APPRAOXIMATE NONLINEAR ANALYSIS OF SOLID ROCKET MOTORS AND T-BURNERS

Volume I

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FOREWORD

The present report consists of two volumes which describe an approximate nonlinear analysis of solid rocket motors and T-burners and the associated computer programs. Volume I contains the analytical basis for the computer programs and the results of the parametric studies, while Volume II describes the computer programs and serves as a user's manual.

The investigation is entitled APPROXIMATE NONLINEAR ANALYSIS OF SOLID ROCKET MOTORS AND T-BURNERS. The two volumes are additionally subtitled as follows:

Volume I - Analysis and Results

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This technical report has been reviewed and is approved.
**Title:** APPROXIMATE NONLINEAR ANALYSIS OF SOLID ROCKET MOTORS AND T-BURNERS: VOLUME I

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**Abstract:**
An approximate theory based on the Galerkin method was developed to describe the nonlinear behavior of axial-mode combustion instability in solid-propellant rocket motors and T-burners. For motors with linear combustion driving and linear particle damping (gasdynamic nonlinearities only), growth rates, limiting amplitudes and waveforms were in reasonable agreement with available "exact" numerical solutions and experimental data. For T-burners the predicted limiting amplitudes were considerably higher than the experimentally measured values.
To assess the importance of combustion nonlinearities, a heuristic nonlinear combustion response model was introduced into the approximate analysis. Results obtained with both the approximate model and the "exact" analysis showed that nonlinear combustion effects may be important for moderate amplitudes and may account for pulsed instabilities in some cases.

The higher Reynolds number correction to the Stokes Drag Law was also included in both the approximate and "exact" analyses. Results show that nonlinear particle effects become increasingly important as particle size and/or frequency increases. Also, particle nonlinearities may have a significant effect on optimum particle size for maximum damping and may account for pulsed instabilities in some cases.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>FOREWORD</td>
<td>1</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>ii</td>
</tr>
<tr>
<td>NOMENCIATURE</td>
<td>v</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>ix</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>xiii</td>
</tr>
<tr>
<td>1. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>2. APPROXIMATE ANALYSIS</td>
<td>5</td>
</tr>
<tr>
<td>2.1 Basic Assumptions and Conservation Equations</td>
<td>5</td>
</tr>
<tr>
<td>2.2 Order of Magnitude Analysis and Approximate Wave Equations</td>
<td>9</td>
</tr>
<tr>
<td>2.3 Application of the Galerkin Method</td>
<td>14</td>
</tr>
<tr>
<td>2.4 Method of Averaging</td>
<td>20</td>
</tr>
<tr>
<td>2.5 Application to Motors</td>
<td>25</td>
</tr>
<tr>
<td>2.6 Application to T-Burners</td>
<td>26</td>
</tr>
<tr>
<td>2.7 Nonlinear Combustion Driving</td>
<td>39</td>
</tr>
<tr>
<td>2.8 Nonlinear Particle Damping</td>
<td>41</td>
</tr>
<tr>
<td>3. EXACT ANALYSIS</td>
<td>45</td>
</tr>
<tr>
<td>3.1 Modification for Quasi-Steady Nozzle</td>
<td>47</td>
</tr>
<tr>
<td>3.2 Linearized Burning Rate Model</td>
<td>48</td>
</tr>
<tr>
<td>3.3 Nonlinear Particle Damping</td>
<td>51</td>
</tr>
<tr>
<td>4. RESULTS AND DISCUSSION</td>
<td>54</td>
</tr>
<tr>
<td>4.1 Typical Nonlinear Solutions</td>
<td>54</td>
</tr>
<tr>
<td>4.2 Parametric Studies of Mode-Coupling</td>
<td>59</td>
</tr>
<tr>
<td>4.3 Effect of Nonlinear Combustion Driving</td>
<td>95</td>
</tr>
</tbody>
</table>
Table of Contents (cont'd)

<table>
<thead>
<tr>
<th>Section</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.4 Effect of Nonlinear Particle Damping</td>
<td>112</td>
</tr>
<tr>
<td>4.5 Solutions for T-Burners</td>
<td>127</td>
</tr>
<tr>
<td>4.6 Comparisons with Experimental Data</td>
<td>135</td>
</tr>
<tr>
<td>4.7 Application of the Method of Averaging</td>
<td>146</td>
</tr>
<tr>
<td>4.8 Computation Time</td>
<td>156</td>
</tr>
<tr>
<td>5. CONCLUSIONS AND RECOMMENDATIONS</td>
<td>158</td>
</tr>
<tr>
<td>5.1 Conclusions</td>
<td>158</td>
</tr>
<tr>
<td>5.2 Recommendations</td>
<td>161</td>
</tr>
<tr>
<td>6. REFERENCES</td>
<td>163</td>
</tr>
<tr>
<td>APPENDIX A Derivation of Approximate Equations</td>
<td>165</td>
</tr>
<tr>
<td>A-1 Derivation of Equations (13) and (14)</td>
<td>165</td>
</tr>
<tr>
<td>A-2 Derivation of Equation (17)</td>
<td>171</td>
</tr>
<tr>
<td>APPENDIX B Use of Complex Variables in the Solution of Nonlinear</td>
<td>174</td>
</tr>
<tr>
<td>Differential Equations</td>
<td></td>
</tr>
<tr>
<td>APPENDIX C Coefficients Appearing in the Approximate Mode Amplitude</td>
<td>177</td>
</tr>
<tr>
<td>Equations</td>
<td></td>
</tr>
<tr>
<td>C-1 Coefficients Appearing in Equations (22) and (23)</td>
<td>177</td>
</tr>
<tr>
<td>C-2 Coefficients Appearing in Equations (25)</td>
<td>179</td>
</tr>
<tr>
<td>C-3 Methodology for Calculating Coefficients in Equations (39)</td>
<td>180</td>
</tr>
<tr>
<td>C-4 Mode Amplitude Equations for T-Burners</td>
<td>184</td>
</tr>
<tr>
<td>APPENDIX D Derivation of Steady-State Solutions</td>
<td>188</td>
</tr>
<tr>
<td>D-1 Steady-State Solutions for Motors</td>
<td>189</td>
</tr>
<tr>
<td>D-2 Steady-State Solutions for T-Burners</td>
<td>189</td>
</tr>
</tbody>
</table>

iv
NOMENCLATURE

\( A, B \) combustion response parameters, Equation (10)

\( A \) dimensionless cross sectional area, \( A^*/A_{ch}^* \)

\( A_{ch}^* \) chamber reference area

\( A_j(t), B_j(t) \) time-dependent amplitudes, Equations (19)

\( A_v^* \) area of T-burner center vent, \( A_v^*/A_{ch}^* \)

\( b_j^* \) complex axial acoustic eigenvalue

\( c \) velocity of sound

\( C_{l}(j,m) \) coefficients of linear terms in Equations (22), (23), and (25)

\( C_{m}(j,m) \) particle to gas mass flux ratio, \( \vec{\bar{m}}_p/\vec{\bar{m}}_g \)

\( c_{p}^* \) specific heat at constant pressure of gaseous combustion products

\( c_{s}^* \) specific heat of solid propellant material

\( c_{v}^* \) specific heat at constant volume of gaseous combustion products

\( D_{l}(j,m,n) \) coefficients of nonlinear terms in Equations (22), (23), and (25)

\( e \) specific internal energy of gas, \( e^*/c_v^*T_r^* \)

\( E_f^* \) activation energy of Arrhenius flame reaction

\( E_s^* \) activation energy of Arrhenius surface reaction

\( f_e \) equilibrium frequency of gas-particle mixture

\( f_g \) pure gas frequency of gas-particle mixture

\( F \) momentum exchange term between gas and particles

\( g_j(t), h_j(t) \) slowly varying time-dependent amplitudes, Equations (38)

\( h \) specific enthalpy of gas, \( h^*/c_p^*T_r^* \)

\( h_c \) enthalpy of combustion products entering chamber from flame zone

\( i \) imaginary unit, \( \sqrt{-1} \)

\( k \) particle drag constant

\( k_g^* \) thermal conductivity of combustion products
thermal conductivity of solid propellant material

chamber length

T-burner cup-grain length

Effective plug flow length for T-burner vent, \( L_{\text{eff}}/L^* \)

T-burner vent length

steady-state Mach number at nozzle entrance

mass flow rate of gas per unit surface area, \( \dot{m}_g/\rho_c^* \)

mass flow rate of particles per unit surface area, \( \dot{m}_p/\rho_c^* \)

pressure exponent in propellant steady-state burning law

dimensionless pressure, \( p^*/p_r^* \)

exothermic heat release in flame reaction

endothermic heat release in surface reaction

propellant surface regression rate, \( r^*/c_r^* \)

dimensionless chamber radius, \( R^*/L^* \)

combustion response function, Equation (9)

Reynolds number, Equation (80)

universal gas constant

cross sectional area of T-burner grain

cross sectional area of T-burner

dimensionless time, \( t^*/(L^*/c_r^*) \)

temperature, \( T^*/T_r^* \)

flame temperature

surface temperature of burning solid propellant

dimensionless axial gas velocity, \( u^*/c_r^* \)

steady state velocity of gases leaving burning solid propellant

velocity of flow out of T-burner vent

dimensionless axial particle velocity, \( u_p^*/c_r^* \)
vent effect parameter

$x$ dimensionless axial coordinate, $x^*/L^*$

$X_j(x)$ axial acoustic eigenfunction, Equation (20)

$y$ coordinate normal to regressing solid propellant surface, $y^*/L^* \hat{y}_c$

$Y$ complex nozzle admittance

$\hat{y}_c \left( k_s/\rho_s c_s c_r L \right)^{1/2}$

$\alpha$ growth rate

$\beta$ dimensionless T-burner total cylindrical grain length, $2L_b^*/L^*$

$\beta_v$ dimensionless T-burner vent length, $L_v^*/L^*$

$\gamma$ specific heat ratio for gas, $c_v/c_p$

$\mu$ gas viscosity

$\rho$ dimensionless gas density, $\rho^*/\rho_r$

$\rho_m$ density of solid particle material

$\rho_p^*$ dimensionless particle density, $\rho_p^*/\rho_r^*$

$\rho_s^*$ solid propellant density

$\sigma$ average particle diameter

$\phi$ gas velocity potential

$\phi_p$ particle velocity potential

$\phi$ real part of $\hat{\phi}$

$\omega$ dimensionless frequency, $\omega^* L^*/c_r^*$

$\omega_o^*$ reaction rate in gas phase flame

$\Omega$ frequency parameter, $\omega/\tau^2$

Superscripts

( )' perturbation quantity

( )' steady-state quantity

( )' dimensional quantity, complex conjugate

( )' approximate solution
r,i real and imaginary parts of a complex quantity

Subscripts

e evaluated at the nozzle entrance
g gas phase
p particle phase
r reference state
v evaluated at T-burner vent
x,t partial differentiation with respect to x,t
o stagnation quantity
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Solid Rocket Motor Configuration</td>
</tr>
<tr>
<td>2.</td>
<td>Typical Linear Propellant Response</td>
</tr>
<tr>
<td>3.</td>
<td>T-Burner Geometry</td>
</tr>
<tr>
<td>4.</td>
<td>T-Burner Steady-State Properties</td>
</tr>
<tr>
<td>5.</td>
<td>Comparison of Pressure Waveforms for Conventional and Quasi-Steady Nozzles</td>
</tr>
<tr>
<td>6.</td>
<td>Comparison of Linearized and Nonlinear Combustion Models with the Two Parameter Response Function</td>
</tr>
<tr>
<td>7.</td>
<td>Moderate Response Curve with Positions of First Five Axial Modes</td>
</tr>
<tr>
<td>8.</td>
<td>Amplitudes of Individual Modes for an Unstable Motor</td>
</tr>
<tr>
<td>9.</td>
<td>Head-End Pressure Waveform for Unstable Motor</td>
</tr>
<tr>
<td>10.</td>
<td>Effect of Number of Modes Used in the Approximate Analysis Upon Resulting Solution</td>
</tr>
<tr>
<td>11.</td>
<td>Effect of Number of Modes Used in the Approximate Analysis Upon Calculated Waveforms</td>
</tr>
<tr>
<td>12.</td>
<td>Effect of Initial Disturbance Amplitude Upon Approach to Limiting Amplitude</td>
</tr>
<tr>
<td>13.</td>
<td>Effect of Harmonic Content of Initial Disturbance</td>
</tr>
<tr>
<td>14.</td>
<td>Decay of Oscillations Due to Mean Flow, Flow Turning, and Nozzle</td>
</tr>
<tr>
<td>15.</td>
<td>Decay of Oscillations in a Stable Motor Without Particles</td>
</tr>
<tr>
<td>16.</td>
<td>Growth of Oscillations to Limiting Amplitude for a Motor Without Particles</td>
</tr>
<tr>
<td>17.</td>
<td>Pressure Waveforms at Limiting Amplitude for a Motor Without Particles</td>
</tr>
<tr>
<td>18.</td>
<td>Pressure Waveforms for Low Frequency Oscillations in a Motor Without Particles</td>
</tr>
</tbody>
</table>
LIST OF FIGURES (continued)

Figure Number | Description                                                                 | Page No.
---------------|-----------------------------------------------------------------------------|--------
19.            | Pressure Waveforms for High Frequency Oscillations in a Motor Without Particles | 76     
20.            | Effect of Particle Size on Decay Rate and Frequency for a Gas/Particle Mixture in a Box | 79     
21.            | Decay Rates of 3% Disturbances Due to 2.5μm Particles in a Box               | 81     
22.            | Decay Rates of 15% Disturbances Due to 2.5μm Particles in a Box              | 82     
23.            | Decay Rates of 15% Disturbances Due to 2.5μm Particles in a Box              | 84     
24.            | Effect of Particle Size on Decay Rate and Frequency for Motor Without Combustion Driving | 85     
25.            | Effect of Particle Concentration on Decay Rate and Frequency for Motor Without Combustion Driving | 87     
26.            | Growth of Oscillations to Limiting Amplitude for a Motor with 2.5μm Particles | 88     
27.            | Pressure Waveforms for a Motor with 2.5μm Particles                         | 90     
28.            | Effect of Particle Size on Pressure Waveforms for $C_m = 0.1$              | 91     
29.            | Effect of Particle Concentration on Pressure Waveform for $\sigma = 2.5\mu$ | 93     
30.            | Influence of Particle Concentration Upon Limiting Pressure Amplitude for Motor with 2.5μm Particles | 94     
31.            | Effect of Combustion Nonlinearities Upon Limiting Amplitude Using the Approximate Model | 97     
32.            | Dependence of Limiting Amplitude Upon Nonlinear Combustion Parameter $b$ | 99     
33.            | Effect of Combustion Nonlinearities Upon Limiting Amplitude Using the "Exact" Analysis | 100    

x
<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Description</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>34.</td>
<td>Effect of Combustion Nonlinearities Upon Pressure and Burning-Rate Waveforms Using the &quot;Exact&quot; Analysis</td>
<td>102</td>
</tr>
<tr>
<td>35.</td>
<td>Effect of Combustion Nonlinearities Upon Pressure Waveforms Using Approximate Analysis</td>
<td>103</td>
</tr>
<tr>
<td>36.</td>
<td>Effect of Combustion Nonlinearities Upon Decay Rate for Motor with 2.5μ Particles Using &quot;Exact&quot; Analysis</td>
<td>105</td>
</tr>
<tr>
<td>37.</td>
<td>Effect of Combustion Nonlinearities Upon Decay Rate for Motor with 2.5μ Particles Using Approximate Analysis</td>
<td>106</td>
</tr>
<tr>
<td>38.</td>
<td>Effect of Combustion Nonlinearities on Decay Rate for 1079 Hz Oscillations in a Motor with 2.5μ Particles</td>
<td>108</td>
</tr>
<tr>
<td>39.</td>
<td>Pulsed Instability Due to Combustion Nonlinearities by Approximate Analysis</td>
<td>110</td>
</tr>
<tr>
<td>40.</td>
<td>Decay Rate vs Amplitude for Nonlinear Combustion with b = 3.0</td>
<td>111</td>
</tr>
<tr>
<td>41.</td>
<td>Effect of Particle Drag Nonlinearities Upon Decay Rates for a Hypothetical Motor Without Combustion Driving</td>
<td>114</td>
</tr>
<tr>
<td>42.</td>
<td>Effect of Particle Drag Nonlinearities Upon Limiting Amplitude for Motor With 2.5μ Particles by &quot;Exact&quot; Analysis</td>
<td>115</td>
</tr>
<tr>
<td>43.</td>
<td>Effect of Particle Drag Nonlinearities Upon Decay Rates for Motor With 8μ Particles by &quot;Exact&quot; Analysis</td>
<td>117</td>
</tr>
<tr>
<td>44.</td>
<td>Effect of Particle Drag Nonlinearities Upon Decay Rates for Motor With 20μ Particles by &quot;Exact&quot; Analysis</td>
<td>118</td>
</tr>
<tr>
<td>45.</td>
<td>Pulsed Instability Due to Particle Drag Nonlinearities for Motor With 8μ Particles by &quot;Exact&quot; Analysis</td>
<td>120</td>
</tr>
<tr>
<td>Figure Number</td>
<td>Description</td>
<td>Page No.</td>
</tr>
<tr>
<td>---------------</td>
<td>------------------------------------------------------------------------------</td>
<td>----------</td>
</tr>
<tr>
<td>46.</td>
<td>Strong Effect of Particle Drag Nonlinearities Upon Motor Stability for 20μm</td>
<td>122</td>
</tr>
<tr>
<td></td>
<td>Particles</td>
<td></td>
</tr>
<tr>
<td>47.</td>
<td>Effect of Particle Drag Nonlinearities on Pressure Waveforms for 20μm</td>
<td>123</td>
</tr>
<tr>
<td></td>
<td>Particles</td>
<td></td>
</tr>
<tr>
<td>48.</td>
<td>Decay Rate vs Amplitude for Motor With 8μm Particles Using Approximate</td>
<td>124</td>
</tr>
<tr>
<td></td>
<td>Nonlinear Drag Model</td>
<td></td>
</tr>
<tr>
<td>49.</td>
<td>Decay Rate vs Amplitude for Motor With 20μm Particles Using Approximate</td>
<td>126</td>
</tr>
<tr>
<td></td>
<td>Nonlinear Drag Model</td>
<td></td>
</tr>
<tr>
<td>50.</td>
<td>Growth Rates for End-Burning T-Burner</td>
<td>129</td>
</tr>
<tr>
<td>51.</td>
<td>Growth Rates for T-Burners with Cup Grains</td>
<td>130</td>
</tr>
<tr>
<td>52.</td>
<td>Pressure Waveform and Mode-Amplitude Functions for End-Burning T-Burner</td>
<td>133</td>
</tr>
<tr>
<td></td>
<td>Without Particles</td>
<td></td>
</tr>
<tr>
<td>53.</td>
<td>Pressure Waveform and Mode Amplitude Functions for Cup Grain T-Burner with</td>
<td>134</td>
</tr>
<tr>
<td></td>
<td>2.5μm Particles</td>
<td></td>
</tr>
<tr>
<td>54.</td>
<td>Comparison of Approximate and &quot;Exact&quot; T-Burner Solutions</td>
<td>136</td>
</tr>
<tr>
<td>55.</td>
<td>Comparison of Approximate Solutions with Experimental Data for T-Burners</td>
<td>144</td>
</tr>
<tr>
<td>56.</td>
<td>Approach to Limiting Amplitude for T-Burners by Approximate Analysis</td>
<td>145</td>
</tr>
<tr>
<td>57.</td>
<td>Waveforms for Motor Without Particles by Galerkin Method and Method of</td>
<td>147</td>
</tr>
<tr>
<td></td>
<td>Averaging</td>
<td></td>
</tr>
<tr>
<td>58.</td>
<td>Comparison of Galerkin, MOA, and &quot;Exact&quot; Solutions for Growth Rate of Small</td>
<td>154</td>
</tr>
<tr>
<td></td>
<td>Amplitude Disturbances</td>
<td></td>
</tr>
<tr>
<td>59.</td>
<td>Comparison of Galerkin, MOA, and &quot;Exact&quot; Solutions for Growth to Limiting</td>
<td>155</td>
</tr>
<tr>
<td></td>
<td>Amplitude</td>
<td></td>
</tr>
</tbody>
</table>
### LIST OF TABLES

<table>
<thead>
<tr>
<th>Table Number</th>
<th>Description</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>ANB 3066 Propellant Data</td>
<td>60</td>
</tr>
<tr>
<td>2.</td>
<td>Response Functions for the First Six Axial Modes</td>
<td>63</td>
</tr>
<tr>
<td>3.</td>
<td>Drag Constants for Approximate and &quot;Exact&quot; Models</td>
<td>78</td>
</tr>
<tr>
<td>4.</td>
<td>Linear and Nonlinear Drag Constants Versus $\sigma$</td>
<td>119</td>
</tr>
<tr>
<td>5.</td>
<td>Effect of Center Vent</td>
<td>131</td>
</tr>
<tr>
<td>6.</td>
<td>Motor Parameters for Laboratory Pulse Motor</td>
<td>137</td>
</tr>
<tr>
<td>7.</td>
<td>Transient Burn Rate Parameters</td>
<td>138</td>
</tr>
<tr>
<td>8.</td>
<td>Approximate Solutions for Cases 1 and 4</td>
<td>139</td>
</tr>
<tr>
<td>9.</td>
<td>Approximate Solutions for Cases 2 and 3</td>
<td>141</td>
</tr>
<tr>
<td>10.</td>
<td>T-Burner Parameters for Comparison Study</td>
<td>142</td>
</tr>
<tr>
<td>11.</td>
<td>Comparison of Single-Mode Solutions with Measured Data</td>
<td>143</td>
</tr>
<tr>
<td>12.</td>
<td>Comparison of Approximate Solutions for Particles in a Box</td>
<td>151</td>
</tr>
<tr>
<td>13.</td>
<td>Comparison of Approximate Solutions for Motor with $R = 0$</td>
<td>153</td>
</tr>
<tr>
<td>14.</td>
<td>Comparison of Computation Times</td>
<td>157</td>
</tr>
</tbody>
</table>
1. INTRODUCTION

Experience with solid propellant rocket motors indicates that the full-thrust operation of such motors is never completely steady. A certain degree of rough combustion (i.e., low amplitude random flow oscillations) is usually present which does not seriously affect system performance. On occasion, larger amplitude organized oscillations may develop in the combustion chamber and nozzle flow field during the ignition transient or after full thrust has been obtained¹. Such organized oscillations are evidence of a positive feedback between the unsteady propellant combustion process and the flow field disturbance. These instabilities may be classified as either spontaneous instabilities, which originate from flow or combustion noise, or pulsed instabilities, which result from the introduction of a sufficiently large disturbance in a motor which is stable with respect to small amplitude disturbances. It is observed that, after passing through a transient period, both spontaneous and pulsed instabilities reach a limiting amplitude (or limit-cycle) at which they oscillate with a frequency that is close to the frequency of one of the chamber's acoustic modes. These limit-cycle oscillations are often characterized by nonsinusoidal pressure waveforms that have sharp peaks and flattened minima. The existence of limit-cycles, pulsed instability, and nonsinusoidal waveforms are all caused by the nonlinearities of the system.

Small amplitude combustion instability may occur without detrimental effects to the system while large amplitude oscillations may lead to increase in mean chamber pressure and burning rate, excessive heat transfer rates, and severe vibration levels. The occurrence of any one of these may result in malfunction or destruction of the rocket motor. Thus, it is important for the rocket designer to be able to estimate the limiting amplitude of the pressure oscillations as well as the conditions under which pulsed instability may occur. To do this, a theoretical approach capable of determining the nonlinear stability characteristics of solid propellant rocket motors is required.

Reliable theoretical approaches capable of predicting the characteristics of combustion instabilities and the conditions under which they are most likely to occur are badly needed. To be of practical use, these analyses should be conceptually simple and easily adaptable for use by engine designers. In addition, these techniques should be capable of solving combustion instability problems without exceeding memory core limitations of current computers and without requiring excessive computation time. These considerations are particularly relevant to multi-dimensional combustion instability problems. As the above requirements cannot be satisfied by any of the "exact" numerical solution techniques, (e.g., finite-difference or finite-element methods) one must resort to the use of one or more approximate solution techniques. The development of such solution techniques for the analysis of nonlinear axial combustion instabilities in solid rockets is the subject of this report.

In recent years, several nonlinear combustion instability theories have been developed. Kooker and Zinn\textsuperscript{2,3} and Levine and Culick\textsuperscript{4,5} used finite-difference techniques to obtain numerical solutions to the conservation equations describing the two-phase flow of gases and particles in solid rocket combustors. However, limitations on available computer core size and computational time make it impractical to use these so-called "exact" analyses in parametric studies and engine design. Hence, efforts have been undertaken in recent years to develop reliable approximate solution techniques that are free from the above-mentioned limitations. In the first known attempt at the development of such an approximate technique, Zinn,


Powell and Lores have used the Galerkin method in the analysis of nonlinear combustion instabilities in liquid rockets with quasi-steady nozzles experiencing longitudinal\textsuperscript{6} and transverse\textsuperscript{7} instabilities. Powell and Zinn\textsuperscript{8} later extended these theories to situations in which the instabilities are three-dimensional and the rocket combustors are attached to conventional nozzles. More recently, a solution procedure utilizing the Galerkin method together with the method of averaging has been applied by Culick\textsuperscript{9,10} in the analysis of nonlinear instabilities in solid rockets. While the above-mentioned approximate theories appear promising, more work is needed to determine their reliability and range of applicability, since these theories are limited by the assumptions upon which they are based.

The main objective of this investigation is to evaluate the usefulness of approximate nonlinear analysis techniques for predicting the stability behavior of solid rocket motors. This objective has been accomplished through a comparison of the predictions of the approximate model, based on the methodology developed by Zinn, Powell, Lores\textsuperscript{6-8} and Culick\textsuperscript{9,10} with the more exact analysis developed by Kooker and Zinn\textsuperscript{2,3} and Levine and Culick\textsuperscript{4,5}. Therefore, extensive development of nonlinear models, such as developing new theory or programming a large analysis, was not necessary. However, modifications to both the approximate and "exact" analyses were made in order to accomplish the objectives of this program. These modifications will be discussed in Sections 2 and 3 of this report.


The investigation is divided into four tasks. In the first task the influence of the gasdynamic nonlinearities, which result in mode-coupling, was assessed. For this task all processes except gasdynamical mode-coupling, were described by linear models. In the second task the approximate and "exact" analyses were extended to include the effects of nonlinear particle damping and nonlinear combustion driving. In the third task the approximate analysis was applied to determine the nonlinear stability characteristics of T-burners. Finally, the restrictions and limitations on the application of the method of averaging to the solution of solid rocket instability problems was investigated in the fourth task.

The development of the approximate analysis is given in Section 2, while the modifications to the "exact" analysis are discussed in Section 3. The results obtained during the performance of the four tasks described above are presented and discussed in Section 4. These results include an extensive parametric study to assess the importance of gasdynamic mode coupling, particle size and concentration, propellant response function, and particle and combustion nonlinearities upon growth and decay rates, frequencies, limiting amplitudes and waveforms for motors and T-burners. In many cases, the approximate solutions are compared with the corresponding "exact" solutions and with experimental data. Conclusions drawn from the results of this investigation are presented in Section 5, along with recommendations for improvement of the approximate model.
2. APPROXIMATE ANALYSIS

2.1 Basic Assumptions and Conservation Equations

The solid rocket configuration investigated herein is described in Figure 1. A full-length cylindrically-perforated solid propellant grain containing aluminum particles burns unsteadily inside a cylindrical combustor-nozzle combination. The combustion process takes place inside a thin region immediately adjacent to the propellant. The products of combustion are assumed to consist of a mixture of a single gaseous species and aluminum oxide particles which enter the engine core flow at the outer edge of the combustion zone and acquire axial velocity through interaction with the engine core flow. During instability, the interaction between the wave motion and the combustion process results in unsteady burning which provides the energy needed to sustain the instability. At the same time the interaction between the wave motion and the particles, the core flow, and the nozzle results in wave energy dissipation. Limit-cycle conditions are achieved when the wave energy addition over a cycle is balanced by the wave energy removal over a cycle.

The following assumptions are used in the development of the theoretical model: (1) the flow in the engine is one-dimensional and it consists of a single gaseous species and spherical particles that can be characterized by a single average diameter; (2) the gas phase is thermally and calorically perfect; (3) the gas phase is inviscid and non-heat conducting; (4) the thermal energy transfer between gas and particle phases is neglected; (5) the momentum exchange between the gas phase and the particles due to viscous interaction can be described by Stokes' Drag Law\textsuperscript{11}; (6) the burning rate responds to pressure oscillations only, and it can be described by a linear burning response function; (7) the interaction of the oscillations in the combustor with the flow in the nozzle can be adequately described by using an appropriate nozzle admittance\textsuperscript{8}; (8) the Mach number of the mean flow is small; and (9) only moderate-amplitude disturbances are considered in this analysis.

Assumptions (1), (2), (3), (7), (8) and (9) are commonly used in

Solid Propellant

Burned Gases and Particles

\[ \bar{u}(x) \]

Nozzle

Figure 1. Solid Rocket Motor Configuration
combustion instability analyses 6-10. Assumption (4) could be relaxed, as was done in Reference 10, at the expense of other assumptions, but it is introduced here to simplify the analysis. The use of Stokes' Law and linear combustion response (assumptions (5) and (6)) are consistent with the requirement that only the effect of gasdynamical nonlinearities is considered in the analysis of nonlinear mode-coupling (first task). The effect of particle and combustion nonlinearities upon the resulting instability are considered in the second task, and the results are presented later in the report.

Using assumptions (1) through (4) above, the system of nondimensional conservation equations that describe the unsteady behavior of the combustor two-phase flow can be expressed in the following form 3:

Mass Conservation (gas and particulate phases)

\[
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} \left( \rho u \right) = \frac{2 \dot{m}_g}{R} 
\]  

\[
\frac{\partial \rho_p}{\partial t} + \frac{\partial}{\partial x} \left( \rho_p u_p \right) = \frac{2 \dot{m}_p}{R} 
\]  

Momentum Conservation (gas and particulate phases)

\[
\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + \frac{1}{\gamma} \frac{\partial p}{\partial x} = F_p - \frac{2 \dot{m}_g}{R} u 
\]  

\[
\rho_p \frac{\partial u_p}{\partial t} + \rho_p u_p \frac{\partial u_p}{\partial x} = - F_p - \frac{2 \dot{m}_p}{R} u_p 
\]
Energy (gas phase)

\[
\frac{\partial (\rho e)}{\partial t} + \frac{\partial}{\partial x} (\rho u e) + p(\gamma - 1) \frac{\partial u}{\partial x}
\]

\[
= \frac{2 \gamma_h c}{R} - \frac{\gamma(\gamma - 1)}{\rho_p} (u - u_p)
\]

\[
+ \frac{\gamma(\gamma - 1)}{R} (m_g u^2 + m_p u_p^2)
\]

Equation of State (gas phase)

\[
p = \rho_e
\]

In the above equations the quantity \( F_p \) describes the momentum exchange between the gas phase and the particulate matter. For linear particle damping, it is derived from the Stokes Drag Law for a single sphere in a steady flow; this \( F_p \) is given by

\[
F_p = K \rho_p (u_p - u)
\]

where

\[
K = \frac{18\mu^*}{\rho_m^* \sigma^2 \left( \frac{L^*}{c_0} \right)}
\]

The case of nonlinear particle damping is discussed later in this section.
The unsteady mass burning rate is described by the following linear response function law \textsuperscript{12} (pressure coupling only):

\[
\frac{\dot{m}_g'}{\dot{m}_g} = \mathcal{R} \frac{p'}{p}
\]  

(9)

where \( \mathcal{R} \) is the complex response function that must be provided from either experimental data or an analysis of the unsteady combustion process. In this report the response function \( \mathcal{R} \) is taken to be the well-known, two-parameter (i.e., A,B) expression given by \textsuperscript{12}

\[
\mathcal{R} = \frac{n A B}{\lambda + \frac{A}{\lambda} - (1+A) + AB}
\]  

(10)

where A and B are related to the steady-state combustion of the propellant \textsuperscript{3}, n is the pressure exponent in the steady-state burning law and \( \lambda \) is a complex solution of the quadratic equation

\[
\lambda^2 = \lambda - i \Omega
\]  

(11)

where \( \Omega \) is a nondimensional frequency parameter. A typical curve of real part of the response function versus frequency parameter \( \Omega \) is shown in Figure 2. Nonlinear expressions for unsteady mass burning rate are given later in this section.

2.2 Order of Magnitude Analysis and Approximate Wave Equations

To proceed with the analysis, each dependent variable is expressed as a sum of a space-dependent steady-state quantity and a perturbation which is both time and space dependent. Following the methodology introduced in

A = 7.54
B = 0.686
n = 0.81

Figure 2. Typical Linear Propellant Response
References (6), (7) and (8) each perturbation quantity and the mean flow Mach number are assumed to be of \( O(\varepsilon) \) where \( \varepsilon \) is a small ordering parameter that is a measure of the wave amplitude. Substituting the assumed expressions for the dependent variables into Equations (1) through (6), neglecting all terms of order higher than \( \varepsilon^2 \) and subtracting out the steady-state equations result in the derivation of the desired second order perturbation equations. Introducing the velocity potentials \( \phi \) and \( \phi_p \) defined by

\[
\begin{align*}
u' &= \frac{\partial \phi}{\partial x} \\
u_p' &= \frac{\partial \phi_p}{\partial x}
\end{align*}
\] (12a)

(12b)

into the resulting system of second order wave equations, using the substitution principle\(^\text{13,14}\) to eliminate variables, the system of second order equations can be combined to yield the following nonlinear wave equation describing the unsteady two-phase flow inside the motor:

\[
\ddot{\phi}_{xx} - \ddot{\phi}_{tt} = 2 \ddot{u} \phi_{xt} + (\gamma + 1) \frac{du}{dx} \phi_t + 2 \phi_x \phi_{xt} + (\gamma - 1) \phi_t \phi_{xx} + K_p \left[ \phi_t - \phi_p' \right] - (\gamma - 1) \rho_p \ddot{u}_p \phi_{xt}/\gamma \rho c \frac{du}{dx} \phi_t
\] (13)

Considering the nature of the problem under consideration, an inspection of the above equation suggests that it has the proper form. The left-hand side of the equation is the wave equation operator; the first two terms on the right-hand side describe the effects of the mean flow; the third and fourth terms describe gasdynamical nonlinearities while the fifth and sixth terms describe the gas-particle interaction; finally, the seventh term represents the "driving" provided by the unsteady combustion process.

\(^\text{13}\) Lighthill, M. J., Surveys in Mechanics, p. 250, 1956.

Applying the order of magnitude analysis to the particle momentum equation yields the following equation for the particle potential, $\frac{\ddot{\phi}}{p}$:

$$
\left( \frac{\ddot{\phi}}{p} \right)_t + u_p \left( \frac{\ddot{\phi}}{p} \right)_x + \frac{1}{2} \left( \frac{\ddot{\phi}}{p} \right)_x^2 = \kappa \left( \frac{\phi}{p} - \frac{\ddot{\phi}}{p} \right) - \frac{\ddot{u}_p}{p} \frac{\ddot{\phi}}{p} \tag{14}
$$

A detailed derivation of Equations (13) and (14) is given in Appendix A.

In the analysis for a solid rocket motor, the head end ($x=0$) of the chamber is assumed to consist of a rigid wall requiring zero velocities for both the gas and particles at this location. The influence of the exhaust nozzle on the wave motion in the combustion chamber is introduced through the boundary condition at $x=1$, which is the location of the nozzle entrance plane. As is customary in combustion instability analyses, the nozzle effect is introduced through an appropriate nozzle admittance relation. Thus, the solutions to Equations (13) and (14) are required to satisfy the following boundary conditions.

$$
\begin{align*}
\left. \frac{\ddot{\phi}}{p} \right|_x &= 0 \quad x = 0 \\
\left. \left( \frac{\ddot{\phi}}{p} \right)_x \right|_x &= 0
\end{align*}
$$

$$
\frac{\ddot{\phi}}{p} + \gamma Y \frac{\ddot{\phi}}{p} = 0 \quad x = 1 \tag{15b}
$$

where $Y = Y_r^* + i Y_i^* = (u'/p')_{x=1}$ is the nozzle admittance coefficient. The boundary conditions at the ends of a T-burner are described later in this section.

By making an additional approximation, the wave equation for the gas phase (i.e., Equation (13)) and the particle potential equation (i.e., Equation
can be combined to obtain a single equation describing the gas oscillations. This approach is similar to that employed by Culick in which the spatial derivatives of the particle properties are neglected in the particle momentum equation. The resulting first order linear equation for $u'$ is solved analytically to obtain

$$u'_p \approx K e^{-Kt} \int_0^t u'(x,t') e^{Kt'} dt'$$  \hspace{1cm} (16)$$

where short-term transient effects have been neglected. This solution is substituted into the gas phase equation, eliminating the need to numerically solve an additional equation describing the particle oscillations (i.e., Equation (14)). The resulting wave equation for the gas phase then becomes:

$$\dddot{\phi} + \dddot{\phi} = 2u_0\dddot{\phi} + (\gamma+1) \frac{du}{dx} \dddot{\phi} + 2\dddot{\phi} \dddot{\phi} + (\gamma-1) \dddot{\phi} \dddot{\phi}$$

$$+ K_p \dddot{\phi} - (\gamma-1)K_p u \dddot{\phi} + K^2_p \dddot{\phi}$$

$$+ K_p e^{-Kt} \int_0^t \dddot{\phi} e^{Kt'} dt'$$

$$+ (\gamma-1)K^2_p u \dddot{\phi} e^{-Kt} \int_0^t \dddot{\phi} e^{Kt'} dt'$$

$$- \frac{\rho h}{c} \frac{du}{dx} \dddot{\phi}$$  \hspace{1cm} (17)$$

A detailed derivation of Equation (17) is also given in Appendix A.

Most of the results presented in this report were obtained by an approximate solution of Equations (13) and (14) for which fewer assumptions were necessary. Approximate solutions of Equation (17), however, are also presented in order to determine the conditions under which solving the single
equation yields accurate results, since a considerable saving in computer
time is expected by using this approach.

2.3 Application of the Galerkin Method

The approximate solutions of Equations (13) and (14) or Equation (17)
are obtained by means of the Galerkin method, which is a special case of the
method of weighted residuals. Before proceeding with the application of
the Galerkin method to this problem, a brief description of the method will be
given.

According to the "classical" Galerkin method the dependent variables
are expanded in terms of a set of functions \( \psi_n \) that identically satisfy the
imposed boundary conditions. The proper choice of the functions \( \psi_n \) is aided
by information from such sources as experimental data, the solution of a
linearized version of the same problem, or from solutions of closely related
problems. Each of the \( \psi_n \) is multiplied by an arbitrary constant or function
yet to be determined. These expansions are then substituted into the governing
differential equations to form residuals, and the arbitrary constants (or
functions) are determined by imposing the condition that the residuals of the
differential equations be orthogonal to all the functions \( \psi_n \).

When the boundary conditions which the solution must satisfy are compli-
cated, it is usually impossible to find a set of functions, \( \psi_n \), that can
identically satisfy these boundary conditions. It has been shown that
when this occurs it is possible to obtain approximate solutions by properly
combining the boundary residuals with the differential equations' residuals
when applying the orthogonality conditions. Thus, the resulting orthogonality
conditions have the following form:

15 Finlayson, B. A. and Scriven, L. E., "The Method of Weighted Residuals--
735-744.

16 Ames, W. F., Nonlinear Partial Differential Equations in Engineering,

17 Zinn, B. T. and Powell, E. A., "Application of the Galerkin Method in the
Solution of Combustion Instability Problems," Proceedings of the 19th

18 Zinn, B. T. and Powell, E. A., "The Galerkin Method and its Use in the
Solution of Combustion Instability Problems", Proceedings of the 5th ICRPG
Combustion Conference, December 1968, pp. 138-144.
where $E_i$ and $B_i$ respectively represent the differential equation and boundary residuals. The residuals are obtained when the approximate expansions for the density, velocity vector, and pressure are substituted into the equations and boundary conditions expressing conservation of mass, momentum, and energy. The above method of solution accounts for the presence of mass, momentum, and energy sources both within the flow field and on the boundaries of the rocket combustor.

In order to obtain approximate solutions of Equations (13) and (14) by the Galerkin method, the velocity potentials are expressed as series of acoustic modes with unknown time-dependent amplitudes as shown below:

\[
\tilde{\phi} = \sum_{j=1}^{N} A_j(t) X_j(x) \quad (19a)
\]

\[
\tilde{\phi}_p = \sum_{j=1}^{N} B_j(t) X_j(x) \quad (19b)
\]

where $N$ is the number of modes included in the series expansion and $X_j(x)$ are the acoustic eigenfunctions for the axial modes. For a rocket motor, the $X_j$'s are the axial acoustic modes for the chamber with no mean flow, a rigid wall boundary condition at the head-end, and a nozzle admittance boundary condition at the other end. These functions (complex) are given by:

\[
X_j(x) = \cosh (ib_jx) \quad (20)
\]
where $b_j$ are the corresponding axial acoustic eigenvalues (complex). The unknown time-dependent amplitude functions (i.e., $A_j(t)$ and $B_j(t)$) are complex functions of time. The assumed form for the chosen expansions has been guided by related work in this area and experimental evidence showing that rocket instabilities usually involve the acoustic modes of the combustor.

The assumed expansions are substituted into Equations (13) and (14) and boundary condition (15b) to form the residuals $E_{p} \{ \hat{\phi}, \hat{\phi}\}$, $E_{p} \{ \hat{\phi}, \hat{\phi}\}$, and $B \{ \hat{\phi}\}$; any of these residuals becomes identically zero when the assumed solutions identically satisfy the corresponding differential equations or boundary conditions. To determine the unknown time-dependent coefficients, these residuals are required to satisfy the following Galerkin orthogonality conditions:

\begin{align}
\int_{0}^{\pi} E_{p} \{ \hat{\phi}, \hat{\phi}\} X^*_{j}(x) \, dx - B \{ \hat{\phi}\} X^*_{j}(1) &= 0 \\
\int_{0}^{\pi} E_{p} \{ \hat{\phi}, \hat{\phi}\} X^*_{j}(x) \, dx &= 0 \\
\end{align}

where $X^*_{j}(x)$ are the complex conjugates of the axial acoustic eigenfunctions in Equations (19). Performing the integrations indicated in Equation (21) results in a system of coupled second-order nonlinear ordinary differential equations describing the behavior of the time-dependent amplitudes. These equations are similar in form to those developed in References 6 through 10, and they can be expressed as follows:
\[
\sum_{m=1}^{2N} \left\{ C_0(j,m) \frac{d^2A_m}{dt^2} + C_1(j,m) A_m + \left[ C_2(j,m) + KC_3(j,m) \right] \right\}
\]
\[
+ \sum_{m=1}^{N} \sum_{n=1}^{N} \left\{ D_1(j,m,n) A_m \frac{dA_n}{dt} \right\} + D_2(j,m,n) A_m \frac{dA_n^*}{dt}
\]
\[
+ D_3(j,m,n) A_m \frac{dA_n^*}{dt} + D_4(j,m,n) A_m \frac{dA_n^*}{dt} \right\} = 0 \quad j = 1, 2, \ldots, N \quad (22)
\]

\[
\sum_{m=1}^{2N} \left\{ C_5(j,m) \frac{dA_m}{dt} + C_6(j,m) A_m \right\} = 0 \quad j = 1, 2, \ldots, N \quad (23)
\]

In the above equations, the \( B_j(t) \) functions for \( j = 1, 2, \ldots, N \) have been denoted by \( A_j(t) \) for \( j = N + 1, \ldots, 2N \); there are 2N unknown time dependent functions \( A_j(t) \) and 2N equations (N of Equation (22) and N of Equation (23)).

The nonlinear terms involving the complex conjugates appearing in Equation (22) arise because the wave equation (i.e., Equation (13)) must be modified for use with the assumed complex solution given by Equations (19). This modification is necessary because only the real part of the assumed solution is physically meaningful. It can easily be shown that if \( \psi = \phi + i \psi \) is a solution to Equation (13), the real part, \( \phi \), is not a solution to Equation (13). This failure of \( \phi \) to satisfy Eq. (13) is due to the presence of the nonlinear terms in this equation. It can also be shown, however, that a modified wave equation can be constructed for which the real part of its solution satisfies the original wave equation (i.e., Equation (13)). This modified wave equation is derived in Appendix B and is given by Equation (B-8). Thus Equation (22) was actually derived by applying the Galerkin method to Equation (B-8) rather than Equation (13).
The coefficients of the various linear and nonlinear terms appearing in Equations (22) and (23) arise from the spatial integrations indicated in Equations (21). Some of these coefficients are functions of the steady state properties \( \bar{u}(x) \), \( \bar{u}_p(x) \), and \( \bar{\rho}_p(x) \) which are obtained by solving the steady state conservation equations. All of the coefficients involve integrals of products of two or three axial acoustic eigenfunctions with various weighting functions, integrated over the length of the chamber. Expressions for these coefficients are given in Appendix C, and the steady state solutions are presented in Appendix D.

In deriving Equation (23) by applying the Galerkin method to Equation (14), the nonlinear term \( 1/2(\bar{\phi}_p)^2 \) was dropped in order to be consistent with the assumption of linear particle damping. As a result of neglecting this term, Equation (23) is a linear equation.

To determine the stability characteristics of a solid rocket motor, the form of the initial disturbance is specified and the subsequent behavior of the individual modes is determined by numerically integrating Equation (22) and (23). Once the time-dependence of the individual modes (i.e., \( A_j(t)'s \)) is known, the gas and particle potentials, \( \bar{\phi} \) and \( \bar{\phi}_p \), are calculated from Equations (19). The pressure perturbation at any axial location within the chamber is related to the real parts of \( \bar{\phi} \) and \( \bar{\phi}_p \) (i.e., \( \varphi \) and \( \varphi_p \)) by the following approximate momentum equation (Appendix A):

\[
p' = -\gamma \left[ \varphi_t + \bar{u}(x) \varphi_x + \frac{1}{2} \varphi_x^2 - \frac{1}{2} \varphi_t^2 + \bar{\rho}_p(x)(\varphi - \varphi_p) + \frac{du}{dx} \varphi \right]
\]  

(24)

The Galerkin method was also used to obtain approximate solutions of Equation (17) which was derived by neglecting the spatial derivatives of the particle velocity in Equation (14). The resulting differential equations governing the behavior of the unknown amplitude functions, \( A_j(t) \) is given in this case as follows:

\[
\sum_{m=1}^{N} \left\{ C_0(j,m) \frac{d^2A_m}{dt^2} + C_1(j,m) A_m + \left[ C_2(j,m) + h \Theta_m C_4(j,m) \right] \frac{dA_m}{dt} + \right\}
\]

18
\[ + C_3(j,m) e^{-Kt} \int_0^t A_m(t') e^{Kt'} dt' \}

\[ + \sum_{m=1}^{N} \sum_{n=1}^{N} \left\{ D_1(j,m,n) A_m \frac{dA_n}{dt} + D_2(j,m,n) A_m \frac{dA_n^*}{dt} \right\} \]

\[ + D_3(j,m,n) A_m^* \frac{dA_n}{dt} + D_4(j,m,n) A_m \frac{dA_n^*}{dt} \right\} = 0 \quad (25) \]

\[ j = 1, 2, \ldots N \]

which is a system of \( N \) equations in \( N \) unknowns. The coefficients \( C_0, C_1, C_2, C_3, \) and \( C_4 \) differ from those in Equations (22) and are given in Appendix C; the coefficients \( D_1, D_2, D_3, \) and \( D_4 \) are the same as those appearing in Equations (22). In order to handle the integral terms in Equations (25), auxiliary variables \( G_m(t) \) are defined by

\[ G_m(t) = e^{-Kt} \int_0^t A_m(t') e^{Kt'} dt' \quad (26) \]

The \( G_m(t) \)'s are described by the following equations

\[ \frac{dG_m}{dt} = A_m(t) - KG_m(t) \quad m = 1, 2, \ldots N \quad (27) \]

which are obtained by differentiating Equations (26). Solutions for the \( A_j(t) \)'s are then obtained by numerically solving Equations (25) simultaneously with Equations (27). The pressure perturbation is then obtained from the appropriate form of the momentum equation (Appendix A):

\[ p' = -\gamma \left[ \phi_t + \bar{u}(x)\phi_x + \frac{1}{2} \phi_x^2 - \frac{1}{2} \phi_t^2 + \frac{d\bar{u}}{dx} \phi \right. \]

\[ + K_{\phi p}^- (x) \phi - K_{\phi p}^2 (x) e^{-Kt} \int_0^t \phi e^{Kt'} dt' \right] \quad (28) \]
where φ is the real part of \( \hat{\phi} \) given by Equation (19a) and the integral term is given by:

\[
e^{-K_t} \int_0^t \varphi e^{K_t} dt = \text{Re} \left\{ \sum_{m=1}^N G_m(t) X_m(x) \right\}
\]

(29)

2.4. Method of Averaging

The theory presented thus far represents a two-stage simplification of the original problem. In the first stage the problem has been reduced to the solution of a pair of nonlinear, partial differential equations (i.e., Equations (13) and (14)). In the second stage the solution was expanded in a series of acoustic modes with time-dependent amplitudes and the Galerkin Method was used to replace the solution of the nonlinear partial differential equations with the solution of a system of nonlinear ordinary differential equations (i.e., Equations (22) and (23) or (25)). As demonstrated in Reference 10 a further simplification of the problem (and reduction in computation time) may be obtained by using the Method of Averaging (referred to hereafter as MOA) to reduce Equations (22) and (23) or Equations (25) into a system of first order, coupled differential equations describing the growth or decay of the unknown amplitudes \( A_j(t) \). A brief description of the application of the MOA to the present problem is given below; further details are available in References 10 and 19.

To apply the MOA the governing equations must be expressed in the following form:

\[
\frac{d^2 A_j}{dt^2} + \omega_j^2 A_j = F_j
\]

(30)

where \( F_j \) is a nonlinear function of the various \( A_j \) and \( \frac{dA_j}{dt} \). According to the MOA each unknown amplitude \( A_j(t) \) is expressed as

\[
A_j(t) = a_j(t) \sin (\omega_j t) + b_j(t) \cos (\omega_j t)
\]

(31)

where \( a_j(t) \) and \( b_j(t) \) are assumed to be slowly-varying functions of time (i.e., their fractional changes during one period are small). The functions \( a_j(t) \) and \( b_j(t) \) are then determined from the following first order equations

\[
\frac{da_j}{dt} = \frac{1}{\omega_j T^1} \int_t^{t+T^1} F_j \cos(\omega_j t') dt' \\
\frac{db_j}{dt} = -\frac{1}{\omega_j T^1} \int_t^{t+T^1} F_j \sin(\omega_j t') dt'
\]

(32a)  

(32b)

where \( T^1 = 2\pi/\omega_1 \).

The solutions of Equations (32) require less computation time as these equations only describe the slowly-varying parts of the unknown amplitudes \( A_j(t) \), as the oscillatory, rapidly-varying parts of the solutions are specified by \( \sin \omega_j t \) and \( \cos \omega_j t \) in Equation (31).

The MOA was first applied to Equations (22) and (23), but the resulting solutions were unstable when particles were present in the combustor flow. Thus Culick's approach\(^\text{10}\) was adopted and the MOA was applied to Equations (25). These equations are a system of coupled differential equations to be solved for the unknown complex amplitude functions, \( A_j(t) \). In order to apply the MOA to this system, they must first be separated into their real and imaginary parts. This is done by assuming that \( A_j(t) = F_j(t) + iG_j(t) \), substituting into Equations (25), and separating real and imaginary parts to obtain the equivalent system of real differential equations that describe the behavior of the \( F_j \)'s and \( G_j \)'s. Since these equations contain twice as many unknown functions (i.e., \( F_j \) and \( G_j \)) as Equations (25) it is convenient to re-index the unknown functions and their coefficients as follows:

\[
F_p(t) = B_{2p-1}(t) \\
G_p(t) = B_{2p}(t)
\]

(33)

Thus the \( B \)'s with odd indices correspond to the real parts, \( F_p(t) \), and the \( B \)'s with even indices correspond to the imaginary parts, \( G_p(t) \). The corresponding set of differential equations is of the following form:
\[
\sum_{p=1}^{2N} \left\{ C'_o(j,p) \frac{d^2B_p}{dt^2} \right\} = g_j(B_1, B_2, \ldots B_{2N}, \frac{dB_1}{dt}, \frac{dB_2}{dt}, \ldots \frac{dB_{2N}}{dt}) \quad (34)
\]

\( j = 1, 2, \ldots 2N \)

The above equations are coupled in the second derivatives; that is, there are two or more \( C'_o \) terms in each equation. This coupling results from the non-orthogonality of the axial eigenfunctions. In order to apply the MOA to Equations (34), they must be decoupled by transforming to the form:

\[
\frac{d^2B_j}{dt^2} = f_j(B_1, B_2, \ldots B_{2N}, \frac{dB_1}{dt}, \frac{dB_2}{dt}, \ldots \frac{dB_{2N}}{dt}) \quad (35)
\]

This transformation is accomplished by a matrix inversion technique described in Appendix C. Finally Equations (35) can be written in the form:

\[
\frac{d^2B_j}{dt^2} + \omega^2_j B_j = F_j \quad (36)
\]

where

\[
F_j = \omega^2_j B_j - \sum_{m=1}^{2N} \left\{ C_1(j,m) B_m + \left[ C_2(j,m) + h c_m \tilde{E}_1(j,m) \right] \frac{dB_m}{dt} + \tilde{C}_3(j,m) e^{-Kt} \int_0^t B_m(t') e^{Kt'} dt' \right\}
\]

\[
+ h c m \tilde{E}_2(j,m) \frac{dB_m}{dt} + \tilde{D}(j,m,n) B_m \frac{dB_n}{dt} \quad j = 1, 2, \ldots 2N \quad (37)
\]
Thus Equations (25) have been transformed to the required form given by Equations (30). The real coefficients $C_1, C_2, C_3, E_1, E_2,$ and $D$ are derived from the complex coefficients appearing in Equation (25) during the process of separating real and imaginary parts and decoupling the second derivatives (Appendix C).

The MOA is then applied to Equations (36) and (37) by assuming that

$$B_j(t) = g_j(t) \sin \left(\omega_j t\right) + h_j(t) \cos \left(\omega_j t\right) \quad (38)$$

where the $g_j(t)$ and $h_j(t)$ are slowly-varying functions of time. In performing the integrations indicated in Equations (32) the $g_j$ and $h_j$ are assumed constant (the basic assumption is that $g_j$ and $h_j$ do not vary significantly during the interval of averaging). The resulting equations describing the time-variation of the functions $g_j$ and $h_j$ are given by:

$$\frac{dg_j}{dt} = \frac{1}{2\omega_j} \left[ \omega_j^2 h_j - \sum_{m=1}^{2N} h_m \delta_{p,\lambda} \left[ \tilde{C}_1(j,m) + \frac{K}{K^2 + \omega_m^2} \tilde{C}_3(j,m) \right] \right]$$

$$- \sum_{m=1}^{2N} \omega_m g_m \delta_{p,\lambda} \left[ \tilde{C}_2(j,m) + h_c \tilde{E}_1(j,m) + h_c \tilde{E}_2(j,m) - \frac{\tilde{C}_3(j,m)}{K^2 + \omega_m^2} \right]$$

$$e^{-Kt} \left( e^{-\frac{2\pi K}{w_1} - 1} \right) \left[ \omega_j \sin \left(\omega_j t\right) - K \cos \left(\omega_j t\right) \right] \frac{\omega_1}{\pi} \times$$

$$\times \sum_{m=1}^{2N} \frac{\tilde{C}_3(j,m)}{K^2 + \omega_m^2} \left[ \omega_m g_m(0) - K h_m(0) \right]$$

$$- \sum_{m=1}^{2N} \sum_{n=1}^{2N} \tilde{\beta}^{(j,m,n)} \left[ - \frac{\omega_n}{\pi} g_m h_n \beta_2(\lambda, p, q) + \frac{\omega_n}{\pi} h_m g_n \beta_1(p, q, \lambda) \right] \right] \quad (39a)$$
\[
\frac{dh_j}{dt} = -\frac{1}{2\omega_j} \left\{ \omega_j^2 g_j - \sum_{m=1}^{2N} \omega_m \delta_{p\ell} \left[ \tilde{C}_1(j,m) + \frac{K}{K^2 + \omega_m^2} \tilde{C}_3(j,m) \right] \right. \\
+ \sum_{m=1}^{2N} \omega_m h_m \delta_{p\ell} \left[ \tilde{C}_2(j,m) + h_c \tilde{E}_1(j,m) + h_c \tilde{E}_2(j,m) + \frac{\tilde{C}_3(j,m)}{K^2 + \omega_m^2} \right] \\
- e^{-Kt} \left[ e^{-\frac{2\pi K}{\omega_1} - 1} \left[ K \sin(\omega_j t) + \omega_j \cos(\omega_j t) \right] \frac{\omega_1}{\pi} \left( \sum_{m=1}^{2N} \frac{\tilde{C}_2(j,m)}{K^2 + \omega_m^2} \left[ \omega_m g_m(0) - Kh_m(0) \right] \right) \\
- \sum_{m=1}^{2N} \sum_{n=1}^{2N} \tilde{D}(j,m,n) \left[ \frac{\omega_m}{\pi} \delta_m \delta_n \beta_2(q,p,\ell) - \frac{\omega_m}{\pi} h_m h_n \beta_2(p,q,\ell) \right] \right\}, \quad j = 1, 2, \ldots, 2N
\] 

(39b)

In these equations \( \delta_{p\ell} \) is zero for \( p \neq \ell \) and unity for \( p = \ell \) and \( \beta_1 \) and \( \beta_2 \) are integrals of products of three trigonometric functions as defined in Appendix C. The indices \( \ell, p, q \) are the axial mode numbers corresponding to the indices, \( j, m, n \) respectively. This notation is necessary because two \( B_j(t) \) functions are needed to describe each mode.

To obtain nonlinear solutions for a rocket motor, initial amplitudes \( g_j(0) \) and \( h_j(0) \) are specified and Equations (39) are integrated numerically. The resulting solutions for \( g_j(t) \) and \( h_j(t) \) yield the growth or decay rates of the various modes directly. To obtain the pressure waveforms Equation (38) is used to compute the corresponding \( B_j(t) \) which are then combined to obtain \( \phi \). Once \( \phi(x,t) \) is known the pressure perturbation, \( p' \), is calculated using Equation (28).

A few comments regarding computation time will now be made. Applying the Galerkin method (without using the MOA) to Equation (17) yields a set of \( 2N \) second order equations and \( 2N \) first order equations after the real and imaginary parts have been separated. This is equivalent to a set of \( 6N \) first order equations. On the basis of the number of equations alone the MOA yields only a modest decrease in computation time. The advantage of the MOA lies in the fact that the \( g_j \) and \( h_j \) functions are slowly
varying, therefore a much larger time increment can be used in performing
the numerical integrations resulting in a substantial saving in computation
time. In order to evaluate the accuracy of the MOA, solutions obtained
by the Galerkin method (without averaging) and solutions obtained by the
MOA will be compared in Section IV.

2.5 **Application to Motors**

The approximate analyses (Galerkin method and MOA) may be applied
to a variety of motor and T-burner configurations. The application of
the approximate methods to determine the nonlinear stability behavior of
a motor with a full-length tubular propellant grain and a quasi-steady
nozzle (Figure 1) is discussed in this subsection. The application to
T-burners is covered in the next subsection.

As mentioned previously, the coefficients that appear in Equations
(22) and (23), (25), and (39) are dependent upon the steady-state proper-
ties \( \bar{u}(x) \), \( \bar{u}_p(x) \), and \( \bar{p}_p(x) \). Therefore, the steady-state solutions must
be obtained before the approximate analysis can be applied. Although these
steady-state quantities could be obtained by numerically integrating the
steady-state versions of Equations (1) through (6), it is more convenient
to use an approximate analytical solution for the steady-state properties.
In deriving the approximate unsteady equations, it was assumed that the
steady-state values of \( \rho \), \( p \), and \( h \) (or \( T \)) were constants (the variation
in these properties is of order \( \bar{u}_{e}^{2} \)). To this order of approximation the
steady-state gas and particle velocities, \( \bar{u}(x) \) and \( \bar{u}_p(x) \), are linear func-
tions of the axial coordinate \( x \), and the particle density \( \bar{p}_p(x) \) is a con-
stant. The following expressions were determined by satisfying the steady-
state continuity and momentum equations with \( \bar{p}(x) = 1 \) and \( \frac{d\bar{p}_p}{dx} = 0 \):

\[
\bar{u}(x) = \left[ \frac{2m}{K} \right] x = \bar{u}_e x
\]

\[
\bar{u}_p(x) = \frac{2\bar{u}_e x}{1 + \sqrt{1 + \frac{8\bar{u}_e}{K}}} = \left( \frac{2}{1 + \sqrt{1 + \frac{8\bar{u}_e}{K}}} \right) \bar{u}(x)
\]
\[ \rho_p = \frac{\dot{m}_p}{2m} \left[ 1 + \sqrt{1 + \frac{8u_e}{K}} \right] = \left\{ \frac{1 + \sqrt{1 + \frac{8u_e}{K}}}{2} \right\} \]  

The derivation of these equations is given in Appendix D.

In order to obtain approximate stability solutions for motors, the nozzle admittance \( Y \) must also be specified. For conventional nozzles \( Y \) is a complex number which is dependent on nozzle geometry and oscillation frequency. For short nozzles the unsteady nozzle flow is nearly quasi-steady and \( Y \) becomes a real number which is independent of frequency. The value of \( Y \) for a quasi-steady nozzle is determined by the condition that the Mach number at the nozzle entrance is a constant. Assuming small perturbations and neglecting the effect of particles yields the following expression for \( Y \):

\[ Y_r = \frac{\gamma - 1}{2\gamma} u_e \]

\[ Y_i = 0 \]  \hspace{1cm} (43)

2.6 Application to T-Burners

The approximate methods described in this section were also used to analyze the nonlinear behavior of T-burners. Since the T-burner geometry differs in several important respects from that of motors, a detailed discussion of these differences is given in this subsection.

The T-burner geometry considered in this investigation is shown in Figure 3. The T-burner consists of a cylindrical tube of length \( L^* \) with a vent of length \( L_v^* \) at the center. In the simplest configuration, propellant disks are placed at each end of the T-burner (end-burning only). For metalized propellants it is often necessary to include short tubular grains of length \( L_b^* \) at the ends in order to increase the burning propellant surface area (cup grains). The cross-sectional area of the burner is \( S_{co}^* \), while the cross-sectional area of the cylindrical grain is \( S_c^* \). As in the case
Figure 3. T-Burner Geometry
of motors, the axial variable \( x^* \) is normalized by the chamber length, thus \( x = x^* / L^* \) and the dimensionless lengths of the cylindrical grains and the center vent are given by

\[
\beta = 2L_b^* / L, \quad \beta_v = L_v^* / L
\]  \hspace{1cm} (44)

For convenience in the analysis, the T-burner is divided into five regions as shown in Figure 3. In Regions 1 and 5 at the ends of the burner, the flux of burned gases and particles at the lateral propellant surface is represented by mass and energy source terms in the conservation equations. Thus these regions are similar to the interior of a motor, except for the boundary fluxes at the ends \((x = 0 \text{ and } x = 1)\). In Regions 2 and 4 there is no combustion at the lateral boundary, so the corresponding source terms are zero. Finally in Region 3, the mass flux through the center vent is represented by mass and energy sinks in the governing equations. For the case of end burning disks only, \( \beta = 0 \) and Regions 1 and 5 vanish.

In order to apply the approximate analysis techniques to T-burners, the analysis must be modified to account for the following: (1) the presence of burning propellant at the ends of the chamber, (2) the presence of partial length lateral propellant surfaces, and (3) the presence of the center vent. These points will now be discussed separately.

**End Burning.** The burning propellant grains at the ends of the T-burner must be treated as boundary conditions rather than source terms as in the case of lateral grains. These burning surface boundary conditions replace the rigid wall boundary condition (head-end) and nozzle admittance boundary condition used in the motor analysis. The boundary conditions at the ends of the T-burner can be described most conveniently in terms of the response function defined by Equation (9):

\[
B_0 = \frac{\partial \psi}{\partial x} + \gamma \overline{u}_b (R - 1) \overline{\psi}_t = 0 \quad \text{at } x = 0
\]

\[
B_1 = \frac{\partial \psi}{\partial x} - \gamma \overline{u}_b (R - 1) \overline{\psi}_t = 0 \quad \text{at } x = 1
\]  \hspace{1cm} (45)

where \( \overline{u}_b \) is the steady-state velocity of the burned gases leaving the propellant surfaces. In deriving Equation (45) from Equation (9) the assumption that
\( \ddot{p} = 1 \) and \( \ddot{p} = 1 \) was used, and \( p' \) was replaced by its first order equivalent \(-\gamma \dot{\phi}_t\). The difference in the signs for \( \dot{B}_0 \) and \( \dot{B}_1 \) arises because a positive pressure perturbation \( (\dot{\phi}_t < 0) \) yields a positive velocity perturbation at the left end \( (\ddot{\phi}_x > 0) \) and a negative velocity perturbation at the right end \( (\ddot{\phi}_x < 0) \). In other words, a positive flux (i.e., leaving the surface) gives positive velocities at \( x = 0 \) and negative velocities at \( x = 1 \), since the positive direction for velocities is taken to be the direction of increasing \( x \).

In applying the Galerkin method to the T-burner, a boundary residual arises at both ends of the chamber, rather than just at one end as in the motor case. Thus the Galerkin orthogonality conditions given by Equations (21) are replaced by the following expressions:

\[
\int_0^1 E_p \left\{ \ddot{\phi}_x, \ddot{\phi}_t \right\} X_j^*(x) \, dx + B_0 \left\{ \ddot{\phi}_t \right\} X_j^*(0) - B_1 \left\{ \ddot{\phi}_t \right\} X_j^*(1) = 0 \tag{46a}
\]

\[
\int_0^1 E_p \left\{ \ddot{\phi}_x, \ddot{\phi}_t \right\} X_j^*(x) \, dx = 0 \tag{46b}
\]

\( j = 1, 2, \ldots, N \)

where the \( E \) and \( E_p \) are the residuals of Equations (13) and (14) and the boundary residuals \( B_0 \) and \( B_1 \) are given by Equations (45). The eigenfunctions \( X_j \) are still given by Equation (20), but the corresponding \( b_j \)'s are different from those used previously for the analysis of motors. The proper selection of the \( b_j \)'s of course, should be the acoustic eigenvalues for a chamber with boundary conditions given by Equations (45). In the case of the T-burner, however, the velocities of the gases leaving the propellant surfaces are sufficiently small that these eigenvalues can be approximated by the eigenvalues for a closed-ended chamber. Thus the eigenvalues \( b_j \) are given by

\[
b_j = j \pi \quad j = 1, 2, 3, \ldots, N \tag{47}
\]

**Partial Length Cylindrical Grains.** The lateral burning surfaces are treated as source terms as was done previously for motors, but the approximate analysis has been extended to handle partial length grains. Two grain configurations are considered in this study: (1) disk grains only
and (2) cup grains (Figure 3). In the latter case flush grains \( S_{co} = S_c \) are assumed. Variable cross-sectional area has an important effect on mode shape and frequency.\(^{20}\) Thus, to accommodate protruding or recessed grains, the mode shapes used in the assumed series expansion must be those given by Derr,\(^{20}\) which are much more complicated than those presently used. Thus a major modification of the present analysis would be necessary to accommodate T-burners of variable cross-sectional area, which is beyond the scope of this project. Thus the approximate T-burner analysis is restricted to flush grain configurations.

In applying the Galerkin method to analyze T-burners with cup grains, the spatial integrations indicated in Equations (46) must be performed over three distinct types of regions. In the first type of region, the source terms present in the wave equation are due to lateral surface burning (Regions 1 and 5) and Equation (13) is used to obtain the residual \( E(\tilde{\psi}) \). In the second type of region, the source terms are zero (Regions 2 and 4), and the residual \( E(\tilde{\psi}) \) is obtained by dropping the \( \frac{du}{dx} \) terms in Equations (13). The third type of region, the vent region, will be discussed later. These differences between the first two types of regions are automatically taken into account by introducing the appropriate steady-state solutions when the integrals are performed.

The steady-state solutions for Regions 1 and 2 (as well as Regions 4 and 5) were obtained from the steady-state continuity and momentum equations. In order to obtain analytical expressions for \( \bar{u}(x) \), \( \bar{u}_p(x) \), and \( \bar{p}_p(x) \) a number of simplifying assumptions were made. These assumptions are consistent with the order of magnitude approximations used in deriving Equations (13) and (14). A discussion of these approximations and the derivation of the steady-state relations are given in Appendix D.

In the regions of lateral surface burning, the steady-state solutions are similar to those for motors, except for differences caused by the finite velocity at the ends of the burner. In Region 1 \( (0 \leq x \leq \beta/2) \) the steady-state solutions are:

---

\[ u(x) = u_b \left( 1 + \frac{2}{R} x \right) \quad (48) \]

\[ \tilde{u}_p(x) = \frac{u_b}{1+\eta} \left( 1 + \frac{2}{R} x + \eta \left[ 1 + \frac{2}{R} x \right]^{-\frac{\eta+2}{\eta}} \right) \quad (49) \]

\[ \rho_p(x) = \frac{1+\eta}{1+\eta \left[ 1 + \frac{2}{R} x \right]^{-\frac{2\eta+2}{\eta}}} C_m \quad (50) \]

where \( \eta = 4u_b/RK \). For \( x = 0 \) these relations yield \( \tilde{u}(0) = u_b, \tilde{u}_p(0) = u_b \), and \( \rho_p(0) = C_m \), which agree with the boundary values imposed on the steady state solutions. The gas velocity is seen to vary linearly with \( x \) with a slope given by:

\[ \frac{d\tilde{u}}{dx} = \frac{2\tilde{u}_b}{R} \quad (51) \]

while the particle velocity approaches a linear variation with \( x \) after a short transitional distance \( \delta x \). Similarly, the particle density approaches a constant value for \( x > \delta x \). These asymptotic values are given as follows:

\[ \tilde{u}_p(x) = \frac{\tilde{u}(x)}{1+\eta} \quad (52) \]

\[ \tilde{\rho}_p(x) = (1+\eta)C_m \quad (53) \]

For particle sizes of a few microns, \( \eta \) is a small positive number and the exponents appearing in Equations (49) and (50) are large, thus Equations (52) and (53) are very good approximations for all but very small values of \( x \). In Region 5, the directions of the velocities \( \tilde{u}(x) \) and \( \tilde{u}_p(x) \) are opposite from those in Region 1, and the magnitudes of the steady-state properties are obtained from equations (48) through (50) by replacing \( x \) with \( 1-x \). For \( 1 - \beta/2 \leq x \leq 1 - \delta x \) Equations (52) and (53) are also
valid, with $\bar{u}(x)$ given by:

$$\bar{u}(x) = -\bar{u}_b \left[ 1 + \frac{2}{R} (1-x) \right] \quad (54)$$

It should be noted that the slope $du/dx$ is the same in both Regions 1 and 5, since the steady-state mass fluxes at the lateral boundaries are the same in both regions.

In Regions 2 and 4 the steady state source terms vanish, and the gas velocity becomes a constant given by:

$$\bar{u}(x) = \pm \bar{u}_b \left( 1 + \frac{\beta}{R} \right) \quad (55)$$

where the upper sign refers to Region 2 and the lower sign refers to Region 4. The magnitude of $\bar{u}(x)$ was obtained by substituting $x = \beta/2$ into Equation (48). The particle velocity in Region 2 is then given by (Appendix D):

$$\bar{u}_p(x) = \bar{u}(\beta/2) \left[ 1 - \frac{\eta}{1+\eta} e^{-\alpha(x-\beta/2)} \right] \quad (56)$$

where $\alpha = K/\bar{u}(\beta/2)$. For $x = \beta/2$ the exponential term is unity and Equation (56) yields $\bar{u}_p(\beta/2) = \bar{u}(\beta/2)/(1 + \eta)$ which is the velocity of the particles leaving Region 1. The exponential term decays rapidly with small increases in $x$ to yield

$$\bar{u}_p(x) = \bar{u}(\beta/2) = \bar{u}_b (1 + \beta/R) \quad (57)$$

which is the same as the gas velocity in Region 2. The particle density quickly relaxes to the constant value given by

$$\rho_p(x) = \frac{C}{m} \quad (58)$$

Again, the steady-state particle properties in Region 4 are obtained from the Region 2 properties by symmetry.

**Center Vent.** The effect of a subsonic exhaust vent at the center of the T-burner is modeled following the methodology of Levine and Culick. Here, two effects are considered: (1) the mean flow/acoustics interaction, which vanishes in the absence of the mean flow, and (2) the acoustic radiation
through the vent, which remains finite in the absence of mean flow. For odd modes, the radiation effect is weak because the vent is located at a pressure node, but the mean flow/acoustics effect is strong due to the maximum acoustic velocity at this location. The opposite is true for even modes; the mean flow/acoustics effect is weak (velocity node) while the radiation effect is strong (pressure anti-node).

In considering the mean flow/acoustics vent effect, Levine and Culick \(^5\) argued that the net effect is nearly zero. This result is obtained because the vent gain effect predicted by formal application of linear theory is cancelled by a vent loss due to viscous effects and the axial force exerted by the walls of the vent on the flow. Due to the possibility of a nonlinear mean flow/acoustics vent effect, however, Levine and Culick \(^5\) retained the ability to vary the mean flow/acoustics vent effect in the nonlinear differential equations. This has also been done in the present approximate analysis.

In the vent region (Region 3 for \((1 - \beta_v) / 2 \leq x \leq (1 + \beta_v / 2)\)), the conservation equations (i.e., Equations (1) - (5)), are modified as follows. The source terms \(2\mathbf{m}_g / R\) and \(2\mathbf{m}_p / R\) are replaced by \(\omega\) and \(\omega_p\), respectively which are defined by:

\[
\omega = -\frac{A_v}{\beta_v} u_n
\]

\[
\omega_p = -\frac{A_v}{\beta_v} \rho_p u_n
\]

where \(A_v\) is the ratio of vent area to chamber cross-sectional area, and \(u_n\) is the dimensionless velocity of the flow out the vent. The minus sign indicates that \(\omega\) and \(\omega_p\) are negative for flow out of the burner \((u_n\) positive), and it has been assumed that both gas and particles exit with the same velocity \(u_n\). In the gas and particle momentum equations (i.e., Equations (2) and (3)) the axial velocities in the source terms arising from mean flow/acoustic interactions are multiplied by the factor \(1 - V_L\). Setting \(V_L = 0\) yields vent gain which corresponds to the combustion products losing their axial momentum as they exit through the vent. For \(V_L = 1\) the combustion products retain their axial momentum and the net vent effect is zero, and \(V_L = 2\) gives a vent loss. Using the order of magnitude analysis given in
Appendix A, the modified conservation equations are combined to yield the following nonlinear equations, valid in the vent region:

\[
\begin{align*}
\ddot{\phi}_{xx} - \ddot{\phi}_{tt} &= 2\ddot{u}\dot{\phi}_{xt} + (\gamma + 1 - V_{L}) \frac{d\ddot{u}}{dx} \ddot{\phi}_t \\
+ 2\dot{\phi}_{x} \dot{\phi}_{xt} + (\gamma - 1) \phi_t \ddot{\phi}_{xx} \\
+ K_{p} (\ddot{\phi}_t - \ddot{\phi}_p) - (\gamma - 1) \rho p \ddot{u} (\ddot{\phi}_p)_{xt} + \omega' h_c &= 0 \quad (61)
\end{align*}
\]

\[
\begin{align*}
(\ddot{\phi}_p)_{t} + \ddot{u}_p (\ddot{\phi}_p)_x + \frac{1}{2} (\ddot{\phi}_p)_x^2 &= K(\ddot{\phi}_p - \ddot{\phi}_t) - (1 - V_{L}) \frac{d\ddot{u}}{dx} \ddot{\phi}_p \\
(\ddot{p})_{t} &= \gamma \left[ \ddot{\phi}_t + \ddot{u}_x + \frac{1}{2} \phi_{xx} - \frac{1}{2} \dot{\phi}_t^2 + K_{p} (\ddot{e} - \ddot{\phi}_p) + (1 - V_{L}) \frac{d\ddot{u}}{dx} \ddot{\phi}_p \right] \quad (62)
\end{align*}
\]

where

\[
\omega' = - \frac{A_v}{\rho_v} u'_n 
\]

The acoustic radiation vent effect is described by the term \(\omega' h_c\) appearing in Equation (61). This introduces the additional unknown quantity, \(u'_n\) into the equations. Assuming plug flow in the pipe connecting the burner with the surge tank, the following transient vent equation is obtained from the unsteady, incompressible, momentum equation:\(^5\)
In this equation, \( L_{\text{eff}} \) is the effective plug length, \( \rho_0 \) is the steady state density, \( P_s \) is the surge tank pressure (assumed constant), and \( P \) is the average pressure over the vent; that is,

\[
\frac{1+\beta_v}{2} \bar{p} = \int_{1-\beta_v}^{1+\beta_v} p(x',t) dx'
\]  

(66)

To obtain the corresponding equation for the perturbation \( u' \), \( u_n = \bar{u} + u' \) is introduced into Equation (65) and the steady state terms are subtracted out to give:

\[
\frac{1+\beta_v}{2} \frac{d u_n}{L_{\text{eff}} dt} + u_n u_n + \frac{1}{2} (u_n')^2 = \int_{1-\beta_v}^{1+\beta_v} p'(x',t) dx'
\]  

(67)

where \( \rho_0 = 1 \) has been used. Neglecting the nonlinear term as higher order and replacing \( p' \) by its first order equivalent \(- \gamma \phi_t \) gives

\[
\frac{d u_n}{L_{\text{eff}} dt} + u_n u_n = -\gamma \int_{1-\beta_v}^{1+\beta_v} \phi_t(x',t) dx'
\]  

(68)

In the vent region (Region 3) the steady-state quantities are also obtained by integrating the steady-state continuity and momentum equations, where the steady-state mass source, \( \bar{w} \), is given by:
The steady-state gas velocity is then given by:

\[ \tilde{\omega} = -\frac{2\tilde{u}_b}{\beta_v} (1 + \beta/R) \]  \hfill (69)

for \((1 - \beta_v)/2 \leq x \leq (1 + \beta_v)/2\). This expression shows that \(\tilde{u}(x)\) decreases linearly from \(\tilde{u}_b(1 + \beta/R)\) at the left boundary of Region 3 to \(-\tilde{u}_b(1 + \beta/R)\) at the right boundary. The steady-state gas velocity vanishes at the center of the burner (\(x = 1/2\)). The particle velocity and density in the vent region are described by the following expressions (Appendix D):

\[ \tilde{u}_p(x) = \frac{\tilde{u}_b(1+\beta/R)}{1 - \eta_v} \left\{ \begin{array}{l} \frac{1 - 2x}{\beta_v} - \eta_v \left[ \frac{1 - 2x}{\beta_v} \right]^q \\ \frac{1 - \eta_v}{1 - \eta_v \left[ \frac{1 - 2x}{\beta_v} \right]^q} \end{array} \right\} \]  \hfill (71)

\[ \rho_p(x) = c_m \left\{ \begin{array}{l} \frac{1 - \eta_v}{1 - \eta_v \left[ \frac{1 - 2x}{\beta_v} \right]^q} \\ \frac{1 - \eta_v}{1 - \eta_v \left[ \frac{1 - 2x}{\beta_v} \right]^q} \end{array} \right\} \]  \hfill (72)

where \(q = 2/\eta_v + (V_L - 1)\)

and \(\eta_v = \frac{(4 - 2V_L)\tilde{u}_b(1 + \beta/R)}{K\beta_v}\)

These expressions are valid for \((1 - \beta_v)/2 \leq x \leq 1/2\), while the corres-
ponding values for \(1/2 \leq x \leq (1 + \beta_v)/2\) are obtained from symmetry. As in Region 1 the exponent \(q\) is large for particle sizes of a few microns; thus, except in thin zones adjacent to the boundaries of Region 3, \(\bar{u}_p(x)\) and \(\bar{\rho}_p(x)\) are well approximated by

\[
\bar{u}_p(x) = \frac{\bar{u}(x)}{1 - \eta_v} \tag{73}
\]

\[
\bar{\rho}_p(x) = (1 - \eta_v)C_m \tag{74}
\]

which are similar to Equations (52) and (53) for Region 1. Steady-state values for a typical T-burner configuration with cup grains are shown in Figure 4.

An additional steady-state value is needed in the vent region; that is, the steady-state velocity of the flow out the vent, \(\bar{u}_n\). This quantity appears in Equation (68) and is given by:

\[
\bar{u}_n = \frac{2\bar{u}_b (1 + \beta/R)}{A_v} \tag{75}
\]

Amplitude Equations for T-Burners. To summarize the solution procedures for T-burners, the application of the Galerkin method in the five regions previously discussed will now be outlined. The series expansions for \(\tilde{\xi}\) and \(\tilde{\xi}_p\) given by Equations (19) with \(b_j = j\pi\) are substituted into the appropriate differential equations and boundary conditions to form the residuals \(E\{\bar{\xi}, \tilde{\xi}_p\}\), \(E_p\{\bar{\xi}, \tilde{\xi}_p\}\), \(B_o(\tilde{\xi})\), and \(B_1(\tilde{\xi})\). In Regions 1, 2, 4, and 5, Equations (13) and (14) are used to obtain \(E\) and \(E_p\), respectively, while Equations (61) and (62) are used to obtain these residuals in Region 3. The Galerkin orthogonality conditions given by Equations (46) are then applied to obtain a system of second-order ordinary differential equations for the mode-amplitudes \(A_j(t)\). The spatial integrations indicated in Equations (46) are evaluated piecewise in each of the five regions, using the appropriate residuals in each region.

The resulting system of differential equations is similar to that
Figure 4. T-Burner Steady-State Properties
obtained for motors (i.e., Equations (22) and (23)), however, some differences should be noted. Due to the acoustic radiation term (i.e., $w'/h_c$) appearing in Equation (61) for the vent region, additional terms of the form $C_7(j)u'_n$ appear in the T-burner mode-amplitude equations. Since $u'_n$ is an additional unknown quantity, another equation is needed in order to obtain solutions. The required equation is supplied by substituting the series expansion for $\ddot{w}$ into the unsteady flow vent equation (i.e., Equation (68)) and performing the indicated integration over the length of the vent. The resulting equation is of the form:

$$\frac{du'_n}{dt} + \left(\frac{\bar{u}_n}{L_{eff}}\right)u'_n = -\left(\frac{\gamma}{L_{eff}}\right) \sum_{j=1}^{N} C_7(j) \frac{dA_j}{dt}$$

(76)

where $\bar{u}_n$ is given by Equation (75). The coefficients of the linear terms in the T-burner equations also differ from the corresponding coefficients (i.e., $C_1$ through $C_6$) in the motor equations, due primarily to the differences in the steady-state properties and the presence of burning propellant disks at the ends of the burner. Expressions for the linear coefficients in the T-burner equations are given in Appendix C. The coefficients of the nonlinear terms in the T-burner equations are the same as the corresponding coefficients in the motor equations.

2.7 Nonlinear Combustion Driving

In the analysis developed in the preceding sections, the response of the burning rate to pressure oscillations was described by a linear response function (see Equations (9) through (11)). Based on intuition and studies conducted at Georgia Tech\(^2\),\(^3\) and elsewhere\(^5\), it is expected that as the amplitude of the oscillation grows, the true combustion response departs from the linear value predicted by Equation (9). Thus, a nonlinear expression relating the perturbations in burning rate to the gas flame pressure fluctuations is needed, if accurate stability predictions are to be obtained.

A nonlinear pressure coupled response is presently included in the "exact" analysis developed by Kooker and Zinn (in fact one of the modifications of the "exact" theory required for the study of gasdynamic non-linearities was to replace this nonlinear response with a linear one).
This nonlinear response is obtained by solving the governing equations describing the thermal wave in the solid, the surface pyrolysis reactions, and the gas phase reactions simultaneously with the conservation equations describing the unsteady flow in the chamber and nozzle. In order to use the same approach in the approximate analysis, it would be necessary to apply the Galerkin Method to obtain the behavior of the temperature in the solid propellant and the flame zone. In principle, this solution for the combustion response would have to be obtained simultaneously with the solutions of the combustor conservation equations, as was done in the "exact" analyses\(^2,3\). Attempts to use the Galerkin Method to obtain the nonlinear solid propellant combustion response were made during the development of the Kooker-Zinn model; however, these studies have shown that the approximate technique was unsuitable for properly predicting the nonlinear propellant response. Based upon this experience, it is expected that the same difficulties would also be encountered when attempting to incorporate the nonlinear combustion response into the present approximate analysis. Thus, in the absence of any other nonlinear models for pressure coupling, a heuristic combustion model has been used in the studies to assess the importance of modelling the nonlinear combustion response.

A number of possible expressions for describing the nonlinear burning rate could be obtained by adding various nonlinear terms to the linear burning response, such as terms proportional to \(|p'|\), \((p')^2\), \(|p'|(p')\), and \((p')^3\). Since the primary concern here is determining the effect of nonlinearities in the combustion response upon the nonlinear stability characteristics of a motor, rather than determining the correct form of the nonlinear combustion response, the simplest of these heuristic models was used. Thus a nonlinear combustion response was incorporated into the approximate analysis by replacing the linear response functions \(R_m\) appearing in Equations (22) with nonlinear response functions \(R_{NL_m}\) given by:

\[
R_{NL_m} = R_m \left[ 1 + b_m \left| \frac{dA}{dt} \right| \right]
\]  

In Equation (77) \(R_m\) is the linear response factor for the \(m^{th}\) mode, \(b_m\) is a complex constant, and \(\left| \frac{dA}{dt} \right|\) is the magnitude of the complex ampli-
where the superscripts \( r \) and \( i \) denote real and imaginary parts of \( A_m \), respectively. Since the pressure amplitude is proportional to \( |dA/dt| \), the effective nonlinear response factor, \( \gamma_{NL} \), is a complex number whose magnitude and phase are linear functions of the amplitude of the oscillation. This expression reduces to the linear burning rate expression in the limit of small amplitude oscillations. At larger amplitudes the magnitude of the response factor will be increased or decreased from its linear value according to the value of \( b \); also, the phase will be shifted if the imaginary part of \( b \) is nonzero. Calculated solutions obtained using this heuristic nonlinear combustion model are presented in section 4.3, where they are compared with approximate solutions obtained with the linear combustion response (\( b_m = 0 \)) as well as with "exact" solutions obtained using the Kooker-Zinn nonlinear combustion model.

