s)$ in the closed right half plane. We now can state the following

(3.1) THEOREM. Suppose $P_0(s)$ has no blocking zeros in the open right half plane. Suppose that the roots of det D(s) in the open right half plane have multiplicity no greater than one. Then given any $b > 1 > a > 0$, there exists a compensator C(s) such that the closed loop system is internally asymptotically stable for each k in [a, b].

The above result shows that if the nominal plant $P_0(s)$ has no blocking zeros and distinct right half plane poles, then the maximal achievable gain margin is ∞. This result is similar to the known results on systems with no right half plane transmission zeros. ZAMES [1981], ZAMES and BENSOUSSAN [1982], FRANCIS and ZAMES [1984], and Helton [1983] show that for systems with no right half plane zeros, perfect tracking is possible. Our result is in a similar spirit and shows that the achievable gain margin is unbounded, when there are no blocking zeros

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