2.8 Nonlinear Particle Damping

In the preceding sections it was assumed that the viscous interaction between the particles and gas was described by the Stokes Drag Law (Equations (7) and (8)) for which the drag force is proportional to the relative velocity between the particles and gas. This linear drag law is valid for small Reynolds numbers up to about unity. For many situations encountered in solid rocket motors, however, the particle sizes and frequencies are sufficiently large that higher Reynolds numbers can occur. In these situations the Stokes law is no longer applicable and a nonlinear drag law must be used. The nonlinear particle drag effects are described by a higher order correction to the Stokes law given by\(^{21}\)

\[ F_p = K_p (u - u_p) \left[ 1 + \frac{\text{Re}}{6} \right]^{2/3} \]  

(79)

where Re is the Reynolds number given by

\[ \text{Re} = \frac{\rho^* c^* \sigma^*}{\mu^*} \left| u - u_p \right| \]  

(80)

The straightforward, rigorous method of introducing the nonlinear particle drag law (Equation (79)) into the approximate analysis is to substitute Equation (79) for \( F_p \) where it appears in the gas and particle momentum equations (i.e., Equations (3) and (4)) and the energy equation (i.e., Equation (5)) and to follow the same order of magnitude analysis outlined in Section 2.2 and Appendix A. This procedure leads to insurmountable difficulties due to the highly nonlinear form of the drag correction term (i.e., the absolute value of the relative velocity raised to the \( 2/3 \) power). The first difficulty arises when attempting to separate the steady-state component from the equations. The relative velocity term can be written as \( |\vec{u} - \vec{u}_p|^{2/3} \left[ 1 + \frac{(u' - u'_p)}{(\vec{u} - \vec{u}_p)} \right]^{2/3} \), but a binomial expansion of this term is valid only if \( \frac{(u' - u'_p)}{(\vec{u} - \vec{u}_p)} << 1 \). Since the relative velocity perturbation \( u' - u'_p \) may often be of the same order of magnitude as the steady state relative velocity \( \vec{u} - \vec{u}_p \), the binomial expansion can not be used and the steady state and perturbation quantities can not be separated. Even if this difficulty did not exist, other problems arise when attempting to combine the conservation equations to obtain the gas and particle potential equations by the procedure followed in Appendix
A. Finally, the absolute value in the drag correction terms make it impossible to separate the time and space variables when applying the Galerkin method, a crucial step needed in order to evaluate the spatial integrations involved.

Because of these difficulties in obtaining equations for the gas and particle potentials with \( F_p \) given by Equation (79), a heuristic approach is followed. It is tacitly assumed that the effect of particle drag nonlinearities can be described by the previously derived mode-amplitude equations (i.e., Equations (22) and (23)) in which the linear drag constant \( K \) is replaced by a nonlinear drag coefficient \( \tilde{K} \) which is amplitude dependent. It is postulated that \( K \) has the following form:

\[
\tilde{K} = K \left[ 1 + K_{ss} + CK_{NL} \left| A - A_p \right|^{2/3} \right]
\] (81)

where \( K \) is the original linear drag constant given by Equation (8), \( K_{ss} \) is a steady-state contribution to the nonlinear drag correction, \( K_{NL} \) is given by

\[
K_{NL} = \frac{1}{6} \left\{ \frac{\rho^* c^* \sigma^*}{\mu^*} \right\}^{2/3}
\] (82)

and C is an adjustable parameter. The steady-state contribution is obtained by averaging the nonlinear drag term corresponding to the steady-state relative velocity, \( \bar{u}(x) - \bar{u}_p(x) \), over the length of the motor. Since the steady-state relative velocity is proportional to \( x \) (Equations (40) and (41)), the value of \( K_{ss} \) is given by:

\[
K_{ss} = \frac{1}{6} \left\{ \frac{\rho^* c^* \sigma^*}{\mu^*} \right\}^{2/3} (\bar{u}_e - \bar{u}_p)^{2/3} \int_0^1 x^{2/3} dx
\] (83)

Evaluating the integral and using Equations (41) and (82) give...
\[
K_{ss} = \frac{3}{5} K_{NL} \left\{ \frac{\sqrt{1 + \frac{8\bar{u}_e}{K} - 1}}{1 + \sqrt{1 + \frac{8\bar{u}_e}{K}}} \right\}^{\frac{2}{3}}
\]

The unsteady contribution to the drag correction is assumed to be proportional to the quantity \( |A - A_p|^{2/3} \) where \( A \) and \( A_p \) are the complex gas and particle mode-amplitude functions for the fundamental mode (the effect of the higher modes upon this term has been neglected). The constant of proportionality for this term contains the \( \sigma^{2/3} \) dependence expected from Equation (79), and the adjustable parameter accounts for the axial variation of the relative velocity perturbation \( u' - u'_p \).

The heuristic nonlinear particle drag model described above has been incorporated into the approximate computer program. Calculated solutions obtained with this model are presented in Section 4.4 where they are compared with the available "exact" solutions. On the basis of these comparisons the validity of the heuristic nonlinear particle drag model is assessed.
3. EXACT ANALYSIS

The "exact" analysis developed by Kooker and Zinn, which was used in the present investigation is based on a finite difference solution of the quasi-one-dimensional conservation equations for a solid rocket motor and nozzle combination. These equations are given as follows:

Continuity (gas)

\[
\frac{\partial \rho}{\partial t} + \rho \frac{\partial u}{\partial x} + u \frac{\partial \rho}{\partial x} = \frac{2\dot{m}_g}{R} - \rho u \frac{\partial \ln A}{\partial x} \tag{85}
\]

Continuity (particles)

\[
\frac{\partial \rho_p}{\partial t} + \rho_p \frac{\partial u_p}{\partial x} + u_p \frac{\partial \rho_p}{\partial x} = \frac{2\dot{m}_p}{R} - \rho_p u_p \frac{\partial \ln A}{\partial x} \tag{86}
\]

Momentum (gas)

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{1}{\gamma_p} \frac{\partial p}{\partial x} = - \left\{ K_p \left( u - u_p \right) + \frac{2\dot{m}_g u_p}{R \rho_p} \right\} \tag{87}
\]

Momentum (particles)

\[
\frac{\partial u_p}{\partial t} + u_p \frac{\partial u_p}{\partial x} = K(u - u_p) - \frac{2\dot{m}_p u_p}{R \rho_p} \tag{88}
\]

Energy (gas)

\[
\frac{\partial S}{\partial t} + u \frac{\partial S}{\partial x} = \gamma (\gamma - 1) \left\{ \frac{2}{R} \left[ \frac{\dot{m}_g \left( \frac{c - h}{\gamma - 1} \right)}{p} \right] + \frac{1}{2} \left( \frac{\dot{m}_g u^2}{p} + \frac{\dot{m}_p u^2}{p_p} \right) \right\} + K_p (u - u_p)^2 \tag{89}
\]

where

\[
S = \ln p - \gamma \ln p \tag{90}
\]

The quantities \(\dot{m}_g\) and \(\dot{m}_p\) respectively represent the mass flow rates of gas and particles per unit surface area leaving the burning propellant surface. Here all mass addition due to the burning propellant occurs at the combustor.
boundaries and the volume sources due to evaporation or combustion of particles are neglected. The source terms in the momentum equations result from the application of Stokes' Law to describe the particle drag and the force required to accelerate the incoming gas and particles to their local velocities. The source terms in the energy equation arise from the local addition of hot combustion products (with enthalpy $h_c$) entering the control volume from the flame zone and particle drag within the control volume. Including the area variation term in the continuity equation allows a quasi-one-dimensional solution for the unsteady flow in the nozzle as well as in the combustion chamber. A more complete description of the combustor model and method of numerical integration of the equations is given in Chapter II of Reference 3.

To complete the analysis, an unsteady combustion model is needed to determine the time-dependent quantities $\dot{m}_g$ and $\dot{m}_p$ which appear as mass and energy sources at the boundary of the combustor. This is done in the Kooker-Zinn model by solving the transient burning rate response to the imposed chamber pressure oscillations simultaneously with the chamber conservation equations. The combustion model used at each burning station along the propellant is based on the following four simplifying assumptions which have been employed in nearly all combustion instability studies to date: 12

1. the unburned propellant is homogeneous and one-dimensional, 
2. the conversion of the solid phase to gas is represented by a simple Arrhenius type pyrolysis law, 
3. condensed phase reactions are neglected, 
4. the behavior of the gas-phase flame zone is quasi-steady. 

Under these assumptions the problem separates into three distinct regions which are analyzed individually: (1) the nonreacting solid phase region, (2) the solid-gas interface where pyrolysis occurs, and (3) the gas-phase flame zone. The conservation equations describing the behavior of this model and their numerical solutions are discussed in Chapter III of Reference 3.

For the case of a constant area chamber, the above equations are identical to Equations (1) through (6) upon which the approximate analysis is based. Thus both the "exact" and approximate analyses are attempting to solve the same set of equations. However, the combustion response model and the treatment of the nozzle used in the original Kooker-Zinn model are different from those used in the approximate analysis. In order to facilitate comparisons with the approximate analysis, the original Kooker-
3.1 Modification for Quasi-Steady Nozzle

In the original Kooker-Zinn model the nonlinear governing equations were solved numerically throughout the entire system consisting of both the combustion chamber and the nozzle. The downstream boundary condition was imposed at an arbitrary location downstream of the nozzle throat where the flow is supersonic. The effect of the nozzle obtained from such an analysis is inherently nonlinear and would be expected to differ significantly from the nozzle effect obtained from the linear nozzle admittance boundary condition used in the approximate analysis. Therefore the "exact" analysis was modified by eliminating the nozzle from the flowfield and replacing it with a quasi-steady linear nozzle admittance condition to be satisfied at the nozzle-end of the chamber. The method of characteristics is used to obtain the boundary values at the nozzle-end in a manner similar to that used at the head-end. This modification allows the nozzle effect to be the same in both the approximate and "exact" analyses.

At the nozzle-end of the chamber, the velocity and pressure must satisfy the following relation:

\[ u_e = \tilde{u}_e + Y (p_e - \tilde{p}_e) \]  

where \( Y \) is the nozzle admittance and \( \tilde{u}_e \) and \( \tilde{p}_e \) are the corresponding steady-state quantities. For a quasi-steady nozzle (neglecting the effect of particles) the nozzle admittance \( Y \) is given by:

\[ Y = \frac{\gamma - 1}{2\gamma} \frac{\tilde{u}_e}{\tilde{p}_e} \]  

These relations have been incorporated into the method of characteristics solution for the dependent variables at the nozzle-end boundary. The nozzle calculations are performed by a new subroutine (NOZMOC) which was added to the existing "exact" computer program.

A test case was run at this point to check out the quasi-steady nozzle modification. Pressure waveforms calculated with a conventional (i.e., nonlinear) nozzle and the quasi-steady nozzle are compared in Figure 47.
5. Here it is seen that substituting the quasi-steady nozzle for the conventional nozzle results in an increase in the frequency of the oscillation and a decrease in the decay rate. The increased frequency results from the decrease in length over which the wave travels, and the decrease in damping results because the quasi-steady nozzle is more like a hard-wall termination than the conventional nozzle.

3.2 Linearized Burning Rate Model

The other modification of the "exact" analysis concerns the transient burning rate model. In the original "exact" analysis solutions were obtained for the thermal profiles in the solid propellant, which yield the surface temperature and regression rate at each increment of time. These profiles were coupled to the pressure oscillations in the chamber through the reaction kinetics in the flame zone and at the solid surface. These processes yielded a nonlinear response of the burning rate to the pressure oscillations.

Since the above model cannot be readily incorporated into the approximate analysis, the approximate solutions were obtained using a linear combustion response function Equation (9). In order to compare the results obtained with the approximate and "exact" theories, a linear combustion response model was incorporated into the "exact" analysis. Since the frequency and waveshape of the pressure oscillation are not known a priori, the two-parameter (A,B) form of the linearized response function (see Equation (10)) could not be introduced directly into the "exact" program. Instead, a perturbation analysis was performed on the nonlinear equations governing the response of the burning solid propellant to pressure disturbances, and the resulting higher order (i.e., nonlinear) terms were neglected. The resulting linear equations and boundary conditions are given below:

Energy Equation in Solid Propellant

\[
\frac{\partial T}{\partial t} + \frac{\partial}{\partial y} \left( \frac{\partial T}{\partial y} \right) - \frac{\partial T}{\partial y} \frac{\partial r}{\partial y} = - \bar{T}(y)r'
\]  
\quad (93)

Surface Regression Law

\[
\frac{r'}{r} = \frac{E_s}{T_s^{2}} T_s'
\]  
\quad (94)
Figure 5. Comparison of Pressure Waveforms for Conventional and Quasi-Steady Nozzles
Boundary Conditions

\[ T'(-y_0, t) = 0 \] (95)

\[ T'_y (0,t) = -Z_1 r' - \frac{r'}{r} \left( \frac{c_s^*}{c_p} - 1 \right) T_s' + \frac{Z_2}{r} \left[ 2 \frac{p'}{p} - \frac{r'}{r} \right] \] (96)

In the above equations the barred quantities are steady state values, while the primed quantities are perturbations. The quantities \( p, r, \) and \( T(y) \) respectively denote pressure at the propellant surface, surface regression rate, and temperature in the solid propellant at depth \( y \). The quantities \( E_s^*, Z_1, \) and \( Z_2 \) are constants which depend on the propellant and gas properties.

A computer subroutine (LINTHW) was developed to solve the linearized equations given above. This subroutine, which is based on the method of invariant imbedding used in the original nonlinear program developed by Kooker, calculates the linearized response of the surface regression rate \( r \) (and hence the mass flux \( \dot{m}_g \)) to small amplitude pressure disturbances of arbitrary waveform. For sinusoidal pressure disturbances the linearized response computed by LINTHW is equivalent to the two-parameter \((A,B)\) model.

In order to compare "exact" and approximate solutions, values of \( A, B, \) and \( n \) corresponding to a given set of propellant properties are needed. These are obtained by an analytical solution of the above linearized combustion response model for assumed sinusoidal pressure disturbances. Explicit relations for \( A \) and \( B \) in terms of the solid propellant activation energy \( E_s^* \), the steady state surface temperature \( T_s^* \), and the surface heat of reaction \( Q_s^* \) are easily derived and are given by:

\[ A = E \left( \frac{T_s^* - T_s^*}{T_s^*} \right) \] (97)

\[ B = \frac{1}{A} \left\{ A + \frac{E Q_s^*}{c_s^* T_s^*} + \frac{c_p^*}{c_s^*} \left( E \Lambda_o^2 + 1 \right) \right\} \] (98)
where

\[ E = E_s^* / R_o T_s^* \]

\[ \Lambda_o^2 = \frac{Q_f^* k_s^* \omega_o^*}{(p_s^* T_s^* c_p^*)^{2/3}} \]

\[ \omega_o^* = \omega_o^* \cos \frac{2\pi}{p} \exp \left( - \frac{E_f^*}{R_o T_f^*} \right) \]

In the linear analysis, the value of \( n \) cannot be obtained explicitly in terms of propellant properties but must be obtained from burning rate calculations at low frequencies. An auxiliary program which calculates the burning rate corresponding to a sinusoidal pressure input has been developed to determine the complex response factor \( \mathbb{R} \) by comparing the computed amplitude and phase of the burning rate perturbation with that of the pressure perturbation. This program is then used to determine the value of \( n \), since the computed values of \( \mathbb{R} \) approach \( n \) as frequency approaches zero. After \( n \) is determined, values of \( \mathbb{R} \) are obtained for several different frequencies using both the linearized and nonlinear burning rate models. These values are compared with the response curve obtained analytically from the two-parameter linear response function described by Equations (10) and (11). An example of such a comparison is shown in Figure 6 where the close agreement obtained is an indication of the validity of both the linearized and nonlinear combustion models for small amplitude oscillations.

3.3 Nonlinear Particle Damping

In the original Kooker-Zinn "exact" model the particle drag was described by the Stokes Drag Law, which is a linear expression valid for Reynolds numbers of order unity. In order to include nonlinear particle drag effects which occur at higher Reynolds numbers (larger particle sizes and higher frequencies), the higher order correction to the Stokes Law was incorporated into the "exact" analysis. Thus the drag term in the
Figure 6. Comparison of Linearized and Nonlinear Combustion Models with the Two Parameter Response Function
The momentum equation is given by Equations (79) and (80). A more convenient form for numerical calculations is given by

\[ F_p = K_p (u-u_p) \left[ 1 + K_{NL}^0 \frac{2}{3} |u-u_p|^{2/3} \right] \]  

(99)

where \( K_{NL} \) is given by Equation (82). These expressions are readily incorporated into the appropriate conservation equations (i.e., Equations (87) through (89)) by replacing the drag constant \( K \) with the variable drag coefficient \( \tilde{K} \) defined by:

\[ \tilde{K} = K \left[ 1 + K_{NL}^0 \frac{2}{3} |u-u_p|^{2/3} \right] \]  

(100)

The above nonlinear drag law has been incorporated into the Kooker-Zinn "exact" computer program; the linear drag law option is available by specifying \( K_{NL} = 0 \).
4. RESULTS AND DISCUSSION

In this section, numerical calculations of nonlinear axial mode instabilities in solid rocket motors and T-burners are presented and discussed. Most of the approximate solutions presented herein were calculated using the Galerkin method without averaging; that is, by obtaining numerical solutions of the system of second order differential equations given by Equations (22) and (23). These approximate solutions are compared with "exact" solutions obtained by the Kooker-Zinn model in many cases. The approximate solutions are also compared with Culick's approximate solutions (Method of Averaging) and Levine's "exact" solutions when available.

Most of the approximate solutions presented in this section were obtained in a parametric study to assess the importance of gasdynamic nonlinearities (mode-coupling) on the stability of solid rocket motors, a major objective of this program. In this study, the only nonlinear process considered was gasdynamical mode-coupling, while other processes such as combustion driving and particle damping were described by linear models. The following parameters were considered: (1) the number of modes used to describe the total wave, (2) the magnitude and harmonic content of the initial disturbance, (3) the characteristics of the propellant response function, (4) the oscillation frequency, and (5) the size and concentration of the aluminum oxide particles. The approximate and exact analyses are compared on the basis of the predicted growth or decay rates for the transient solutions and the final limiting amplitude and waveform. In addition, the transient behavior and relative amplitudes of the individual modes are determined.

In addition to the above parametric study approximate solutions are presented for (1) motors with a nonlinear combustion response to pressure oscillations, (2) motors with nonlinear particle damping, and (3) T-burners. Also solutions obtained using the Method of Averaging (MOA) are compared with the Galerkin method solutions for motors with and without particles. Finally approximate solutions are compared with available experimental data for both motors and T-burners.

4.1 Typical Nonlinear Solutions

Before presenting the results of the parametric studies, the applica-
tion of the approximate nonlinear analysis will be illustrated by an example.

To obtain an approximate solution for a given engine configuration and operating condition, the following must be specified: (1) the number of modes \( N \) present in the approximate series solution; (2) the structure of the initial disturbance (i.e., \( A_j(0) \) and \( dA_j(0)/dt \) for \( j = 1, 2, \ldots, 2N \)); (3) the combustor and propellant grain lengths; (4) the nozzle admittance \( Y \) for each of the modes in the solution; (5) the combustion parameters \( A, B, \) and \( n \) or the values of \( \bar{g} \) for each mode; (6) the concentration and size of the aluminum oxide particles; (7) the mean flow Mach number at the nozzle entrance; (8) the steady-state temperature in the combustor and (9) the ratio of the specific heats of the gaseous phase. For the example considered here a five-mode series was employed, the initial disturbance was a pure fundamental mode (1L) oscillation of about 15% pressure amplitude, and the nozzle admittances for each of the modes were obtained by the quasi-steady nozzle relation (i.e., Equation (43)). The specific heat ratio was \( \gamma = 1.2 \) and the mean flow Mach number at the nozzle entrance was \( \bar{M}_e = 0.10 \). The response function was described by \( A = 7.54, B = 0.686, \) and \( n = 0.81 \) (Figure 7). This is the moderately strong response considered by Kooker in the "exact" analysis. Here the frequency \( \Omega \) was chosen such that all of the modes lay on the descending branch of the response curve; thus, the real parts of the response function for the higher frequency modes were all smaller than the corresponding value for the fundamental (1L) mode. For such cases the series expansion in terms of the acoustic modes was expected to converge fairly rapidly, and the mode-coupling should tend to limit the amplitude. Finally, it was assumed that no particles were present in the combustor.

The approximate solutions (Galerkin method) for this case are shown in Figures 8 and 9. Figure 8 shows the relative magnitudes and transient development of the individual modes (i.e., the real parts of the functions \( A_j(t) \)) following the introduction of the pure 1L mode initial disturbance. It is seen that the amplitudes of the higher harmonics are much smaller than the amplitude of the fundamental mode. The transient development of the head-end pressure perturbation shown in Figure 9 was determined from the individual modes using Equations (19) and (24). The initially sinusoidal pressure waveform quickly becomes distorted due to the excitation of the higher harmonics.
Figure 7. Moderate Response Curve with Positions of First Five Axial Modes

A = 7.54
B = 0.686
n = 0.81
Figure 8. Amplitudes of Individual Modes for an Unstable Motor
Figure 9. Head-End Pressure Waveform for Unstable Motor

A = 7.54, B = 0.686, n = 0.81, \( \Omega = 8.25 \), No Particles
It should be noted that the individual mode-amplitude functions in Figure 8 are the real parts of the $A_j(t)$'s for the gas-phase velocity potential; thus, they are the harmonic components of the velocity waveform rather than the pressure waveform. From Equations (19a) and (24) and the nearly sinusoidal shape of the $A_j(t)$'s, it is easily seen that the amplitudes of the corresponding harmonics of the pressure waveform are approximately obtained by multiplying the amplitudes of the $A_j(t)$'s by $\gamma j \omega_1$ where $\omega_1$ is the frequency of the $1L$ mode. This relationship is valid for oscillations of sufficiently small amplitude that the second order terms in equation (24) are negligible. For larger oscillation amplitudes the individual pressure harmonics may be obtained by Fourier analysis of the waveform obtained by Equation (24).

4.2 Parametric Studies of Mode-Coupling

The basic set of motor parameters used in this study was based on the data given by Levine and Culick which correspond to a small laboratory pulse motor. This motor has a grain length of about .597 m (1.958 ft) and an initial bore of about 50 mm (2 in). The physical properties of the propellant (ANB 3066), and its gaseous and particulate combustion products, are given in Table 1.

The data along with the endothermic heat release at the propellant surface ($Q_s^*$) is used as input to the Kooker-Zinn "exact" analysis, which then computes the corresponding values of $A$, $B$, and $n$ which describe the transient propellant response. A reference state is also needed in order to evaluate some of the constants which appear in the burning rate model; in this case a surface regression rate of 1.17 cm/sec (0.459 in/sec) and a surface temperature of 800°K (1440°R) is assumed for a pressure of 10810 knt/m$^2$ (1568 psi). For most cases considered here, the mean chamber pressure is assumed equal to the above reference pressure. For this case the total flux of burned gases leaving the tubular propellant grain is 2.338 kg/sec which corresponds to a Mach number at the nozzle entrance of $M_e = 0.0780$. For this combination of chamber length and steady-state temperature ($T_f^*$), the chamber sonic speed is 1278 m/sec (4194 ft/sec) giving a fundamental mode frequency of 1071 Hz for the pure gas (without particles). When comparing the approximate solutions with the "exact" solutions, the values of $A$, $B$, and $n$ computed from the above data are used as input.
Table 1. ANB 3066 Propellant Data

**Propellant**

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>density: $\rho^*$</td>
<td>1.766 gm/cm$^3$</td>
</tr>
<tr>
<td>specific heat: $c^*$</td>
<td>0.329 cal/gm-°K</td>
</tr>
<tr>
<td>thermal conductivity: $k^*$</td>
<td>$1.791\times10^{-4}$ cal/cm-sec-°K</td>
</tr>
<tr>
<td>activation energy: $E^*$</td>
<td>15,250 cal/mole</td>
</tr>
</tbody>
</table>

**Gas-Phase Flame**

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>specific heat ratio: $\gamma$</td>
<td>1.23</td>
</tr>
<tr>
<td>specific heat $c^*_P$</td>
<td>0.483 cal/gm-°K</td>
</tr>
<tr>
<td>thermal conductivity $k^*_g$</td>
<td>$1.658\times10^{-4}$ cal/cm-sec-°K</td>
</tr>
<tr>
<td>activation energy: $E^*_f$</td>
<td>30,000 cal/mole</td>
</tr>
<tr>
<td>flame temperature: $T^*_f$</td>
<td>3525°K (6345°R)</td>
</tr>
</tbody>
</table>

**Particles**

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>density: $\rho^*_m$</td>
<td>4.0 gm/cm$^3$</td>
</tr>
<tr>
<td>diameter: $\sigma$</td>
<td>2.5 $\mu$m</td>
</tr>
<tr>
<td>particle loading: $C^*_m$</td>
<td>0.36</td>
</tr>
</tbody>
</table>
to the approximate analysis. In order to completely define the values of \( R^2 \) and \( R^1 \) for each of the modes, the frequency parameters \( \Omega_j \) are also needed for each mode. These are obtained from the computed frequency of the 1L mode and the steady state burning rate obtained from the "exact" analysis, and are given by:

\[
\Omega_j = j\omega_1/r^2 \quad j = 1, 2, \ldots, N
\]  

(101)

The most useful parameter to vary to obtain different response functions is \( Q_s^* \); making \( Q_s^* \) more negative (exothermic) decreases \( B \) and yields larger values of the peak response. The frequency parameter is varied by changing the chamber length or by changing the steady state burning rate.

The results of the parametric studies will now be presented in the following order: (1) effect of number of modes in series, (2) effect of initial disturbance, (3) effect of propellant response function, (4) effect of frequency, and (5) effect of particles.

Effect of Number of Modes Used to Represent the Solution. Figures 10 and 11 show the effects of the number of modes \( N \) used to represent the approximate solutions for a case in which no particles are present. Solutions were obtained using a basic series consisting of the three lowest frequency axial modes (i.e., the first, second and third longitudinal modes). Higher harmonics were then added one at a time to the basic series and the resulting solutions were compared on the basis of wave shape and limiting amplitude. This comparison was used to determine the minimum number of modes needed to adequately describe the behavior of the unstable rocket motor with a minimum expenditure of computer time.

Figure 10 shows the development of the peak-to-peak head-end pressure amplitude with time for an initial 7% fundamental mode disturbance. The combustion parameters are \( A = 6.00, B = 0.590, n = 0.583 \) and \( \Omega_1 = 4.907 \). The growth of the pressure oscillation amplitude is shown for one, three, four, five, and six mode series expansions. For the one-mode series (\( N = 1 \)) expansion, there is no nonlinear mode-coupling to limit the amplitude, so the oscillations grow at the linear rate of about 80 sec\(^{-1}\). For \( 4 \leq N \leq 6 \), increasing the number of modes decreases the predicted limit-cycle amplitude as a result of the nonlinear coupling of the fundamental mode with increasingly stable higher modes. The increased stability of
Figure 10. Effect of Number of Modes Used in the Approximate Analysis Upon Resulting Solution
these higher modes results from the decreased combustion response at higher frequencies for this case (Table 2). As the number of modes is increased, the approximate solutions also approach the "exact" solution also shown in Figure 10, but it is not clear from this figure that the approximate solutions converge to the "exact" solution in the limit of an infinite number of modes. Indeed, it appears that the approximate solutions will converge to a larger limiting amplitude than the "exact" solutions, probably due to the order of magnitude approximations made in deriving Equations (13) and (14) upon which the approximate technique is based. Also the convergence of the approximating series as \( N \to \infty \) appears to occur in an irregular or oscillatory fashion, as shown by the three-mode series which yields nearly the same growth curve as the six-mode series.

Table 2. Response Functions for the First Six Axial Modes

<table>
<thead>
<tr>
<th>Mode</th>
<th>( R^T )</th>
<th>( \frac{i}{R} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1L</td>
<td>3.596</td>
<td>-1.221</td>
</tr>
<tr>
<td>2L</td>
<td>0.779</td>
<td>-1.407</td>
</tr>
<tr>
<td>3L</td>
<td>0.459</td>
<td>-.934</td>
</tr>
<tr>
<td>4L</td>
<td>0.357</td>
<td>-.718</td>
</tr>
<tr>
<td>5L</td>
<td>0.306</td>
<td>-.594</td>
</tr>
<tr>
<td>6L</td>
<td>0.273</td>
<td>-.512</td>
</tr>
</tbody>
</table>

A comparison of the pressure waveforms obtained with the four, five, and six mode expansions is shown in Figure 11. The principal effect of the higher harmonics is apparent in the secondary "wiggles" which are dependent upon the number of modes used to obtain the solution. The overall shape of the pressure waveforms (i.e., the steep rise to maximum and more gradual decline to minimum), the frequency, and the amplitude are less strongly affected by the number of modes.

Figures 10 and 11 indicate that for motors without particles, more than six modes (or possibly as few as three modes) may be necessary to adequately represent the solution. The effect of the number of modes used to describe the solution for cases in which particles are present will be discussed later in this section.

**Effect of Initial Disturbance.** This study was designed to determine the effect of the amplitude of the initial disturbance and its harmonic
Figure 11. Effect of Number of Modes Used in the Approximate Analysis Upon Calculated Waveforms
content upon the resulting solutions. Of particular interest are cases in which growth to limit-cycle amplitude occurs. It has been shown in References (6), (7), and (8) for liquid rockets that the final limiting amplitude is independent of the form of the initial disturbance. On the other hand, results presented by Levine and Culick for solid rockets indicate a significant dependence of limit-cycle amplitude upon initial disturbance amplitude for amplitudes in the range 5% - 20%. Thus additional data is needed to clarify this issue.

To determine the effect of initial disturbance amplitude upon the resulting limit-cycle solution, the case considered previously (no particles, $A = 6.00$, $B = 0.590$, $n = 0.583$, $\Omega = 4.907$) was run for initial disturbances of 3%, 7%, and 20%. Plots of head-end pressure amplitude versus time for these cases are shown in Figure 12 (using a five mode series). The 3% and 7% solutions appear to smoothly approach a limiting amplitude of 12.5% after about 36 cycles. The 20% disturbance also approaches the same limiting amplitude but in an oscillatory fashion. Although the paths are different, it appears that if the calculations are continued for a sufficiently long time, all three solutions approach the same limiting amplitude. This result is consistent with previous results for liquid rockets, in which limit-cycle amplitude is independent of the initial disturbance.

For a similar case for which 2.5 particles are present, 3% and 10% initial amplitudes yield 3.1% and 9.0% amplitude oscillations after 12 cycles. It is difficult to determine whether this indicates a true dependence of limit-cycle amplitude upon initial amplitude, or whether the approach to limit-cycle amplitude is just extremely slow. Such a slow approach to limiting amplitude is expected for cases in which the linear losses due to the mean flow, particles, and the nozzle are in nearly exact balance with the gain due to combustion driving. Such a motor is said to be operating near the point of linear neutral stability.

The effect of harmonic content of the initial disturbance was studied by introducing initial amplitudes of two or more modes simultaneously. Figure 13 shows the pressure waveform obtained for an initial disturbance composed of a combination of 1L and 2L modes for the case of no particles. Due to nonlinear coupling of modes, the amplitude of the 1L mode increases while the amplitude of the 2L mode decreases. As time progresses the waveform assumes more of a 1L mode character.
Figure 12. Effect of Initial Disturbance Amplitude Upon Approach to Limiting Amplitude

No Particles

\[ A = 6.00 \]
\[ B = 0.59 \]
\[ n = 0.583 \]
\[ \Omega = 4.91 \]
\[ \bar{M}_e = 0.078 \]
A = 7.54, B = 0.686, n = 0.81, \( \Omega = 8.25 \)

No Particles

1L and 2L Modes Initially Present

Figure 13. Effect of Harmonic Content of Initial Disturbance
Effect of Propellant Response Function. The effect of propellant response function upon the stability of solid rocket motors was investigated using both approximate and "exact" methods. The most important parameter influencing the amount of combustion driving was found to be the real part of the response function for the fundamental mode, $R_1^r$. This depends on the values of $A$, $B$, and $n$ for the propellant, and the frequency parameter $\Omega$. Values of $A$, $B$, and $n$ were selected to give values of $R_1^r$ in the range 0 to 4. In this study the effect of particles was neglected.

In order to be certain that the propellant response function is the same for both approximate and "exact" solutions, the following procedure is used. First a value of $Q^*_s$ is selected for input into the "exact" analysis; the remaining propellant and gas properties are those given in Table 1. The corresponding values of $A$, $B$, and $n$ are then calculated. The "exact" solutions are then obtained for a given initial disturbance and the resulting frequency is computed from the pressure waveforms after several cycles. From this frequency and the steady state burning rate, the frequency parameter $\Omega$ is obtained for input into the approximate analysis. The approximate solutions are then obtained for the calculated values of $A$, $B$, $n$, and $\Omega$ for the same initial disturbance amplitude, and they are then compared with the corresponding "exact" solutions.

The first case considered was a hypothetical motor in which the propellant is insensitive to the pressure oscillations; that is $R = 0$ for all modes. Steady-state combustion was included, however, giving acoustic losses due to mean flow, flow turning, and the quasi-steady nozzle. Exact and approximate solutions for the decay of the pressure amplitude with time are shown in Figure 14 for $\tilde{M}_e = 0.0847$. Excellent agreement between the approximate and "exact" solutions was obtained for both small (2.9%) and moderate (10.4%) amplitude initial disturbances. It can be shown analytically that the damping rate (dimensionless) approaches a limiting value at small amplitudes given by:

$$\alpha = -\frac{2\gamma+1}{2} \frac{\tilde{M}}{\tilde{M}_e}$$  (102)

For this case ($f = 1070$ Hz) the "exact" and approximate solutions both yield a damping rate of about $-308$ sec$^{-1}$. 

68
Figure 14. Decay of Oscillations Due to Mean Flow, Flow Turning, and Nozzle
In the second case, a value of $Q_s^* = -131.8 \text{ cal/gm}$ was assumed yielding $A = 6.00$, $B = 0.643$, and $n = 0.616$. The exact calculations gave $\Omega_1 = 4.91$ for which $R_1^r = 2.73$. Plots of pressure amplitude versus time for a 3.0% initial disturbance is shown in Figure 15 where $M_e = 0.0780$. The initial vertical displacement between the approximate and "exact" solutions results from a mean pressure shift obtained with the "exact" analysis during the first cycle of oscillation and is not significant. The decay rate is much less than in the previous case due to the effect of combustion driving; the approximate analysis yields a decay rate of $-14 \text{ sec}^{-1}$ while the "exact" analysis gives $-31 \text{ sec}^{-1}$. Considering that the net damping in the motor is the difference between two relatively large quantities, the "exact" and approximate solutions are in quite good agreement. In this case the discrepancy in the damping predicted by the two theories is $17 \text{ sec}^{-1}$, which is only about 6% of the damping in the absence of combustion driving ($-288 \text{ sec}^{-1}$ for $M_e = 0.078$).

Increasing $Q_s^*$ to $-134.7 \text{ cal/gm}$ gave $A = 6.00$, $B = 0.607$, and $n = 0.594$. For a 3% initial disturbance the "exact" analysis gave a growth to a limiting amplitude of about 2.8% with a corresponding $\Omega_1 = 4.888$ and $R_1^r = 3.26$. For the approximate analysis, however, the oscillations grew to 4.5% after 12 cycles and were still growing at the rate of $36 \text{ sec}^{-1}$. For this case solutions for a 10% initial disturbance were also calculated; here the frequency was slightly higher giving $\Omega_1 = 4.932$ and $R_1^r = 3.23$. Both solutions were still decaying after 12 cycles, the "exact" solution at 7.5% amplitude with $\alpha = -42 \text{ sec}^{-1}$ and the approximate solution at 9.5% amplitude with $\alpha = -31 \text{ sec}^{-1}$. A limit cycle of about 7% for the approximate analysis is consistent with these results.

Finally, a value of $Q_s^* = -136.1 \text{ cal/gm}$ was chosen for which $A = 6.00$, $B = 0.590$, and $n = 0.583$. Amplitude-time plots are given for 3% and 7% initial amplitudes in Figure 16, for which $R_1^r = 3.6$. This figure shows that again the approximate analysis predicts a somewhat greater initial growth rate and larger limit-cycle amplitude than the "exact" analysis. In this case the approximate analysis yields a growth rate of $73 \text{ sec}^{-1}$ for the 3% solutions and a final amplitude of 11.3%, while the corresponding values obtained with the "exact" analysis are $38 \text{ sec}^{-1}$ and 8.0% respectively. The frequencies obtained are in close agreement at about 1050 Hz. Head-end and mid-chamber waveforms are shown in Figure 17 for limit-cycle con-
Figure 15. Decay of Oscillations in a Stable Motor Without Particles

No Particles

$A = 6.00$
$B = 0.643$
$n = 0.616$
$\Omega = 4.91$
$\gamma = 1.23$
$\bar{M} = 0.078$
Figure 16. Growth of Oscillations to Limiting Amplitude for a Motor Without Particles
Figure 17. Pressure Waveforms at Limiting Amplitude for a Motor Without Particles.
ditions (after 11 cycles of oscillation). The approximate waveforms show considerably more harmonic content than the "exact" waveforms, but the qualitative features of the waveforms are in good agreement (i.e., waveform steepening and phase relation between even harmonics and the fundamental).

The Effect of Oscillation Frequency. The dependence of unstable solid rocket limit-cycle behavior upon the oscillation frequency was studied by varying the frequency parameter $\Omega$ for a given propellant. Figures 18 and 19 describe the head-end pressure oscillations for two rockets having identical propellants ($A = 7.54$, $B = 0.880$, $n = 0.85$) but different nondimensional frequencies $\Omega$. The magnitudes of the combustion response "driving" for the five modes present in these two cases are also included in these figures which clearly show that for the case $\Omega_1 = 1.64$ (Figure 18) the magnitudes of the combustion responses of the higher harmonics are considerably larger than in the case when $\Omega_1 = 10.0$ (Figure 19). This difference in driving is directly responsible for the observed difference in the head-end pressure wave forms. Furthermore, these figures suggest that more than five modes are probably needed to adequately describe the solution in the $\Omega_1 = 1.64$ case.

Effect of Particles. In the last phase of the parametric study the effect of particle size and concentration upon the nonlinear stability characteristics of solid rocket motors was investigated. Three basic cases were considered: (1) attenuation of waves in a particle-gas mixture in a closed-ended chamber (i.e., the case of particles in a box treated by Culick and Levine and Culick), (2) attenuation of waves in a motor without combustion driving ($\mathcal{R} = 0$), and (3) limit-cycle solutions in an unstable motor. In all cases approximate solutions obtained with the Galerkin method (six modes) are compared with "exact" solutions obtained with the Kooker-Zinn model. For the case of particles in a box, solutions obtained with Culick's MOA and Levine's "exact" analysis are also included.

For the case of particles in a box, the input parameters are the same as those given by Culick. These are needed in order to calculate the particle drag constant $K$ for use in the "exact" and approximate analyses. Since Culick specifies the equilibrium frequency for the particle-gas mixture rather than the chamber length and speed of sound, the value of $K$ for a given particle size was calculated from the following expression:

74
A = 7.54
B = 0.880
n = 0.85
Ω₁ = 10.0

Figure 19. Pressure Waveforms for High Frequency Oscillations in a Motor Without Particles
where \( f_g^* = c_0^* / 2L^* \) is the frequency for the pure gas. The gas viscosity was obtained from the formula given in Reference 10:

\[
\mu^* = 8.834 \times 10^{-5} \left( \frac{T}{3485} \right)^{0.66} \text{ kg/m-sec} \tag{104}
\]

for \( T = 3416^\circ \text{K} \) used by Culick. In the approximate analysis the ratio of specific heats \( \gamma \) for the mixture replaces \( \gamma \) for the gas. The value of \( \gamma \) depends on the specific heats of the gas and particle material and the concentration of the particles and is given by:

\[
\tilde{\gamma} = \gamma \left( \frac{\rho_P C_P}{\rho C} \right) \left/ \left( 1 + \gamma \frac{\rho_P C_P}{\rho C} \right) \right. \tag{105}
\]

where \( C \) and \( C_P \) are the specific heats of the particles and gas respectively. Also in specifying the initial disturbance, the equilibrium frequency is needed; it is related to the pure gas frequency by:

\[
f_e = f_g \sqrt{\frac{\tilde{\gamma}}{\gamma \left( 1 + \frac{\rho_P}{\rho} \right)}} \tag{106}
\]

To evaluate the accuracy of the approximate analysis in describing the effects of linear particle damping, "exact" and approximate solutions were obtained for the case of a particle-gas mixture in a closed-ended box. Assuming an initial disturbance amplitude of 3%, values of damping and frequency...
were computed for several values of the particle diameter \( \sigma \). The results were calculated for a pure-gas frequency of \( f_g = 1071 \text{ Hz} \) and a particle concentration of \( C_m = \frac{\rho_p}{\rho} = 0.20 \). The values of \( K \) corresponding to the particle sizes considered in this study are given in Table 3.

<table>
<thead>
<tr>
<th>Particle Size, ( \sigma ) (( \mu ))</th>
<th>( K ) (Approximate)</th>
<th>( K' ) (Exact)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>46.72</td>
<td>160.2</td>
</tr>
<tr>
<td>2.5</td>
<td>29.90</td>
<td>102.5</td>
</tr>
<tr>
<td>3.0</td>
<td>20.76</td>
<td>71.18</td>
</tr>
<tr>
<td>4.0</td>
<td>11.68</td>
<td>40.04</td>
</tr>
<tr>
<td>6.0</td>
<td>5.191</td>
<td>17.80</td>
</tr>
<tr>
<td>8.0</td>
<td>2.920</td>
<td>10.01</td>
</tr>
<tr>
<td>9.0</td>
<td>2.307</td>
<td>7.909</td>
</tr>
<tr>
<td>10.0</td>
<td>1.869</td>
<td>6.406</td>
</tr>
<tr>
<td>15.0</td>
<td>0.8306</td>
<td>2.847</td>
</tr>
<tr>
<td>20.0</td>
<td>0.4672</td>
<td>1.602</td>
</tr>
<tr>
<td>25.0</td>
<td>0.2990</td>
<td>1.025</td>
</tr>
<tr>
<td>30.0</td>
<td>0.2076</td>
<td>0.7118</td>
</tr>
<tr>
<td>40.0</td>
<td>0.1168</td>
<td>0.4004</td>
</tr>
</tbody>
</table>

The value of \( K' \) used in the "exact" analysis differs from the value \( K \) used in the approximate analysis due to the difference in the reference state used in nondimensionalizing the "exact" equations. In the Kooker-Zinn analysis standard atmospheric conditions are used as the reference state, while the chamber stagnation conditions are used in the approximate analysis. Thus the two particle drag constants are related by

\[
K' = K \left( \frac{c^*}{c_{\text{ref}}} \right)
\]

where \( c^* \) and \( c_{\text{ref}} \) are the speeds of sound at chamber stagnation conditions and standard atmospheric conditions respectively.

Curves of decay rate and frequency as a function of particle diameter are presented in Figure 20 for particles in a box (i.e., \( \bar{K}_s = 0 \)). The damping and frequency shift arise solely from the particle-gas interaction,
Figure 20. Effect of Particle Size on Decay Rate and Frequency For a Gas/Particle Mixture in a Box
since mean flow, combustion, and nozzle effects are absent. The damping is low for particles smaller than 2μ and for particles larger than 30 μ, and the maximum damping occurs at about 8.5μ for the frequency considered. The frequency varies from the equilibrium value of 985 Hz for small particles (σ < 4μ) to the pure-gas value of 1080 Hz (σ > 25μ).

This figure shows very good agreement between the decay rates and frequencies predicted by the approximate and "exact" models over the size range from 2μ to 40μ. This indicates that the approximate analysis correctly models the particle-gas interaction for small amplitude disturbances in the absence of mean flow and combustion effects.

To further investigate the particle-gas interaction, several of the cases considered by Culick were calculated using the Galerkin Method and the Kooker-Zinn "exact" analysis. These solutions were then compared with Culick's MOA solutions and Levine's "exact" solutions for the same cases. The case of 2.5μ particles with C_m = 0.36 and f_e = 800 Hz was first considered (K = 32.9). Plots of damping versus number of cycles for 3% initial disturbances are presented in Figure 21, while similar plots for 15% initial disturbances are shown in Figure 22. Figure 21 shows very good agreement between the Galerkin solutions and the Kooker-Zinn solutions, while the agreement between the Galerkin method and Culick's and Levine's solutions is not as good. For 15% initial disturbances, Figure 22 shows that the Galerkin solutions approach the "exact" solutions asymptotically, although significant quantitative differences in the computed decay rates occur during the initial cycles of oscillation. This figure also shows the effect of number of modes upon the computed decay rates; in this case the solution is adequately described by a five-mode series. The Galerkin solutions for the 15% disturbance are also compared with Culick's (MOA) and Levine's ("exact") solutions in Figure 22. Again the agreement with Culick's and Levine's solutions is not as good as with Kooker's solutions.

Some of the discrepancies between the Galerkin solutions and Culick's MOA solutions (as well as Levine's "exact" solutions) may be due to differences in the physical models involved. For example, the Galerkin method results shown were obtained using a linear drag law to describe the particle damping, while the results of Culick and Levine were obtained using a nonlinear drag law. Furthermore, the analyses of Culick and Levine included a thermal equation for the particles, while this equation was ignored in both the Galerkin and the Kooker-Zinn analyses.
Figure 21. Decay Rates of 3% Disturbances Due to 2.5μ Particles in a Box.
Figure 22. Decay Rates of 15% Disturbances Due to 2.5μ Particles in a Box

\[
\begin{align*}
\sigma &= 2.5\mu \\
C_m &= 0.36 \\
f_e &= 800\text{ Hz}
\end{align*}
\]
Two additional cases were considered for particles in a box: (1) \( \sigma = 2.5\mu \), \( f_e = 1500 \text{ Hz} \) (\( K = 17.55 \)) and (2) \( \sigma = 10\mu \), \( f_e = 1500 \text{ Hz} \) (\( K = 1.097 \)). For both cases \( C_m = 0.36 \) and a 15\% initial disturbance was considered. Approximate and "exact" solutions for these cases are shown in Figure 23. For the 2.5\( \mu \) particles the agreement between approximate and "exact" solutions at 1500 Hz is better than at 800 Hz (Figure 22), and a four-mode series is adequate to describe the solution. For the 10\( \mu \) case both theories predict large variations in decay rate from cycle to cycle, and significant differences between four, five, and six mode solutions are apparent. Although for any given cycle the decay rates predicted by the approximate and "exact" analyses can differ significantly, the mean decay rates during the first 14 cycles are in fairly good agreement.

The second case considered in the study of particle damping effects was a motor for which \( R = 0 \), that is the combustion process does not respond to the pressure oscillations. Thus the effects of mean flow and nozzle damping are considered along with the particle damping effects. Calculations of decay rate and frequency were made for a motor with \( \bar{M}_e = 0.0780 \) and \( f_g = 1071 \text{ Hz} \) for several values of \( \sigma \) and \( C_m \).

The effect of particle size on the "exact" and approximate solutions for \( C_m = 0.20 \) is shown in Figure 24 for 3\% initial disturbances. These curves are similar to those shown in Figure 20 for particles in a box, except that the damping curves are shifted upwards due to the large damping due to mean flow and nozzle effects (\( \omega = -289 \text{ sec}^{-1} \) in the absence of particles). The decay rate curves show that the approximate and "exact" analyses are in excellent agreement for particle diameters larger than about 15\( \mu \), and both analyses predict practically the same optimum particle size for maximum damping. For particles smaller than 15\( \mu \) the approximate analysis predicts a smaller linear decay rate than the "exact" analysis with a maximum error of about 13\%. This discrepancy is probably due to certain terms proportional to the particle drag constant \( K \) and the mean flow Mach number (see Appendix A) that were neglected as higher order in the approximate analysis. Since \( K \) is inversely proportional to the frequency of oscillation and the square of the particle diameter, better agreement is obtained in the high-frequency and large-particle size range. The frequencies predicted by the two models are in best agreement for small (\( \sigma < 6 \mu \)) and large particles (\( \sigma > 20\mu \)).

It should be noted that the good agreement between "exact" and appro-
Figure 23. Decay Rates of 15% Disturbances Due to 2.5 µ and 10 µ Particles in a Box
Figure 24. Effect of Particle Size on Decay Rate and Frequency for Motor Without Combustion Driving
ximate analyses for large particles has been obtained under the assumption that the particle-gas interaction is described by the Stokes Drag Law (Equations (7) and (8)). For large particle sizes and high frequencies (i.e., large Reynolds numbers), however, the Stokes Drag Law is no longer valid and the nonlinear drag law given by Equation (79) should be used. The case of nonlinear particle damping is considered in a later subsection.

The dependence of damping and frequency upon the particle concentration $C_m$ is shown in Figure 25 for $10\mu$ particles. Both approximate and "exact" analyses predict a nearly linear dependence of frequency and damping upon particle concentration for small amplitude initial disturbances (3%). Here the decay rate increases from $-289$ sec$^{-1}$ for no particles ($C_m = 0$) to about $-800$ sec$^{-1}$ for $C_m = 0.35$, while the frequency decreases from the pure-gas value of 1071 Hz to about 1010 Hz at $C_m = 0.35$. For this particle size, the approximate analysis yields smaller decay rates than the "exact" analysis, with a discrepancy of about 5% at $C_m = 0.35$. The predicted frequencies are also lower for the approximate analysis, but the discrepancies are all less than 1%.

The last case considered in the particle damping study was a motor with linear combustion driving. The same basic motor parameters were used as for the cases considered previously when particles were absent. As in Levine's cases, a particle diameter of 2.5$\mu$ was selected at a concentration of 0.36. In order to obtain a limiting amplitude, a larger propellant response than that of Figure 16 was needed to overcome the increased damping due to the particles. Thus a value of $Q_s^* = -137.0$ cal/gm was chosen which gave $A = 6.00$, $B = 0.580$, $n = 0.575$. The corresponding response curve has a peak of $R_r = 4.2$ at $\Omega = 4.5$.

Plots of head-end pressure amplitude versus time for this case are presented in Figure 26 for 3% and 10% initial disturbances. The agreement between the approximate and "exact" analyses is seen to be fairly good. Both analyses yield a limit-cycle of about 9% amplitude for the 10% initial disturbance and a relatively slow growth for the 3% initial disturbance. For the low amplitude disturbance, the approximate analysis gives the smaller growth rate, which is in contrast to the case when particles are absent (Figure 16) where the approximate analysis predicted a larger growth rate than the "exact" analysis. This result was unexpected, since the approximate analysis underestimates the damping for the case of 2.5$\mu$ particles.
Figure 25. Effect of Particle Concentration on Decay Rate and Frequency for Motor Without Combustion Driving
Figure 26. Growth of Oscillations to Limiting Amplitude for a Motor with 2.5\(\mu\) Particles
without combustion driving (Figure 24). The corresponding head-end and mid-chamber pressure waveforms for limit-cycle conditions are shown in Figure 27. Both analyses yield nearly sinusoidal waveforms; the only evidence of higher harmonics is the steepening of the rising branch of the waveform and the double frequency oscillation at the center of the chamber. This is due to the presence of particles which suppress the higher harmonics.

The importance of using the proper frequency parameter in the approximate analysis when making comparisons with the "exact" analysis is readily illustrated by this case. From the "exact" analysis, the frequency parameter for the 10% initial amplitude is \( \Omega_1 = 4.244 \) for which \( R_1^p = 4.17 \). If, however, the pure-gas frequency is assumed, \( \Omega_1 = 4.94 \) for which \( R_1^p = 3.78 \), and the approximate analysis would predict a decaying oscillation. Thus the approximate analysis would appear to give the wrong results, because the response function was not the same in both cases. For this reason the "exact" program must always be run first to obtain the proper \( \Omega_1 \) for input into the approximate program. Of course, if the calculations are being made with the approximate analysis only, the values of A, B, n, and \( \Omega_1 \) must be estimated or the values of \( R \) for the various modes may be obtained from experimental data.

Some parametric variations on the case considered above will now be considered. The effect of variations of particle size upon the limit-cycle amplitude and waveform are shown in Figure 28 for \( C_m = 0.10 \). Here head-end pressure waveforms are shown for \( \sigma = 2.5, 3.5, \) and \( 5\mu \); the corresponding amplitudes are 19.3%, 15.3%, and 8.2%. This decrease in limiting amplitude with increasing particle size is consistent with the increasing particle damping with increasing \( \sigma \) as shown in Figure 20. The distortion of the waveform also decreases with increasing \( \sigma \) due to the decreasing amplitude and the increased attenuation of the higher harmonics by the particles. For particles with diameters somewhat greater than \( 5\mu \) (for the given propellant response and particle concentration), limit-cycles are not obtained and the oscillations decay. For continued increases in particle size the decay rate reaches a maximum, then declines, and limit-cycles are again obtained for large particles (\( \sigma \) greater than roughly \( 15\mu \)). For large particles, the opposite trend is obtained, that is, limit-cycle amplitude increases with increasing particle size.

The influence of particle concentration \( C_m \) upon the limit-cycle
Figure 27. Pressure Waveforms for a Motor with 2.5μ Particles
Figure 28. Effect of Particle Size on Pressure Waveforms for $C_m = 0.1$
amplitude and pressure waveform is shown in Figures 29 and 30 for 2.5 μm particles. Figure 29 gives limit-cycle pressure waveforms for \( C_m = 0.10, 0.15, \) and 0.20 for which the corresponding amplitudes are 19.3%, 16.5%, and 14.3% respectively. The increased attenuation of the higher harmonics with increasing particle concentration is evident from these waveforms. A plot of limiting amplitude as a function of particle concentration is presented in Figure 30 for \( 0.05 \leq C_m \leq 0.03. \) For \( C_m > 0.1 \) the decline in amplitude with increasing \( C_m \) is seen to be nearly linear, while a sharp rise in amplitude occurs for \( C_m < 0.1. \)

In studying the effects of \( \sigma \) and \( C_m \) upon limiting amplitude and waveform, the frequency parameter was held constant at \( \Omega_1 = 4.2 \) in order to isolate the effects of particle damping. In an actual motor the situation is more complicated. The oscillation frequency is dependent on particle size and concentration (Figures 24 and 25), thus changing \( \sigma \) or \( C_m \) also changes the propellant response function \( \mathcal{R} \) through the effect of frequency on \( \Omega_1. \) Depending on the values of \( A, B, \) and \( n \) for the given propellant, allowing \( \Omega_1 \) to vary will yield higher or lower limit-cycle amplitudes than would be obtained for \( \Omega_1 = \text{constant}. \) It should be noted from Figure 24 that for small and large particles the equilibrium and pure-gas frequencies, respectively, are good approximations for use in calculating \( \Omega_1. \)

**Summary of Mode-Coupling Results.** The principal results of this study to assess the importance of gasdynamic mode-coupling upon axial mode instabilities in solid rocket motors will now be summarized. It has been shown that the solutions obtained with the approximate and "exact" analyses are in fairly good agreement regarding growth/decay rates, frequencies, limiting amplitudes and pressure waveforms. These results indicate that the approximate analysis (Galerkin Method) accounts for mean flow, flow-turning, and nozzle damping effects, as well as the effects of particle damping, combustion driving, and nonlinear gasdynamic mode-coupling. These studies have shown that gasdynamic mode-coupling is probably the most important nonlinear process which influences unstable motor behavior. Gasdynamic mode-coupling appears to limit the oscillation amplitude in unstable motors by the transfer of energy from the unstable fundamental mode to the stable higher frequency modes. On the other hand, the lack of pulsed instabilities in the results presented in this section indicates that gasdynamic nonlinearities (at least to second order) can not account for pulsed or triggered instabilities. These observations on the role of gasdynamic mode-coupling
Figure 29. Effect of Particle Concentration on Pressure Waveform for $\sigma = 2.5 \mu$
Figure 30. Influence of Particle Concentration Upon Limiting Pressure Amplitude for Motor with 2.5 µm Particles
in solid rocket motors are in agreement with the results of previous studies for liquid rockets \(6,7\).

Concerning the effect of number of modes upon the approximate solutions, the following results are notable. The number of modes necessary to adequately represent the solution depends upon the relative stability of the higher frequency modes with respect to the fundamental mode. For cases in which the higher modes are heavily damped, as in the case when small particles are present, a three or four mode series expansion appears to be adequate (Figures 22 and 23). For motors with large particles or for motors without particles, the higher modes are less strongly damped and six or more modes may be required (Figures 10 and 23). For cases in which the propellant response for the higher modes are comparable to or greater than the propellant response for the fundamental mode (Figure 18), the convergence of the series expansion may be so slow that an excessive number of modes are required. In the latter case, the approximate technique may require more computation time than the "exact" analysis.

Other results of the parametric studies are of interest. For the cases considered, limiting amplitude appeared to be independent of initial disturbance amplitude and harmonic content. The linear combustion response factor \(\mathcal{R}_1^r\) is a major factor influencing growth/decay rates and limiting amplitudes. For cases without particles, oscillations decay for \(\mathcal{R}_1^r < 3.3\) while for \(\mathcal{R}_1^r > 3.3\) limiting amplitude increases with increasing \(\mathcal{R}_1^r\). For cases with particles, larger values of \(\mathcal{R}_1^r\) are needed for instability. The oscillation frequency, determined by the chamber length and sound speed, affects the stability characteristics of the motor primarily through its influence on combustion driving and particle damping. Finally, particle size and concentration are both important parameters influencing growth and decay rates, limiting amplitudes and waveforms; there is an optimum value of the particle drag constant \(K\) (determined by particle size and frequency) for maximum damping.

4.3 Effect of Nonlinear Combustion Driving

In the studies presented so far, it was assumed that the only nonlinearity present in the solid rocket system was gasdynamic mode-coupling. In the next phase of this investigation, the effects of additional nonlinearities were included in the analysis. The effects of nonlinearities in
the pressure-coupled transient combustion process upon the stability characteristics of solid rocket motors are presented and discussed in this section. Velocity-coupled nonlinear combustion processes, which are also known to be important, were not considered in this study due to the time and economic limitations of this project.

The results presented in this section were obtained by the "exact" Kooker-Zinn model using the nonlinear combustion option and by the approximate analysis (Galerkin method) using the heuristic nonlinear combustion model described in Section 2.7. Kooker and Zinn\(^3\) and Levine and Culick\(^5\) have both considered the effects of combustion nonlinearities using "exact" numerical methods, and both have shown that the effect of a nonlinear combustion response is significant for amplitudes greater than about 7 - 10%. On the other hand, this study is the first attempt at incorporating burning rate nonlinearities into an approximate instability model. One of the major objectives of this study, therefore, is to assess the feasibility of modeling combustion nonlinearities using the approximate technique.

The approximate model was first used to study the basic case considered previously in the mode-coupling investigations. The linear combustion parameters for this case are \(A = 6.00, B = 0.59, n = 0.583\); there are no particles present in the flow; and the mean flow Mach number at the nozzle entrance is 0.0780. The acoustic frequency and the steady-state burn rate yield \(\Omega_l = 4.907\). The nonlinear combustion parameter \(b_m\) appearing in the heuristic nonlinear combustion model (Equation (77)) is not known a priori. In general, \(b_m\) is a complex number which is expected to be different for each acoustic mode. In order to reduce the number of parameters to be considered in this study, it was assumed that the \(b_m\)'s are all real numbers and that they are the same for all modes. Thus the effect of combustion nonlinearities was studied by varying the single parameter \(b\) in the approximate analysis.

Approximate solutions for the growth of a 7% initial disturbance to limiting amplitude are shown in Figure 31 for both linear and nonlinear combustion models. For the nonlinear combustion response, both positive \((b = 0.5)\) and negative \((b = -0.5)\) values of \(b\) were considered. This figure shows a significant effect of combustion nonlinearities upon the initial growth rate and limit-cycle amplitude of the pressure oscillations in the motor. For the positive value of \(b\), both the initial growth rate and limit-cycle amplitude are greater than the corresponding values obtained with
Figure 31. Effect of Combustion Nonlinearities Upon Limiting Amplitude Using the Approximate Model
the linear combustion response \(b = 0\), while the negative value of \(b\) yields reduced values of the initial growth rate and limiting amplitude. The limiting amplitudes obtained were as follows: 11.3% for linear combustion, 10.1% for \(b = -0.5\), and 12.9% for \(b = 0.5\). This behavior is expected from the form of the assumed nonlinear combustion model (Equation (77)), since for \(b > 0\) the effective combustion response \(R_{NL}\) increases with increasing amplitude and for \(b < 0\) the combustion response decreases with increasing amplitude.

Figure 32 shows the dependence of limit-cycle pressure amplitude upon the nonlinear combustion parameter \(b\) for the case considered above. For \(-1.0 \leq b \leq 1.0\) it is seen that limiting amplitude increases nonlinearly with increasing \(b\) and that the strongest effect of nonlinear combustion occurs for positive values of \(b\). Two factors contribute to this nonlinear dependence of limiting amplitude upon \(b\): (1) limiting amplitude is a nonlinear function of \(R_{NL}\) and (2) \(R_{NL}\) is proportional to the pressure amplitude as well as the value of \(b\).

In order to determine realistic values of \(b\) to use in the approximate analysis, comparisons were made with "exact" solutions obtained with the nonlinear burning rate. "Exact" solutions obtained with linear and nonlinear burning rates are compared in Figure 33 for the same case \((Q_s^* = -136.1 \text{ cal/gm})\) considered above. Here it is seen that nonlinear combustion effects yield a larger limiting amplitude (8.7% nonlinear versus 7.9% linear). It should be noted that for this case the approximate analysis yields a significantly larger limiting amplitude (11.3%) than the "exact" analysis for linear combustion, therefore the appropriate value of \(b\) cannot be obtained by direct comparison of approximate and "exact" solutions. Instead, a value of \(b\) is chosen which gives the same relative increase in limiting amplitude due to combustion nonlinearities as the "exact" analysis. For this case, nonlinear combustion increases the limiting amplitude by a factor of 1.10 according to the "exact" analysis. Thus the corresponding approximate solution should yield a limit-cycle amplitude of 12.4% corresponding to \(b = 0.32\) (Figure 32).

The effects of a nonlinear burning rate upon the pressure and burning rate waveforms are shown in Figure 34 for the "exact" analysis, while the corresponding approximate solutions for the pressure waveforms \((b = 0.32)\) are given in Figure 35. Figure 34 shows that a nonlinear combustion response modifies significantly the burning rate waveforms; the positive
Figure 32. Dependence of Limiting Amplitude Upon Nonlinear Combustion Parameter $b$
Figure 33. Effect of Combustion Nonlinearities Upon Limiting Amplitude Using the "Exact" Analysis.
part of the waveform becomes steeper with higher peak burning rates and the negative part becomes flatter. On the other hand, both approximate and "exact" models predict that, for the moderate amplitudes obtained in this case, combustion nonlinearities have very little effect on the shape of the pressure wave (Figures 34 and 35). The only noticeable effect of nonlinear burning rate upon the pressure waveform is a small mean pressure shift which is obtained with the "exact" analysis but not with the approximate model.

The steepening of the burning rate waveforms and the insensitivity of the pressure wave shape to combustion nonlinearities are in agreement with previously reported "exact" solutions obtained by Levine and Culick. However, Levine and Culick considered a different set of combustion parameters \((A = 6.0, B = 0.53, n = 0.3)\) and included 2\(\mu\) particles in the flow, which gave a reduction in limiting amplitude from 31\% to 15\% when burning rate nonlinearities were included in the analysis. This trend is opposite from that obtained above, and it indicates that negative values of \(b\) may be appropriate in some cases.

To further investigate the effect of combustion nonlinearities on the stability of motors, a second case was considered which was obtained by adding 2.5\(\mu\) diameter particles \((C_m = 0.36)\) to the case considered above. The linear combustion parameters \((A = 6.00, B = 0.59, n = 0.583)\) and the steady-state Mach number at the nozzle entrance \((\dot{M}_e = 0.0780)\) remain the same as before. The oscillation frequency, however, is lowered from 1071 Hz to 918 Hz due to the effect of the particles. This decrease in frequency results in an increase in the linear combustion response factor for the fundamental mode; that is, \(\tilde{R}_1\) increases from 3.60 to 3.81. The increased damping due to the presence of the particles offsets the increased combustion driving to yield a system which is stable to moderate amplitude disturbances. Therefore, in the results to follow the effects of combustion nonlinearities upon the decay rate, rather than limiting amplitude, are presented.

The effect of the burning rate nonlinearities upon the decay rates for 3\% and 10\% initial disturbances is shown in Figure 36 as calculated by the "exact" analysis. Here the nonlinear combustion response yields larger decay rates, which reflects a decrease in the amount of combustion driving with the addition of nonlinear combustion effects. For the 3\% disturbance, combustion nonlinearities increase the damping after 10 cycles
Figure 34. Effect of Combustion Nonlinearities Upon Pressure and Burning-Rate Waveforms Using the "Exact" Analysis
Figure 35. Effect of Combustion Nonlinearities Upon Pressure Waveform Using Approximate Analysis
from $-2.8 \text{ sec}^{-1}$ to $-11.1 \text{ sec}^{-1}$ ($-8.3 \text{ sec}^{-1}$ increase), while an increase in damping from $-16.7 \text{ sec}^{-1}$ to $-28.4 \text{ sec}^{-1}$ ($-11.7 \text{ sec}^{-1}$ increase) occurs for the 10% disturbance. This trend of decreased combustion driving with the addition of combustion nonlinearities is opposite to the trend obtained earlier for the motor without particles and in agreement with the results of Levine and Culick$^5$ for a motor with $2\mu$-diameter particles. It is believed that this change in the behavior of the nonlinear combustion response when particles are included is simply a result of the decreased frequency of oscillation, which apparently affects the nonlinear characteristics of the combustion response as well as the linear burning characteristics. These results also show that combustion nonlinearities can be important for amplitudes as small as 3%.

The results shown in Figure 36 suggest that a negative value of $b$ should be used in the approximate analysis. Plots of decay rate versus time obtained with the approximate model are shown in Figure 37 for linear combustion ($b = 0$) and nonlinear combustion ($b = -0.9$). Comparison with Figure 36 shows that for the case of linear combustion driving the approximate analysis yields larger values of the decay rate than the "exact" analysis. In order to assess the usefulness of the heuristic nonlinear combustion model, the change in decay rates due to combustion nonlinearities rather than the actual values of the decay rates, will be used as a basis for comparing the approximate and "exact" solutions. For $b = -0.9$ the decay rate increases by $-4.6 \text{ sec}^{-1}$ for the 3% disturbance and by $-14.5 \text{ sec}^{-1}$ for the 10% disturbance. This increase in decay rate is roughly in proportion to the disturbance amplitude, as expected from the form of the heuristic nonlinear combustion model (see Equation (77)). In contrast, the "exact" analysis yields a $-8.3 \text{ sec}^{-1}$ increase in the decay rate for the 3% disturbance and a $-11.7 \text{ sec}^{-1}$ increase for the 10% disturbance where the nonlinear combustion model is used. This result suggests that the nonlinear combustion effect is not proportional to the first power of the oscillation amplitude, as has been assumed in the heuristic model, but is proportional to some positive power $p$ of the amplitude, where $p < 1$.

It was postulated above that frequency is the primary factor responsible for the differences in the nonlinear combustion characteristics observed in the two cases considered in this study. To check this hypothesis, a third case was also considered in which the motor was shortened from $0.597 \text{ m}$
Figure 36. Effect of Combustion Nonlinearities Upon Decay Rate for Motor with 2.5μm Particles Using "Exact" Analysis
Figure 37. Effect of Combustion Nonlinearities Upon Decay Rate for Motor with 2.5 μm Particles Using Approximate Analysis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>6.00</td>
</tr>
<tr>
<td>σ</td>
<td>2.5μm</td>
</tr>
<tr>
<td>B</td>
<td>0.59</td>
</tr>
<tr>
<td>C</td>
<td>0.36</td>
</tr>
<tr>
<td>n</td>
<td>0.583</td>
</tr>
<tr>
<td>Ω</td>
<td>4.2</td>
</tr>
<tr>
<td>$\bar{N}$</td>
<td>0.078</td>
</tr>
</tbody>
</table>

$f = 918$ Hz

- Linear Combustion
- Nonlinear Combustion ($b = -0.9$)
(1.958 ft) to 0.501 m (1.642 ft) for the case with 2.5μ particles. This gave an oscillation frequency of 1079 Hz which is only slightly higher than the frequency obtained in the longer motor when particles are absent. The corresponding frequency parameter was $\Omega = 5.00$ which gave a linear response factor for the fundamental mode of $\Theta_1 = 3.505$. In addition the mean flow Mach number at the nozzle entrance was reduced to 0.0654 due to the decrease in the surface area of the burning propellant. Both approximate and "exact" models were used to obtain plots of decay rate versus time for this case, which are shown in Figure 38. In the approximate calculations, the same value of b was used as in the previous case in which particles were absent (i.e., $b = 0.32$). In contrast to the results presented in Figure 36 and 37 for $f = 918$ Hz, Figure 38 shows a decrease in damping when the nonlinear combustion model is used for $f = 1079$ Hz. Thus combustion nonlinearities increase the amount of combustion driving in this case, which agrees with the trend obtained in the first case considered (no particles, 1071 Hz). On the basis of the "exact" solutions, it appears that a somewhat larger value of b should be used in the approximate analysis; this may be due to the slightly higher frequency obtained in this case (1079 Hz as compared to 1071 Hz obtained previously). These results indicate that the effects of combustion nonlinearities upon motor stability are strongly dependent upon the frequency of oscillation.

It has been shown in this section that for some cases combustion nonlinearities increase the amount of combustion driving as the amplitude increases. In the first case considered ($f = 1071$ Hz, no particles), the motor was unstable to small amplitude disturbances, and the burning rate nonlinearities led to a larger limiting amplitude of the resulting oscillations. In the third case ($f = 1079$ Hz, 2.5μ particles) the motor was stable to moderate amplitude disturbances, and the increased combustion driving due to burning rate nonlinearities resulted in smaller decay rates. The latter case suggests the possibility that, if the combustion nonlinearities are sufficiently strong, larger amplitude disturbances will grow in a motor which is stable for small amplitude oscillations. Such a motor is said to be subject to pulsed or triggered instabilities.

The possibility of pulsed instability was investigated with the approximate analysis using the heuristic nonlinear combustion model. For the second case considered above (918 Hz, 2.5μ particles), a hypothetical combustion response was considered by assuming that $b = 3.0$ (rather than
Figure 38. Effect of Combustion Nonlinearities on Decay Rate for 1079 Hz Oscillations in a Motor with 2.5μ Particles
the negative value indicated by the "exact" model). Solutions were then generated for several different initial disturbance amplitudes using both linear and nonlinear burning rate models. The results of these computations are shown in Figure 39. For the 1% and 3% disturbances, combustion nonlinearities yield smaller decay rates as expected, while the 10% and 20% disturbances grow when the nonlinear combustion model is used. For the 20% disturbance it appears that the oscillations are approaching a limiting amplitude of about 30%. This is clearly an example of pulsed instability in a linearly stable motor.

To further explore this case, a plot of growth/decay rate, \( \alpha \), versus amplitude was constructed for these solutions. This graph is presented in Figure 40 which shows that \( \alpha \) increases monotonically from negative values (decay) at small amplitudes, vanishes at about 4% amplitude, and reaches maximum positive values (growth) at about 20% amplitudes. For amplitudes larger than 20%, \( \alpha \) falls rapidly to zero. The vanishing of \( \alpha \) at 4% amplitude is very significant, since it indicates a threshold amplitude below which all oscillations decay and above which all oscillations grow; this amplitude is referred to as an unstable limit-cycle or triggering limit. The rapid drop in \( \alpha \) for large amplitude oscillations indicates approach of the solution to a final limiting amplitude or stable limit cycle at which \( \alpha = 0 \). At this point the increased combustion driving at higher amplitudes is balanced by the increased losses due to gasdynamic mode-coupling which limits the final amplitude attained. Although this case is purely hypothetical, it illustrates the essential features of pulsed instability in actual rocket motors and demonstrates the feasibility of using the approximate nonlinear combustion model to investigate this phenomenon.

In summary, this study has shown that pressure coupled combustion nonlinearities are important in many cases; the "exact" calculations indicate that they should be included whenever the oscillation amplitudes exceed the 7 - 10% range (and sometimes for amplitudes as small as 3%). Depending on the frequency of oscillation and the various propellant properties, combustion nonlinearities may increase or decrease the combustion driving as the wave amplitude increases thereby affecting the growth/decay rates and limiting amplitudes. It has been shown that the heuristic nonlinear combustion model used in the approximate analysis accounts for many of the effects of nonlinear combustion driving, but additional information is needed.
Figure 39. Pulsed Instability Due to Combustion Nonlinearities by Approximate Analysis
Nonlinear Combustion $b = 3.0$

$A = 6.00 \quad \sigma = 2.5 \mu$

$B = 0.59 \quad C_m = 0.36$

$n = 0.583 \quad \bar{M}_e = 0.078$

$\Omega = 4.22$

Initial Amplitudes

- $1\%$
- $3\%$
- $6\%$
- $10\%$
- $15\%$
- $20\%$

Figure 40. Decay Rate vs Amplitude for Nonlinear Combustion with $b = 3.0$
to determine the nonlinear combustion parameter b. Finally it has been shown that burning rate nonlinearities may be an important factor contributing to the development of pulsed instabilities, a phenomenon which is not predicted on the basis of gasdynamic nonlinearities (mode-coupling) alone.

The treatment of combustion nonlinearities presented in this section is by no means exhaustive; much work remains to be done which is beyond the scope of this project. In the first place, the effects of combustion nonlinearities should be investigated over a greater range of parameters (i.e., frequency, mean pressure, and propellant properties) than was possible here. The heuristic nonlinear combustion model should be improved to account for the nonlinear dependence of combustion driving upon amplitude noted earlier. Finally, the most difficult, and probably the most important, work needed in this area is to incorporate the effects of velocity-coupled combustion nonlinearities into the "exact" and approximate models.

4.4 Effect of Nonlinear Particle Damping

In the two preceding sections, the effects of nonlinearities in the gasdynamic mode-coupling and the pressure-coupled combustion response were investigated. In this section the effects of nonlinearities in the viscous interaction between the particles and gas are considered. Using the Kooker-Zinn "exact" model with the nonlinear particle drag law (Equation (99)), solutions were calculated for several particle sizes for cases with and without combustion driving. The nonlinear particle damping solutions were then compared with the corresponding solutions obtained with the linear Stokes drag law (Equation (7)) to assess the importance of particle damping nonlinearities. In these studies, nonlinear gasdynamic mode-coupling was also included in the analysis, but the pressure-coupled combustion response was assumed to be linear. Approximate solutions were also obtained using the heuristic nonlinear particle damping law given by Equation (81), and these solutions were compared with the corresponding "exact" solutions in order to assess the validity of the heuristic approach.

"Exact" Solutions. The first case considered was a hypothetical motor for which the propellant is insensitive to the pressure oscillations (i.e., $\mathcal{R} = 0$), but steady-state combustion is present. This case was chosen in order to retain the mean flow contribution to the nonlinear particle damping, an effect which is not present in the case of a particle/gas
mixture in a closed-ended box. Using the "exact" computer program with
the nonlinear particle drag option, solutions were calculated for $2.5 \mu \leq \sigma 
\leq 40 \mu$ for $C_m = 0.20$, $M_e = 0.078$, and $f_g = 1071$ Hz. Plots of decay rate 
versus particle diameter for nonlinear drag (2.5% and 5% amplitude) are 
compared with the corresponding curve for linear drag in Figure 41.

Figure 41 shows that the dependence of decay rate upon particle 
size for the nonlinear drag calculations is similar to that obtained with 
the linear drag law; both exhibit an optimum particle size for maximum 
damping. However, the particle drag nonlinearities shift the peaks to 
larger particle sizes. For the nonlinear drag law, the maximum damping 
occur at $\sigma = 13 \mu$ for 2.5% amplitude and at $\sigma = 16 \mu$ for 5.0% amplitude 
as compared to $\sigma = 8.5 \mu$ for the Stokes drag law. The nonlinear particle 
drag effect increases as the amplitude of the oscillation increases, and 
it also increases with increasing particle size. Both of these effects 
are a consequence of the higher Reynolds numbers resulting from the in-
crease in particle size and relative velocity. For $\sigma < 10 \mu$ the decay rate 
decreases with increasing amplitude, while damping increases with increasing 
amplitude for $\sigma > 10 \mu$. For $\sigma < 3 \mu$ the effect of particle nonlinearities 
is small and can be neglected, but the nonlinear drag effects become very 
important for $\sigma > 15 \mu$.

The next case considered was the basic set of motor parameters used 
in the mode-coupling study: $Q_s = -137.0$ cal/gm ($A = 6.0$, $B = 0.58$, $n = 
0.575$), $M_e = 0.078$, and $f_g = 1071$ Hz. Calculations were made for $\sigma = 2.5 \mu$, 
$8.0 \mu$, and $20.0 \mu$ for a particle loading of $C_m = 0.36$ using both linear 
and nonlinear particle drag models. The results of these calculations are 
shown in Figures 42, 43, and 44.

For $2.5 \mu$ particles the growth to limiting amplitude of a 10% initial 
disturbance is shown in Figure 42 for both linear and nonlinear drag laws. 
Particle nonlinearities increase the limiting amplitude from about 9.2% 
to about 10.5%, a modest effect. For small particles this result is expected 
from the small decrease in particle damping due to nonlinear drag effects 
at higher amplitudes (Figure 41).

For $8.0 \mu$ particles the motor is stable due to the greatly increased 
particle damping. To determine the effect of particle drag nonlinearities 
upon the decaying oscillations, decay rate was plotted as a function of 
amplitude (zero-to-peak) for each cycle during the decay. These plots for
\[ \text{Decay Rate, } -\alpha_p \text{ (sec}^{-1}) \]

"Exact" Analysis

No Combustion Response

\[ \begin{align*}
C_m &= 0.20 \\
f &= 1071 \text{ Hz} \\
\gamma &= 1.23 \\
\bar{M}_e &= 0.078
\end{align*} \]

Nonlinear Drag

- 5% Amplitude
- 2.5% Amplitude

Linear Drag

(Stokes)

Particle Diameter, \( \sigma \) (\( \mu \))

Figure 41. Effect of Particle Drag Nonlinearities Upon Decay Rates for a Hypothetical Motor Without Combustion Driving
Figure 42. Effect of Particle Drag Nonlinearities Upon Limiting Amplitude for Motor With 2.5\(\mu\) Particles by "Exact" Analysis

A = 6.00 \quad C_m = 0.36
B = 0.58 \quad f_g = 1071 \text{ Hz}
n = 0.575 \quad \bar{M}_e = 0.078
10% and 20% initial amplitudes for nonlinear drag and for 10% initial amplitude for linear drag are shown in Figure 43. For nonlinear drag, the decay rate increases as the oscillation decays, approaching a constant value of about -273 sec$^{-1}$ for small amplitudes. This limiting value is significantly smaller than the nearly constant value of -323 sec$^{-1}$ obtained with the linear drag law. There is very good overlap between the nonlinear solutions for 10% and 20% initial amplitudes, indicating only a slight dependence of decay rate on the history of the oscillation.

Similar plots are shown in Figure 44 for 20μ particles, which also yields a stable motor. Here the particle drag nonlinearities have an effect on the decay rate opposite to that observed for the 8μ particles. The decay rate decreases as the wave decays and the limiting value of the decay rate at small amplitude (-293 sec$^{-1}$) is considerably larger than the value (-137 sec$^{-1}$) obtained with the linear drag law. In addition there is a significant dependence of decay rate upon the prior history of the oscillation, indicated by the differences between the 10% and 20% initial amplitude solutions.

The nonlinear behavior described above can be explained on the basis of the variation of linear particle damping with particle diameter $\sigma$ and the fact that the nonlinear contribution to $K$ is always positive (equation (100)). Thus for all particle sizes the effect of including the nonlinear drag term is similar to the effect obtained by increasing $K$ without including the nonlinear drag term. For small particles (large $K$) increasing $K$ decreases the damping, while for large particles (small $K$) increasing $K$ increases the damping. For this reason including the nonlinear drag term decreases the decay rate for small particles and increases the decay rate for large particles. The increasing importance of particle drag nonlinearities with increasing particle size arises because $K \propto \sigma^{-2}$ and $K_{NL} \propto \sigma^{2/3}$. Thus for sufficiently large $\sigma$, the nonlinear contribution $K_{NL}$ becomes larger than the linear contribution $K$ (Table 4).

The principal factor influencing the nonlinear drag term is the absolute value of the relative velocity between particles and gas. The relative speed is composed of a steady-state part and a fluctuating part. Due to the steady-state lag of the particles relative to the gas in a motor with burning at the lateral boundary (Equation (41)), the steady-state part is significant. The steady-state contribution to the relative velocity explains the fact that the damping at vanishingly small amplitude with non-
Nonlinear Drag

- 10% Initial Amplitude
- 20% Initial Amplitude

Linear Drag

10% Initial Amplitude

\[ Q_S^* = -137.0 \text{ cal/gm} \quad \bar{M}_c = 0.078 \quad C_m = 0.36 \]

Figure 43. Effect of Particle Drag Nonlinearities Upon Decay Rates for Motor With 8\(\mu\) Particles by "Exact" Analysis
$\sigma = 20 \mu$

$Q_s^* = -137.0 \text{ cal/gm}$

$\bar{M}_e = 0.078$

$C_m = 0.36$

**Linear Drag**

10% Initial Amplitude

**Nonlinear Drag**

- 10% Initial Amplitude
- 20% Initial Amplitude

*Figure 44. Effect of Particle Drag Nonlinearities Upon Decay Rates for Motor With 20\(\mu\) Particles by "Exact" Analysis*
linear drag differs from the corresponding damping with linear drag. Thus nonlinear particle drag is important even for infinitesimally small amplitude oscillations. The fluctuating contribution to the nonlinear drag accounts for most of the variation of decay rate with amplitude when the drag nonlinearities are taken into account.

To determine the effect of particle drag nonlinearities upon the limit-cycle amplitudes for motors with particles much larger than 2.5µ, it was necessary to increase the combustion response and decrease the particle concentration so that instability could occur. Thus $Q_s^*$ was increased to $-139.4$ cal/gm giving $A = 6.00$, $B = 0.55$ and $n = 0.552$, and $C_m$ was decreased to 0.20. Results calculated with the "exact" analysis for 8.0µ and 20µ particles are shown in Figures 45, 46, and 47.

Figure 45 shows growth rate versus number of cycles for 3% and 10% initial disturbances for a motor with 8.0µ particles. This case is interesting because it shows that particle drag nonlinearities, like combustion nonlinearities, may lead to pulsed instabilities. In this example 3% initial disturbances decay at a rate of roughly $-25$ sec$^{-1}$, while 10% initial disturbances grow at a rate of about $15$ sec$^{-1}$. This behavior is a consequence of the nonlinear drag law which causes a decrease in particle damping with increasing amplitude (Figure 43) allowing the combustion process to drive oscillations for sufficiently large amplitudes.
Figure 45. Pulsed Instability Due to Particle Drag Nonlinearities for Motor With 8 µm Particles by "Exact" Analysis
The particle nonlinearities have the opposite effect for 20 µ particles as shown in Figure 46. Using the linear drag law, the analysis predicts growth of oscillations to a limiting amplitude exceeding 8%. For the nonlinear drag law, the greatly increased particle damping causes the oscillations to decay to zero (or at least to a limiting amplitude less than 1.5%). The corresponding pressure waveforms for this case are presented in Figure 47, which shows the dramatic effect of particle drag nonlinearities upon the amplitude of the oscillations but little or no effect on the shape of the pressure waveform.

Approximate Solutions. In order to assess the validity of the heuristic nonlinear particle drag law described in Section 2.8, approximate solutions were obtained for 8 µ and 20 µ particles and were compared with the corresponding "exact" solutions. These results are presented as plots of decay rate versus amplitude in Figures 48 and 49 which correspond to the "exact" solution presented in Figures 43 and 44.

Figure 48 shows approximate solutions for decay rate as a function of amplitude for a motor with 8 µ particles. For nonlinear drag these curves are shown for several values of the parameter C which appears in Equation (81). For the case of C = 0 the amplitude-dependent part of the nonlinear drag vanishes and only the steady-state contribution remains. For positive values of C the decay rate increases as the amplitude of the wave decreases, which agrees with the "exact" calculations. In addition the approximate solutions agree with the "exact" solutions regarding the prediction that the particle damping at low amplitudes is smaller when the nonlinear particle drag law is used (due to the steady-state contribution to the Reynolds number).

In comparing the approximate and "exact" solutions shown in Figures 43 and 48, it should be noted that the two analyses predict different decay rates for linear particle drag (i.e., $\alpha = -323$ sec$^{-1}$ for the "exact" analysis compared to $\alpha = -400$ sec$^{-1}$ for the approximate analysis). Thus quantitative assessment of the validity of the postulated nonlinear drag model must be made by comparing the "exact" and approximate results on the basis of the change in decay rate due to the addition of particle drag nonlinearities rather than on the basis of the actual values of the decay rates. Therefore two quantities, $\Delta \alpha_1$ and $\Delta \alpha_2$ will be considered in comparing
"Exact" Analysis

\[ \sigma = 20\mu \]
\[ C_m = 0.20 \]
\[ f = 1071 \text{ Hz} \]
\[ \frac{g}{g} \]
\[ M_e = 0.078 \]

Linear Drag

Nonlinear Drag

\[ A = 6.00 \]
\[ B = 0.55 \]
\[ n = 0.552 \]

Figure 46. Strong Effect of Particle Drag Nonlinearities Upon Motor Stability for 20\(\mu\) Particles
Figure 47. Effect of Particle Drag Nonlinearities on Pressure Waveforms for 20μm Particles
Approximate Analysis  
5 Modes

\( \sigma = 8 \mu \) \( A = 6.00 \)
\( C_m = 0.36 \) \( B = 0.58 \)
\( \bar{M}_e = 0.078 \) \( n = 0.575 \)
\( \Omega = 4.32 \)

Nonlinear Drag

\( C = 1.5 \)
\( C = 1.0 \)
\( C = 0.5 \)
\( C = 0 \)

Linear Drag

Zero-to-Peak Pressure Amplitude (%)

Figure 48. Decay Rate vs Amplitude for Motor With 8\( \mu \) Particles Using
Nonlinear Drag Model.
the approximate and "exact" solutions. The first quantity, \( \Delta \alpha_1 \) is the difference between the decay rate at small amplitude computed using the nonlinear drag law and the corresponding decay rate obtained using the linear drag law; thus, \( \Delta \alpha_1 = \alpha(\text{nonlinear drag, small amplitude}) - \alpha(\text{linear drag}) \). By comparing the approximate values of \( \Delta \alpha_1 \) with the "exact" values, the validity of the steady-state term, \( K_{ss} \), in Equation (81) can be assessed. The second quantity, \( \Delta \alpha_2 \), is the difference between the decay rate at finite amplitude and the decay rate at vanishingly small amplitude as calculated using the nonlinear drag models; thus, \( \Delta \alpha_2 = \alpha(\text{finite amplitude}) - \alpha(\text{small amplitude}) \). Comparing the approximate and "exact" values of \( \Delta \alpha_2 \) for a given amplitude can be used to determine the proper value of the parameter \( C \) appearing in the unsteady contribution to the nonlinear particle drag (Equation (81)).

For the case of 8.0\( \mu \) particles shown in Figure 48, values of \( \Delta \alpha_1 \) and \( \Delta \alpha_2 \) were calculated using both approximate and "exact" analyses. For \( \Delta \alpha_1 \) the approximate analysis yields 35 sec\(^{-1}\) compared to 50 sec\(^{-1}\) for the "exact" analysis. Thus the heuristic nonlinear drag model underestimates the steady-state contribution to the nonlinear drag for 8\( \mu \) particles. At 4% amplitude the approximate analysis yields \( \Delta \alpha_2 = 87 \text{ sec}^{-1} \) for \( C = 1 \) and \( \Delta \alpha_2 = 48 \text{ sec}^{-1} \) for \( C = 0.5 \), while the "exact" analysis yields \( \Delta \alpha_2 = 63 \text{ sec}^{-1} \). This result indicates that a value of \( C = 0.7 \) would yield better agreement with the "exact" analysis for the case of 8\( \mu \) particles.

Figure 49 shows similar decay rate versus amplitude curves obtained with the approximate analysis for a motor with 20\( \mu \) particles. Again the qualitative agreement with the "exact" solutions (Figure 44) is good; the approximate solutions (\( C = 1 \)), also exhibit a decreasing decay rate as the amplitude increases and a limiting decay rate at low amplitude which is considerably larger than the value obtained with the linear drag law. For this case the approximate method gave \( \Delta \alpha_1 = -205 \text{ sec}^{-1} \) as compared to \( \Delta \alpha_1 = -156 \text{ sec}^{-1} \) from the "exact" analysis. Thus the approximate analysis overestimates the steady-state contribution to the nonlinear drag for 20\( \mu \) particles, which is opposite from the discrepancy obtained for 8\( \mu \) particles. Also the amplitude dependent part of the nonlinear drag is overestimated for \( C = 1 \); at 4% amplitude the approximate analysis yielded \( \Delta \alpha_2 = -160 \text{ sec}^{-1} \) while the "exact" analysis (10% initial amplitude) gave \( \Delta \alpha_2 = -117 \text{ sec}^{-1} \).
Figure 49. Decay Rate vs Amplitude for Motor With 20 μm Particles Using

Approximate Analysis  5 Modes

\[ \sigma = 20 \mu m \]
\[ C_m = 0.36 \]
\[ M_e = 0.078 \]

Linear Drag

Nonlinear Drag

\[ A = 6.0 \]
\[ B = 0.58 \]
\[ n = 0.575 \]
\[ \Omega = 4.66 \]

Zero-to-Peak Pressure Amplitude (%)
As in the case of 8\mu particles, these results indicate that C = 0.7 will give better agreement regarding the amplitude dependence of the nonlinear particle damping.

These results presented above indicate that the heuristic nonlinear drag model used in the approximate analysis gives a reasonable qualitative description of the effects of particle drag nonlinearities on the decay rates for stable solid rocket motors. To obtain better quantitative agreement between the approximate and "exact" nonlinear particle damping solutions, a more extensive parametric study is needed to determine how the heuristic nonlinear particle drag model can be improved. Such a study was beyond the scope of this project and is recommended for future research.

4.5 Solutions for T-Burners

In this subsection the Galerkin method is used to obtain approximate solutions for a few selected T-burner configurations in order to demonstrate the applicability of the approximate analysis to T-burners. For most of the cases considered here, the length, bore, propellant properties, burning rate and combustion-product properties (gaseous and particulate) for the T-burner are assumed to be the same as those for the laboratory pulse motor considered in Section 4.2. Thus the dimensionless radius for the T-burner is R = 0.051 and the dimensionless velocity of the combustion products leaving the burning propellant surfaces is \( \bar{u}_b = 0.002 \). The transient combustion parameters are the same as those used for the motor solutions with particles; that is, A = 6.00, B = 0.58, and n = 0.575. The dimensionless frequency parameter is assumed to be \( \Omega = 4.2 \) which yields a response factor of \( R^r_{11} = 4.14 \). The dimensionless length of the center vent is given by \( \beta_v = 0.1 \), while the dimensionless plug flow length for the pipe connecting the burner with the surge tank is assumed the value \( L_{\text{eff}} = 0.166 \). In accordance with the theoretical limitations discussed in Section 2.6, flush grains (\( S_c/S_{\text{co}} = 1.00 \)) are assumed for all cases considered in this study.

In obtaining the approximate T-burner solutions the following parameters were varied: (1) the length of the cylindrical grains, (2) the combustion response \( R^r_{11} \), (3) the particle concentration \( C_m \), and (4) the vent factor \( \beta_v \). Grain configurations considered were end-burning only (\( \beta = 0 \)) and cup grains (\( \beta = 0.1 \) and 0.3). The effect of combustion
driving on the T-burner solutions was determined by comparing the growth rate for \( R_1^r = 4.14 \) with the decay rate obtained when the burning rate is insensitive to pressure oscillations (\( R = 0 \)). Solutions for the case of no particles (\( C_m = 0 \)) were also compared with those obtained for the case of 2.5\( \mu \) -diameter particles with \( C_m = 0.36 \). The effect of the center vent was assessed by considering the case of vent gain (\( V_L = 0 \)) and no vent effect (\( V_L = 1 \)).

The case of end-burning (disk grains, \( \beta = 0 \)) was considered first. Figure 50 shows growth/decay rates for 3\% initial IL-mode disturbances in end-burning T-burners for four cases: (1) no particles, \( R = 0 \), (2) no particles, \( R_1^r = 4.14 \), (3) 2.5\( \mu \) particles, \( R = 0 \), and (4) 2.5\( \mu \) particles, \( R_1^r = 4.14 \). In each case both linear (1L mode only) and nonlinear (5 modes) solutions are presented. For the first case, the nonlinear solutions decay at approximately the linear rate of \(-1.8 \text{ sec}^{-1}\) during the first few cycles; the slight amount of damping is due to the effect of the mean flow and the center vent. Similarly the initial growth rate of the nonlinear solutions for the second case (with combustion driving) is close to the linear value of \(41 \text{ sec}^{-1}\). In both cases the rapid rise in growth rate after seven cycles is a result of strong waveform steepening due to nonlinear gasdynamical mode-coupling (generation of higher harmonics). When 2.5\( \mu \) particles are present, the T-burner pressure oscillations decay at a nearly constant rate of \(-68 \text{ sec}^{-1}\) for \( R = 0 \) (case 3) and \(-37 \text{ sec}^{-1}\) for \( R_1^r = 4.14 \) (case 4) due to the increased attenuation of the higher harmonics by the particles.

In order to obtain growth of oscillations to limiting amplitude for T-burners with metallized propellants it is often necessary to increase the surface area of the burning propellant by using cup grains. This is the situation in the case selected for this study, for a cup-length of \( \beta = 0.1 \) is necessary to obtain spontaneous growth of oscillations when the 2.5\( \mu \) particles are present. Growth rates for cup-lengths of \( \beta = 0 \) (end-burning only), \( \beta = 0.1 \), and \( \beta = 0.3 \) are compared in Figure 51. For \( \beta = 0.1 \), the growth rate of the nonlinear solutions rapidly decreases from the linear value of \(12 \text{ sec}^{-1}\) to nearly zero, indicating approach to limiting amplitude (about 7\% in this case). For \( \beta = 0.3 \), the initial growth rate is much larger (about 100 \text{ sec}^{-1}) than for \( \beta = 0.1 \), but it decreases rapidly as the amplitude increases. For the latter case, the
End - Burning Only ($\beta = 0$)

- $R = 0.051$
- $u_b = 0.002$
- $\beta_v = 0.10$
- $L_{eff} = 0.166$
- $V_L = 1$

3% Initial Disturbance

Symbols: 5 Modes

---

1L Mode Only

---

No Particles

$\sigma = 2.5 \mu, \ C_m = 0.36$

$\theta = 0$

Number of Cycles

Figure 50. Growth Rates for End-Burning T-Burner
Figure 51. Growth Rates for T-Burners with Cup Grains

R = 0.051, \( u_b = 0.002 \), \( \beta_v = 0.10 \), \( L_{\text{eff}} = 0.166 \)
A = 6.00, B = 0.580, n = 0.575, \( \Omega = 4.2 \)

Symbols: 5 Modes

\( \beta = 0.3 \)
10% Initial Amplitude

\( \beta = 0.1 \)
7.5% Initial Amplitude

\( \beta = 0 \)
3% Initial Amplitude
solution was not carried out to limiting amplitude, but it probably exceeds 20%.

The importance of the center vent for this case was determined by comparing solutions for $V_{VL} = 0$ (vent gain) with the corresponding solutions for $V_{VL} = 1$ (no vent effect). These comparisons were made for both end-burning and cup-grain configurations, for T-burners without combustion driving ($\varphi = 0$) or particle damping. The results of these calculations are given in Table 5. The quantity $\Delta \alpha_v$ is the difference between the decay rates obtained with $V_{VL} = 0$ and $V_{VL} = 1$; thus, $\Delta \alpha_v$ represents the vent gain due to the mean-flow/acoustic interaction. From Table 5 it is seen that $\Delta \alpha_v$ is a rather small quantity that increases as $\beta$ increases due to the resulting increase in steady-state flow velocity in the vent region. These results indicate, that, at least for the cases considered, the mean-flow/acoustic vent effect is relatively unimportant and may be neglected in the nonlinear analysis of T-burners.

Table 5. Effect of Center Vent

<table>
<thead>
<tr>
<th>Case</th>
<th>$\beta$</th>
<th>$\alpha$, (sec$^{-1}$)</th>
<th>$\Delta \alpha_v$, (sec$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$V_{VL} = 1$</td>
<td>$V_{VL} = 0$</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.0</td>
<td>-1.83</td>
<td>-1.76</td>
</tr>
<tr>
<td>2</td>
<td>0.1</td>
<td>-22.18</td>
<td>-21.98</td>
</tr>
<tr>
<td>3</td>
<td>0.3</td>
<td>-59.03</td>
<td>-58.55</td>
</tr>
</tbody>
</table>
Pressure waveforms and mode-amplitude functions were obtained for two of the T-burner cases considered in this study. These results are shown in Figure 52 for an end-burning T-burner without particles and in Figure 53 for a cup grain configuration ($\beta = 0.1$) with $2.5 \mu -$ diameter particles; in both cases a five-mode series was used and the response function was $R_1 = 4.14$.

In the first case (Figure 52), a 10% amplitude $1L$-mode initial disturbance rapidly grows and steepens after two wave cycles as higher harmonics are generated by nonlinear gasdynamic mode-coupling. As in the case of solid rocket motors (Figure 8), the amplitude of the fundamental mode is considerably larger than the amplitudes of the higher harmonics (the response functions for the higher modes are considerably smaller than the response function for the fundamental mode). It is also seen that the amplitudes of the higher modes decrease as the axial mode number increases, except that the $5L$ mode has a larger amplitude than the $4L$ mode. The latter result is probably due to the finite number of modes (i.e., five) used in the series, and it accounts for the fifth harmonic "wiggles" apparent in the pressure waveform.

In the second case (Figure 53) when particles are present, a 7.5% amplitude $1L$-mode initial disturbance is shown developing into a nearly sinusoidal limiting-amplitude pressure waveform. In contrast to the previous case the amplitudes of the higher harmonics decrease rapidly as the mode number increases; in fact it appears that the $3L$, $4L$ and $5L$ modes are negligible and that a two or three mode series is adequate to represent the solution.

In order to adequately evaluate the applicability of the approximate technique to the nonlinear analysis of T-burners, the approximate T-burner solutions were compared with available "exact" solutions as well as with experimental data. Since time did not permit extension of the Kooker-Zinn "exact" analysis to analyze T-burner configurations, Levine's T-burner calculations $^5$ were used in these comparisons. The case selected for comparison with the "exact" analysis was for $S_b/S_{co} = 7.06$ (Table A-1 of Reference 5) for which $\beta = 0.216$ (tubular grains without end-burning disks). The combustion parameters used by Levine were $A = 8.8$, $B = 0.67$, $n = 0.3$, and $\Omega = 7.80$ which gave a response function of $R_1 = 2.03$. The particles were characterized by an average diameter of $3.0 \mu$ and a
Figure 52: Pressure Waveform and Mode-Amplitude Functions for End-Burning T-Burner Without Particles
Figure 53. Pressure Waveform and Mode Amplitude Functions for Cup Grain T-Burner with 2.5μm Particles
concentration of $C_m = 0.36$.

T-burner pressure-time histories obtained with the Galerkin method and Levine's "exact" analysis are compared in Figure 54 for 3% initial disturbances. The resulting solutions are seen to disagree regarding the initial growth rate and limiting amplitude. The approximate solutions grow very slowly as they appear to be near limiting amplitude (probably about 4%), while the "exact" solutions are still growing rapidly after 12 wave cycles (eventually reaching 37% limiting amplitude). Both solutions agree, however, regarding the nearly sinusoidal waveshape and the double frequency small amplitude oscillation at the center vent. Comparisons with experimental data for T-burners are presented in the next section.

The results presented in this subsection indicate that the Galerkin method can be used to obtain approximate solutions for growth/decay rates and limiting amplitudes for flush-grain T-burner configurations. The limited parametric study performed indicates that the approximate solutions exhibit the expected trends regarding the effect of cup-grain length, combustion response factor, particle concentration, and center vent upon the behavior of the pressure oscillations. It is difficult to draw conclusions regarding the accuracy of the approximate solutions from the limited comparison with "exact" theory given above, since Levine's model differs significantly from the present approximate model (i.e., it accounts for the thermal particle/gas interaction as well as nonlinear combustion and particle drag nonlinearities). Furthermore, the limiting amplitude predicted by the "exact" analysis is far higher than that obtained from the measured data (37% as compared to 8% for the data). It is evident that much more work needs to be done to further develop and improve both the approximate and "exact" T-burner models, and that more extensive comparisons of the approximate T-burner solutions with "exact" solutions and experimental data are needed.

4.6 Comparisons With Experimental Data

In the preceding subsections, the approximate analysis has been evaluated on the basis of comparisons with available "exact" solutions for both motors and T-burners. Since the usefulness of an analytical method lies in its ability to predict observed behavior, the approximate
Figure 54. Comparison of Approximate and "Exact" T-Burner Solutions
solutions were also compared with available T-burner and motor test data. The results of this study are presented in this subsection.

Comparison With Motor Test Data. Due to the unavailability of any other motor test data, the experimental data considered by Levine and Culick was also used in this study. This data was obtained by Aerojet several years ago for ANB 3066 propellant using a small laboratory pulse motor. This motor has a grain length of about 0.597 m (23.5 in) and an initial bore of about 50 mm (2 in); it is the same motor configuration considered previously in Sections 4.2, 4.3, and 4.4. The physical properties of the ANB 3066 propellant and its gaseous and particulate combustion products have already been given in Table 1 of Section 4.2. The motor parameters for the four cases considered by Levine and Culick are given in Table 6.

Table 6. Motor Parameters for Laboratory Pulse Motor

<table>
<thead>
<tr>
<th>Nominal Throat Diameter (cm)</th>
<th>Pulse Number</th>
<th>Port Area (cm²)</th>
<th>Throat Area (cm²)</th>
<th>$\frac{P}{(\text{knt/m}^2)}$</th>
<th>$\frac{M_e}{(\text{knt/m}^2)}$</th>
<th>$T_f$ (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.97</td>
<td>1</td>
<td>21.5</td>
<td>2.83</td>
<td>10810</td>
<td>0.0782</td>
<td>3526</td>
</tr>
<tr>
<td>1.97</td>
<td>3</td>
<td>36.3</td>
<td>2.95</td>
<td>14800</td>
<td>0.0481</td>
<td>3556</td>
</tr>
<tr>
<td>2.20</td>
<td>1</td>
<td>24.4</td>
<td>3.57</td>
<td>8330</td>
<td>0.0869</td>
<td>3503</td>
</tr>
<tr>
<td>2.20</td>
<td>2</td>
<td>30.5</td>
<td>3.63</td>
<td>9740</td>
<td>0.0703</td>
<td>3518</td>
</tr>
</tbody>
</table>

The Mach number at the nozzle entrance, which is needed in the approximate analysis, was determined from the ratio of port area to throat area using the isentropic flow relations (the low flow coefficient of the nozzle was already taken into account by Levine and Culick by adjusting the throat area). The flame temperature $T_f$ was taken from Figure 7-17.
of Reference 5. For 2.0μ particles the drag constant K was about 46.7 for all four cases.

In order to obtain approximate solutions for the test conditions given above, the transient burning rate parameters (i.e., A and B) must be estimated. Levine and Culick 5 made such estimates based on the following criteria: (1) the response function must be greater than 3 for instability, (2) the values of A and B must not result in nonlinear intrinsic instability, and (3) the corresponding values of $E_s^*$ and $Q_s^*$ should be reasonable. For Cases 1 and 4 this procedure was relatively straightforward, but for Cases 2 and 3 the considerable spread in mean pressure caused difficulties. For the latter two cases Levine and Culick postulated a pressure dependence of $Q_s^*$ above and beyond that which is accounted for by the Levine-Culick nonlinear transient burning rate model.

In addition to A and B, the dimensionless frequency parameter $\Omega$ was calculated using the steady-state burning rate given by

$$F = 0.813 \left(\frac{P}{3448}\right)^{0.3}$$

(108)

where F is the surface regression rate in units of cm/sec and P is the steady-state pressure in knt/m$^2$. The transient burning rate parameters assumed by Levine and Culick were used as the starting point in this investigation; they are given in Table 7.

<table>
<thead>
<tr>
<th>Case</th>
<th>A</th>
<th>B</th>
<th>$\Omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.00</td>
<td>0.55</td>
<td>4.3</td>
</tr>
<tr>
<td>2</td>
<td>5.98</td>
<td>0.53</td>
<td>3.65</td>
</tr>
<tr>
<td>3</td>
<td>6.02</td>
<td>0.54</td>
<td>5.06</td>
</tr>
<tr>
<td>4</td>
<td>6.00</td>
<td>0.55</td>
<td>4.3</td>
</tr>
</tbody>
</table>
Approximate solutions were obtained for each of the four cases described above using the Galerkin method with a five-mode series. In each case, using Levine and Culick's estimates of \(A\), \(B\), and \(\Omega\) along with \(n = 0.3\), the approximate analysis predicted a decaying oscillation. Therefore the parameters \(A\) and \(B\) were varied slightly in order to increase the propellant response to the level necessary to sustain oscillations. The results of these calculations are now presented and discussed for each of these cases in the following paragraphs.

The results for Case 1 (1.97-cm throat, Pulse #1) are summarized in Table 8. The measured limiting amplitude for this case was about 3%.

### Table 8. Approximate Solutions for Cases 1 and 4

<table>
<thead>
<tr>
<th>(B)</th>
<th>Initial Amplitude</th>
<th>(\frac{\alpha}{\Omega_1})</th>
<th>Oscillation after 12 cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Case 1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Ampl. (%)</td>
</tr>
<tr>
<td>0.550</td>
<td>5%</td>
<td>3.33</td>
<td>3.2</td>
</tr>
<tr>
<td>0.545</td>
<td>5%</td>
<td>3.66</td>
<td>4.3</td>
</tr>
<tr>
<td>0.545</td>
<td>2%</td>
<td>3.66</td>
<td>1.8</td>
</tr>
<tr>
<td>0.543</td>
<td>5%</td>
<td>3.82</td>
<td>5.0</td>
</tr>
<tr>
<td>0.543</td>
<td>2%</td>
<td>3.82</td>
<td>2.1</td>
</tr>
<tr>
<td>0.540</td>
<td>5%</td>
<td>4.09</td>
<td>6.3</td>
</tr>
</tbody>
</table>
while the approximate analysis predicts a limiting amplitude slightly larger than 5% for $B = 0.543$. For comparison Levine and Culick obtained an amplitude of 4.2% with $B = 0.550$.

Case 4 (2.20-cm throat, Pulse #2) is considered next because the mean pressure is nearly the same as for Case 1, allowing the same values of $A$ and $B$ to be used (Table 7). Case 4 differs from Case 1 due to the smaller value of $\bar{M}_e$ (0.0703 as compared to 0.0782 for Case 1). Using $A = 6.0$ and $B = 0.543$ as before, the approximate analysis yielded a limiting amplitude of a little over 2% which is in excellent agreement with the measured value of slightly over 2%. For this case Levine and Culick obtained a 3% oscillation with $B = 0.55$. This comparison shows that the approximate analysis correctly predicts the experimentally observed trend of decreasing limiting amplitude with decreasing steady state Mach number.

The approximate solutions for Case 2 (1.97-cm throat, Pulse #3) and Case 3 (2.20-cm throat, Pulse #1) are presented in Table 9. In each case, the first set of parameters corresponds to Levine's values, which yield decaying oscillations. Since the values of $\Omega$ for these cases yield values of $\hat{R}_1$ considerably below the peak value, variations of the parameter $B$ alone are ineffective in raising $\hat{R}_1$ sufficiently to drive instability. Since $n$ is not a strong function of pressure and $\Omega$ is fairly well determined for the given chamber pressures, the only parameter left which can be varied is $A$. Thus the parameter $A$ was changed (and $B$ as well for Case 2) to yield a response factor of about 3.8. For Case 2 the second set of parameters yields a limiting amplitude of about 1.5% in good agreement with the measured value of 1-3% (modulated). For Case 3 the second set of parameters gives an amplitude in excess of 6% compared to measured values of 2-2.5%, but a slight decrease in $A$ should give better agreement. It is difficult to assess the validity of the approximate solutions on the basis of these last two cases, since the values of the transient combustion parameters $A$ and $B$ used in obtaining these solutions are uncertain.
Table 9. Approximate Solutions for Cases 2 and 3

<table>
<thead>
<tr>
<th>Case</th>
<th>A</th>
<th>B</th>
<th>Initial Amplitude</th>
<th>Oscillation after 12 cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Amplitude (%)</td>
<td>Amplitude (%)</td>
</tr>
<tr>
<td>2</td>
<td>5.98</td>
<td>0.530</td>
<td>5%</td>
<td>2.74</td>
</tr>
<tr>
<td>2</td>
<td>5.60</td>
<td>0.517</td>
<td>5%</td>
<td>3.90</td>
</tr>
<tr>
<td>2</td>
<td>5.60</td>
<td>0.517</td>
<td>2%</td>
<td>3.90</td>
</tr>
<tr>
<td>3</td>
<td>6.02</td>
<td>0.540</td>
<td>5%</td>
<td>2.36</td>
</tr>
<tr>
<td>3</td>
<td>6.35</td>
<td>0.540</td>
<td>5%</td>
<td>3.81</td>
</tr>
<tr>
<td>3</td>
<td>6.35</td>
<td>0.54</td>
<td>2%</td>
<td>3.81</td>
</tr>
</tbody>
</table>

*Modulated

Comparison With T-Burner Test Data. The T-burner test data considered in this study are the same three cases considered by Levine and Culick\(^5\) for comparison with the "exact" analysis. This data was selected from the ANB 3066 (3.8-cm bore) data reported in Reference 22 on the basis of two requirements: the measurements were taken near the flush grain conditions and limiting amplitudes were reported. In these cases, the T-burner length was 62.2 cm (24.5 in), the steady-state pressure was 3448 knt/m$^2$ (500 psi), and the propellant configuration consisted of

tubular propellant grains at each end of the burner without end-burning disks. The principal parameter of interest in this study was the area of the burning propellant surface which was specified by the ratio $S_b/S_{co}$ and determined by the length of the propellant grains $L_b$. The pertinent data for the three values of $S_b/S_{co}$ selected is given in Table 10; for all cases $R = 0.0306$, $V_L = 1.0$ (no vent effect), and $L_{eff} = 0.163$. The particle diameter was assumed to be $3 \mu$ and the particle concentration was assumed to be $C_m = 0.36$. Following Levine and Culick, the combustion response was assumed to be given by $A = 8.8$, $B = 0.67$, and $n = 0.3$.

Table 10. T-Burner Parameters for Comparison Study

<table>
<thead>
<tr>
<th>Case</th>
<th>$S_b/S_{co}$</th>
<th>$\beta$</th>
<th>$f$ (Hz)</th>
<th>$\bar{r}$ (cm/sec)</th>
<th>$\Omega_1$</th>
<th>$\bar{u}_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.48</td>
<td>0.0465</td>
<td>772</td>
<td>0.856</td>
<td>6.62</td>
<td>0.00322</td>
</tr>
<tr>
<td>2</td>
<td>5.58</td>
<td>0.1714</td>
<td>816</td>
<td>0.815</td>
<td>7.71</td>
<td>0.00307</td>
</tr>
<tr>
<td>3</td>
<td>7.06</td>
<td>0.2163</td>
<td>820</td>
<td>0.813</td>
<td>7.80</td>
<td>0.00306</td>
</tr>
</tbody>
</table>

Approximate solutions were obtained for the above three cases using both single-mode (1L) and five-mode series expansions. The single-mode results are presented in Table 11; they correspond to exponential growth or decay of oscillations at small amplitudes. Since the approximate analysis predicts more stable behavior than that indicated by the measured data for $B = 0.67$ (Levine's value), approximate solutions for $B = 0.635$ are also shown in Table 11 which give better agreement with the experimental data. The additional case of $S_b/S_{co} = 0$ (no burning, no mean flow) was also considered; for this case the approximate analysis predicted a
decay rate of $-88.2$ sec$^{-1}$ as compared to a measured value of $-77$ sec$^{-1}$ (extrapolated).

### Table 11. Comparison of Single-Mode Solutions with Measured Data

<table>
<thead>
<tr>
<th>Case</th>
<th>$S_b/S_{co}$</th>
<th>$B = 0.670$</th>
<th>$B = 0.635$</th>
<th>Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\dot{R}_1^r$</td>
<td>$\alpha$ (sec$^{-1}$)</td>
<td>$\dot{R}_1^r$</td>
<td>$\alpha$ (sec$^{-1}$)</td>
</tr>
<tr>
<td>1</td>
<td>1.48</td>
<td>2.15</td>
<td>-64.9</td>
<td>3.25</td>
</tr>
<tr>
<td>2</td>
<td>5.58</td>
<td>2.06</td>
<td>-4.9</td>
<td>2.83</td>
</tr>
<tr>
<td>3</td>
<td>7.06</td>
<td>2.03</td>
<td>15.1</td>
<td>2.75</td>
</tr>
</tbody>
</table>

Both five-mode and single-mode solutions for $B = 0.635$ are compared with the measured data in Figure 55, which shows plots of growth rate versus number of wave cycles for 3% amplitude initial disturbances. For $S_b/S_{co} = 1.48$ the approximate solutions are in very good agreement with the measured data, while reasonably good agreement is obtained for $S_b/S_{co} = 5.58$ and $S_b/S_{co} = 7.06$. The decline in growth rate for the latter two cases after six cycles indicates that the oscillations in the T-burner are approaching limiting amplitude. Figure 56 shows the continuation of the calculations for 40 cycles for these two cases, which yields limiting peak-to-peak amplitudes of about 31% for $S_b/S_{co} = 5.58$ and about 43% for $S_b/S_{co} = 7.06$, corresponding to measured values of 9.0% and 15.6% respectively.

Three significant observations regarding the applicability of the approximate T-burner analysis can be made from the comparisons presented above. First, the linear growth rates predicted by the approximate
Figure 55. Comparison of Approximate Solutions with Experimental Data for T-Burners
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>8.8</td>
</tr>
<tr>
<td>$B$</td>
<td>0.635</td>
</tr>
<tr>
<td>$n$</td>
<td>0.3</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>3.0 $\mu$</td>
</tr>
<tr>
<td>$C_{m}$</td>
<td>0.36</td>
</tr>
</tbody>
</table>

Approximate Solutions
5 Modes

$S_{b} / S_{co} = 7.06$

$S_{b} / S_{co} = 5.58$

15% Initial Disturbance

Figure 56. Approach to Limiting Amplitude for T-Burners by Approximate Analysis
T-burner analysis do not agree well with those obtained by Levine's "exact" analysis (Levine obtained good agreement with the measured data for $B = 0.67$). Second, for a somewhat stronger propellant response ($B = 0.635$) the approximate analysis agrees well with the measured data regarding the effect of $S_b/S_{co}$ upon the linear growth rates. Finally, the approximate analysis predicts limiting amplitudes which are far higher than the measured values indicate, even though the linear growth rates are in good agreement (Levine's "exact" model also predicts excessive limiting amplitudes$^5$). The results of this limited comparison study indicate that, although the present approximate T-burner analysis may be useful in predicting trends, further improvement and development of the T-burner model is needed before it becomes an accurate indicator of nonlinear T-burner behavior.

4.7 Application of the Method of Averaging

As discussed in Section 2.4, the Method of Averaging (MOA) can be used to reduce the computation time required to obtain approximate solutions at the cost of introducing additional error into the analysis. In order to determine whether the savings in computation time obtained with the MOA compensates for the resulting loss of accuracy, a comparison study was conducted. In this investigation, approximate solutions obtained using the MOA were compared with the corresponding solutions obtained with the Galerkin method and the "exact" analysis in order to determine the range of applicability of the MOA. The results of this study are presented in this subsection.

For cases in which particles are not present, the MOA and the Galerkin method are in excellent agreement, as illustrated by the waveforms shown in Figure 57. For cases with particle damping, however, the MOA predicts a larger decay rate than the Galerkin method. For instance, solutions were obtained by both methods for the case of the attenuation of 800 Hz waves in a gas-particle mixture enclosed in a box with a particle loading of $C_m = 0.36$ and a particle diameter of $2.5\mu$. In this case, the linear decay rate obtained using the Galerkin method was $-56 \text{ sec}^{-1}$, while the linear decay rate obtained using the MOA was found to be $-72 \text{ sec}^{-1}$.

Since previously published results by Culick$^{10}$ showed good agreement between the MOA solutions and Levine's "exact" solutions for cases
Figure 57. Waveforms for Motor Without Particles by Galerkin Method and Method of Averaging

- Galerkin Method
- Method of Averaging

No Particles
\[ \bar{M}_e = 0.0780 \]

A = 6.00
B = 0.59
n = 0.583
\( \Omega = 4.91 \)

Dimensionless Time, t
with particle damping, Culick's equations were also examined to aid in determining the source of discrepancy. Unlike the present analysis where a wave equation for the velocity potential is derived, Culick has formulated the problem by developing an inhomogeneous wave equation for the pressure perturbation, \( p' \). Considering a single mode expansion consisting of the 1L mode, an amplitude function \( \eta_1(t) \) is defined such that

\[
P'/P_0 = \eta_1(t) \psi_1(x) \quad (109)
\]

\[
u' = \left( \frac{1}{\gamma K_1^2} \right) \frac{d\eta_1}{dt} \frac{d\psi_1}{dx} \quad (110)
\]

where \( \psi_1 \) describes the mode structure and \( K_1 = \omega_1/c_1 \) is the wave number. To determine \( \eta_1 \) the inhomogeneous wave equation was multiplied by \( \psi_1 \) and integrated over the length of the chamber (this step is equivalent to the Galerkin method) to obtain the following second order equation for \( \eta_1 \):

\[
\frac{d^2 \eta_1}{dt^2} + \omega_1^2 \eta_1 = F_1(p) \quad (111a)
\]

where

\[
F_1(p) = \gamma \int_0^L \left[ \delta F' p \frac{d\psi_1}{dx} + \delta u' p \psi_1 \right] \, dx \quad (111b)
\]

In this equation,

\[
F_p = \frac{\rho_p}{\tau_d} (u'_p - u')
\]

\[
\tau_d = \frac{\rho g \sigma^2}{18 \mu}
\]

and \( \delta u'_p \) and \( \delta F'_p \) are defined in Reference 10 (i.e., Equations (8.15) and
Using the referenced equations in Equations (111), the equation describing the behavior of \( \eta_1 \) becomes:

\[
\frac{d^2 \eta_1}{dt^2} + \omega_1^2 \eta_1 = \frac{C_m}{1+C_m} \left[ \frac{d^2 \eta_1}{dt^2} - \frac{1}{\tau_d} \frac{d\eta_1}{dt} + \frac{e^{-t/\tau_d}}{\tau_d^2} \int_0^t e^{t'/\tau_d} \frac{d\eta_1}{dt'} dt' \right]
\]  

(112)

Since the right-hand-side of Equation (112) contains a second derivative term, it is not in the proper form for solution by the MOA. This equation can be simplified and rewritten in the proper form in several ways. The most straightforward procedure is to bring the term \( \frac{C_m}{1+C_m} \frac{d^2 \eta_1}{dt^2} \) to the left-hand-side and simplify to obtain the following expression:

\[
\frac{d^2 \eta_1}{dt^2} + \omega_1^2 \eta_1 = \frac{C_m}{1+C_m} \left\{ \frac{1}{\tau_d} \frac{d\eta_1}{dt} - \omega_1 \eta_1 + \frac{e^{-t/\tau_d}}{\tau_d^2} \int_0^t e^{t'/\tau_d} \frac{d\eta_1}{dt'} dt' \right\}
\]  

(113)

Another way of handling Equation (112) is to use an order of magnitude argument and substitute \( \frac{d^2 \eta_1}{dt^2} = -\omega_1^2 \eta_1 \) on the right-hand-side. Along with applying the MOA, this substitution has been made by Culick in deriving Equations (8.23) and (8.25). With this substitution, Equation (112) becomes:

\[
\frac{d^2 \eta_1}{dt^2} + \omega_1^2 \eta_1 = \frac{C_m}{1+C_m} \left\{ -\frac{1}{\tau_d} \frac{d\eta_1}{dt} - \omega_1 \eta_1 + \frac{e^{-t/\tau_d}}{\tau_d^2} \int_0^t e^{t'/\tau_d} \frac{d\eta_1}{dt'} dt' \right\}
\]  

(114)

It is now apparent that the right-hand-side of Equation (114) differs from the right-hand-side of Equation (113) by the factor \( 1/(1 + C_m) \), thus it
is expected that the solutions of these two equations will differ by a similar amount.

Equations (113) and (114) respectively describe the Galerkin Equations obtained using Culick's formulation without and with the above-mentioned approximations. The MOA can now be applied to solve each of these equations. Letting

$$\eta_1(t) = g_1(t) \sin(\omega_1 t) + h_1(t) \cos(\omega_1 t)$$

where $g_1$ and $h_1$ are slowly-varying functions, and using Equations (32), equations describing the behavior of $g_1$ and $h_1$ can be derived. Thus with Culick's formulation, four sets of results were obtained for a given case: two solutions using the Galerkin method both with (Equation (114)) and without (Equation (113)) the substitution $\frac{d^2 \eta_1}{dt^2} = -\omega_1^2 \eta_1$ in $F_1(p)$ and the two corresponding solutions obtained using the MOA.

The decay rates obtained with Culick's formulation for $\sigma = 2.5 \mu$, $c_m = 0.36$, and $f_e = 800$ Hz are presented in Table 12 along with the decay rates obtained with the present analysis (i.e., using Equations (25) for the Galerkin solutions and Equations (39) for the MOA). In these computations the thermal energy transfer between the particles and the gas phase was ignored. For this case the "exact" calculations using the Kooker-Zinn model yield $\alpha = -57$ sec$^{-1}$.

A comparison of the results presented in Table 12 leads to several observations. In every case, the MOA yields considerably higher decay rates than the Galerkin method. The substitution $\frac{d^2 \eta_1}{dt^2} = -\omega_1^2 \eta_1$ results in a significant decrease in the computed decay rates. The solution presented in Reference 10 (i.e., with MOA and making the substitution $\frac{d^2 \eta_1}{dt^2} = -\omega_1^2 \eta_1$ agrees well with numerical results, but the corresponding solution using the Galerkin method does not agree well. Finally, the Galerkin equations as derived by the present analysis and that of Reference 10 without the substitution $\frac{d^2 \eta_1}{dt^2} = -\omega_1^2 \eta_1$ give good results, but in both cases the MOA leads to incorrect results.

The results reported above raise serious questions regarding the applicability of the MOA over the wide range of solid rocket operating conditions experienced in practice. The example considered above demonstrated that good results could be obtained with the MOA in a relatively
simple situation only after the approximation \( \frac{d^2 \eta_1}{dt^2} / \frac{d^2}{dt^2} = -\omega^2 \eta_1 \) was introduced into the Galerkin equation while erroneous results were obtained when the MOA was applied to the "exact" Galerkin equation. At this point, in view of the absence of contrary evidence, one cannot help the feeling that at least in this example the success of the MOA was fortuitous.

Table 12. Comparison of Approximate Solutions for Particles in a Box

<table>
<thead>
<tr>
<th>Present Approximate Analysis</th>
<th>Galerkin Method</th>
<th>Method of Averaging</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analysis of Reference (10)</td>
<td>-55.7 sec^{-1}</td>
<td>-72.1 sec^{-1}</td>
</tr>
<tr>
<td>Without the Approximation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \frac{d^2 \eta_1}{dt^2} = -\omega \eta_1 )</td>
<td>-53.2 sec^{-1}</td>
<td>-71.9 sec^{-1}</td>
</tr>
</tbody>
</table>

The applicability of the MOA in the analysis of solid rocket motors cannot be judged solely on the basis of the above comparisons for particles in a box. Therefore more extensive comparisons were made for a hypothetical motor in which the propellant is insensitive to the pressure oscillations (i.e., \( \omega = 0 \)). These calculations, therefore, include the effects of mean flow, flow turning, and nozzle damping as well as the effects of particle damping. Four particle diameters were considered, 2\( \mu \), 6\( \mu \), 10\( \mu \), and 20\( \mu \) for the same motor geometry and steady-state properties considered in Section 4.2 (Figure 24). Approximate solutions were calculated using the following three techniques: (1) the Galerkin method applied to Equations (13) and (14) (i.e., numerical solution of Equations (22)
and (23)), (2) the Galerkin method applied to Equation (17) in which gas and particle equations have been combined (i.e., numerical solution of Equations (25)), and (3) the MOA applied to Equations (25) (i.e., numerical solution of Equations (39)). Decay rates and frequencies obtained with the three approximate techniques are compared with the Kooker-Zinn "exact" solutions in Table 13 for 3% initial disturbances.

Several interesting observations can be made from the data presented in Table 13. For the larger particle diameters of 10μ and 20μ (i.e., smaller values of K), all three approximate solutions are in good agreement with the "exact" solutions. On the other hand, the approximate solutions for 2μ and 6μ diameter particles (large K) differ from the "exact" solutions by various amounts. In both cases the Galerkin methods yield decay rates which are smaller than the "exact" decay rates, with Equations (25) yielding the greatest error (it was necessary to neglect the spatial derivatives of the particle velocity in order to combine the gas and particle equations). The MOA, surprisingly, yielded decay rates which were in better agreement with the "exact" decay rates than the Galerkin solutions, in spite of the fact that additional approximations were made in applying the MOA to Equations (25). Apparently the error associated with applying the MOA to Equation (25) partially compensated for the errors associated with the order of magnitude approximations and the Galerkin method used in deriving Equations (25). Since it is impossible to guarantee that this fortuitous circumstance will occur in every situation, the applicability of the MOA to more general solid rocket instability problems is still open to question.

To further investigate this question, a more realistic case was considered in which both combustion driving and particle damping were present. Thus, the three approximate solutions were compared with the "exact" solutions for the case shown in Figures 26 and 27 of Section 4.2 (i.e., σ = 2.5μ, C_m = 0.36, M_e = 0.078, M^R_1 = 4.17). The results of these calculations are shown in Figures 58 and 59. Figure 58 shows growth rate versus number of wave cycles for 3% amplitude 1L mode initial disturbances, while Figure 59 shows the growth toward limiting amplitude for 10% initial disturbances. From Figure 58 it is evident that the Galerkin method (Equations (22) and (23)) underestimates the growth rate and the MOA overestimates the growth rate by about the same amount with respect to the
Table 13. Comparison of Approximate Solutions for Motor with R = 0

<table>
<thead>
<tr>
<th>Particle Size, μ</th>
<th>2.0</th>
<th>6.0</th>
<th>10.0</th>
<th>20.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drag Constant, K</td>
<td>46.72</td>
<td>5.191</td>
<td>1.869</td>
<td>0.4672</td>
</tr>
<tr>
<td>Damping, sec⁻¹</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Galerkin, Eqs. 22 and 23</td>
<td>-282</td>
<td>-502</td>
<td>-582</td>
<td>-408</td>
</tr>
<tr>
<td>Galerkin, Eqs. 25</td>
<td>-260</td>
<td>-488</td>
<td>-582</td>
<td>-407</td>
</tr>
<tr>
<td>M.O.A., Eqs. 39</td>
<td>-312</td>
<td>-538</td>
<td>-579</td>
<td>-408</td>
</tr>
<tr>
<td>Exact</td>
<td>-322</td>
<td>-525</td>
<td>-589</td>
<td>-411</td>
</tr>
<tr>
<td>Frequency, Hz</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Galerkin, Eqs. 22 and 23</td>
<td>975</td>
<td>989</td>
<td>1031</td>
<td>1071</td>
</tr>
<tr>
<td>Galerkin, Eqs. 25</td>
<td>975</td>
<td>987</td>
<td>1029</td>
<td>1071</td>
</tr>
<tr>
<td>M.O.A., Eqs. 39</td>
<td>979</td>
<td>1000</td>
<td>1041</td>
<td>1074</td>
</tr>
<tr>
<td>Exact</td>
<td>976</td>
<td>990</td>
<td>1038</td>
<td>1068</td>
</tr>
</tbody>
</table>
Figure 58. Comparison of Galerkin, MOA, and "Exact" Solutions for Growth Rate of Small Amplitude Disturbances

A = 6.00, B = 0.58, n = 0.575, Ω = 4.24

σ = 2.5 \mu, \ c_m = 0.36, \ \bar{M_e} = 0.0780

3% Initial Amplitude

Method of Averaging

Galerkin - Eqs. 25

"Exact"

Galerkin - Eqs. 22 and 23

Number of Cycles
Figure 59. Comparison of Galerkin, MOA, and "Exact" Solutions for Growth to Limiting Amplitude
"exact" solution, while the Galerkin method (Equations (25)) yields growth rates between the "exact" and MOA values. Figure 59 shows that the MOA also predicts a considerably higher limiting amplitude than either the Galerkin methods or the "exact" analysis. These results show that the fortuitous agreement between the decay rates predicted by the MOA and the "exact" analysis for motors without combustion driving does not occur when combustion driving is present.

In conclusion, the considerable reduction in computation time obtained with the MOA clearly warrants its use in the nonlinear analysis of solid rocket motors without particles. For cases in which particles are present, however, the MOA should be used with caution, particularly for small particles, since significant errors may result.

4.8 Computation Time

Table 14 shows typical computation times obtained with the approximate and "exact" programs under various conditions using the CDC CYBER 70/74 computer. If two values are given for the computation time, the first value refers to the case when particles are absent. These results show that for equal time step sizes the two Galerkin approaches are about equally fast and that the MOA is much slower. The MOA, however, can be used with much larger time step sizes (one or two steps per wave cycle) resulting in much shorter computation times than the Galerkin method. The T-burner program runs about three times faster than the corresponding motor program due to the simpler eigenfunctions used in the T-burner analysis. For equal time step sizes, the approximate programs (excluding the MOA) yield from a two-fold to a ten-fold increase in computational speed over the "exact" programs. For cases where the MOA yields accurate results, computational speeds obtained with the MOA may approach a hundred times that obtained with the "exact" analysis.
Table 14. Comparison of Computation Times

<table>
<thead>
<tr>
<th>Method</th>
<th>Number of Modes</th>
<th>Time Step Size</th>
<th>Computation Time Seconds/wave cycle</th>
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5. CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

The primary objective of this research program was to evaluate the usefulness of approximate nonlinear analysis techniques for predicting the stability characteristics of solid rocket motors and T-burners. This work involved the adaptation of a previous liquid-rocket approximate analysis for the solution of solid-rocket instability problems. This technique, which was based on the solution of an approximate wave equation by means of the Galerkin method, was also further simplified by application of the Method of Averaging (MOA). The usefulness of the approximate techniques (Galerkin and MOA) in the nonlinear stability analysis of solid rocket motors was then investigated for cases in which gasdynamic mode-coupling was the only nonlinearity considered. For this case an extensive parametric study was performed in which the approximate solutions were compared with available "exact" solutions. Later the effects of nonlinearities in the combustion driving and particle damping mechanisms were assessed using both "exact" and approximate models, which required the development of heuristic nonlinear combustion response and particle drag laws for the approximate analysis. Finally, the applicability of the approximate model to the nonlinear analysis of T-burners was determined.

The objective of this program has been reached. The predictions of the approximate techniques have been compared with the "exact" solutions for a wide variety of motor operating conditions. On the basis of the predicted growth/decay rates, frequencies, limiting amplitudes, and pressure waveforms, the Galerkin technique (without averaging) was found to be sufficiently accurate for most of the cases considered. For cases in which particle effects were important, these studies also showed that the MOA was generally less reliable than the Galerkin method. These findings were also supported by comparisons with experimental motor data. In the T-burner studies, it was found that the approximate T-burner analysis failed to accurately predict limiting amplitudes, although growth rates at low amplitudes were in fairly good agreement with measured data.

Additional conclusions reached during this investigation are summarized below in four parts: accuracy of the approximate methods, number of modes required to satisfactorily model the instability, effects of the improvements (i.e., incorporation of nonlinear burning rate and nonlinear particle drag laws into the models), and computation time.
Accuracy of the Approximate Method. The accuracy of the Galerkin method (without averaging) was found to be dependent on the case under consideration. The Galerkin method was in very good agreement with the "exact" solutions for the two simplest cases considered: (1) a motor without particle damping or combustion driving (mean flow, flow turning, and nozzle damping only) and (2) a particle-gas mixture in a closed-ended box. For a motor with particle damping but without combustion driving, the approximate analysis underestimated the damping by as much as 13% for particles smaller than 15 microns. For motors with combustion driving the approximate analysis tended to underestimate both the growth rates at low amplitudes and the limiting amplitudes for cases without particles, while these quantities were generally underestimated for cases with particles. For these cases the approximate analysis accurately predicted the expected trends regarding the effect of particle size and concentration and propellant response upon limiting amplitude. For T-burners the approximate analysis also correctly predicted the effect of various particle properties, propellant response, and cup-grain length, but for given values of these parameters the growth rates and limiting amplitudes did not agree well with the available "exact" solutions.

The accuracy of the MOA was also found to be case dependent. For small to moderate amplitude disturbances in motors without particles the MOA and Galerkin method were equally accurate. For small particles in a box, however, the MOA predicted excessive damping compared to the "exact" analysis and the Galerkin method. Although the MOA gave better agreement with the "exact" analysis than the Galerkin method for a motor with small particles but no combustion driving, the MOA did not yield accurate results for growth rates and limiting amplitudes in the same motor when combustion driving was included.

Comparison of the approximate (Galerkin) solutions with available motor test data gave reasonable agreement with the measured limiting amplitudes considering the difficulties encountered in estimating the transient combustion parameters. The approximate analysis correctly predicted the experimentally observed trend of increasing limiting amplitude with increasing steady-state Mach number. For T-burners the predicted small amplitude growth/decay rates were in good agreement with the measured values for reasonable values of the transient burning rate parameters; however, the predicted limiting amplitudes were far higher than the measured values.
Number of Modes Required. The number of modes required to adequately model the instability depended principally upon the relative stability of the higher frequency modes with respect to the fundamental mode. A three or four mode series appeared to be adequate for cases in which the higher modes were heavily damped, such as cases in which small particles are present. When the higher modes were less strongly damped, six or more modes were necessary (i.e., motors with large particles or without particles). The most unfavorable situations arose when the propellant response for the higher modes was comparable to or greater than the propellant response for the fundamental mode; here a large number of modes were required and the approximate solutions required more computer time than the "exact" solutions.

Effects of the Improvements. Results obtained with the "exact" and approximate models indicate that combustion nonlinearities should be included in the analysis of solid rocket instabilities whenever the oscillation amplitudes exceed 7-10% and sometimes for amplitudes as small as 3%. Depending upon various motor and propellant parameters, particularly frequency, combustion nonlinearities may either decrease or increase combustion driving with increasing amplitude; in the latter case pulsed instabilities may result. Although the heuristic nonlinear combustion model developed in this study accounts for the above effects, it needs to be improved to account for the nonlinear dependence of the combustion driving upon amplitude obtained with the "exact" analysis.

On the basis of the nonlinear particle damping studies, the nonlinear particle drag law should be used for the higher frequencies and larger particle sizes. Depending on the particle size, nonlinear particle drag effects may either increase or decrease particle damping with increasing amplitude; in the latter case pulsed instabilities may result. The nonlinear drag effect is important even for small amplitude disturbances due to the steady-state contribution to the relative velocity between particles and gas. Although the heuristic nonlinear particle drag model used in the approximate analysis predicts the above trends, more parametric studies are needed in order to guide further improvements of this model.

Computation Time. For equal integration time-step sizes, the Galerkin method program with a six-mode series runs about twice as fast as the "exact" program. The six-mode series requires about 20 seconds per wave cycle central processor time on the CDC CYBER 70/74 computer used in these studies. For cases
in which a four mode series is adequate, a further four-fold increase in speed can be obtained. The MOA program runs about ten times faster than the Galerkin program because a much larger time increment can be used. Due to the simpler eigenfunctions used to represent the T-burner solutions, the T-burner program runs about three times faster than the corresponding motor program.

Under the most favorable circumstances the Galerkin method may yield a ten-fold saving in computer time as compared to the "exact" analysis, while the MOA (when applicable) may yield a hundred-fold increase in computational speed over the "exact" analysis. Thus the approximate solution techniques may be used for extensive parametric studies of nonlinear solid rocket and T-burner stability behavior which would be impractical with the "exact" analytical techniques.

5.2 Recommendations

A number of investigations aimed at improving and generalizing the approximate models are recommended for future research. Some of these can be readily accomplished with only a modest expenditure of time and effort, while others will require a major research effort for their completion.

One of the basic assumptions used in the development of the present approximate and "exact" analyses was that the exchange of thermal energy between the particles and gas is negligible. Since it is known that particle thermal effects may contribute significantly to the total particle damping, they should be incorporated into the approximate and "exact" models.

The present approximate program for motors is restricted to full-length, cylindrical propellant grains. The approximate analysis should be extended to more general motor geometries such as partial length grains, gaps in grain, end-burning grains, variable port area, and arbitrary grain cross-sections. Two of these features, partial length grains and end-burning grains, have already been incorporated into the T-burner model and could easily be included in the analysis of motors.

More extensive parametric studies are needed to adequately assess the effects of nonlinear combustion driving and nonlinear particle damping upon the stability characteristics of motors and T-burners. These studies should be conducted using both the "exact" model and the approximate model. The results of these studies should then be used as a guide toward improvements of the heuristic nonlinear combustion response and particle drag laws.
In view of the limited number of comparisons with experimental data obtained during this research effort, more extensive comparisons between the approximate solutions and motor test data are badly needed. This will only be possible as more motor test data becomes available.

More studies are needed to determine the cause of the poor agreement between T-burner limiting amplitudes predicted by the approximate model and the values obtained by the "exact" analysis or from T-burner test data. This will probably involve the development of a nonlinear vent model and the inclusion of wall heat transfer effects.

It is well known that the combustion response of solid propellants depends upon both the pressure and velocity oscillations. While considerable work has been done to date on the investigation of the pressure-coupled response, relatively little work has been done on the elucidation of the velocity-coupled response. It is therefore recommended that the available information on the velocity-coupled response be used to develop analytical expressions capable of describing the velocity-coupled response of solid propellants, and that these expressions should be incorporated into the approximate and "exact" nonlinear instability models. Comparisons should then be made between the predictions of the available models, with and without the velocity-coupled response. Such comparisons should be useful in determining the role of velocity-coupled response in the behavior of unstable rockets, particularly regarding the development of pulsed instabilities.

With the completion of the above recommended improvements, the value of the approximate analysis as a useful engineering tool will be greatly enhanced.
6. REFERENCES


APPENDIX A

DERIVATION OF APPROXIMATE EQUATIONS

A-1. Derivation of Equations (13) and (14).

Under the assumption stated in Section 2, the system of nondimensional conservation equations that describe the unsteady behavior of the combustor two-phase flow are given by Equations (1) through (6). The unsteady flow in the T-burner vent region can also be described by these equations if the axial velocities in the source terms arising from the mean-flow/acoustic interactions are multiplied by the factor 1 - Vt (Section 2.6). Thus the gas and particle momentum equations become:

\[
\rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \cdot \frac{\partial \mathbf{u}}{\partial x} + \frac{1}{\gamma} \frac{\partial p}{\partial x} = - \left[ K_p (u-u_p) + \frac{2\dot{m}}{R} (1-V_t)u \right] \quad (A-1)
\]

\[
\rho_p \frac{\partial \mathbf{u}_p}{\partial t} + \rho_p \mathbf{u}_p \cdot \frac{\partial \mathbf{u}_p}{\partial x} = k_p (u-u_p) - \frac{2\dot{m}}{R} (1-V_t)u_p \quad (A-2)
\]

Writing the energy equation in terms of the enthalpy, Equation (5) yields:

\[
\rho \frac{\partial h}{\partial t} + (\gamma-1) \frac{\partial}{\partial t} \left[ \frac{\rho u^2}{2} + \frac{\rho_p u_p^2}{2} \right] - \frac{\gamma-1}{\gamma} \frac{\partial p}{\partial x} + \rho u \frac{\partial h}{\partial x} = \frac{2\dot{m}}{R} (h_c - h) \quad (A-3)
\]

Using order of magnitude approximations, Equations (1), (2), (A-1), (A-2), and (A-3) will now be combined to obtain approximate equations governing the gas and particle velocity potentials.

Each of the dependent variables is first expressed as the sum of a steady-state quantity and a perturbation quantity, thus

\[
\rho = \rho + \rho' \\
\rho_p = \rho_p + \rho_p'
\]

165
\( u = \bar{u} + u' \)

\( u = \bar{u}_p + u_p' \)

\( p = \bar{p} + p' \)

\( h = \bar{h} + h' \)

(A-4)

where \( (\cdot) \) denotes a steady-state quantity and \((\cdot)\)' denotes a perturbation quantity. Equations (A-4) are then substituted into the conservation equations and the equation of state. Subtracting out the steady-state equations (obtained by setting the time derivatives equal to zero in the conservation equations) yields a system of equations describing the perturbation quantities. These equations are too complicated to be solved by the Galerkin method and must be further simplified by order of magnitude approximations.

It is now assumed that each perturbation quantity and the steady-state Mach number are of \( O(\varepsilon) \) where \( \varepsilon \) is a small ordering parameter that is a measure of the wave amplitude. Thus the perturbation equations are simplified by neglecting all terms of order higher than \( \varepsilon^2 \), which includes products of three perturbations, products of the mean flow Mach number (i.e., \( \bar{u} \)) and two perturbations, products of \( \bar{u}^2 \) and a perturbation quantity, and terms proportional to \( \bar{u}^3 \). As a result of the small Mach number assumption, the steady-state pressure, density, and enthalpy are nearly constant throughout the chamber, thus \( \bar{\rho} = 1, \bar{p} = 1, \) and \( \bar{h} = 1 \) (the error introduced by this approximation is \( O(\varepsilon^3) \)). Furthermore the steady-state continuity equations for the gas and particles show that the steady-state source terms \( m_g \) and \( m_p \) are also of \( O(\varepsilon) \), while the linear response function (i.e., Equation (9)) indicates that for \( R \) of \( O(1) \) the unsteady source terms \( \dot{m}_g' \) and \( \dot{m}_p' \) are of \( O(\varepsilon^2) \). Introducing these approximations into Equations (1), (2), (A-1), (A-2), and (A-3) give:

\[
\frac{\partial p'}{\partial t} + \frac{\partial}{\partial x} \left( \rho' \bar{u} + u' + \rho' u' \right) = \frac{2\dot{m}'_g}{R} \tag{A-5}
\]

\[
\frac{\partial p'}{\partial t} + \frac{\partial}{\partial x} \left( \rho_p' \bar{u} + \rho_p' u' + \rho_p' u' \right) = \frac{2\dot{m}'_p}{R} \tag{A-6}
\]
\[
\frac{\partial u'}{\partial t} + p' \frac{\partial u'}{\partial t} + u \frac{\partial u'}{\partial x} + u' \frac{\partial u'}{\partial x} + u' \frac{\partial u'}{\partial x} + \frac{1}{\gamma} \frac{\partial p'}{\partial x} = \\
- \left\{ K p (u'-u'_p) + K p (u'-u'_p) + K p (u'-u'_p) \right\} \\
- \frac{2}{R} \tilde{m}_p (1-V_p) u' \tag{A-7}
\]

\[
\tilde{p} \frac{\partial u'}{\partial t} + p \frac{\partial u'}{\partial t} + u \frac{\partial u'}{\partial x} + p \frac{\partial u'}{\partial x} + u \frac{\partial u'}{\partial x} + \frac{1}{\gamma} \frac{\partial p}{\partial x} = \\
+ K p (u'-u'_p) + K p (u'-u'_p) + K p (u'-u'_p) \\
- \frac{2}{R} \tilde{m}_p (1-V_p) u'_p \tag{A-8}
\]

\[
\frac{\partial h'}{\partial t} + p' \frac{\partial h'}{\partial t} + (\gamma-1) \frac{\partial}{\partial t} \left[ u u' + \frac{1}{2} (u')^2 \right] + (\gamma-1) \frac{\partial}{\partial t} \left[ u u' + \frac{1}{2} \right] \\
- \frac{\gamma-1}{\gamma} \frac{\partial p}{\partial t} + u \frac{\partial h'}{\partial x} + u' \frac{\partial h'}{\partial x} = \frac{2}{R} \left[ - \tilde{m}_p h' + \tilde{m}_p (h_p - 1) \right] \tag{A-9}
\]

while the equation of state becomes:

\[
p' = h' + p' + \rho' h' \tag{A-10}
\]

In order to further simplify these equations, the following velocity potentials are defined:

\[
u' = \frac{\partial \tilde{\phi}}{\partial x}
\]

\[
u'_p = \frac{\partial \tilde{\phi}_p}{\partial x} \tag{A-11}
\]

Further order of magnitude considerations are also used to simplify the particle drag terms in Equations (A-7) and (A-8). Here it is assumed that the particle loading of the propellant is sufficiently low that \( \tilde{p} \) is \( O(\epsilon) \) which implies that
\( \rho_p' \) is \( O(\varepsilon^2) \). Since the relative velocities \( \bar{u} - \bar{u}_p' \) and \( u' - u_p' \) are of \( O(\varepsilon) \) the second and third particle drag terms on the right-hand-sides of Equations (A-7) and (A-8) are of \( O(\varepsilon^3) \) and can be neglected. Introducing these approximations and the velocity potentials into Equations (A-5) through (A-9) and using subscript notation for the partial derivatives gives the following second order conservation equations:

\[
\begin{align*}
\rho' & \frac{dx}{dt} + \bar{u}' \frac{dx}{dt} + \rho_p' \frac{d\bar{u}}{dx} + \bar{\phi}' \frac{d\bar{u}}{dx} + \rho_p' \frac{d\bar{\phi}}{dx} + \rho_p' \frac{d\bar{\phi}}{dx} = \frac{2 \cdot \bar{m}'}{R} \\
(\rho_p') & \frac{dx}{dt} + \bar{u}_p(\rho_p') \frac{dx}{dt} + \rho_p' \frac{d\bar{u}}{dx} + (\rho_p'(\bar{\phi}_p) \frac{dx}{dt} + \rho_p' \frac{d\bar{\phi}_p}{dx} + (\rho_p)' (\bar{\phi}_p)' = \\
\bar{\phi}_x + \rho_p' \bar{\phi}_x + \bar{u}_p \bar{\phi}_x + \frac{d\bar{u}}{dx} \bar{\phi}_x + \frac{d\bar{u}}{dx} \bar{\phi}_x + \frac{1}{\gamma} \bar{p}_x' &= \\
- \bar{K}_p' \left[ \bar{\phi}_x - (\bar{\phi}_p)' \right] - \frac{2 \bar{m}}{R} \bar{g} (1 - V_c) \bar{\phi}_x
\end{align*}
\]

(A-13)

(A-14)

(A-15)

(A-16)

In order to combine Equations (A-12) through (A-16) to obtain Equations (13) and (14), the substitution principle used by Lighthill\textsuperscript{13} and Blackstock\textsuperscript{14} is invoked. This principle states that any factor of a second order term may be replaced by its equivalent first order expression, because any more precise
substitution would result in the appearance of higher order terms. The substitution principle is used here to replace $\rho'$ and $h'$ in the second order terms with equivalent first order expressions involving $\phi$. The first order expressions which are needed in this analysis are as follows:

$$
\rho' = -\phi_t
$$

$$
p' = -\gamma \phi_t
$$

$$
h' = -(\gamma-1) \phi_t
$$

which are derived in Reference (23). In addition the steady-state mass source $2\bar{m}_g/R$ may be replaced by the equivalent quantity $\partial u/\partial x$ as shown by the steady-state continuity equation. Making these substitutions into Equations (A-12), (A-14), (A-16), and (A-10) and rearranging terms yields the following expressions:

$$
\rho_t + \phi_{xx} = u \phi_{xt} + \frac{\partial u}{\partial x} \phi_t + \phi_t \phi_{xx} + \phi_x \phi_{xt} + \frac{2m'}{R}
$$

$$
\left[ \frac{p'}{\rho} + \frac{\partial}{\partial x} \left( \frac{\phi}{\phi_t} \right) \right] + \left( 1 - \frac{\gamma}{\gamma-1} \right) \frac{\partial u}{\partial x} = 0
$$

$$
h_t + (\gamma-1) \phi_t \phi_{tt} + (\gamma-1) \left[ \frac{\partial}{\partial x} \left( \phi_{p} \phi_{p} \right) + \frac{1}{2} \frac{\partial}{\partial x} \left( \phi_{p} \phi_{p} \right) \right]_t
$$

$$
- \frac{\gamma-1}{\gamma} p' = (\gamma-1) \frac{\partial u}{\partial x} \phi_t + \frac{2m'}{R} (h_{c-1})
$$

$$
h' = p' - \rho' - (\gamma-1) \phi_t^2
$$

---

Equations (A-18) through (A-21) will now be combined to obtain a non-linear wave equation for the gas velocity potential. Substituting Equation (A-21) into (A-20) and collecting terms, the energy equation becomes:

\[
\frac{1}{\gamma} \rho_t' - \rho_t' - (\gamma - 1) \phi_t \phi_{tt} - (\gamma - 1) \frac{\partial \phi}{\partial x} \phi_t = \frac{2m'}{R} (h - 1) - (\gamma - 1) \left[ \frac{\rho_t \phi_t' \phi_p}{\rho_p} + \frac{1}{2} \frac{\partial \rho}{\partial x} \phi_p^2 \right]_t
\]

(A-22)

The gas momentum equation (i.e., Equation (A-19)) is integrated with respect to \(x\) (the constant of integration must be zero according to the argument given in Reference (22)) to give:

\[
p' = -\gamma \left[ \phi_t + \bar{u} \phi_x + \frac{1}{2} \phi_x^2 - \frac{1}{2} \phi_{xx}^2 + K_{p,phi} (\phi - \phi_p) + (1 - V_t) \frac{\partial \phi}{\partial x} \phi_t \right]
\]

(A-23)

which is also Equation (63) of Section 2.6. Setting \(V_t = 0\) gives the corresponding equation for motors; that is, Equation (24) of Section 2.3. To proceed with the derivation of Equation (13), Equation (A-23) is differentiated with respect to \(t\) and the resulting expression is substituted for \((1/\gamma) \rho_t'\) in Equation (A-22). Furthermore the continuity equation (i.e. Equation (A-18)) is substituted for \(\rho_t'\) in Equation (A-22). Combining terms, neglecting the nonlinear particle term in Equation (A-22), and using the first order relation, \(\phi_{tt} = \phi_{xx}\), yields

\[
\phi_{xx} - \phi_{tt} = 2\bar{u} \phi_{xt} + (\gamma + 1 - V_t) \frac{\partial \phi}{\partial x} \phi_t
\]

\[
+ 2\phi_x \phi_{xt} + (\gamma - 1) \phi_t \phi_{xx}
\]

\[
+ K_p (\phi_t - \phi_{\rho}) - (\gamma - 1) \rho_t \frac{\partial \phi}{\partial x} \phi_{xt} + \frac{2m'}{R} h_c
\]

(A-24)

If \(2m'/R\) is replaced by \(w'\), Equation (A-24) becomes Equation (61) of Section 2.6 which describes the unsteady flow in the T-burner vent region. For motors the source term in Equation (A-24) is related to the pressure perturbation through the linear response function (i.e., Equation (9)). Substituting Equation (9) into the source term and using \(2m'/R = \partial \bar{u}/\partial x\) and the first order re-
Substituting Equation (A-25) into Equation (A-24) and setting \( V_L = 0 \) yields Equation (13) of Section 2.2.

The particle potential equation (i.e., Equation (14)) is readily derived from the particle momentum equation (i.e., Equation (A-15)). Using the steady-state particle continuity equation to replace \( \frac{2m}{R} \) with \( \frac{\bar{p}}{p} \frac{du}{dx} \) in Equation (A-15), dividing by \( \bar{p} \), and rearranging terms gives:

\[
\frac{\partial}{\partial x} \left[ (\bar{\phi}_p) t + \bar{u}_p (\bar{\phi}_p) x + \frac{1}{2} (\bar{\phi}_p)^2 - K (\bar{\phi} - \bar{\phi}_p) + (1-V_L) \frac{du_p}{dx} \bar{p} \right] = 0
\]

where \( \bar{u}_p / dx \) is assumed to be constant (the resulting error is of \( O(\epsilon^3) \)). As in the case of the gas momentum equation (i.e., Equation (A-19)) Equation (A-26) can be integrated with respect to \( x \) (the same argument holds regarding the constant of integration). Performing this integration and rearranging terms gives the desired particle potential equation:

\[
(\bar{\phi}_p) t + \bar{u}_p (\bar{\phi}_p) x + \frac{1}{2} (\bar{\phi}_p)^2 = K (\bar{\phi} - \bar{\phi}_p) - (1-V_L) \frac{du_p}{dx} \bar{p}
\]

which is the same as Equation (62) of Section 2.6. Setting \( V_L = 0 \) gives Equation (14) of Section 2.2.


The basic assumption used in deriving Equation (17) is that the terms involving the spatial derivatives of the particle velocity (both steady-state and perturbation) are negligible compared to the remaining terms in the particle momentum equation. Neglecting these terms as well as those involving \( \rho' \) (since \( \rho' \) is \( O(\epsilon^2) \)) in Equation (A-8) yields:

\[
\frac{du_p}{dt} + Ku' = Ku'
\]

Using the integrating factor \( e^{Kt} \) and the initial condition \( u_p'(0) = u_p' \), the solution to Equation (A-28) is

\[
u_p' = Ke^{-Kt} \int_0^t e^{Kt} u_p' \, dt' + u_p' e^{-Kt}
\]
Since the second term decays rapidly as time increases, it can be neglected to give:

$$u'(t) = K e^{-Kt} \int_0^t u' e^{K't'} dt'$$  \hspace{1cm} (A-30)

Introducing the velocity potentials given by Equations (A-11), Equation (A-30) becomes:

$$\frac{\partial \phi_p}{\partial x} = K e^{-Kt} \int_0^t \frac{\partial \phi}{\partial x} e^{K't'} dt'$$  \hspace{1cm} (A-31)

To derive Equation (17) the analytical solution for $\phi_p$ given by Equation (A-31) is substituted into the second order gas momentum equation (i.e., Equation (A-19)) with $V = 0$ to obtain:

$$\frac{\partial}{\partial x} \left[ \frac{p'}{\gamma} + \phi_t + \bar{u}_x \phi + \frac{1}{2} \phi^2_t - \frac{1}{2} \phi_x^2 + K_p \phi + \frac{du}{dx} \phi \right] - K_{p_p} \left[ K e^{-Kt} \int_0^t \phi e^{K't'} dt' \right] = 0$$  \hspace{1cm} (A-32)

while interchanging the order of differentiation and integration in the last term gives:

$$\frac{\partial}{\partial x} \left[ \frac{p'}{\gamma} + \phi_t + \bar{u}_x \phi + \frac{1}{2} \phi_t^2 - \frac{1}{2} \phi_x^2 + \frac{du}{dx} \phi + K_p \phi \right] - K_{p_p} e^{-Kt} \int_0^t \phi e^{K't'} dt' = 0$$  \hspace{1cm} (A-33)

Integrating with respect to $x$ and rearranging terms gives:

$$p' = -\gamma \left[ \phi_t + \bar{u}_x \phi + \frac{1}{2} \phi_t^2 - \frac{1}{2} \phi_x^2 + \frac{du}{dx} \phi + K_p \phi \right] - K_{p_p} e^{-Kt} \int_0^t \phi e^{K't'} dt'$$  \hspace{1cm} (A-34)

which is the same as Equation (28) in Section 2.3.

Next, Equation (A-31) is substituted into the energy equation (i.e., Equation (A-22)) and the nonlinear particle term is neglected to obtain:
\[
\frac{1}{\gamma} p'_t - \rho'_t - (\gamma - 1) \phi'_t \phi_{tt} - (\gamma - 1) \frac{d\phi}{dx} \phi_t = \\
\frac{2m'}{R} \frac{g}{(h_c - 1)} - (\gamma - 1) \frac{d}{dt} \left[ \rho_p \bar{u}_p Ke^{-Kt} \int_0^t \phi_x e^{Kt'} dt' \right]
\]

(A-35)

As in the previous derivation for Equation (A-24), the momentum equation (i.e., Equation (A-34)) is differentiated with respect to time, which yields:

\[
\frac{1}{\gamma} p'_t = - \left\{ \phi_{tt} + \bar{u}_p \phi_{xt} + \phi_x \phi_{xt} - \phi_t \phi_{tt} + \frac{d\phi}{dx} \phi_t + K_p \phi \right\}
\]

(A-36)

Substituting Equation (A-36) for \((1/\gamma)p'_t\) and Equation (A-18) for \(\rho'_t\) in Equation (A-35), combining terms, and using the first order relation, \(\phi_{tt} = \phi_{xx}\), gives the desired equation:

\[
\phi_{xx} - \phi_{tt} = 2\bar{u}_p \phi_{xt} + (\gamma + 1) \frac{d\phi}{dx} \phi_t + 2\phi_x \phi_{xt} + (\gamma - 1) \phi_t \phi_{xx}
\]

+ \(K_p \phi_t\) - (\gamma - 1) \(K_p \bar{u}_p \phi_x - K_p^2 \phi\)

+ \(K^3 \bar{u}_p e^{-Kt} \int_0^t \phi_x e^{Kt'} dt'\)

+ \((\gamma - 1) K_p^2 \bar{u}_p e^{-Kt} \int_0^t \phi_x e^{Kt'} dt'\)

+ \(\frac{2m'}{R} \frac{g}{h_c}\)

(A-37)

Finally replacing \(2m'/R\) with Equation (A-25) yields Equation (17) of Section 2.2.
APPENDIX B

USE OF COMPLEX VARIABLES IN THE SOLUTION OF NONLINEAR DIFFERENTIAL EQUATIONS

It is often convenient to use complex variables in the solution of the linear equations which arise in acoustics, combustion instability and related fields. In this case the solution is expressed in complex form, and the real part represents the physically meaningful solution. However, care must be used when applying this technique in the solution of nonlinear equations. The difficulties that are encountered in applying the complex variable technique to nonlinear problems will be illustrated by analyzing the following simplified example. Consider the nonlinear wave equation given by:

\[ \ddot{\phi}_{xx} - \ddot{\phi}_{tt} = \phi_t \]  \hspace{1cm} (B-1)

A complex solution of Equation (B-1) of the form \( \phi = \varphi + i\psi \) would be useful only if its real part, \( \varphi \), satisfies Equation (B-1), which would be the case if the equation were linear. However, straightforward substitution of \( \phi = \varphi + i\psi \) into Equation (B-1) and separating its real and imaginary parts yields the following equation for \( \varphi \):

\[ \varphi_{xx} - \varphi_{tt} = \psi_t \]  \hspace{1cm} (B-2)

indicating that the real part, \( \varphi \), does not satisfy Equation (B-1) because of the extra term, \( -\psi_t \), appearing on the right hand side. In order to eliminate this extra term, the form of the original differential equation (i.e., Equation (B-1) must be modified.

Since Equation (B-1) supposedly describes some physical phenomenon, and since only the real part of the complex solution is physically meaningful, then the nonlinear term \( \phi_t \) should really be expressed as the product Re(\( \ddot{\phi} \)) x Re(\( \ddot{\phi}_t \)) which is equivalent to \( (\ddot{\phi}_t + \ddot{\phi}_t^* + \dot{\phi}_t^* + \dot{\phi}_t^* )/4 \). Substituting this expression into Equation (B-1) yields:

\[ \ddot{\phi}_{xx} - \ddot{\phi}_{tt} = [(\ddot{\phi}_t + \ddot{\phi}_t^* + \dot{\phi}_t^* + \dot{\phi}_t^*)]/4 \]  \hspace{1cm} (B-3)

174
Substituting $\psi = \varphi + i\psi$ into Equation (B-3) and separating its real and imaginary parts yield:

$$\varphi_{xx} - \varphi_{tt} = \varphi\psi_t$$

$$\psi_{xx} - \psi_{tt} = 0$$ \hspace{1cm} (B-4)

which shows that the real part of the solution of Equation (B-3) satisfies the desired equation (i.e., Equation (B-1)) and the imaginary part satisfies a homogeneous linear wave equation. This technique was applied to the solution of nonlinear combustion instability problems (i.e., to Equation (13)), and the resulting modified wave equation was solved using the Galerkin Method. Due to the approximate nature of the Galerkin Method, however, the resulting solution contained an error term which grew without limit. Consequently, the above procedure had to be modified in order to obtain satisfactory solutions of Equation (13) using the Galerkin Method.

An alternate technique is to modify Equation (B-1) such that both the real and imaginary parts satisfy the original equation. This can be done by replacing terms of the form $\varphi\psi_t$ with $\text{Re}(\psi)\text{Re}(\psi_t) + i\text{Im}(\psi)\text{Im}(\psi_t)$; using the relations:

$$\text{Re}(\psi)\text{Re}(\psi_t) = \left( \frac{\psi + \psi^*}{2} \right) \left( \frac{\psi_t + \psi_t^*}{2} \right) = \left[ \frac{\psi\psi_t}{4} + \frac{\psi^*\psi_t^*}{4} + \frac{\psi^*\psi_t}{4} + \frac{\psi\psi_t^*}{4} \right]$$ \hspace{1cm} (B-5)

$$i\text{Im}(\psi)\text{Im}(\psi_t) = -i \left( \frac{\psi - \psi^*}{2} \right) \left( \frac{\psi_t - \psi_t^*}{2} \right) = -\left[ \frac{\psi\psi_t}{4} - \frac{\psi^*\psi_t^*}{4} - \frac{\psi^*\psi_t}{4} + \frac{\psi\psi_t^*}{4} \right]$$

in Equation (B-1) gives:

$$\xi_{xx} - \xi_{tt} = \left[ (1 - i)(\psi\psi_t + \psi^*\psi_t^*) + (1 + i)(\psi^*\psi_t + \psi\psi_t^*) \right] /4 \hspace{1cm} (B-6)$$

Substituting $\xi = \varphi + i\psi$ into Equation (B-6) and separating into its real and imaginary parts gives:

$$\varphi_{xx} - \varphi_{tt} = \varphi\psi_t$$

$$\psi_{xx} - \psi_{tt} = \psi\psi_t$$ \hspace{1cm} (B-7)
which shows that both $\varphi$ and $\Upsilon$ satisfy Equation (B-1). Applying this method to the solution of Equation (13) yields the following modified wave equation:

$$
\frac{\partial^2 \phi}{\partial x^2} - \frac{\partial^2 \phi}{\partial t^2} = 2\tilde{u} \frac{\partial \phi}{\partial x} + (\gamma+1) \frac{\partial \tilde{u}}{\partial x} \frac{\partial \phi}{\partial t} - \gamma \tilde{u} \frac{\partial \phi}{\partial x} \frac{\partial \phi}{\partial t} \\
+ K_p \left[ \frac{\partial \phi}{\partial t} - (\phi_p)' \right] - (\gamma-1) \tilde{u} \frac{\partial \phi_p}{\partial x} (\phi_p)_{xt} \\
+ \frac{1-i}{2} \left[ \frac{\partial \phi}{\partial x} \frac{\partial \phi}{\partial t} + \phi^* \phi^* \right] + \frac{1+i}{2} \left[ \phi^* \frac{\partial \phi}{\partial x} + \phi \frac{\partial \phi^*}{\partial x} \right] \\
+ \frac{\gamma-1}{4} \left\{ (1-i) \left[ \frac{\partial \phi}{\partial x} \frac{\partial \phi}{\partial x} + \phi^* \phi^* \right] + (1+i) \left[ \phi^* \frac{\partial \phi}{\partial x} + \phi \frac{\partial \phi^*}{\partial x} \right] \right\} 
$$

(B-8)

Application of the Galerkin Method to Equation (B-8) yields Equations (22) of Section 2.3.
APPENDIX C

COEFFICIENTS APPEARING IN THE APPROXIMATE MODE AMPLITUDE EQUATIONS

C-1. Coefficients Appearing in Equations (22) and (23).

The coefficients of the linear terms in Equations (22) are:

\[ C_0(j,m) = \int_0^1 X_m X_j^* \, dx \quad \text{for } 1 \leq m \leq N \]  
(C-1)

\[ C_0(j,m) = 0 \quad \text{for } N+1 \leq m \leq 2N \]

\[ C_1(j,m) = -\int_0^1 \frac{d^2X}{dx^2} X_j^* \, dx + \frac{dX}{dx} (1)X_j^*(1) \quad \text{for } 1 \leq m \leq N \]  
(C-2)

\[ C_1(j,m) = 0 \quad \text{for } N+1 \leq m \leq 2N \]

\[ C_2(j,m) = 2\int_0^1 u(x) \frac{dX}{dx} X_j^* \, dx \]

\[ + (\gamma+1) \int_0^1 \frac{dx}{dx} X_m X_j^* \, dx \]

\[ + \gamma Y X_m(1) X_j^*(1) \quad \text{for } 1 \leq m \leq N \]  
(C-3)

\[ C_2(j,m) = -((\gamma-1)\rho\int_0^1 u(x) \frac{dX}{dx} X_j^* \, dx \quad \text{for } N+1 \leq m \leq 2N \]

\[ C_3(j,m) = \rho \int_0^1 X_m X_j^* \, dx \quad \text{for } 1 \leq m \leq N \]  
(C-4)

\[ C_3(j,m) = -\rho \int_0^1 X_m X_j^* \, dx \quad \text{for } N+1 \leq m \leq 2N \]
The coefficients of the linear terms in Equations (23) are:

\[ C_4(j,m) = \begin{cases} \int_0^1 \frac{du}{dx} X_m^* X_j^* \, dx & \text{for } 1 \leq m \leq N \\ 0 & \text{for } N+1 \leq m \leq 2N \end{cases} \]  \hfill (C-5)

The coefficients of the nonlinear terms in Equations (22) are given below for \( 1 \leq j \leq N, 1 \leq m \leq N, \) and \( 1 \leq n \leq N. \) For all other values of \( j, m, \) and \( n, \) \( D_1 = D_2 = D_3 = D_4 = 0. \)

\[ D_1(j,m,n) = \frac{1-i}{2} \left( \int_0^1 \frac{dx_m}{dx} \frac{dx_n}{dx} X^*_j \, dx + \frac{(\gamma-1)(1-i)}{4} \int_0^1 \frac{dx_m^2}{dx^2} X_n^* X_j^* \, dx \right) \]  \hfill (C-8)

\[ D_2(j,m,n) = \frac{1+i}{2} \left( \int_0^1 \frac{dx_m}{dx} \frac{dx_n}{dx} X_j^* \, dx + \frac{(\gamma-1)(1+i)}{4} \int_0^1 \frac{dx_m^2}{dx^2} X_n^* X_j^* \, dx \right) \]  \hfill (C-9)
\[ D_3(j,m,n) = \frac{1+i}{2} \int_0^1 \frac{dX_j^*}{dx} \frac{dX_n}{dx} X_j^* \, dx \]

\[ + \frac{(\gamma-1)(1+i)}{4} \int_0^1 \frac{d^2X_j^*}{dx^2} X_n X_j^* \, dx \]  

\[ (C-10) \]

\[ D_4(j,m,n) = \frac{1-i}{2} \int_0^1 \frac{dX_j^*}{dx} \frac{dX_n^*}{dx} X_j^* \, dx \]

\[ + \frac{(\gamma-1)(1-i)}{4} \int_0^1 \frac{d^2X_j^*}{dx^2} X_n^* X_j^* \, dx \]  

\[ (C-11) \]

In the integrals appearing in the coefficients given above, the eigenfunctions \( X_j(x) \) are defined by Equation (20) and \( X_j^*(x) \) denotes their complex conjugates.

C-2. Coefficients Appearing in Equations (25).

The linear coefficients in Equations (25) are given below for \( 1 \leq j \leq N \) and \( 1 \leq m \leq N \). The nonlinear coefficients in Equations (25) are identical to those in Equations (22) and are given by Equations (C-8) through (C-11).

\[ C_0(j,m) = \int_0^1 X_m X_j^* \, dx \]  

\[ (C-12) \]

\[ C_1(j,m) = -\int_0^1 \frac{d^2X_j^*}{dx^2} X_j^* \, dx - \int_0^1 X_m X_j^* \, dx \]

\[ - K(\gamma-1)\tilde{\beta}_p \int_0^1 u_p(x) \frac{dX_m}{dx} X_j^* \, dx + \frac{dX_m}{dx} (1)X_j^*(1) \]  

\[ (C-13) \]

\[ C_2(j,m) = 2\int_0^1 \tilde{u}(x) \frac{dX_m}{dx} X_j^* \, dx + (\gamma+1) \int_0^1 \frac{d\tilde{u}}{dx} X_m X_j^* \, dx \] 

\[ + K\tilde{\beta}_p \int_0^1 X_m X_j^* \, dx + \gamma YX_m (1)X_j^*(1) \]  

\[ (C-14) \]

Separation of Real and Imaginary Parts. Equations (39) were derived by applying the Method of Averaging (MOA) to Equations (25). This procedure requires that Equations (25) be separated into their real and imaginary components. Assuming that $A_j(t) = F_j(t) + iG_j(t)$, substituting into Equations (25), and separating real and imaginary parts yields:

$$
2N \sum_{m=1}^{2N} \left[ C'_0(j,m) \frac{d^2 B_m}{dt^2} + C'_1(j,m) B_m + \left[ C'_2(j,m) + h e^{\mu m E'_1(j,m)} + h e^{\mu m E'_2(j,m)} \right] \frac{dB_m}{dt} \right] + C'_3(j,m) e^{-Kt} \int_0^t B_m(t') e^{Kt'} dt' + \sum_{m=1}^{2N} \sum_{n=1}^{2N} \left[ D'(j,m,n) B_m \frac{dB_n}{dt} \right] = 0
$$

$$
\quad j = 1, 2, 3 \ldots 2N \quad (C-17)
$$

where the $B_m$'s are related to the $F_m$'s and $G_m$'s by Equations (33) of Section 2.4.

The real coefficients $C'_0$, $C'_1$, $C'_2$, $C'_3$, $E'_1$, $E'_2$ and $D'$ in Equation (C-17) are related to the original complex coefficients (i.e., $C_0$, $C_1$, $C_2$, $C_3$, $D_1$, $D_2$) appearing in Equations (25) as follows:

$$
C'_k(2j-1, 2m-1) = \text{Re} \left[ C_k(j,m) \right]
$$

$$
C'_k(2j-1, 2m) = -\text{Im} \left[ C_k(j,m) \right]
$$

$$
C'_k(2j, 2m-1) = \text{Im} \left[ C_k(j,m) \right]
$$

$$
C'_k(2j, 2m) = \text{Re} \left[ C_k(j,m) \right]
$$

(C-18)
for $k = 0,1,2,3$, $j = 1,2,\ldots N$, $m = 1,2,\ldots N$,

\[ E_1(2j-1, 2m-1) = \text{Re} \left[ C_4(j,m) \right] \]
\[ E_1(2j-1, 2m) = -\text{Im} \left[ C_4(j,m) \right] \]
\[ E_1(2j, 2m-1) = \text{Im} \left[ C_4(j,m) \right] \]
\[ E_1(2j, 2m) = \text{Re} \left[ C_4(j,m) \right] \]
\[ E_2(2j-1, 2m-1) = -\text{Im} \left[ C_4(j,m) \right] \]
\[ E_2(2j-1, 2m) = -\text{Re} \left[ C_4(j,m) \right] \]
\[ E_2(2j, 2m-1) = \text{Re} \left[ C_4(j,m) \right] \]
\[ E_2(2j, 2m) = -\text{Im} \left[ C_4(j,m) \right] \] (C-19)

for $j = 1,2,\ldots N$, $m = 1,2,\ldots N$, and:

\[ D'(2j-1, 2m-1, 2n-1) = \text{Re} \left[ D_1(j,m,n) + D_2(j,m,n) + D_3(j,m,n) + D_4(j,m,n) \right] \]
\[ D'(2j-1, 2m-1, 2n) = \text{Im} \left[ -D_1(j,m,n) + D_2(j,m,n) - D_3(j,m,n) + D_4(j,m,n) \right] \]
\[ D'(2j-1, 2m, 2n-1) = \text{Im} \left[ -D_1(j,m,n) - D_2(j,m,n) + D_3(j,m,n) + D_4(j,m,n) \right] \]
\[ D'(2j-1, 2m, 2n) = \text{Re} \left[ -D_1(j,m,n) - D_2(j,m,n) + D_3(j,m,n) - D_4(j,m,n) \right] \]
\[ D'(2j, 2m-1, 2n-1) = \text{Im} \left[ D_1(j,m,n) + D_2(j,m,n) + D_3(j,m,n) + D_4(j,m,n) \right] \]
\[ D'(2j, 2m-1, 2n) = \text{Re} \left[ D_1(j,m,n) - D_2(j,m,n) + D_3(j,m,n) - D_4(j,m,n) \right] \]
\[ D'(2j, 2m, 2n-1) = \text{Re}\left[D_1(j,m,n) + D_2(j,m,n) - D_3(j,m,n) - D_4(j,m,n)\right] \]

\[ D'(2j, 2m, 2n) = \text{Im}\left[-D_1(j,m,n) + D_2(j,m,n) + D_3(j,m,n) - D_4(j,m,n)\right] \]

\[(C-20)\]

for \(j = 1,2,\ldots N, \ m = 1,2,\ldots N, \ n = 1,2\ldots N.\)

**Transformation to Uncoupled Form.** Equations (C-17) are coupled in the second derivatives; that is, there are two or more \(C'_o\) terms in each equation. This coupling results from the non-orthogonality of the axial eigenfunctions. In order to apply the MOA to Equations (C-17), they must be decoupled to the form:

\[
\frac{d^2B_j}{dt^2} = f_j\left(B_1, B_2, \ldots, B_{2N}, \frac{dB_1}{dt}, \frac{dB_2}{dt}, \ldots, \frac{dB_{2N}}{dt}\right) \quad (C-21)
\]

in which only one second derivative appears in each equation. Using Equation (C-21), it is seen that Equation (34) (or Equation (C-17)) can be expressed as

\[
C'_o f = g \quad (C-22)
\]

where \(C'_o\) is the \(2N \times 2N\) matrix of coefficients of the coupled system, \(f\) is the column matrix corresponding to the right-hand-side of the decoupled system, and \(g\) is the column matrix corresponding to the right-hand-side of the coupled system. To decouple Equations (C-17), therefore, Equation (C-22) is solved for \(f\), thus

\[
f = C^{-1}_o \ g \quad (C-23)
\]

where \(C^{-1}_o\) is the inverse of the matrix \(C'_o\). Performing these operations and equating the coefficients of like terms in \(f\) and \(C^{-1}_o \ g\) gives the following relations:

\[
\mathcal{C}_i(j,m) = \sum_{k=1}^{2N} C^{-1}_o(j,k) C'_i(k,m) \quad i = 1,2,3 \quad (C-24)
\]

\[
\mathcal{E}_i(j,m) = \sum_{k=1}^{2N} C^{-1}_o(j,k) E'_i(k,m) \quad i = 1,2 \quad (C-25)
\]
\[ \mathcal{C}(j,m,n) = \sum_{k=1}^{2N} \mathcal{C}_{o}^{-1}(j,k) \mathcal{D}'(k,m,n) \]

where \( \mathcal{C}_{i}, \mathcal{E}_{i}, \) and \( \mathcal{D} \) are the corresponding coefficients of the decoupled system.

Due to the complicated nature of the relationships given by Equations (C-24), explicit expressions for the coefficients \( \mathcal{C}_{i}, \mathcal{E}_{i}, \) and \( \mathcal{D} \) in terms of the spatial integrals can not be given. The coefficients must be calculated numerically by first computing the complex coefficients using Equations (C-12) through (C-16) and Equations (C-8) through (C-11), next calculating the coefficients of the equivalent coupled real system using Equations (C-18), (C-19), and (C-20), and finally computing the coefficients of the decoupled system using Equations (C-24). A similar procedure is used in the numerical solution of the equations obtained using the Galerkin method without averaging (i.e., Equations (22) and (23) or Equations (25)).

**Application of MOA.** The MOA procedure is described in Section 2.4 which results in Equations (39). In performing the time integrations indicated by Equations (32), two integrals of the products of three trigonometric functions arise. These appear in Equations (39) and are defined by:

\[
\beta_{1}(p,q,t) = \frac{2\pi}{w_{1}} \int_{0}^{2\pi/w_{1}} \cos(w_{p}t) \cos(w_{q}t) \cos(w_{d}t) \, dt
\]

\[
= \pi/2 \quad \text{if} \quad p = q + \ell
\]

\[
\text{or} \quad q = p + \ell
\]

\[
\text{or} \quad \ell = p + q
\]

\[
= 0 \quad \text{for all other integer values of } p,q,\ell \quad \text{(C-25)}
\]

\[
\beta_{2}(p,q,t) = \frac{2\pi}{w_{1}} \int_{0}^{2\pi/w_{1}} \cos(w_{p}t) \sin(w_{q}t) \sin(w_{d}t) \, dt
\]

\[
= \pi/2 \quad \text{if} \quad \ell = p + q
\]

\[
\text{or} \quad q = p + \ell
\]

\[
= -\pi/2 \quad \text{if} \quad p = q + \ell
\]

\[
= 0 \quad \text{for all other integer values of } p,q,\ell \quad \text{(C-26)}
\]
The mode-amplitude equations for T-burners are similar to Equations (22) and (23) with an additional equation for $u'_n$ (Equation (76)). The T-burner equations are:

\[
2N \sum_{m=1}^{2N} \left\{ C_0(j,m) \frac{d^2 A_m}{dt^2} + C_1(j,m) A_m + \left[ C_2(j,m) + KC_3(j,m) \right] + h_c \beta_m C_4(j,m) \frac{dA_m}{dt} \right\} + \frac{A_v h_c}{\beta_v} C_7(j) u'_n \\
+ \sum_{m=1}^{2N} \sum_{n=1}^{2N} \left\{ D_1(j,m,n) \left[ A_m \frac{dA_n}{dt} + A_m^{*} \frac{dA_n^{*}}{dt} \right] \\
+ D_2(j,m,n) \left[ A_m \frac{dA_n^{*}}{dt} + A_m^{*} \frac{dA_n}{dt} \right] \right\} = 0 \quad j = 1, 2, \ldots, N \tag{C-27}
\]

\[
2N \sum_{m=1}^{2N} \left\{ C_5(j,m) \frac{dA_m}{dt} + C_6(j,m) A_m \right\} = 0 \quad j = 1, 2, \ldots, N \tag{C-28}
\]

\[
\frac{du'_n}{dt} + (\bar{u}_n/L_{eff})u'_n = (\gamma/L_{eff}) \sum_{m=1}^{N} C_7(m) \frac{dA_m}{dt} \tag{C-29}
\]

The linear coefficients appearing in Equations (C-27) are:

\[
C_0(j,m) = \int_0^1 X_m X_j dx \quad \text{for } 1 \leq m \leq N \tag{C-30}
\]

\[
C_0(j,m) = 0 \quad \text{for } N+1 \leq m \leq 2N
\]
\[ C_1(j,m) = -\int_0^1 \frac{d^2x}{dx^2} m X_j \, dx \quad \text{for } 1 \leq m \leq N \]  
(C-31)

\[ C_1(j,m) = 0 \quad \text{for } N+1 \leq m \leq 2N \] 

\[ C_2(j,m) = 2\int_0^1 u(x) \frac{dx}{dx} m X_j \, dx \]

\[ + (\gamma+1) \int_0^1 \frac{\tilde{u}}{\rho} m X_j \, dx = \gamma \frac{(1+\beta_\gamma)}{2} \int_0^1 \frac{d\tilde{u}}{dx} m X_j \, dx - \frac{1}{V_L} \int_0^1 \frac{d\tilde{u}}{dx} m X_j \, dx \]

\[ + \gamma \tilde{u}_b \left[ 1 + X_m(1) X_j(1) \right] \quad \text{for } 1 \leq m \leq N \]  
(C-32)

\[ C_2(j,m) = -(\gamma-1) \int_0^1 \frac{1}{\rho^p} m X_j \, dx \quad \text{for } N+1 \leq m \leq 2N \]  

\[ C_3(j,m) = \int_0^1 \rho^p m X_j \, dx \quad \text{for } 1 \leq m \leq N \]  
(C-33)

\[ C_3(j,m) = -\int_0^1 \rho^p m X_j \, dx \quad \text{for } N+1 \leq m \leq 2N \]  

\[ C_4(j,m) = -\gamma \left[ \frac{\beta}{2} \int_0^1 \frac{\tilde{u}}{\rho} m X_j \, dx + \int_0^1 \frac{d\tilde{u}}{dx} m X_j \, dx \right] \]

\[ -\gamma \tilde{u}_b \left[ 1 + X_m(1) X_j(1) \right] \quad \text{for } 1 \leq m \leq N \]  
(C-34)

\[ C_4(j,m) = 0 \quad \text{for } N+1 \leq m \leq 2N \]
\[ C_7(j) = \frac{1}{j\pi} \sin \left[ j\pi \left( \frac{1+\beta_v}{2} \right) \right] - \sin \left[ j\pi \left( \frac{1-\beta_v}{2} \right) \right] \]  

\hspace{1cm} \text{(C-35)}

The coefficients of the linear terms in Equations (C-28) are:

\[ C_5(j,m) = 0 \quad \text{for } 1 \leq m \leq N \]  

\hspace{1cm} \text{(C-36)}

\[ C_5(j,m) = \int_0^1 X_m X_j \, dx \quad \text{for } N+1 \leq m \leq 2N \]

\[ C_6(j,m) = -K \int_0^1 X_m X_j \, dx \quad \text{for } 1 \leq m \leq N \]

\[ C_6(j,m) = \int_0^1 u_p(x) \frac{dX}{dx} X_j \, dx + \int_0^1 \frac{d\bar{u}_p}{dx} X_m X_j \, dx \]  

\[ \frac{(1+\beta_v)}{2} \int_0^1 \frac{d\bar{u}_p}{dx} X_m X_j \, dx + K \int_0^1 X_m X_j \, dx \]  

\[ \frac{(1-\beta_v)}{2} \]  

\hspace{1cm} \text{(C-37)}

\[ \text{for } N+1 \leq m \leq 2N \]

The nonlinear coefficients appearing in Equations (C-27) are similar to those appearing in Equations (22) and are given by:

\[ D_1(j,m,n) = \frac{(1-i)}{2} \int_0^1 \frac{dX_m}{dx} \frac{dX_n}{dx} X_j \, dx \]  

\[ + \frac{(\gamma-1)(1-i)}{4} \int_0^1 \frac{d^2X_m}{dx^2} X_n X_j \, dx \]  

\[ \text{(C-38)} \]

\[ D_2(j,m,n) = \frac{1+i}{2} \int_0^1 \frac{dX_m}{dx} \frac{dX_n}{dx} X_j \, dx \]  

\[ + \frac{(\gamma-1)(1+i)}{4} \int_0^1 \frac{d^2X_m}{dx^2} X_n X_j \, dx \]  

\[ \text{(C-39)} \]

for \( 1 \leq j \leq N, \ 1 \leq m \leq N, \ 1 \leq n \leq N \)
The axial eigenfunctions $X_j(x)$ appearing in the above expressions are given by:

$$X_j(x) = \cosh(i\pi x) = \cos j\pi x$$  \hspace{1cm} (C-40)

thus

$$\frac{dX_j}{dx} = -j\pi \sin j\pi x$$ \hspace{1cm} (C-41)

and

$$\frac{d^2X_j}{dx^2} = -j^2\pi^2 \cos j\pi x$$ \hspace{1cm} (C-42)

The steady-state solutions for gas and particle velocity and particle density for motors with full-length, cylindrically perforated grains (i.e., Equations (40), (41), and (42) of Section 2.5) will now be derived. For small steady-state Mach numbers the gas density and mass burning rate are assumed to be constant (the variation in $\rho$ and $\dot{m}$ is of $O(M_e^2)$); thus for $\rho(x) = 1$ the steady-state continuity equation becomes:

$$\frac{du}{dx} = \frac{2\dot{m}}{R}$$  \hspace{2cm} (D-1)

Integrating Equation (D-1) and using the head-end boundary condition $\tilde{u}(0) = 0$ gives:

$$\tilde{u}(x) = \left(\frac{2\dot{m}}{R}\right)x = \tilde{u}_e x$$  \hspace{2cm} (D-2)

which is Equation (40) of Section 2.5.

The steady-state particle density and velocity are obtained from the steady-state particle continuity and momentum equations. Neglecting variations of $\rho_p$ with $x$ as being of $O(M_e^2)$, $\partial\rho_p/\partial x \approx 0$ and Equations (2) and (4) of Section 2.1 become:

$$\frac{d\tilde{u}_p}{dx} = \frac{2}{\rho_p R} \tilde{m}_p$$  \hspace{2cm} (D-3)

$$\rho_p \tilde{u}_p \frac{d\tilde{u}_p}{dx} = \rho_p K(\tilde{u} - \tilde{u}_p) - 2\frac{\dot{m}}{R} \tilde{u}_p$$  \hspace{2cm} (D-4)

Integrating Equation (D-3) with $\tilde{u}_p(0) = 0$ gives:

$$\tilde{u}_p(x) = \left(\frac{2\dot{m}_p}{\rho_p R}\right)x$$  \hspace{2cm} (D-5)

To determine $\rho_p$, Equations (D-2), (D-3) and (D-5) are substituted into Equation (D-4) to obtain the following quadratic equation for $\rho_p$:  

188
which is independent of \(x\). Solving Equation (D-6) and taking the positive root yields:

\[
\ddot{\rho}_p = \left( \frac{\dot{m}_p}{\dot{m}_g} \right) \left[ 1 + \sqrt{1 + \frac{8\ddot{u}_e}{\ddot{u}_K}} \right]
\]

which is Equation (42) of Section 2.5. The particle velocity is then obtained by substituting Equation (D-7) for \(\ddot{\rho}_p\) in Equation (D-5) to obtain:

\[
\ddot{u}_p(x) = \frac{2\ddot{u}_e x}{1 + \sqrt{1 + \frac{8\ddot{u}_e}{\ddot{u}_K}}}
\]

which is Equation (41) of Section 2.5.

D-2. **Steady-State Solutions for T-Burners.**

The steady-state solutions for gas and particle velocities and particle density for a cup-grain T-burner will now be derived. Solutions are obtained for Regions 1, 2, and 3; the solutions in Regions 4 and 5 are readily obtained by symmetry.

**Region 1.** The steady-state solutions in the region of lateral surface burning (i.e., \(0 \leq x \leq \beta/2\)) for the T-burner differ from the solutions given previously in Section D-1 for motors only if end-burning is present. For this case the steady-state velocity of the gases leaving both the lateral propellant surfaces and the end surfaces is denoted by \(\ddot{u}_b\); hence, \(\ddot{m}_g = \ddot{u}_b\) for \(\ddot{\rho} = 1\). Integrating the steady-state continuity equation (i.e., Equation (D-1)) and using the boundary condition \(\ddot{u}(0) = \ddot{u}_b\) gives:

\[
\ddot{u}(x) = \ddot{u}_b \left( 1 + \frac{2}{R} x \right) \quad (0 \leq x \leq \beta/2)
\]

which is Equation (48) of Section 2.6.

As before, the particle properties are obtained by solving the steady-state particle continuity and momentum equations. If the approach used in Section D-1 is
employed, the resulting solution for \( \tilde{\rho}_p \) is dependent on \( x \) which violates the assumption that \( \tilde{\rho}_p \) is constant. Furthermore \( \tilde{\rho}_p \rightarrow \infty \) as \( x \rightarrow 0 \) which is physically meaningless. Thus for the case of end burning, the assumption of a uniform particle density is inconsistent with the particle continuity and momentum equations.

To obtain solutions for \( \tilde{\rho}_p (x) \) and \( \tilde{u}_p (x) \) an alternative solution procedure is used. Allowing \( \tilde{\rho}_p \) to vary but assuming that \( \tilde{\beta}_p \) is a constant, the particle continuity equation becomes:

\[
\frac{d}{dx} (\tilde{\rho}_p \tilde{u}_p) = \frac{2\tilde{m}_p}{R}
\]

Using the boundary condition, \( \tilde{\rho}_p (0) \tilde{u}_p (0) = \tilde{m}_p \), Equation (D-10) is integrated to obtain:

\[
\tilde{\rho}_p \tilde{u}_p = \tilde{m}_p \left( 1 + \frac{2}{R} x \right)
\]

The steady-state particle velocity is then obtained from Equation (D-4) by substituting Equation (D-11) for \( \tilde{\rho}_p \tilde{u}_p \), substituting Equation (D-9) for \( \tilde{u}_p \), and approximating \( \tilde{\rho}_p \) in the particle drag term by \( C_m = \tilde{m}_p / \tilde{m}_g \). Simplifying the resulting expression yields a linear first order differential equation of the form:

\[
\frac{d\tilde{u}_p}{dx} + \frac{B}{1+Ax} \tilde{u}_p = K
\]

where \( B = K/\tilde{u}_b + 2/R \) and \( A = 2/R \). This equation has the general solution:

\[
\tilde{u}_p (x) = C_2 (1 + Ax)^{-B/A} + \frac{K}{A+B} (1 + Ax)
\]

Using the boundary condition \( \tilde{u}_p (0) = \tilde{u}_b \) (the particles and gas are assumed to leave the propellant surface with the same velocity), introducing the expressions for \( A \) and \( B \), and introducing \( \eta = 4\tilde{u}_b/RK \), Equation (D-13) becomes:

\[
\tilde{u}_p (x) = \frac{\tilde{u}_b}{1 + \eta} \left\{ 1 + \frac{2}{R} x + \eta \left[ 1 + \frac{2}{R} x \right]^{-\frac{\eta+2}{\eta}} \right\}
\]

The steady-state solution for particle density is obtained by substituting
Equation (D-14) into Equation (D-11) and solving for \( \bar{p}_p \), thus:

\[
\bar{p}_p(x) = \left\{ \frac{1 + \eta}{1 + \eta \left[ 1 + \frac{2}{R} \frac{x}{\eta} \right]^{-(2\eta+2)/\eta}} \right\} C_m
\]  \hspace{1cm} (D-15)

Equations (D-14) and (D-15) correspond to Equations (49) and (50) of Section 2.6 respectively.

Region 2. In Region 2 the source terms \( \bar{m}_g \) and \( \bar{m}_p \) vanish and the steady-state continuity equation for the gas phase becomes:

\[
\frac{d}{dx} (\bar{p}u) = 0 \hspace{1cm} (D-16)
\]

which is integrated to obtain \( \bar{p} \bar{u} \) = constant. The constant is obtained by matching the solutions at the boundary between Regions 1 and 2. Evaluating Equation (D-9) at \( x = \beta/2 \) yields the constant value of \( \bar{u} \) in Region 2, thus:

\[
\bar{u}(x) = \bar{u}_b (1 + \beta/R) \hspace{1cm} (\beta/2 \leq x \leq (1-\beta_s)/2) \hspace{1cm} (D-17)
\]

which is Equation (55) of Section 2.6.

The particle conservation equations in Region 2 become:

\[
\frac{d}{dx} (\bar{p}_p \bar{u}_p) = 0 \hspace{1cm} (D-18)
\]

\[
\bar{p}_p \frac{d\bar{u}_p}{dx} = \bar{p}_p K (\bar{u} - \bar{u}_p) \hspace{1cm} (D-19)
\]

Integrating Equation (D-18) and matching solutions at the boundary between Regions 1 and 2 gives:

\[
\bar{p}_p \bar{u}_p = \bar{p}_p (\beta/2) \bar{u}_p (\beta/2) \hspace{1cm} (D-20)
\]

Substituting Equation (D-20) into Equation (D-19), approximating \( \bar{p}_p \) by \( C_m \) in the right-hand-side of Equation (D-19), and substituting \( \bar{u} = \bar{u}(\beta/2) \) yields a linear first order differential equation of the form:
\[
\frac{d\bar{u}_p}{dx} + A \bar{u}_p = Q
\]

where

\[
A = \frac{C_m K}{[\bar{\beta}_p(\beta/2) \bar{u}_p(\beta/2)]}
\]

and

\[
Q = \frac{C_m K\bar{u}(\beta/2)/[\bar{\beta}_p(\beta/2) \bar{u}_p(\beta/2)]}
\]

Equation (D-21) has the general solution given by:

\[
\bar{u}_p(x) = \frac{R}{A} + C_1 e^{-Ax}
\]

Matching particle velocities at the boundary between Regions 1 and 2 gives the constant \(C_1\). Introducing the values of \(C_1\), \(A\), and \(Q\) into Equation (D-22) gives the desired steady-state solution for the particle velocity in Region 2:

\[
\bar{u}_p(x) = \bar{u}(\beta/2) \left\{ 1 - \frac{\eta}{1+\eta} e^{-\alpha(x-\beta/2)} \right\}
\]

\((\beta/2 \leq x \leq (1-\beta_v)/2)\)

where \(\alpha = K/\bar{u}(\beta/2)\) and \(\eta\) is the same as in Region 1. Equation (D-23) corresponds to Equation (56) of Section 2.6.

Substituting Equation (D-23) for \(\bar{u}_p\) in Equation (D-20) and solving for \(\bar{p}_p\) yields:

\[
\bar{p}_p(x) = \frac{\bar{p}_p(\beta/2)\bar{u}_p(\beta/2)}{\bar{u}(\beta/2)} \left\{ 1 - \frac{\eta}{1+\eta} e^{-\alpha(x-\beta/2)} \right\}^{-1}
\]

\((\beta/2 \leq x \leq (1-\beta_v)/2)\)

Region 3. In the vent region the source terms appearing in the continuity and momentum equations are determined by the flow out the vent. Assuming uniform flow across the vent, the steady-state continuity equation for the gas phase becomes:

\[
\frac{d}{dx} (\bar{\rho}\bar{u}) = -2\bar{\rho}_b (1 + \beta/R) / \beta_v
\]

\[(D-25)\]
Again assuming that \( \rho = 1 \) Equation (D-25) is integrated to obtain

\[
\tilde{u}(x) = \tilde{u}_b (1 + \beta/R)(1 - 2x)/\beta_v \quad ((1-\beta_v)/2 \leq x \leq (1 + \beta_v)/2)
\]  

(D-26)

where the constant of integration was obtained by matching the solutions at the boundary between Regions 2 and 3. Equation (D-26) is the same as Equation (70) of Section 2.6.

Assuming that the particle sink at the vent is simply the product of \( C_m \) and the gas sink, the particle conservation equations become:

\[
\frac{d}{dx} (\rho_p \tilde{u}_p) = -2 C_m \tilde{u}_b (1 + \beta/R)/\beta_v \quad (D-27)
\]

\[
\frac{\rho_p \tilde{u}_p}{dx} = \rho_p K(\tilde{u}_p - \tilde{u}_b) + 2 C_m \tilde{u}_b (1 + \beta/R) (1 - \tilde{V}_v) \tilde{u}_p / \beta_v 
\]  

(D-28)

Following the method used in Region 1, Equation (D-27) is integrated to obtain:

\[
\rho_p \tilde{u}_p = C_m \tilde{u}_b (1 + \beta/R)(1 - 2x)/\beta_v 
\]  

(D-29)

which is substituted into Equation (D-28) to obtain a linear first order differential equation of the same form as Equation (D-12) where \( A = -2 \) and:

\[
B = \frac{K \beta_v}{\tilde{u}_b (1 + \beta/R)} - 2(1 - \tilde{V}_v) 
\]

Substituting these values of \( A \) and \( B \) into the general solution given by Equation (D-13) and matching solutions at \( x = (1 - \beta_v)/2 \) gives the desired solution for \( \tilde{u}_p \):

\[
\tilde{u}_p(x) = \frac{\tilde{u}_b (1 + \beta/R)}{1 - \eta_v} \left\{ \frac{1 - 2x}{\beta_v} - \eta_v \left[ \frac{1 - 2x}{\beta_v} \right]^q \right\} 
\]  

(D-30)

\( (1 - \beta_v)/2 \leq x \leq 1/2 \)

where \( \eta_v = \frac{(4 - 2V_v)}{K \beta_v} \tilde{u}_b (1 + \beta/R) \)
and \[ q = \frac{2}{\eta_v} + (V_e - 1) \]

Substituting Equation (D-30) into Equation (D-29) and solving for \( \rho_p \) gives:

\[
\tilde{\rho}_p(x) = \left\{ \begin{array}{ll}
\frac{1 - \eta_v}{1 - \eta_v \left[ \frac{1 - 2x}{\beta_v} \right]^{q-1}} & \text{for } (1 - \beta_v)/2 < x < 1/2
\end{array} \right. \quad \text{(D-31)}
\]

Equations (D-30) and (D-31) correspond to Equations (71) and (72) of Section 2.6. These equations are valid only in the left half of the vent region \[ ((1 - \beta_v)/2 \leq x \leq 1/2) \], while the corresponding values in the right half of Region 3 are obtained by symmetry.
FOREWORD

The present report consists of two volumes which describe an approximate nonlinear analysis of solid rocket motors and T-burners and the associated computer programs. Volume I contains the analytical basis for the computer programs and the results of the parametric studies, while Volume II describes the computer programs and serves as a user's manual.

The investigation is entitled APPROXIMATE NONLINEAR ANALYSIS OF SOLID ROCKET MOTORS AND T-BURNERS. The two volumes are additionally subtitled as follows:

Volume I - Analysis and Results

This investigation was sponsored by the Air Force Rocket Propulsion Laboratory, Edwards AFB, California 93523 under contract number F04611-75-C-0036 with Capt. Jack Donn as technical monitor. Program management was provided by B. T. Zinn (Co-principal Investigator) while project engineering was provided by E. A. Powell (Co-principal Investigator) and M. S. Padmanabhan (Post Doctoral Fellow).

This technical report has been reviewed and is approved.
### REPORT DOCUMENTATION PAGE

<table>
<thead>
<tr>
<th>Field</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. REPORT NUMBER</td>
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</tr>
<tr>
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<td>Nonlinear Combustion Analysis Galerkin Method Solid Rocket Motor T-Burner</td>
</tr>
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<td>20. ABSTRACT (Continue on reverse side if necessary and identify by block number)</td>
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</tr>
</tbody>
</table>
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>FOREWORD</td>
<td>i</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>ii</td>
</tr>
<tr>
<td>1. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>2. GENERAL STRUCTURE OF APPROXIMATE PROGRAMS</td>
<td>2</td>
</tr>
<tr>
<td>3. PROGRAMS FOR SOLID ROCKET MOTORS</td>
<td>3</td>
</tr>
<tr>
<td>3.1 PROGRAM SOLID1</td>
<td>3</td>
</tr>
<tr>
<td>Program Structure</td>
<td>3</td>
</tr>
<tr>
<td>Description of Input</td>
<td>4</td>
</tr>
<tr>
<td>Description of Subroutines</td>
<td>5</td>
</tr>
<tr>
<td>Description of Output</td>
<td>8</td>
</tr>
<tr>
<td>Sample Case</td>
<td>9</td>
</tr>
<tr>
<td>FORTRAN Source Code</td>
<td>34</td>
</tr>
<tr>
<td>3.2 PROGRAM SOLID2</td>
<td>59</td>
</tr>
<tr>
<td>Program Structure</td>
<td>59</td>
</tr>
<tr>
<td>Description of Input</td>
<td>60</td>
</tr>
<tr>
<td>Description of the Subroutines</td>
<td>63</td>
</tr>
<tr>
<td>Description of Output</td>
<td>66</td>
</tr>
<tr>
<td>Sample Case</td>
<td>68</td>
</tr>
<tr>
<td>FORTRAN Source Code</td>
<td>93</td>
</tr>
<tr>
<td>3.3 PROGRAM MA1</td>
<td>122</td>
</tr>
<tr>
<td>Program Description</td>
<td>122</td>
</tr>
<tr>
<td>Sample Case</td>
<td>122</td>
</tr>
<tr>
<td>FORTRAN Source Code</td>
<td>140</td>
</tr>
<tr>
<td>Section</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>-------------</td>
</tr>
<tr>
<td>3.4</td>
<td>PROGRAM MA2</td>
</tr>
<tr>
<td>4.</td>
<td>APPROXIMATE T-BURNER PROGRAMS</td>
</tr>
<tr>
<td>4.1</td>
<td>PROGRAM TB1</td>
</tr>
<tr>
<td>4.2</td>
<td>PROGRAM TB2</td>
</tr>
<tr>
<td>5.</td>
<td>&quot;EXACT&quot; PROGRAM FOR MOTORS</td>
</tr>
</tbody>
</table>

**Program Description**

<table>
<thead>
<tr>
<th>Program</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>MA2</td>
<td>151</td>
</tr>
<tr>
<td>TB1</td>
<td>195</td>
</tr>
<tr>
<td>TB2</td>
<td>239</td>
</tr>
</tbody>
</table>

**Sample Case**

<table>
<thead>
<tr>
<th>Program</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>MA2</td>
<td>151</td>
</tr>
<tr>
<td>TB1</td>
<td>196</td>
</tr>
<tr>
<td>TB2</td>
<td>239</td>
</tr>
</tbody>
</table>

**FORTRAN Source Code**

<table>
<thead>
<tr>
<th>Program</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>MA2</td>
<td>176</td>
</tr>
<tr>
<td>TB1</td>
<td>223</td>
</tr>
<tr>
<td>TB2</td>
<td>265</td>
</tr>
</tbody>
</table>

**Description of Input**

<table>
<thead>
<tr>
<th>Program</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>TB1</td>
<td>196</td>
</tr>
<tr>
<td>TB2</td>
<td>239</td>
</tr>
</tbody>
</table>

**Description of Subroutines**

<table>
<thead>
<tr>
<th>Program</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>TB1</td>
<td>197</td>
</tr>
<tr>
<td>TB2</td>
<td>241</td>
</tr>
</tbody>
</table>

**Description of Output**

<table>
<thead>
<tr>
<th>Program</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>TB1</td>
<td>199</td>
</tr>
<tr>
<td>TB2</td>
<td>242</td>
</tr>
</tbody>
</table>

**Sample Case**

<table>
<thead>
<tr>
<th>Program</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>TB1</td>
<td>200</td>
</tr>
<tr>
<td>TB2</td>
<td>242</td>
</tr>
</tbody>
</table>

**FORTRAN Source Code**

<table>
<thead>
<tr>
<th>Program</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>TB1</td>
<td>223</td>
</tr>
<tr>
<td>TB2</td>
<td>265</td>
</tr>
<tr>
<td>Table of Contents (cont'd)</td>
<td>Page No.</td>
</tr>
<tr>
<td>---------------------------</td>
<td>----------</td>
</tr>
<tr>
<td>Sample Case</td>
<td>295</td>
</tr>
<tr>
<td>FORTRAN Source Code</td>
<td>321</td>
</tr>
</tbody>
</table>
1. INTRODUCTION

Volume II of this report is a user's manual for the computer programs based on the approximate and "exact" nonlinear axial-mode combustion instability analyses developed in Volume I. Three approximate programs were developed as follows: (1) nonlinear analysis of solid rocket motors by the Galerkin method, (2) nonlinear analysis of solid rocket motors by the Method of Averaging (MOA), and (3) nonlinear analysis of T-burners (Galerkin method). These programs numerically solve the systems of nonlinear ordinary differential equations which arise on the application of the Galerkin method or MOA to the governing one-dimensional, two-phase partial differential equations. The details of the approximate and "exact" instability models upon which these programs are based are available in Volume I of this report.

This volume contains a description of all the main programs and subroutines used in the instability programs. Descriptions of the inputs needed to operate the programs and the outputs generated are given. Furthermore a sample case is presented to illustrate the text and to facilitate checkout of the program. The entire program has been written in the FORTRAN IV programming language. While the programs have been checked out on a CDC CYBER 70/74-28 system operating under NOS 1.1 at the Georgia Institute of Technology, the structure of the programs has been kept simple enough to be run on any modern digital computer. The program includes an option to obtain plots using a CALCOMP plotter, but the absence of a plotting capability in a computing system does not affect the functioning of the rest of the program.
2. GENERAL STRUCTURE OF APPROXIMATE PROGRAMS

The computations for the solid rocket motor and the T-burner are performed by two separate sets of programs. Programs SOLID1 and SOLID2 compute instability in a solid rocket motor whereas programs TB1 and TB2 refer to the T-burner. Both these sets of programs use the Galerkin method which is described in Volume I. Programs SOLID1 and TB1 compute the coefficients which appear in the differential equations that govern the mode-amplitude functions for a solid rocket motor and a T-burner respectively. The coefficients generated by these programs must be supplied as input to SOLID2 and TB2. Using these coefficients, SOLID2 and TB2 integrate the system of ordinary differential equations for the mode amplitudes in a solid rocket motor and a T-burner respectively. An option is provided for SOLID1 and TB1 to write the generated coefficients onto a disk from which these coefficients can be conveniently recovered by SOLID2 and TB2. The advantage of dividing the computations into two separate programs will become obvious in the program description which follows. With this segmentation of tasks, coefficients can be generated just once using SOLID1 (or TB1) and can then be repeatedly run with different combustion responses and different initial conditions as desired using SOLID2 (or TB2), thereby considerably reducing the computer time.

The above two sets of programs utilize the Galerkin method without application of the method of averaging to the resulting mode-amplitude equations. Programs MA1 and MA2 obtain numerical solutions of the equations derived by application of the method of averaging after using the Galerkin method as described in Volume I to calculate the stability behavior of solid rocket motors. Program MA1 corresponds to programs SOLID1 and TB1 since it computes the coefficients appearing in the governing equations. Program MA2 performs the numerical integration of the system of differential equations like SOLID2 and TB2.

Section 3 gives a description of the programs used to investigate the stability of solid rocket motors. The various subroutines, inputs, and outputs are described, and a sample case is presented for each program. Section 4 presents a similar description for the T-burner programs. Equations and appendices referenced in these sections refer to equations and appendices in Volume I.
3. PROGRAMS FOR SOLID ROCKET MOTORS

As pointed out earlier, the motor stability computations are performed by SOLID1 and SOLID2 or MA1 and MA2. These programs are described in the following sections.

3.1 PROGRAM SOLID1

Program SOLID1 calculates the coefficients of both the linear and nonlinear terms which appear in Equations (22) and (23). The coefficients to be calculated are functions of various integrals of complex hyperbolic functions as defined in Appendix C.

**Program Structure.** The program can be divided functionally into five major sections: (1) input, (2) calculation of the complex linear coefficients, (3) calculation of the complex nonlinear coefficients, (4) obtaining coefficients of the equivalent uncoupled real system, and (5) output.

The inputs to the program include the parameters describing the motor geometry and the nozzle boundary condition, the modes included in the approximating series expansion, particle characteristics and various control numbers. All of the inputs are supplied to the main program. The next subsection gives a description of the necessary inputs.

In the second section of the program, the axial acoustic eigenvalues are calculated by means of subroutines EIGVAL and FCNS, and the integrals of the product of two axial eigenfunctions are computed by means of subroutines AXIAL1 and UBAR. The complex linear coefficients are then calculated according to Equations (C-1) through (C-7) and normalized by dividing by the coefficient of the highest derivative (i.e. \( C_0(j,j) \) in Equations (22)).

In the third section the integrals of products of three axial eigenfunctions are computed using the subroutine AXIAL2. The complex nonlinear coefficients are obtained from Equations (C-8) through (C-11) and are then normalized.

In the fourth section the normalized complex coefficients are used to obtain coefficients for the equivalent system of real differential equations obtained by separating the real and imaginary parts of the complex equations. Since the axial eigenfunctions are not orthogonal, the resulting system of equations may be coupled in the highest derivative terms. Therefore a matrix inversion procedure is used to obtain the coefficients of an equivalent system which is not coupled in the highest derivatives. The subroutine GJR
performs the matrix inversion.

In the last section, the computed values of the coefficients are either printed out or stored on disk or both as desired.

**Description of Input.** The input data consists of the steady-state Mach numbers, the type of nozzle used, the various particle and gas constants, information about the modes used in the series expansion, and various control numbers. The input must be supplied to the program by means of punched cards. All real numbers must be punched with a decimal point in F10.0 format, while all integers must be punched in I5 format in the rightmost locations of the allocated field of 5 columns. For instance, in the second card, described below, the format is (2F10.0, I5).

A list of necessary input data is described below. The units of all the dimensional data are also indicated in this description. As can be observed from this description, these data must be specified in metric units. A sample input is also given at the end of this section.

The first card gives the title of the case, in columns 1 through 70.

Second card: GAM, UE, NOZZLE

GAM is the specific heat ratio.

UE is the steady state Mach number at the nozzle entrance.

NOZZLE specifies the type of nozzle used:

- NOZZLE = 0 quasi-steady.
- NOZZLE = 1 conventional nozzle.

Third card: NJMAX, NONLIN, NEGL, NOUT, NPRTKL

NJMAX is the number of mode-amplitude functions in the assumed series solution.

The coefficients computed are determined by NONLIN as follows:

- NONLIN = 0 linear coefficients only.
- NONLIN = 1 both linear and nonlinear coefficients.

Coefficients to be neglected are determined by NEGL as follows:

- NEGL = 0 terms smaller than 0.00001 are neglected.
- NEGL = 1 linear terms smaller than SM1 and nonlinear terms smaller than SM2 are neglected.

The output is determined by NOUT as follows:

- NOUT = 0 printed output only.
- NOUT = 1 write into a file and print output.
NOUT = 2 write into a file only.

NPRTKL determines whether the particles are present:
 NPRTKL = 0 particles not present.
 NPRTKL = 1 particles present.

Next card (necessary only if NPRTKL = 1): DIA, RHOM, SP, TEMP, FREQ, CM
 DIA is the particle diameter, in microns.
 RHOM is the density of the particle material, in kg/m$^3$.
 SP is the ratio of the specific heats of particle material and gas.
 TEMP is the chamber temperature, in degrees Kelvin.
 FREQ is the frequency of oscillation in pure gas, in Hertz.
 CM is the particle loading.

Next card (necessary only if NEGL = 1): SM1, SM2
 SM1 and SM2 are as defined above.

Next NJMAX cards (necessary only if NOZZLE = 1): J, AMPL(J), PHASE(J)
 AMPL(J) is the magnitude of the nozzle admittance for the J$^{th}$ mode.
 PHASE(J) is the phase of the nozzle admittance for the J$^{th}$ mode.

Next NJMAX cards: J, L(J), NAME(J)
 Each mode-amplitude is assigned an integer J.
 The mode is specified by the index L(J).
 L(J) is the axial mode number and must not exceed NJMAX.
 NAME(J) is a four-character name for the J$^{th}$ mode.

**Description of the Subroutines.** The different tasks performed by the program SOLIDI were outlined earlier in this section. This subsection explains the different subroutines which are involved in performing these tasks.

**SUBROUTINE EIGVAL (L, SMN, GAMMA, ZE, YAMPL, YPHASE, RESULT).** This subroutine, called from the main program, computes the complex axial acoustic eigenvalues for a cylindrical chamber with a hard wall at one end and a nozzle at the other end. The amplitude and phase of the nozzle admittance is specified in the argument list by the variables YAMPL and YPHASE respectively. L specifies the axial mode number for which the acoustic eigenvalue is required. SMN indicates the transverse frequency and is set zero in the current program since only axial modes are considered. ZE is the nondimensional length of the chamber and equals 1. The computed value of the complex axial acoustic eigenvalue is obtained from this subroutine through the argument variable, RESULT.
This subroutine computes the real and imaginary parts of the eigenvalue (\(\varepsilon\) and \(\eta\) respectively) by solving a pair of simultaneous transcendental equations, \(f(x,y)=0\) and \(g(x,y)=0\), which are obtained from the wave equation for the case of no mean flow, combustion or particles. The roots \(\varepsilon\) and \(\eta\) of this system of transcendental equations are calculated by a method of successive approximations using Newton's method.

**SUBROUTINE FCNS(X,Y,ZE,F,G,FX,FY,GX,GY).** This subroutine is called by the subroutine EIGVAL at every iteration in the successive approximation scheme. It computes the functions \(F\) and \(G\), and their partial derivatives \(FX\), \(FY\), \(GX\), and \(GY\) with respect to \(X\) and \(Y\). Here \(F\) and \(G\) are the functions whose roots are the desired axial eigenvalues. The value of these functions \(F\) and \(G\) and their partial derivatives are needed in the recursion formulas

\[
\varepsilon_{i+1} = \varepsilon_i - \frac{f \frac{\partial g}{\partial \eta} - g \frac{\partial f}{\partial \eta}}{\frac{\partial f}{\partial \varepsilon} \frac{\partial g}{\partial \eta} - \frac{\partial g}{\partial \varepsilon} \frac{\partial f}{\partial \eta}} i
\]

\[
\eta_{i+1} = \eta_i - \frac{g \frac{\partial f}{\partial \varepsilon} - f \frac{\partial g}{\partial \varepsilon}}{\frac{\partial f}{\partial \varepsilon} \frac{\partial g}{\partial \eta} - \frac{\partial g}{\partial \varepsilon} \frac{\partial f}{\partial \eta}} i
\]

where \(\varepsilon_{i+1}\) and \(\eta_{i+1}\) are the \((i+1)\)th approximations to the real and imaginary parts of the eigenvalue.

**SUBROUTINE AXIAL1 (N\(\eta\),NP,NJ,UE,ZE,RESULT).** This subroutine calculates the different integrals which appear in the expressions for the coefficients of the linear terms in Equations (22) and (23) according to the value of \(N\(\eta\)\). The computed value of the desired integral is obtained as the argument variable RESULT. The different integrals that can be computed from this subroutine are:

\[
N\eta = 1 \quad \text{RESULT} = \int_{0}^{X} X_j^* \, dx
\]

\[
N\eta = 2 \quad \text{RESULT} = \int_{0}^{\frac{d^2 X}{dx^2}} X_j^* \, dx
\]
The subscripts \( p(=NP) \) and \( j(=NJ) \) denote the axial mode numbers, and \( X_p \) and \( X_j \) are the axial eigenfunctions for the \( p^{th} \) and \( j^{th} \) mode. The asterisk denotes the complex conjugate of the quantity. The eigenvalues, which are required to compute the eigenfunctions, are obtained from the main program through blank common. The integrals for \( \text{NOPT}=1 \) and \( \text{NOPT}=2 \) are evaluated analytically, but the last two integrals, which involve the mean flow velocity and its derivative, are evaluated numerically using Simpson's Rule. For these cases the value of the mean flow velocity and its derivative are obtained by calling the subroutine UBAR.

**SUBROUTINE AXIAL2 (NOPT, NCNJ, NP, NQ, NJ, ZE, RESULT).** This subroutine computes the integrals which appear in expressions for the coefficients of the nonlinear terms in Equations (22). The combination of integers \( \text{NOPT} \) and \( \text{NCNJ} \) determines the integral that is computed and is available as \( \text{RESULT} \). The three basic forms of the integral are specified by \( \text{NOPT} \) as follows:

\[
\text{NOPT} = 1 \quad \text{RESULT} = \int_{0}^{\text{ZE}} X_p X_q X_j^* \, dx
\]

\[
\text{NOPT} = 2 \quad \text{RESULT} = \int_{0}^{\text{ZE}} \frac{dX_p}{dx} \frac{dX_q}{dx} X_j^* \, dx
\]

\[
\text{NOPT} = 3 \quad \text{RESULT} = \int_{0}^{\text{ZE}} \frac{d^2 X_p}{dx^2} X_q X_j^* \, dx
\]

When \( \text{NCNJ} = 1 \), these basic forms are calculated. For \( \text{NCNJ} = 2 \) the second function in the above integrands is replaced by its complex conjugate, while for \( \text{NCNJ} = 3 \) the first function in the above integrands is replaced by its complex conjugate. For \( \text{NCNJ} = 4 \), both the first and the second functions in these integrands are replaced by their complex conjugates. In these expressions \( X_p, X_q \) and
are the axial acoustic eigenfunctions for the $p^{th}$, $q^{th}$ and $j^{th}$ axial modes ($p(=NP)$, $q(=NQ)$ and $j(=NJ)$ are the axial mode numbers). The subroutine obtains the value of the acoustic eigenvalues from the main program through blank common space.

All the above integrals are evaluated in this subroutine analytically.

SUBROUTINE UBAR (N OPT, UE, ZE, Z, RESULT). This subroutine is called by subroutine AXIAL1 and it calculates the steady-state velocity and its derivative at the axial location $Z$ in a motor whose exit Mach number is $UE$. For $N\text{OPT} = 1$ the velocity is calculated, while for $N\text{OPT} = 2$ the derivative of the velocity is calculated. In the present program, a linear steady-state velocity distribution is assumed, which varies from zero at $Z = 0$ to $UE$ at $Z = ZE$. While this computation could have been easily carried out in the calling program itself, a separate subroutine is provided for the steady-state velocity distribution so that more realistic velocity distributions can be conveniently incorporated in the motor program if desired.

SUBROUTINE GJR(A", NC, NR, N, MC, JC, V). This is a matrix inversion routine. Here NC and NR are dimensions of the matrix $A$ to be inverted, but only the first $N \times N$ elements of $A$ are inverted, and the inverted matrix replaces the original matrix under the name $A$. $V(1)$ should be set equal to 1 in the calling program to achieve the matrix inversion. If there is trouble (singularity) in the matrix inversion, a message is printed out. This subroutine is taken from the MATHPACK library for the UNIVAC 1100 series computers with suitable modifications to run on CDC 6000 series computers.

COMPLEX FUNCTION CCOSH(X). This function calculates the hyperbolic cosine (i.e., cosh) of a complex number $X$.

COMPLEX FUNCTION CSINH(X). This function calculates the hyperbolic sine (i.e., sinh) of a complex number $X$.

Both CCOSH(X) and CSINH(X) can be omitted in computing facilities where complex arguments are acceptable in the hyperbolic functions provided in their library.

Description of Output. The coefficients calculated by program SOLID1 are printed or stored on disk according to the value of the control numbers $N\text{OUT}$ as indicated in the section on inputs. These two output modes will now be discussed.

Printed output is the only output obtained when $N\text{OUT} = 0$, which is used for checkout purposes only. Printed output can be obtained in conjunction with
disk storage mode by giving the value 1 to NOUT.

The first page of the printed output gives a restatement of the input parameters. The page is headed by the title of the case, and it gives the values of all the particle and gas constants. This is followed by information about the modes included in the series expansion, their eigenvalues and nozzle admittance values. In the following pages, the decoupled linear and nonlinear coefficients are printed out in the matrix format.

A sample printed output for the five term series used in the sample case is given in the next subsection.

Disk storage is the most convenient means of storing the output of Program SOLID1. This mode of output is obtained by setting NOUT = 1 or NOUT = 2. The output is written in a format which corresponds to the input format for SOLID2 so that the two programs can be run in tandem with ease. The disk into which the output is written is given the I/O device number 9. The control statement needed to request this number for I/O depends on the computer facilities used.

Sample Case. A sample case is presented in this section to facilitate checkout and to illustrate the I/O procedure. This case refers to a motor with exit Mach number $\tilde{M}_e = 0.078$. The average diameter of the particles is 2.5 microns and particle loading is 0.1. The pure gas frequency of oscillation is 1071 Hertz, and the mean temperature in the chamber is 3525 °K. A quasi-steady nozzle is employed and the specific heat ratio is 1.23. It is desired to consider the first five longitudinal modes in the series expansion for the velocity potential. A printed output as well as disk storage of the output is required so that they may be used for checking out Program SOLID2 later.

An input deck needed for fulfilling the above conditions is illustrated in Table 1. The printed output generated by the program with this output deck is shown in the following pages.
TEST CASE FOR SOLID

GAMMA = 1.23000     UE = .0780

QUASI-STEADY NOZZLE.

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PROGRAM SOLI D1 (INPUT, OUTPUT, DATA,
1 TAPES=INPUT, TAPE6=OUTPUT, TAPE9=DATA)

*************** PROGRAM SOLI D1 ********************************

THIS PROGRAM COMPUTES THE COEFFICIENTS WHICH APPEAR
IN THE DIFFERENTIAL EQUATIONS WHICH GOVERN THE MODE-AMPLITUDE
FUNCTIONS. THESE COEFFICIENTS CAN BE WRITTEN INTO A FILE
FOR INPUT TO PROGRAM SOLI D2.

THE FOLLOWING INPUTS ARE REQUIRED:

THE FIRST CARD GIVES THE TITLE OF THE CASE.

SECOND CARD: GAM, UE, NOZZLE
GAM IS THE SPECIFIC HEAT RATIO.
UE IS THE STEADY STATE MACH NUMBER AT THE NOZZLE ENTRANCE.
NOZZLE SPECIFIES THE TYPE OF NOZZLE USED:
NOZZLE = 0 QUASI-STEADY
NOZZLE = 1 CONVENTIONAL NOZZLE

THIRD CARD: NJMAX, NONLIN, NEGL, NOUT, NPRTKL
NJMAX IS THE NUMBER OF MODE-AMPLITUDE FUNCTIONS IN THE ASSUM
SERIES SOLUTION.
THE COEFFICIENTS COMPUTED ARE DETERMINED BY NONLIN AS FOLLOW:
NONLIN = 0 LINEAR COEFFICIENTS ONLY
NONLIN = 1 BOTH LINEAR AND NONLINEAR COEFFICIENTS
COEFFICIENTS TO BE NEGLECTED ARE DETERMINED BY NEGL
AS FOLLOWS:
NEGL = 0 TERMS SMALLER THAN 0.00001 ARE NEGLECTED.
NEGL = 1 LINEAR TERMS SMALLER THAN SM1 AND NONLINEAR
TERMS SMALLER THAN SM2 ARE NEGLECTED.

THE OUTPUT IS DETERMINED BY NOUT AS FOLLOWS:
NOUT = 0 PRINTED OUTPUT ONLY
NOUT = 1 WRITE INTO A FILE AND PRINT OUTPUT.
NOUT = 2 WRITE INTO A FILE ONLY.

NPRTKL DETERMINES WHETHER THE PARTICLES ARE PRESENT:
NPRTKL = 0 PARTICLES NOT PRESENT.
NPRTKL = 1 PARTICLES PRESENT.

NEXT CARD (ONLY IF NPRTKL=1): DIA, RHOM, SP, TEMP, FREQ, CM
DIA IS THE PARTICLE DIAMETER, IN MICRONS.
RHOM IS THE DENSITY OF THE PARTICLE MATERIAL, IN KG/M**3.
SP IS THE RATIO OF THE SPECIFIC HEATS OF PARTICLE MATERIAL
AND GAS.
TEMP IS THE CHAMBER TEMPERATURE, IN DEGREES KELVIN.
FREQ IS THE FREQUENCY OF OSCILLATION IN PURE GAS, IN HERTZ.
CM IS THE PARTICLE LOADING.
**COMPLEX**

*SM1* AND *SM2* ARE AS DEFINED ABOVE.

**NEXT NJMAX CARDS** (ONLY IF NOZZLE = 1): J, AMPL(J), PHASE(J)

AMPL(J) IS THE MAGNITUDE OF THE NOZZLE ADMITTANCE
FOR THE JTH MODE.

PHASE(J) IS THE PHASE OF THE NOZZLE ADMITTANCE
FOR THE JTH MODE.

**NEXT NJMAX CARDS**

J, L(J), NAME(J)

EACH MODE-AMPLITUDE IS ASSIGNED AN INTEGER J.

THE MODE IS SPECIFIED BY THE INDEX L(J).

L(J) IS THE AXIAL MODE NUMBER AND MUST NOT EXCEED NJMAX.

NAME(J) IS A FOUR-CHARACTER NAME FOR THE JTH MODE.

******************************************************************

**DIMENSION**

L(6), NAME(6), TITLE(7), AMPL(6), PHASE(6),

I V(2), C(3, 12, 24), CI(12, 12), JC(12),

3 EX(12, 12, 2), D(12, 12, 12),

4 KMAX(6), TS(12, 24), TSQ(12, 12), TS(3, 24),

C1PAR(12, 12), CPAR(2, 12, 24), TS1PAR(2, 12), KMAXPR(2)

**COMPLEX**

CRSLT, CI, ZEJ, ZEP1, ZEP2, AX(5), AXINT(4, 4),

1 DC0EF, B(6), BC(6),

2 YNOZ(6), CC(5, 6, 12), CNORM(12),

3 CD1(6, 6, 6), CD2(6, 6, 6),

4 CD3(6, 6, 6), CD4(6, 6, 6),

5 CCPAR(3, 6, 12), CCOSH, CSINH

**COMMON**

B

**DATA INPUT.**

MAXMD = 6

MAXMD2 = 12

MAXMD4 = 24

PI = 3.1415926536

SM1 = 0.00001

SM2 = 0.00001

CI = (0.0, 1.0)

**INPUT PARAMETERS**

4 READ (5, 5000) TITLE

IF (EOF(5)) 600, 1

1 CONTINUE

READ (5, 5001) GAM, UE, NOZZLE

READ (5, 5004) NJMAX, NONLIN, NEGL, NOUT, NPRTKL

IF (NPRTKL * EQ. 1) READ (5, 5006) DIA, RHOM, SP, TEMP, FREQ, CM

GAMMA = GAM * (1.0 + SP*CM) / (1.0 + GAM*SP*CM)

IF (NEGL * EQ. 1) READ (5, 5005) SM1, SM2

IF (NOZZLE * EQ. 1) GO TO 5
COMPUTE ADMITTANCE FOR QUASI-STEADY NOZZLE.

\[ Y = (\Gamma - 1.0) \times \frac{U_e}{2.0 \times \Gamma} \]

DO 3 J = 1, NJMAX
  AMPL(J) = Y
  PHASE(J) = 0.0
3 CONTINUE

DO 6 I = 1, N
  READ (5,5003) J, AMPL(J), PHASE(J)
6 CONTINUE

DO 10 I = 1, NJMAX
  READ (5,5002) J, L(J), NAME(J)
10 CONTINUE

DO 12 J = 1, NJMAX
  THETA = PHASE(J) \times \frac{\pi}{180.0}
  YR = AMPL(J) \times \cos(\theta)
  YI = AMPL(J) \times \sin(\theta)
  YNOZ(J) = CMPLX(YR, YI)
12 CONTINUE

NJMAX2 = NJMAX
IF (NPRTKL EQ 1) NJMAX2 = 2 * NJMAX
ZE = 1.0
ZCOMB = 1.0
CAX = \Gamma + 1.0

RHOP = 0.0
IF (NPRTKL EQ 0) GO TO 14
VISC = 8.834 \times 0.00001 \times (TEMP/3485)^{0.66}
PARTKL = (9.0 \times VISC) / (RHOM \times FREQ \times \text{DIA} \times \text{DIA} \times 10^{12})
UPBYU = 2.0 / (1.0 + \sqrt{1.0 + 8.0 \times U_e / PARTKL})
RHOP = CM/UPBYU
14 CONTINUE

*****************************************************************

CALCULATE AXIAL ACOUSTIC EIGENVALUES.

DO 40 J = 1, NJMAX
  LL = L(J)
  SMN = 0.0
  YAMPL = AMPL(J)
  YPHASE = PHASE(J)
  CALL EIGVAL(LL, SMN, GAMMA, ZE, YAMPL, YPHASE, CRSLT)
  B(J) = CRSLT
  BC(J) = CONJG(CRSLT)
40 CONTINUE
CALCULATE LINEAR COEFFICIENTS.

DO 100 NJ = 1, NJMAX
DO 100 NP = 1, NJMAX2

ZERO COEFFICIENT ARRAYS.
DO 105 KC = 1, 5
CC(KC,NJ,NP) = (0.0, 0.0)
105 CONTINUE

NPM = NP
NJMJ = NJ
IF (NP GT NJMAX) NPM = NP - NJMAX

CALCULATE AXIAL INTEGRALS.
127 DO 130 NOPT = 1, 4
CALL AXIAL1(NOPT, NPM, NJMJ, UE, ZE, CRSLT)
AX(NOPT) = CRSLT
130 CONTINUE

EVALUATE FUNCTIONS AT NOZZLE END.
ZEJ = CCOSH(CI * BC(NJM) * ZE)
ZEP1 = CCOSH(CI * B(NPM) * ZE)
ZEP2 = CI * B(NPM) * CSINH(CI*B(NPM)*ZE)
IF (NP GT NJMAX) GO TO 704

COEFFICIENT OF THE SECOND DERIVATIVE OF A(P).
CC(1,NJ,NP) = AX(1)

COEFFICIENT OF A(P).
CC(2,NJ,NP) = - AX(2) + ZEP2*ZEJ

COEFFICIENT OF THE FIRST DERIVATIVE OF A(P).
CC(3,NJ,NP) = (CAX*AX(3) + (2.0, 0.0)*AX(4)
1 + GAMMA*YNOZ(NP)*ZEP1*ZEJ)
CC(4,NJ,NP) = RHOP * AX(1)
CC(5,NJ,NP) = - GAMMA * AX(3)
GO TO 100

704 CC(3,NJ,NP) = - (GAMMA - 1.0) * RHOP * UBYU * AX(4)
CC(4,NJ,NP) = - RHOP * AX(1)

100 CONTINUE

NORMALIZE LINEAR COEFFICIENTS.
DO 140 NJ = 1, NJMAX
CNORM(NJ) = CC(1,NJ,NJ)
DO 140 NP = 1, NJMAX2
DO 140 KC = 1, 5
CC(KC,NJ,NP) = CC(KC,NJ,NP)/CNORM(NJ)
140 CONTINUE
IF (NPRTKL .EQ. 0) GO TO 1005
DO 1010 NJ = 1, NJMAX
DO 1010 NP = 1, NJMAX2
DO 1015 KC = 1, 3
CCPAR(KC,NJ,NP) = (0.0,0.0)
1015 CONTINUE
NPM = NP
IF (NP .GT. NJMAX) NPM = NP - NJMAX
NJM = NJ

CALCULATE AXIAL INTEGRALS.
DO 1020 NOPT = 1, 4
CALL AXIALI(NOPT,NPM,NJM,UE,ZE,CRSLT)
1020 AX(NOPT) = CRSLT
IF (NP .GT. NJMAX) GO TO 1025
CCPAR(3,NJ,NP) = - AX(1)
GO TO 1010
1025 CCPAR(1,NJ,NP) = AX(1)
CCPAR(2,NJ,NP) = UPBYU*(AX(4) + AX(3))
CCPAR(3,NJ,NP) = AX(1)
1010 CONTINUE
DO 1030 NJ = 1, NJMAX
NJM = NJ + NJMAX
CNORM(NJM) = CCPAR(1,NJ,NJM)
DO 1030 NP = 1, NJMAX2
DO 1030 KC = 1, 3
CCPAR(KC,NJ,NP) = CCPAR(KC,NJ,NP)/CNORM(NJM)
1030 CONTINUE
1005 CONTINUE

*******************************************************************************

COMPUTE NONLINEAR COEFFICIENTS.
IF (NONLIN .EQ. 0) GO TO 402
G1 = (GAMMA - 1.0) * 0.5

170 DO 200 NJ = 1, NJMAX
NJM = NJ + NJMAX
DCOEF = 0.5 / CNORM(NJ)
DO 200 NP = 1, NJMAX2
DO 200 NQ = 1, NJMAX
CD1(NJ,NP,NQ) = (0.0,0.0)
CD2(NJ,NP,NQ) = (0.0,0.0)
CD3(NJ,NP,NQ) = (0.0,0.0)
CD4(NJ,NP,NQ) = (0.0,0.0)
244 DO 240 J = 2, 3
DO 240 NC = 1, 4
CALL AXIAL2(J,NC,NP,NQ,NJ,ZE,CRSLT)
AXINT(NC,J) = CRSLT
240 CONTINUE
\[
\begin{align*}
CD1(NJ,NP,NQ) &= AXINT(1,2) + G1*AXINT(1,3) \\
CD2(NJ,NP,NQ) &= AXINT(2,2) + G1*AXINT(2,3) \\
CD3(NJ,NP,NQ) &= AXINT(3,2) + G1*AXINT(3,3) \\
CD4(NJ,NP,NQ) &= AXINT(4,2) + G1*AXINT(4,3) \\
CD1(NJ,NP,NQ) &= CD1(NJ,NP,NQ) * DCOEF * (1.0,-1.0) \\
CD2(NJ,NP,NQ) &= CD2(NJ,NP,NQ) * DCOEF * (1.0,1.0) \\
CD3(NJ,NP,NQ) &= CD3(NJ,NP,NQ) * DCOEF * (1.0,1.0) \\
CD4(NJ,NP,NQ) &= CD4(NJ,NP,NQ) * DCOEF * (1.0,-1.0)
\end{align*}
\]

200 CONTINUE

******************************************************************
CALCULATE COEFFICIENTS FOR EQUIVALENT REAL SYSTEM.

402 DO 350 NJ = 1, NJMAX
NEWJ = (2 * NJ) - 1
NEWJ1 = NEWJ + 1
DO 360 NP = 1, NJMAX2
NEWP = (2 * NP) - 1
NEWP1 = NEWP + 1

COEFFICIENTS OF LINEAR TERMS:
IF (NP GT NJMAX) GO TO 1040
CCR = REAL(CC(1,NJ,NP))
CCI = AIMAG(CC(1,NJ,NP))
C1(NEWJ,NEWP) = CCR
C1(NEWJ,NEWP1) = -CCI
C1(NEWJ1,NEWP) = CCI
C1(NEWJ1,NEWP1) = CCR
1040 CONTINUE

DO 360 KC = 1, 3
CCR = REAL(CC(KC+1,NJ,NP))
CCI = AIMAG(CC(KC+1,NJ,NP))
C(KC,NEWJ,NEWP) = CCR
C(KC,NEWJ,NEWP1) = -CCI
C(KC,NEWJ1,NEWP) = CCI
C(KC,NEWJ1,NEWP1) = CCR
360 CONTINUE

COEFFICIENTS OF THE COMBUSTION TERM:
DO 350 NP = 1, NJMAX
NEWP = 2*NP + 1
NEWP1 = NEWP + 1
CCR = REAL(CC(5,NJ,NP))
CCI = AIMAG(CC(5,NJ,NP))
E(NEWJ,NEWP,1) = CCR
E(NEWJ,NEWP,2) = -CCI
E(NEWJ,NEWP1,1) = -CCI
E(NEWJ,NEWP1,2) = -CCR
E(NEWJ1,NEWP1, 1) = CCR
E(NEWJ1,NEWP1, 2) = CCR
E(NEWJ1,NEWP1, 1) = CCR
E(NEWJ1,NEWP1, 2) = - CCI

371 CONTINUE

C
C COEFFICIENTS OF NONLINEAR TERMS.
IF (NONLIN .EQ. 0) GO TO 350
DO 370 NQ = 1, NJMAX
NEWQ = (2 * NQ) - 1
NEWQ1 = NEWQ + 1
CD1R = REAL( CD1(NJ,NP,NQ) )
CD1I = AIMAG( CD1(NJ,NP,NQ) )
CD2R = REAL( CD2(NJ,NP,NQ) )
CD2I = AIMAG( CD2(NJ,NP,NQ) )
CD3R = REAL( CD3(NJ,NP,NQ) )
CD3I = AIMAG( CD3(NJ,NP,NQ) )
CD4R = REAL( CD4(NJ,NP,NQ) )
CD4I = AIMAG( CD4(NJ,NP,NQ) )
DC(NEWJ,NEWP,NEWQ) = CD1R + CD2R + CD3R + CD4R
DC(NEWJ,NEWP,NEWQ1) = -CD1I + CD2I - CD3I + CD4I
DC(NEWJ1,NEWP1,NEWQ) = -CD1I - CD2I + CD3I + CD4I
DC(NEWJ1,NEWP1,NEWQ1) = -CD1R + CD2R + CD3R - CD4R
DC(NEWJ,NEWP,NEWQ) = CD1I + CD2I + CD3I + CD4I
DC(NEWJ1,NEWP1,NEWQ1) = -CD1I + CD2I + CD3I + CD4I

370 CONTINUE

350 CONTINUE

C
IF (NPRTKL .EQ. 0) GO TO 1035
DO 1050 NJ = 1, NJMAX
NJM = NJ + NJMAX
NEWJ = 2*NJ - 1
NEWJ1 = NEWJ + 1
DO 1050 NP = 1, NJMAX2
NEWP = 2*NJ - 1
NEWP1 = NEWP + 1
DO 830 KC = 1, 2
CCR = REAL( CCPAR(KC+1,NJ,NP) )
CCI = AIMAG( CCPAR(KC+1,NJ,NP) )
CPAR(KC,NEWJ,NEWP) = CCR
CPAR(KC,NEWJ1,NEWP1) = -CCI
CPAR(KC,NEWJ1,NEWP) = CCI
CPAR(KC,NEWJ1,NEWP1) = CCR

830 CONTINUE
IF (NP .LE. NJMAX) GO TO 1050
NEWP = NEWP - NJMAX2
NEWP1 = NEWP + 1
CCR = REAL( CCPAR(1,NJ,NP) )
CCI = AIMAG( CCPAR(1,NJ,NP) )
CPAR(NEWJ,NEWP) = CCR
CPAR(NEWJ,NEWP1) = -CCI
CPAR(NEWJ,NEWP) = CCI
CPAR(NEWJ,NEWP1) = CCR

1050 CONTINUE
1035 CONTINUE
COMPUTE COEFFICIENTS FOR THE EQUATIONS WHICH ARE DECOUPLED IN THE SECOND DERIVATIVES.

DO 405 KC = 1, 6
KMAX(KC) = 0
405 CONTINUE

CALCULATE INVERSE OF THE MATRIX C(I,J).

JMAX = NJMAX
NJMAX = 2 * NJMAX
JMAX2 = NJMAX2
NJMAX2 = 2 * NJMAX2

V(1) = 1
CALL GJR(C1,MAXMD2,MAXMD2,NJMAX,0,JC,V)

USE INVERSE TO CALCULATE DECOUPLED COEFFICIENTS.

LINEAR COEFFICIENTS.

DO 430 NP = 1, NJMAX2
DO 420 NJ = 1, NJMAX
DO 420 KC = 1, 3
TS(KC,NJ) = 0.0
DO 420 K = 1, NJMAX
420 CONTINUE

DO 430 NJ = 1, NJMAX
DO 430 KC = 1, 3
C(KC,NJ,NP) = TS(KC,NJ)
ABSVAL = ABS(C(KC,NJ,NP))
IF (ABSVAL .GE. SM1) KMAX(KC) = KMAX(KC) + 1
430 CONTINUE

COEFFICIENTS OF THE COMBUSTION RESPONSE TERM.

DO 720 NP = 1, NJMAX
DO 725 NJ = 1, NJMAX
TSR(1,NJ) = 0.0
TSR(2,NJ) = 0.0
DO 725 K = 1, NJMAX
TSR(1,NJ) = TSR(1,NJ) + C1(NJ,K) * E(K,NP,1)
TSR(2,NJ) = TSR(2,NJ) + C1(NJ,K) * E(K,NP,2)
725 CONTINUE

DO 730 NJ = 1, NJMAX
E(NJ,NP,1) = TSR(1,NJ)
ABSVAL = ABS(E(NJ,NP,1))
IF (ABSVAL .GE. SM1) KMAX(4) = KMAX(4) + 1
E(NJ,NP,2) = TSR(2,NJ)
ABSVAL = ABS(E(NJ,NP,2))
IF (ABSVAL .GT. SM1) KMAX(5) = KMAX(5) + 1
730 CONTINUE

720 CONTINUE
C
IF (NPRTKL .EQ. 0) GO TO 1060
KMAXPR(1) = 0
KMAXPR(2) = 0
V(1) = 1
CALL GJRC1PAR,MAXMD2,MAXMD2,NJMAX,0,JC,V)
DO 1065 NP = 1, NJMAX
DO 1070 NJ = 1, NJMAX
DO 1070 KC = 1, 2
TSPAR(KC,NJ) = 0.0
DO 1070 K = 1, NJMAX
TSPAR(KC,NJ) = TSPAR(KC,NJ) + C1PAR(NJ,K) * CPAR(KC,K,NP)
1070 CONTINUE
DO 1065 NJ = 1, NJMAX
DO 1065 KC = 1, 2
CPAR(KCNJ,NP) = TSPAR(KC,NJ)
ABSVAL = ABS(CPAR(KCNJ,NP))
IF (ABSVAL .GE. SM1) KMAXPR(KC) = KMAXPR(KC) + 1
1065 CONTINUE
1060 CONTINUE
C
C
C
IF (NONLIN .EQ. 0) GO TO 410
DO 735 NP = 1, NJMAX
DO 735 NQ = 1, NJMAX
DO 440 NJ = 1, NJMAX
TSQ(NJ) = 0.0
DO 440 K = 1, NJMAX
TSQCNJ) = TSQ(NJ) + C1(NJ,K) * D(K,NP,NQ)
440 CONTINUE
DO 445 NJ = 1, NJMAX
D(NJa NPNQ) = TSQ(NJ)
ABSVAL = ABS(D(NJ, NP,NQ))
IF (ABSVAL .GT. SM2) KMAX(6) = KMAX(6) + 1
445 CONTINUE
735 CONTINUE
410 CONTINUE
C
C
C
******************************************************************************
OUTPUT:
C
IF (NOUT .EQ. 2) GO TO 455
WRITE (6,6001) TITLE
WRITE (6,6002) GAM, UE
IF (NOZZLE .EQ. 0) WRITE (6,6012)
IF (NPRTKL .EQ. 0) WRITE (6,6022)
IF (NPRTKL .EQ. 1) WRITE (6,6021) DIAl, CM, FREQ,
1 TEMP, SP, NHOM, PARTKL
WRITE (6,6004)
DO 310 J = 1, JMAX
WRITE (6,6003) NAME(J), J, L(J), B(J), YNOZ(J)
310 CONTINUE
IF (NONLIN .EQ. 0) WRITE (6,6013)
OUTPUT OF LINEAR COEFFICIENTS.
DO 320 KC = 1, 3
NJS = 0
NJF = 0
KOUNTJ = 1
758 NJS = NJF + 1
NJF = 10 * KOUNTJ
IF (NJF * GT* NJMAX) NJF = NJMAX
NPS = 0
NPF = 0
KOUNTP = 1
754 NPS = NPF + 1
NPF = 10 * KOUNTP
IF (NPF * GT* NJMAX2) NPF = NJMAX2
IF (KC * EQ* 1) WRITE (6,6005)
IF (KC * EQ* 2) WRITE (6,6006)
IF (KC * EQ* 3) WRITE (6,6007)
WRITE (6,6008) (NP, NP = NPS, NPF)
WRITE (6,6014)
DO 750 NJ = NJS, NJF
WRITE (6,6009) NJ, (C(KC, NJ, NP), NP = NPS, NPF)
750 CONTINUE
IF (NPF EQ NJMAX2) GO TO 752
KOUNTP = KOUNTP + 1
GO TO 754
752 IF (NJF EQ NJMAX) GO TO 756
KOUNTJ = KOUNTJ + 1
GO TO 758
756 CONTINUE
320 CONTINUE

OUTPUT OF THE COMBUSTION RESPONSE TERM.
DO 770 KC = 1, 2
NJS = 0
NJF = 0
KOUNTJ = 1
760 NJS = NJF + 1
NJF = 10 * KOUNTJ
IF (NJF * GT* NJMAX) NJF = NJMAX
NPS = 0
NPF = 0
KOUNTP = 1
762 NPS = NPF + 1
NPF = 10 * KOUNTP
IF (NPF * GT* NJMAX) NPF = NJMAX
IF (KC * EQ* 1) WRITE (6,6019)
IF (KC * EQ* 2) WRITE (6,6020)
WRITE (6,6008) (NP, NP = NPS, NPF)
WRITE (6,6014)
DO 764 NJ = NJS, NJF
WRITE (6,6009) NJ, (E(NJ,NP,KC), NP = NPS, NPF)
764 CONTINUE
IF (NPF .EQ. NJMAX) GO TO 766
KOUNTP = KOUNTP + 1
GO TO 762

766 IF (NJF .EQ. NJMAX) GO TO 768
KOUNTJ = KOUNTJ + 1
GO TO 760

768 CONTINUE
770 CONTINUE

C
IF (NPRTL .EQ. 0) GO TO 835
DO 1080 KC = 1, 2
NJS = 0
NJF = 0
KOUNTJ = 1

1072 NJS = NJF + 1
NJF = 10*KOUNTJ
IF (NJF .GT. NJMAX) NJF = NJMAX
NPS = 0
NPF = 0
KOUNTP = 1

1074 NPS = NPF+1
NPF = 10*KOUNTP
IF (NPF .GT. NJMAX2) NPF = NJMAX2
WRITE (6,6023)
WRITE (6,6008) (NP, NP = NPS,NPF)
WRITE (6,6014)
DO 1076 NJ = NJS,NJF
WRITE (6,6009) NJ, (CPAR(KC,NJ,NP),NP = NPS,NPF)

1076 CONTINUE
IF (NPF .EQ. NJMAX2) GO TO 1078
KOUNTP = KOUNTP + 1
GO TO 1074

1078 IF (NJF .EQ. NJMAX) GO TO 1080
KOUNTJ = KOUNTJ + 1
GO TO 1072

1080 CONTINUE
835 CONTINUE

C
OUTPUT OF NONLINEAR COEFFICIENTS.
IF (NONLIN .EQ. 0) GO TO 452
DO 400 NJ = 1, NJMAX
NPS = 0
NPF = 0
KOUNTP = 1

780 NPS = NPF + 1
NPF = 10 * KOUNTP
IF (NPF .GT. NJMAX) NPF = NJMAX
NQS = 0
NQF = 0
KOUNTQ = 1

776 NQS = NQF + 1
NQF = 10 * KOUNTQ
IF (NQF .GT. NJMAX) NQF = NJMAX
WRITE (6,6010) NJ
WRITE (6,6011) (NQ, NQ = NQS, NQF)
WRITE (6,6015)
DO 772 NP = NPS, NPF
WRITE (6,6009) NP, (D(NJ,NP,NQ), NQ = NQS, NQF)
    772 CONTINUE
    771 CONTINUE
IF (NQF .EQ. NJMAX) GO TO 774
KOUNTQ = KOUNTQ + 1
GO TO 776
774 IF (NPF .EQ. NJMAX) GO TO 778
KOUNTP = KOUNTP + 1
GO TO 780
778 CONTINUE
400 CONTINUE
452 IF (NOUT .EQ. 0) GO TO 4
C WRITE COEFFICIENTS ON FILE.
C 455 WRITE (9,7001) GAMMA, UE, ZE, NJMAX, NPRTKL
    IF (NPRTKL .EQ. 1) WRITE (9,7007) DIA, RHOM, SP, TEMP, FREQ, 1
        PARTKL, CM
    DO 450 J = 1, JMAX
        WRITE (9,7002) J, L(J), NAME(J)
    450 CONTINUE
    DO 457 J = 1, JMAX
        WRITE (9,7006) J, YNOZ(J), B(J)
    457 CONTINUE
    DO 460 KC = 1, 3
        WRITE (9,7003) KMAX(KC)
    DO 460 NJ = 1, NJMAX
    DO 460 NP = 1, NJMAX2
        ABSVAL = ABS( C(KC,NJ,NP) )
        IF (ABSVAL .GE. SM1) WRITE (9,7004) NJ, NP, C(KC,NJ,NP)
    460 CONTINUE
    DO 820 KC = 4, 5
        WRITE (9,7003) KMAX(KC)
        KCMIN3 = KC - 3
    DO 820 NJ = 1, NJMAX
    DO 820 NP = 1, NJMAX
        ABSVAL = ABS(E(NJ,NP,KCMIN3))
        IF (ABSVAL .GT. SM1) WRITE (9,7004) NJ, NP, E(NJ,NP,KCMIN3)
    820 CONTINUE
DO 1082 KC = 1, 2
WRITE (9, 7003) KMAXPR(KC)
DO 1082 NJ = 1, NJMAX
DO 1082 NP = 1, NJMAX
ABSVAL = ABS(CPAR(KC, NJ, NP))
IF (ABSVAL .GE. SM1) WRITE (9, 7004) NJ, NP, CPAR(KC, NJ, NP)
1082 CONTINUE
C
WRITE (9, 7003) KMAX(6)
IF (NONLIN =EQ. 0) GO TO 4
DO 470 NJ = 1, NJMAX
DO 470 NP = 1, NJMAX
DO 470 NQ = 1, NJMAX
ABSVAL = ABS(D(NJ, NP, NQ))
IF (ABSVAL .GE. SM2) WRITE (9, 7005) NJ, NP, NQ, D(NJ, NP, NQ)
470 CONTINUE
GO TO 4
C
600 CONTINUE
C
C  *****************************************************************
C  FORMAT SPECIFICATIONS.
C  5000 FORMAT (7A10)
C  5001 FORMAT (2F10.0, I 5)
C  5002 FORMAT (2I5, 1X, A4)
C  5003 FORMAT (I 5, 2F10.0)
C  5004 FORMAT (6I5)
C  5005 FORMAT (2F10.0)
C  5006 FORMAT (6F10.0)
C  6001 FORMAT (1H1, 1X, 7A10//)
C  6002 FORMAT (2X, 8HGAMMA = F8.5, 5X, 4HUE =, F6.4, //)
C  6003 FORMAT (2X, A4, 2X, J, 2X, 8HLPS, 2X, 2HYR, 2HYI //)
C  6004 FORMAT (2X//2X, 14HNAME = J, L, 6X, 3HEPS, 7X, 3HETA, 1
C  8X, 2HYR, 7X, 2HYI //)
C  6005 FORMAT (1H1, 4H DECOPLED COEFFICIENT OF B(P), C(J, P)///)
C  6006 FORMAT (1H1, 4H DECOPLED COEFFICIENT OF THE DERIVATIVE OF,
C  1 6H B(P), 5X, 8HC(J, P)///)
C  6007 FORMAT (1H1, 3H DECOPLED COEFFICIENT OF THE TER,
C  1 15H MULTIPLYING Ks, 5X, 8HC(3, J, P)///)
C  6008 FORMAT (7X, 1HP, I8, 9112)
C  6009 FORMAT (2X//2X, I3, 3X, 10F12.6)
C  6010 FORMAT (1H1, 47HD6COUPL D NONLINEAR COEFFICIENT IN GAS EQUATION,
C  1 7H FOR B(12, 1H)///)
C  6011 FORMAT (7X, 1HP, I8, 9112)
C  6012 FORMAT (2X, 20HQUASI-STEADY NOZZLE///)
C  6013 FORMAT (2X//2X, 2A4LINEAR COEFFICIENTS ONLY)
C  6014 FORMAT (4X, 1HJ)
C  6015 FORMAT (4X, 1HP)
C  6019 FORMAT (1H1, 4H DECOPLED COEFFICIENT OF THE REAL PART,
C  1 24H OF THE COMBUSTION TER, 5X, 8HE(J, P, 1)///)
6020 FORMAT (1H1, 45H DECOPLED COEFFICIENT OF THE IMAGINARY PART, 1 24H OF THE COMBUSTION TERM, 5X, 8HE(J, P,2)///)
6021 FORMAT (///, 10X, 27HPARTICLE DIA (IN MICRONS) = , F5.2, 10X,
1 4HCN = , F6.4, 10X, 18HFREQ (IN HERTZ) = , F6.1,///,
2 10X, 26HCHAMBER TEMP (IN DEG K) = , F6.1, 10X, 4HSP = ,
3 F6.4, 10X, 27HRHOM (IN KG/CUBIC METER) = , F6.1,///, 10X,
4 30HPARTICLE DRAG COEFFICIENT, K = , F8.4,///)
6022 FORMAT (2X, 22HPARTICLES NOT PRESENT///)
6023 FORMAT (1H1, 39HCOEFFICIENTS IN THE PARTICLE EQUATIONS:
1 12H CPAR(J, P)///)
7001 FORMAT (3F10.5, 3I5)
7002 FORMAT (2I5, 1X, A4)
7003 FORMAT (I5)
7004 FORMAT (2I5, F15.8)
7005 FORMAT (3I5, F15.8)
7006 FORMAT (I5, 4F12.8)
7007 FORMAT (7F15.8)
END
SUBROUTINE AXIAL1(NOPT, NP, NJ, UE, ZE, RESULT)

THIS SUBROUTINE CALCULATES THE INTEGRAL OVER THE INTERVAL (0, ZE) OF THE FOLLOWING FUNCTIONS ACCORDING TO THE VALUE OF NOPT:

\[ \begin{align*}
\text{NOPT} = 1 & \quad Z(NP) \cdot ZC(NJ) \\
\text{NOPT} = 2 & \quad ZPP(NP) \cdot ZC(NJ) \\
\text{NOPT} = 3 & \quad UP \cdot Z(NP) \cdot ZC(NJ) \\
\text{NOPT} = 4 & \quad U \cdot ZP(NP) \cdot ZC(NJ)
\end{align*} \]

IN THE ABOVE EQUATIONS:
- \( Z(NP) \) is the axial acoustic eigenfunction of index NP.
- \( Z(NJ) \) is the axial acoustic eigenfunction of index NJ.
- \( ZC \) is the complex conjugate of the axial eigenfunction.
- \( ZP \) and \( ZPP \) are the first and second derivatives of the axial eigenfunctions respectively.
- \( U \) is the steady state velocity distribution and \( UP \) is its axial derivative.
- The velocity distribution is computed by the subroutine UBAR.

REAL MAG
COMPLEX CI, CZE, BP, BJ, T1, T2, CH, F1, F2, F3, CZ, ARG, S1, S2, S3, RESULT, FUNCT(500), B(6), CCOSH, CSINH
COMMON B

MAXMD = 6
CI = (0.0, 1.0)
CZE = CMPLX(ZE, 0.0)
BP = B(NP)
BJ = CONJG(B(NJ))

IF (NOPT GT 2) GO TO 50
CALCULATE INTEGRALS BY MEANS OF ANALYTICAL EXPRESSIONS FOR NOPT = 1 AND NOPT = 2:
ARG = (BP + BJ) * CI
MAG = CABS(ARG)
IF (MAG) 20, 25, 20
20 T1 = CSINH(ARG*CZE)/ARG
GO TO 30
25 T1 = CZE
30 ARG = (BP - BJ) * CI
MAG = CABS(ARG)
IF (MAG) 35, 40, 35
35 T2 = CSINH(ARG*CZE)/ARG
GO TO 45
40 T2 = CZE
45 RESULT = (T1 + T2) * (0.5, 0.0)
IF (NOPT EQ 2) RESULT = -B(NP) * B(NP) * RESULT
GO TO 100
NUMERICAL EVALUATION OF INTEGRALS FOR NOPT = 3 AND NOPT = 4.

COMPUTE STEP SIZE FOR SIMPSON INTEGRATION.

50 N = 50
RN = N
RESULT = (0.0, 0.0)
NOPT2 = NOPT - 2

H = ZE/RN
ZO = 0.0
NP1 = N + 1
CH = CMPLX(H, 0.0)

COMPUTE INTEGRANDS.
DO 60 I = 1, NP1
STEP = I - 1
Z = (STEP * H) + ZO
CZ = CMPLX(Z, 0.0)
ARG = CI * BP
IF (NOPT2 .EQ. 2) GO TO 120
CALL UBAR(2, UE, ZE, Z, F)
F2 = CCOSH(ARG*CZ)
GO TO 170

120 CALL UBAR(1, UE, ZE, Z, F)
F2 = ARG * CSINH(ARG*CZ)

170 CONTINUE
F1 = CMPLX(F, 0.0)
ARG = CI * BJ
F3 = CCOSH(ARG*CZ)
FUNCT(I) = F1 * F2 * F3

60 CONTINUE

PERFORM SIMPSON INTEGRATION.
NM1 = N - 1
S1 = FUNCT(1) + FUNCT(NP1)
S2 = (0.0, 0.0)
S3 = (0.0, 0.0)
DO 70 I = 2, NM1, 2
S2 = S2 + FUNCT(I)
70 CONTINUE
DO 80 I = 3, NM1, 2
S3 = S3 + FUNCT(I)
80 CONTINUE
RESULT = RESULT +
1 CH * (S1 + (4.0, 0.0)*S2 + (2.0, 0.0)*S3)/(3.0, 0.0)

100 CONTINUE
RETURN
END
SUBROUTINE AXIAL2(NOPT, NCONJ, NP, NQ, NJ, ZE, RESULT)

THIS SUBROUTINE CALCULATES THE INTEGRAL OVER THE INTERVAL
(0, ZE) OF THE FOLLOWING FUNCTIONS ACCORDING TO THE VALUES
OF NOPT AND NCONJ

FOR NCONJ = 1 AND
NOPT = 1 Z(NP) * Z(NQ) * ZC(NJ)
NOPT = 2 ZP(NP) * ZP(NQ) * ZC(NJ)
NOPT = 3 ZPP(NP) * Z(NQ) * ZC(NJ)
NOPT = 4 ZP(NP) * Z(NQ) * ZC(NJ)

FOR NCONJ = 2 AND
NOPT = 1 Z(NP) * ZC(NQ) * ZC(NJ)
NOPT = 2 ZP(NP) * ZPC(NQ) * ZC(NJ)
NOPT = 3 ZPPC(NP) * Z(NQ) * ZC(NJ)

FOR NCONJ = 3 AND
NOPT = 1 ZC(NP) * Z(NQ) * ZC(NJ)
NOPT = 2 ZPC(NP) * ZP(NQ) * ZC(NJ)
NOPT = 3 ZPPC(NP) * Z(NQ) * ZC(NJ)

FOR NCONJ = 4 AND
NOPT = 1 ZC(NP) * ZC(NQ) * ZC(NJ)
NOPT = 2 ZPC(NP) * ZPC(NQ) * ZC(NJ)
NOPT = 3 ZPPC(NP) * ZC(NQ) * ZC(NJ)

IN THE ABOVE EQUATIONS:
Z(NP), Z(NQ), AND Z(NJ) ARE THE AXIAL ACOUSTIC EIGENFUNCTIONS
AND NP, NQ, AND NJ ARE THEIR INDICES.
ZP IS THE FIRST DERIVATIVE OF THE AXIAL EIGENFUNCTIONS.
ZPP IS THE SECOND DERIVATIVE OF THE AXIAL EIGENFUNCTIONS.
ZC AND ZPC ARE COMPLEX CONJUGATES OF Z AND ZP RESPECTIVELY.

REAL MAG
COMPLEX CI, CF, CZE, BP, BQ, BJ, SUM, RESULT,
1 ARG(4), FUNCT(4), B(6), CCOSH, CSINH
COMMON B
MAXMD = 6

CALCULATE INTEGRALS BY MEANS OF ANALYTICAL EXPRESSIONS.
CI = (0.0, 1.0)
CF = (0.25, 0.0)
CZE = CMPLX(ZE, 0.0)
BP = B(NP)
BQ = B(NQ)
BJ = CONJG(B(NJ))
IF ((NCONJ EQ 2) OR (NCONJ EQ 4)) BQ = CONJG(BQ)
IF (NCONJ GT 2) BP = CONJG(BP)
ARG(1) = (BP + BQ + BJ) * CI
ARG(2) = (BP + BQ - BJ) * CI
ARG(3) = (BP - BQ + BJ) * CI
ARG(4) = (BP - BQ - BJ) * CI
DO 10 J = 1, 4
MAG = CABS(ARG(J))
IF (MAG) 12, 15, 12
12 FUNCT(J) = CSINH(ARG(J)*CZE)/ARG(J)
IF (NOPT EQ 4) FUNCT(J) = (CCOSH(ARG(J)*CZE) - 1.0)/ARG(J)
GO TO 10
15 FUNCT(J) = CZE
IF (NOPT EQ 4) FUNCT(J) = 0.0
10 CONTINUE
IF (NOPT EQ 2) GO TO 30
SUM = FUNCT(1) + FUNCT(2) + FUNCT(3) + FUNCT(4)
RESULT = CF * SUM
IF (NOPT EQ 3) RESULT = -BP * BP * RESULT
IF (NOPT EQ 4) RESULT = CI * BP * RESULT
GO TO 50
30 SUM = FUNCT(1) + FUNCT(2) - FUNCT(3) - FUNCT(4)
RESULT = -CF * BP * BQ * SUM
50 CONTINUE
RETURN
END
SUBROUTINE EIGVAL(L, SMN, GAMMA, ZE, YAMPL, YPHASE, RESULT)

C

COMPLEX RESULT
COMMON /BLK1/ GS0, ABSQ, ALBET, SMNSQ
C
C
C
C
C
C
C
C
C
C
C

*******************************************************************************

THIS SUBROUTINE COMPUTES THE COMPLEX AXIAL ACOUSTIC EIGENVALUES FOR A CYLINDRICAL CHAMBER WITH A NOZZLE AND STORES THEM IN RESULT.

THE EIGENVALUES ARE COMPUTED BY MEANS OF NEWTON'S METHOD.

THE INPUT PARAMETERS ARE AS FOLLOWS

L IS THE AXIAL MODE NUMBER.

SMN IS THE DIMENSIONLESS ACOUSTIC FREQUENCY.

GAMMA IS THE SPECIFIC HEAT RATIO.

ZE IS THE DIMENSIONLESS LENGTH OF THE CHAMBER.

YAMPL IS THE NOZZLE AMPLITUDE FACTOR.

YPHASE IS THE NOZZLE PHASE SHIFT IN DEGREES.

********************************************************************************

PI = 3.1415926536
ERR = 0.0000001

IF (YAMPL) 5, 60, 5
CALCULATE CONSTANTS

5 PHASE = YPHASE * PI/180.0
ALPHA = YAMPL * COS(PHASE)
BETA = YAMPL * SIN(PHASE)
GSQ = GAMMA * GAMMA
ABSQ = (ALPHA * ALPHA) - (BETA * BETA)
ALBET = ALPHA * BETA
SMNSQ = SMN * SMN

ASSIGN INITIAL GUESS FOR EIGENVALUE
IF (L .EQ. 0) GO TO 45
RL = L
PHI = PI/2.0 + PHASE
XM = RL * PI/ZE
A = YAMPL/ZE
XO = XM + A*COS(PHI)
YO = A*SIN(PHI)
GO TO 47

45 PHI = PI/4.0 + 0.5*PHASE
A = YAMPL * 10.0/ZE
XO = A * COS(PHI)
YO = A * SIN(PHI)
ITERATION USING NEWTON'S METHOD FOR A SYSTEM OF TWO EQUATIONS
IN TWO unknowns.

47 L1 = 0
   X = X0
   Y = Y0
40 CALL FCNS(X, Y, ZE, F, G, FX, FY, GX, GY)
   IF (L1 * 40) GO TO 50
   RJFG = (FX * GY) - (GX * FY)
   IF (RJFG < 20) 30, 20
20 DELTAX = (-F * GY + G * FY) / RJFG
   DELTAY = (-G * FX + F * GX) / RJFG
   LI = LI + 1
   X = X + DELTAX
   Y = Y + DELTAY

TEST FOR CONVERGENCE:
   IF (ABS(DELTAX) * GE. ERR OR ABS(DELTAY) * GE. ERR) GO TO 40
   GO TO 10

WARNING MESSAGES
30 WRITE (6, 6005)
   GO TO 10
50 WRITE (6, 6006)
   GO TO 10

CASE OF HARD WALL (YAMPL = 0).
60 RL = L
   X = RL * PI/ZE
   Y = 0.0
10 RESULT = CMPLX(X, Y)

FORMAT SPECIFICATIONS.
6005 FORMAT (2X/6HJACOBIAN IS ZERO//)
6006 FORMAT (2X/6HFAILED TO CONVERGE IN 40 ITERATIONS//)
RETURN
END
SUBROUTINE FCNS(X,Y,Z,E,F,G,FX,FY,GX,GY)

C

C THIS SUBROUTINE COMPUTES THE FUNCTIONS F(X,Y) AND G(X,Y)
AND THEIR PARTIAL DERIVATIVES WITH RESPECT TO X AND Y.

C

COMMON /BLK1/ GSQ, ABSQ, ALBET, SMNSQ

C

C COMPUTE THE TRIGONOMETRIC FUNCTIONS, THE HYPERBOLIC FUNCTIONS
AND THEIR SQUARES.

C

I = 1
ARGX = ZE * X
ARGY = ZE * Y
10 SX = SIN(ARGX)
CX = COS(ARGX)
SHY = SINH(ARGY)
CHY = COSH(ARGY)
IF (I .EQ. 2) GO TO 20
SX SQ = SX * SX
CXSQ = CX * CX
SHYSQ = SHY * SHY
CHYSQ = CHY * CHY
ARGX = 2.0 * ARGX
ARGY = 2.0 * ARGY
I = 2
GO TO 10

C

C COMPUTE TRANSCENDENTAL FUNCTIONS AND THEIR DERIVATIVES

C

20 FF = (SXSQ * CHYSQ) - (CXSQ * SHYSQ)
GG = (CXSQ * CHYSQ) - (SXSQ * SHYSQ)
HH = 0.25 * SX * SHY
FFX = ZE * SX * CHY
GGY = ZE * CX * SHY
FFY = -GGY
GGX = -FFX
HHX = 0.5 * GGY
HHY = 0.5 * FFX

C

C COMPUTE FACTORS

XYSQ = (X * X) - (Y * Y)
XY = X * Y
SMNX = SMNSQ + XYSQ
F1 = (ABSQ * SMNX) - (4.0 * ALBET * XY)
F2 = (ALBET * SMNX) + (ABSQ * XY)
G1 = (ABSQ * SMNX) + (4.0 * ALBET * XY)
FX1 = (2.0 * X * ABSQ) - (4.0 * ALBET * Y)
FX2 = (2.0 * X * ALBET) + (ABSQ * Y)
FY1 = (-2.0 * Y * ABSQ) - (4.0 * ALBET * X)
FY2 = (-2.0 * Y * ALBET) + (ABSQ * X)
GX1 = (2.0 * X * ABSQ) + (4.0 * ALBET * Y)
GY1 = (-2.0 * Y * ABSQ) + (4.0 * ALBET * X)
COMPUTE F(X, Y) AND G(X, Y)

\[
F = (XYSQ \times FF) - (4.0 \times XY \times HH)
1 + GSQ \times ((F1 \times GG) + (4.0 \times F2 \times HH))
\]

\[
G = (XYSQ \times HH) + (XY \times FF)
1 + GSQ \times ((F2 \times GG) - (G1 \times HH))
\]

COMPUTE THE PARTIAL DERIVATIVES OF F AND G

\[
FX = (2.0 \times X \times FF) + (XYSQ \times FFX)
1 -4.0 \times ((Y \times HH) + (XY \times HHX))
2 + GSQ \times ((FX1 \times GG) + (F1 \times GGX))
3 + (4.0 \times FX2 \times HH) + (4.0 \times F2 \times HHX))
\]

\[
FY = (-2.0 \times Y \times FF) + (XYSQ \times FFY)
1 -4.0 \times ((X \times HH) + (XY \times HHY))
2 + GSQ \times ((FY1 \times GG) + (F1 \times GGY))
3 + (4.0 \times FY2 \times HH) + (4.0 \times F2 \times HHY))
\]

\[
GX = (2.0 \times X \times HH) + (XYSQ \times HHX)
1 + (Y \times FF) + (XY \times FFX)
2 + GSQ \times ((FX2 \times GG) + (F2 \times GGX))
3 - (GX1 \times HH) - (G1 \times HHX))
\]

\[
GY = (-2.0 \times Y \times HH) + (XYSQ \times HHY)
1 + (X \times FF) + (XY \times FFY)
2 + GSQ \times ((FY2 \times GG) + (F2 \times GGY))
3 - (GY1 \times HH) - (G1 \times HHY))
\]

RETURN
END
SUBROUTINE GJR(A,NC, NR, NaMC, JC, V)
DIMENSION A(NC, NR), JC(NR), V(2)
MW=V(1)
M=1
S=1.
L=NC+(MC-N)*(MW/4)
KD=2-MOD(MW/2, 2)
IF(KD.EQ.1) V(2)=0.
KI=2-MOD(MW, 2)
IF (KI .EQ. 2) GO TO 20
5 DO 10 I=1,N
10 JC(I)=I
20 DO 91 I=1..N
IF (KI .EQ. 1) GO TO 22
21 M=I
22 IF(I .EQ. N) GO TO 60
X=-1
DO 30 J=IoN
IF (X .GT. ABS( A(J, I) ) ) GO TO 30
X=ABS(A(J, I) )
K=J
30 CONTINUE
IF(K .EQ. I) GO TO 60
Scs—S
V(1)=—V(1)
IF (KI .EQ. 2) GO TO 40
35 MU=JC(I)
JC(I)=JC(K)
JC(K)=MU
40 DO 50 J=M,L
X=A(I, J)
A(I, J)=A(K, J)
50 A(K, J)=X
60 IF(ABS(A(I, I)) .GT. 0. ) GO TO 70
IF(KD.EQ.1) V(1)=0.
JC(1)=I—1
WRITE (6, 1000)
1000 FORMAT (1X, 29HTROUBLE WITH MATRIX INVERSION))
WRITE (6, 1001) JC(1)
1001 FORMAT (1X, 6HJC(1)=, I 4)
STOP
70 IF (KD .EQ. 2) GO TO 72
71 IF(A(I, I)+LT.0. ) S=-S
V(2)=V(2)+ALOG(ABS(A(I, I)))
72 X=A(I, I)
A(I, I)=1.
DO 80 J=M,L
A(I, J)=A(I, J)/X
80 CONTINUE
DO 91 K=1,N
IF(K.EQ.I) GO TO 91
X=A(KsI)
A(KsI)=0.
DO 90 J=M,L
A(KsJ)=A(KsJ)-X*A(I,J)
90 CONTINUE
91 CONTINUE
IF (Kl .EQ. 2) GO TO 140
95 DO 130 J=1,N
IF(JC(J) .EQ. J) GO TO 130
JJ=J+1
DO 100 I=JJ,N
IF(JC(I) .EQ. J) GO TO 110
100 CONTINUE
110 JC(I)=JC(J)
DO 120 K=1,N
X=A(KsI)
A(KsI)=A(KsJ)
120 A(KsJ)=X
130 CONTINUE
140 JC(1)=N
IF(KD .EQ. 1) V(1)=S
RETURN
END
SUBROUTINE UBAR(NOPT, UE, ZE, Z, RESULT)

C
C THIS SUBROUTINE CALCULATES THE STEADY STATE VELOCITY DISTRIBUTION.
C UE IS THE EXIT MACH NUMBER.
C ZE IS THE DIMENSIONLESS LENGTH.
C Z IS THE AXIAL COORDINATE.
C
C IF NOPT = 1 THE DISTRIBUTION IS CALCULATED.
C IF NOPT = 2 THE DERIVATIVE IS CALCULATED.
C
IF (NOPT .EQ. 1) RESULT = UE * Z / ZE
IF (NOPT .EQ. 2) RESULT = UE / ZE
RETURN
END

COMPLEX FUNCTION CCOSH(X)
COMPLEX X
CCOSH = 0.5 * (CEXP(X) + CEXP(-X))
RETURN
END

COMPLEX FUNCTION CSINH(X)
COMPLEX X
CSINH = 0.5 * (CEXP(X) - CEXP(-X))
RETURN
END
3.2 PROGRAM SOLID2

In conjunction with Program SOLIDI, Program SOLID2 calculates the non-linear stability characteristics of a cylindrical combustor according to the approximate theory developed in Volume I. Using the coefficients computed by SOLIDI, this program integrates the system of differential equations for the mode amplitudes (i.e., Equations (22) and (23)) and computes the time-history of a pressure disturbance in the motor.

**Program Structure.** This program performs the following operations: (1) reads the input data, (2) calculates the initial values, (3) numerically integrates the differential equations, and (4) plots and prints the solutions.

The inputs to the program include the data generated by SOLIDI, the combustion response parameters, various control numbers, plotting information and a description of the initial disturbance. The data from SOLIDI is read first and then printed out. Next the space dependent coefficients appearing in the series expansions for $\hat{\xi}_t$ and $\hat{\xi}_x$ are computed and printed. These coefficients are calculated by subroutine PHICFS for use in the computation of the pressure perturbation. The remaining input data is then read, and following program execution, control is returned to this point so that several cases may be run for a given set of coefficients generated by SOLIDI.

The initial disturbance may be specified as a purely fundamental mode (1L) disturbance or as an arbitrary combination of modes. The number of modes for which initial disturbances are specified is NTERMS, and for each of those modes the amplitudes AST and ACT are specified. Thus the initial waveform of the real part of the amplitude for each specified mode is given by:

$$B_{2j-1}(t) = AST \sin(\omega_j t) + ACT \cos(\omega_j t)$$

where $B_{2j-1}$ is the real part of the amplitude of the $j$th mode and $\omega_j$ is the acoustic frequency of the $j$th mode. The amplitudes, $B_{2j}$, of the imaginary parts are calculated in the program by requiring the individual modes to satisfy the nozzle admittance condition. The initial values of the amplitudes of those modes whose initial disturbance is specified to be non-zero are then printed out. It must be noted that the specified inputs AST and ACT refer to the amplitude of the velocity potential and not to the pressure amplitude.

After the starting values are calculated, numerical integration of the system of differential equations is performed using a fourth-order Runge-Kutta scheme with the specified step size. A step size of about 0.025 is recommended.
Subroutine RHS is used to compute the differential increments needed in the Runge-Kutta integration. From the computed amplitude functions and the coefficients from the subroutine PHICFS the pressure perturbation at each step is calculated using the subroutine PRSVEL. The computed value of the pressure perturbation is then printed out if the pressure history printout is desired.

If plotting of the pressure history is required, subroutine GRAPHS is used to obtain CALCOMP plots for the desired pressure waveforms.

The program then computes the dimensional frequency of oscillation and the pressure growth rate at the end of each oscillation cycle using the subroutine GROWTH.

Description of Input. The input data required to run this program consists of three parts: (1) the control numbers NOUTCF and NHISTR which determine the extent of desired printed output, as explained later in this section, (2) the parameters and coefficients generated by SOLID1, and (3) data describing the case to be run. For each case, the following information must be provided: the combustion response parameters, description of the initial disturbance, various control numbers and plotting information.

A list of necessary input data is described below. The description of format for integer and real constants, given previously for SOLID1, apply here also; i.e., each integer is allotted five columns and each real constant is allotted a field of ten columns.

The three parts of the input are:

(1) The control numbers, NOUTCF and NHISTR.
(2) The coefficients from Program SOLID1.
(3) The data deck.

The first card gives the control numbers, NOUTCF and NHISTR.

NOUTCF determines printout of coefficients:
If NOUTCF = 0 coefficients are not printed out.
If NOUTCF = 1 only linear coefficients are printed out.
If NOUTCF = 2 all coefficients are printed out.

NHISTR determines if pressure history is to be printed:
If NHISTR = 0 printed
If NHISTR = 1 not printed.

The coefficients are obtained from program SOLID1 by putting NOUT = 1 or NOUT = 2 in that program, thereby writing the coefficients into a disk.
This disk has been given the device number 9.

The data deck consists of the following cards:

First card: Title of the case.

Second card: H, TSTART, TQUIT, FREQ, BCOMB
    H is the integration step size.
    TSTART is the time at which output starts.
    TQUIT is the time at which computations are terminated.
    FREQ is the motor frequency (in pure gas) in Hertz.
    BCOMB is the combustion response nonlinearity factor.

Third card: A2PARA, B2PARA, EN, OMEGA
    A2PARA and B2PARA are the combustion parameters in the A-B model.
    EN is the pressure exponent in the burning rate law.
    OMEGA is the frequency nondimensionalized by the square of the steady-state burning rate.

Fourth card: NLOC, NTERMS, NOUT, NCOMB, NNPRT
    NLOC determines the location of the wall pressure maxima and minima:
        If NLOC = 1 location is x = 0.0
        If NLOC = 2 location is x = 1.0
        If NLOC = 3 location is x = 0.5
    NTERMS is the number of terms given initial values.
    NOUT is the output control number.
        If NOUT = 0 printed output only.
        If NOUT > 0 both printed and plotted output;
        If NOUT = 1 plot of pressure at x = 0.0 only.
        If NOUT = 2 plot of pressure at x = 0.0 and x = 1.0
        If NOUT = 3 plot of pressure at x = 0.0, 1.0 and 0.5.
    NCOMB determines if combustion nonlinearities are considered:
        If NCOMB = 0 neglected.
        If NCOMB = 1 included.
    NNPRT determines if nonlinear particle damping is considered:
        If NNPRT = 0 not considered.
        If NNPRT = 1 considered.

Next card (necessary only if NNPRT = 1): REFPRS, CPGAS, CNLP
    REFPRS is the chamber pressure, in psi.
CPGAS is the specific heat at constant pressure of the gas phase in cal/gm-deg K.
CNLP is the constant C in the amplitude dependence of the nonlinear particle drag.

Next card (necessary only if plots are required): YHI, YLAB, ITICY
YHI is the maximum ordinate for pressure plots.
Note: the ordinate scales for pressure and amplitude plots are symmetric about zero.
YLAB is the interval for ordinate labeling for above plots.
ITICY is the number of ordinate tick marks for above plots.
Note: ITICY should be negative for pressure and amplitude plots to obtain centerline.

Next card (necessary only if plots are required): MDPLOT
MDPLOT determines if plots of individual modes are required:
If plot of $J^{th}$ mode is required, punch "1" in the $5 \times J^{th}$ column.
If plot of $J^{th}$ mode is not required, punch "0" in the $5 \times J^{th}$ column.

Next card (necessary only if plot of any mode amplitude is required):
YHIMD, YLABMD, ITICMD
YHIMD is the maximum ordinate.
YLABMD is the interval for ordinate labelling.
ITICMD is the number of ordinate tick marks for mode plots.
Note: ITICMD should be negative to obtain centerline.

Remaining cards (NTERMS in number): J, AST, ACT
AST is the amplitude of the sine term of the $J^{th}$ mode.
ACT is the amplitude of the cosine term of the $J^{th}$ mode.

It must be noted that AST and ACT are the amplitudes of the mode-amplitude functions and not the pressure. If the initial pressure disturbance is given for only one mode (say, the $j^{th}$ longitudinal mode) then the values of AST and ACT which yield the desired initial pressure disturbance are given as follows:

$$\text{ACT} = 0.0, \quad \text{AST} = \sqrt{\frac{1 - 2|p'|}{\gamma}} \omega_j$$
where $|p'|$ is the desired initial head-end pressure amplitude, $\gamma$ is the specific heat ratio of the gas-particle mixture given by Equation (105), and $w_j$ is the acoustic frequency of the $j^{th}$ longitudinal mode given by

$$w_j = j \pi \frac{\gamma}{\gamma(1+C_m)}$$

If no particles are present, $\gamma = \gamma$ and $w_j = j \pi$. The above equation for AST was derived from the momentum equation (i.e., Equation (24)).

**Description of the Subroutines.** The various subroutines utilized in program SOLID2 are described below:

**SUBROUTINE PHICFS(NP, Z, CT, CZ).** This subroutine computes the space-dependent coefficients appearing in the $NP^{th}$ mode of the series expansion for $\phi_t$ and $\phi_x$. $Z$ is the axial location (i.e., $x$) at which these coefficients are needed, and CT and CZ are these coefficients. CT is just the eigenfunction for the $NP^{th}$ mode and CZ is the axial derivative of the eigenfunction (both evaluated at the location $Z$). The eigenvalues $B$ are supplied through the labeled common block BLK2.

**SUBROUTINE PRSVEL (UBAR, UMS, Y, P, VZGAS, VZPAR).** This subroutine computes the pressure ($P$) and axial velocity perturbations ($VZGAS$ and $VZPAR$) of gas and particles using the supplied mode-amplitude functions and their derivatives ($Y$). UBAR is the steady-state velocity and UMS is its derivative at the axial location where pressure is to be computed. Pressure is computed from the second order momentum equation (i.e., Equation (24)) and velocity is computed as the axial derivative of the velocity potential. The space-dependent coefficients ($COEF$) of $\phi_t$ and $\phi_x$ are computed by subroutine PHICFS and are supplied through the common block BLK3.

**SUBROUTINE RHS (U,UP).** This subroutine calculates the right-hand-sides of the equations for the mode-amplitude functions (written as an equivalent first order system); i.e., $f_j$ in the system of equations:

$$\frac{dB_j}{dt} = B'_j$$

$$\frac{dB'_j}{dt} = f_j(B_1, B_2, \ldots, B'_1, B'_2, \ldots)$$

$U$ is the array containing the mode-amplitude functions and their derivatives.
which is supplied to the subroutine, and UP is the array containing the computed values of \( f_j \). The coefficients of the linear and nonlinear terms in the equations for the mode-amplitude functions are supplied from the blank common space. If nonlinear combustion response (using the heuristic model) is to be considered, the necessary additional terms are calculated using the quantities in common block BLK5. Similarly, if nonlinear particle damping is to be considered, common block BLK7 contains the information necessary to calculate the additional terms.

**SUBROUTINE GROWTH (MAXP, TIMAX, PMAX, FREQ).** This subroutine computes the pressure growth rate and frequency of oscillation at the end of each cycle of oscillation. PMAX is an array containing the values of the pressure maxima and minima which are computed in the calling (main) program, TIMAX is an array containing the values of the dimensionless times at which these maxima and minima occur and MAXP gives the total number of maxima and minima. FREQ is the pure-gas acoustic frequency (dimensional) which is needed for converting the nondimensional growth rate and frequency values into dimensional quantities. For computing the growth rate, only pressure maxima are considered, and the exponential growth rate (negative if there is decay) and frequency during each cycle are calculated and stored under the array names ALPHA and F respectively. The subroutine itself prints out these values; the only output of subroutine GROWTH is this printout.

**SUBROUTINE RESPNS (EN, A, B, OMEGA, RES).** This subroutine calculates the complex combustion response RES according to the two-parameter (A-B) model (Equations (10) and (11)). A and B are the two parameters appearing in Equation (10), EN is the pressure exponent in the steady-state burning law, and OMEGA is the frequency nondimensionalized by the square of the steady-state burning rate as described in Volume I.

**COMPLEX FUNCTION CCOSH (X).** This function calculates the hyperbolic cosine (i.e., \( \cosh \)) of a complex number X.

**COMPLEX FUNCTION CSINH(X).** This function calculates the hyperbolic sine (i.e., \( \sinh \)) of a complex number X.

Both CCOSH(X) and CSINH(X) can be omitted in computing facilities where complex arguments are permitted in the hyperbolic functions provided in their library.
This is the principal plotting subprogram. This subroutine sets up the CALCOMP plotter, calls all other plotting routines and produces the desired plots. First, the different variables to be supplied to this subroutine through the argument list are explained.

**IBUT**: address of buffer area for plot output; has dimension NLOC.

**NLOC**: number of locations in the buffer area. The most economical value depends on the computation facility being used; on CDC CYBER 70/74 it is 512.

**LDEV**: logical device number for plot; 4 in the present program.

**NTOT**: number of points to be plotted.

**NTICX**: number of tick marks on abscissa.

**NTICY**: number of tick marks on ordinate.

**XMAX**: upper limit of abscissa domain.

**YMAX**: upper limit of ordinate range.

**XMIN**: lower limit of abscissa domain.

**YMIN**: lower limit of ordinate range.

**ITITLX**: abscissa label (alphanumeric).

**ITITLY**: ordinate label (alphanumeric).

**LTITLX**: number of characters in ITITLX.

**LTITLY**: number of characters in ITITLY.

**XARRAY**: array containing abscissa values of points to be plotted, in terms of XMIN-XMAX coordinates.

**YARRAY**: array containing ordinate values of points to be plotted, in terms of XMIN-XMAX coordinates.

**DELEX**: interval of abscissa tick mark labelling in terms of XMIN-XMAX coordinates.

**DELEY**: interval of ordinate tick mark labelling in terms of YMIN-YMAX coordinates.
This subroutine first sets up the plotter variables like height of characters (0.105 in), symbol for plotting a point, length of axes on plots, width of margins, etc. It then makes the initial PLOTS call for the CALCOMP plotter. Next, it calls the subroutine MYAXIS to draw the axes and label them. Finally, it calls the subroutine MYLINE to plot the points, draw the optional centerlines, and the page scissorlines. The plotting parameters are supplied to these subroutines through the argument lists and through the common block BLK6.

SUBROUTINE MYAXIS (NTICX, NTICY, ITITLX, ITITLY, XMAX, YMAX, XMIN, YMIN, LITILX, LITILY, DELX, DELY). This subroutine draws the X and Y axes and plots the tick marks on these axes. It calls the subroutine AXLAB to label these axes and the subroutine DENDEC to determine the number of decimal places needed on the tick mark labelling of the axes. The meaning of the argument variables is the same as given above.

SUBROUTINE MYLINE (XARRAY, YARRAY, XMAX, YMAX, XMIN, YMIN, NTOT, NTICY). This subroutine plots the NTOT points whose abscissae and ordinates are given by XARRAY and YARRAY. If NTICY is negative, it draws the centerline. Finally, it draws the scissorlines for trimming the plots.

SUBROUTINE AXLAB (ANGLE, IBCD, NCHARX). This subroutine labels the axes. IBCD is the label which is printed and ANGLE is the angle between the line of printing and the direction of movement of the paper on the plotter. NCHARX is the number of characters in the label.

SUBROUTINE DENDEC (QMAX, DELQ, NDEC). This subroutine determines the number of decimal places NDEC needed on the tick mark labelling of an axis. DELQ is the interval of the axis tick mark labelling and QMAX is the range.

Description of Output. There are two modes of output from this program: (1) printed output, and (2) plotted output. These are described below. The printed output produced by SOLID2 consists of seven sections described below.

Section (1) is a restatement of the input from Program SOLID1. It includes the following information: (a) the mean flow Mach number at the combustor exit (UE), the number of modes considered, the ratio of specific heats (GAMMA) in pure gas and (if particles are present) the ratio of specific heats in the mixture (GAMMABAR); (b) information about the particles (if present); (c) the parameters which describe and identify each term in the series expansion;
(d) the nozzle admittance (YR and YI) and the axial acoustic eigenvalue (EPS and ETA) for each series term; (e) the nonzero linear coefficients if NOUTCF = 1 or 2; and (f) the nonzero nonlinear coefficients if NOUTCF = 2.

Section (2) gives the coefficients needed for computation of the pressure perturbation; i.e., the coefficients in the series for each of the series terms at the two ends and the middle of the chamber.

Section (3) prints out the combustion parameters and the real and imaginary parts of the response functions. If a nonlinear combustion response is considered, the parameter b in the heuristic nonlinear combustion model is given.

Section (4) gives the acoustic frequency and initial amplitudes of all the series terms included in the assumed initial disturbance.

Section (5) is printed only if NHISTR = 0. In this section, the time histories of (a) the pressure disturbance at the two ends and the middle of the chamber and (b) the gas and particle velocity perturbations at the middle of the chamber are printed out, starting at time t = TSTART and ending at time t = TQUIT (TSTART and TQUIT having been specified in the input).

Section (6) gives the pressure maxima and minima values at the chamber location specified by NLOC. This information is printed as an array of number pairs giving the value of the pressure maximum or minimum (upper numbers) and the corresponding time of maximum or minimum (lower number).

In section (7), average pressure growth rate and frequency of oscillation in each cycle are printed out. This information is printed as an array of number triads giving the value of the pressure growth rate (upper number), frequency of oscillation (middle number) and the dimensionless time at the mid-point of the cycle (lower number).

According to the value of NOUT the pressure waveforms at the ends and middle of the chamber may be plotted. Furthermore, waveforms of individual mode amplitudes may also be plotted depending on the specified values of elements in the array MDPLT (see the description of inputs for details). All the plots have dimensionless time as the abscissa. The range of the abscissa for each plot is 10 units, and the first plot starts with t = TSTART. Thus for each quantity whose plot is desired, N plots are produced where N is the largest multiple of 10 contained in the interval TSTART to TQUIT. All quantities to be plotted for a given time interval are plotted before proceeding to the next time interval. All of the plots are scaled to fit on standard (8½ in x 11 in) paper and scissor-lines are plotted for trimming plots to this size. The data
is plotted as individual points using a centered circle symbol. Before the first plot is produced the identifying title is printed.

Sample Case. A sample case is now presented to facilitate checkout. Pressure-time histories are obtained for the motor considered in the sample case for SOLID1. The data generated by SOLID1 is used for the second part of the necessary input for SOLID2 through a data file stored on disk. Since the purpose of this run is only to illustrate the operation of SOLID2, \texttt{TQUIT = 10.0} and only linear combustion response (\texttt{NCOMB = 0}) and linear particle damping (\texttt{NNPRT = 0}) are considered. The combustion parameters are \( A = 5.996, \ B = 0.58, \ n = 0.575, \) and \( \Omega = 4.20. \) A step size of 0.025 is chosen. A 1L-mode initial disturbance of 10% pressure amplitude is considered, for which the formula given previously yields \( ACT = 0.0 \) and \( AST = -0.0266818. \)

Pressure maxima and minima are obtained at the head end; hence \( NL1C = 1. \) To check out the plotting routine, plots of the pressure disturbance are obtained at the two ends of the chamber as well as at the middle, hence \( NOU T = 3. \) Plots of the first five longitudinal modes are obtained by punching 1 at the 5\textsuperscript{th}, 10\textsuperscript{th}, 15\textsuperscript{th}, 20\textsuperscript{th}, and 25\textsuperscript{th} columns of the appropriate card in the input deck.

An input deck with the above data is illustrated on the next page. The printed output and the plotted output generated by Program SOLID2, using this deck and the coefficients generated by Program SOLID1, are presented in the following pages.
UE = .3780

NUMBER OF MODES = 5

GAMMA = 1.23

GAMMA_BAR = 1.212250

PARTICLE DIA (IN MICRONS) = 2.5

CHAMBER TEMP (IN DEG K) = 3525.0

PARTICLE DRAG CONSTANT, K = 29.918E

CM = .10

FREQ (IN HERTZ) = 1071.0

SP = .68

RHOST (IN KG/CUBIC METER) = 4300.0

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## COMBUSTION PARAMETERS

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### Linear Combustion Response

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### Linear Particle Damping
INITIAL CONDITIONS ARE OF THE FORM:

\[ U(I,J) = AC(J) \cos(FREQ \cdot T) + AS(J) \sin(FREQ \cdot T) \]

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THIS RUN PRODUCES PLOTTED OUTPUT.
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PRESSURE MAXIMA AND MINIMA AT:  Z = 0.00
VALUES COMPUTED:  11

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PRESSURE GROWTH RATE AND FREQUENCY.
TOTAL NUMBER OF CYCLES: 3

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HEAD-ENO PRESSURE PERTURBATION

DIMENSIONLESS TIME, T
AMPLITUDE OF IL MODE

DIMENSIONLESS TIME, T
FORTRAN Source Code.

```
PROGRAM SOLID2 (INPUT, OUTPUT, DATA,
1 TAPE5=INPUT, TAPE6=OUTPUT, TAPE9=DATA)

*************** PROGRAM SOLID2 ********************

THIS PROGRAM INTEGRATES THE SYSTEM OF DIFFERENTIAL EQUATIONS
FOR MODE AMPLITUDES USING THE COEFFICIENTS COMPUTED BY THE
PROGRAM SOLID1. TIME-HISTORY OF A PRESSURE DISTURBANCE IN THE
ROCKET IS COMPUTED, AND THE DESIRED PLOTS & PRINTOUTS ARE
PRODUCED.

THE FOLLOWING INPUTS ARE REQUIRED:
(1) THE CONTROL NUMBERS, NOUTCF AND NHISTR.
(2) THE COEFFICIENTS FROM PROGRAM SOLID1.
(3) THE DATA DECK.

THE FIRST CARD GIVES THE CONTROL NUMBERS, NOUTCF AND NHISTR.
NOUTCF DETERMINES PRINTOUT OF COEFFICIENTS:
IF NOUTCF = 0 COEFFICIENTS ARE NOT PRINTED OUT.
IF NOUTCF = 1 ONLY LINEAR COEFFICIENTS ARE PRINTED OUT.
IF NOUTCF = 2 ALL COEFFICIENTS ARE PRINTED OUT.
NHISTR DETERMINES IF PRESSURE HISTORY IS TO BE PRINTED:
IF NHISTR = 0 PRINTED
IF NHISTR = 1 NOT PRINTED.

THE COEFFICIENTS ARE OBTAINED FROM PROGRAM SOLID1
BY PUTTING NOUT = 1 OR NOUT = 2, THEREBY WRITING THE COEFFICIENTS
INTO A DISK. THIS DISK HAS BEEN GIVEN THE DEVICE NUMBER 9.

THE DATA DECK CONSISTS OF THE FOLLOWING CARDS:
FIRST CARD: TITLE OF THE CASE.
SECOND CARD: H, TSTART, QTUIT, FREQ, BCOMB
H IS THE INTEGRATION STEP SIZE.
TSTART IS THE TIME AT WHICH OUTPUT STARTS.
QTUIT IS THE TIME AT WHICH COMPUTATIONS ARE TERMINATED.
FREQ IS THE MOTOR FREQUENCY (IN PURE GAS), IN Hertz.
BCOMB IS THE COMBUSTION RESPONSE NONLINEARITY FACTOR.
THIRD CARD: A2PARA, B2PARA, EN, OMEGA
A2PARA AND B2PARA ARE THE COMBUSTION PARAMETERS IN THE A-B MODEL.
EN IS THE PRESSURE EXPONENT IN THE BURNING RATE LAW.
OMEGA IS THE FREQUENCY NONDIMENSIONALIZED BY THE SQUARE OF
THE STEADY-STATE BURNING RATE.
FOURTH CARD: NLOC, NTERMS, NOUT, NCORB, NNPRT
```
NLOC DETERMINES THE LOCATION OF THE WALL PRESSURE MAXIMA AND MINIMA:

- If NLOC = 1, location is $Z = 0.0$
- If NLOC = 2, location is $Z = 1.0$
- If NLOC = 3, location is $Z = 0.5$

NTERMS IS THE NUMBER OF TERMS GIVEN INITIAL VALUES.

NOUT IS THE OUTPUT CONTROL NUMBER:

- If NOUT = 0, printed output only.
- If NOUT > 0, both printed and plotted output.
- If NOUT = 1, plot of pressure at $Z = 0.0$ only.
- If NOUT = 2, plot of pressure at $Z = 0.0$ and $Z = 1.0$
- If NOUT = 3, plot of pressure at $Z = 0.0$, $1.0$ and $0.5$.

NCOMB DETERMINES IF COMBUSTION NONLINEARITIES ARE CONSIDERED:

- If NCOMB = 0, neglected.
- If NCOMB = 1, included.

NNPRT DETERMINES IF NONLINEAR PARTICLE DAMPING IS CONSIDERED:

- If NNPRT = 0, not considered.
- If NNPRT = 1, considered.

NEXT CARD (NECESSARY ONLY IF NNPRT = 1): REFPRS, CPGAS, CNLP

- REFPRS IS THE CHAMBER PRESSURE, IN PSI.
- CPGAS IS THE SPECIFIC HEAT AT CONSTANT PRESSURE OF THE GAS PHASE, IN CAL/GM-DEG K.
- CNLP IS THE COEFFICIENT IN THE AMPLITUDE-DEPENDENT PART OF THE NONLINEAR PARTICLE DRAG.

NEXT CARD (NECESSARY ONLY IF PLOTS ARE REQUIRED): YHI, YLAB, ICTIC

- YHI IS THE MAXIMUM ORDIinate FOR PRESSURE PLOTS.
- NOTE: THE ORDIinate SCALES FOR PRESSURE AND AMPLITUDE PLOTS ARE SYMMETRIC ABOUT ZERO.
- YLAB IS THE INTERVAL FOR ORDIinate LABELLING FOR ABOVE PLOTS.
- ICTIC IS THE NUMBER OF ORDIinate TIC MARKS FOR ABOVE PLOTS.
- NOTE: ICTIC SHOULD BE NEGATIVE FOR PRESSURE AND AMPLITUDE PLOT TO OBTAIN CENTERLINE.

NEXT CARD (NECESSARY ONLY IF PLOTS ARE REQUIRED): MDPLOT

- MDPLOT DETERMINES IF PLOTS OF INDIVIDUAL MODES ARE REQUIRED:
  - If plot of $J$th mode is required, punch "1" in the $5*J$th column.
  - If plot of $J$th mode is not required, punch "0" in the $5*J$th column.

NEXT CARD (NECESSARY ONLY IF PLOT OF ANY MODE AMPLITUDE IS REQUIRED): YHIMD, YLABMD, ICTICMD

- YHIMD IS THE MAXIMUM ORDIinate.
- YLABMD IS THE INTERVAL FOR ORDIinate LABELLING.
- ICTICMD IS THE NUMBER OF ORDIinate TIC MARKS FOR MODE PLOTS.
- NOTE: ICTICMD SHOULD BE NEGATIVE TO OBTAIN CENTERLINE.

REMAINING CARDS (NTERMS IN NUMBER): J, AST, ACT

- AST IS THE AMPLITUDE OF THE SINE TERM OF THE $J$TH MODE.
- ACT IS THE AMPLITUDE OF THE COSINE TERM OF THE $J$TH MODE.
COMPLEX YNOZ(6), B(6), C1, C3, RES(6), CRES
DIMENSION L(6), NAME(6), AA(4), FROQ(24),
1 YI(12), YR(12), E(12, 12, 2),
2 CFT(3, 12), CFZ(3, 12), ASC(24),
3 AC(24), U(5, 36), Y(36), PRESS(3),
4 YP(36), FZ(4, 36), UZ(36), Z(3), TIMAX(500),
5 TPL(500), YFLOT(3, 500), DUMY(500), DUMMY(500),
6 IBUF(512), ITT(3), ITY1(3), ITY2(3), ITY3(4),
7 TITLE(7), PRS(500), TI(500), PMAX(500),
8 MDPLOT(6), UPILOT(6, 500), MTI TL1(2), MTI TL2(2),
9 MTI TL3(2), MTI TL4(2), MTI TL5(2), MTI TL6(2), MTI TL(2)

COMMON C(2, 12, 24), D(12, 144), CP(3, 12, 24),
1 KPMAX(3, 12), IC(2, 12, 24), CPPAR(2, 12, 24),
2 COMMON /BLK2/ B
COMMON /BLK3/ NQRTKL, NJMAX, NLMAX, GAMMA,
1 COMMON /COE(2, 12), NJMAX2
COMMON /BLK4/ CM, PARTKL, RHOP
COMMON /BLK5/ RES, NCOMB, BCOMB, E
COMMON /BLK7/ NNPRT, CNPRT

DATA ITT/"DIMENSIONL", "ESS TIME", "T" "/,
1 ITY1/"HEAD-END P", "RESSURE PE", "RTURBATION" "/,
2 ITY2/"NOZZLE PRE", "SSURE PERT", "URBATION" "/,
3 ITY3/"PRESSURE P", "ERTURBATION", "N AT THE C", "ENTER" "/,
4 MTI TL1/"AMPLITUDE", "OF 1L MODE" "/,
5 MTI TL2/"AMPLITUDE", "OF 2L MODE" "/,
6 MTI TL3/"AMPLITUDE", "OF 3L MODE" "/,
7 MTI TL4/"AMPLITUDE", "OF 4L MODE" "/,
8 MTI TL5/"AMPLITUDE", "OF 5L MODE" "/,
9 MTI TL6/"AMPLITUDE", "OF 6L MODE" "/

MAXMD = 6
MAXMD2 = 12
MAXMD4 = 24
MAXMD4 = 144
LAST = 5
ERR = 0.001
TDEL = 10.0
NPT = 0
AA(1) = 0.0
AA(2) = 0.5
AA(3) = 0.5
AA(4) = 1.0
PI = 3.1415926536
HC = 1.0
READ (5, 5003) NOUTCF, NHIST
*************** COEFFICIENT INPUT SECTION ***********************

THIS VERSION OF SOLID2 READS THE COEFFICIENT DATA FROM A FILE GENERATED BY PROGRAM SOLIDI. TO READ THIS DATA FROM CARDS, USE READ (5,XXXX) INSTEAD OF READ (9,XXXX) IN THIS SECTION.

INPUT OF MOTOR PARAMETERS AND NUMBER OF TERMS:

READ (9,5001) GAMMA, UE, ZE, NJMAX, NPRTKL
RHOP = 0.0
PARTKL = 0.0
PRTSS = 0.0
UPBYU = 0.0
JMX = NJMAX/2
NJMAX2 = NJMAX
NU = 2 * NJMAX
GAM = GAMMA
FRATI 0
IF (NPRTKL .EQ. 0) GO TO 14
READ (9,5011) DIA, RHOM, SP, TEMP, FREQ, PARTKL, CM
UPBYU = 2.0 / (1.0 + SQRT(1.0 + 8.0*UE/PARTKL))
RHOP = CM / UPBYU
GAM = GAMMA / (1.0 + 0.68*CM - 0.68*CM*GAMMA)
FRATI = SQRT(GAMMA / (GAM * (1.0 + CM)))
NJMAX2 = 2*NJMAX
NU = NJMAX2 + NJMAX
14 CONTINUE

WRITE (6,6001) UE, JMX, GAM
IF (NPRTKL .EQ. 0) WRITE (6,6033)
IF (NPRTKL .EQ. 1) WRITE (6,6009) GAMMA
IF (NPRTKL .EQ. 1) WRITE (6,6030) DIA, CM, FREQ, TEMP, 1
   SP, RHOM, PARTKL
WRITE (6,6002)

INPUT OF DESCRIPTION OF SERIES EXPANSION:

DO 10 K = 1, JMX
READ (9,5002) NJ, L(NJ), NAME(NJ)
WRITE (6,6003) NAME(NJ), NJ, L(NJ)
10 CONTINUE

WRITE (6,6010)
DO 15 K = 1, JMX
READ (9,5010) J, YNOZ(J), B(J)
WRITE (6,6015) J, YNOZ(J), B(J)
NJ = (2 * J) - 1
YR(NJ) = REAL(YNOZ(J))
YI(NJ) = AIMAG(YNOZ(J))
YR(NJ+1) = YR(NJ)
YI(NJ+1) = YI(NJ)
15 CONTINUE
ZERO LINEAR COEFFICIENT ARRAYS:
DO 20 KC = 1, 3
DO 20 NJ = 1, MAXMD2
DO 20 NP = 1, MAXMD4
CP(KC,NJ,NP) = 0.0
20 CONTINUE

C C ZERO NONLINEAR COEFFICIENT ARRAY:
C
DO 30 NJ = 1, MAXMD2
DO 30 NPQ = 1, MAXMD4
D(NJ,NPQ) = 0.0
30 CONTINUE

C C INPUT OF LINEAR COEFFICIENTS:
C
DO 40 KC = 1, 3
READ (9,5003) KMAX
IF (NOUTCF.GT.0) WRITE (6,6004) KC, KMAX
IF (KMAX.EQ.0) GO TO 40
DO 45 K = 1, KMAX
READ (9,5004) NJ, NP, CP(KC,NJ,NP)
IF (NOUTCF.GT.0) WRITE (6,6005) KC, NJ, NP, CP(KC,NJ,NP)
45 CONTINUE
40 CONTINUE

C C INPUT OF NONLINEAR COEFFICIENTS:
C
DO 305 KC = 4, 5
READ (9,5003) KMAX
KCMIN3 = KC - 3
IF (NOUTCF.GT.0) WRITE (6,6031) KCMIN3, KMAX
IF (KMAX.EQ.0) GO TO 305
DO 310 K = 1, KMAX
READ (9,5004) NJ, NP, E(NJ,NP,KCMIN3)
IF (NOUTCF.GT.0) WRITE (6,6032) NJ, NP, KCMIN3, E(NJ,NP,KCMIN3)
310 CONTINUE
305 CONTINUE

C C INPUT OF NONLINEAR COEFFICIENTS:
C
READ (9,5003) NLMAX
IF (NOUTCF.EQ.2) WRITE (6,6006) NLMAX
IF (NLMAX.EQ.0) GO TO 50
DO 52 NJ = 1, MAXMD2
KPQMAX(NJ) = 0
52 CONTINUE

97
DO 55 K = 1, NLMAX
READ (9, 5005) NJ, NP, NQ, DT
IF (NOUTCF * EQ. 2) WRITE (6, 6007) NJ, NP, NQ, DT
KPQMAX(NJ) = KPQMAX(NJ) + 1
KPQ = KPQMAX(NJ)
IDP(NJ, KPQ) = NP
IDQ(NJ, KPQ) = NQ
D(NJ, KPQ) = DT
55 CONTINUE
50 CONTINUE

************* PRESSURE COEFFICIENT SECTION ***********************
C
C CALCULATE SPATIAL COORDINATES FOR PRESSURE COMPUTATION.
Z(1) = 0.0
Z(2) = ZE
Z(3) = 0.5 * ZE
C
C CALCULATE COEFFICIENTS FOR PRESSURE TIME HISTORIES.
DO 53 NPRES = 1, 3
DO 53 J = 1, JMX
NP = (2 * J) - 1
Z1 = Z(NPRES)
CALL PHICFS(J, Z1, C1, C3)
CFT(NPRES, NP) = REAL(C1)
CFT(NPRES, NP+1) = -AIMAG(C1)
CFZ(NPRES, NP) = REAL(C3)
CFZ(NPRES, NP+1) = -AIMAG(C3)
53 CONTINUE
C
C OUTPUT OF COEFFICIENTS FOR PRESSURE TIME HISTORIES.
WRITE (6, 6020)
DO 56 NPRES = 1, 3
WRITE (6, 6014)
DO 56 J = 1, NJMAX
WRITE (6, 6021) J, Z(NPRES), CFT(NPRES, J), CFZ(NPRES, J)
56 CONTINUE
C
C ************* DATA INPUT SECTION ***********************
C
READ (5, 5000) TITLE
C
ZERO INITIAL VALUE AND FREQUENCY ARRAYS.
5 DO 57 K = 1, NJMAX2
AS(K) = 0.0
AC(K) = 0.0
FRQ1(K) = 0.0
57 CONTINUE
C READ COMBUSTION AND CONTROL PARAMETERS.
READ (5, 5006) H, TSTART, TQUIT, FREQ, BCOMB
IF (EOF(5)) 300, 1
1 CONTINUE
READ (5, 5013) A2PARA, B2PARA, EN, OMEGA
WRITE (6, 6034) A2PARA, B2PARA, EN, OMEGA
DO 46 K = 1, JMX
OMEGAK = OMEGA * K
CALL RESPNS(EN, A2PARA, B2PARA, OMEGAK, CRES)
RES(K) = CRES
WRITE (6, 6035) K, RES(K)
46 CONTINUE
C C
C READ CONTROL NUMBERS.
READ (5, 5008) NLOC, NTERMS, NOUT, NCOMB, NNPRT
IF (NOUT .GT. 0) NPT = 1
IF (NCOMB .EQ. 0) WRITE (6, 6039)
IF (NCOMB .EQ. 1) WRITE (6, 6040) BCOMB
IF (NNPRT .EQ. 0) WRITE (6, 6041)
IF (NNPRT .EQ. 0) GO TO 11
READ (5, 5013) REFRPS, CPGAS, CNLP
VISCC = 8.834 * 0.00001 * (TEMP/3485)**0.66
REYN = ((0.00010655765 * GAM * REFRPS * DIA) / (VISCC
1 * SQRT((GAM-1.0) * CPGAS * TEMP)))**(2./3.) / 6.0
WRITE (6, 6038) REFRPS, CPGAS, REYN
11 CONTINUE
C C
C IF (NOUT .EQ. 0) GO TO 9
C READ DATA FOR SETTING UP PLOTS.
READ (5, 5009) YHI, YLAB, ITCY
READ (5, 5014) MDPLOT
MDPLTL = 0
DO 320 K = 1, JMX
320 MDPLTL = MDPLTL + MDPLOT(K)
IF (MDPLTL .EQ. 0) GO TO 9
READ (5, 5015) YHIMD, YLABMD, ITCMD
YLOMD = - YHIMD
C C
C ************* INITIAL AMPLITUDES SECTION *********************
C 9 DO 54 K = 1, NTERMS
C C INPUT INITIAL AMPLITUDES FOR F-FUNCTIONS.
READ (5, 5007) J, AST, ACT
NJ = (2 * J) - 1
AS(NJ) = AST
AC(NJ) = ACT
C C CALCULATE FREQUENCY.
AX = L(J) * PI * FRATIO/ZE
FRQ1(NJ) = AX
FRQ1(NJ+1) = FRQ1(NJ)
CALCULATE INITIAL AMPLITUDES FOR G-FUNCTIONS.

IF (FRQ1(NJ)) 58, 58, 581

581 GYRU = GAMMA*YR(NJ)*UE
GYIF = GAMMA*YI(NJ)*FRQ1(NJ)
GYRF = GAMMA*YR(NJ)*FRQ1(NJ)
GYIU = GAMMA*YI(NJ)*UE

NPRES = 2

A1 = (1.0 + GYRU)*CFZ(NPRES,NJ+1)
1 - GYIF*CFT(NPRES,NJ+1)
A2 = GYRF*CFT(NPRES,NJ+1) + GYIU*CFZ(NPRES,NJ+1)
A3 = -(1.0 + GYRU)*CFZ(NPRES,NJ) + GYIF*CFT(NPRES,NJ)
A4 = GYRF*CFT(NPRES,NJ) + GYIU*CFZ(NPRES,NJ)

DET = A1*A1 + A2*A2
IF (DET LT 0.0000001) GO TO 583
R1 = A3*AC(NJ) - A4*AS(NJ)
R2 = -A4*AC(NJ) - A3*AS(NJ)

AC(NJ+1) = (R1*A1 + R2*A2)/DET
AS(NJ+1) = -(R2*A1 - R1*A2)/DET
GO TO 58

583 AC(NJ+1) = -AS(NJ)
AS(NJ+1) = AC(NJ)

58 CONTINUE

IF (NPRTKL = EQ. 0) GO TO 54
AS(NJ+NJMAX) = AS(NJ)
AC(NJ+NJMAX) = AC(NJ)
AS(NJ+1+NJMAX) = AS(NJ+1)
AC(NJ+1+NJMAX) = AC(NJ+1)
54 CONTINUE

OUTPUT OF INITIAL AMPLITUDES.
WRITE (6,6016)
DO 590 J = 1, NJMAX
IF (AS(J)) 591, 592, 591
592 IF (AC(J)) 591, 590, 591
591 WRITE (6,6017) J, FRQ1(J), AC(J), AS(J)
590 CONTINUE

IF (NOUT = GE. 1) WRITE (6,6027)

*************** LINEAR COEFFICIENTS SECTION **********************
DO 315 KC = 1, 2
DO 315 NJ = 1, MAXM2
DO 315 NP = 1, MAXM4
C(KC,NJ,NP) = 0.0
315 CONTINUE

IF (NNPRT *EQ* 0) GO TO 410
UMINUP = UE * (1.0 - UPBYU)
IF (UMINUP *LT* 0.0000001) GO TO 410
PRRTSS = 0.6 * REYN * UMINUP **(2./3.)
CNNPRT = PARTKL * REYN * CNLP
410 CONTINUE

COMPUTE LINEAR COEFFICIENTS FOR GIVEN VALUES OF HC AND RESPONSE FUNCTION.
DO 60 NJ = 1, NJMAX
DO 60 NP = 1, NJMAX2
CT = CP(1,NJ,NP)
IF (CT) 61, 62, 61
KPMAX(1,NJ) = KPMAX(1,NJ) + 1
KP = KPMAX(1,NJ)
IC(1,NJ,KP) = NP
C(1,NJ,KP) = CT
62 CONTINUE
IF (NP GT NJMAX OR NJ GT NJMAX) GO TO 316
NP12 = (NP+1)/2
RESR = REAL (RES(NP12))
RESI = AIMAG (RES(NP12))
CT = CP(2,NJ,NP) + PARTKL * (1.0 + PRRTSS) * CP(3,NJ,NP)
1 + HC*RESR*E(NJ,NP,1) + HC*RESI*E(NJ,NP,2)
GO TO 318
316 CONTINUE
CT = CP(2,NJ,NP) + PARTKL * (1.0 + PRRTSS) * CP(3,NJ,NP)
318 CONTINUE
IF (CT) 63, 60, 63
KPMAX(2,NJ) = KPMAX(2,NJ) + 1
KP = KPMAX(2,NJ)
IC(2,NJ,KP) = NP
C(2,NJ,KP) = CT
60 CONTINUE

IF (NPRTKL *EQ* 0) GO TO 415
DO 420 NJ = 1, NJMAX
DO 420 NP = 1, NJMAX2
CPPAR(NJ,NP) = CPPAR(1,NJ,NP) +
1 PARTKL * (1.0 + PRRTSS) * CPPAR(2,NJ,NP)
420 CONTINUE
415 CONTINUE

101
************** INITIAL VALUES SECTION *********************

NSTEP = 0
NP1 = 3
H6 = H/6
TIME = 0.0
I = NP1
TI(I) = TIME

DO 75 J = 1, NJMAX2
  JP = J + NJMAX2
  IF (AC(J)) 751, 753, 751
  IF (AS(J)) 751, 752, 751
  U(I,J) = 0.0
  IF (JP .GT. NU) GO TO 75
  U(I,JP) = 0.0
  GO TO 75

75 ARG = FRQ1(J) * TIME
  FSIN = SIN(ARG)
  FCOS = COS(ARG)
  U(I,J) = ASCJ) * FSIN + AC(J) * FCOS
  IF (JP .GT. NU) GO TO 75
  U(I,JP) = (AS(J) * FCOS) - (AC(J) * FSIN) * FRQ1(J)
75 CONTINUE

CALCULATE INITIAL VALUES OF PRESSURE AND VELOCITY.

DO 704 NPRES = 1, 3
  DO 702 J = 1, NJMAX
    COEF(1,J) = CFT(NPRES,J)
    COEF(2,J) = CFZ(NPRES,J)
  702 CONTINUE
  DO 703 J = 1, NU
    Y(J) = UCI,J)
  703 CONTINUE
  UBAR = UE * Z(NPRES)
  UMS = UE
  CALL PRSVEL(UBAR, UMS, Y, P, VZGAS, VZPAR)
  PRESS(NPRES) = P
  704 CONTINUE
  PR5(I) = PRESS(NLOC)
70 CONTINUE

IF (NHISTR .EQ. 0) WRITE (6,6008) GAM, UE
IF (NHISTR .EQ. 0) WRITE (6,6022)

************** INITIALIZE CONTROL NUMBERS **********************

LINE = 8
K = 0
MAXNO = 0
MAXP = 0
IF (NOUT .EQ. 0) GO TO 100
JPLT = 0
**NUMERICAL CALCULATIONS SECTION**

100  \( I = NP1 \)

105  \( NSTEP = I - NP1 + (LAST - NP1) * K \)

RSTEPL NSTEP
TIME = RSTEP * H
\( TI(I) = TIME \)
DO 120 J = 1, NU
Y(J) = U(I,J)
120  CONTINUE
CALL RHS(Y,YP)
DO 130 J = 1, NU
FZ(1,J) = YP(J)
130  CONTINUE
DO 140 II = 2, 4
DO 144 J = 1, NU
UZ(J) = Y(J) + AA(II) * H * FZ(II-1,J)
144  CONTINUE
CALL RHS(UZ,YP)
DO 148 J = 1, NU
FZ(II,J) = YP(J)
148  CONTINUE
140  CONTINUE
150  CONTINUE

CALCULATE PRESSURE TIME HISTORIES.
DO 154 NPRES = 1, 3
DO 158 J = 1, NJMAX
COEF(1,J) = CFT(NPRES,J)
COEF(2,J) = CFZ(NPRES,J)
152  CONTINUE
UBAR = UE * Z(NPRES)
UMS = UE
CALL PRSVEL(UBAR, UMS, Y, P, VZGAS, VZPAR)
PRESS(NPRES) = P
154  CONTINUE

IF \( K < EQ. 0 \) GO TO 175

DETERMINE MAXIMUM AND MINIMUM PRESSURE AT LOCATION SPECIFIED

DPL = PRS(I) - PRS(I-1)
DPS = PRS(I-1) - PRS(I-2)
IF (DPL*DPS) 173, 173, 175
173 PNUM = PRS(I-2) - PRS(I)
PDEN = 2.0 * (PRS(I-2) + PRS(I) - 2.0*PRS(I-1))
IF (PDEN) 174, 175, 174
174 PP = PNUM/PDEN
PA = (PP - 1.0) * PP * 0.5
PB = 1.0 - (PP * PP)
PC = (PP + 1.0) * PP * 0.5
MAXP = MAXP + 1
PMAX(MAXP) = PA*PRS(I-2) + PB*PRS(I-1) + PC*PRS(I)
TIMAX(MAXP) = TI(I-1) + PP*H
IF (MAXP + GE* 500) GO TO 250
175 CONTINUE
C
IF (TIME .LT. TSTART) GO TO 155
IF (NOUT .EQ. 0) GO TO 156
C
************* TIME HISTORY PLOTTING SECTION ******************
C
IF (TMAX .GT. TSTART) GO TO 156
IF ((TIME + GT. TMAX) .OR. (JPLOT + GE. 500)) GO TO 1000
C
JPLOT = JPLOT + 1
C
FILL TIME ARRAY FOR PLOTTING:
TPLOT(JPLOT) = TIME
C
FILL PRESSURE ARRAYS FOR PLOTTING:
DO 1001 J = 1, 3
YPLOT(J,JPLOT) = PRESS(J)
1001 CONTINUE
C
IF (MDPLOL .EQ. 0) GO TO 156
C
FILL MODE AMPLITUDE ARRAYS FOR PLOTTING:
DO 322 J = 1, JMX
IF (MDPLOT(J) .EQ. 0) GO TO 322
J12 = 2*J - 1
UPLOT(J,JPLOT) = U(I,J12)
322 CONTINUE
C
GO TO 156
C
1000 NUM = JPLOT
C
PLOT TIME HISTORIES:
C
DO 1020 NPlot = 1, NOUT
C
JPLOT = 0
C
C ASSIGN PLOTTING PARAMETERS.
YMIN = YLO
YMAX = YHI
NTICY = ITICY
DELY = YLAB
C
C ELIMINATE POINTS THAT ARE OUT OF THE ORDINATE RANGE.
DO 1010 J = 1, NUM
IF ((YPLOT(NPLOT,J) .LT. YMIN) .OR. (YPLOT(NPLOT,J) .GT. YMAX))
1 GO TO 1010
JPLOT = JPLOT + 1
DUMMY(JPLOT) = TPLOT(J)
DUMMY(JPLOT) = YPLOT(NPLOT,J)
1010 CONTINUE
C
C IF (JPLOT .EQ. 0) GO TO 1020
GO TO (1011, 1014, 1015, NPLOT)
C
C PLOT HEAD-END PRESSURE.
1011 CALL GRAPHS(IBUF, 512, 4, JPLOT, 11, NTICY, TMAX, YMAX, TMIN, YMIN,
1 ITTY1, ITTY2, 21, 30, DUMMY, DUMMYY, 2.0, DELY, TITLE)
GO TO 1020
C
C PLOT NOZZLE PRESSURE.
1014 CALL GRAPHS(IBUF, 512, 4, JPLOT, 11, NTICY, TMAX, YMAX, TMIN, YMIN,
1 ITTY1, ITTY2, 21, 28, DUMMY, DUMMYY, 2.0, DELY, TITLE)
GO TO 1020
C
C PLOT PRESSURE AT THE CENTER (X = 0.5).
1015 CALL GRAPHS(IBUF, 512, 4, JPLOT, 11, NTICY, TMAX, YMAX, TMIN, YMIN,
1 ITTY1, ITTY2, 21, 35, DUMMY, DUMMYY, 2.0, DELY, TITLE)
C
1020 CONTINUE
C
DO 324 NPLOT = 1, JMX
IF (MDPLOT(NPLOT) .EQ. 0) GO TO 324
JPLOT = 0
DO 328 J123 = 1, 2
IF (NPLOT .EQ. 1) MTITL(J123) = MTITL1(J123)
IF (NPLOT .EQ. 2) MTITL(J123) = MTITL2(J123)
IF (NPLOT .EQ. 3) MTITL(J123) = MTITL3(J123)
IF (NPLOT .EQ. 4) MTITL(J123) = MTITL4(J123)
IF (NPLOT .EQ. 5) MTITL(J123) = MTITL5(J123)
IF (NPLOT .EQ. 6) MTITL(J123) = MTITL6(J123)
328 CONTINUE
C
DO 326 J = 1, NUM
IF ((UPLOT(NPLOT,J) .LT. YLO MD) .OR. (UPLOT(NPLOT,J) .GT. YHIMD))
1 GO TO 326
JPLOT = JPLOT + 1
DUMMY(JPLOT) = TPLOT(J)
DUMMY(JPLOT) = UPLOT(NPLOT,J)
326 CONTINUE
IF (JPLLOT EQ 0) GO TO 324
CALL GRAPHS(I BUF, 512, 4, JPLLOT, 11, ITI CMD, Tmax, YHI MD, TMIN,
1 YLOMD, ITT, MTI TL, 21, 20, DUMMY T, DUMMYY, 2, 0, YLAB M, TIT LE)
324 CONTINUE
C C REASSIGN PLOTTING PARAMETERS FOR NEXT SET OF PLOTS:
JPLLOT = 0
TMIN = Tmax
TMAX = Tmax + TDEL
C *************** TIME HISTORY PRINTED OUTPUT SECTION ***************
C 156 IF (NHISTR EQ 0)
1 WRITE (6,6011) NSTEP, TIME, (PRESS(J), J = 1, 3), VZGAS, VZPA
LINE = LINE + 1
157 IF (TIME GT TQUIT) GO TO 250
IF (LINE LT 52) GO TO 155
IF (NHISTR EQ 0) WRITE (6,6013)
IF (NHISTR EQ 0) WRITE (6,6022)
LINE = 4
C 155 I = I + 1
IF (I LT LAST) GO TO 105
K = K + 1
C C RE-ASSIGN ARRAYS:
DO 200 I = 1, NP1
ILAST = LAST - NP1 + I
PRS(I) = PRS(ILAST)
TI(I) = TI(ILAST)
DO 200 J = 1, NU
U(I,J) = U(ILAST,J)
200 CONTINUE
GO TO 100
C C *************** PRESSURE MAXIMA AND MINIMA PRINTOUT ***************
C 250 WRITE (6,6023) Z(NLOC), MAXP
LINE = 4
DO 255 JST = 1, MAXP, 8
JSTART = JST
JSTOP = JST + 7
IF (JSTOP GT MAXP) JSTOP = MAXP
WRITE (6,6024) (PMAX(J), J = JSTART, JSTOP)
WRITE (6,6024) (TIMAX(J), J = JSTART, JSTOP)
WRITE (6,6014)
LINE = LINE + 3
IF (LINE LT 52) GO TO 255
LINE = 0
WRITE (6,6013)
255 CONTINUE
CALL GROWTH(MAXP, TIMAX, PMAX, FREQ)
GO TO 5
300 CONTINUE

C TURN OFF PLOTTING ROUTINE.
IF (NPT = EQ 1) CALL PLOT(0.0, 0.0, 0.999)

C ************* READ FORMAT SPECIFICATIONS **************

C 5000 FORMAT (7A10)
5001 FORMAT (3F10.0, 31.5)
5002 FORMAT (2I5, 1X, A5)
5003 FORMAT (2I5)
5004 FORMAT (2I5, F15.8)
5005 FORMAT (3I5, F15.8)
5006 FORMAT (7F10.0)
5007 FORMAT (I5, 2F10.0)
5008 FORMAT (6I5)
5009 FORMAT (2F10.0, I5)
5010 FORMAT (I5, 4F12.8)
5011 FORMAT (7F15.8)
5012 FORMAT (6I5)
5013 FORMAT (4F10.0)
5014 FORMAT (6I5)
5015 FORMAT (2F10.0, I5)
5016 FORMAT (2F10.0)

C ************* WRITE FORMAT SPECIFICATIONS ***************

C 6001 FORMAT (1H1, 6X, 4HUE =, F6.4, 6X, 17HNUMBER OF MODES =, I5)
6002 FORMAT (14HNAME J L/)
6003 FORMAT (6X, A4, 21.5)
6004 FORMAT (1HO, 26H NUMBER OF COEFFICIENTS C(I1, 10H, NJ, NP) I5, I5/)
6005 FORMAT (2X, 2HC(I1, 10H, I2, 1H, I2, 4H) =, F10.5)
6006 FORMAT (1HO, 38H NUMBER OF COEFFICIENTS D(NJ, NP, NQ) I5, I5/)
6007 FORMAT (2X, 2HD(I2, 1H, I2, 1H, I2, 4H) =, F10.5)
6008 FORMAT (1H1, 2X, 17HMOTOR PARAMETERS, 15K,
18HGAMMA =, F4.2, 10X, 19HEXIT MACH NUMBER =, F7.5/)
6009 FORMAT (16X, 10HGAMMABAR =, F9.6/)n
6010 FORMAT (1HO, 6X, 1HJ, 7X, 2HYR, 8X, 2HYI, 7X, 3EPS, 7X, 3ETA//)
6011 FORMAT (2X, I5, F12.5, 5F22.5)
6012 FORMAT (1H1)
6013 FORMAT (1H1)
6014 FORMAT (1H1)
6015 FORMAT (1H1)
6016 FORMAT (1H1, 1X, 36H INITIAL CONDITIONS ARE OF THE FORM:
1) 2X, 47HU(I,J) = AC(J)*COS(FREQ*T) + AS(J)*SIN(FREQ*T),
2) //, 6X, 1HJ, 6X, 9HFREQUENCY, 10X, 5HAC(J), 10X, 5HAS(J)///)
6017 FORMAT (2X, I5, 4F15.8/)n
6020 FORMAT (1H1, //, 2X, 45HCoefficients FOR COMPUTATION OF WALL PRESSURE,
1) 10H WAVEFORMS//34X, 27HCoefficients IN SERIES FOR///
2) 37X, 4HTIME, 21X, 5AXIAL/6X, 1HJ, 7X, 1H2, 19X, 10HDERIVATIVE,
3) 15X, 10HDERIVATIVE///)
6021 FORMAT (2X, I5, F10.3, 12X, F15.7, 10X, F15.7)
FORMAT (3X, 4HSTEP, 8X, 4HTIME, 15X, 8HPRESSURE, 14X, 8HPRESSURE, 14X,
    1 8HPRESSURE, 14X, 7HGAS VEL, 15X, 7HPAR VEL, /3, 34X,
    2 8HAT Z=0.0, 14X, 8HAT Z=1.0, 14X, 8HAT Z=0.5, 13X,
    3 8HAT Z=0.0, 14X, 8HAT Z=0.5, /)

FORMAT (1H1) 38H PRESSURE MAXIMA AND MINIMA AT: Z = .F5.2,
    1 /19H VALUES COMPUTED: ,I3//)

FORMAT (1H ,7X, 8F13.6)

FORMAT (1H0, 32H NUMBER OF COEFFICIENTS E(NJ, NP, , I1, 4H) I S, I5/)

FORMAT (2X, 2HE(I2, 1H,, I2, 1H,, I1, 4H) = ,F10,, 5)

FORMAT (1H0, 3HPARTICLE DIA (IN MICRONS) = ,YF5,, 1, 10X,
    1 4HCBM =, F4,, 2, 10X, 18HFREQ (IN Hertz) = ,F6,, 1, /,
    2 6X, 26HCHAMBER TEMP (IN DEG K) = ,F6,, 1, 10X, 4HSP =,
    3 F4,, 2, 10X, 27HRHOM (IN KG/CUBIC METER) = ,F6,, 1, /, 6X,
    4 27HPARTICLE DRAG CONSTANT, K =, F8,, 4, / /)

FORMAT (1H0, 32H NUMBER OF COEFFICIENTS E(NJ, NP, , I1, 4H) I S, I5/)

FORMAT (2X, 2HE(I2, 1H,, I2, 1H,, I1, 4H) = ,F10,, 5)

FORMAT (1H0, 3HPARTICLE DIA (IN MICRONS) = ,YF5,, 1, 10X,
    1 4HCBM =, F4,, 2, 10X, 18HFREQ (IN Hertz) = ,F6,, 1, /,
    2 6X, 26HCHAMBER TEMP (IN DEG K) = ,F6,, 1, 10X, 4HSP =,
    3 F4,, 2, 10X, 27HRHOM (IN KG/CUBIC METER) = ,F6,, 1, /, 6X,
    4 27HPARTICLE DRAG CONSTANT, K =, F8,, 4, / /)

FORMAT (1H0, 3HNUMBER OF COEFFICIENTS IN PARTICLE EQUATIONS I S, I5/)

FORMAT (2X, 5HCPAR(I2, 1H, I2, 4H) = ,F10, 5)

FORMAT (1H0, 3HNUMBER OF COEFFICIENTS IN PARTICLE EQUATIONS I S, I5/)

FORMAT (2X, 5HCPAR(I2, 1H, I2, 4H) = ,F10, 5)

FORMAT (3X, 27HLINEAR COMBUSTION RESPONSE )

FORMAT (3X, 27HLINEAR COMBUSTION RESPONSE )

FORMAT (3X, 27HLINEAR COMBUSTION RESPONSE )

END
COMPLEX FUNCTION CCOSH(X)
COMPLEX X
CCOSH = 0.5 * (CEXP(X) + CEXP(-X))
RETURN
END

COMPLEX FUNCTION CSINH(X)
COMPLEX X
CSINH = 0.5 * (CEXP(X) - CEXP(-X))
RETURN
END

SUBROUTINE PHICFS(NP,Z,CT,CZ)
   C
   THIS SUBROUTINE COMPUTES THE COEFFICIENTS NEEDED TO
   CALCULATE THE PRESSURE PERTURBATION.
   C
   NP IS THE INDEX OF THE COMPLEX SERIES TERM.
   C
   Z IS THE AXIAL LOCATION.
   C
   CT IS THE COEFFICIENT IN THE SERIES FOR THE TIME DERIVATIVE OF
   THE VELOCITY POTENTIAL.
   C
   CZ IS THE COEFFICIENT IN THE SERIES FOR THE AXIAL DERIVATIVE
   OF THE VELOCITY POTENTIAL.
   C
   COMPLEX CI, CZ, B(6), CT, CCOSH, CSINH
   COMMON /BLK2/ B
   CI = (0.0,1.0)
   CT = CCOSH(CI * B(NP) * Z)
   CZ = CI * B(NP) * CSINH(CI * B(NP) * Z)
   RETURN
END
SUBROUTINE PRSVEL(UBAR, UMS, Y, P, VZGAS, VZPAR)

C
C THIS SUBROUTINE COMPUTES THE PRESSURE AND VELOCITY.
C
C UBAR IS THE LOCAL AXIAL STEADY STATE VELOCITY.
C UMS IS THE DERIVATIVE OF THE VELOCITY.
C Y IS THE ARRAY CONTAINING VALUES OF THE MODE-AMPLITUDE
C FUNCTIONS AND THEIR DERIVATIVES.
C P IS THE VALUE OF THE PRESSURE PERTURBATION.
C VZGAS IS THE AXIAL COMPONENT OF VELOCITY OF GAS.
C VZPAR IS THE AXIAL COMPONENT OF PARTICLE VELOCITY.
C
C DIMENSION Y(36), SUMS(5), SUMSQ(2)
COMMON /BLK3/ NPRTKL, NJMAX, NLMAX, GAMMA,
1 COEF(2, 12), NJMAX2
COMMON / BLK4/ CM, PARTKL, RHOP
C
DO 10 I = 1, 5
SUM(I) = 0.0
10 CONTINUE
C
DO 20 J = 1, NJMAX
JY = J + NJMAX2
20 SUM(1) = SUM(1) + Y(JY) * COEF(1, J)
DO 50 J = 1, NJMAX
SUM(2) = SUM(2) + Y(J) * COEF(2, J)
SUM(3) = SUM(3) + Y(J) * COEF(1, J)
IF (NPRTKL * EQ 0) GO TO 50
JP = J + NJMAX
SUM(4) = SUM(4) + (Y(J) - Y(JP)) * COEF(1, J)
SUM(5) = SUM(5) + Y(JP) * COEF(2, J)
50 CONTINUE
PLIN = SUM(1) + UBAR * SUM(2) + UMS * SUM(3)
1 + PARTKL * RHOP * SUM(4)
PNL = 0.0
IF (NLMAX * EQ 0) GO TO 40
DO 30 I = 1, 2
SUMSQ(I) = SUM(I) * SUM(I)
30 CONTINUE
PNL = 0.5 * (SUMSQ(2) - SUMSQ(1))
C
40 P = -GAMMA * (PLIN + PNL)
VZGAS = SUM(2)
VZPAR = SUM(5)
C
RETURN
END
SUBROUTINE RHS(U, UP)

COMPLEX RES(6), RESNL(6)
DIMENSION U(36), UP(36), E(12, 12, 2)
COMMON C(2, 12, 24), D(12, 144), CP(3, 12, 24),
1 KPMAX(3, 12), IC(2, 12, 24), CPPAR(2, 12, 24),
2 COMMON /BLK3/ NPRTKL, NJMAX, NLMAX, GAMMA,
1 COMMON /BLK5/ RES, NCOMB, BCOMB, E
COMMON /BLK7/ NNPRT, CNNPRT

IF (NPRTKL .EQ. 0) GO TO 110
NJS = NJMAX + 1
DO 112 NJ = NJS, NJMAX2
NJPAR = NJ - NJMAX
SLP = 0.0
DO 114 KP = 1, NJMAX2
SLP = SLP + (CPAR(NJPAR, KP) * U(KP))
114 CONTINUE
UP(NJ) = - SLP
112 CONTINUE

IF (NNPRT .EQ. 0) GO TO 110
DIFF = (U(1) - U(NJS))**2 + (U(2) - U(NJS+1))**2
IF (DIFF .LT. 0.000000001) GO TO 110
DIFF13 = CNNPRT * DIFF **(1/3.)
DO 120 NJ = NJS, NJMAX2
NJPAR = NJ - NJMAX
SLNP = 0.0
DO 125 KP = 1, NJMAX2
SLNP = SLNP + (CPPAR(NJPAR, KP) * DIFF13 * U(KP))
125 CONTINUE
UP(NJ) = UP(NJ) - SLNP
120 CONTINUE
110 CONTINUE

IF (NCOMB .EQ. 0) GO TO 116
JMX = NJMAX/2
DO 118 NJ = 1, JMX
NJPLNJ = 2*NJ
NJ2MN1 = NJPLNJ - 1
RESNL(NJ) = RES(NJ) * BCOMB * SQRT (UP(NJ2MN1)**2 + UP(NJPLNJ)**2)
118 CONTINUE
116 CONTINUE

DO 10 NJ = 1, NJMAX
NJP = NJ + NJMAX2
UP(NJ) = U(NJP)
SL1 = 0.0
SL2 = 0.0
SL3 = 0.0
SNL1 = 0.0
SNLC = 0.0
MAX = KPMAX(1,NJ)
IF (MAX .EQ. 0) GO TO 25
DO 20 KP = 1, MAX
   NP = IC(1,NJ,KP)
   SL1 = SL1 + (C(1,NJ,KP) * U(NP))
20 CONTINUE

25 MAX = KPMAX(2,NJ)
IF (MAX .EQ. 0) GO TO 45
DO 30 KP = 1, MAX
   NP = IC(2,NJ,KP)
   SL2 = SL2 + (C(2,NJ,KP) * U(NP))
30 CONTINUE

45 IF (NJMAX .EQ. 0) GO TO 55
MAX = KPGMAX(NJ)
IF (MAX .EQ. 0) GO TO 55
DO 50 KPQ = 1, MAX
   NP = IDP(NJ,KPQ)
   NQP = IDQ(NJ,KPQ) + NJMAX2
   SNL1 = SNL1 + (D(NJ,KPQ) * U(NP) * U(NQP))
50 CONTINUE

55 CONTINUE
IF (NCOMB .EQ. 0) GO TO 65
DO 60 KP = 1, NJMAX
   KP12 = (KP+1)/2
   SNLC = SNLC + (REAL(RESNL(KP12)) * E(NJ,KP,1) +
   AIMAG(RESNL(KP12)) * E(NJ,KP,2)) * UP(KP)
60 CONTINUE

65 UP(NJP) = -(SL1 + SL2 + SNL1 + SNLC)
10 CONTINUE

C
IF (NNPRT .EQ. 0) RETURN
IF (DIFF .LT. 0.0000000001) RETURN
DIFF13 = CNNPRT * DIFF **(1./3.)
DO 140 NJ = 1, NJMAX
   NJP = NJ + NJMAX2
   SLNP = 0.0
   DO 145 KP = 1, NJMAX2
      SLNP = SLNP + DIFF13 * CP(3,NJ,KP) * UP(KP)
145 CONTINUE
   UP(NJP) = UP(NJP) - SLNP
140 CONTINUE
C
RETURN
END
SUBROUTINE RESPNS (EN, A, B, OMEGA, RES)
COMPLEX RES, LAMBDA
C = (1.0 + 16.*OMEGA*OMEGA)**0.5
C1 = 0.5 * (1.0 + ((C+1.0)/2.0)**0.5)
C2 = 0.5 * ((C-1.0)/2.0)**0.5
LAMBDA = CMPLX(C1, C2)
RES = (EN*A*B) / (LAMBDA + A/LAMBDA - (1.0+A) + A*B)
RETURN
END
SUBROUTINE GRAPHS(I BUF, NLOC, LDEV, NTOT, NTICX, NTICY,
1 XMAX, YMAX, XMIN, YMIN, I TI TLX, I TI TLY, L TI TLX, L TI TLY, XARRAY,
2 YARRAY, DELX, DELY, TITLE)

C
C IDENTIFIER MEANING TYPE
C
C I BUF: ADDRESS OF BUFFER AREA FOR PLOT OUTPUT INTEGER
C NLOC: NUMBER OF LOCATIONS IN BUFFER AREA INTEGER
C LDEV: LOGICAL DEVICE NUMBER FOR PLOT INTEGER
C NTOT: NUMBER OF POINTS TO BE PLOTTED INTEGER
C NTICX: NUMBER OF TIC MARKS ON ABSCISSA (> OR = 2) INTEGER
C NTICY: NUMBER OF TIC MARKS ON ORDINATE (> OR = 2) INTEGER
C XMAX: UPPER LIMIT OF ABSCISSA DOMAIN REAL
C YMAX: UPPER LIMIT OF ORDINATE RANGE REAL
C XMIN: LOWER LIMIT OF ABSCISSA DOMAIN REAL
C YMIN: LOWER LIMIT OF ORDINATE RANGE REAL
C I TI TLX: ABSCISSA LABEL FIELD DATA ARRAY
C I TI TLY: ORDINATE LABEL FIELD DATA ARRAY
C LTI TLX: NUMBER OF CHARACTERS IN I TI TLX INTEGER
C LTI TLY: NUMBER OF CHARACTERS IN I TI TLY INTEGER
C XARRAY: ABSCISSA POINTS IN TERMS OF XMIN XMAX COORD'S REAL ARRAY
C YARRAY: ORDINATE POINTS IN TERMS OF YMIN YMAX COORD'S REAL ARRAY
C DELX: INTERVALS OF ABSCISSA TIC MARK LABELING REAL
C DELY: INTERVALS OF ORDINATE TIC MARK LABELING REAL
C TITLE: LABEL FOR THE WHOLE RUN FIELD DATA ARRAY
C
C DIMENSION I BUF(NLOC), XARRAY(NTOT), YARRAY(NTOT), I TI TLX(1),
1 I TI TLY(1), YDI T(100)
DIMENSION TITLE(1)
REAL LEFMAR, LABSEP
COMMON / BLK6/ HEIGHT, J, INTEQ, ABSCIS, ORDINA, I CODE, TOPMAR, BOTMAR,
1 LEFMAR, RYMAR, FACT, MAXI S, MINES, Tickle, STARTL, SEPLA
2 SYMHL, LABSEP, ASTART, YDI T, ROTFAC
C
C FIXED BASIC PARAMETERS
C
C DATA J/1/
DATA HEIGHT/ 105/
DATA INTEQ/1/
DATA ABSCIS/7.5/
DATA ORDINA/4.5/
DATA I CODE/ - 1/
DATA TOPMAR/1.6/
DATA BOTMAR/2.4/
19 INITIAL COMPUTATION OF DERIVED PARAMETERS  
AND INITIAL PLOTS CALL  

20 SKIPS PRELIMINARIES FOR 2ND AND SUBSEQUENT CALLS

IF (J .GT. 1) GO TO 20  
19  
  YDIT(1) = 3./19.  
  TICKLE = HEIGHT/2.  
  ROTFAC = -11.0/14.0 * HEIGHT  
  STARTL = 6 * HEIGHT + ROTFAC + TICKLE  
  SEPLAB = STARTL + 1.5 * HEIGHT  
  SYMBLH = 0.070  
  LABSEP = 4. * HEIGHT  
  ASTART = 2. * HEIGHT  
  LEFMAR = LEFMAR + 0.5  
  RYTMAR = RYTMAR - 0.5  
  DO 1 I = 2,100  
  1  
  YDIT(I) = YDIT(I - 1) + (2 * MOD(I,2) + 1)/19.  
  YDIT(100) = YDIT(100) + .5  
  CALL PLOTS(IBUF,NLOC,LDEV,00)  
  CALL FACTOR(1.)  
  J = 2  
  CALL SYMBOL(HEIGHT,36 * HEIGHT + 5.5*HEIGHT, Ti TLE,270.,70)  
  CALL PLOT(1.,0.0,3)  
  3  
  DO 3 I = 1,100  
  2  
  CALL PLOT(0.,YDIT(I),3 - MOD(I,2))  
  DO 33 I = 1,100  
  33  
  YDIT(I) = YDIT(I) - ABSCI S - RYTMAR  

RESET ORIGIN  

XPAGE = BOTMAR + ORDINA  
GO TO 2019  
20  
XPAGE = BOTMAR + ORDINA + TOPMAR  
2019  
CALL WHERE(RXPAGE,RYPAGE,FACT)  
YPAGE = RYPAGE - LEFMAR  
CALL PLOT(XPAGE,YPAGE,3)  
CALL FACTOR(FACT)  

DRAW AXES AND LABELING MAXIS TIMES
DO 100 I = 1, MAXI S
100 CALL MYAXI S(N TI CX, N TI CY, I TI TLX, I TI TLY, XMAX, YMAX, XMIN, YMIN,
L TI TLX, L TI TLY, DELX, DELY)

C
C
C
C

DRAW POINTS, OPTIONAL CENTERLINE, AND PAGE SCISSORLINE
C
MLINE TIMES

C
C
C

DO 200 I = 1, MLINE
200 CALL MYL I NE(XARRAY, YARRAY, XMAX, YMAX, XMIN, YMIN, NTOT, NTCY)
RETURN
END

SUBROUTINE MYAXI S(N TI CX, N TI CY, I TI TLX, I TI TLY, XMAX, YMAX, XMIN, YMIN,
L TI TLX, L TI TLY, DELX, DELY)
COMMON /BLK6/HEIGHT, J, INTEG, ABSCI S, ORDINA, I CODE, TOPMAR, BOTMAR,
LEFMAR, RY TMA, FACT, MAXI S, MLINE, TI CKL E, STARTL, SEPL,
SYMBLH, LABSEP, ASTART, YDI T, ROTFAC
REAL LABSEP, LEFMAR
DIMENSION I TI TLX(1), I TI TLY(1), YDI T(100)
STARTL = 6 * HEIGHT + ROTFAC + TI CKL E
IMAX = IFIX((YMAX - YMIN) /DELY + 0.5)
TICSEP = ORDINA/(IABS(NTICY) - 1)
CALL DENDEC(YMAX, DELY, NDEC)
K = 1
N = (IABS(NTICY)/IMAX) - 1 + MOD(IABS(NTICY), 2)
IMAX1 = IMAX + 1
DO 9 ILOOP = 1, IMAX1
9 I = ILOOP - 1
NN DEC = 0
IF (K .NE. 1) GO TO 12
11 IF(2 * I .LT. IMAX) GO TO 12
CALL AXLAB(0., I TI TLY, L TI TLY)
K = 2
12 FPN = YMAX - I * DELY
IF (NDEC .LT. 0 .AND. ABS(FPN) .LT. 0.5) FPN = 0.0
IF (NDEC .GT. 0 .AND. ABS(FPN) .LT. 5.0*10.0**(NDEC-1)) FPN = 0
TMID = 1.
XPAGE = - I * ORDINA/IMAX - .5 * HEIGHT
IF(FPN)113, 122, 118
113 IF(NDEC - 2)115, 114, 112
114 YPAGE = STARTL
GO TO 112
115 IF(NDEC - 1)117, 116, 112
116 YPAGE = STARTL - HEIGHT
GO TO 112
117 IF(ABS(FPN) - 100.)119, 116, 116
119 IF(ABS(FPN) - 10.120, 121, 121
120 YPAGE = STARTL - 3 * HEIGHT
GO TO 112
121 YPAGE = STARTL - 2 * HEIGHT  
122 YPAGE = STARTL - 4 * HEIGHT  
123 GO TO 112  
118 IF(NDEC - 2)123, 116, 112  
123 IF(NDEC - 1)125, 124, 112  
124 IF(FPN - 10.)121, 116, 116  
125 IF(FPN - 10.)122, 120, 126  
126 IF(FPN - 100.)120, 121, 127  
127 IF(FPN - 1000.)121, 116, 128  
128 IF(FPN - 10000.)116, 114, 114  
112 NNDEC = -1  
119 IF (FPN NE 0.0) NNDEC = NDEC  
120 CALL NUMBER(XPAGE, YPAGE, HEIGHT, FPN, 270., NNDEC)  
121 XPAGE = - I * (ORDINA/IMAX)  
122 DO 10 JJ = 1, N  
123 YPAGE = TICKLE * TMID  
124 CALL PLOT(XPAGE, YPAGE, 3)  
125 YPAGE = YPAGE * ( - 1 + I/IMAX * .5)  
126 CALL PLOT(XPAGE, YPAGE, 2)  
127 IF(I/IMAX) 110, 110, 9  
110 YPAGE = 0  
128 CALL PLOT(XPAGE, YPAGE, 3)  
129 XPAGE = XPAGE - TICSEP  
130 CALL PLOT(XPAGE, YPAGE, 2)  
131 TMID = .5  
10 CONTINUE  
9 CONTINUE  
121 K = 1  
122 IMAX = I/FIX((XMAX-XMIN)/DELX + 0.5)  
123 TICSEP = ABSCI S/(NTICX - 1)  
124 XPAGE = - ASTART - ORDINA  
125 CALL DENDEC(XMAX, DELX, NDEC)  
126 IMAX1 = IMAX + 1  
127 DO 28 ILOOP = 1, IMAX1  
128 I = ILOOP - 1  
129 NNDEC = 0  
130 STARTL = - I * ABSCI S/IMAX  
131 IF (K GT 1) GO TO 25  
132 IF(2 * I LT IMA X) GO TO 25  
133 CALL AXLAB(270., I TILX, LTI TLX)  
134 K = 2  
25 FPN = XMIN + I * DELX  
26 IF (NDEC LT 0 AND ABS(FPN) LT 0.5) FPN = 0.0  
27 IF (NDEC GT 0 AND ABS(FPN) LT 5.*0**NDEC-1)) FPN = 0.0  
28 IF(FPN)813, 822, 818  
813 IF(NDEC = 2)815, 814, 23  
814 YPAGE = STARTL + 16.*7. * HEIGHT  
815 GO TO 23
IF(NDEC = 1) YPAGE = STARTL + 25./14. * HEIGHT
GO TO 23
IF(ABS(FPN) = 100.) YPAGE = STARTL + 9./7. * HEIGHT
GO TO 23
IF(NDEC = 2) YPAGE = STARTL + 2./7. * HEIGHT
GO TO 23
IF(NDEC = 1) YPAGE = STARTL + 11./14. * HEIGHT
GO TO 23
IF(ABS(FPN) = 10.) YPAGE = STARTL + 2.17 * HEIGHT
GO TO 23
IF(NDEC = 2) YPAGE = STARTL + 25./14. * HEIGHT
GO TO 23
IF(NDEC = 1) YPAGE = STARTL + 11./14. * HEIGHT
GO TO 23
IF(ABS(FPN) = 100.) YPAGE = STARTL + 9./7. * HEIGHT
GO TO 23
IF(ABS(FPN) = 1000.) YPAGE = STARTL + 2.17 * HEIGHT
GO TO 23
IF(FPN = 10000.) YPAGE = STARTL + 9./7. * HEIGHT
GO TO 23

IF (FPN .NE. 0.0) NNDEC = NDEC
CALL NUMBER(XPAGE,YPAGE,HEIGHT,FPN,270.,NNDEC)
N = (NTICX/IMAX) - 1 + MOD(NTICX,2)
IMAX1 = IMAX + 1
DO 26 ILOOP = 1, IMAX1
  I = IMAX1 - ILOOP
  TMID = 1.
  YPAGE = - I * ABSCI S/IMAX
  DO 27 JJ = 1,N
    XPAGE = - ORDINA - TICKLE * TMID
    CALL PLOT(XPAGE,YPAGE,3)
    XPAGE = XPAGE + 2.*TICKLE*TMID
    CALL PLOT(XPAGE,YPAGE,2)
    IF (I .EQ. 0) GO TO 26
  111 XPAGE = - ORDINA
  CALL PLOT(XPAGE,YPAGE,3)
  YPAGE = YPAGE + TICSEP
  CALL PLOT(XPAGE,YPAGE,2)
  TMID = .5
  CONTINUE
  CONTINUE
  RETURN
END
SUBROUTINE MYLINE(XARRAY, YARRAY, XMAX, YMAX, XMIN, YMIN, NTOT, NTICY)
COMMON /BLK6/HEIGHT, J, INTEG, ABSCI, ORDINA, I CODE, TOPMAR, BOTMAR,
        LEMAR, RYMAR, FACT, MAXS, MLINE, TICKLE, STARTL, SEPLAB,
        SYMBLH, LABSEP, ASTART, YDI T, ROTFAC
REAL LEMAR, LABSEP
DIMENSION XARRAY(NTOT), YARRAY(NTOT), YDI T(100)
I TOP = IFIX((ABSCI + RYMAR + 0.5)*9.0 + 0.5)
I BOT = IFIX(RYMAR*9.0 + 0.5)
DO 17 I = 1, NTOT
XPAGE = (YARRAY(I) - YMAX)/(YMAX - YMIN) * ORDINA
YPAGE = (XMIN - XARRAY(I))/(XMAX - XMIN) * ABSCI
17 CALL SYMBOL(XPAGE, YPAGE, SYMBLH, INTEG, 270., I CODE)
IF(NTICY GE 0) GO TO 22
XPAGE = 0.25*ORDINA
YPAGE = 0.25*ABSCI
CALL PLOT(XPAGE, YPAGE, 3)
DO 18 I = I BOT, I TOP
18 CALL PLOT(XPAGE, YDI T(I), 3 - MOD(I, 2))
22 XPAGE = TOPMAR
YPAGE = 0.25*ABSCI - RYMAR + 0.5
CALL PLOT(XPAGE, YPAGE, 3)
DO 21 I = 1, 100
21 CALL PLOT(XPAGE, YDI T(I), 3 - MOD(I, 2))
RETURN
END

SUBROUTINE AXLAB(ANGLE, I BCD, NCHAR)
COMMON /BLK6/HEIGHT, J, INTEG, ABSCI, ORDINA, I CODE, TOPMAR, BOTMAR,
        LEMAR, RYMAR, FACT, MAXS, MLINE, TICKLE, STARTL, SEPLAB,
        SYMBLH, LABSEP, ASTART, YDI T, ROTFAC
REAL LEMAR, LABSEP
DIMENSION I BCD(1), YDI T(100)
K = 2
NCHAR = NCHAR
IF (ABS(ANGLE) GT 0.1) GO TO 30
XPAGE = - 0.25*ORDINA - 0.5 - NCHAR*HEIGHT*0.5
YPAGE = 0.25*SEPLAB
GO TO 31
30 XPAGE = - 0.25*ORDINA - LABSEP
YPAGE = 0.25*ABSCI + 0.5 + NCHAR*HEIGHT*0.5
31 CALL SYMBOL(XPAGE, YPAGE, HEIGHT, I BCD, ANGLE, NCHAR)
RETURN
END

SUBROUTINE DENDEC(QMAX, DELQ, NDEC)
IF( INT(ABS(QMAX)) GE 10) GO TO 5
IF( AMOD(ABS(QMAX - DELQ), 1) GE 0.01) GO TO 7
NDEC = 1
RETURN
5 NDEC = - 1
RETURN
7 NDEC = 2
RETURN
END

119
SUBROUTINE GROWTH(MAXP, TIMAX, PMAX, FREQ)
DIMENSION TIMAX(1), PMAX(1), TIME(100), PEAK(100), ALPHA(100),
T(100), F(100)
C
DO 100 I = 1, MAXP
TIME(I) = 0.0
PEAK(I) = 0.0
ALPHA(I) = 0.0
T(I) = 0.0
F(I) = 0.0
100 CONTINUE
C
K = 0
J = 0
110 K = K + 1
IF (K .GT. MAXP) GO TO 120
IF (PMAX(K) .LT. 0.0) GO TO 110
J = J + 1
TIME(J) = TIMAX(K)
PEAK(J) = PMAX(K)
140 IF (PMAX(K+1) .LT. PEAK(J)) GO TO 130
K = K + 1
TIME(J) = TIMAX(K)
PEAK(J) = PMAX(K)
GO TO 140
C
130 IF (PMAX(K+1) .LE. 0.0) GO TO 110
K = K + 1
GO TO 140
120 CONTINUE
C
NCYCLE = J - 1
DO 150 I = 1, NCYCLE
T(I) = 0.5 * (TIME(I) + TIME(I+1))
F(I) = 2.0 * FREQ / (TIME(I+1) - TIME(I))
ALPHA(I) = ALOG(PEAK(I+1)/PEAK(I)) * F(I)
150 CONTINUE
C
WRITE (6,6001) NCYCLE
LINE = 4
DO 255 IST = 1, NCYCLE, 8
ISTART = IST
ISTOP = IST + 7
IF (ISTOP .GT. NCYCLE) ISTOP = NCYCLE
WRITE (6,6024) (ALPHA(I), I = ISTART, ISTOP)
WRITE (6,6024) (F(I), I = ISTART, ISTOP)
WRITE (6,6024) (T(I), I = ISTART, ISTOP)
WRITE (6,6014)
LINE = LINE + 4
IF (LINE .LT. 52) GO TO 255
LINE = 0
WRITE (6,6013)
255 CONTINUE
6001 FORMAT (1H1,4X,35HPRESSURE GROWTH RATE AND FREQUENCY., 1  /*5X,23HTOTAL NUMBER OF CYCLES: I3///)
6013 FORMAT (1H1)
6014 FORMAT (1H )
6024 FORMAT (1H ,7X,8F13.6)
C
   RETURN
END
3.3 **PROGRAM MA1**

Program MA1 calculates the coefficients of both the linear and nonlinear terms in the equations describing the behavior of the mode amplitudes (i.e., Equations (25)) to be solved using the Method of Averaging (MQA).

*Program Description.* This program is very similar to Program SOLID1 in its structure, subroutines, input and output. Hence it is unnecessary to describe them in detail; a description of MA1 is obtained by substituting "MA1" for "SOLID1" and "MA2" for "SOLID2" in Section 3.1.

*Sample Case.* Here, a sample case utilizing Program MA1 is given. All of the motor parameters and specifications are the same as described in Section 3.1. In the following pages, printed output of MA1 for this sample case is presented.
TEST CASE FOR MA1

GAMMA = 1.23000       UE = .0780

QUASI-STEADY NOZZLE.

PARTICLE DIA (IN MICRONS) = 2.50   CM = .1000   FREQ (IN HERTZ) = 1071.0
CHAMBER TEMP (IN DEG K) = 3525.0   SP = .6000   RHOM (IN KG/CUBIC METER) = 4000.0
PARTICLE DRAG COEFFICIENT, K = 29.9186

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**CECOUFLEC COEFFICIENT OF B(F) * CB(C)/DT IN EQUATION FOR B(10)**

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FORTRAN Source Code.

PROGRAM MA1(INPUT, OUTPUT, DATA,  
1 TAPES=INPUT, TAPE6=OUTPUT, TAPE9=DATA)

*************** PROGRAM MA1 *********************************

THIS PROGRAM COMPUTES THE COEFFICIENTS WHICH APPEAR  
IN THE DIFFERENTIAL EQUATIONS WHICH GOVERN THE MODE-AMPLITUDE  
FUNCTIONS. THESE COEFFICIENTS CAN BE WRITTEN INTO A FILE  
FOR INPUT TO PROGRAM MA2.

THE FOLLOWING INPUTS ARE REQUIRED:

THE FIRST CARD GIVES THE TITLE OF THE CASE.

SECOND CARD: GAM, UE, NOZZLE  
GAM IS THE SPECIFIC HEAT RATIO.  
UE IS THE STEADY STATE MACH NUMBER AT THE NOZZLE ENTRANCE.  
NOZZLE SPECIFIES THE TYPE OF NOZZLE USED:

NOZZLE = 0 QUASI-STeadY  
NOZZLE = 1 CONVENTIONAL NOZZLE

THIRD CARD: NJMAX, NONLIN, NEGL, NOUT, NPRTKL  
NJMAX IS THE NUMBER OF MODE-AMPLITUDE FUNCTIONS IN THE ASSUM  
SERIES SOLUTION.  
THE COEFFICIENTS COMPUTED ARE DETERMINED BY NONLIN AS FOLLOW:

NONLIN = 0 LINEAR COEFFICIENTS ONLY  
NONLIN = 1 BOTH LINEAR AND NONLINEAR COEFFICIENTS.

COEFFICIENTS TO BE NEGLECTED ARE DETERMINED BY NEGL  
AS FOLLOWS:

NEGL = 0 TERMS SMALLER THAN 0.00001 ARE NEGLECTED.  
NEGL = 1 LINEAR TERMS SMALLER THAN SM1 AND NONLINEAR  
TERMS SMALLER THAN SM2 ARE NEGLECTED.

THE OUTPUT IS DETERMINED BY NOUT AS FOLLOWS:

NOUT = 0 PRINTED OUTPUT ONLY  
NOUT = 1 WRITE INTO A FILE AND PRINT OUTPUT.  
NOUT = 2 WRITE INTO A FILE ONLY.

NPRTKL DETERMINES WHETHER THE PARTICLES ARE PRESENT:

NPRTKL = 0 PARTICLES NOT PRESENT.  
NPRTKL = 1 PARTICLES PRESENT.

NEXT CARD (ONLY IF NPRTKL=1): DIA, RHOM, SP, TEMP, FREQ, CM  
DIA IS THE PARTICLE DIAMETER, IN MICRONS.  
RHOM IS THE DENSITY OF THE PARTICLE MATERIAL, IN KG/M**3.  
SP IS RATIO OF SPECIFIC HEATS OF PARTICLE MATERIAL AND GAS.  
TEMP IS THE CHAMBER TEMPERATURE, IN DEGREES KELVIN.  
FREQ IS THE FREQUENCY OF OSCILLATION IN PURE GAS, IN Hertz.  
CM IS THE PARTICLE LOADING.

NEXT CARD (NECESSARY ONLY IF NEGL = 1): SM1, SM2  
SM1 AND SM2 ARE AS DEFINED ABOVE.
NEXT NJMAX CARDS (ONLY IF NOZZLE = 1): J, AMPL(J), PHASE(J)
AMPL(J) IS THE MAGNITUDE OF THE NOZZLE ADMITTANCE
FOR THE J TH MODE.
PHASE(J) IS THE PHASE OF THE NOZZLE ADMITTANCE
FOR THE J TH MODE.

NEXT NJMAX CARDS: J, L(J), NAME(J)
EACH MODE-AMPLITUDE IS ASSIGNED AN INTEGER J-
THE MODE IS SPECIFIED BY THE INDEX L(J).
L(J) IS THE AXIAL MODE NUMBER AND MUST NOT EXCEED NJMAX.
NAME(J) IS A FOUR-CHARACTER NAME FOR THE J TH MODE.

******************************************************************
DIMENSION L(6), NAME(6), TITLE(7),
1 AMPL(6), PHASE(6), V(2), C(3),
2 12, 12, C112, 12, JC(12),
3 E(12, 12, 2), D(12, 12, 12),
4 KMAX(6), TSR(2, 12), TSQ(12), TS(3, 12)
COMPLEX
CERST, CI, ZEJ, ZEP1, ZEP2, AX(4, 4), AXINT(4, 3),
1 DCOEF, B(6), BC(6),
2 YNOZ(6), CC(5, 6, 6), CNORM(6),
3 CD1(6, 6, 6), CD2(6, 6, 6),
4 CD3(6, 6, 6), CD4(6, 6, 6), CCOSH, CSINH
COMMON
B

DATA INPUT.

MAXMD = 6
MAXMD2 = 12
MAXMD4 = 24
PI = 3.1415926536
SM1 = 0.00001
SM2 = 0.00001
CI = (0.0, 1.0)

INPUT PARAMETERS.
4 READ (5, 5000) TITLE
IF (EOF(5)) 600, 1
1 CONTINUE
READ (5, 5001) GAM, UE, NOZZLE
IF (GAM) 600, 600, 1
1 CONTINUE
READ (5, 5001) GAM, UE, NOZZLE
IF (GAM) 600, 600, 8
8 READ (5, 5004) NJMAX, NONLIN, NEGL, NOUT, NPRTKL
IF (NPRTKL .EQ. 1) READ (5, 5006) DIA, RHOM, SP, TEMP, FREQ, CM
IF (NEGL .EQ. 1) READ (5, 5005) SM1, SM2
GAMMA = GAM * (1.0 + SP * CM) / (1.0 + SP * GAM * CM)
IF (NOZZLE .EQ. 1) GO TO 5
C COMPUTE ADMITTANCE FOR QUASI-STEADY NOZZLE.
  Y = (GAMMA - 1.0) * UE/(2.0 * GAMMA)
DO 3 J = 1, NJMAX
  AMPL(J) = Y
  PHASE(J) = 0.0
3 CONTINUE
DO 6 I = 1, NJMAX
READ (5, 5003) J, AMPL(J), PHASE(J)
6 CONTINUE
7 DO 10 I = 1, NJMAX
 READ (5, 5002) J, L(J), NAME(J)
10 CONTINUE
C DO 12 J = 1, NJMAX
  THETA = PHASE(J) * PI/180.0
  YR = AMPL(J) * COS(THETA)
  YI = AMPL(J) * SIN(THETA)
  YNOZ(J) = CMPLX(YR, YI)
12 CONTINUE
C NJMAX2 = NJMAX
IF (NPRTKL =EQ. 1) NJMAX2 = 2 * NJMAX
ZE = 1.0
ZCOMB = 1.0
CAX = GAMMA + 1.0
RHOP = 0.0
IF (NPRTKL =EQ. 0) GO TO 14
VISC = 8.834 * 0.00001 * (TEMP/3485)**0.66
PARTKL = (9.0*VISC) / (RHOM * FREQ * DIA * DIA * 10**(-12))
UPBYU = 2.0 / (1.0 + SQRT(1.0 + 8.0*UE/PARTKL))
RHOP = CM/UPBYU
14 CONTINUE
C ****************************************************************
C CALCULATE AXIAL ACOUSTIC EIGENVALUES.
C
C COMPUTE EIGENVALUES.
DO 40 J = 1, NJMAX
  LL = L(J)
  SMN = 0.0
  YAMPL = AMPL(J)
  YPHASE = PHASE(J)
  CALL EIGVAL(LL, SMN, GAMMA, ZE, YAMPL, YPHASE, CRSLT)
  B(J) = CRSLT
  BC(J) = CONJG(CRSLT)
40 CONTINUE
CALCULATE LINEAR COEFFICIENTS.

DO 100 NJ = 1, NJMAX
DO 100 NP = 1, NJMAX

ZERO COEFFICIENT ARRAYS.
DO 105 KC = 1, 5
CC(KC,NJ,NP) = (0.0,0.0)
105 CONTINUE
NPM = NP
NJM = NJ

CALCULATE AXIAL INTEGRALS.
127 DO 130 NOPT = 1, 4
CALL AXIAL1(NOPT,NPM,NJM,UE,ZE,CRSLT)
AX(NOPT) = CRSLT
130 CONTINUE

EVALUATE FUNCTIONS AT NOZZLE END.
ZEJ = CCOSH(CI * BC(NJM) * ZE)
ZEP1 = CCOSH(CI * B(NPM) * ZE)
ZEP2 = CI * B(NPM) * CSINH(CI*B(NPM)*ZE)

COEFFICIENT OF THE SECOND DERIVATIVE OF A(P).
CC(1,NJ,NP) = AX(1)

COEFFICIENT OF A(P).
CC(2,NJ,NP) = - AX(2) + ZEP2*ZEJ

COEFFICIENT OF THE FIRST DERIVATIVE OF A(P).
CC(3,NJ,NP) = (CAX*AX(3) + 2.0*AX(4))
  + GAMMA*YN0Z(NP)*ZEP1*ZEJ
IF (NPRTKL .EQ. 0) GO TO 100
CC(2,NJ,NP) = CC(2,NJ,NP) - PARTKL **2 *RHOP * AX(1)
1
CC(3,NJ,NP) = CC(3,NJ,NP) + PARTKL * RHOP * AX(1)
CC(4,NJ,NP) = RHOP * PARTKL**3 * AX(1)
  + (GAMMA - 1.0) * PARTKL**2 * CM * AX(4)
100 CONTINUE

NORMALIZE LINEAR COEFFICIENTS.
DO 140 NJ = 1, NJMAX
CNORM(NJ) = CC(1,NJ,NJ)
DO 140 NP = 1, NJMAX
DO 140 KC = 1, 5
CC(KC,NJ,NP) = CC(KC,NJ,NP)/CNORM(NJ)
140 CONTINUE
**COMPUTE NONLINEAR COEFFICIENTS.**

IF (NONLIN .EQ. 0) GO TO 402

G1 = (GAMMA - 1.0) * 0.5

DO 200 NJ = 1, NJMAX
DCOEF = 0.5/CNORM(NJ)
DO 200 NP = 1, NJMAX
DO 200 NQ = 1, NJMAX

CD1(NJ,NP,NQ) = (0.0, 0.0)
CD2(NJ,NP,NQ) = (0.0, 0.0)
CD3(NJ,NP,NQ) = (0.0, 0.0)
CD4(NJ,NP,NQ) = (0.0, 0.0)

244 DO 240 J = 2, 3
DO 240 NC = 1, 4
CALL AXIAL2(J,NC,NP,NQ,NJ,ZE,CRSLT)
AXINT(NC,J) = CRSLT
240 CONTINUE

CD1(NJ,NP,NQ) = AXINT(1,2) + Gl*AXINT(1,3)
CD2(NJ,NP,NQ) = AXINT(2,2) + Gl*AXINT(2,3)
CD3(NJ,NP,NQ) = AXINT(3,2) + Gl*AXINT(3,3)
CD4(NJ,NP,NQ) = AXINT(4,2) + Gl*AXINT(4,3)

200 CONTINUE

**CALCULATE COEFFICIENTS FOR EQUIVALENT REAL SYSTEM.**

402 DO 350 NJ = 1, NJMAX
NEWJ = (2 * NJ) - 1
NEWJ1 = NEWJ + 1
DO 360 NP = 1, NJMAX
NEWP = (2 * NP) - 1
NEWP1 = NEWP + 1

COEFFICIENTS OF LINEAR TERMS:
CCR = REAL(CC(1,NJ,NP))
CCI = AIMAG(CC(1,NJ,NP))
Cl(NEWJ,NEWP) = CCR
Cl(NEWJ,NEWP1) = -CCI
Cl(NEWJ1,NEWP) = CCI
Cl(NEWJ1,NEWP1) = CCR

CONTINUE
DO 360 KC = 1, 3
CCR = REAL(CC(KC+1,NJ,NP))
CCI = AIMAG(CC(KC+1,NJ,NP))
C(KC,NEWJ,NEWP) = CCR
C(KC,NEWJ,NEWP1) = -CCI
C(KC,NEWJ1,NEWP) = CCI
C(KC,NEWJ1,NEWP1) = CCR

CONTINUE

CONTINUE

COEFFICIENTS OF THE COMBUSTION TERM:
DO 370 NP = 1, NJMAX
NEWP = 2*NP - 1
NEWP1 = NEWP + 1
CCR = REAL(CC(5,NJ,NP))
CCI = AIMAG(CC(5,NJ,NP))
E(NEWJ,NEWP,1) = CCR
E(NEWJ,NEWP,2) = -CCI
E(NEWJ,NEWP1,1) = -CCI
E(NEWJ,NEWP1,2) = -CCR
E(NEWJ1,NEWP,1) = CCI
E(NEWJ1,NEWP,2) = CCR
E(NEWJ1,NEWP1,1) = CCR
E(NEWJ1,NEWP1,2) = -CCI

COEFFICIENTS OF NONLINEAR TERMS:
IF (NONLIN EQ 0) GO TO 350
DO 370 NQ = 1, NJMAX
NEWQ = (2*NQ) - 1
NEWQ1 = NEWQ + 1
CD1R = REAL(CD1(NJ,NP,NQ))
CD1I = AIMAG(CD1(NJ,NP,NQ))
CD2R = REAL(CD2(NJ,NP,NQ))
CD2I = AIMAG(CD2(NJ,NP,NQ))
CD3R = REAL(CD3(NJ,NP,NQ))
CD3I = AIMAG(CD3(NJ,NP,NQ))
CD4R = REAL(CD4(NJ,NP,NQ))
CD4I = AIMAG(CD4(NJ,NP,NQ))
D(NEWJ,NEWP,NEWQ) = CD1R + CD2R + CD3R + CD4R
D(NEWJ,NEWP1,NEWQ1) = -CD1I + CD2I - CD3I + CD4I
D(NEWJ,NEWP1,NEWQ) = -CD1I - CD2I + CD3I + CD4I
D(NEWJ1,NEWP1,NEWQ1) = -CD1R + CD2R + CD3R - CD4R
D(NEWJ1,NEWP1,NEWQ) = CD1I + CD2I + CD3I + CD4I
D(NEWJ1,NEWP1,NEWQ1) = CD1R - CD2R + CD3R - CD4R
D(NEWJ1,NEWP1,NEWQ) = CD1R + CD2R - CD3R - CD4R
D(NEWJ1,NEWP1,NEWQ1) = -CD1I + CD2I + CD3I - CD4I

CONTINUE
350 CONTINUE
**COMPUTE COEFFICIENTS FOR THE EQUATIONS WHICH ARE DECOUPLED IN THE SECOND DERIVATIVES.**

```
DO 405 KC = 1, 6
KMAX(KC) = 0
405 CONTINUE
```

**CALCULATE INVERSE OF THE MATRIX C1(I,J).**

```
JMAX = NJMAX
NJMAX = 2 * NJMAX
```

```
V(1) = 1
CALL GJR(C1,MAXMD2,MAXMD2,NJMAX,0,JC,V)
```

**USE INVERSE TO CALCULATE DECOUPLED COEFFICIENTS.**

**LINEAR COEFFICIENTS.**

```
DO 430 NP = 1, NJMAX
DO 420 NJ = 1, NJMAX
DO 420 KC = 1, 3
TS(KC,NJ) = 0.0
DO 420 K = 1, NJMAX
TS(KC,NJ) = TS(KC,NJ) + CI(NJ,K) * C(KC,K,NP)
420 CONTINUE
DO 430 NJ = 1, NJMAX
DO 430 KC = 1, 3
C(KC,NJ,NP) = TS(KC,NJ)
ABSVAL = ABS(C(KC,NJ,NP))
IF (ABSVAL .GE. SM1) KMAX(KC) = KMAX(KC) + 1
430 CONTINUE
```

**COEFFICIENTS OF THE COMBUSTION RESPONSE TERM.**

```
DO 720 NP = 1, NJMAX
DO 725 NJ = 1, NJMAX
TSR(1,NJ) = 0.0
TSR(2,NJ) = 0.0
DO 725 K = 1, NJMAX
TSR(1,NJ) = TSR(1,NJ) + CI(NJ,K) * E(K,NP,1)
TSR(2,NJ) = TSR(2,NJ) + CI(NJ,K) * E(K,NP,2)
725 CONTINUE
```

```
E(NJ,NP,1) = TSR(1,NJ)
ABSVAL = ABS(E(NJ,NP,1))
IF (ABSVAL .GE. SM1) KMAX(4) = KMAX(4) + 1
E(NJ,NP,2) = TSR(2,NJ)
ABSVAL = ABS(E(NJ,NP,2))
IF (ABSVAL .GT. SM1) KMAX(5) = KMAX(5) + 1
730 CONTINUE
720 CONTINUE
```
NONLINEAR COEFFICIENTS.
IF (NONLIN *EQ* 0) GO TO 410
DO 735 NP = 1, NJMAX
DO 735 NQ = 1, NJMAX
DO 440 NJ = 1, NJMAX
TSQ(NJ) = 0.0
DO 440 K so 1, NJMAX
TSQ(NJ) = TSQ(NJ) + C 1 (NJ, K) * D(K, NP, NQ)
440 CONTINUE
DO 445 NJ = 1, NJMAX
D(NJ, NP, NQ) = TSQ(NJ)
ABSVAL = ABS(D(NJ, NP, NQ))
IF (ABSVAL GT SM2) KMAX(6) = KMAX(6) + 1
445 CONTINUE
735 CONTINUE

****************************************************
OUTPUT

IF (NOUT *EQ* 2) GO TO 455
WRITE (6,6001) TITLE
WRITE (6,6002) GAM, UE
IF (NOZZLE *EQ* 0) WRITE (6,6012)
IF (NPRTKL *EQ* 0) WRITE (6,6022)
IF (NPRTKL *EQ* 1) WRITE (6,6021) DIAs CM, FREQ,
1 TEMP, SP, RHOM, PARTKL
WRITE (6,6004)
DO 310 J = 1, JMAX
WRITE (6,6003) NAME(J), JS L(J), B(J), YNOZ(J)
310 CONTINUE
IF (NONLIN *EQ* 0) WRITE (6,6013)

OUTPUT OF LINEAR COEFFICIENTS.
DO 320 KC = 1, 3
NJ5 = 0
NJF = 0
KOUNTJ = 1
758 NJ5 = NJF + 1
NJF = 10 * KOUNTJ
IF (NJF GT NJMAX) NJF = NJMAX
NPS = 0
NPF = 0
KOUNTP = 1
754 NPS = NPF + 1
NPF = 10 * KOUNTP
IF (NPF GT NJMAX) NPF = NJMAX
IF (KC *EQ* 1) WRITE (6,6005)
IF (KC *EQ* 2) WRITE (6,6006)
IF (KC *EQ* 3) WRITE (6,6007)
WRITE (6,6008) (NP, NP = NPS, NPF)
WRITE (6,6014)
DO 750 NJ = NJS, NJF
WRITE (6,6009) NJ, (C(KC,NJ, NP), NP = NPS, NPF)
750 CONTINUE
IF (NPF =EQ. NJMAX) GO TO 752
KOUNTP = KOUNTP + 1
GO TO 754
752 IF (NJF =EQ. NJMAX) GO TO 756
KOUNTJ = KOUNTJ + 1
GO TO 758
756 CONTINUE
320 CONTINUE
C
C OUTPUT OF THE COMBUSTION RESPONSE TERM.
DO 770 KC = 1, 2
NJS = 0
NJF = 0
KOUNTJ = 1
760 NJS = NJF + 1
NJF = 10 * KOUNTJ
IF (NJF = GT. NJMAX) NJF = NJMAX
NPS = 0
NPF = 0
KOUNTP = 1
762 NPS = NPF + 1
NPF = 10 * KOUNTP
IF (NPF = GT. NJMAX) NPF = NJMAX
IF (KC = EQ. 1) WRITE (6,6019)
IF (KC = EQ. 2) WRITE (6,6020)
WRITE (6,6008) (NP, NP = NPS, NPF)
WRITE (6,6014)
DO 764 NJ = NJS, NJF
WRITE (6,6009) NJ, (E(NJ, NP, KC), NP = NPS, NPF)
764 CONTINUE
IF (NPF = EQ. NJMAX) GO TO 766
KOUNTP = KOUNTP + 1
GO TO 762
766 IF (NJF = EQ. NJMAX) GO TO 768
KOUNTJ = KOUNTJ + 1
GO TO 760
768 CONTINUE
770 CONTINUE
C
C OUTPUT OF NONLINEAR COEFFICIENTS.
IF (NONLIN = EQ. 0) GO TO 452
DO 400 NJ = 1, NJMAX
NPS = 0
NPF = 0
KOUNTP = 1
780 NPS = NPF + 1
NPF = 10 * KOUNTP
IF (NPF = GT. NJMAX) NPF = NJMAX
NQS = 0
NQF = 0
KOUNTQ = 1
776 NQS = NQF + 1
NQF = 10 * KOUNTQ
IF (NQF .GT. NJMAX) NQF = NJMAX
WRITE (6,6010) NJ
WRITE (6,6011) (NQ, NQ = NQS, NQF)
WRITE (6,6015)
DO 772 NP = NPS, NPF
WRITE (6,6009) NP, (D(NJ,NP,NQ), NQ = NQS, NQF)
772 CONTINUE
IF (NQF .EQ. NJMAX) GO TO 774
KOUNTQ = KOUNTQ + 1
GO TO 776
774 IF (NPF .EQ. NJMAX) GO TO 778
KOUNTP = KOUNTP + 1
GO TO 780
778 CONTINUE
400 CONTINUE
452 IF (NOUT .EQ. 0) GO TO 4
C WRITE COEFFICIENTS ON FILE.
C 455 WRITE (9,7001) GAMMA, UE, ZE, NJMAX, NPrTKL
IF (NPRTKL .EQ. 1) WRITE (9,7007) DIA, RHOM, SP,
1 TEMP, FREQ, PARTKL, CM
DO 450 J = 1, JMAX
WRITE (9,7002) J, L(J), NAME(J)
450 CONTINUE
DO 457 J = 1, JMAX
WRITE (9,7006) J, YNOZ(J), B(J)
457 CONTINUE
DO 460 KC = 1, 3
WRITE (9,7003) KMAX(KC)
DO 460 NJ = 1, NJMAX
DO 460 NP = 1, NJMAX
ABSVAL = ABS(C(KC,NJ,NP))
IF (ABSVAL .GE. SM1) WRITE (9,7004) NJ, NP, C(KC,NJ,NP)
460 CONTINUE
DO 820 KC = 4, 5
WRITE (9,7003) KMAX(KC)
KCMIN3 = KC - 3
DO 820 NJ = 1, NJMAX
DO 820 NP = 1, NJMAX
ABSVAL = ABS(E(NJ,NP,KCMIN3))
IF (ABSVAL .GT. SM1) WRITE (9,7004) NJ, NP, E(NJ,NP,KCMIN3)
820 CONTINUE
WRITE (9,7003) KMAX(6)
IF (NONLIN .EQ. 0) GO TO 4
DO 470 NJ = 1, NJMAX
DO 470 NP = 1, NJMAX
DO 470 NQ = 1, NJMAX
ABSVAL = ABS(D(NJ,NP,NQ))
IF (ABSVAL .GE. SM2) WRITE (9,7005) NJ, NP, NQ, D(NJ,NP,NQ)
470 CONTINUE
600 CONTINUE
C
C **********************************************************
C FORMAT SPECIFICATIONS
5000 FORMAT (7A10)
5001 FORMAT (2F10.0, 15)
5002 FORMAT (2I5, 1X, 8A4)
5003 FORMAT (I5, 2F10.0)
5004 FORMAT (6I5)
5005 FORMAT (2F10.0)
5006 FORMAT (6F10.0)
6001 FORMAT (1H1, 1X, 7A10//)
6002 FORMAT (2X, 8HGAMMA =, F7.5, 5X, 4HUE =, F6.4//)
6003 FORMAT (2X, 4A4, 2I5, 4F10.5/
1 8X, 2HYR, 7X, 2HYI//)
6004 FORMAT (1H1, 45H DECOUPLED COEFFICIENT OF B(P): C(1,J,P)///)
6005 FORMAT (1H1, 44H DECOUPLED COEFFICIENT OF THE DERIVATIVE OF,
1 6H B(P): 5X, 8HC(2,J,P)///)
6006 FORMAT (1H1, 43H DECOUPLED COEFFICIENT OF G(P): C(3,J,P)///)
6007 FORMAT (1H1, 42H DECOUPLED COEFFICIENT OF THE IMAGINARY PART,
1 24H OF THE COMBUSTION TERM: 5X, 8HE(J,P)///)
6008 FORMAT (7X, 1HP, 18)91 12)
6009 FORMAT (2X, 20HQUASI-STEADY NOZZLE///)
6010 FORMAT (2X, 24HLINEAR COEFFICIENTS ONLY)
6011 FORMAT (4X, 1HJ)
6012 FORMAT (4X, 1HP)
6013 FORMAT (1HLS, 40H DECOUPLED COEFFICIENT OF THE REAL PART:,
1 24H OF THE COMBUSTION TERM: 5X, 8HE(J,P)///)
6014 FORMAT (1H1, 45H DECOUPLED COEFFICIENT OF THE IMAGINARY PART:
1 24H OF THE COMBUSTION TERM: 5X, 8HE(J,P)///)
6015 FORMAT (1H1, 40H DECOUPLED COEFFICIENT OF THE REAL PART:
1 24H OF THE COMBUSTION TERM: 5X, 8HE(J,P)///)
6016 FORMAT (1X, 10X, 27HPARTICLE DIA (IN MICRONS) =, F5.2, 10X,
1 4HCM =, F6.4, 10X, 18HFREQ (IN Hertz) =, F6.1//)
2 10X, 26HCHAMBER TEMP (IN DEG K) =, F6.1, 10X, 4HSP =,
3 27HRHOM (IN KG/CUBIC METER) =, F6.1///10X,
4 30HPARTICLE DRAG COEFFICIENT K =, F8.4///)
6017 FORMAT (2X, 26HPARTICLES ARE NOT PRESENT///)
7001 FORMAT (3F10.5, 315)
7002 FORMAT (215, 1X, 4A4)
7003 FORMAT (I5)
7004 FORMAT (2I5, F15.8)
7005 FORMAT (3I5, F15.8)
7006 FORMAT (I5, 4F12.8)
7007 FORMAT (7F15.8)
END
3.4 PROGRAM MA2

Program MA2 carries out a numerical integration of the mode-amplitude equations resulting from the application of the Galerkin method in combination with the method of averaging (i.e., Equations (39)).

Program Description. Just as Program MA1 is similar to Program SOLID1, Program MA2 is similar to Program SOLID2 in its structure, subroutines, input and output. Hence one is referred to Section 3.2 where a detailed description of Program SOLID2 is given.

Sample Case. In the following example, the same case considered in Section 3.2 is treated using Program MA2. The printed and plotted output of this case is presented in the following pages.
UE = .0780

NUMBER OF MODES = 5

\( \text{GAMMA} = 1.23 \)

\( \text{GAMMA}_{\text{EAFR}} = 1.212250 \)

\( \text{PARTICLE DIA (IN MICRONS)} = 2.5 \)

\( \text{CHAMBER TEMP (IN DEG K)} = 3525.0 \)

\( \text{PARTICLE DRAG CONSTANT, K} = 29.9186 \)

\( \text{CM} = .10 \)

\( \text{FREQ (IN HERTZ)} = 1071.0 \)

\( \text{SP} = .68 \)

\( \text{RHOM (IN KG/CUBIC METER)} = 4000.0 \)

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### COEFFICIENTS FOR COMPUTATION OF WALL PRESSURE WAVEFORMS

#### COEFFICIENTS IN SERIES FOR:

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COMBUSTION PARAMETERS: \( A = 5.996 \), \( B = 0.580 \), \( EN = 0.575 \), \( OMEGA = 4.200 \)

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LINEAR COMBUSTION RESPONSE.

LINEAR PARTICLE DAMPING.
INITIAL CONDITIONS ARE OF THE FORM:

\[ \ell(i,j) = AC(j) \cdot \cos(freq \cdot t) + AS(j) \cdot \sin(freq \cdot t) \]

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THIS RUN PRODUCES PLOTTED OUTPUT.
Rt.n
00

STEP

TIME

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2.20000
2.22500
2.25000
2.27500

84

85
86
87
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89
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91

PRESSURE
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-.10573
-.10358
-.10162
-.09862
-.09498
-.09068
-.08570
-.08004
-.07369
-.06666
-.05898
-.05065
-.04174
-.03228
-.02235
-.01204
-.00145
.00932
.02014
.03089
.04142
.05163
.06138
.07056
.07508
.08687
.09386
.10002
.10532
.10977
.11336
.11613
.11809
.11929
.11975
.11951
.11861
.11707
.11494
.11224
.10902
.10530
.10114
.09656
.09160

PRESSURE
AT Z=1.0

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.10542
.10179
.09767
.09309
.08808
.08271
.07699
.07097
.06469
.05818
.05148
.04462
.03762
.03054
.02338
.01617
.00896
.00176

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4■ 01953
-.02643

-.03320
-.03983
...04628

-.05254
-.05860
..06443
-.07003
-.07537
-.08045
-.08523
• .08970
-.09383
-.09762

-.10102
-.10403
-.10662

-.10876
..11045
.11164
-.11233
•.011249
-.11208
.11107
-.10943
-

PRESSURE
AT Z=0.5

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.00426
.00551
.00663
.06/60
.00839
.00898
.00935
.00951
.00943
.00913
.00861
.00789
.00698
.00590
.00468
.00334
.00191
.00042
-.00110
-.00263
6,00414
.•'.00560
• .00698
• .00825
.■ .00940
-.01039
-.01119
-.01178
-.01214
-.01225
-.01209
• .01164
-.01092
.00.393
.•..00867
-.00718
■ .00549
-.00365
.•.00172
.00026
.00221
.00408
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GAS VEL
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PAR VEL
AT Z=0.5

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• .03438

• .00552
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-.02586
..•.03226

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-.04643
-.05204
• .05732

-.04443

-.06224

-.05547

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PRESSURE MAXIMA AND MINIMA AT: Z = 0.00
VALUES COMPUTED: 14

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PRESSURE GROWTH RATE AND FREQUENCY.
TOTAL NUMBER OF CYCLES: 4

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<th>Pressure Growth Rate</th>
<th>Cycle Frequency</th>
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<tr>
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<tr>
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<tr>
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<td>5.155200</td>
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<td>5.155200</td>
<td>7.282092</td>
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HEAD END PRESSURE PERTURBATION

DIMENSIONLESS TIME, T

-0.15 -0.10 -0.05 0 0.05 0.10 0.15

0 1 2 3 4 5 6 7 8 9 10
NOZZLE PRESSURE PERTURBATION

DIMENSIONLESS TIME, T

NOZZLE PRESSURE PERTURBATION

0.15
0.1
0.05
0.05
0.0
0.0
0.0
0.0
AMPLITUDE OF 1L MODE

DIMENSIONLESS TIME, T
AMPLITUDE OF 3L MODE

DIMENSIONLESS TIME, T
PROGRAM MA2 (INPUT, OUTPUT, DATA,
1 TAPS = INPUT, TAPE6 = OUTPUT, TAPE9 = DATA)

*************** PROGRAM MA2 ********************

THIS PROGRAM INTEGRATES THE SYSTEM OF DIFFERENTIAL EQUATIONS
FOR MODE AMPLITUDES USING THE COEFFICIENTS COMPUTED BY THE
PROGRAM MA1. TIME-HISTORY OF A PRESSURE DISTURBANCE IN THE
ROCKET IS COMPUTED, AND THE DESIRED PLOTS & PRINTOUTS ARE
PRODUCED.

THE FOLLOWING INPUTS ARE REQUIRED:
(1) THE CONTROL NUMBERS, NOUTCF AND NHISTR.
(2) THE COEFFICIENTS FROM PROGRAM MA1.
(3) THE DATA DECK.

THE FIRST CARD GIVES THE CONTROL NUMBERS: NOUTCF AND NHISTR.
NOUTCF DETERMINES PRINTOUT OF COEFFICIENTS:
IF NOUTCF = 0 COEFFICIENTS ARE NOT PRINTED OUT.
IF NOUTCF = 1 ONLY LINEAR COEFFICIENTS ARE PRINTED OUT.
IF NOUTCF = 2 ALL COEFFICIENTS ARE PRINTED OUT.
NHISTR DETERMINES IF PRESSURE HISTORY IS TO BE PRINTED:
IF NHISTR = 0 NOT PRINTED.
IF NHISTR = 1 PRINTED.

THE COEFFICIENTS ARE OBTAINED FROM PROGRAM MA1
BY PUTTING NOUT = 1 OR NOUT = 2, THEREBY WRITING THE COEFFICIENTS
INTO A DISK. THIS DISK HAS BEEN GIVEN THE DEVICE NUMBER 9.

THE DATA DECK CONSISTS OF THE FOLLOWING CARDS:

FIRST CARD: TITLE OF THE CASE.

SECOND CARD: H, TSTART, TQUIT, FREQ, BCOMB
H IS THE INTEGRATION STEP SIZE.
TSTART IS THE TIME AT WHICH OUTPUT STARTS.
QUIT IS THE TIME AT WHICH COMPUTATIONS ARE TERMINATED.
FREQ IS THE MOTOR FREQUENCY (IN PURE GAS), IN HERTZ.
BCOMB IS THE COMBUSTION RESPONSE NONLINEARITY FACTOR.

THIRD CARD: A2PARA, B2PARA, EN, OMEGA
A2PARA AND B2PARA ARE THE COMBUSTION PARAMETERS
IN THE A-B MODEL.
EN IS THE PRESSURE EXPONENT IN THE BURNING RATE LAW.
OMEGA IS THE FREQUENCY NONDIMENSIONALIZED BY THE SQUARE OF
THE STEADY-STATE BURNING RATE.
FOURTH CARD: NLOC, NTERMS, NOUT, NCOMB
NLOC DETERMINES THE LOCATION OF THE WALL PRESSURE MAXIMA AND MINIMA:
   IF NLOC = 1  LOCATION IS Z = 0.0
   IF NLOC = 2  LOCATION IS Z = 1.0
   IF NLOC = 3  LOCATION IS Z = 0.5
NTERMS IS THE NUMBER OF TERMS GIVEN INITIAL VALUES.
NOUT IS THE OUTPUT CONTROL NUMBER:
   IF NOUT = 0  PRINTED OUTPUT ONLY.
   IF NOUT > 0  BOTH PRINTED AND PLOTTED OUTPUT.
   IF NOUT = 1  PLOT OF PRESSURE AT Z = 0.0 ONLY.
   IF NOUT = 2  PLOT OF PRESSURE AT Z = 0.0 AND Z = 1.0
   IF NOUT = 3  PLOT OF PRESSURE AT Z = 0.0, 1.0 AND 0.5.
NCOMB DETERMINES IF COMBUSTION NONLINEARITIES ARE CONSIDERED:
   IF NCOMB = 0  NEGLECTED.
   IF NCOMB = 1  INCLUDED.

NEXT CARD (NECESSARY ONLY IF PLOTS ARE REQUIRED): YHI, YLAB, ITICY
YHI IS THE MAXIMUM ORDIATE FOR PRESSURE PLOTS.
NOTE: THE ORDIATE SCALES FOR PRESSURE AND AMPLITUDE PLOTS ARE SYMMETRIC ABOUT ZERO.
YLAB IS THE INTERVAL FOR ORDIATE LABELING FOR ABOVE PLOTS.
ITICY IS THE NUMBER OF ORDIATE TIC MARKS FOR ABOVE PLOTS.
NOTE: ITICY SHOULD BE NEGATIVE FOR PRESSURE AND AMPLITUDE PLOTS TO OBTAIN CENTERLINE.

NEXT CARD (NECESSARY ONLY IF PLOTS ARE REQUIRED): MDPLOT
MDPLOT DETERMINES IF PLOTS OF INDIVIDUAL MODES ARE REQUIRED:
   IF PLOT OF J TH MODE IS REQUIRED, PUNCH "1" IN THE 5*J TH COLUMN.
   IF PLOT OF J TH MODE IS NOT REQUIRED, PUNCH "0" IN THE 5*J TH COLUMN.

NEXT CARD (NECESSARY ONLY IF PLOT OF ANY MODE AMPLITUDE IS REQUIRED): YHIMD, YLABMD, ITICMD
YHIMD IS THE MAXIMUM ORDIATE.
YLABMD IS THE INTERVAL FOR ORDIATE LABELLING.
ITICMD IS THE NUMBER OF ORDIATE TIC MARKS FOR MODE PLOTS.
NOTE: ITICMD SHOULD BE NEGATIVE TO OBTAIN CENTERLINE.

REMAINING CARDS (NTERMS IN NUMBER): J, AST, ACT
AST IS THE AMPLITUDE OF THE SINE TERM OF THE J TH MODE.
ACT IS THE AMPLITUDE OF THE COSINE TERM OF THE J TH MODE.

******************************************************************************
COMPLEX YNOZ(6), B(6), C1, C3, RES(6), CRES
DIMENSION L(6), NAME(6), AA(4), DELTA(6,6), YI(12),
1 YR(12), CP(3,12,12), E(12,12,2), UMA(24),
2 CFT(3,12), CFZ(3,12), AS(12), BETA1(6,6,6),
3 AC(12), U(5,24), Y(24), PRESS(3), BETA2(6,6,6),
4 YF(24), FZ(4,24), UZ(24), Z(3), TIMAX(500),
5 TPL0T(500), YPLOT(3,500), DUMMY(500), DUMMY(500),
6 IBUF(512), ITT(3), IYT1(3), IYT2(3), IYT3(4),
7 ITP(2), TITL(7), PRSC(500), T(500), PMAX(500),
8 MDPLOT(6), UPL0T(6,500), MTI(L1(2), MTI(L2(2),
9 MTI(L3(2), MTI(L4(2), MTI(L5(2), MTI(L6(2), MTI(L(2)

COMMON C(2,24,12), D(4,12,144), KPMAX(2,24), IC(2,24,12),
1 KPNMAX(12), IDP(12,144), IDQ(12,144)
COMMON /BLK2/ B
COMMON /BLK3/ NPT(12), NLMAX, LGMAX, GAMMA, COEF(2,12)
COMMON /BLK4/ PARTK1, HHOP, FRQ(12)
COMMON /BLK5/ RES, NCOMB, BCOMB, E

DATA ITT/"DIMENSIONL","ESS TIME","T"/
1 IYT1/"HEAD END P","RESSURE PE","RTurbation"/
2 IYT2/"NOZZLE PRE","SSURE PERT","URBATION"/
3 IYT3/"PRESSURE P","ERTURBATION","N AT THE C","ENTER"/
4 ITP/"PRESSURE P","EAKS"/
5 MTI(L1/"AMPLITUDE","OF 1L MODE"/
6 MTI(L2/"AMPLITUDE","OF 2L MODE"/
7 MTI(L3/"AMPLITUDE","OF 3L MODE"/
8 MTI(L4/"AMPLITUDE","OF 4L MODE"/
9 MTI(L5/"AMPLITUDE","OF 5L MODE"/
1 MTI(L6/"AMPLITUDE","OF 6L MODE"/

MAXMD = 6
MAXMD2 = 12
MAXMD4 = 24
MAXMD6 = 36
MAXMDD = 144
LAST = 5
ERR = 0.001
TDEL = 10.0
NPT = 0
AA(1) = 0.0
AA(2) = 0.5
AA(3) = 0.5
AA(4) = 1.0
PI = 3.1415926536
HC = 1.0
READ (5,5003) NOUTCF, NHSTR

************* COEFFICIENT INPUT SECTION **************

THIS VERSION OF MA2 READS THE COEFFICIENT DATA FROM
A FILE GENERATED BY PROGRAM MA1, TO READ
THIS DATA FROM CARDS, USE READ (5,XXX) INSTEAD OF
READ (9,XXX) IN THIS SECTION.
INPUT OF MOTOR PARAMETERS AND NUMBER OF TERMS.
READ (9,5001) GAMMA, UE, ZE, NJMAX, NPRTKL
RHOP = 0.0
PARTKL = 0.0
UPBYU = 0.0
JMX = NJMAX/2
NU = 2 * NJMAX
GAM = GAMMA
FRATIO = 1.0
IF (NPRTKL .EQ. 0) GO TO 14
READ (9,5011) DI, RHOM, SP, TEMP, FREQ, PARTKL, CM
UPBYU = 2.0/(1.0 + SQRT(1.0 + 8.0 * UE/PARTKL))
RHOP = CM/UPBYU
GAM = GAMMA / (1.0 + SP*CM - SP*CM*GAMMA)
FRATIO = SQRT(GAMMA/(GAM*(1.0 + CM)))
14 CONTINUE

WRITE (6,6001) UE, JMX, GAM
IF (NPRTKL .EQ. 0) WRITE (6,6033)
IF (NPRTKL .EQ. 1) WRITE (6,6009) GAMMA
IF (NPRTKL .EQ. 1) WRITE (6,6030) DI, CM, FREQ, TEMP,

WRITE (6,6002)

INPUT OF DESCRIPTION OF SERIES EXPANSION.
DO 10 K = 1, JMX
READ (9,5002) NJ, L(NJ), NAME(NJ)
WRITE (6,6003) NAME(NJ), NJ, L(NJ)
10 CONTINUE

WRITE (6,6010)
DO 15 K = 1, JMX
READ (9,5010) J, YNOZ(J), B(J)
WRITE (6,6015) J, YNOZ(J), B(J)
NJ = (2 * J) - 1
YR(NJ) = REAL(YNOZ(J))
YI(NJ) = AIMAG(YNOZ(J))
YR(NJ+1) = YR(NJ)
YI(NJ+1) = YI(NJ)
15 CONTINUE

DO 402 K = 1, JMX
J = 2*K - 1
AX = L(K) * PI * FRATIO / ZE
FRQ1(J) = AX
FRQ1(J+1) = AX
402 CONTINUE
DO 404 NJ = 1, JMX
DO 404 NP = 1, JMX
DELT(A(NJ, NP) = 0.0
IF (NJ .EQ. NP) DELTA(NJ, NP) = 1.0
404 CONTINUE
PIBY2 = PI/2.
DO 406 NJ = 1, JMX
DO 406 NP = 1, JMX
DO 406 NQ = 1, JMX
IF (NP .NE. NQ+NJ) GO TO 408
BETA1(NP, NQ, NJ) = PIBY2
BETA2(NP, NQ, NJ) = -PIBY2
GO TO 406
408 IF (NQ .NE. NP+NJ .AND. NJ NE. NP+NQ) GO TO 406
BETA1(NP, NQ, NJ) = PIBY2
BETA2(NP, NQ, NJ) = PIBY2
406 CONTINUE
C
ZERO LINEAR COEFFICIENT ARRAYS.
DO 20 KC = 1, 3
DO 20 NJ = 1, MAXMD2
DO 20 NP = 1, MAXMD2
CP(KC, NJ, NP) = 0.0
20 CONTINUE
C
ZERO NONLINEAR COEFFICIENT ARRAY.
DO 30 KC = 1, 4
DO 30 NJ = 1, MAXMD2
DO 30 NPQ = 1, MAXMDD
D(KC, NJ, NPQ) = 0.0
30 CONTINUE
C
INPUT OF LINEAR COEFFICIENTS.
DO 40 KC = 1, 3
READ (9, 5003) KMAX
IF (NOUTCF .GT. 0) WRITE (6, 6004) KC, KMAX
IF (KMAX .EQ. 0) GO TO 40
DO 45 K = 1, KMAX
READ (9, 5004) NJ, NP, CP(KC, NJ, NP)
IF (NOUTCF .GT. 0) WRITE (6, 6005) KC, NJ, NP, CP(KC, NJ, NP)
45 CONTINUE
40 CONTINUE
C
DO 305 KC = 4, 5
READ (9, 5003) KMAX
KCMIN3 = KC - 3
IF (NOUTCF .GT. 0) WRITE (6, 6031) KCMIN3, KMAX
IF (KMAX .EQ. 0) GO TO 305
DO 310 K = 1, KMAX
READ (9, 5004) NJ, NP, E(NJ, NP, KC MIN3)
IF (NOUTCF .GT. 0) WRITE (6, 6032) NJ, NP, KCMIN3, E(NJ, NP, KCMIN3)
310 CONTINUE
305 CONTINUE
INPUT OF NONLINEAR COEFFICIENTS.
READ (9, 5003) NLMAX

IF (NOUTCF * EQ. 2) WRITE (6, 6006) NLMAX
IF (NLMAX * EQ. 0) GO TO 50
DO 52 NJ = 1, MAXMD2
KPQMAX(NJ) = 0
52 CONTINUE
DO 55 K = 1, NLMAX
READ (9, 5005) NJ, NP, NQ, DT
IF (NOUTCF * EQ. 2) WRITE (6, 6007) NJ, NP, NQ, DT
KPQMAX(NJ) = KPQMAX(NJ) + 1
KPQ = KPQMAX(NJ)
IDP(NJ, KPQ) = NP
IDQ(NJ, KPQ) = NQ
NJ12 = (NJ+1)/2
NP12 = (NP+1)/2
NQ12 = (NQ+1)/2
DT = DT * 0.5 * FRQ1(NQ) / (FRQ1(NJ) * PI)
D(1, NJ, KPQ) = DT * BETA(NJ12, NP12, NQ12)
D(2, NJ, KPQ) = - DT * BETA(NP12, NQ12, NJ12)
D(3, NJ, KPQ) = DT * BETA(NQ12, NP12, NJ12)
D(4, NJ, KPQ) = - DT * BETA(NP12, NQ12, NJ12)
55 CONTINUE
50 CONTINUE

************* PRESSURE COEFFICIENT SECTION **************

CALCULATE SPATIAL COORDINATES FOR PRESSURE COMPUTATION.
Z(1) = 0.0
Z(2) = ZE
Z(3) = 0.5 * ZE

CALCULATE COEFFICIENTS FOR PRESSURE TIME HISTORIES.
DO 53 NPRES = 1, 3
DO 53 J = 1, JMX
NP = (2 * J) - 1
Z1 = Z(NPRES)
CALL PHICFS(J, Z1, C1, C3)
CFT(NPRES, NP) = REAL(C1)
CFT(NPRES, NP+1) = -AIMAG(C1)
CFZ(NPRES, NP) = REAL(C3)
CFZ(NPRES, NP+1) = -AIMAG(C3)
53 CONTINUE

OUTPUT OF COEFFICIENTS FOR PRESSURE TIME HISTORIES.
WRITE (6, 6020)
DO 56 NPRES = 1, 3
WRITE (6, 6014)
DO 56 J = 1, NJMAX
WRITE (6, 6021) J, Z(NPRES), CFT(NPRES, J), CFZ(NPRES, J)
56 CONTINUE
*************** DATA INPUT SECTION *****************************************

**READ (5,5000) TITLE**

**ZERO INITIAL VALUE AND FREQUENCY ARRAYS.**

5 DO 57 K = 1, NJMAX
   AS(K) = 0.0
   AC(K) = 0.0
57 CONTINUE

**READ COMBUSTION AND CONTROL PARAMETERS.**

**READ (5,5006) HO, TSTART, TQUIT, FREQ, BCOMB**

IF (EOF(5)) 300, 1
1 CONTINUE

**READ (5,5013) A2PARA, B2PARA, EN, OMEGA**

WRITE (6,6034) A2PARA, B2PARA, EN, OMEGA

DO 46 K = 1, JMX
   OMEGAK = OMEGA * K
   CALL RESPNS(EN,A2PARA,B2PARA,OMEGAK,CRES)
   RES(K) = CRES
   WRITE (6,6035) K, RES(K)
46 CONTINUE

**READ CONTROL NUMBERS.**

**READ (5,5008) NLOC, NTERMS, NOUT, NCOMB**

IF (NOUT GT. 0) NPT = 1
IF (NCOMB EQ. 0) WRITE (6,6039)
IF (NCOMB EQ. 1) WRITE (6,6040) BCOMB
WRITE (6,6041)

C

IF (NOUT EQ. 0) GO TO 9
C

**READ DATA FOR SETTING UP PLOTS.**

**READ (5,5009) YHI, YLAB, ITICY**

**READ (5,5014) MDPLLOT**

**MDPLTL = 0**

DO 320 K = 1, JMX
   MDPLTL = MDPLTL + MDPLLOT(K)
320 IF (MDPLTL EQ. 0) GO TO 9

**READ (5,5015) YHIMD, YLABMD, ITICMD**

**YLMD = - YHIMD**

C

*************** INITIAL AMPLITUDES SECTION *******************************

9 DO 54 K = 1, NTERMS

**INPUT INITIAL AMPLITUDES FOR F-FUNCTIONS.**

**READ (5,5007) J, AST, ACT**

**NJ = (2 * J) - 1**

**AS(NJ) = AST**

**AC(NJ) = ACT**
CALCULATE INITIAL AMPLITUDES FOR G-FUNCTIONS.

IF (FRQ1(NJ)) 58, 58, 581
581 GYRU = GAMMA*YR(NJ)*UE
GYIF = GAMMA*YI(NJ)*FRQ1(NJ)
GYRF = GAMMA*YR(NJ)*FRQ1(NJ)
GYIU = GAMMA*YI(NJ)*UE

NPRES = 2

A1 = (1.0 + GYRU)*CFZ(NPRES,NJ+1)  
1 - GYI F*CFT(NPRES,NJ+1)
A2 = GYRF*CFT(NPRES,NJ+1) + GYIU*CFZ(NPRES,NJ+1)
A3 = -(1.0 + GYRU)*CFZ(NPRES,NJ) + GYIF*CFT(NPRES,NJ)
A4 = GYRF*CFT(NPRES,NJ) + GYI U*CFZ(NPRES,NJ)

DET = A1*A1 + A2*A2
IF (DET .LT. 0.0000001) GO TO 583
R1 = A3*AC(NJ) - A4*AS(NJ)
R2 = -A4*AC(NJ) - A3*AS(NJ)

AC(NJ+1) = (R1*A1 + R2*A2)/DET
AS(NJ+1) = -(R2*A1 - R1*A2)/DET
GO TO 58
583 AC(NJ+1) = -AS(NJ)
AS(NJ+1) = AC(NJ)

58 CONTINUE
54 CONTINUE

OUTPUT OF INITIAL AMPLITUDES.
WRITE (6,6016)
DO 590 J = 1, NJMAX
IF (AS(J)) 591, 592, 591
591 WRITE (6,6017) J, FRQ1(J), AC(J), AS(J)
590 CONTINUE
IF (NOUT .GE. 1) WRITE (6,6027)

************ LINEAR COEFFICIENTS SECTION *******************
COMPUTE LINEAR COEFFICIENTS FOR GIVEN VALUES OF HC AND RESPONSE FUNCTION.

DO 60 NJ = 1, NJMAX
NJ12 = (NJ+1)/2
DO 60 NP = 1, NJMAX
NP12 = (NP+1)/2
FR2K2 = FRQ1(NP)**2 + PARTKL**2
RESR = REAL(RES(NP12))
RESI = AIMAG(RES(NP12))
CT = 0.5 * FRQ1(NP) / FRQ1(NJ) * DELTA(NJ12,NP12) * (CP(2,NJ,NP) + HC * RESR * E(NJ,NP,1) + HC * RESI * E(NJ,NP,2)
2 - CP(3,NJ,NP)/FR2K2)
IF (CT) 61, 61
61 KPMAX(1,NJ) = KPMAX(1,NJ) + 1
KP = KPMAX(1,NJ)
IC(1,NJ,KP) = NP
C(1,NJ,KP) = CT
62 CONTINUE
CT = -0.5 / FRQ1(NJ) * DELTA(NJ12,NP12) * (CP(1,NJ,NP) + PARTKL * CP(3,NJ,NP)/FR2K2)
IF (NJ EQ NP) CT = CT + FRQ1(NJ) * 0.5
IF CT) 63, 64
63 KPMAX(2,NJ) = KPMAX(2,NJ) + 1
KP = KPMAX(2,NJ)
IC(2,NJ,KP) = NP
C(2,NJ,KP) = CT
64 CONTINUE
CT = 0.5/FRQ1(NJ) * DELTA(NJ12,NP12) * (CP(1,NJ,NP) + PARTKL * CP(3,NJ,NP)/FR2K2)
IF (NJ EQ NP) CT = CT - 0.5 * FRQ1(NJ)
IF (CT) 420, 422
420 KPMAX(1,NJ+NJMAX) = KPMAX(1,NJ+NJMAX) + 1
KP = KPMAX(1,NJ+NJMAX)
IC(1,NJ+NJMAX,KP) = NP
C(1,NJ+NJMAX,KP) = CT
422 CONTINUE
CT = -0.5 * FRQ1(NP) / FRQ1(NJ) * DELTA(NJ12,NP12) * (CP(2,NJ,NP) + HC * RESR * E(NJ,NP,1) + HC * RESI * E(NJ,NP,2) - CP(3,NJ,NP)/FR2K2)
IF (CT) 424, 410
424 KPMAX(2,NJ+NJMAX) = KPMAX(2,NJ+NJMAX) + 1
KP = KPMAX(2,NJ+NJMAX)
IC(2,NJ+NJMAX,KP) = NP
C(2,NJ+NJMAX,KP) = CT
410 CONTINUE
60 CONTINUE

184
**INITIAL VALUES SECTION**

NSTEP = 0  
NP1 = 3  
H6 = H/6  
TIME = 0.0  
I = NP1  
TI(I) = TIME

DO 75 J = 1, NJMAX  
JP = J + NJMAX  
U(I,J) = AS(J)  
U(I,J) = AC(J)

75 CONTINUE

**CALCULATE INITIAL VALUES OF PRESSURE AND VELOCITY**

DO 704 NPRES = 1, 3  
DO 702 J = 1, NJMAX  
COEF(1,J) = CFT(NPRES,J)  
COEF(2,J) = CFZ(NPRES,J)  
ARG = FRQ1(J) * TIME  
SINARG = SIN(ARG)  
COSARG  
UMA(J) = U(I,J) * SINARG + U(I,J+NJMAX) * COSARG  
UMA(J+NJMAX) = FRA1(J) * (U(I,J) * COSARG - U(I,J+NJMAX) * SINARG)

702 CONTINUE

DO 703 J = 1, NU  
Y(J) = U(I,J)

703 CONTINUE

UBAR = UE * Z(NPRES)  
UMS = UE  
CALL PRSVEL(UBAR, UMS, UMA, P, VZGAS, VZPAR)  
PRESS(NPRES) = P

704 CONTINUE

PRS(I) = PRESS(NLOC)

70 CONTINUE

**INITIALIZE CONTROL NUMBERS**

LINE = 8  
K = 0  
MAXNO = 0  
MAXP = 0  
IF (NOUT *EQ* 0) GO TO 100  
JPLT = 0  
TMIN = TSTART  
TMAX = TSTART + TDEL  
YLO = -YHI
100 I = NP1

RUNGE-KUTTA INTEGRATION SCHEME.

105 NSTEP = (I - NP1 + (LAST - NP1) * K)
RSTEP = NSTEP
TIME = RSTEP * H
TI(I) = TIME
DO 120 J = 1, NU
   Y(J) = U(I,J)
120 CONTINUE
CALL RHS(Y,YP)
DO 130 J = 1, NU
   FZ(1,J) = YP(J)
130 CONTINUE
DO 140 II = 2, 4
   DO 144 J = 1, NU
      UZ(J) = Y(J) + AA(II) * H * FZ(II-1,J)
144 CONTINUE
CALL RHS(UZ,YP)
DO 148 J = 1, NU
   FZ(II,J) = YP(J)
148 CONTINUE
DO 150 J = 1, NU
   U(II+1,J) = Y(J) + (FZ(1,J) + 2.0*(FZ(2,J)+FZ(3,J)) + FZ(4,J)) * H6
150 CONTINUE

CALCULATE PRESSURE TIME HISTORIES.

DO 154 NPRES = 1, 3
   DO 152 J = 1, NJMAX
      COEF(1,J) = CFT(NPRES,J)
      COEF(2,J) = CFZ(NPRES,J)
      ARG = FRQ1(J) * TIME
      SINARG = SIN(ARG)
      COSARG = COS(ARG)
      UMA(J) = U(I,J) * SINARG + U(I,J+NJMAX) * COSARG
      UMA(J+NJMAX) = FRQ1(J) * (U(I,J)*COSARG - U(I,J+NJMAX)*SINARG)
152 CONTINUE
   UBAR = UE * Z(NPRES)
   UMS = UE
   CALL PRSVEL(UBAR, UMS, UMA, P, VZGAS, VZPAR)
   PRESS(NPRES) = P
154 CONTINUE

PRSI(I) = PRESS(NLOC)
IF (K * EQ. 0) GO TO 175

DETERMINE MAXIMUM AND MINIMUM PRESSURE AT LOCATION SPECIFIED

BY NLOC.

DPL = PRS(I) - PRS(I-1)
DPS = PRS(I-1) - PRS(I-2)
IF (DPL*DPS) > 0, 173, 173s 175
173 PNUM = PRS(I-2) - PRS(I)
PDEN = 2.0 * (PRS(I-2) + PRS(I) - 2.0*PRS(I-1))
IF (PDEN) 174, 175, 174
174 PP = PNUM/PDEN
PA = (PP - 1.0) * PP * 0.5
PB = 1.0 - (PP * PP)
PC = (PP + 1.0) * PP * 0.5
MAXP = MAXP + 1
PMAX(MAXP) = PA*PRS(I-2) + PB*PRS(I-1) + PC*PRS(I)
TIMAX(MAXP) = TI(I-1) + PP*H
IF (MAXP .GE. 500) GO TO 250
175 CONTINUE
C
IF (TIME .LT. TSTART) GO TO 155
IF (NOUT .EQ. 0) GO TO 156
C
*************** TIME HISTORY PLOTTING SECTION *****************
C
IF (TMAX .GT. TQUIT) GO TO 156
IF ((TIME .GT. TMAX) .OR. (JPLOT .GE. 500)) GO TO 1000
C
JPLOT = JPLOT + 1
C
FILL TIME ARRAY FOR PLOTTING.
T PLOT(JPLOT) = TIME
C
FILL PRESSURE ARRAYS FOR PLOTTING.
DO 1001 J = 1,3
YPLOT(J,JPLOT) = PRESS(J)
1001 CONTINUE
C
C
IF (MDPLOT .EQ. 0) GO TO 156
C
FILL MODE AMPLITUDE ARRAYS FOR PLOTTING.
DO 322 J = 1, JMX
IF (MDPLOT(J) .EQ. 0) GO TO 322
J12 = 2*J - 1
UPLOT(J,JPLOT) = U(I,J12)
322 CONTINUE
C
GO TO 156
C
1000 NUM = JPLOT
C
PLOT TIME HISTORIES.
C
DO 1020 N PLOT = 1, NOUT
C
J PLOT = 0
ASSIGN PLOTTING PARAMETERS.
YMIN = YLO
YMAX = YHI
NTICY = ITICY
DELY = YLAB

ELIMINATE POINTS THAT ARE OUT OF THE ORDINATE RANGE.
DO 1010 J = 1, NUM
IF (YPLOT(NPLOT,J) .LT. YMIN) .OR. (YPLOT(NPLOT,J) .GT. YMAX))
1  GO TO 1010
JPLOT = JPLOT + 1
DUMMYT(JPLOT) = TPlot(J)
DUMMYY(JPLOT) = YPLOT(NPLOT,J)
1010 CONTINUE

IF (JPLOT .EQ. 0) GO TO 1020
GO TO (1011,1014,1015) NPlot

PLOT HEAD-END PRESSURE.
1011 CALL GRAPHS(IBUF, 512, 4, JPLOT, 11, NTICY, TMAX, YMAX, TMIN, YMIN,
1    ITT, I1Y1, 21, 30, DUMMYT, DUMMYY, 2.0, DELY, TITLE)
GO TO 1020

PLOT NOZZLE PRESSURE.
1014 CALL GRAPHS(IBUF, 512, 4, JPLOT, 11, NTICY, TMAX, YMAX, TMIN, YMIN,
1    ITT, I1Y2, 21, 28, DUMMYT, DUMMYY, 2.0, DELY, TITLE)
GO TO 1020

PLOT PRESSURE AT THE CENTER (X = 0.5).
1015 CALL GRAPHS(IBUF, 512, 4, JPLOT, 11, NTICY, TMAX, YMAX, TMIN, YMIN,
1    ITT, I1Y3, 21, 35, DUMMYT, DUMMYY, 2.0, DELY, TITLE)

1020 CONTINUE

DO 324 NPlot = 1, JMX
IF (MDPLOT(NPLOT) .EQ. 0) GO TO 324
JPlot = 0
DO 328 J123 = 1, 2
IF (NPLOT .EQ. 1) MTI TL(J123) = MTI TL1(J123)
IF (NPLOT .EQ. 2) MTI TL(J123) = MTI TL2(J123)
IF (NPLOT .EQ. 3) MTI TL(J123) = MTI TL3(J123)
IF (NPLOT .EQ. 4) MTI TL(J123) = MTI TL4(J123)
IF (NPLOT .EQ. 5) MTI TL(J123) = MTI TL5(J123)
IF (NPLOT .EQ. 6) MTI TL(J123) = MTI TL6(J123)
328 CONTINUE
DO 326 J = 1, NUM
IF ((UPLOT(NPLOT,J) .LT. YLOMD) .OR. (UPLOT(NPLOT,J) .GT. YHIMD)) GO TO 326
JPLOT = JPLOT + 1
DUMMYT(JPLOT) = TPlot(J)
DUMMYY(JPLOT) = UPLOT(NPLOT,J)
326 CONTINUE
IF (JPLLOT .EQ. 0) GO TO 324
CALL GRAPHS(IBUF, 512, 4, JPLLOT, 11, ITICMD, TMAX, YHIMD, TMIN,
       YLOMD, ITT, MTITL, 21, 20, DUMMYT, DUMMYY, 20, YLABMD, TITLE)
324 CONTINUE

C
C REASSIGN PLOTTING PARAMETERS FOR NEXT SET OF PLOTS.
JPLLOT = 0
TMIN = TMAX
TMAX = TMAX + TDEL
C
C ************* TIME HISTORY PRINTED OUTPUT SECTION ***************
C
156 IF (NHISTR .EQ. 0)
   1 WRITE (6, 6011) NSTEP, TIME, (PRESS(J), J = 1, 3), VZGAS, VZPAR
   LINE = LINE + 1
157 IF (TIME .GT. TQUIT) GO TO 250
   IF (LINE .LT. 52) GO TO 155
   IF (NHISTR .EQ. 0) WRITE (6, 6013)
   IF (NHISTR .EQ. 0) WRITE (6, 6022)
   LINE = 4
C
155 I = I + 1
   IF (I .LT. LAST) GO TO 105
   K = K + 1
C
C REASSIGN ARRAYS.
DO 200 I = 1, NP1
   ILAST = LAST - NP1 + I
   PRS(I) = PRS(ILAST)
   TI(I) = TI(ILAST)
   DO 200 J = 1, NU
   U(I,J) = U(ILAST,J)
200 CONTINUE
   GO TO 100
C
C ************* PRESSURE MAXIMA AND MINIMA PRINTOUT ***************
C
250 WRITE (6, 6023) Z(NLOC), MAXP
   LINE = 4
   DO 255 JST = 1, MAXP, 8
   JSTART = JST
   JSTOP = JST + 7
   IF (JSTOP .GT. MAXP) JSTOP = MAXP
   WRITE (6, 6024) (FMAX(J), J = JSTART, JSTOP)
   WRITE (6, 6024) (TIMAX(J), J = JSTART, JSTOP)
   WRITE (6, 6014)
   LINE = LINE + 3
   IF (LINE .LT. 52) GO TO 255
   LINE = 0
   WRITE (6, 6013)
255 CONTINUE
CALL GROWTH(MAXP,TIMAX,PMAX,FREQ)
GO TO 5
300 CONTINUE
C TURN OFF PLOTTING ROUTINE.
IF (NPT .EQ. 1) CALL PLOT(0.0,0.0,999)
C
*************** READ FORMAT SPECIFICATIONS ***********************
C
5000 FORMAT (7A10)
5001 FORMAT (3F10.0,3I5)
5002 FORMAT (2I5,1X,A4)
5003 FORMAT (2I5)
5004 FORMAT (2I5,F15.8)
5005 FORMAT (3I5,F15.8)
5006 FORMAT (7F10.0)
5007 FORMAT (I5,2F10.0)
5008 FORMAT (6I5)
5009 FORMAT (2F10.0,I5)
5010 FORMAT (I5,4F12.8)
5011 FORMAT (7F15.8)
5013 FORMAT (4F10.0)
5014 FORMAT (6I5)
5015 FORMAT (2F10.0,I5)
C
*************** WRITE FORMAT SPECIFICATIONS ****************************
C
6001 FORMAT (1H1,6X,4HUE =F6.4,6X,17HNUMBER OF MODES =,I2,6X,7H GAMMA =,F5.2)
6002 FORMAT (6X,14HNAME J L/)
6003 FORMAT (6X,A4,2I5)
6004 FORMAT (1H0,2H NUMBER OF COEFFICIENTS C(I1,10H,NJ,NP) I5,I5/)
6005 FORMAT (2X,2HC(I1,1H,I2,1H,I2,4H) =F10.5)
6006 FORMAT (1H0,3H NUMBER OF COEFFICIENTS D(NJ,NP,NQ) I5,I5/)
6007 FORMAT (2X,2HD(12,1H,I2,1H,I2,4H) =F10.5)
6008 FORMAT (1H0,17H MOTOR PARAMETERS I5,1X,8H GAMMA = F4.2,10X,19HEXIT MACH NUMBER =,F7.5/)
6009 FORMAT (6X,10H GAMMABAR =,F9.6//)
6010 FORMAT (1H0,6X,1HJ,7X,2HYR,8X,2HYI,7X,3HEPS,7X,3HETA//)
6011 FORMAT (2X,I5,F12.5,5F22.5)
6012 FORMAT (1H1)
6013 FORMAT (1H1)
6014 FORMAT (1H1)
6015 FORMAT (2X,I5,4F10.5)
6016 FORMAT (1H1,1X,36H INITIAL CONDITIONS ARE OF THE FORM: // 1 2X,4HYU(I,J) = AC(J)*COS(FREQ*T) + AS(J)*SIN(FREQ*T), 2 ///6X,1HJ,6X,9HFREQ,10X,5HAC(J),10X,5HAS(J)//)
6017 FORMAT (2X,I5,4F15.8/)
6020 FORMAT (1H1,2X,45H COEFFICIENTS FOR COMPUTATION OF WALL PRESSURE, 1 10H WAVEFORMS ///34X,27H COEFFICIENTS IN SERIES FOR: // 2 37X,4HTIME,21X,5HAXIAL,6X,1HJ,7X,1HZ,19X,10HDERIVATIVE, 3 15X,10HDERIVATIVE//)
6021 FORMAT (2X,I5,F10.3,12X,F15.7,10X,F15.7)
END
SUBROUTINE PRSVEL(UBAR, UMS, UMA, P, VZGAS, VZPAR)

C

C THIS SUBROUTINE COMPUTES THE PRESSURE AND VELOCITY.

C UBAR IS THE LOCAL AXIAL STEADY STATE MACH NUMBER.
C UMS IS THE DERIVATIVE OF THE MACH NUMBER.
C UMA IS THE ARRAY CONTAINING VALUES OF THE MODE-AMPLITUDE
C FUNCTIONS AND THEIR DERIVATIVES.
C P IS THE VALUE OF THE PRESSURE PERTURBATION.
C VZGAS IS THE AXIAL COMPONENT OF GAS VELOCITY.
C VZPAR IS THE AXIAL COMPONENT OF PARTICLE VELOCITY.

C

DIMENSION UMA(24), SUM(5), SUMSQ(2)
COMMON /BLK3/ NPRTKL, NJMAX, NLMAX, GAMMA, COEF(2,12)
COMMON /BLK4/ PARTKL, RHOP, FRQ1(12)

C

DO 10 I = 1, 5
SUM(I) = 0.0
10 CONTINUE

DO 20 J = 1, NJMAX
JY = J + NJMAX
20 SUM(1) = SUM(1) + UMA(JY) * COEF(1,J)
DO 50 J = 1, NJMAX
SUM(2) = SUM(2) + UMA(J) * COEF(2,J)
SUM(3) = SUM(3) + UMA(J) * COEF(1,J)
IF (NPRTKL .EQ. 0) GO TO 50
SUM(4) = SUM(4) + COEF(1,J) * (PARTKL * UMA(J) -
1 UMA(J+NJMAX)) / (FRQ1(J)**2 + PARTKL**2)
SUM(5) = SUM(5) + COEF(2,J) * (PARTKL * UMA(J) - UMA(J+NJMAX))
1 / (FRQ1(J)**2 + PARTKL**2)
50 CONTINUE
PLIN = SUM(1) + UBAR * SUM(2) + UMS * SUM(3)
1 + PARTKL * RHOP * (SUM(3) - PARTKL * SUM(4))
PNL = 0.0
IF (NLMAX .EQ. 0) GO TO 40
DO 30 I = 1, 2
SUMSQ(I) = SUM(I) * SUM(I)
30 CONTINUE
PNL = 0.5 * (SUMSQ(2) - SUMSQ(1))

40 P = -GAMMA * (PLIN + PNL)
VZGAS = SUM(2)
VZPAR = PARTKL * SUM(5)

C

RETURN
END
SUBROUTINE RHS(U, UP)

C C
C COMPLEX RES(6), RESNL(6)
DIMENSION U(24), UP(24), E(12,12,2)
COMMON C(2,24,12), D(4,12,144), KPMAX(2,24), IC(2,24,12),
1 KPMAX(12), IDP(12,144), IDQ(12,144)
COMMON /BLK3/ NPRTKL, NJMAX, NLMAX, GAMMA, COEF(2,12)
COMMON /BLK4/ PARTKL, RHOP, FRQ1(12)
COMMON /BLK5/ RES, NCOMB, BCOMB, E

C IF (NCOMB .EQ. 0) GO TO 116
JMX = NJMAX/2
DO 118 NJ = 1, JMX
NJPLNJ = 2*NJ
NJ2MN1 = NJPLNJ - 1
RESNL(NJ) = RES(NJ) * BCOMB * SQRT (UP(NJ2MN1)**2 + UP(NJPLNJ)**2)
118 CONTINUE
116 CONTINUE

DO 10 NJ = 1, NJMAX
NJP = NJ + NJMAX
SL1 = 0.0
SL2 = 0.0
SNL1 = 0.0
SNL2 = 0.0
SNLC = 0.0
MAX = KPMAX(1,NJ)
IF (MAX .EQ. 0) GO TO 25
DO 20 KP = 1, MAX
NP = IC(1,NJ,KP)
SL1 = SL1 + (C(1,NJ,I(P) * U(NP) )
20 CONTINUE
25 MAX = KPMAX(2,NJ)
IF (MAX .EQ. 0) GO TO 45
DO 30 KP = 1, MAX
NP = IC(2,NJ,KP)
SL2 = SL2 + (C(2,NJ,KP) * U(NP+NJMAX))
30 CONTINUE
45 IF (NLMAX .EQ. 0) GO TO 55
MAX = KPMAX(NJ)
IF (MAX .EQ. 0) GO TO 55
DO 50 KPQ = 1, MAX
NP = IDP(NJ,KPQ)
NQ = IDQ(NJ,KPQ)
SNL1 = SNL1 + D(1,NJ,KPQ) * U(NP) * U(NQ+NJMAX) +
1 D(2,NJ,KPQ) * U(NQ) * U(NP+NJMAX)
SNL2 = SNL2 + D(3,NJ,KPQ) * U(NP) * U(NQ) +
1 D(4,NJ,KPQ) * U(NP+NJMAX) * U(NQ+NJMAX)
50 CONTINUE
55 UP(NJP) = SL1 + SL2 + SNL1
   SL1 = 0.0
   SL2 = 0.0
   MAX = KPMAX(1,NJP)
   IF (MAX .EQ. 0) GO TO 65
   DO 60 KP = 1, MAX
      NP = IC(1,NJP,KP)
      SL1 = SL1 + C(1,NJP,KP) * U(NP)
   60 CONTINUE
   65 MAX = KPMAX(2,NJP)
   IF (MAX .EQ. 0) GO TO 75
   DO 70 KP = 1, MAX
      NP = IC(2,NJP,KP)
      SL2 = SL2 + C(2,NJP,KP) * U(NP+NJMAX)
   70 CONTINUE
   75 UP(NJP) = SL1 + SL2 + SNL2
10 CONTINUE
RETURN
END
4. APPROXIMATE T-BURNER PROGRAMS

This chapter describes the programs which determine the T-burner stability characteristics, TB1 and TB2. TB1 and TB2 perform the same tasks for T-burners as SOLID1 and SOLID2 perform for rocket motors. The programs TB1 and TB2 have the same structure as programs SOLID1 and SOLID2, and many subroutines are common between these sets of programs. Hence, the ensuing discussion of programs TB1 and TB2 is kept brief, and wherever possible one is referred to the discussion on programs SOLID1 and SOLID2.

4.1 PROGRAM TB1

Program TB1 calculates the coefficients of both the linear and nonlinear terms which appear in the T-burner mode-amplitude equations. The coefficients to be calculated are functions of various integrals of trigonometric functions (Appendix C).

Program Structure. The structure of this program is similar to that of SOLID1. It can be divided functionally into five sections: (1) input, (2) calculation of the linear coefficients, (3) calculation of the nonlinear coefficients, (4) obtaining the coefficients of the equivalent uncoupled real system, and (5) output.

The inputs to the program include the description of the T-burner geometry (length of the propellant grain, width of the vent, ratio of radius to burner length, effective vent plug-flow length), velocity at the burning surface, information about the modes included in the approximating series, particle characteristics and various control numbers. As in the case of the motor, all the inputs are supplied to the main program.

As explained in Volume I, the axial acoustic eigenvalues for the T-burner are those for a cylinder with hard walls at both ends. In other words, the eigenvalue for the \( n^{th} \) axial mode is simply \( nm \) and the eigenfunctions are real numbers. Hence, program TB1 does not include subroutines EIGVAL and FCNS which were present for program SOLID1. The integrals of the products of two axial eigenfunctions and the steady-state quantities are computed by means of subroutines AXIAL1 and STEADY. The linear coefficients are then calculated and normalized by dividing by the coefficient of the highest derivative (i.e., \( C_0(j,j) \)).

In the third section, the integrals of products of three axial eigenfunctions are computed using the subroutine AXIAL2 and the complex nonlinear coefficients are calculated.
The remaining two sections are the same as in SOLID1.

**Description of Input.** The input deck for Program TB1 is the same as for Program SOLID1, except the second and third cards which give information about burner geometry and vent effect as described below. Furthermore, the nozzle admittance data card is absent from the data deck. A complete list of inputs is given below.

The first card gives the title of the case.

**Second card:** GAM, BETA, BETAV, RRL, UB, VL, EL

GAM is the specific heat ratio.
BETA is the ratio of the length of the two cup grains to the length of the T-burner.
BETAV is the ratio of the vent width to the length of the burner.
RRL is the radius-to-length ratio.
UB is the velocity at the burning surface.
VL determines the vent effect:

\[
VL = 0 \quad \text{vent gain.} \\
VL = 1 \quad \text{no effect.} \\
VL = 2 \quad \text{vent loss.}
\]

EL is the effective vent length.

**Third card:** NJMAX, NONLIN, NEGL, NOUT, NPRTKL, NBURN

NJMAX is the number of mode-amplitude functions in the assumed series solution.
The coefficients computed are determined by NONLIN as follows:

\[
\text{NONLIN} = 0 \quad \text{linear coefficients only.} \\
\text{NONLIN} = 1 \quad \text{both linear and nonlinear coefficients.}
\]

Coefficients to be neglected are determined by NEGL as follows:

\[
\text{NEGL} = 0 \quad \text{terms smaller than 0.00001 are neglected.} \\
\text{NEGL} = 1 \quad \text{linear terms smaller than SM1 and nonlinear terms smaller than SM2 are neglected.}
\]

The output is determined by NOUT as follows:

\[
\text{NOUT} = 0 \quad \text{printed output only.} \\
\text{NOUT} = 1 \quad \text{write into a file and print output.} \\
\text{NOUT} = 2 \quad \text{write into a file only.}
\]

NPRTKL determines whether the particles are present:

\[
\text{NPRTKL} = 0 \quad \text{particles not present.} \\
\text{NPRTKL} = 1 \quad \text{particles present.}
\]
NBURN indicates whether end burning is present:

- NBURN = 0 not present.
- NBURN = 1 present.

Next card (necessary only if NPRTKL = 1): DIA, RHOM, SP, TEMP, FREQ, CM

- DIA is the particle diameter, in microns.
- RHOM is the density of the particle material, in Kg/m³.
- SP is the ratio of the specific heats of particle material and gas.
- TEMP is the chamber temperature, in degrees Kelvin.
- FREQ is the frequency of oscillation in pure gas, in Hertz.
- CM is the particle loading.

Next card (necessary only if NEGL = 1): SM1, SM2

- SM1 and SM2 are as defined above.

Next NJMAX cards: J, L(J), NAME(J)

- Each mode-amplitude is assigned an integer J.
- The mode is specified by the index L(J).
- L(J) is the axial mode number and must not exceed NJMAX.
- NAME(J) is a four-character name for the Jth mode.

Description of the Subroutines. The different subroutines included in program TB1 are described below:

**SUBROUTINE AXIAL1 (NØPT, NP, NJ, RESULT).** This subroutine calculates the different integrals which appear in the expressions for the coefficients of the linear terms in the T-burner amplitude equations. While this subroutine performs the same function for TB1 as its counterpart performs in SOLID1, the two versions of AXIAL1 differ considerably in their details. The T-burner version of AXIAL1 returns the computed value of the desired integral under the name RESULT according to the value of NØPT as follows:

\[
\text{RESULT} = \begin{cases} 
\int_0^1 X P X_j \, dx & \text{if } NØPT = 1 \\
\int_0^1 \frac{d^2 X}{dx^2} X_j \, dx & \text{if } NØPT = 2 
\end{cases}
\]
In the above expressions, $X_p$ and $X_j$ are the axial eigenfunctions for the $p$th and $j$th axial mode, with $p$ and $j$ being equal to $NP$ and $NJ$ respectively, and $\bar{u}_p$ and $\bar{u}_j$ are the local steady-state velocities of the gas and particles respectively. Also $\tilde{\rho}_p$ is the local steady-state density of the particles, and $\beta$ and $\beta_v$ are the dimensionless propellant cup length and vent width as defined in Volume I.
The required eigenvalues $B$ are obtained from the main program through the blank common; $\beta$ and $\beta_v$ are obtained through common block BLK7. The steady-state quantities are obtained from the subroutine STEADY. The integrals for $NOPT = 1$ and $NOPT = 2$ are obtained analytically, but for the other cases a numerical integration scheme using Simpson's rule is employed.

**SUBROUTINE AXIAL2 (NOPT, NP, NQ, NJ, RESULT).** This subroutine returns RESULT with the value of the following integrals:

- $NOPT = 1$
  $$\text{RESULT} = \int_0^1 \frac{dx}{dx} \frac{d}{dx} X_p \, dx$$

- $NOPT = 2$
  $$\text{RESULT} = \int_0^1 \frac{d^2x}{dx^2} X_q X_j \, dx$$

where $X_p$, $X_q$ and $X_j$ are the $p^{th}$, $q^{th}$ and $j^{th}$ eigenfunctions respectively. Subscripts $p(=NP)$, $q(=NQ)$ and $j(=NJ)$ represent the axial mode numbers. The eigenvalues $B$ are supplied to the subroutine through blank common. All the quantities involved in these integrals are real numbers, the eigenfunctions are trigonometric functions, and the integrals are evaluated in the subroutine analytically.

**SUBROUTINE STEADY (X, UBAR, UPBAR, RHOP, DUBAR, DUPBAR).** This subroutine computes the steady-state flow variables at a specified location $X$ in the T-burner. $UBAR$ and $UPBAR$ are the computed steady-state velocities of the gas phase and particle phase respectively, $DUBAR$ and $DUPBAR$ are their axial derivatives, and $RHOP$ is the steady-state value of the particle density. All of these quantities are calculated according to the steady-state equations given in Section 2.6 of Volume I. Common block BLK7 supplies the information necessary to compute these variables.

**SUBROUTINE GJR (A, NC, NR, N, MC, JC, V).** This subroutine is identical to the one used in Program SOLIDI and described in Section 3.1.

**Description of Output.** The arrangement of output and the variables controlling the output are the same as for Program SOLIDI as described in Section 3.1. As before, two modes of output - printed output and disk storage - are available and the disk is given the I/O device number 9. The data is written on the disk by setting $NOUT = 1$ or $NOUT = 2$, and the data on the disk forms part of the input to Program TB2 to be described in Section 4.2.
Sample Case. The sample case which is considered here refers to a T-burner with radius to length ratio of 0.05071 and a vent whose width is 10% of the total length of the burner. The total length of the propellant grain is 10% of the total length of the burner (5% at either end), $\gamma$ is 1.23 and the burning velocity at the propellant surface is 0.001978. The particles are 2.5 microns in diameter with a particle loading of 0.36. The pure-gas frequency of oscillation is 1071 Hertz, and the temperature in the T-burner is 3525°K. The first five longitudinal modes are considered, and it is desired to obtain a printed output as well as disk storage of generated data so that they may be used for checking out program TB2 later.

An input deck needed for running program TB1 with the above T-burner conditions is illustrated on the next page. The printed output generated by the program TB1 for this input is shown in the following pages.
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TEST CASE FOR TBI

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BETAV = .10030
R/L = .09071
UB = .00196
VL = 1.0

END BURNING IS PRESENT.

PARTICLE Dia (IN MICRONS) = 2.50
CHAMBER TEMP (IN DEG K) = 3525.0
FREQ (IN Hertz) = 1071.0

PARTICLE DRAG COEFFICIENT, K = 29.9166

CM = .36
SP = .68
RHOM (IN KG/CUBIC METER) = 4000.0
### Decoupled Coefficient of B(p): C(1, j, p)

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DECOUPLED NONLINEAR COEFFICIENT IN GAS EQUATION FOR 3(9)

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This program computes the coefficients which appear in the differential equations which govern the mode-amplitude functions. These coefficients can be written into a file for input to program TB2.

The following inputs are required:

The first card gives the title of the case.

Second card: \( \Gamma_0, \beta, \beta_0, RRL, UB, VL, EL \)
- \( \Gamma_0 \) is the specific heat ratio.
- \( \beta \) is the ratio of the length of the two cup grains to the length of the T-burner.
- \( \beta_0 \) is the ratio of the vent width to the length of the burner.
- \( RRL \) is the radius-to-length ratio.
- \( UB \) is the velocity at the burning surface.
- \( VL \) determines the vent effect:
  - \( VL = 0 \): Vent gain.
  - \( VL = 1 \): No effect.
  - \( VL = 2 \): Vent loss.
- \( EL \) is the effective vent length.

Third card: \( NJMAX, \text{NONLIN}, \text{NEGL}, \text{NOUT}, \text{NPRTKL}, \text{NBURN} \)
- \( NJMAX \) is the number of mode-amplitude functions in the assumed series solution.
- The coefficients computed are determined by NONLIN as follows:
  - \( \text{NONLIN} = 0 \): Linear coefficients only
  - \( \text{NONLIN} = 1 \): Both linear and nonlinear coefficients.
- Coefficients to be neglected are determined by NEGL as follows:
  - \( \text{NEGL} = 0 \): Terms smaller than 0.00001 are neglected.
  - \( \text{NEGL} = 1 \): Linear terms smaller than \( SM1 \) and nonlinear terms smaller than \( SM2 \) are neglected.
- The output is determined by NOUT as follows:
  - \( \text{NOUT} = 0 \): Printed output only
  - \( \text{NOUT} = 1 \): Write into a file and print output.
  - \( \text{NOUT} = 2 \): Write into a file only.
- NPRTKL determines whether the particles are present:
  - \( \text{NPRTKL} = 0 \): Particles not present.
  - \( \text{NPRTKL} = 1 \): Particles present.
- NBURN indicates whether end burning is present:
  - \( \text{NBURN} = 0 \): Not present.
  - \( \text{NBURN} = 1 \): Present.
NEXT CARD (ONLY IF NPRTKL=1): DIA, RHOM, SP, TEMP, FREQ, CM
DIA IS THE PARTICLE DIAMETER, IN MICRONS.
RHOM IS THE DENSITY OF THE PARTICLE MATERIAL, IN KG/M**3.
SP IS THE RATIO OF THE SPECIFIC HEATS OF PARTICLE MATERIAL
AND GAS.
TEMP IS THE CHAMBER TEMPERATURE, IN DEGREES KELVIN.
FREQ IS THE FREQUENCY OF OSCILLATION IN PURE GAS, IN HERTZ.
CM IS THE PARTICLE LOADING.

NEXT CARD (NECESSARY ONLY IF NEGL = 1): SM1, SM2
SM1 AND SM2 ARE AS DEFINED ABOVE.

NEXT NJMAX CARDS: J, L(J), NAME(J)
EACH MODE-AMPLITUDE IS ASSIGNED AN INTEGER J.
THE MODE IS SPECIFIED BY THE INDEX L(J).
L(J) IS THE AXIAL MODE NUMBER AND MUST NOT EXCEED NJMAX.
NAME(J) IS A FOUR-CHARACTER NAME FOR THE J TH MODE.

*****************************************************************

DIMENSION L(6), NAME(6), TITLE(7), V(2), JC(12), C(4,12,24),
1 C1(12,12), D(12,12,12), KMAX(5), TSW(12), TS(4,12),
2 C1PAR(12,12), CPAR(12,24), TSPAR(12), AXINT(2),
3 AX(11), CC(4,6,12), CNORM(12), CCPAR(2,6,12),
4 CCV(6), CV(12), TSV(12)
COMPLEX CD1(6,6,6), CD2(6,6,6), CD3(6,6,6), CD4(6,6,6), CI
COMMON B(6)
COMMON /BLK7/BETA, BETAV, RRL, UB, PARTKL, CM, NPRTKL, NBURN

DATA INPUT

MAXMD = 6
MAXMD2 = 12
MAXMD4 = 24
PI = 3.1415926536
SM1 = 0.00001
SM2 = 0.00001
CI = (0.0,1.0)

INPUT PARAMETERS.
4 READ (5,5000) TITLE
IF (EOF(5)) 600, 1
1 CONTINUE
READ (5,5001) GAM, BETA, BETAV, RRL, UB, VL, EL
READ (5,5004) NJMAX, NONLIN, NEGL, NOUT, NPRTKL, NBURN
IF (NPRTKL *EQ* 1) READ (5,5006) DIA, RHOM, SP, TEMP, FREQ, CM
GAMMA = GAM * (1.0 + SP*CM) / (1.0 + GAM*SP*CM)
IF (NEGL *EQ* 1) READ (5,5005) SM1, SM2
DO 10 I = 1, NJMAX
READ (5,5002) J, L(J), NAME(J)
B(I) = PI * I
10 CONTINUE

224
NJMAX2 = NJMAX
IF (NPRTKL .EQ. 1) NJMAX2 = 2 * NJMAX
ZE = 1.0
ZCOMB = 1.0
CAX = GAMMA + 1.0
AV = PI * BETAV * BETAV / 4.0
BETAV1 = PI * (BETAV + 1.0) / 2.0
BETAV2 = PI * (BETAV - 1.0) / 2.0
IF (NPRTKL .EQ. 0) GO TO 14
VISC = 8.834 * 0.00001 * (TEMP/3485.)**0.66
PARTKL = (9.0 * VISC) / (RHOME * FREQ * DIA * DIA *10.**(-12))
14 CONTINUE

C
C CALCULATE LINEAR COEFFICIENTS.
C
DO 100 NJ = 1, NJMAX
IF (BETAV .LT. 0.00001) GO TO 110
CCV(NJ) = AV * (SIN(NJ*BETAV1) - SIN(NJ*BETAV2))
   / (BETAV * NJ * PI)
110 DO 100 NP = 1, NJMAX2

C
C ZERO COEFFICIENT ARRAYS.
DO 105 KC = 1, 4
CC(KC,NJ,NP) = (0.0,0.0)
105 CONTINUE
CCPAR(1,NJ,NP) = CCPAR(2,NJ,NP) = 0.0
NPM = NP
NJM = NJ
IF (NP .GT. NJMAX) NPM = NP - NJMAX

C
C CALCULATE AXIAL INTEGRALS.
DO 130 NOPT = 1, 11
CALL AXIAL1(NOPT,NPM,NJM,CRSLT)
AX(NOPT) = CRSLT
130 CONTINUE

C
C EVALUATE FUNCTIONS AT THE END.
ZEJ = COS(B(NJM) * ZE)
ZEPI = COS(B(NPM) * ZE)

C
IF (NP .GT. NJMAX) GO TO 704

C
C COEFFICIENT OF THE SECOND DERIVATIVE OF A(P).
CC(1,NJ,NP) = AX(1)
C
C COEFFICIENT OF A(P).
CC(2,NJ,NP) = - AX(2)
COEFFICIENT OF THE FIRST DERIVATIVE OF $a(p)$:

- $CC(3, NJ, NP) = 2.0*A(4) + CAX*A(3) + PARTKL*A(5) - 1*VL*A(11) + GAMMA*UB*NBURN*(1.0 + ZEJ*ZEP1)$
- $CC(4, NJ, NP) = -GAMMA*(A(9) + UB*NBURN*(1.0 + ZEJ*ZEP1))$
- $CCPAR(2, NJ, NP) = -PARTKL*A(1)$

GO TO 100

- $CC(3, NJ, NP) = PARTKL*A(5) - (GAMMA - 1.0)*A(6)$
- $CCPAR(1, NJ, NP) = A(1)$
- $CCPAR(2, NJ, NP) = PARTKL*A(1) + A(7) + A(8) - VL*A(10)$

GO TO 100

NORMALIZE LINEAR COEFFICIENTS:

```
DO 140 NJ = 1, NJMAX
CNORM(NJ) = CC(1, NJ, NJ)
CV(NJ) = CV(NJ) / CNORM(NJ)
DO 140 NP = 1, NJMAX2
DO 140 KC = 1, 4
CC(KC, NJ, NP) = CC(KC, NJ, NP) / CNORM(NJ)
140 CONTINUE
```

IF (NPRTKL .EQ. 0) GO TO 1005

```
DO 1030 NJ = 1, NJMAX
NJM = NJ + NJMAX
CNORM(NJM) = CCPAR(1, NJ, NJM)
DO 1030 NP = 1, NJMAX2
DO 1030 KC = 1, 2
CCPAR(KC, NJ, NP) = CCPAR(KC, NJ, NP) / CNORM(NJM)
1030 CONTINUE
```

1005 CONTINUE

COMPUTE NONLINEAR COEFFICIENTS:

```
IF (NONLIN .EQ. 0) GO TO 402
G1 = (GAMMA - 1.0) * 0.5
```

```
DO 200 NJ = 1, NJMAX
DCOEF = 0.5 / CNORM(NJ)
DO 200 NP = 1, NJMAX
DO 200 NQ = 1, NJMAX
CD1(NJ, NP, NQ) = (0.0, 0.0)
CD2(NJ, NP, NQ) = (0.0, 0.0)
CD3(NJ, NP, NQ) = (0.0, 0.0)
CD4(NJ, NP, NQ) = (0.0, 0.0)
DO 240 J = 1, 2
CALL AXIAL2(J, NP, NQ, NJ, CRSRT)
AXINT(J) = CRSRT
240 CONTINUE
```
CD1(NJ,NP,NQ) = CD4(NJ,NP,NQ) = (AXINT(1) + G1 * AXINT(2)) * 
DCOEF * (1.0, -1.0)
CD2(NJ,NP,NQ) = CD3(NJ,NP,NQ) = (AXINT(1) + G1 * AXINT(2)) * 
DCOEF * (1.0, 1.0)

200 CONTINUE

***********************************************
CALCULATE COEFFICIENTS FOR EQUIVALENT REAL SYSTEM.

402 DO 350 NJ = 1, NJMAX
   NEWJ = (2 * NJ) - 1
   NEWJ1 = NEWJ + 1
   CVCNEWJ) = CVC(NEWJ1) = CCV(NJ)
   DO 360 NP = 1, NJMAX2
   NEWP = 2 * NP - 1
   NEWP1 = NEWP + 1

   COEFFICIENTS OF LINEAR TERMS.
   IF (NP .GT. NJMAX) GO TO 1040
   C(NEWJ,NEWP) = C1(NEWJ1,NEWP1) = CC(1,NJ,NP)
   1040 CONTINUE
   C(NEWJ,NEWP1) = CC(4, NJ,NP)
   C(4,NEWJ1,NEWP) = CC(4, NJ,NP)
   DO 360 KC = 1, 3
   C(KC,NEWJ,NEWP) = C(KC,NEWJ1,NEWP1) = CCO(1+1,NJ,NP)
   360 CONTINUE

   COEFFICIENTS OF NONLINEAR TERMS.
   IF (NONLIN .EQ. 0) GO TO 350
   DO 370 NP = 1, NJMAX
   NEWP = 2 * NP - 1
   NEWP1 = NEWP + 1
   DO 370 NO = 1, NJMAX
   NEWQ = (2 * NO) - 1
   NEWQ1 = NEWQ + 1
   CD1R = REAL(CD1(NJ,NP,NQ))
   CD1I = AIMAG(CD1(NJ,NP,NQ))
   CD2R = REAL(CD2(NJ,NP,NQ))
   CD2I = AIMAG(CD2(NJ,NP,NQ))
   CD3R = REAL(CD3(NJ,NP,NQ))
   CD3I = AIMAG(CD3(NJ,NP,NQ))
   CD4R = REAL(CD4(NJ,NP,NQ))
   CD4I = AIMAG(CD4(NJ,NP,NQ))
   D(NNEWJ,NEWP,NEWQ) = CD1R + CD2R + CD3R + CD4R
   D(NNEWJ,NEWP,NEWQ1) = -CD1I + CD2I - CD3I + CD4I
   D(NNEWJ,NEWP1,NEWQ) = -CD1I - CD2I + CD3I + CD4I
   D(NNEWJ,NEWP1,NEWQ1) = -CD1R + CD2R + CD3R - CD4R

227
\[ D(NEWJ_1, NEWP, NEWQ) = CD_1I + CD_2I + CD_3I + CD_4I \]
\[ D(NEWJ_1, NEWP, NEWQ_1) = CD_1R - CD_2R + CD_3R - CD_4R \]
\[ D(NEWJ_1, NEWP_1, NEWQ) = CD_1R + CD_2R - CD_3R + CD_4R \]
\[ D(NEWJ_1, NEWP_1, NEWQ_1) = -CD_1I + CD_2I + CD_3I - CD_4I \]

370 CONTINUE
350 CONTINUE

C
IF (NPHTKL .EQ. 0) GO TO 1035
DO 1050 NJ = 1, NJMAX
NEWJ = 2 * NJ - 1
NEWJ1 = NEWJ + 1
DO 1050 NP = 1, NJMAX2
NEWP = 2 * NP - 1
NEWP1 = NEWP + 1
CPAR(NEWJ, NEWP) = CPAR(NEWJ1, NEWP1) = CCPAR(2, NJ, NP)
IF (NP .LE. NJMAX) GO TO 1050
NEWP = NEWP - NJMAX2
NEWP1 = NEWP + 1
CPAR(NEWJ, NEWP) = CPAR(NEWJ1, NEWP1) = CCPAR(1, NJ, NP)
1050 CONTINUE
1035 CONTINUE

C
C
*****************************************************************
C
C
COMPUTE COEFFICIENTS FOR THE EQUATIONS WHICH ARE DECOUPLED
C IN THE SECOND DERIVATIVES.
C
DO 405 KC = 1, 5
KMAX(KC) = 0
405 CONTINUE
C
C
CALCULATE INVERSE OF THE MATRIX C1(I,J).
JMAX = NJMAX
NJMAX = 2 * NJMAX
JMAX2 = NJMAX2
NJMAX2 = 2 * NJMAX2
C
V(1) = 1
CALL 6JR(C1, MAXM D2, MAXMD2, NJMAX, 0, JC, V)
C
USE INVERSE TO CALCULATE DECOUPLED COEFFICIENTS.
C
C
LINEAR COEFFICIENTS.
DO 430 NP = 1, NJMAX2
DO 420 NJ = 1, NJMAX
DO 420 KC = 1, 4
TS(KC, NJ) = 0.0
DO 420 K = 1, NJMAX
420 CONTINUE
DO 430 NJ = 1, NJMAX
DO 430 KC = 1, 4
C(KC,NJ,NP) = TS(KC,NJ)
ABSVAL = ABS(C(KC,NJ,NP))
IF (ABSVAL .GE. SM1) KMAX(KC) = KMAX(KC) + 1
430 CONTINUE

C
DO 720 NJ = 1, NJMAX
TSV(NJ) = 0.0
DO 720 K = 1, NJMAX
TSV(NJ) = TSV(NJ) + C1(NJ,K) * CV(K)
720 CONTINUE
C
DO 730 NJ = 1, NJMAX
CV(NJ) = TSV(NJ)
730 CONTINUE
C
IF (NPRTKL .EQ. 0) GO TO 1060
KMAXPR = 0
V(1) = 1
CALL GJR(C1PAR,MAXMD2,MAXMD2,NJMAX,0,JC,V)
DO 1065 NP = 1, NJMAX2
DO 1070 NJ = 1, NJMAX
TSPAR(NJ) = 0.0
DO 1070 K = 1, NJMAX
TSPAR(NJ) = TSPAR(NJ) + C1PAR(NJ,K) * CPAMK(NP)
1070 CONTINUE
DO 1065 NJ = 1, NJMAX
CPAR(NJ, NP) = TSPAR(NJ)
ABSVAL = ABS(CPAR(NJ, NP))
IF (ABSVAL .GE. SM1) KMAXPR = KMAXPR + 1
1065 CONTINUE

C
C NONLINEAR COEFFICIENTS.
IF (NONLIN .EQ. 0) GO TO 410
DO 735 NP = 1, NJMAX
DO 735 NO = 1, NJMAX
DO 440 NJ = 1, NJMAX
TSQ(NJ) = 0.0
DO 440 K = 1, NJMAX
TSQ(NJ) = TSQ(NJ) + C1(NJ,K) * D(K,NP,NO)
440 CONTINUE
DO 445 NJ = 1, NJMAX
D(NJ,NP,NO) = TSQ(NJ)
ABSVAL = ABS(D(NJ,NP,NO))
IF (ABSVAL .GT. SM2) KMAX(5) = KMAX(5) + 1
445 CONTINUE

735 CONTINUE

410 CONTINUE
**** OUTPUT ****

IF (NOUT .EQ. 2) GO TO 455
WRITE (6, 6001) TITLE
WRITE (6, 6002) GAM, BETA, BETA0, RHL, UB, VL
IF (NBURN .EQ. 0) WRITE (6, 6025)
IF (NBURN .EQ. 1) WRITE (6, 6026)
IF (NPRTKL .EQ. 0) WRITE (6, 6022)
IF (NPRTKL .EQ. 1) WRITE (6, 6021) DIA, CM, FREQ, 1
   TEMP, SP, RHOM, PARTKL
WRITE (6, 6004)
DO 310 J = 1, JM
WRITE (6,6003) NAME(J), JO(J), B(J)
310 CONTINUE
IF (NONLIN .EQ. 0) WRITE (6, 6013)

**** OUTPUT OF LINEAR COEFFICIENTS ****
DO 320 KC = 1, 4
NJS = 0
NJF = 0
KOUNTJ = 1
758 NJS = NJF + 1
NJF = 10 * KOUNTJ
IF (NJF .GT. NJMAX) NJF = NJMAX
NPS = 0
NPF = 0
KOUNTP = 1
754 NPS = NPF + 1
NPF = 10 * KOUNTP
IF (NPF .GT. NJMAX2) NPF = NJMAX2
IF (KC .EQ. 1) WRITE (6, 6005)
IF (KC .EQ. 2) WRITE (6, 6006)
IF (KC .EQ. 3 .OR. KC .EQ. 4) WRITE (6, 6007)
WRITE (6,6008) (NP, NP = NPS, NPF)
WRITE (6,6014)
DO 750 NJ = NJS, NJF
WRITE (6,6009) NJ, (C(KC,J,NP), NP = NPS, NPF)
750 CONTINUE
IF (NPRTKL .EQ. 0) GO TO 752
KOUNTP = KOUNTP + 1
GO TO 754
752 IF (NJF .EQ. NJMAX) GO TO 756
KOUNTJ = KOUNTJ + 1
GO TO 758
756 CONTINUE
320 CONTINUE

IF (NPRTKL .EQ. 0) GO TO 1080
NJS = 0
NJF = 0
KOUNTJ = 1
C

1072 NJS = NJF + 1
NJF = 10*KOUNTJ
IF (NJF GT NJMAX) NJF = NJMAX
NPS = 0
NPF = 0
KOUNTP = 1

1074 NPS = NPF+1
NPF = 10*KOUNTP
IF (NPF GT NJMAX2) NPF = NJMAX2
WRITE (6,6023)
WRITE (6,6008) (NP, NP = NPS,NPF)
WRITE (6,6014)
DO 1076 NJ = NJS,NJF
WRITE (6,6009) NJ, (CPAR(NJ,NP),NP = NPS,NPF)
1076 CONTINUE
IF (NPF EQ NJMAX2) GO TO 1078
KOUNTP = KOUNTP + 1
GO TO 1074

1078 IF (NJF EQ NJMAX) GO TO 1080
KOUNTJ = KOUNTJ + 1
GO TO 1072

1080 CONTINUE

C

OUTPUT OF NONLINEAR COEFFICIENTS.
IF (NONLIN EQ 0) GO TO 452
DO 400 NJ = 1, NJMAX
NPS = 0
NPF = 0
KOUNTP = 1

780 NPS = NPF + 1
NPF = 10 * KOUNTP
IF (NPF GT NJMAX) NPF = NJMAX
NQS = 0
NQF = 0
KOUNTQ = 1

776 NQS = NQF + 1
NQF = 10 * KOUNTQ
IF (NQF GT NJMAX) NQF = NJMAX
IF (NQS GT NJMAX AND NPF LE NJMAX) GO TO 771
IF (NQF LE NJMAX AND NPS GT NJMAX) GO TO 771
WRITE (6,6010) NJ
WRITE (6,6011) (NQ, NQ = NQS, NQF)
WRITE (6,6015)
DO 772 NF = NPS, NPF
WRITE (6,6009) NP, (D(NJ,NP,NQ), NQ = NQS, NQF)
772 CONTINUE

771 CONTINUE
IF (NQF EQ NJMAX) GO TO 774
KOUNTQ = KOUNTQ + 1
GO TO 776
774 IF (NPF .EQ. NJMAX) GO TO 778
  KOUNTP = KOUNTP + 1
  GO TO 780
778 CONTINUE
400 CONTINUE
452 CONTINUE
  IF (NOUT .EQ. 0) GO TO 4
C
C WRITE COEFFICIENTS ON FILE.
C
455 WRITE (9,7001) GAMMA, BETA, BETAU, RRL, UB, VL, EL,
  1 NJMAX, NPRTKL, NBURN
  IF (NPRTKL .EQ. 1) WRITE (9,7007) DIA, RHOM, SP,
  1 TEMP, FREQ, PARTKL, CM
C
DO 450 J = 1, JMAX
  WRITE (9,7002) J, 1.0(J), NAME(J)
450 CONTINUE
C
DO 457 J = 1, JMAX
  WRITE (9,7006) J, B(J)
457 CONTINUE
C
DO 460 KC = 1, 4
  WRITE (9,7003) KMAX(KC)
  DO 460 NJ = 1, NJMAX
    DO 460 NP = 1, NJMAX2
      ABSVAL = ABS(C(KC,NJ,NP))
      IF (ABSVAL .GE. SM1) WRITE (9,7004) NJ, NP, C(KC,NJ,NP)
460 CONTINUE
C
DO 465 NJ = 1, NJMAX
  WRITE (9,7008) NJ, CV(NJ)
465 CONTINUE
C
WRITE (9,7003) KMAXPR
  DO 1082 NJ = 1, NJMAX
    DO 1082 NP = 1, NJMAX2
      ABSVAL = ABS(CPAR(NJ,NP))
      IF (ABSVAL .GE. SM1) WRITE (9,7004)NJ, NP, CPAR(NJ,NP)
1082 CONTINUE
C
WRITE (9,7003) KMAX(5)
  IF (NONLIN .EQ. 0) GO TO 4
  DO 470 NJ = 1, NJMAX
    DO 470 NP = 1, NJMAX
      DO 470 NQ = 1, NJMAX
        ABSVAL = ABS(D(NJ,NP,NQ))
        IF (ABSVAL .GE. SM2) WRITE (9,7005) NJ, NP, NQ, D(NJ,NP,NQ)
470 CONTINUE
  GO TO 4

232
C 600 CONTINUE
C
C ******************************************************************
C
C " FORMAT SPECIFICATIONS"
5000 FORMAT (7A10)
5001 FORMAT (7F10.0)
5002 FORMAT (2I5, 1X, A4)
5004 FORMAT (6I5)
5005 FORMAT (2F10.0)
5006 FORMAT (6F10.0)
6001 FORMAT (1H1, //, 2X, 7A10//)
6002 FORMAT (2X, 8HGAMMA = , F5.3, //, 2X, 6HBETA =, F5.3, //, 2X, 7HBETAV =,
1 F7.5, //, 2X, 5HR/L =, F7.5, //, 2X, 4HUB =, F7.5, //, 2X, 1
2 5HVL = , F4.2, / )
6003 FORMAT (2X, A4, 2I5, 2F10.5/)
6004 FORMAT (2X, 14HNAME J L, 5X, 4HB(J) /)
6005 FORMAT (1H1, 45H DECOUPLED COEFFICIENT OF B(P): C(1,J,P)//)
6006 FORMAT (1H1, 44H DECOUPLED COEFFICIENT OF THE DERIVATIVE OF,
1 6H B(P): 5X, 8HC(2, J, P)//)
6007 FORMAT (1H1, 47H DECOUPLED COEFFICIENT OF THE COMBUSTION TERM
1 5X, 8HC(3, J, P)//)
6008 FORMAT (7X, 1HP, I8, 9I12)
6009 FORMAT (7X, 1HP, I2, 1H)
6010 FORMAT (1H1, 47HDECOUPLED NONLINEAR COEFFICIENT IN GAS EQUATION,
1 7H FOR B(:, 12, 1H)//)
6011 FORMAT (7X, 1HP, I8, 9I12)
6012 FORMAT (2X, 25HLINEAR COEFFICIENTS ONLY. )
6013 FORMAT (2X, 25HLINEAR COEFFICIENTS ONLY. )
6014 FORMAT (4X, 1HJ)
6015 FORMAT (4X, 1HP)
6021 FORMAT (7X, 10X, 27HPARTICLE DIA (IN MICRONS) = , F5.2, 10X,
1 4HC = , F4.2, 10X, 18HFREQ (IN HERTZ) = , F6.1, //,
2 10X, 26CHAMBER TEMP (IN DEG K) = , F6.1, 10X, 4HSP =,
3 F4.2, 10X, 27HHRHOM (IN KG/CUBIC METER) = , F6.1, 10X,
4 30HPARTICLE DRAG COEFFICIENT, K = , F8.4, //)
6022 FORMAT (2X, 26HPARTICLES ARE NOT PRESENT.//)
6023 FORMAT (1H1, 39HCOEFFICIENTS IN THE PARTICLE EQUATIONS:,
1 12H CPAR(J,P)//)
6025 FORMAT (2X, 15HNO END BURNING.//)
6026 FORMAT (2X, 23HEND BURNING IS PRESENT.//)
7001 FORMAT (7F10.5, 3I5)
7002 FORMAT (2I5, 1X, A4)
7003 FORMAT (I5)
7004 FORMAT (2I5, F15.8)
7005 FORMAT (3I5, F15.8)
7006 FORMAT (1I5, 2F12.8)
7007 FORMAT (7F15.8)
7008 FORMAT (1I5, F12.8)
END
SUBROUTINE AXIAL1(NOPT, NP, NJ, RESULT)

C
C
C THIS SUBROUTINE CALCULATES THE INTEGRAL OVER THE INTERVAL 
C (0,1) OF THE FOLLOWING FUNCTIONS ACCORDING TO THE VALUE 
C OF NOPT
C
C NOPT = 1  Z(NP) * Z(NJ)
C NOPT = 2  DDZ(NP) * Z(NJ)
C NOPT = 3  DUBAR * Z(NP) * Z(NJ)
C NOPT = 4  UBAR * DZ(NP) * Z(NJ)
C NOPT = 5  RHOP * Z(NP) * Z(NJ)
C NOPT = 6  RHOP * UPBAR * DZ(NP) * Z(NJ)
C NOPT = 7  UPBAR * DZ(NP) * Z(NJ)
C NOPT = 8  DUPBAR * Z(NP) * Z(NJ)

C IN THE ABOVE EQUATIONS:
C Z(NP) IS THE AXIAL ACOUSTIC EIGENFUNCTION OF INDEX NP.
C Z(NJ) IS THE AXIAL ACOUSTIC EIGENFUNCTION OF INDEX NJ.
C DZ AND DDZ ARE THE FIRST AND SECOND DERIVATIVES OF THE
C AXIAL EIGENFUNCTIONS RESPECTIVELY.
C UBAR IS THE STEADY STATE VELOCITY DISTRIBUTION AND
C DUBAR IS ITS AXIAL DERIVATIVE.
C
C DIMENSION FUNCT(250)
COMMON B(6)
COMMON /BLK7/ BETA, BETAV, RRL, UB, PARTKL, CM, NPKTL, NBURN

C MAXMD = 6
RESULT = 0.0
ZE = 1.0
BP = B(NP)
BJ = B(NJ)

C IF (NOPT GT 2) GO TO 50
C CALCULATE INTEGRALS BY MEANS OF ANALYTICAL EXPRESSIONS FOR
C NOPT = 1 AND NOPT = 2.
C IF (NP EQ NJ) RESULT = 0.5
C IF (NOPT EQ 2) RESULT = BP * BP * RESULT
RETURN

C NUMERICAL EVALUATION OF INTEGRALS FOR NOPT > 2.
C
C COMPUTE STEP SIZE FOR SIMPSON INTEGRATION.
C
50 N = 200
RN = N
RESULT = 0.0
NOPT2 = NOPT - 2
IC = 1
J = 1
H = ZE/RN
ZO = 0.0
NP1 = N + 1
COMPUTE INTEGRANDS
DO 60 I = 1, NP1
Z = ZO + (I - 1) * H
ARG = BP * Z
CALL STEADY(Z, UBAR, UPBAR, RHOP, DUBAR, DUPBAR)
GO TO (110, 120, 130, 140, 150, 155, 160, 170, 180), NOPT2
110 F2 = COS(ARG)
F1 = DUBAR
GO TO 190
120 F1 = UBAR
F2 = -BP * SIN(ARG)
GO TO 190
130 F1 = RHOP
F2 = COS(ARG)
GO TO 190
140 F1 = RHOP + UBAR
F2 = -BP * SIN(ARG)
GO TO 190
150 F1 = UPBAR
F2 = -BP * SIN(ARG)
GO TO 190
155 F1 = DUPBAR
F2 = COS(ARG)
GO TO 190
160 F1 = 0.0
BETA2 = BETA/2.0
IF (Z * GT* BETA2 * AND* Z * LT* 1.0 - BETA2) GO TO 190
F1 = DUBAR
F2 = COS(ARG)
GO TO 190
170 BETAV2 = BETAV/2.0
F1 = 0.0
IF (Z * LT* 0.5 - BETAV2 & OR* Z * GT* 0.5 + BETAV2) GO TO 190
F1 = DUBAR
F2 = COS(ARG)
GO TO 190
180 BETAV2 = BETAV/2.0
F1 = 0.0
IF (Z * LT* 0.5 - BETAV2 & OR* Z * GT* 0.5 + BETAV2) GO TO 190
F1 = DUBAR
F2 = COS(ARG)
GO TO 190
190 CONTINUE
F3 = COS(BJ*Z)
FUNCT(I) = F1 * F2 * F3
60 CONTINUE
PERFORM SIMPSON INTEGRATION.
NM1 = N - 1
S1 = FUNCT(1) + FUNCT(NP1)
S2 = 0.0
S3 = 0.0
DO 70 I = 2, N, 2
    S2 = S2 + FUNCT(I)
70 CONTINUE
DO 80 I = 3, NM1, 2
    S3 = S3 + FUNCT(I)
80 CONTINUE
RESULT = RESULT +
1   H * (S1 + 4.0*S2 + 2.0*S3) / 3.0
90 CONTINUE
C
100 CONTINUE
RETURN
END
SUBROUTINE AXIALZ(NOPT, NP, NQ, NJ, RESULT)

THIS SUBROUTINE CALCULATES THE INTEGRAL OVER THE INTERVAL (0, 1) OF THE FOLLOWING FUNCTIONS ACCORDING TO THE VALUE OF NOPT.

NOPT = 1  DZ(NP) * DZ(NQ) * Z(NJ)
NOPT = 2  DDZ(NP) * Z(NQ) * Z(NJ)

IN THE ABOVE EQUATIONS:
Z(NP), Z(NQ), AND Z(NJ) ARE THE AXIAL ACOUSTIC EIGENFUNCTIONS
AND NP, NQ, AND NJ ARE THEIR INDICES.
DZ IS THE FIRST DERIVATIVE OF THE AXIAL EIGENFUNCTIONS.
DDZ IS THE SECOND DERIVATIVE OF THE AXIAL EIGENFUNCTIONS.

COMMON B(6)

CALCULATE INTEGRALS BY MEANS OF ANALYTICAL EXPRESSIONS.
BP = B(NP)
BQ = B(NQ)
BJ = B(NJ)
RESULT = 0.0
IF (NOPT * Eq. 2) GO TO 10
IF (NP-NQ+NJ * Eq. 0 * Or. NP-NQ-NJ * Eq. 0) RESULT = BP * BQ / 4.0
IF (NP+NQ-NJ * Eq. 0) RESULT = - BP * BQ / 4.0
RETURN
10 CONTINUE
1 IF (NP-NQ+NJ * Eq. 0 * Or.
2 IF (NP-NQ-NJ * Eq. 0) RESULT = - (BP * BP) / 4.0
RETURN
END
SUBROUTINE STEADY(X, UBAR, UPBAR, RHOP, DUBAR, DUPBAR)

COMMON /BLK7/ BETA, BETAV, RRL, UB, PARTKL, CM, NPRTKL, NBURN

C

RLR = 1.0 / RRL
BBY2 = BETA / 2.0
BVBY2 = BETAV / 2.0

IF (NPRTKL .EQ. 1) A5 = (4.0 * UB) / (RRL * PARTKL)
IF (X .GE. BBY2) GO TO 10
UBAR = UB * (NBURN + 2.0 * RLR * X)
UPBAR = UBAR / (1.0 + A5)
RHOP = CM * (1.0 + A5)
DUBAR = 2.0 * UB * RLR
DUPBAR = DUBAR / (1.0 + A5)
RETURN

10 A4 = NBURN + BETA * RLR
IF (X .GE. BBY2) GO TO 20
UBAR = UB * A4
UPBAR = UBAR
RHOP = CM
DUBAR = DUPBAR = 0.0
RETURN

20 IF (X .GT. 0.5 + BVBY2) GO TO 30
X1 = (1.0 - 2.0 * X) / BETAV
UBAR = UB * A4 * X1
A6 = 1.0
IF (NPRTKL .EQ. 1) A6 = 1.0 - (4.0 * UB * A4) / (PARTKL * BETAV)
UPBAR = UBAR / A6
RHOP = CM * A6
DUBAR = - (2.0 * UB * A4) / BETAV
DUPBAR = DUBAR / A6
RETURN

30 IF (X .GT. 1.0 - BBY2) GO TO 40
UBAR = - UB * A4
UPBAR = UBAR
RHOP = CM
DUBAR = DUPBAR = 0.0
RETURN

40 X1 = NBURN + 2.0 * (1.0 - X) * RLR
UBAR = - UB * X1
UPBAR = UBAR / (1.0 + A5)
RHOP = CM * (1.0 + A5)
DUBAR = 2.0 * RLR * UB
DUPBAR = DUBAR / (1.0 + A5)
RETURN
END
4.2 PROGRAM TB2.

Program TB2 calculates the nonlinear stability characteristics of a T-burner according to the approximate analysis (Galerkin method) described in Volume I. Using the coefficients generated by TB1, this program integrates the system of differential equations governing the mode amplitudes and computes the time history of a pressure disturbance in the T-burner.

Program Structure. This program performs the same calculations for the T-burner that SOLID2 performs for the motor, and its structure is similar to the structure of SOLID2. In fact, the description of the structure of SOLID2 given in Section 3.2 adequately describes the structure of TB2 if "SOLID1" and "SOLID2" are replaced by "TB1" and "TB2" respectively.

Description of Input. As in the case of Program SOLID2, the input data required to run this program consists of three parts: (1) the control numbers NOUTCF and NHISTR which determine the extent of desired printed output, (2) the parameters and coefficients generated by Program TB1, and (3) data describing the case to be run. The information to be provided for each case is the same as for Program SOLID2. The general description of format given in Sections 3.1 and 3.2 apply here also. A list of necessary inputs is described below.

The three parts of the input are:
(1) The control numbers, NOUTCF and NHISTR.
(2) The coefficients from Program TB1.
(3) The data deck.

The first card gives the control numbers, NOUTCF and NHISTR.

NOUTCF determines printout of coefficients:
   If NOUTCF = 0 coefficients are not printed out.
   If NOUTCF = 1 only linear coefficients are printed out.
   If NOUTCF = 2 all coefficients are printed out.

NHISTR determines if pressure history is to be printed:
   If NHISTR = 0 printed
   If NHISTR = 1 not printed.

The coefficients are obtained from Program TB1 by putting NOUT = 1 or NOUT = 2, thereby writing the coefficients into a disk. This disk has been given the device number 9.
The data deck consists of the following cards:

First card: Title of the case.

Second card: H, TSTART, TQUIT, FREQ, BCOMB

- H is the integration step size.
- TSTART is the time at which output starts.
- TQUIT is the time at which computations are terminated.
- FREQ is the motor frequency (in pure gas), in Hertz.
- BCOMB is the combustion response nonlinearity factor.

Third card: A2PARA, B2PARA, EN, OMEGA

- A2PARA and B2PARA are the combustion parameters in the A-B model.
- EN is the pressure exponent in the burning rate law.
- OMEGA is the frequency nondimensionalized by the square of the steady-state burning rate.

Fourth card: NLQC, NTERMS, NOUT, NCOMB

- NLQC determines the location of the wall pressure maxima and minima:
  - If NLQC = 1, location is x = 0.0
  - If NLQC = 2, location is x = 1.0
  - If NLQC = 3, location is x = 0.5
- NTERMS is the number of terms given initial values.
  - If NOUT = 0, printed output only.
  - If NOUT > 0, both printed and plotted output:
    - If NOUT = 1, plot of pressure at x = 0.0 only.
    - If NOUT = 2, plot of pressure at x = 0.0 and x = 1.0.
    - If NOUT = 3, plot of pressure at x = 0.0, 1.0, and 0.5.
- NCOMB determines if combustion nonlinearities are considered:
  - If NCOMB = 0, neglected.
  - If NCOMB = 1, included.

Next card (necessary only if plots are required): YHI, YLAB, ITICY

- YHI is the maximum ordinate for pressure plots.
  - Note: the ordinate scales for pressure and amplitude plots are symmetric about zero.
- YLAB is the interval for ordinate labeling for above plots.
- ITICY is the number of ordinate tick marks for above plots.
  - Note: ITICY should be negative for pressure and amplitude plots to obtain centerline.
Next card (necessary only if plots are required): MDPLOT

MDPLOT determines if plots of individual modes are required:
If plot of J\textsuperscript{th} mode is required, punch "1" in the
5xJ\textsuperscript{th} column.
If plot of J\textsuperscript{th} mode is not required, punch "0" in the
5xJ\textsuperscript{th} column.

Next card (necessary only if plot of any mode amplitude is required):
YHIMD, YLABMD, ITICMD

YHIMD is the maximum ordinate.
YLABMD is the interval for ordinate labelling.
ITICMD is the number of ordinate tick marks for mode plots.
Note: ITICMD should be negative to obtain centerline.

Remaining cards (NTERMS in number): J, AST, ACT
AST is the amplitude of the sine term of the J\textsuperscript{th} mode.
ACT is the amplitude of the cosine term of the J\textsuperscript{th} mode.

The comments in Section 3.2 on choosing AST to obtain a desired single-
mode initial pressure disturbance apply here also.

Description of the Subroutines.

SUBROUTINE PHICFS (NP, Z, CT, CZ).
This subroutine performs the same
function and is structurally the same as subroutine PHICFS described in Section
3.2.

SUBROUTINE PRSVEL (AXL\textsuperscript{LOC}, VL, Y, P, VZGAS, VZPAR).
This subroutine computes
the pressure (P) and axial velocity perturbations (VZGAS and VZPAR) of the gas
and particles at a given axial location (AXL\textsuperscript{LOC}) in the T-burner, using the sup-
plied mode-amplitude functions and their derivatives (Y). The steady-state quan-
tities at the given axial location are computed using subroutine STEADY, and VL
represents the required vent effect. Pressure is computed from the second-order
momentum equation and velocity is computed as the axial derivative of the vel-
ocity potential. The space-dependent coefficients (COEF) of \( \hat{v}_x \) and \( \hat{v}_z \) are com-
puted by subroutine PHICFS and are supplied through the common block BLK3.

SUBROUTINE RHS (U, UP, UN, UNP).
This subroutine is similar to subroutine
RHS described in Section 3.2, except that in this case there are two additional
dependent variables (UN) representing the real and imaginary parts of the fluc-
tuating velocity through the vent. The equations for these variables (Equations
are solved simultaneously with the equations for the mode-amplitudes for the T-burner. The differential increments (UNP) of these variables needed for the Runge-Kutta integration are calculated by this subroutine. The calculation of the increments UP is similar to that in program SOLID2.

SUBROUTINE STEADY (X, UBAR, UPBAR, RHOP, DUBAR, DUPBAR). This subroutine, used for calculating the steady-state quantities, is same as that in Program TB1.

Besides the above subroutines, Program TB2 contains the following subroutines which are exactly the same as those used in Program SOLID2: (1) GROWTH, (2) RESPNS, (3) GRAPHS, (4) MYAXIS, (5) MYLINE, (6) AXLAB, and (7) DENDEC. These subroutines have already been described in Section 3.2.

Description of Output. As in the case of Program SOLID2, there are two modes of output: (1) printed output and (2) plotted output. The printed output produced by Program TB2 consists of seven sections.

Section (1) is a restatement of the input from Program TB1. It includes the following information: (a) the propellant grain length (BETA), vent width (BETAV), radius-to-length ratio (RRL), velocity at the burning surface (UB), parameter (VL) indicating the vent effect, effective plug-flow vent length (EL), number of modes considered in the series, specific heat ratio of the pure gas (GAM), and (if particles are present) the specific heat ratio of the mixture (GAMMABAR); (b) information about the particles (if present); (c) the parameters which describe and identify each term in the series expansion; (d) the axial acoustic eigenvalues B(J), for each term; (e) linear coefficients if NOUTCF = 1 or 2; and (f) nonlinear coefficients if NOUTCF = 2.

The other six sections of printed output and the plotted output are the same as described in Section 3.2 for Program SOLID2.

Sample Case. In this sample run, pressure-time histories are obtained for the T-burner considered in Section 4.1. The data generated by Program TB1 in Section 4.1 is used for the second part of the necessary input as described in this section. The same linear combustion response parameters (A, B, n, and Ω) as for the motor are used, and linear particle damping is considered. A step size of 0.025 is chosen, and TQUIT = 10.0. A pure 1L-mode initial disturbance of 7.5% pressure is assumed for which ACT = 0.0 and AST = -0.0234625. Pressure maxima and minima are obtained at X = 0.0, hence NLOC = 1, and plots of the pressure disturbance at z = 0.0 only is desired, hence NOUT = 1. It is also desired to obtain plots of the growth of all of the mode-amplitude functions.
An input deck with these data is illustrated on the next page. The following pages present the printed output and the plotted output, generated by Program TB2 using this deck and the coefficients generated by Program TB1 in Section 4.2.
ENC BURNING IS PRESENT.

BETA = .10000

BETAV = .10000

R/L = .05071

U3 = .00198

VL = 1.0

EL = .16600

NUMBER OF MODES = 5

GAMMA = 1.23

GAMMA BAR = 1.176770

PARTICLE DIA (IN MICRONS) = 2.50

CHAMBER TEMP (IN DEG K) = 3525.0

PARTICLE DRAG CONSTANT, K = 29.9186

NAME | J  | L
---|----|---
1L  | 1  | 1
2L  | 2  | 2
3L  | 3  | 3
4L  | 4  | 4
5L  | 5  | 5

J   | B(J)
---|----
1   | 3.14159
2   | 6.28319
3   | 9.42478
4   | 12.56637
5   | 15.70796

CM = .36

FREQ (IN HERTZ) = 1071.0

SP = .68

RHO M (IN KG/CUBIC METER) = 4000.0
### Coefficients for Computation of Wall Pressure Waveforms

#### Coefficients in Series for:

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<th>Axial Derivative</th>
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{
COMBUSTION PARAMETERS:  A = 5.9960   B = .5600   EN = .575   OMEGA = 4.200

  J  RESR  RESI

1  4.1401  .0926
2  .9605  -1.7016
3  .5013  -1.3372
4  .3777  -.8191
5  .3194  -.5694

LINEAR COMBUSTION RESPONSE.

LINEAR PARTICLE DAMPING.
}
INITIAL CONDITIONS ARE OF THE FORM:

\[ U(x, t) = AC(j) \cdot \cos(FREQ \cdot t) + AS(j) \cdot \sin(FREQ \cdot t) \]

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This run produces plotted output.

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<th>PAR VEL AT Z=0.5</th>
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The values are rounded to the nearest tenth for clarity.
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FORTRAN Source Code.

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PROGRAM TB2(INPUT, OUTPUT, DATA,
TAPE5=INPUT, TAPE6=OUTPUT, TAPE9=DATA)

*************** PROGRAM TB2 ***************

THIS PROGRAM INTEGRATES THE SYSTEM OF DIFFERENTIAL EQUATIONS FOR MODE AMPLITUDES USING THE COEFFICIENTS COMPUTED BY THE PROGRAM TB1. TIME-HISTORY OF A PRESSURE DISTURBANCE IN THE T-BURNER IS COMPUTED AND THE DESIRED PLOTS AND PRINTOUTS ARE PRODUCED.

THE FOLLOWING INPUTS ARE REQUIRED:
(1) THE CONTROL NUMBERS, NOUTCF AND NHI STR.
(2) THE COEFFICIENTS FROM PROGRAM TB1.
(3) THE DATA DECK.

THE FIRST CARD GIVES THE CONTROL NUMBERS, NOUTCF AND NHI STR.
NOUTCF DETERMINES PRINTOUT OF COEFFICIENTS:
IF NOUTCF = 0 COEFFICIENTS ARE NOT PRINTED OUT.
IF NOUTCF = 1 ONLY LINEAR COEFFICIENTS ARE PRINTED OUT.
IF NOUTCF = 2 ALL COEFFICIENTS ARE PRINTED OUT.
NHI STR DETERMINES IF PRESSURE HISTORY IS TO BE PRINTED:
IF NHI STR = 0 PRINTED.
IF NHI STR = 1 NOT PRINTED.

THE COEFFICIENTS ARE OBTAINED FROM PROGRAM TB1 BY PUTTING NOUT = 1 OR NOUT = 2, THEREBY WRITING THE COEFFICIENTS INTO A DISK. THIS DISK HAS BEEN GIVEN THE DEVICE NUMBER 9.

THE DATA DECK CONSISTS OF THE FOLLOWING CARDS:

FIRST CARD: TITLE OF THE CASE.

SECOND CARD: H, TSTART, TQUIT, FREQ, BCOMB
H IS THE INTEGRATION STEP SIZE.
TSTART IS THE TIME AT WHICH OUTPUT STARTS.
QUIT IS THE TIME AT WHICH COMPUTATIONS ARE TERMINATED.
FREQ IS THE MOTOR FREQUENCY (IN PURE GAS), IN HERTZ.
BCOMB IS THE COMBUSTION RESPONSE NONLINEARITY FACTOR.

THIRD CARD: A2PARA, B2PARA, EN, OMEGA
A2PARA AND B2PARA ARE THE COMBUSTION PARAMETERS IN THE A-B MODEL.
EN IS THE PRESSURE EXPONENT IN THE BURNING RATE LAW.
OMEGA IS THE FREQUENCY NONDIMENSIONALIZED BY THE SQUARE OF THE STEADY-STATE BURNING RATE.

265
FOURTH CARD: NLOC, NTERMS, NOUT, NCOMB
NLOC DETERMINES THE LOCATION OF THE WALL PRESSURE MAXIMA
AND MINIMA:
    IF NLOC = 1  LOCATION IS Z = 0.0
    IF NLOC = 2  LOCATION IS Z = 1.0
    IF NLOC = 3  LOCATION IS Z = 0.5
NTERMS IS THE NUMBER OF TERMS GIVEN INITIAL VALUES.
NOUT IS THE OUTPUT CONTROL NUMBER.
    IF NOUT = 0  PRINTED OUTPUT ONLY.
    IF NOUT > 0  BOTH PRINTED AND PLOTTED OUTPUT.
    IF NOUT = 1  PLOT OF PRESSURE AT Z = 0.0 ONLY.
    IF NOUT = 2  PLOT OF PRESSURE AT Z = 0.0 AND Z = 1.0
    IF NOUT = 3  PLOT OF PRESSURE AT Z = 0.0, 1.0 AND 0.5.
NCOMB DETERMINES IF COMBUSTION NONLINEARITIES ARE CONSIDERED:
    IF NCOMB = 0  NEGLCETED.
    IF NCOMB = 1  INCLUDED.

NEXT CARD (NECESSARY ONLY IF PLOTS ARE REQUIRED): YHI, YLAB, ITICY
YHI IS THE MAXIMUM ORGANITE FOR PRESSURE PLOTS.
NOTE: THE ORGANITE SCALES FOR PRESSURE AND AMPLITUDE PLOTS
ARE SYMMETRIC ABOUT ZERO.
YLAB IS THE INTERVAL FOR ORGANITE LABELING FOR ABOVE PLOTS.
ITICY IS THE NUMBER OF ORGANITE TIC MARKS FOR ABOVE PLOTS.
NOTE: ITICY SHOULD BE NEGATIVE FOR PRESSURE AND AMPLITUDE
PLOTS TO OBTAIN CENTERLINE.

NEXT CARD (NECESSARY ONLY IF PLOTS OF INDIVIDUAL MODES ARE REQUIRED):
MDPLOT
MDPLOT DETERMINES IF PLOTS OF INDIVIDUAL MODES ARE REQUIRED:
    IF PLOT OF J TH MODE IS REQUIRED, PUNCH "1" IN THE
    5*J TH COLUMN.
    IF PLOT OF J TH MODE IS NOT REQUIRED, PUNCH "0" IN THE
    5*J TH COLUMN.

NEXT CARD (NECESSARY ONLY IF PLOT OF ANY MODE AMPLITUDE IS
REQUIRED): YHIMD, YLABMD, ITICMD
YHIMD IS THE MAXIMUM ORGANITE.
YLABMD IS THE INTERVAL FOR ORGANITE LABELLING.
ITICMD IS THE NUMBER OF ORGANITE TIC MARKS FOR MODE PLOTS.
NOTE: ITICMD SHOULD BE NEGATIVE TO OBTAIN CENTERLINE.

REMAINING CARDS (NTERMS IN NUMBER): J, AST, ACT
AST IS THE AMPLITUDE OF THE SINE TERM OF THE J TH MODE.
ACT IS THE AMPLITUDE OF THE COSINE TERM OF THE J TH MODE.

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1 YN(2), YNP(2), FZ(4,2), UN(5,2), UZN(2),
2 CFT(3,12), CFZ(3,12), AS(24),
3 AC(24), U(5,36), Y(36), PRESS(3),
4 YP(36), FZ(4,36), UZ(36),Z(3), TIMAX(500),
5 TPL0T(500), YPL0T(3,500), DUMMYT(500), DUMMYY(500),
6 IBUF(512), ITT(3), ITY1(3), ITY2(3), ITY3(4),
7 TITLE(7), PRI(500), TI(500), PMAX(500),
8 MDPLOT(6), UPL0T(6,500), MTITL1(2), MTITL2(2),
9 MTITL3(2), MTITL4(2), MTITL5(2), MTITL6(2), MTITL7(2)

COMMON C(2,12,24), D(12,144),
1 CV1(12), CV2(12), CV3(12), CV4(12),
2 KPMAX(3,12), IC(2,12,24), KPQMAX(12), IDP(12,144),
3 I DO(12,144), CPARK(12,24), UNBAR, EL, CP(4,12,24)

COMMON /BLK2/ B(6)
COMMON /BLK3/ NJMAX, NLMAX, GAMMA, COEF(2,12), NJMAX2
COMMON /BLK5/ RES, NCOMB, BCOMB
COMMON /BLK7/ BETA, BETAV, RRL, UB, PARTKL, CM, N

DATA ITT/"DIMENSION", "ESS TIME", "T"
1 I TY1/"HEAD END P", "RESSURE PE", "RTURBATION"/
2 I TY2/"NOZZLE PRE", "SSURE PERT", "UBURBATION /
3 I TY3/"PRESSURE P", "ERTURBATION", "N AT THE C", "ENTER /
4 MTITL1/"AMPLITUDE ", "OF 1L MODE"/
5 MTITL2/"AMPLITUDE ", "OF 2L MODE"/
6 MTITL3/"AMPLITUDE ", "OF 3L MODE"/
7 MTITL4/"AMPLITUDE ", "OF 4L MODE"/
8 MTITL5/"AMPLITUDE ", "OF 5L MODE"/
9 MTITL6/"AMPLITUDE ", "OF 6L MODE"/

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MAXMD4 = 24
MAXMD6 = 36
MAXMDD = 144
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READ (5,5003) NOUTCF, NHISTR

267
*************** COEFFICIENT INPUT SECTION ***********************

THIS VERSION OF TB2 READS THE COEFFICIENT DATA FROM
A FILE GENERATED BY PROGRAM TB1. TO READ
THIS DATA FROM CARDS, USE READ (5, XXXX) INSTEAD OF
READ (9, XXXX) IN THIS SECTION.

INPUT OF MOTOR PARAMETERS AND NUMBER OF TERMS:
READ (9, 5001) GAMMA, BETA, BETAV, RRL, UB, VL, EL,
1       NJMAX, NPRTKL, NBURN

JMX = NJMAX/2
NJMAX2 = NJMAX
NU = 2 * NJMAX
GAM = GAMMA
FRATIO = 1.0
AV = PI * BETAV * BETAV / 4.0
UNBAR = 2.0 * UB * AV * (NBURN + BETA/RRL)
IF (NPRTKL =EQ= 0) GO TO 14
READ (9, 5011) DI, Al, RHOM, SP, TEMP, FREQ, PARTKL, CM
GAM = GAMMA / (1.0 + SP*CM - SP*CM*GAMMA)
FRATIO = SQRT(GAMMA / (GAM * (1.0 + CM)))
NJMAX2 = 2*NJMAX
NU = NJMAX2 + NJMAX
14 CONTINUE

IF (NBURN =EQ= 0) WRITE (6, 6025)
IF (NBURN =EQ= 1) WRITE (6, 6026)
WRITE (6, 6001) BETA, BETAV, RRL, UB, VL, EL, JMX, GAM
IF (NPRTKL =EQ= 0) WRITE (6, 6033)
IF (NPRTKL =EQ= 1) WRITE (6, 6009) GAMMA
IF (NPRTKL =EQ= 1) WRITE (6, 6030) DI, CM, FREQ,
1       TEMP, SP, RHOM, PARTKL
WRITE (6, 6002)

INPUT OF DESCRIPTION OF SERIES EXPANSION:
DO 10 K = 1, JMX
READ (9, 5002) NJ, L(NJ), NAME(NJ)
WRITE (6, 6003) NAME(NJ), NJ, L(NJ)
10 CONTINUE

WRITE (6, 6010)
DO 15 K = 1, JMX
READ (9, 5010) J, B(J)
WRITE (6, 6015) J, B(J)
15 CONTINUE

ZERO LINEAR COEFFICIENT ARRAYS:
DO 20 KC = 1, 4
DO 20 NJ = 1, MAXMD2
DO 20 NF = 1, MAXMD4
CP(KC, NJ, NP) = 0.0
20 CONTINUE
ZERO NONLINEAR COEFFICIENT ARRAY.
DO 30 NJ = 1, MAXMD2
DO 30 NPQ = 1, MAXMD
D(NJ,NPQ) = 0.0
30 CONTINUE

INPUT OF LINEAR COEFFICIENTS.
DO 40 KC = 1, 4
READ (9,5003) KMAX
IF (NOUTCF .GT. 0) WRITE (6,6004) KC, KMAX
IF (KMAX .EQ. 0) GO TO 40
DO 45 K = 1, KMAX
READ (9,5004) NJ, NP, CP(KC,NJ,NP)
IF (NOUTCF .GT. 0) WRITE (6,6005) KC, NJ, NP, CP(KC,NJ,NP)
45 CONTINUE
40 CONTINUE

DO 305 K = 1, NJMAX
READ (9,5016) J, CPV(J)
305 CONTINUE

READ (9,5003) KMAXPR
IF (NOUTCF .GT. 0) WRITE (6,6036) KMAXPR
IF (KMAXPR .EQ. 0) GO TO 1040
DO 1042 K = 1, KMAXPR
READ (9,5004) NJ, NP, CPAR(NJ,NP)
IF (NOUTCF .GT. 0) WRITE (6,6037) NJ, NP, CPAR(NJ,NP)
1042 CONTINUE
1040 CONTINUE

INPUT OF NONLINEAR COEFFICIENTS.
READ (9,5003) NLMAX
IF (NOUTCF .EQ. 2) WRITE (6,6006) NLMAX
IF (NLMAX .EQ. 0) GO TO 50
DO 52 NJ = 1, MAXMD2
KPQMAX(NJ) = 0
52 CONTINUE
DO 55 K = 1, NLMAX
READ (9,5005) NJ, NP, NQ, DT
IF (NOUTCF .EQ. 2) WRITE (6,6007) NJ, NP, NQ, DT
KPQMAX(NJ) = KPQMAX(NJ) + 1
KPO = KPQMAX(NJ)
IDP(NJ,KPO) = NP
IDQ(NJ,KPO) = NQ
D(NJ,KPO) = DT
55 CONTINUE
50 CONTINUE
******* PRESSURE COEFFICIENT SECTION ***********

**CALCULATE SPATIAL COORDINATES FOR PRESSURE COMPUTATION.**

\[
\begin{align*}
Z(1) &= 0.0 \\
Z(2) &= ZE \\
Z(3) &= 0.5 \times ZE
\end{align*}
\]

**CALCULATE COEFFICIENTS FOR PRESSURE TIME HISTORIES**

```
DO 53 NPRES = 1, 3
  DO 53 J = 1, JMAX
    NP = (2 * J) - 1
    AXLOC = Z(NPRES)
    CALL PHI_CFS(J, AXLOC, C1, C3)
    CFT(NPRES, NP) = C1
    CFZ(NPRES, NP) = C3
    CFT(NPRES, NP+1) = CFZ(NPRES, NP+1) = 0.0
53 CONTINUE
```

**OUTPUT OF COEFFICIENTS FOR PRESSURE TIME HISTORIES**

```
WRITE (6, 6020)
DO 56 NPRES = 1, 3
  WRITE (6, 6014)
    DO 56 J = 1, NJMAX
      WRITE (6, 6021) J, Z(NPRES), CFT(NPRES, J), CFZ(NPRES, J)
56 CONTINUE
```

******* DATA INPUT SECTION ***********

**READ**

```
READ (5, 5000) TITLE
```

**ZERO INITIAL VALUE AND FREQUENCY ARRAYS.**

```
5 DO 57 K = 1, NJMAX2
    AS(K) = 0.0
    AC(K) = 0.0
    FRQ1(K) = 0.0
57 CONTINUE
```

**READ COMBUSTION AND CONTROL PARAMETERS.**

```
READ (5, 5006) HS, TSTART, TQUIT, FREQ, BCOMB
IF (EOF(5)) 300, 1
1 CONTINUE
```

```
READ (5, 5013) A2PARA, B2PARA, EN, OMEGA
WRITE (6, 6034) A2PARA, B2PARA, EN, OMEGA
```

```
DO 46 K = 1, JMX
  OMEGAK = OMEGA * K
  CALL RESPONSE(EN, A2PARA, B2PARA, OMEGA, CRES)
  RES(K) = CRES
  WRITE (6, 6035) K, RES(K)
46 CONTINUE
```
READ CONTROL NUMBERS.
READ (5, 5008) NLOC, NTERMS, NOUT, NCOMB
IF (NOUT .EQ. 0) NPT = 1
IF (NCOMB .EQ. 0) WRITE (6, 6039)
IF (NCOMB .EQ. 1) WRITE (6, 6040) BCOMB
WRITE (6, 6041)

IF (NOUT .EQ. 0) GO TO 9
READ DATA FOR SETTING UP PLOTS.
READ (5, 5009) YHI, YLAB, ITICY
READ (5, 5014) MDPLOT
MDPLTL = 0
DO 320 K = 1, JMX
MDPLTL = MDPLTL + MDPLOT(K)
IF (MDPLTL .EQ. 0) GO TO 9
READ (5, 5015) YHIMD, YLABMD, ITICMD
YLOMD = -YHIMD

************ INITIAL AMPLITUDES SECTION ************

9 DO 54 K = 1, NTERMS
READ (5, 5007) J, AST, ACT
NJ = (2 * J) - 1
AS(NJ) = AST
AC(NJ) = ACT

CALCULATE FREQUENCY.
AX = L(J) * PI * FRATIO/ZE
FRQ1(NJ) = AX
FRQ1(NJ+1) = FRQ1(NJ)
583 AC(NJ+1) = -AS(NJ)
AS(NJ+1) = AC(NJ)

58 CONTINUE
IF (NPRTKL .EQ. 0) GO TO 54
AS(NJ+1+NJMAX) = AS(NJ)
AC(NJ+1+NJMAX) = AC(NJ)
AS(NJ+1+NJMAX) = AS(NJ+1)
AC(NJ+1+NJMAX) = AC(NJ+1)
54 CONTINUE

OUTPUT OF INITIAL AMPLITUDES.
WRITE (6, 6016)
DO 590 J = 1, NJMAX
IF (AS(J)) 591, 592, 591
591 WRITE (6, 6017) J, FRQ1(J), AC(J), AS(J)
592 WRITE (6, 6016)
590 CONTINUE
IF (NOUT .GE. 1) WRITE (6, 6027)
************* LINEAR COEFFICIENTS SECTION ***********************

DO 59 KC = 1, 3
DO 59 NJ = 1, MAXMD2
KPMAX(KC,NJ) = 0
59 CONTINUE

DO 315 KC = 1, 2
DO 315 NJ = 1, MAXMD2
DO 315 NP = 1, MAXMD4
C(KC,NJ,NP) = 0.0
315 CONTINUE

COMPUTE LINEAR COEFFICIENTS FOR GIVEN VALUES OF HC AND RESPONSE FUNCTION.

605 DO 60 NJ = 1, NJMAX
CV1(NJ) = CV2(NJ) = 0.0
IF (NJ .NE. (NJ/2)*2) CV1(NJ) = CPV(NJ) * HC
IF (NJ .E61. (NJ/2)*2) CV2(NJ) = CPV(NJ) * HC
CV3(NJ) = (GAMMA*BETAV) / (EL*AV*HC) * CV1(NJ)
CV4(NJ) = (GAMMA*BETAV) / (EL*AV*HC) * CV2(NJ)
DO 60 NP = 1, NJMAX2
CT = CP(1,NJ,NP)
IF (CT) 61, 62, 61
61 KPMAX(1,NJ) = KPMAX(1,NJ) + 1
KP = KPMAX(1,NJ)
IC(1,NJ,KP) = NF
C(1,NJ,KP) = CT
62 CONTINUE
IF (NP .GT. NJMAX .OR. NJ GT. NJMAX) GO TO 316
NP12 = (NP+1)/2
RESR = REAL (RES(NP12))
RESI = AIMAG (RES(NP12))
CT = CP(2,NJ,NP) + HC*RESR*CP(3,NJ,NP) + HC*RESI*CP(4,NJ,NP)
60 CONTINUE

************* INITIAL VALUES SECTION **********************************

NSTEP = 0
NP1 = 3
H6 = H/6
TIME = 0.0
I = NP1
TI(I) = TIME

272
DO 75 J = 1, NJMAX2
JP = J + NJMAX2
IF (AC(J)) 751, 753, 751
753 IF (AS(J)) 751, 752, 751
752 U(I,J) = 0.0
IF (JP GT NU) GO TO 75
U(I,JP) = 0.0
GO TO 75
751 ARG = FRQ1(J) * TIME
FSIN = SIN(ARG)
FCOS = COS(ARG)
U(I,J) = AS(J)*FSIN + AC(J)*FCOS
IF (JP GT NU) GO TO 75
U(I,JP) = ((AS(J) * FCOS) - (AC(J) * FSIN)) * FRQ1(J)
75 CONTINUE
C CALCULATE INITIAL VALUES OF PRESSURE AND VELOCITY:
DO 704 NPRES = 1, 3
DO 702 J = 1, NJMAX
COEF(1,J) = CFTC(NPRES,J)
COEF(2,J) = CFZ(NPRES,J)
702 CONTINUE
DO 703 J = 1, NU
Y(J) = U(I,J)
703 CONTINUE
AXLOC = Z(NPRES)
CALL PRSVEL(AXLOC, U, Y, P, VZGAS, VZPAR)
PRESS(NPRES) = P
704 CONTINUE
PRS(I) = PRESS(NLOC)
70 CONTINUE
C
IF (NHI STR *EQ* 0) WRITE (6,6022)
C
C ************* INITIALIZE CONTROL NUMBERS **********************
C
LINE = 8
K = 0
MAXNO = 0
MAXP = 0
IF (NOUT *EQ* 0) GO TO 100
JPLT = 0
TMIN = TSTART
TMAX = TSTART + TDEL
YLO = -YHI
C
C ************* NUMERICAL CALCULATIONS SECTION **********************
C
100 I = NP1
RUNGE-KUTTA INTEGRATION SCHEME.

105 NSTEP = I - NP1 + (LAST - NP1) * K
RSTEP = NSTEP
TIME = RSTEP * H
TI(I) = TIME
DO 120 J = 1, NU
  Y(J) = U(I,J)
120 CONTINUE

YN(1) = UN(I,1)
YN(2) = UN(I,2)
CALL RHS(Y,YP,YN,YNP)
DO 130 J = 1, NU
  FZ(I,J) = YP(J)
130 CONTINUE

YNP(1) = UN(I,1)
YNP(2) = UN(I,2)
CALL RHS(Y,YP,YN,YNP)
DO 140 J = 1, NU
  KZ(1,J) = YP(J)
140 CONTINUE

FZN(1,1) = YNP(1)
FZN(1,2) = YNP(2)
DO 144 J = 1, NU
  UZ(J) = Y(J) + AA(I) * H * FZ(I,J)
144 CONTINUE

UZ(1) = YN(1) + AA(I) * H * FZ(I,1)
UZ(2) = YN(2) + AA(I) * H * FZ(I,2)
CALL RHS(UZ,YF,UZN,YNP)
DO 148 J = 1, NU
  FZ(I,J) = YP(J)
148 CONTINUE

FZN(1,1) = YNP(1)
FZN(1,2) = YNP(2)
DO 150 J = 1, NU
  U(I+1,J) = Y(J) + (FZ(1,J)+2.0*(FZ(2,J)+FZ(3,J)) + FZ(4,J)) * H6
150 CONTINUE

UN(1+1,1) = YN(1) + (FZN(1,1) + 2.0*(FZN(2,1)+FZN(3,1)) + FZ(4,1)) * H6
UN(1+1,2) = YN(2) + (FZN(1,2) + 2.0*(FZN(2,2)+FZN(3,2)) + FZ(4,2)) * H6

C CALCULATE PRESSURE TIME HISTORIES.

DO 154 NPRES = 1, 3
  DO 152 J = 1, NJMAX
    COEF(1,J) = CIF(NPRES,J)
  152 CONTINUE

AXLOC = Z(NPRES)
CALL PRSVEL(AXLOC,YL,YL,P,VZGAS,VZPAR)
PRESS(NPRES) = P

C

154 CONTINUE

PRS(I) = PRESS(NLOC)
IF (K.EQ.0) GO TO 175

C
DETERMINE MAXIMUM AND MINIMUM PRESSURE AT LOCATION SPECIFIED

BY NLOC:

DPL = PRS(I) - PRS(I-1)
DPS = PRS(I-1) - PRS(I-2)
IF (DPL*DPS) > 173, 173, 175

173 PNUM = PRS(I-2) - PRS(I)
PDEN = 2.0 * (PRS(I-2) + PRS(I) - 2.0*PRS(I-1))
IF (PDEN) > 174, 175, 174

174 PP = PNUM/PDEN
PA = (PP - 1.0) * PP * 0.5
PB = 1.0 - (PP * PP)
PC = (PP + 1.0) * PP * 0.5
MAXP = MAXP + 1
PMAX(MAXP) = PA*PRS(I-2) + PB*PRS(I-1) + PC*PRS(I)
TIMAX(MAXP) = TI(I-1) + PP*H
IF (MAXP > 500) GO TO 250

175 CONTINUE

IF (TIME < TSTART) GO TO 155
IF ((NOUT EQ. 0) OR (NOUT > 6)) GO TO 156

*************** TIME HISTORY PLOTTING SECTION ***************

IF (TMAX > TQUIT) GO TO 156
IF ((TIME = TMAX) OR (JPLT > 500)) GO TO 1000

JPLT = JPLT + 1

FILL TIME ARRAY FOR PLOTTING:
TPTL(JPLT) = TIME

FILL PRESSURE ARRAYS FOR PLOTTING:
DO 1001 J = 1, 3
YPLOT(J, JPLT) = PRESS(J)
1001 CONTINUE

IF (MDPLTL = EQ. 0) GO TO 156

FILL MODE AMPLITUDE ARRAYS FOR PLOTTING:
DO 322 J = 1, JMX
IF (MDPLOT(J) = EQ. 0) GO TO 322
J12 = 2*J - 1
UPLOT(J, JPLT) = U(I, J12)
322 CONTINUE

GO TO 156

1000 NUM = JPLT
PLOT TIME HISTORIES.

DO 1020 NPLOT = 1, NOUT

JPLLOT = 0

ASSIGN PLOTTING PARAMETERS.
YMIN = YLO
YMAX = YHI
NTICY = ITICY
DELY = YLAB

ELIMINATE POINTS THAT ARE OUT OF THE ORDINATE RANGE.
DO 1010 J = 1, NUM
IF (((YPLOT(NPLOT,J) ,LT, YMIN) ,OR, (YPLOT(NPLOT,J) ,GT, YMAX))
 1  GO TO 1010
JPLLOT = JPLLOT + 1
DUMMY T(JPLLOT) = TPLLOT(J)
DUMMYY(JPLLOT) = YPLOT(NPLOT,J)
1010 CONTINUE

IF (JPLLOT .EQ. 0) GO TO 1020
GO TO (1011, 1014, 1015)* NPLOT

PLOT HEAD-END PRESSURE.
1011 CALL GRAPHS(IBUF, 512, 4, JPLLOT, 11, NTICY, TMAX, YMAX, TMIN, YMIN,
 1  I TT, I TY 1, 21, 30, DUMMY T, DUMMY Y, 2.0, DELY, TI TLE)
GO TO 1020

PLOT NOZZLE PRESSURE.
1014 CALL GRAPHS(IBUF, 512, 4, JPLLOT, 11, NTICY, TMAX, YMAX, TMIN, YMIN,
 1  I TT, I TY 2, 21, 28, DUMMY T, DUMMY Y, 2.0, DELY, TI TLE)
GO TO 1020

PLOT PRESSURE AT THE CENTER (X = 0.5).
1015 CALL GRAPHS(IBUF, 512, 4, JPLLOT, 11, NTICY, TMAX, YMAX, TMIN, YMIN,
 1  I TT, I TY 3, 21, 35, DUMMY T, DUMMY Y, 2.0, DELY, TI TLE)

1020 CONTINUE

DO 324 NPLOT = 1, JMX
IF (MDPLOT(NPLOT) .EQ. 0) GO TO 324
JPLLOT = 0
DO 328 J123 = 1, 2
IF (NPLOT .EQ. 1) MTITL(J123) = MTI TL1(J123)
IF (NPLOT .EQ. 2) MTITL(J123) = MTI TL2(J123)
IF (NPLOT .EQ. 3) MTITL(J123) = MTI TL3(J123)
IF (NPLOT .EQ. 4) MTITL(J123) = MTI TL4(J123)
IF (NPLOT .EQ. 5) MTITL(J123) = MTI TL5(J123)
IF (NPLOT .EQ. 6) MTITL(J123) = MTI TL6(J123)
328 CONTINUE
DO 326 J = 1, NUM
   IF ((UPLOT(NPLOT,J) .LT. YLOMD) .OR. (UPLOT(NPLOT,J) .GT. YHIMD)) GO TO 326
   JPLT = JPLT + 1
   DUMMYT(JPLT) = TPLT(J)
   DUMMYY(JPLT) = UPLT(NPLOT,J)
326 CONTINUE
   IF (JPLT .EQ. 0) GO TO 324
   CALL GRAPHS(IBUF, 512, JPLT, 11, ITCMD, TMAX, YHIMD, TMIN,
        YLOMD, ITT, MTI TL, 21, 20, DUMMYT, DUMMYY, 20, YLABMD, TITLE)
324 CONTINUE
C
C REASSIGN PLOTTING PARAMETERS FOR NEXT SET OF PLOTS.
   JPLT = 0
   TMIN = TMAX
   TMAX = TMAX + TDEL
C
C ************* TIME HISTORY PRINTED OUTPUT SECTION *************
C
156 IF (NHISTR .EQ. 0)
    1 WRITE (6, 6011) NSTEP, TIME, (PRESS(J), J = 1, 3), VZGAS, VZPAR
    LINE = LINE + 1
157 IF (TIME .GT. TQUIT) GO TO 250
   IF (LINE .LT. 52) GO TO 155
   IF (NHISTR .EQ. 0) WRITE (6, 6013)
   IF (NHISTR .EQ. 0) WRITE (6, 6022)
   LINE = 4
155 I = I + 1
   IF (I .LT. LAST) GO TO 105
   K = K + 1
C
C RE-ASSIGN ARRAYS.
190 DO 200 I = 1, NP1
   ILAST = LAST - NP1 + I
   PRS(I) = PRS(ILAST)
   TI(I) = TI(ILAST)
200 CONTINUE
   DO 200 J = 1, NU
   U(I,J) = U(ILAST,J)
200 CONTINUE
   GO TO 100
C
C ************* PRESSURE MAXIMA AND MINIMA PRINTOUT *************
C
250 WRITE (6, 6023) Z(NLOC), MAXP
   LINE = 4
   DO 255 JST = 1, MAXP, 8
   JSTART = JST
   JSTOP = JST + 7
   IF (JSTOP .GT. MAXP) JSTOP = MAXP
   WRITE (6, 6024) (P[MAXP), J = JSTART, JSTOP)
   WRITE (6, 6024) (TIMAX(J), J = JSTART, JSTOP)
WRITE (6, 6014)
LINE = LINE + 3
IF (LINE LT 52) GO TO 255
LINE = 0
WRITE (6, 6013)
255 CONTINUE
CALL GROWTH(MAXP, TIMAX, PMAX, FREQ)
C
GO TO 5
300 CONTINUE
C
C TURN OFF PLOTTING ROUTINE.
IF (NPT EQ 1) CALL PLOT(0, 0, 0, 0, 0, 999)
C
C ************* READ FORMAT SPECIFICATIONS ***********************
C
5000 FORMAT (7A10)
5001 FORMAT (7F10.3, 315)
5002 FORMAT (215, 1X, A4)
5003 FORMAT (215)
5004 FORMAT (215, F15.8)
5005 FORMAT (315, F15.8)
5006 FORMAT (7F10.0)
5007 FORMAT (I5, 2F10.0)
5008 FORMAT (6I5)
5009 FORMAT (2F10.0, I5)
5010 FORMAT (I5, 2F12.8)
5011 FORMAT (7F15.8)
5012 FORMAT (I5, 2F10.0)
5013 FORMAT (4F10.0)
5014 FORMAT (6I5)
5015 FORMAT (2F10.0, I5)
5016 FORMAT (I5, F12.8)
C
C ************* WRITE FORMAT SPECIFICATIONS ***********************
C
6000 FORMAT (6X, 6HBETA =, F7.5, //, 6X, 7HBETAV =, F7.5, //,
1 6X, 5HR/L =, F7.5, //, 6X, 4HUB =, F7.5, //, 6X, 4HVL =,
2 F4.1, //, 6X, 4HEL =, F8.5, //, 6X, 17HNUMBER OF MODES =,
3 I2, //, 6X, 7HGamma =, F5.2)
6001 FORMAT (6X, 14HNAME J L/) 278
6013 FORMAT (1H1)
6014 FORMAT (1H )
6015 FORMAT (2X, I5, 4X, F10. 5)
6016 FORMAT (1H1, 36H INITIAL CONDITIONS ARE OF THE FORM: //
  1 2X, 47HUC(I,J) = AC(J)*COS(FREQ*T) + AS(J)*SIN(FREQ*T),
  2 ///6X, 1HJ, 6X, 9HFREQUENCY, 10X, 5HAC(J), 10X, 5HAS(J) ///)
6017 FORMAT (2X, I5, 4F15.8/ )
6020 FORMAT (1H1, 46H COEFFICIENTS FOR COMPUTATION OF WALL PRESSURE//
  1 10H WAVEFORMS///43X, 27HCOEFFICIENTS IN SERIES FOR: //
  2 37X, 4HTIME, 21X, 5SHAI XL/6X, 1HJ, 9X, 1HZ, 17X, 10HDERI VATIVE,
  3 15X, 10HDERI VATIVE///)
6021 FORMAT (2X, I5, F10.3, 12X, F15.7 , 10X, F15.7)
6022 FORMAT (3X, 4HSTEP, 8X, 4HTIME, 15X, 8HPRESSURE, 14X, 8HPRESSURE, 14X,
  1 8HPRESSURE, 14X, 7HGAS VEL, 15X, 7HPAR VEL, / 34X,
  2 8HAT Z=0.0, 14X, 8HAT Z=0.5, 14X, 8HAT Z=0.5///)
6023 FORMAT (1H1, 38H PRESSURE MAXIMA AND MINIMA AT: Z =, F5.2,
  1 /19H VALUES COMPUTED: I ///)
6024 FORMAT (1H, 7X, 8F13.6)
6025 FORMAT (1H1// 6X, 15NO END BURNING,///)
6026 FORMAT (1H1// 6X, 23END BURNING IS PRESENT,///)
6027 FORMAT (2X//2X, 33THI S RUN PRODUCES PLOTTED OUTPUT.)
6030 FORMAT (///, 6X, 27HPARTICLE DIA (IN MICRONS) =, F5.2, 10X,
  1 4HCM =, F4.2, 10X, 18HFREQ (IN Hertz) =, F6.1///,
  2 6X, 26HCHAMBER TEMP (IN DEG K) =, F6.1, 10X, 4HSP =,
  3 F4.2, 10X, 27HRHOM (IN KG/CUBIC METER) =, F6.1///, 6X,
  4 27HPARTICLE DIA DRAG CONSTANT, K =, F8.4, ///)
6032 FORMAT (2X, 8HE, I2, 1H, I2, 1H, I2, 4H) =, F10.5)
6033 FORMAT (//////, 6X, 26HPARTICLES ARE NOT PRESENT///)
6034 FORMAT (///, 3X, 22HCOMBUSTION PARAMETERS: 5X, 3HA =, F7.4, 6X,
  1 3HB =, F6.4, 5X, 4HEN =, F5.3, 5X, 7HOMEGA =, F6.3,
  2 //////, 25X, 1HJ, 16X, 4HRESR, 15X, 4HRESI ///)
6035 FORMAT (1H0, 20X, I5, 10X, F10.4, 9X, F10.4)
6036 FORMAT (1H0, 47HNUMBER OF COEFFICIENTS IN PARTICLE EQUATIONS IS,
  1 I5/)  
6037 FORMAT (2X, 8HEPAR(I2, I2, I2, 4H) =, F10.5)
6039 FORMAT (///, 3X, 27HLINEAR COMBUSTION RESPONSE///)
6040 FORMAT (///, 3X, 30HNONLINEAR COMBUSTION RESPONSE:
  1 5X, 8HBCOMB =, F6.3)
6041 FORMAT (//////, 3X, 24HLINEAR PARTICLE DAMPING///)
END
SUBROUTINE PHICFS(NP, Z, CT, CZ)

C THIS SUBROUTINE COMPUTES THE COEFFICIENTS NEEDED TO
C CALCULATE THE PRESSURE PERTURBATION.
C
NP IS THE INDEX OF THE COMPLEX SERIES TERM.
Z IS THE AXIAL LOCATION.
CT IS THE COEFFICIENT IN THE SERIES FOR THE TIME DERIVATIVE OF
THE VELOCITY POTENTIAL.

CZ IS THE COEFFICIENT IN THE SERIES FOR THE AXIAL DERIVATIVE
OF THE VELOCITY POTENTIAL.

COMMON /HLK2/ B(6)

CT = COS(B(NP)*Z)
CZ = - B(NP) * SIN(B(NP)*Z)

RETURN
END
SUBROUTINE FRUEL(AXLOC,VL,Y,P,VZGAS,VZPAR)
C
C THIS SUBROUTINE COMPUTES THE PRESSURE AND VELOCITY.
C
AXLOC IS THE AXIAL LOCATION IN THE BURNER WHERE PRESSURE AND
VELOCITY ARE TO BE DETERMINED.
Y IS THE ARRAY CONTAINING VALUES OF THE MODE-AMPLITUDE
FUNCTIONS AND THEIR DERIVATIVES.
P IS THE VALUE OF THE PRESSURE PERTURBATION.
VZGAS IS THE AXIAL COMPONENT OF GAS VELOCITY.
VZPAR IS THE AXIAL COMPONENT OF PARTICLE VELOCITY.
C
DIMENSION Y(36), SUM(5), SUMSQ(2)
COMMON /BLK3/ NJMAX, NLMAX, GAMMA, COEF(2,12), NJMAX2
COMMON /BLK7/ BETA, BETAV, RRL, UB, PARTKL, CM,
1 NPRTKL, NBU

BETAV2 = BETAV/2.0
CALL STEADY(AXLOC,UBAR,UPBAR,RHOP,DUBAR,DUFBAR)
DO 10 I = 1, 5
SUM(I) = 0.0
10 CONTINUE

DO 20 J = 1, NJMAX
JY = J + NJMAX2
20 SUM(1) = SUM(1) + Y(JY) * COEF(1,J)
DO 50 J = 1, NJMAX
SUM(2) = SUM(2) + Y(J) * COEF(2,J)
SUM(3) = SUM(3) + Y(J) * COEF(1,J)
IF (NPRTKL * EQ* 0) GO TO 50
JP = J + NJMAX
SUM(4) = SUM(4) + (Y(J)-Y(JP)) * COEF(1,J)
SUM(5) = SUM(5) + COEF(2,J) * Y(JP)
50 CONTINUE

PLIN = SUM(1) + UBAR * SUM(2) + DUBAR * SUM(3)
1 + PARTKL * RHOP * SUM(4)
IF (AXLOC * GT* 0.5-BETAV2 * AND* AXLOC * LT* 0.5+BETAV2)
1 PLIN = PLIN - VL * DUBAR * SUM(3)
PNL = 0.0
IF (NLMAX * EQ. 0) GO TO 40
DO 30 I = 1, 2
SUMSQ(I) = SUM(I) * SUM(I)
30 CONTINUE
PNL = 0.5 * (SUMSQ(2) - SUMSQ(1))

40 P = -GAMMA * (PLIN + PNL)
VZGAS = SUM(2)
VZPAR = SUM(5)
C
RETURN
END
SUBROUTINE RHS(U, UP, UN, UNF)

COMPLEX RES(6), RESNL(6)
DIMENSION U(36), UP(36), UN(2), UNP(2)
COMMON C(2,12,24), D(12,144),
1 CV1(12), CV2(12), CV3(12), CV4(12),
2 KFMAX(3,12), IC(2,12,24), KPMAX(12), IDP(12,144),
3 IDQ(12,144), CPAR(12,24), UNBAR, EL, CP(4,12,24)
COMMON /BLK3/ NJMAX, NLMAX, GAMMA, COEF(2,12), NJMAX2
COMMON /BLK5/ RES, NCOMB, BCOMB
COMMON /BLK7/ BETA, BETAV, RHL, UB, PARTKL, CM,
1 NPRTKL, NBURN

IF (NPRTKL * EQ. 0) GO TO 110
NJS = NJMAX + 1
DO 112 NJ = NJS, NJMAX2
NJPAR = NJ - NJMAX
SLP = 0.0
DO 114 KP = 1, NJMAX2
SLP = SLP + (CPAR(NJPAR,KP) * U(KP))
114 CONTINUE
UP(NJ) = - SLP
112 CONTINUE
110 CONTINUE
IF (NCOMB * EQ. 0) GO TO 116
JMX = NJMAX/2
DO 118 NJ = 1, JMX
NJPLNJ = 2*NJ
NJ2MN1 = NJPLNJ - 1
RESNL(NJ) = RES(NJ) * BCOMB * SQRT (UP(NJ2MN1)**2 + UP(NJPLNJ)**2)
118 CONTINUE
116 CONTINUE
DO 10 NJ = 1, NJMAX
NJP = NJ + NJMAX2
UP(NJ) = U(NJP)
SL1 = 0.0
SL2 = 0.0
SLV = 0.0
SNL1 = 0.0
SNLC = 0.0
MAX = KFMAX(1,NJ)
IF (MAX * EQ. 0) GO TO 25
DO 20 KP = 1, MAX
NP = IC(1,NJ,KP)
SL1 = SL1 + (C(1,NJ,KP) * U(NP))
20 CONTINUE
25 MAX = KFMAX(2,NJ)
IF (MAX * EQ. 0) GO TO 45
DO 30 KP = 1, MAX
NP = IC(2,NJ,KP)
SL2 = SL2 + (C(2,NJ,KP) * UP(NP))
30 CONTINUE
I F (NLMAX .EQ. 0) GO TO 55
MAX = KPGMAX(NJ)
I F (MAX .EQ. 0) GO TO 55
D0 50 KPO = 1, MAX
NP = I DP(NJ,KPO)
NQP = I D0(NJ,KPO) + NJMAX2
SNL1 = SNL1 + (D(NJ,KPO) * U(NP) * U(NQP))
50 CONTINUE
55 CONTINUE
SLV = CV1(NJ) * UN(1) + CV2(NJ) * UN(2)
I F (NCOMB .EQ. 0) GO TO 65
D0 60 KP = 1, NJMAX
KP12 = (KP+1)/2
SNLC = SNLC + (REAL(RESNL(KP12)) * CF(3,NJ,KP) + 1
AIMAG(RESNL(KP12)) * CP(4,NJ,KP)) * UP(KP)
60 CONTINUE
65 UP(NJP) = -(SL1 + SL2 + SLV + SNL1 + SNLC)
10 CONTINUE
C
SLU = 0.0
D0 220 KP = 1, NJMAX
220 SLU = SLU + CV3(KP) * U(KP)
UNP(1) = -(UNBAR/EL) * UN(1) + SLU
SLU = 0.0
D0 230 KP = 1, NJMAX
230 SLU = SLU + CV4(KP) * U(KP)
UNP(2) = -(UNBAR/EL) * UN(2) + SLU
RETURN
END
5. "EXACT" PROGRAM FOR MOTORS

Program KZM calculates "exact" instability solutions for solid rocket motors with full-length tubular propellant grains by numerically integrating the conservation equations for a gas-particle mixture (i.e., Equations (85) through (90)). The main purpose of this program is to generate "exact" solutions for comparison with the approximate solutions obtained with SOLID2 or MA2. For this reason, program KZM has the following characteristics: (1) it is restricted to motors with quasi-steady nozzles; (2) a linear combustion option is included in addition to the basic nonlinear pressure-coupled combustion response; and (3) a linear particle drag (Stokes Law) option is included in addition to the nonlinear particle drag law. The theoretical basis for KZM is given in Section 3 of Volume I of this report and in Reference 3.

5.1 Program Description of KZM.

Program KZM consists of a calling program (MAIN) and several subroutines. The main program reads two control parameters and calls subroutines INPUT and START which in turn call other subroutines.

Subroutine INPUT reads the parameters necessary to describe the solid rocket motor under consideration. These include motor length, diameter, and Mach number; chamber pressure and temperature; propellant and gas properties; particle properties; and reference quantities. Subroutine INPUT also prints out the values of the above quantities, sets up the initial thermal profiles in the solid propellant, calculates and prints out the combustion parameters A and B (for linear combustion option only), and returns to MAIN.

Subroutine START reads additional parameters (description of initial disturbance and number of integration time steps performed), calculates the steady-state properties for the motor (uses subroutine COMBNT), calculates the initial disturbance, prints out the steady-state and initial disturbance profiles (by calling subroutine OUTPUT), and calls subroutine LOGIC.

Subroutine LOGIC controls the integration of the conservation equations by calculating the time-step-size, calling subroutines BNDMOC and NOZMOC to calculate the boundary values at the head-end and nozzle-end respectively, and calling subroutine TAYLOR to calculate the dependent variables at the interior grid points using a Taylor series expansion (Reference 3). The burning rate at each grid point is calculated using subroutines NUTEMP (which calls THRMWV or LINTHW) and COMBNT. Subroutine LOGIC also calculates pressure maxima and minima, tests for shock development (using subroutine SHDETL), and produces printed
(subroutine OUTPUT) and plotted (subroutine GRAPHS) output. After these tasks are completed by LOGIC, control is returned to MAIN (via START) and the job is completed.

5.2 Description of Input.

The inputs are divided into four sections according to the subroutines in which the READ statements appear: MAIN, INPUT, START, and LOGIC. These input sections are described in sequence below, where the format is the same as given previously for the approximate programs (five columns for integers and ten columns for real numbers).

Inputs in MAIN. The MAIN program reads the first card of the data deck which gives the two integer control numbers described below:

\[ \text{NOPT} = \begin{cases} 
0 & \text{no burning response} \\
1 & \text{linear burning response} \\
2 & \text{nonlinear burning response} 
\end{cases} \]

\[ \text{NPRNT} = \begin{cases} 
0 & \text{print time histories and profiles} \\
1 & \text{time histories and profiles not printed} 
\end{cases} \]

Inputs in INPUT. Subroutine INPUT reads the geometrical motor parameters; steady-state parameters; propellant, gas, and particle properties; and reference quantities needed to describe the motor under consideration. This information is given on the nine cards described below:

Card 1:
HEADER    Alphanumeric heading

Card 2:
PIN    = initial chamber pressure (atmospheres)
TFSTR    = adiabatic flame temperature (deg R)

Card 3:
ELSTR    = chamber length (ft)
RCHSTR    = chamber radius (ft)
UT    = Mach number at nozzle entrance

Card 4:
PRSTR    = reference pressure in psi (normally 14.7)
TRSTR    = reference temperature in deg R (normally 540.0)
RGAS    = gas constant (ft-lbf)/(lbm-R)
GAM    = specific heat ratio
Card 5:
SIGMA = particle diameter (microns)
SMDPC = ratio of particle mass flux to gas mass flux
NLDG = 0 linear drag law
= 1 nonlinear drag law

Card 6:
RHSSTR = density of the solid in lbm/ft^3
CSTR = specific heat of the solid in BTU/(lbm-R)
SKSSTR = conductivity of the solid in BTU/(ft-sec-R) x 10^{-5}
AES = activation energy of surface reaction in cal/mole

Card 7:
QWSTR = endothermic heat release at the surface in BTU/lbm

Card 8:
CPSTR = specific heat of the gas in BTU/(lbm-R)
SKGSTR = conductivity of the gas in BTU/(ft-sec-R) x 10^{-5}
AEG = activation energy of the gas phase reaction in cal/mole

Card 9:
TSSTRR = reference temperature of surface in deg Rankine
SRS = surface regression rate in ft/sec

Note-- 0.0328 ft/sec = 1 cm/sec
PSURS = pressure (psi) at surface when TS = TSR and rate = SRS
CTSI = initial guess of surface temperature (dimensionless)

The reference quantities PRSTR and TRSTR are normally associated with standard atmospheric conditions (14.7 psi, 540 °R) which is the reference state used in nondimensionalizing the governing equations in the Kooker-Zinn analysis. The reference quantities TSSTRR, SRS, and PSURS are needed to define various constants in the equations describing the nonlinear transient combustion response; these must be obtained from experimental data.

Inputs in START. Subroutine START reads three additional control numbers, the initial disturbance amplitude, and the number of integration time-steps desired. These inputs are given on the three cards described below:

Card 1:
NSHOCK = 0 does not test for shocks
= 1 tests for shocks
ICTYPE = 1 Kookers continuous disturbance
= 2 initial first longitudinal mode

286
ISEN = 0 isothermal disturbance
= 1 isentropic disturbance
AMPL = half-amplitude of disturbance
Card 2:
NTIMES = number of integration time steps
Card 3:
HEADER = alphanumeric heading

In the description of ICTYPE, Kooker's continuous disturbance is a pulse-type perturbation with a maximum positive value at the head-end which smoothly declines to zero at about x = 1/3. For all of the cases considered in Volume I, the initial disturbance was an isentropic first longitudinal mode (ICTYPE = 2, ISEN = 1). Also the test for shocks was suppressed by choosing NSHOCK = 0.

Inputs in LOGIC. Subroutine LOGIC reads control parameters necessary for generating plotted output. These inputs are given on the four cards described below:

Card 1:
LPLØT = 0 no plots produced
= 1 plotted output produced
NLAST = number of axial locations plotted
Card 2:
YMAX = maximum ordinate for pressure plots
DELY = interval of ordinate labeling for pressure plots
NTICY = number of ordinate tick marks for pressure plots
Card 3:
TMAX = maximum abscissa for pressure plots
DELT = interval of abscissa labeling for pressure plots
NTICT = number of abscissa tick marks for pressure plots
Card 4:
TMAX4 = maximum abscissa for amplitude plot
DELT4 = interval of abscissa labeling for amplitude plot
NTICT4 = number of abscissa tick marks for amplitude plot

If LPLØT = 0 the last three cards are omitted. The pressure plots are produced for the head-end, mid-chamber, and nozzle-end in that order. The number of plots desired is given by NLAST; thus, if all three plots are desired NLAST = 3, while if only the first plot is desired NLAST = 1. The ordinate for the pressure plots
is specified in terms of the normalized pressure perturbation \( \frac{p'}{\bar{p}} \), and the corresponding abscissa is dimensionless time (using chamber sonic speed in normalizing); thus these units are used in specifying \( Y_{MAX} \), \( \text{DELY} \), \( T_{MAX} \), \( \text{DELT} \), \( T_{MAX4} \), and \( \text{DELT4} \). A centerline \( (p'/\bar{p} = 0) \) is provided if \( \text{NTICY} \) is specified as negative, and the ordinate range is \(-Y_{MAX} < p'/\bar{p} < Y_{MAX}\). For the pressure amplitude versus time plot, the ordinate is the logarithm (base 10) of the normalized pressure perturbation; thus the ordinate range, labeling, and tick marks are fixed for all cases and are not specified in the input.

5.3 Description of the Subroutines

A brief description of each of the subroutines used in program KZM is given in this section, except GRAPHS, MYAXIS, MYLINE, AXLAB, and DENDEC which are the same as in the approximate programs.

SUBROUTINE INPUT (N\text{OPT}, N\text{T}, T). The primary purpose of this subroutine is to read the motor parameters given above in the description of the input. After these parameters are read, several constants used in the linear and non-linear transient combustion models are evaluated. Next, a surface temperature \( TS \) is calculated which is compatible with the chamber pressure \( PIN \); this computation is necessary if \( PIN \) differs from the reference pressure for the propellant \( PSURS \). The solution for \( TS \) involves an iteration procedure using the subroutine ITSUB. Once the surface temperature is known, the initial temperature profiles and burning rates for the 11 burning stations are calculated using the steady-state equations. For linear combustion \( (N\text{OPT} = 1) \) the parameters \( A \) and \( B \) are calculated using Equations (97) and (98). This subroutine also prints out the input parameters, calculates and prints out the particle drag constants \( DGK(K) \) and \( DGKNL(K_{NL}) \), and prints out other calculated constants (see description of output).

SUBROUTINE START (N\text{OPT}, N\text{T}, T, N\text{PRNT}). This subroutine first calculates the steady-state values of the chamber properties at each of the 25 equally spaced grid points. The following properties are calculated based on a constant-pressure, linear-velocity profile: gas density \( R(M,KS) \), pressure \( P(M,KS) \), entropy \( S(M,KS) \), sonic speed \( SSP(M,KS) \), axial gas velocity \( U(M,KS) \), particle density \( RP(M,KS) \), and particle velocity \( UP(M,KS) \). Also the gas and particle mass fluxes at the burning propellant surface \( (SMDG(M,KS) \text{ and } SMDP(M,KS) \text{ respectively}) \) are calculated for each of the 25 grid points using subroutine COMBNT. In the above variables \( M \) is the index of the grid point and \( KS = 1 \) in this program.
After reading the parameters NSHOCK, ICTYPE, ISEN, AMPL, and NTIMES; the nozzle admittance (ADM), the mass balance (SMOUT and SMIN), and the frequency parameter (OMEGA) are calculated. The steady-state profiles calculated previously are printed out (by calling subroutine OUTPUT) along with ADM, SMOUT, SMIN, and OMEGA. Next the initial disturbance is calculated (based on the parameters ICTYPE, ISEN, and AMPL) and added to the steady-state solutions to obtain the initial values of the chamber properties which are then printed out (again using subroutine OUTPUT). Finally subroutine START calls subroutine LOGIC to perform the numerical solutions of the chamber conservation equations.

**SUBROUTINE OUTPUT (NT, T, KST, NPRNT).** This subroutine produces most of the printed output generated by KZM. Two types of output are produced: (1) values of the calculated chamber properties at each of the 25 grid points for certain values of the dimensionless time and (2) values of the chamber properties at frequent time intervals for head-end, mid-chamber, and nozzle-end locations. These are described in more detail in the next section. The input variables NT and T are the time step index (integer) and the dimensionless time, respectively, while KST = 1 in this program. This subroutine also stores data in plotting arrays (YPLOT) for later plotting by GRAPHS (called by LOGIC). The data to be printed or stored for plotting is transferred by COMMON blocks (VARG and PLTVAR).

**SUBROUTINE LOGIC (KST, NOPT, NPRNT).** Subroutine LOGIC controls the numerical integration of the chamber conservation equations. The following operations are performed at each integration time-step (by means of a DO loop):

1. the integration time step-size DT is calculated using the Courant-Friedrichs-Lewy stability condition (Reference 3);
2. a test for pressure maxima or minima is performed for NT > 3, if a maximum or minimum is found the peak-to-peak amplitude is calculated
3. for NSHOCK ≠ 0, a test for shock-wave development is performed (subroutine SHDETL), if a shock-wave is detected current values are printed out and the job is terminated;
4. boundary values are calculated at the head-end and nozzle end for the next time increment (subroutines BNDMOC and NOZMOC);
5. values of the chamber variables are calculated at the interior grid points for the next time increment using subroutine TAYLOR;
(6) new values of the gas and particle fluxes at the burning propellant surface are calculated by subroutines NUTEMP and COMBNT; and
(7) the calculated values of the chamber variables are printed out by subroutine OUTPUT.
The numerical integrations are terminated when NT = NTIMES and the plotted output is produced by subroutine GRAPHS.

**SUBROUTINE BNDMOC (KST, MBC).** Subroutine BNDMOC uses the method of characteristics to calculate the rigid wall boundary values at the head-end for a motor or at both ends for a closed-ended chamber (particles in a box). For MBC = -1 a left-hand boundary (head-end) is considered, while a right-hand boundary is considered for MBC = 1. In either case the rigid wall boundary conditions require that the gas and particle velocities are zero; these values are assigned and stored in UUD(KM,1) and UPUD(MM,1) respectively. The method of characteristics is then used to calculate the remaining variables: pressure, gas density, particle density, entropy, and sonic speed and stores them in PUD(MM,1), RUD(KM,1), RPUD(MM,1), SUD(MM,1), and SSPUD(KM,1) respectively. In these variables MM = 1 for a left-hand boundary and MM = 2 for a right-hand boundary. The boundary variables are transferred to subroutine LOGIC through COMMON block VARUPD.

**SUBROUTINE NOZMOC (KST).** This subroutine uses the method of characteristics to calculate the boundary values at the nozzle-end of the rocket motor. In these calculations the gas velocity and pressure at the nozzle entrance are related by the quasi-steady nozzle admittance condition (i.e., Equations (91) and (92)). The calculated values of the pressure, gas density, particle density, gas velocity, particle velocity, entropy, and sonic speed are stored in the arrays PUD(2,1), RUD(2,1), RPUD(2,1), UUD(2,1), UPUD(2,1), SUD(2,1), and SSPUD(2,1) respectively. These values are transferred to subroutine LOGIC through COMMON block VARUPD.

**SUBROUTINE TAYLOR (KS).** Subroutine TAYLOR calculates new values of the gas and particle densities, gas and particle velocities, and the entropy for all interior grid points by means of a Taylor series expansion in time. All spatial derivatives (first and second) needed in these calculations are determined using central differences. The pressure and sonic speed are calculated using the equation of state. The calculated values are returned to subroutine LOGIC through COMMON block VARG.

290
**SUBROUTINE NUTEMP (KST, NOPT, NT).** Subroutine NUTEMP controls the calculation of the gas and particle mass fluxes at the burning propellant surface. To save computation time, the burning rate calculations are performed at fewer locations (11) than the number of grid points (25) used in the chamber calculations. Thus NUTEMP first determines the pressure at each of the 11 burning stations from the previously calculated values at the 25 grid points by means of linear interpolation. These values are stored in APLOC(IBS) where IBS is the burning station index. If the linear combustion option is selected (NOPT=1), the pressure perturbation PDEL is calculated from APLOC(IBS) and subroutine LINTHW is called to compute the burning rate. For nonlinear combustion (NOPT=2), the burning rate is calculated from APLOC(IBS) using subroutine THRMWV.

**SUBROUTINE THRMWV (IBS, PLOC, TFIL, NT).** This subroutine determines the surface regression rate RGR(IBS) and the mass flux of gaseous combustion products FSMDG(IBS) at burning station IBS using the nonlinear Kooker-Zinn combustion model. For a given local pressure PLOC, the temperature profile in the solid is determined by solving the energy equation (i.e., Equation (3.20) of Reference 3) subject to the time-dependent heat transfer boundary condition (i.e., Equation (3.20b) of Reference 3) at the burning surface. Once the temperature profile is determined, the surface temperature is known and the regression rate and mass flow rate are obtained from the Arrhenius rate equation (Equations (3.21) and (3.22) of Reference 3).

The solution for the temperature profiles is accomplished by means of the method of invariant imbedding described in pp. 88-96 in Reference 3. The numerical solution is obtained using the algorithm given on pp. 95-96 of Reference 3, where the following nomenclature is used in THRMWV: \( \Theta(y) \) is denoted by CNU(J,IBS), \( v(y) \) by SV(J,IBS), \( w(y) \) by SW(J,IBS), and \( T(y) \) by CT(J,IBS). The time-dependent boundary condition at the propellant surface is satisfied by using an iterative procedure which calls subroutine ITSUB.

**SUBROUTINE LINTHW (IBS, PDEL).** This subroutine determines the surface regression rate RGR(IBS) and the mass flux FSMDG(IBS) using the linearized version of the Kooker-Zinn combustion model described in Section 3.2 of Volume I. In this case the temperature profiles in the solid propellant are obtained by using the method of invariant imbedding to solve Equations (93) through (96). The numerical calculations are performed using an algorithm similar to that used by THRMWV, and the nomenclature is the same as in THRMWV. The pressure PDEL which must be specified when LINTHW is called is the perturbation obtained
by subtracting the steady-state pressure form the local pressure.

**SUBROUTINE COMBNT (XND, KS, KST, HAFL, SMDGL, SMDPL).** Subroutine COMBNT calculates the gas mass flow rate SMDGL and the particle mass flow rate SMDPL emerging from the burning propellant surface at the axial location specified by XND. The value of SMDGL is obtained by linear interpolation from the previously calculated values of FSMDG(IBS) at the 11 burning stations. The corresponding value of SMDPL is determined by multiplying SMDGL by the particle loading SMDPC. The axial location XND is specified in terms of the dimensionless axial coordinate $\xi$ which is -1 at the head-end and zero at the nozzle end. The dimensionless adiabatic flame enthalpy HAFL is also calculated (in this program it is simply the constant value HAFC). The values of FSMDG, SMDPC, and HAFC needed for these calculations are obtained through COMMON blocks MSFSLD and COMFIX.

**SUBROUTINE MOCINT (Y1, Y2, Y3, DXN, YO).** This subroutine determines the value of the function YO at the intermediate location specified by DXN from the values of the function Y1, Y2, Y3 at three equally spaced grid points. This is done using the standard three-point interpolation formula, where DXN is measured from the left-most point (i.e., Y1) in terms of the grid-spacing DX. Subroutine MOCINT is called by subroutines BNDMOC and NOZMOC to obtain interpolated values of various chamber properties needed in the method of characteristics solutions.

**SUBROUTINE SHDETL (U1, U2, U3, U4, SHDET, NYES).** This subroutine tests for shock-wave formation using the values of the pressure at four grid points specified by U1, U2, U3, and U4. This calculation is performed using a polynomial fitting technique described on pp. 64-67 of Reference 3, where an infinite gradient indicates shock-wave formation. If a shock-wave is detected, SHDETL returns NYES = 1, otherwise NYES = 0. The value of the discriminant $\delta$ given by Equation (2.50) of Reference 3 is also calculated as SHDET.

**SUBROUTINE ITSUB(FOFY, Y, SAVE, CONV, NTIMES).** This subroutine obtains the solution of the equation $F(y) = 0$ by an iterative technique. Here FOFY is the value of the function $F$ corresponding to a guess of the solution $Y$, and SAVE is a one-dimensional array of eight locations. Beginning with an initial guess for $Y$, ITSUB changes $Y$ in subsequent iterations in such a manner as to drive FOFY to zero (within a specified tolerance). Thus the final value of $Y$ calculated is the desired root of the equation $F(y) = 0$. The maximum number of iterations permitted is specified by NTIMES. Furthermore SAVE(1) = 1 and SAVE(2) is the initial increment used in changing Y. Convergence to the desired root is
indicated by the value of SAVE(1) returned: $1 \leq \text{SAVE}(1) \leq 4$ indicates that further iterations are required; \text{SAVE}(1) = 5 indicates convergence to the required root, and \text{SAVE}(1) = 6 indicates that convergence did not occur within the specified number of iterations.

5.4 Description of Output

Program KZM produces printed output with an option to produce plotted output if desired.

Printed Output. The printed output is given in six major sections: (1) restatement of input parameters, (2) motor geometry and burning rates, (3) steady-state profiles, (4) initial disturbance profiles, (5) time histories of pressures, gas and particle velocities, and burning rates, and (6) profiles of chamber properties at selected time intervals.

Section (1), which occupies the first page, gives the following input parameters: (1) chamber pressure in atmospheres, (2) chamber length and radius in feet, (3) nozzle entrance Mach number, (4) surface heat release in BTU/lbm, (5) reference surface properties (temperature in °R, regression rate in ft/sec, and pressure in psi), and (6) particle properties. The last of these includes the calculated values of the linear and nonlinear drag constants.

In addition a number of calculated quantities are given. Most of these relate to the Kooker-Zinn transient combustion model (i.e., QFSTR, BIGK, YHC, BRAY, BR, WCÖN, WSTRR, EÖRGEÖRS, ETF, ACÖN, QMC, Z1H, Z2H, WSTRS, Z1, and Z2P) and are not of interest to the general user. The remaining quantities in this group are defined as follows: TF is the normalized flame temperature, TS is the normalized surface temperature, R is the dimensionless surface regression rate (used to calculate $\Omega$), DXSBS is the axial spacing between burning stations, ARSTR is the reference speed of sound in ft/sec, and RHRSTR is the reference density in lbm/ft$^3$. Finally the linear combustion parameters A and B are given if the linear combustion option (NOPT = 1) is selected.

Section (2) is given on the next two pages. The first of these describes the motor geometry and coordinate system, while the second gives the initial burning rates. These are tabulated according to the index M for each of the 25 grid points (M=1 at the head-end and M=25 at the nozzle-end). The variables tabulated on the first page are described as follows: CAP X is the dimensionless axial coordinate (i.e., X of Reference 3), XI is X-1, X STR is the axial location of the grid point in feet (measured from the nozzle-end of the chamber),
RWALL is the local chamber radius in feet, DRWDX is the axial derivative of RWALL, AREA is the chamber cross-sectional area in ft\(^2\), DLNADX is the logarithmic derivative of AREA, and RW2 is \(2L^*/R^*\) which is four times the length-to-diameter ratio. The second page gives the dimensionless mass burning rates of the gas, SMDG, and particles, SMDP, as well as the dimensionless chamber enthalpy (or temperature), HAF.

Section (3) gives the steady-state profiles in tabular form. For each of the 25 grid points (index M) the following variables are given: the axial coordinate (X STR), Mach number, pressure, density, axial velocity, entropy, enthalpy, local sound speed (A(LOC)), particle density, particle velocity, mass burning rate of gas (SMDG) and particles (SMDP), and time derivative of the gas burning rate (SMDDG). This is followed by a tabulation of the following properties at each of the 11 burning stations (index IBS): mass burning rate for gas (F SMDG), dimensionless regression rate (R), surface temperature, and local pressure. Finally the mass balance, dimensionless frequency parameter \(\Omega\) and the nozzle parameters are given. Here MASS IN is the total flux of burned gases leaving the propellant surface, and MASS OUT is the mass flux through the nozzle entrance plane; the steady-state solutions must be chosen such that these mass fluxes are equal. UBAR and PBAR are the axial velocity and pressure at the nozzle entrance plane, which are used to compute the quasi-steady nozzle admittance ADM. TCONV is the factor relating the dimensionless time used in the "exact" analysis (based on standard atmospheric conditions) to the dimensionless time used in the approximate analysis (based on chamber conditions); it is the ratio of sound speeds for these two reference states.

Section (4) gives the initial values of the chamber properties in the same format as Section (3). These values are the sum of the steady-state values given in Section (3) and the initial disturbance perturbations calculated in subroutine START.

Section (5) gives the time histories of the most important chamber properties in tabular form. The first column gives the index of the integration time step, where only even numbered steps are printed. The second column gives the dimensionless time, where the upper value is based on standard atmospheric conditions and the lower value is based on chamber stagnation conditions. The next three columns give the pressure at the head-end (P(1)), the mid-chamber (P(13)), and the nozzle-end (P(25)). The upper value is the total pressure (i.e., steady-state plus perturbation) in atmospheres, and the lower value is the pressure.
perturbation normalized by the steady-state pressure. The sixth column gives the dimensionless axial gas velocity (sum of steady-state and perturbation) at the nozzle-end ($U(25)$) and at the chamber midpoint ($U(13)$), while the Mach number at the nozzle entrance ($M(25)$) and the particle velocity at the midpoint ($UP(13)$) are given in the seventh column. The last column gives the burning rate perturbation normalized by the steady-state burning rate for both the head-end ($MG(1)$) and the nozzle-end ($MG(25)$).

Information about the head-end pressure maxima and minima is also given in Section (5). For each maximum or minimum the following information is given: the index of the maximum or minimum ($N_{P_{\text{MAX}}}$), the dimensionless time (based on chamber conditions) at which the extremum occurs ($T_{\text{MAX}}$), the corresponding value of the head-end pressure perturbation ($P_{\text{MAX}}$), the peak-to-peak pressure amplitude, the mean time for the half-cycle ($T$), and the mean pressure perturbation ($P_{\text{MEAN}}$). The last two quantities are arithmetic averages of the time and pressure at the current extremum and the immediately preceding extremum. If the printout of the time-histories is suppressed ($N_{P_{\text{RNT}}} = 1$), only the maxima and minima are printed in Section (5).

Finally Section (6) gives the profiles of the chamber properties at selected time intervals in the same format as Sections (3) and (4). These profiles are printed every 100 time-steps. For $N_{P_{\text{RNT}}} = 1$ these profiles are not printed.

Plotted Output. For $L_{P_{\text{LOT}}} = 1$ plotted output is produced using a Calcomp plotter. The format of the plots is the same as that described for the approximate programs (i.e., SOLID2, MA2 and TB2). The first three plots give the normalized pressure perturbation versus dimensionless time (chamber conditions) for the head-end, mid-chamber, and nozzle-end. These plots can be compared directly with similar plots produced by the approximate programs. The last plot produced gives peak-to-peak pressure amplitude versus dimensionless time, where the amplitude is plotted on a logarithmic scale.

Sample Case. A test case is given to illustrate the operation of Program KZM and facilitate check-out of the program. This case corresponds to the non-linear drag case ($N_{\text{LDG}} = 1$) shown in Figure 46 of Volume I: 20 micron particles at 36% loading. The linear combustion option is selected ($N_{\text{OPT}} = 1$) with $Q_{\text{WSTR}} = -250.9$ BTU/lbm ($Q^*_{\text{s}} = -139.4$ cal/gm) which corresponds to $A = 5.996$, $B = 0.550$, and $n = 0.552$. The gas phase and propellant properties (i.e., $T_{\text{FSTR}}$, $\text{GAM}$, $R_{\text{HSSTR}}$, $295$
CSTR, SKSSTR, AES, CPSTR, SKGSTR, and AEG) are taken from Table 1 of Volume I. The motor geometry considered in Volume I is also specified; the motor length (ELSTR) is 1.958 ft and the chamber radius (RCHSTR) is 0.099285 ft. The reference quantities used in non-dimensionalizing the various chamber properties are the standard atmospheric quantities (PRSTR = 14.7 psi and TRSTR = 540 °R), while the reference properties needed to characterize the propellant are TSSTTR = 1440 °R, SRS = 0.03827 ft/sec and PSURS = 1568 psi. The initial guess of the normalized surface temperature is based on the above value, thus CTS1 = 2.67. The initial disturbance is assumed to be a fundamental mode, isentropic disturbance (ICTYPE = 2, ISEN = 1) of 5% amplitude (AMPL = 0.05). The program is run for 400 integration time steps which gives printed output for dimensionless times (based on chamber conditions) of 0 ≤ t ≤ 12.3 and plotted output for 0 ≤ t ≤ 10.0.

The input for this case is shown in the proper format on the following page. This is followed by the resulting printed and plotted output.
TEST CASE FOR KZM

INITIAL CHAMBER PRESSURE = 106.7

CHAMBER LENGTH = 1.958
CHAMBER RADIUS = .099285
NOZZLE ENTRANCE MACH NO. = .07804

SURFACE HEAT RELEASE = -250.9

REF. SURFACE TEMPERATURE = 1440.0
REF. SURFACE REGRESSION RATE = .03627
REF. SURFACE PRESSURE = 1968.0

PARTICLE DIAMETER = 20.00 MICRONS
NO. L. DRAG CONSTANT = 2.969
SMOP/SMOG = .3600

OFSTR(8TJ/LBM) = .24143150E+04 BASEC ON (QWSTR,ICSSTR,CFSTR,CSR,TF,TSR) OF
-.25090000E+03 .54060000E+03 .43300000E+03 .32900000E+03 .11750000E+02 .26666667E+01
BIGC = .4427912E-09 THC = .21144556E-04 BRAY = .21659956E+05 BR = .26144818E+03 HCON = .28402314E+02
EDRG = .5032172E+02 EDRS = .25522956E+02 TF = .11750000E+02 TS = .26666667E+01
ETF = .13798993E+01 R = .14767069E+01 ACON = .16666667E+01 QMC = .41707685E+01
Z1H = -.14122481E+01 Z2PH = .12436448E+03 WSTR = .44620019E+04
Z2P = .30589246E+05 Z2P = .57417670E+08 DXSBS = .19580000E+00 ARSTR = .12235632E+04
RMRSTR = .56000000E-01
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ETF = .13798993E+01 R = .14767069E+01 ACON = .16666667E+01 QMC = .41707685E+01
Z1H = -.14122481E+01 Z2PH = .12436448E+03 WSTR = .44620019E+04
Z2P = .30589246E+05 Z2P = .57417670E+08 DXSBS = .19580000E+00 ARSTR = .12235632E+04
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- **Mass Out:** 5.1547

**Omega:** 4.9383

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**Notes:**
- The data is organized in a table format with columns for step number, temperature (T), pressure (P), mean gauge (M), NPMAX, TMAX, and PMAX.
- The table shows the progression of these values over different steps, with each step having its own set of values for temperature, pressure, and various other parameters.
- The NPMAX, TMAX, and PMAX values indicate the maximum values for each parameter across all steps.
- The PK-PK Amplitude and PMEAN values provide a measure of the amplitude and mean of the pressure fluctuations, respectively.
STEP (148) $T = 1.263276$ $P(1) = 105.785683$ $P(13) = 105.24673$ $P(25) = 105.220779$ $U(13) = .170714$ $U(13) = .110599$ $M(25) = .111919$ $M(25) = .111919$

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16 |  7.4242E+00  |  5.7921E+02  |  9.1209E+01  |  1.9867E+00  |  1.9556E+01  |  1.1754E+02  
17 |  6.5626E+00  |  6.0654E+02  |  9.1209E+01  |  2.0765E+00  |  1.9556E+01  |  1.1754E+02  
18 |  5.7193E+00  |  6.3256E+02  |  9.1209E+01  |  2.1615E+00  |  1.9556E+01  |  1.1754E+02  
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21 |  3.3633E+00  |  7.0783E+02  |  9.1209E+01  |  2.3959E+00  |  1.9556E+01  |  1.1754E+02  
22 |  2.4475E+00  |  7.1828E+02  |  9.1209E+01  |  2.4656E+00  |  1.9556E+01  |  1.1754E+02  
23 |  1.6316E+00  |  7.3942E+02  |  9.1209E+01  |  2.5362E+00  |  1.9556E+01  |  1.1754E+02  
24 |  8.1583E+00  |  7.5933E+02  |  9.1209E+01  |  2.6062E+00  |  1.9556E+01  |  1.1754E+02  
25 |  0.00  |  7.8827E+02  |  9.1209E+01  |  2.6762E+00  |  1.9556E+01  |  1.1754E+02  

INDEX OF DISCONTINUITY DIRECTION = 0 AND DISCONTINUITY LOCATION = 0.
<table>
<thead>
<tr>
<th>I0S</th>
<th>XSTPPS</th>
<th>F SMG</th>
<th>TEMP SLR</th>
<th>PRESS (LOG)</th>
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<tr>
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<td>-1.9503E+01</td>
<td>-6.0537E+01</td>
<td>1.4514E+01</td>
<td>2.66197E+01</td>
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<td>1.50414E+01</td>
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<td>0.00</td>
<td>-6.26507E+01</td>
<td>1.50729E+01</td>
<td>2.67154E+01</td>
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</tbody>
</table>

STEP(400) T = 3.603253 P(11) = 107.67745 P(13) = 107.39012 P(25) = 106.569353 U(25) = 0.267400 M(25) = 0.078384 MG(11) = 0.17082
12.351320 .002161 .006467 .00022 U(13) = 0.101914 UP(13) = 0.114296 MG(25) = 0.015816
MID-CHAMBER PRESSURE PERTURBATION
LOG HEAD-END PRESSURE AMPLITUDE (PK-PK)

-3.00
-2.00
-1.00
0

DIMENSIONLESS TIME, T
FORTRAN Source Code.

PROGRAM MAIN(INPUT, OUTPUT, TAPE5=INPUT, TAPE6=OUTPUT)

**** KZM VERSION 14JAN77 ****
**** INCLUDES PLOTTING ROUTINES ****
**** EXPLANATION OF INPUT ****

**** INPUTS IN MAIN PROGRAM ****
CARD 1
NOPT = 0 NO BURNING RESPONSE
      = 1 LINEAR BURNING RESPONSE
      = 2 NONLINEAR BURNING RESPONSE
NPRNT = 0 PRINT TIME HISTORIES AND PROFILES
       = 1 TIME HISTORIES AND PROFILES NOT PRINTED

**** INPUTS IN SUBROUTINE INPUT ****
CARD 1
HEADER ALPHANUMERIC HEADING
CARD 2
PIN = INITIAL CHAMBER PRESSURE (ATMOSPHERES)
TFSTR = ADIABATIC FLAME TEMPERATURE (DEG R)
CARD 3
ELSTR = CHAMBER LENGTH (FT)
RCHSTR = CHAMBER RADIUS (FT)
UT = MACH NUMBER AT NOZZLE ENTRANCE
CARD 4
PRSTR = REFERENCE PRESSURE IN PSI
TRSTR = REFERENCE TEMPERATURE IN DEG R
RGAS = GAS CONSTANT (FT-LBF)/(LBM-R)
GAM = SPECIFIC HEAT RATIO
CARD 5
SIGMA = PARTICLE DIAMETER (MICRONS)
SMDPC = RATIO OF PARTICLE MASS FLUX TO GAS MASS FLUX
NLDG = 0 LINEAR DRAG LAW
       = 1 NONLINEAR DRAG LAW
CARD 6
RHSSTR = RHO OF THE SOLID IN LBM/FT**3
CSTR = SPECIFIC HEAT OF THE SOLID IN BTU/(LBM-R)
SKSSTR = CONDUCTIVITY OF THE SOLID IN BTU/(FT-SEC-R)
AES = ACTIVATION ENERGY OF SURFACE REACTION IN CAL/MOLE
CARD 7
OWSTR = ENDOTHERMIC HEAT RELEASE AT THE SURFACE IN BTU/LBM
CARD 8
CPSTR = SPECIFIC HEAT OF THE GAS IN BTU/(LBM-R)
SKGSTR = CONDUCTIVITY OF THE GAS IN BTU/(FT-SEC-R)
AEG = ACTIVATION ENERGY OF THE GAS PHASE REACTION IN CAL/MOLE
CARD 9
TSSTR = REFERENCE TEMPERATURE OF SURFACE IN DEG RANKINE
SRS = SURFACE REGRESSION RATE IN FT/SEC
NOTE -- 3.28E-02 FT/SEC = 1 CM/SEC
PSURS = PRESSURE (PSI) AT SURFACE WHEN TS=TSR AND RATE = SRS
CTS1 = INITIAL GUESS OF SURFACE TEMPERATURE

321
**** INPUTS IN SUBROUTINE START ****
C
CARD 1
NSHOCK = 0 DOES NOT TEST FOR SHOCKS
NSHOCK = 1 TESTS FOR SHOCKS
ICTYPE = 1 HOOKER'S CONTINUOUS DISTURBANCE
ICTYPE = 2 INITIAL FIRST LONGITUDINAL MODE
ISEN = 0 ISOThermal DISTURBANCE
ISEN = 1 ISENTROPIC DISTURBANCE
AMPL = HALF-AMPLITUDE OF DISTURBANCE
CARD 2
NTIMES = NUMBER OF INTEGRATION TIME STEPS
CARD 3
HEADER = ALPHANUMERIC HEADING
**** INPUTS IN SUBROUTINE LOGIC ****
PLOTTING PARAMETERS
CARD 1
LPlot = 0 NO PLOTS PRODUCED
LPlot = 1 PLOTTED OUTPUT PRODUCED
NLAST = NUMBER OF AXIAL LOCATIONS PLOTTED
CARD 2
YMAX, DELY, NTICY
CARD 3
TMAX, DELT, NTICT
CARD 4
TMAX4, DELT4, NTICT4 (AMPLITUDE PLOT).

10 FORMAT(2I5)
READ(5,10) NOPT, NPRNT
CALL INPUT(NOPT, NT, T)
CALL START(NOPT, NT, NPRNT)
STOP
END

SUBROUTINE INPUT(NOPT, NT, T)
COMMON/HEAD/ HEADER(7), LPRINT, NTIMES, NSHOCK, JPlot, PIN
COMMON/FRXPR/TOTA(5), GM1, GM12, GM12G, G2, GM1, DGK, DGKNL
COMMON/TI MTRN/DT, DT22, DXA(5), XI SW(5), IDDA(5), ACM(5), ASMB(5),
1 1 ASMC(5)
COMMON/COMFIX/ HAF(200, 5), SMDG(200, 5), SMDP(200, 5), XIFC, HAFC, SMDG
1 , REACTN, SDPC
COMMON/NOZDAT/ELSTR, RCHSTR, UT
COMMON/MFSLD/ SMXS(100), APLOC(100), FMDG(100), DXSBS, NBS, AFC(100)
COMMON/TM WAVE/CT(101, 20), CNUX(101, 20), SW(101, 20), SV(101, 20), HRGR(20)
COMMON/BRCON/ DYA(3), NTY(3), ALFTA(3), TYLOC(50), YO, Z1, Z2P, WCON,
1 EORG, EORS, ETF, QMC, BRAY, JTOT, JTOTM1, ARSTR, RRHSR, Z1T, TS
DIMENSION SAVE(8)
1 FORMAT(7A10)
2 FORMAT(2X, I6, E14.8, I6)
5 FORMAT(3F10.0)
6 FORMAT(5F10.0)
7 FORMAT(2F10.0, I5)
8 FORMAT(/,,/,,30X,7A10)
9 FORMAT(/,,10X,9E13.5)
200 FORMAT (1H1)
201 FORMAT (6X, 7E16.8)
202 FORMAT (2X, 22H**********************)
203 FORMAT (/* I TSUB W.N.C. FOR THE INITIAL SURFACE TEMP(AFTER 150 I TE
1RATIONS) */,, 3E15.8)
204 FORMAT (* BI GK=*, E148,* YHC=*, E148,* BRAY=*, E148,* BR=*,
1E14+8,* WCON=*, E148,* WSTRR=*, E148)
205 FORMAT (* EORG=*, E148,* EORS=*, E148,* TF=*, E148,* TS=*,
1E148,* ETF=*, E148)
206 FORMAT (* Re=*, E148,* ACON=*, E148,* QMC=*, E148,* Z1H=*, E148
1*, Z2PH=*, E148,* WSTRR=*, E148)
207 FORMAT (* Z1=*, E148,* Z2F=*, E148,* DX SBS=*, E148,* ARSTR=*,
1E148,* RHRSTR=*, E148)
210 FORMAT (/* QFSTR(BTU/LBM)=*, E148,* BASED ON (QWSTR, TCSTR, CPST
1R, CSTR, TF, TSH) OF */,, 2X, 6E15.8)
211 FORMAT (/* INITIAL CHAMBER PRESSURE = *, F10.1/)
212 FORMAT (40X,*CHAMBER LENGTH = *, F10.3/)
1 40X,*CHAMBER RADIUS = *, F10.6/
2 30X,*NOZZLE ENTRANCE MACH NO. = *, F10.5/)
213 FORMAT (34X,*SURFACE HEAT RELEASE = *, F10.1/)
214 FORMAT (30X,*SURFACE TEMPERATURE = *, F10.1/)
1 26X,*SURFACE REGRESSION RATE = *, F10.5/
2 33X,*SURFACE PRESSURE = *, F10.1/)
215 FORMAT (2X/10X,*LINEAR COMBUSTION PARAMETERS*, 10X,
1 4HA = F7.3, 5X, 4HB = F7.3//)
216 FORMAT (37X,*PARTICLE DIAMETER = *, F10.2*, MICRONS//)
1 41X,*DRAG CONSTANT = *, F10.3/
2 36X,*N+L. DRAG CONSTANT = *, F10.3/
3 45X,*SMDP/SMDG = *, F10.4/)
217 FORMAT (1H1, 3X,*SMB = *, F9.5, 5X,*SMO = *, F9.5,
1 5X,*CMB = *, F9.5//)

C
READ(5,1)(HEADER(I), I=1,7)
WRITE(6,200)
WRITE(6,8)(HEADER(I), I=1,7)
READ(5,5) PIN, TFSTR
WRITE(6,211) PIN
PI=3.141592654
MTOTA(1)=25
SMDGC=0.100000000
READ (5, 5) ELSTRA, RCHSTRA, UT
WRITE (6, 212) ELSTRA, RCHSTRA, UT
DXA(1)=1.00/FLOAT(MTOTA(1)-1)
SMB=-1.0000000
SMC=0.0
CMB=SMB-SMB
KS=1
ASMC(KS)=SMC
ASMB(KS)=SMB
ACMB(KS)=CMB
READ (5, 6) PRSTRA, TRSTRA, RGAS, GAM
HAF=TFSTR/TRSTRA
ARSTRA=SQRT(GAM*RGAS*TRSTRA*32.2)
RHRSTRA=(PRSTRA*144.00)/(RGAS*TRSTRA)
READ (5,7) SIGMA, SMDPC, NLDG
SIG = SIGMA * 1.0E-06
RMU = (8.834E-05)*TFSTR/6273.0)**0.66
RHOM = 4000
DGK = 18.0*RMU*ELSTR/(RHOH*SIG*SIG*ARSTR)
IF (NLDG =EQ= 0) GO TO 10
DGKNL = 0.4797 * RHRSTR*ARSTRA SIG/RMU)**0.6667
GO TO 15
10 DGKNL = 0.0
C ********** UNSTABLE FORMULATION **********
C
C *** PROPELLANT CONSTANTS ***
15 READ (5,6) RHSSTR, CSTR, SKSSTR, AES
SKSSTR = SKSSTR * 1.0E05
C EORS= AES/(R*TCS) ASSUMING R=1.987 CAL/MOLE-K AND TCS=300-K
EORS=AES/(1.987*300.00)
C QWSTR= ENDOThERMIC HEAT RELEASE AT THE SURFACE IN BTU/LEM
READ (5,5) QWSTR
WRITE (6,213) QWSTR
C TCSSTR= TEMPERATURE OF THE COLD SOLID PROPELLANT IN DEG RANKINE
TCSSTR = TRSTR
C *** PROPERTIES OF THE GAS PHASE FLAME ***
READ (5,6) CPSTR, SKGSTR, AEG
SKGSTR = SKGSTR * 1.0E.05
C EORG= AEG/(R*TCS) ASSUMING R=1.987 CAL MOLE-K AND TCS=300-K
EORG=AEG/(1.987*300.00)
TF=TFSTR/TCSSTR
ETF=EXP( -EORG/TF)

C *** REFERENCE STATE AT WHICH CONSTANTS ARE EVALUATED ***
C
C TSSTRR= REFERENCE TEMPERATURE OF SURFACE IN DEG RANKINE
C ASSUME SURFACE REGRESSION RATE IS SRS IN FT/SEC
C NOTE-- 3.28E-02 FT/SEC = 1 CM/SEC
C PSURS=PRESSURE(PSI) AT SURFACE WHEN TS=TSR AND RATE = SRS
READ (5,6) TSSTRR, SRS, PSURS, CTS1
TSR=TSSTRR/CTSSTR
EXS=EXP(-EORS/TSR)
C NOTE-- SRS=BR*EXP(-EORS/TSR)
BR=SRS/EXS
PSUR=PSURS/PRSTR
WRITE (6,214) TSSTRR, SRS, PSURS
WRITE (6,216) SIGMA, DGK, DGKNL, SMDPC
C *** EVALUATION OF CONSTANTS ***
BIGK=SKSSTR/(RHSSTR*CSTR*ARSTR*ELSTR)
YHC=SQRT(BIGK)
BRAY=BR/(ARSTR*YHC)
C QFS TR= EXOTHERMIC HEAT RELEASE IN GAS PHASE REACTION IN BTU/LEM
QFS TR=QWSTR+TCSSTR*CPSTR*(TF-TSR)+CSTR*(TSTR-1.00))
Z1H=QWSTR/(CSTR*TCSSTR)
Z2PH=(SKGSTR/SKSSTR)*(QFS TR/(CPSTR*TCSSTR))*(ELSTR/(RHSSTR*ARSTR))
Z1 = Z1H * BRAY
Z1T = (CPSTR/CSTR - 1.0) * BRAY
Z2P = Z2PH / BRAY
QMC = (RHSSTR * YHC) / RHRSTR

*** SOLUTION FOR WCON BASED ON GIVEN VALUES OF PSUR AND TSR ***

SRR = SRS / (YHC * ARSTR)
ACONR = TSR - 1.00
TYM = ACONR * SRR
WSTRR = (TYM + Z1H * SRR) * (SRR / Z2PH)
WCON = WSTRR / (PSUR * PSUR * ETF)

*** ITERATIVE SOLUTION FOR A TS COMPATIBLE WITH INITIAL PRESSURE ***

WSTRS = WCON * PIN * PIN * ETF
Z2 = Z2P * WSTRS
CTS = CTS1
SAVE(1) = 1.00
SAVE(2) = 0.01 * CTS

20 EETS = EXP(EORS / CTS)
GH = (-Z1 / EETS) + Z2 * EETS
CALL ITSUB(CAPF, CTS, SAVE, 0.000001, 150)
KBR = SAVE(1)
GO TO (20, 20, 20, 20, 22, 21), KBR

21 WRITE(6, 203) CTS, GH, CAPF

22 TS = CTS
ACON = TS - 1.00
R = BRAY * EXP(-EORS / TS)
YO = +7.000
NBS = 11
DXSBS = ELSTR / FLOAT(NBS - 1)
WRITE(6, 210) QFSTR, QWSTR, TCSSTR, CPSTR, CSTR, TF, TSR
WRITE(6, 204) BIGK, YHC, BRAY, BR, WCON, WSTRR
WRITE(6, 205) EORG, EORS, TF, TS, ETF
WRITE(6, 206) R, ACON, QMC, Z1H, Z2PH, WSTRS
WRITE(6, 207) Z1, Z2P, DXSBS, ARSTR, RHRSTR

DYA(1) = 0.50
DYA(2) = 0.20
DYA(3) = 0.10
NTY(1) = 8
NTY(2) = 10
NTY(3) = 10
JTOT = NTY(1) + NTY(2) + NTY(3) + 1
JTOTM = JTOT - 1
TYLOC(1) = -YO
NF = 1
DO 46 I = 1, 3
NI = NF + 1
NF = NF + NTY(I)

DY = DYA(I)
DO 45 J=NI,NF
TYLOC(J)=TYLOC(J-1)+DY
45 CONTINUE
46 CONTINUE
IF (UT LE 0.0) R = 0.0
DO 30 IBS=1,NBS
SMXBS(IBS)=1.000+FLOAT(IBS-1)*(DXSBS/ELSTR)
APLOC(IBS)=PIN
RGR(IBS)=R
FSMDG(IBS)=R*GMC
DO 25 J=1,JTOT
Y=TYLOC(J)
CT(J,IBS)=ACON*EXP(R*Y)+1.000
25 CONTINUE
30 CONTINUE
G2=2.00*GAM
GM1=GAM-1.000
GM12=GM1/2.00
GM12G=GM1/(2.00*GAM)
GGM1=GAM1*GM1
C
C COMPUTE LINEAR COMBUSTION PARAMETERS A, B, AND N.
IF (NOPT NE 1) GO TO 160
E = EORS/TS
A = E * (TS - 1.0)/TS
RSTR = R * ARSTR * YHC
RMSTR = RHSSTR * RSTR
RMCP = RMSTR * CPSTR
SOLAM = QFSTR*SKGSTR*WSTRS/(RMCP*RMCP*TS*TCSSTR)
ELLCC = (E * SOLAM + 1.0) * CPSTR/CSTR
EQCT = E * QWSTR/(CSTR*TS*TCSSTR)
B = (A + ELLCC + EQCT)/A
WRITE (6,215) A, B
160 LPRINT=0
WRITE(6,217) ASMB(1),ASMC(1),ACMB(1)
IDDA(1)=0
XI SW(1)=ASMC(1)
RETURN
END

SUBROUTINE START(NOPT,NT,T,NPRINT)

COMMON/HEAD/ HEADER(7),LPRINT,NTIMES,NSHOCK,JPLOT,PIN
COMMON/FIXPR/MTOA(5),GAM,GM1,GM12,GM12G,G2,GGM1,DGK,DGKNL
COMMON/TIMTRN/DT,DT22,DXA(5),XI SW(5),IDDA(5),ACMB(5),ASMB(5),
1 ASMC(5)
COMMON/SPCTR/ CAPX(200,5),XI(200,5),SMX(200,5),
1 RP(200,5),DLNA(200,5)
COMMON/COMP/ HAF(200,5),SMDG(200,5),SMDP(200,5),XIFC,HAFC,SMDGC
1 ,REACTN,SMDPC
COMMON/VARG/R(200,5),UP(200,5),U(200,5),UP(200,5),S(200,5)

326
DIMENSION AREA(100), SAVE(8)

1 FORMAT(7A10)
100 FORMAT(2X, I3, 10E12.5)
102 FORMAT(1H1, 3X, *SMDG*, 8X, *SMDP*, 8X, *CAP HAF*)
103 FORMAT(2X, *THE FOLLOWING FLOW FIELD WAS READ IN ON CARDS FROM
1 A PREVIOUS COMPUTATION*)
105 FORMAT(2X, 5E14.8)
107 FORMAT(2X/* UBAR = *, F9.6, * PBAR = *, F9.4,
2 2X, *THE RESTARTED FLOW FIELD CONTAINING*,
3 * THE ADD DE DISTURBANCE FOLLOWS *)
108 FORMAT(2X, * ITS IN START W.N.C. (WLST, WCHOK, UT, RHON, ARL) ARE*, /
16E14.8)
109 FORMAT(3I5, F10.0)
110 FORMAT(2X/*MASS BALANCE (LBM/SEC) =*, 10X,
1 *MASS IN = *, F9.4, 5X, *MASS OUT = *, F9.4/)
111 FORMAT(9X, *OMEGA = *, F7.4/)

NT = 0
T = 0.0
PI = 3.1415927
KS = 1
KST = 1
MTO T = MTOA(KS)
CMB = ACMB(KS)
SMB = ASMB(KS)
DX = DXA(KS)
IDDA(1) = 0
XI SW(1) = ASMC(1)
CX = 0.00
WRITE(6, 101)
RW = RCHSTR
DRWDX = 0.0
AW = PI * RW * RW
DO 10 I = 1, MTO T
CAPX(I, KS) = CX
XI(I, KS) = CMB * CX + SMB
SMX(I, KS) = XI(I, KS)
SMXI = SMX(I, KS)
DLNA(I,KS) = 0.0
RW2(I,KS) = 2.0*ELSTR/RW
AREA(1) = AW
SMXP = SMXI*ELSTR
WRITE(6,100) I,CX,XI(I,KS),SMXP,RW,DRWDX,AW,
1 DLNA(I,KS),RW2(I,KS)
CALL COMBNT(SMXI,KS,KST,HAF(I,KS),SMDG(I,KS),SMDP(I,KS))
CX = DX*FLOAT(I)
10 CONTINUE
WRITE(6,102)
DO 15 M=1,MTOT
WRITE(6,100) M,SMDG(M,KS),SMDP(M,KS),HAF(M,KS)
15 CONTINUE
HIN = HAFC
RIN = PIN/HIN
SIN = ALOG(PIN) - GAM*ALOG(RIN)
AINT = SQRT(PIN/RIN)
UIN = UT*AINT
OOG = 1.00/GAM
DLO = SMX(1,KS)
DDL = SMX(MTOT,KS) - DLO
DUDX = UIN/DDL
CPART = 1.0 + SQRT(1.0 + 8.0*DUDX/DGK)
DO 40 M = 1,MTOT
R(M,KS) = RIN
P(M,KS) = PIN
S(M,KS) = SIN
SSP(M,KS) = AINT
U(M,KS) = (SMX(M,KS) - DLO) * DUDX
UP(M,KS) = 2.0*U(M,KS)/CPART
RP(M,KS) = 0.5*SMDPC*CPART*R(M,KS)
40 CONTINUE
LPRINT=0
READ (5,109) NSHOCK,ICTYPE,ISEN,AMPL
READ (5,109) NTIMES
P11 = P(1,1)
SMDG11 = SMDG(1,1)
UBAR = U(MTOT,1)
PBAR = P(MTOT,1)
ADM = GM120 * UBAR/PBAR
LPRINT=0
JPLT = 0
TCONV = SSP(1,1)/(SMX(MTOT,1) - SMX(1,1))
WRITE(6,103)
DO 176 IBS = 1,NBS
APC(IBS) = APLOC(IBS)
176 CONTINUE
CALL OUTPUT(NTOT,KST,0)
LPRINT = 0
C
C CALCULATE MASS BALANCE
SMOUT = R(MTOT, 1) * RHRSTR * PI * RCHSTR * RCHSTR * U(MTOT, 1) * ARSTR
SUM = 0.0
DO 173 M = 2, MTOT
DELX = ELSTR * DX
AVG = ( SMDG(M, 1) + SMDG(M-1, 1) ) / 2.0
SUM = SUM + AVG * DELX
173 CONTINUE
SMIN = 2.0 * PI * RCHSTR * RHRSTR * ARSTR * SUM
WRITE (6, 110) SMIN, SMOUT
C IF (RGR(1) .LE. 0.0) GO TO 169
FREQ = PI * SSP(1, 1) / (SMX(MTOT, 1) - SMX(1, 1))
OMEGA = FREQ / (RGR(1) * RGR(1))
WRITE (6, 111) OMEGA
C CALL INITIAL DISTURBANCE
169 AMDIST = AMPL * P(1, 1)
DLO = SMX(1, 1)
IF (ICTYPE .EQ. 2) GO TO 174
AMSTR = AMDIST
MSTOP = 9
DDL = SMX(9, 1) - DLO
GO TO 175
174 AMSTR = 0.0
MSTOP = MTOT
DDL = SMX(MTOT, 1) - DLO
175 DO 180 M = 1, MSTOP
PINC = AMSTR + AMDIST * COS(P(1, 1) - DLO) / DDL
IF (ISEN .EQ. 1) GO TO 177
TLOCL = P(M, 1) / R(M, 1)
P(M, 1) = P(M, 1) + PINC
R(M, 1) = P(M, 1) / TLOCL
S(M, 1) = ALOG(P(M, 1)) - GAM * ALOG(R(M, 1))
GO TO 179
177 P(M, 1) = P(M, 1) + PINC
R(M, 1) = (P(M, 1) / EXP(S(M, 1))) ** 00G
179 SSP(M, 1) = SQRT(P(M, 1) / R(M, 1))
180 CONTINUE
READ(5, 1) (HEADER(KK), KK=1, 7)
WRITE(6, 107) UBAR, PBAR, ADM, TCONV
NT=0
T=0.00
C IF (NOPT .NE. 1) GO TO 200
DO 185 N = 1, NBS
DO 185 J = 1, JTOT
CTBAR(J,N) = CT(J,N)
CTYBAR(J,N) = (CT(J,N) - 1.0) * RGR(N)
185 CONTINUE
C 200 CALL NUTEMP(KST, NOPT, NT)
JPLOT = 0
CALL OUTPUT(NT, T, KST, 0)
CALL LOGIC(KST, NOPT, NPRNT)
RETURN
END
SUBROUTINE OUTPUT(NT, TKST, NPRNT)

COMMON/HEAD/ HEADER(7), LPRINT, NTM, SCHOK, JPL, PIN
COMMON/VARG/RCR(200,5), RP(200,5), UC(200,5), UP(200,5), S(200,5)
* P(200,5), SPP(200,5)
COMMON/FIX/MTOTA(5), GM, G1, G12, GM12, G2, GGM, DGK, DKNL
COMMON/TIM/DT, DTT2, DXA(5), XI(5), IDD(5), ACMB(5), ASMB(5),
* ASMC(5)
COMMON/SP/CP, CAPE(200,5), XI(200,5), SMX(200,5),
1 RW(200,5), DLN(200,5)
COMMON/COM/HAF(200,5), SMG(200,5), SMDP(200,5), XI, HAF, SMG, SMD
1, REACTN, SMDP
COMMON/NOZ/EL STR, RCHSTR, UT
COMMON/MSFL/SMXBS(100), APLC(100), FSDG(100), DXSB, NBS, APC(100)
COMMON/TEM/CT(101,20), CNU(101,20), SW(101,20), SW(101,20), RGR(20)
COMMON/BRN/DA(3), NTY(3), ALPA(3), TYLOC(50), Y0, L, Z2P, WCON,
1 E0R, EORS, ET, QMC, DRA, PAT, JTO, JTM, ARST, RHR, Z1T, TS
COMMON/DDG/SMDDG(200,5)
COMMON/NOZ/EL STR, RCHSTR, UT
COMMON/PL/TCOV, P11, TPLTO(1000), YPL(3,1000), SMDG11

1 FORMAT(1H1,25X,7A10)
2 FORMAT(/,15X,*COMPUTATION STEP(*, I5,*) WHERE TIME(NON-DIMENSIONAL
1)=**, E11.4,*, DT=**, E11.4,*, T(MIL-SEC)=**, E11.4
3 FORMAT(7X,*INDEX M*, 6X,*X STR*, 8X,*MACH NO.*, 5X,*PRESSURE*, 5X,
* DENSITY*, 6X,*VELOCITY*, 5X,*ENTROPY*, 5X,*ENTRALPY*, 5X,*A(LOC)*)
4 FORMAT(7X, I4,4X,8E13.6)
5 FORMAT(/,9X,*REGION(*, I2,*) WHERE SMB=**,
1 E15.5,*, SMC=**, E15.5,*, CMB=**, E15.5,*)
6 FORMAT(/,32X,*PARTICLE*, 5X,*PARTICLE*/7X,*INDEX M*,
1 6X,*X STR*, 8X,*DENSITY*, 5X,*VELOCITY*, 6X,*SMDG*, 9X,
2 *SMDP*, 8X,*SMDDG*)
7 FORMAT(/,9X,*INDEX OF DISCONTINUITY DIRECTION=**, I3,
1 AND DISCONTINUITY LOCATION=**, E12.5,/) 8 FORMAT(/,8X,*IBS*, 9X,*XSTRBS*, 7X,*F SDG*, 7X,
1 1R*11X,*TEMP SUR*, 5X,*PRESS(LCC)*)
9 FORMAT(2X,*STEP(*, I4,*) T=**, F10.6,*, P(*, I2,*)=*, F10.6,
*, P(*, I2,*)=*, F10.6,*, P(*, I2,*)=*, F10.6,
2 * M(*, I2,*)=*, F10.6,*, MG(*, I2,*)=*, F10.6)
10 FORMAT(16X,F10.6,7X,F10.6,8X,F10.6** U(*, I2,*)=*, F10.6,
1 	* U(*, I2,*)=*, F10.6,*, MG(*, I2,*)=*, F10.6)

LPRINT=LPRINT+1
IF(LPRINT.LT.0) GO TO 200
LPRINT=-100
IF(NPRNT .GT. 0) GO TO 200
TELSTR=2.00*ELSTR
TACT=T*(ELSTR/ARSTR)*1000000
WRITE(6,1) (HEADER(JJ), JJ=1,7)
WRITE(6,2) NT, TKST, TACT

330
DO 100 KS=1,KST
SMB=ASMB(KS)
CMB=ACMB(KS)
SMC=ASMC(KS)
WRITE(6,5) KS, SMB, SMC, CMB
WRITE(6,3)
MTOT=MTOTAL(KS)
MID = (MTOT + 1)/2
DO 50 M=1,MTOT
RHOG = R(M,KS)
SG = S(M,KS)
PG = P(M,KS)
UG = U(M,KS)
HLOC = PG/RHOG
ALOC = SSP(M,KS)
EMACH = UG/ALOC
SMXSTR = SMX(M,KS)*ELSTR
RWALL = TELSTR/RW2(M,KS)
WRITE(6,4) M, SMXSTR, EMACH, PG, RHOG, UG, SG, HLOC, ALOC
50 CONTINUE
WRITE(6,7) IDDA(KS), XISW(KS)
WRITE(6,6)
DO 60 M=1,MTOT
SMXSTR = SMX(M,KS)*ELSTR
WRITE(6,4) M, SMXSTR, RP(M,KS), UP(M,KS), SMDG(M,KS),
1 SMDP(M,KS), SMDDG(M,KS)
60 CONTINUE
WRITE(6,8)
DO 150 IBS=1,NBS
SMXBSS = SMXBS(IBS)*ELSTR
WRITE(6,4) IBS, SMXBSS, FSMDG(IBS)*RGR(IBS), CT(JTOT,IBS), APLOC(IBS)
150 CONTINUE
100 CONTINUE
200 NTR = (NT/2) * 2
IF (NT * NE. NTR) GO TO 210
EMACH = U(MTOT,1)/SSP(MTOT,1)
IF (SMDG11 * LE. 0.0) GO TO 204
YSMDG = (SMDG(1,1) - SMDG11)/SMDG11
YSMDGN = (SMDG(MTOT,1) - SMDG11)/SMDG11
GO TO 206
204 YSMDG = 0.0
YSMDGN = 0.0
206 IF (NPRNT * GT. 0) GO TO 207
WRITE(6,9) NT, T, P(1,1), MID, P(MID,1), MTOT, P(MTOT,1),
1 MTOT, U(MTOT,1), MID, UP(MID,1)
207 JPLOT = JPLOT + 1
TPlot(JPLOT) = TCONV * T
YPLOT(1, JPLOT) = (P(1,1) - P11)/P11
YPLOT(2, JPLOT) = (P(MID,1) - P11)/P11
YPLOT(3, JPLOT) = (P(MTOT,1) - P11)/P11
IF (NPRNT * GT. 0) GO TO 210
WRITE (6,10) TPlot(JPLOT), (YPLOT(KK,JPLOT), KK = 1,3),
1 MID, U(MID,1), MID, UP(MID,1), MTOT, YSMDG
210 RETURN
END
SUBROUTINE LOGIC(KST,NOPT,NPRNT)

COMMON/HEAD/ HEADER(7),LPRINT,TIMES,NSHOCK,JPLOT,PIN
COMMON/FIXPR/MOTA(5),GAM,GM1,GM2,GM12,G2,GGM1,GDK,DGKNL
COMMON/TIMTRN/DT,DT22,DXA(5),XSW(5),IINDA(5),ACMB(5),ASMB(5),1
ASM(5)
COMMON/SPCTRNI/CAPX(200,5),XI(200,5),SMX(200,5),
1RWX(200,5),DLNA(200,5)
COMMON/COMFIX/ HAF(200,5),SMDG(200,5),SMDP(200,5),XIFC,HAFC,SMDGC
COMMON/FIXPR/MTOTA(5),GAM,GM1,GM2,GM12,G2,GGM1,GDK,DGKNL
COMMON/TIMTRN/DT,DT22,DXA(5),XSW(5),IINDA(5),ACMB(5),ASMB(5),1
ASM(5)
COMMON/SPCTRNI/CAPX(200,5),XI(200,5),SMX(200,5),
1RWX(200,5),DLNA(200,5)
COMMON/COMFIX/ HAF(200,5),SMDG(200,5),SMDP(200,5),XIFC,HAFC,SMDGC
COMMON/FIXPR/MTOTA(5),GAM,GM1,GM2,GM12,G2,GGM1,GDK,DGKNL
COMMON/TIMTRN/DT,DT22,DXA(5),XSW(5),IINDA(5),ACMB(5),ASMB(5),1
ASM(5)
COMMON/SPCTRNI/CAPX(200,5),XI(200,5),SMX(200,5),
1RWX(200,5),DLNA(200,5)
COMMON/COMFIX/ HAF(200,5),SMDG(200,5),SMDP(200,5),XIFC,HAFC,SMDGC

DIMENSION IBUF(512),ITTT(3),ITY1(3),ITY2(4),ITY3(4),ITY4(4),1
DUMMY(T(500),DUMMYY(500),PRSC(3),TME(3),2
PMAX(100),TMMAX(100),PKPK(100),TMEAN(100)

2 FORMAT(2X,3I10,2E15.8)
3 FORMAT(2X,5E15.8)
202 FORMAT(2X/* CMB FOR REGION KS=*,I2, I S=*,E14.6,5X,(NT=*,I8,1
1 WHERE T=*,E14.6,**)*)
203 FORMAT(* THE LAST REGION(KS=*,I2,*) IS BEING DELETED*)
204 FORMAT(* THE SHOCK WAVE IS TO BE REPLIED ON THIS STEP*)
205 FORMAT(* IN REGION(KS=*,I2,*) SHDET WAS FOUND TO BE=*,E14.
 18, FOR JS=*,I3, WITH DIRECTION=*,I3, (NT=*,I8, WHERE T=*,E
24.6,**))
206 FORMAT(* THE NEW FLOW FIELD REGIONS APPEAR AS FOLLOWS, (NO
1TE, THE ADVANCE AT THIS TIME STEP IS NOT YET COMPLETE)*)
207 FORMAT(* THIS FLOW FIELD APPEARS AS FOLLOWS *)
210 FORMAT(2X/2X,*NMAX = *,I3,5X,*TMAX = *,F8.4,5X,
 1 *PMAX = *,F8.5,5X,*PK-PK AMPLITUDE = *,F8.5,
 2 *AT T = *,F8.4,5X,*PMEAN = *,F8.5)
500 FORMAT(/* IN LOGIC(CNT=*,I6, AT T=*,E14.8,*) THE REGION(+
1S=*,I2,*) WHERE CMB=*,E14.8, HAS FAILED THE CRITERIA FOR A SMALL
2SECTION*)
501 FORMAT(2F10.0, I5)
502 FORMAT(2I15)
503 FORMAT(2X/2X,*NEGATIVE REGRESSION RATE AT M = *,I2)

332
DATA

1  ITY1/30HEAD-END PRESSURE PERTURBATION/
2  ITY2/33CHAMBER PRESSURE PERTURBATION/
3  ITY3/32NOZZLE-END PRESSURE PERTURBATION/
4  ITY4/39LOG HEAD-END PRESSURE AMPLITUDE (PK-PK)/

READ (5,502) LPLLOT,NLAST
IF (LPLLOT .EQ. 0) GO TO 6
READ (5,501) YMAX, DELY, NTICY
READ (5,501) TMAX, DELT, NTICT
READ(5,501) TMAX4, DELT4, NTICT4
YMIN = -YMAX
TMIN = 0.0

6
KI = 1
LCHEK = 0
DXMIN = 1.00
DO 5 KS = 1,KST
SMX1 = SMX(1,KS)
SMX2 = SMX(2,KS)
DXTR = SMX2 - SMX1
IF (DXTR .LT. DXMIN) DXMIN = DXTR
5 CONTINUE
IF (NOPTNE 1) GO TO 8
DO 7 IBS = 1,NBS
RBAR(IBS) = RGR(IBS)
Z2PH = Z2P*BRAY
PC = APC(IBS)
Z2RP(IBS) = 2.0*Z2BAR/(RBAR(IBS)*PC)
Z1H = Z1/BRAY
GHBAR = Z1H*RBAR(IBS) + Z2BAR/RBAR(IBS)
CTS = CT(JTOT,IBS)
ETT = EORS/(CTS*CTS)
ETGH(IBS) = GHBAR*ETT + Z1T*RBAR(IBS)/BRAY
ERTS(IBS) = RBAR(IBS)*ETT
7 CONTINUE
8
NPAX = 1
PMAX(1) = (P(1,1) - P11)/P11
TIMAX(1) = 0.0
PMEAN = 0.0
DO 200 NT=1,NTIMES
AMAX = 1.00
UMAX = 0.0
DO 15 KS=1,KST
MTOT=MTOTA(KS)
DO 10 M=1,MTOT
UT1 = ABS(U(M,KS))
APOS = SSP(M,KS)
IF (UT1 .GT. UMAX) UMAX = UT1
IF (APOS .GT. AMAX) AMAX = APOS
10 CONTINUE
15 CONTINUE
20 DT=0.800*DXMIN/(UMAX + AMAX)
DT22=DT*DT/2.00
TWDT=2.00*DT
ALFTA(1)=DYA(1)/TWDT
ALFTA(2)=DYA(2)/TWDT
ALFTA(3)=DYA(3)/TWDT

**** CALCULATE PRESSURE MAXIMA AND MINIMA ****
IF (NT GT 3) GO TO 30
PRS(NT) = (P(1,1) - P11)/P11
TIME(NT) = T * TCONV
GO TO 35
30 PRS(1) = PRS(2)
PRS(2) = PRS(3)
PRS(3) = (P(1,1) - P11)/P11
TIME(1) = TIME(2)
TIME(2) = TIME(3)
TIME(3) = T * TCONV
35 IF (NT .LT. 3) GO TO 40
DPL = PRS(3) - PRS(2)
DPS = PRS(2) - PRS(1)
IF (DPL*DPS) 45, 45, 48
45 DPMX1 = PMAX(NPMAK) - PMEAN
DPMX2 = PRS(2) - PMEAN
IF (DPMX1*DPMX2) 46, 46, 48
48 PMX1 = ABS(DPMX1)
PMX2 = ABS(DPMX2)
IF (PMX2 .GE. PMX1) GO TO 49
GO TO 40
46 IF (NPMAK = EQ. 1) GO TO 47
SMALL = 0.5 * ABS(DPMX1)
PMX2 = ABS(DPMX2)
IF (PMX2 .LT. SMALL) GO TO 40
PMX1 = PMAX(NPMAK-1)
PMX2 = PMAX(NPMAK)
PKPK(NPMAK) = ABS(PMX2 - PMX1)
TMEAN(NPMAK) = (TIMAX(NPMAK-1) + TIMAX(NPMAK))/2.0
PMEAN = (PMAX(NPMAK-1) + PMAX(NPMAK))/2.0
WRITE(6,210) NPMAK, TIMAX(NPMAK), PMAX(NPMAK), PKPK(NPMAK),
1 TMEAN(NPMAK), PMEAN
47 NPMAK = NPMAK + 1
49 PMAX(NPMAK) = PRS(2)
TIMAX(NPMAK) = TIME(2)
40 CONTINUE

**** TEST FOR SHOCK DEVELOPMENT ****
IF (NSHOCK = EQ. 0) GO TO 101
DO 100 KS=K1,KST
Y1=P(1,KS)
Y2=P(2,KS)
Y3=P(3,KS)
MTOT=MTOTA(KS)
SHDETM=0.000
100 CONTINUE
DO 95 MO=4,MTOT
Y4=P(MO,KS)
CALL SHDETL(Y1,Y2,Y3,Y4,SHDET,NYES)
IF(NYES.EQ.0) GO TO 92
IF(SHDET.LT.SHDETM) GO TO 92
SHDETM=SHDET
MOH=MO
Y1H=Y1
Y4H=Y4
C
92 Y1=Y2
Y2=Y3
Y3=Y4
95 CONTINUE
IF(NYES.EQ.0) GO TO 100
INDD=-1
IF(Y1H.GT.Y4H) INDD=+1
JS=MOH-1
IF(INDD.EQ.1) JS=MOH-2
WRITE(6,205) KS, SHDETM, JS, INDD, NT, T
WRITE(6,206)
LPRINT=0
CALL OUTPUT(NT,T,KST,NPRINT)
LPRINT=0
STOP
C
100 CONTINUE
C
101 CALL BNDMOC(KST,-1)
IF(UT.GT.0.0) GO TO 103
CALL BNDMOC(KST,1)
GO TO 104
103 CALL NOZMOC(KST)
104 DO 105 KS=1,KST
CALL TAYLOR(KS)
105 CONTINUE
DO 106 KS=KI,KST
MTOT=MTOTA(KS)
U(1,KS)=UUD(1,KS)
UP(1,KS)=UPUD(1,KS)
P(1,KS)=PUD(1,KS)
R(1,KS)=RUD(1,KS)
RP(1,KS)=RPUD(1,KS)
S(1,KS)=SUD(1,KS)
SSP(1,KS)=SSPUD(1,KS)
U(MTOT,KS)=UUD(2,KS)
UP(MTOT,KS)=UPUD(2,KS)
P(MTOT,KS)=PUD(2,KS)
R(MTOT,KS)=RUD(2,KS)
RP(MTOT,KS)=RPUD(2,KS)
S(MTOT,KS)=SUD(2,KS)
SSP(MTOT,KS)=SSPUD(2,KS)
106 CONTINUE
121 CONTINUE
IF (NOPT.EQ.0) GO TO 198
193 CALL NUTEMP(KST,NOPT,NT)
194 DO 196 KS=1,KST
    M TO T=M TO TA(KS)
    DO 195 MI=1,MTOT
        SM DG = SM DG (MI,K S)
        CALL COMMIT(SMX(M,KS),KS,KST,HAF(M,KS),SMDGN,SMDP(M,KS))
        SMDDG(M,KS)=(SMDGN-O SMDG)/DT
        SMDG(M,KS)=SMDGN
    IF (SMDGN .GE. 0.0) GO TO 195
    WRITE(6,503) M
    LPRINT = 0
    CALL OUTPUT(NT,T,KST,NPRNT)
    STOP
195 CONTINUE
196 CONTINUE
198 CALL OUTPUT(NT,T,KST,NPRNT)
    IF (LPRINT .EQ. 0) GO TO 200
    T1 = TCONV * T
    IF ((T1.GT.TMAX) .OR. (JPLOT.GE.500)) GO TO 300
    GO TO 200
C    **** PLOTTED OUTPUT ****
C
300 IF (NLAST .EQ. 0) GO TO 330
    NUM = JPLOT
    DO 320 NPLOT = 1,NLAST
        JPLOT = 0
        DO 310 J = 1,NUM
            IF ((YPLOT(NPLOT,J).LT.YMIN) .OR. (YPLOT(NPLOT,J).GT.YMAX))
                GO TO 310
        J PLOT = JPLOT + 1
        DUMMY T(JPLOT) = TPLOT(J)
        DUMMYY(JPLOT) = YPLOT(NPLOT,J)
    310 CONTINUE
    IF (JPLOT .EQ. 0) GO TO 320
    GO TO (311,312,313), NPLOT
311 CALL GRAPHS(I BUF,512,4,JPLOT,NTICT,NTICY,TMAX,YMAX,
    1 TMIN,YMIN,ITY1,21,30,DUMMY T,DUMMYY,DELT,DELY,
    2 HEADER)
    GO TO 320
312 CALL GRAPHS(I BUF,512,4,JPLOT,NTICT,NTICY,TMAX,YMAX,
    1 TMIN,YMIN,ITY2,21,33,DUMMY T,DUMMYY,DELT,DELY,
    2 HEADER)
    GO TO 320
313 CALL GRAPHS(I BUF,512,4,JPLOT,NTICT,NTICY,TMAX,YMAX,
    1 TMIN,YMIN,ITY3,21,32,DUMMY T,DUMMYY,DELT,DELY,
    2 HEADER)
320 CONTINUE
C
330 JPLOT = 0
    TMIN = TMAX
    TMAX = TMAX + 10.0
200 CONTINUE
IF (LPLT EQ 0) GO TO 400
JPLT = 0
NPMAX = NPMAX - 1
DO 350 J = 1, NPMAX
IF (PKPK(J) LE 0.0) GO TO 350
PMXLG = ALOG10(PKPK(J))
IF ((PMXLG LT -3.0) OR (PMXLG GT 0.0)) GO TO 350
JPLT = JPLT + 1
DUMMY T(JPLOT) = TMEN(J)
DUMMY Y(JPLOT) = PMXLG
350 CONTINUE
CALL GRAPHS(IBUF, 512, 4, JPLT, NTICT4, 31, Tmax4, 0.0, 0.0, -3.0, 1, I T,T, I TY4, 21, 39, DUMMY T, DUMMY Y, DEL T4, 1.0, HEADER)
C
IF (LPLT EQ 1) CALL PLOT(0, 0, 999)
C
400 RETURN
C
SUBROUTINE BNDMOC(KST, MBC)
C
** THIS SUBROUTINE IS CORRECT FOR TWO PHASE FLOW - THE
PARTICLE FLOW IS COMPUTED AND DCLN (AREA)/DX = 0.0 **
COMMON/FIXPR/MTOTA(5), GAM, GM1, GM12, GM12G, G2, GGM1, DGK, DGKNL
COMMON/TIMTRN/DT, DT22, DXA(5), XI SW(5), IDDA(5), ACMB(5), ASMB(5),
1 ASMC(5)
COMMON/SPCTRNX/ CAPX(200, 5), XI(200, 5), SMX(200, 5),
1 RW2(200, 5), DLNA(200, 5)
COMMON/COMFIX/ HAF(200, 5), SMDG(200, 5), SMDP(200, 5), XI FC, HAFC, SMDGC
1, REACTN, SMDPC
COMMON/VARG/R(200, 5), RP(200, 5), U(200, 5), UP(200, 5), S(200, 5)
1, P(200, 5), SSP(200, 5)
COMMON/VARUPD/ RUD(2, 5), UUD(2, 5), SU(2, 5), UPD(2, 5), SSPUD(2, 5),
1 RPUD(2, 5), UPUD(2, 5)
200 FORMAT(/,** ERROR IN BNDMOC **,14, 4E14.8)
C
C
***STREAMLINED (1AUG72) VERSION OF BNDMOC***
C
*** MBC = -1 LEFT-HAND BOUNDARY ***
C
*** MBC = 1 RIGHT-HAND BOUNDARY ***
C
CMB = ACMB(1)
SMB = ASMB(1)
RMBC = MBC
DX = DXA(1)
IF (MBC EQ 1) GO TO 5
M1 = 1
M2 = 2
M3 = 3
MM = 1
GO TO 8

337
5 MTOT = MTOTA(1)
   M1 = MTOT
   M2 = MTOT-1
   M3 = MTOT-2
   MM = 2
8 R1=R(M1,1)
   S1=S(M1,1)
   P1=P(M1,1)
   P20LD=P1
   RP1 = RP(M1,1)
   RP2 = RP1
   RW2F=RW2(M1,1)
   GMRF=GAM*SMDG(M1,1)*RW2F
   H1=P1/R1
   A1=SSP(M1,1)
   HAFF=HAF(M1,1)
   SINC1=GMRF*(HAFF-H1)/P1
   S2=S1+SINC1*DT
   CAPU2 = RMBC*A1/CMB
   DXO=RMBC*DT*CAPU2
   DXNO=DXO/DX
   UG2=U(M2,1)
   UG3=U(M3,1)
   RG2=R(M2,1)
   RG3=R(M3,1)
   PG2=P(M2,1)
   PG3=P(M3,1)
   UPG2 = UP(M2,1)
   UPG3 = UP(M3,1)
   RPG2 = RP(M2,1)
   RPG3 = RP(M3,1)
   DUPDX = -RMBC*UPG2/DX
   ADC = DUPDX/CMB
   GPA2=GAM*P1/A1
   C22=GMRF*HAFF
   C31 = RW2F*SMDP(M1,1) - RP1*ADC
   RP2 = RP1 + C31*DT
   NTH=1
10 CALL MOCINT(0.0,UG2,UG3,DXNO,UP)
   CALL MOCINT(P1,PG2,PG3,DXNO,PO)
   CALL MOCINT(R1,RG2,RG3,DXNO,RO)
   CALL MOCINT(0.0,UPG2,UPG3,DXNO,UP)
   CALL MOCINT(RP1,RPG2,RPG3,DXNO,RO)
   IF (MBC.EQ.1) SMXO = DXO*CMB + SMB
   IF (MBC.EQ.1) SMXO = (1.0 - DXO)*CMB + SMB
   CALL COMBNT(SMXO,1,KST,HaFO,SMDGO,SMDPO)
   AO=SQRT(PO/RO)
   CAPU0 = (UO + RMBC*AO)/CMB
   GPAAVE=(GPA2+GAM*PO/A0)/2.00
   UMUP = UO - UPO
   SMU = SMDGO*UO
   RUUP = ABS(RO*UMUP)
   DGKVAR = DGK*(1.0 + DGKNL*RUUP**0.66666667)
   DKRUUP = DGKVAR*RPO*UMUP

338
C2AVE = (C22 - RMBC*GAM*A0*(DKRUUP + RW2F*SMU) + 
1 GGM1*(RW2F*(SMDGO*HAFF/GM1 + (SMU*U0 + SMDPO*UPO*UPO)/2.0) 
2 + DKRUUP*UMUP))/2.0
P2 = P0 + RMBC*GPAAVE*U0 + C2AVE*DT
ERR = (P2 - P2OLD)/P2OLD
IF(ABS(ERR) LT 1.0E-07) GO TO 15
IF(NTH .GT. 20) GO TO 14
P2OLD = P2
NTH = NTH + 1
PG = P2**GM12G
A2 = PG*EXP(S2/G2)
S2 = S1 + DT*(SINC1 + GMRF*(HAFF - A2*A2)/P2)/2.00
A2 = PG*EXP(S2/G2)
GPA2 = GAM*P2/A2
RP2 = RP1 + DT*(C31 + RW2F*SMDPO - RP2*ADC)/2.0 
CAPU2 = RMBC*A2/CMB
DX0 = RMBC*DT*(CAPU2 + CAPU0)/2.00
DXNO = DX0/DX
GO TO 10
14 WRITE(6,200) NTH, P2, P2OLD
15 CONTINUE
PUD(MM,1) = P2
SUD(MM,1) = S2
RNEW = EXP((ALOG(P2) - S2)/GAM)
RUD(MM,1) = RNEW
SSPUD(MM,1) = SQRT(P2/RNEW)
UUD(MM,1) = 0.00
RPUD(MM,1) = RP2
UPUD(MM,1) = 0.0
RETURN
END

SUBROUTINE NOZMOC(KST)

** THIS SUBROUTINE IS CORRECT FOR TWO PHASE FLOW - THE PARTICLE FLOW IS COMPUTED AND D(LN(AREA))/DX=0.0**

COMMON/FIXPR/MTOTA(5), GAM, GM1, GM12, GM12G, G2, GGM1, DGK, DGKNL
COMMON/TIMTRN/DT, DT22, DXA(5), XI SW(5), IDDA(5), ACMB(5), ASMB(5), 1 ASMC(5)
COMMON/SPCTR/ CAPX(200,5), XI(200,5), SMX(200,5), 1 RW2(200,5), DLANA(200,5)
COMMON/COMFIX/ HAF(200,5), SMDG(200,5), SMDP(200,5), XI FC, HAFC, SMDGC 1, REACTN, SMDPC
COMMON/VARG/RC(200,5), RP(200,5), U(200,5), UP(200,5), S(200,5)
1 F(200,5), SSPC(200,5)
COMMON/VARUDP/ RUD(2,5), UUD(2,5), SUD(2,5), PUD(2,5), SSPUD(2,5), 1 RPUD(2,5), UPUD(2,5)
COMMON/NOZADM/ UBAR, PBAR, ADM
200 FORMAT(* ERROR IN NOZMOC *, I4, 4E14.8)

339
CMB = ACMB(1)
SMB = ASMB(1)
DX = DXA(1)
MTOT = MTO(1)
RO = R(MTOTA, 1)
PO = P(MTOT, 1)
P1OLD = PO

U1 = U(MTOT, 1)
RP1 = RP(MTOT, 1)
UP1 = UP(MTOT, 1)
A1 = SSP(MTOT, 1)
SMDG1 = SMDG(MTOT, 1)
SMDP1 = SMDP(MTOT, 1)
SLOPE2 = U1/CMB
SLOPE3 = (U1 + A1)/CMB
SLOPE4 = UP1/CMB

MTM1 = MTOT - 1
MTM2 = MTOT - 2
UG2 = U(MTM1, 1)
UG3 = U(MTM2, 1)
UPG2 = UP(MTM1, 1)
UPG3 = UP(MTM2, 1)
RG2 = R(MTM1, 1)
RG3 = R(MTM2, 1)
RPG2 = RP(MTM1, 1)
RPG3 = RP(MTM2, 1)
PG2 = P(MTM1, 1)
PG3 = P(MTM2, 1)

DUPDX = (UP1 - UPG2)/DX

RW2F = RW2(MTOT, 1)
GMRF1 = GAM * SMDG(MTOT, 1) * RW2F
H1 = PO/RO
HAFF = HAF(MTOT, 1)
UMUP = U1 - UP1
RUUF = ABS(RO*UMUP)
DGKVAR = DGK*(1.0 + DGKNL*RUUF**0.66666667)
SMU = SMDG1*U1
DKRUUP = DGKVAR*RP1*UMUP
C2PART = RW2F*(SMDG1*HAFF/GM1 + (SMU*U1 + SMDP1*UP1*UP1)/2.0) / (1 + DKRUUP*UMUP)
C21 = (GGM1*C2PART - GMRF1*H1)/PO
C31 = -GAM*A1*(DKRUUP + RW2F*SMU) + GGM1*C2PART
IF (SMDPC <LE< 0.0) GO TO 9
CU41 = DGKVAR*UMUP - RW2F*SMDP1*UP1/RP1
CR41 = RP1*DUPDX/CMB - RW2F*SMDP1
9 GPA1 = GAM * PO/A1
NTH = 1

10 DX2 = SLOPE2 * DT
DX3 = SLOPE3 * DT
DX4 = SLOPE4 * DT
DXN2 = DX2 / DX
DXN3 = DX3 / DX
DXN4 = DX4 / DX
CALL MOCINT(U1, UG2, UG3, DXN2, U2)
CALL MOCINT(U1, UG2, UG3, DXN3, U3)
CALL MOCINT(U1, UG2, UG3, DXN4, U4)

SMX2 = (1.0 - DX2) * CMB + SMB
SMX3 = (1.0 - DX3) * CMB + SMB
SMX4 = (1.0 - DX4) * CMB + SMB
CALL COMBNT(SMX2, KST, KST, HAF2, SMDG2, SMDP2)
CALL COMBNT(SMX3, KST, KST, HAF3, SMDG3, SMDP3)
CALL COMBNT(SMX4, KST, KST, HAF4, SMDG4, SMDP4)

H2 = P2 / R2

C22 = GM1 * (RW2F * (SMDG2 * HAF/H2) / GM1 + SMU * U2 + SMDP2 * UP2 * UP2) / 2.0 + DKRUUP * UMUP / P2
A3 = SQRT(P3 / R3)

C2AVE = (C21 + C22) / 2.0
C3AVE = (C31 + C33) / 2.0

IF (SMDPC LE 0.0) GO TO 12

CU4AVE = (CU41 + DGKVAR * (U4 - UP4) - RW2F * SMDP4 * UP4 / RP4) / 2.0
CR4AVE = (CR41 + RP4 * DUPDX / CMB - RW2F * SMDP4) / 2.0
GO TO 13

12 CU4AVE = 0.0
CR4AVE = 0.0
\[ S_2 = \text{ALOG}(P_2) - \text{GAM*ALOG}(R_2) \]

\[ S_1 = S_2 + C_2\text{AVE*DT} \]

\[ P_{NUM} = P_3 + G_{PAAVE} \times (U_3 - U_{BAR} + A_{DM*PBAR}) + C_3\text{AVE*DT} \]

\[ P_1 = \frac{P_{NUM}}{(1.0 + G_{PAAVE*ADM})} \]

\[ U_1 = U_{BAR} + A_{DM*PBAR} \]

\[ R_1 = \exp((\text{ALOG}(P_1) - S_1)/\text{GAM}) \]

\[ A_1 = \sqrt{(P_1/R_1)} \]

\[ U_{P1} = U_{P4} + C_{U4\text{AVE*DT}} \]

\[ R_{P1} = R_{P4} - C_{R4\text{AVE*DT}} \]

\[ \text{ERR} = \frac{(P_1 - P_{10LD})}{P_{10LD}} \]

\[ \text{IF} (\text{ABS(ERR)} < 1.0E-07) \text{ GO TO 15} \]

\[ \text{IF} (NTH > 20) \text{ GO TO 14} \]

\[ P_{10LD} = P_1 \]

\[ NTH = NTH + 1 \]

\[ H_1 = \frac{P_1}{R_1} \]

\[ U_{MUP} = U_1 - U_P \]

\[ R_{UUP} = \text{ABS}(R_1*U_{MUP}) \]

\[ \text{DGKVAR} = \text{DGK}(1.0 + \text{DGKNL*R}_{UUP}**0.66666667) \]

\[ \text{SMU} = \text{SMDG1*U_1} \]

\[ R_{KRUUP} = \text{DGKVAR*R}_{P1*U_{MUP}} \]

\[ C_{2PARD} = \text{RW2F*(SMDG1*HAFF/GM1 + (SMU*U_1 + SMDP1*U_P*U_{UP1})/2.0)} \]

\[ 1 + R_{KRUUP*U_{MUP}} \]

\[ C_{21} = (G_{GM1*C{2PART} - GMRF1*H_1})/P_1 \]

\[ C_{31} = \text{-GAM*A1*(DKRUUP} + \text{RW2F*SMU}) + G_{GM1*C_{2PART}} \]

\[ G_{PA1} = \text{GAM} \times \frac{P_1}{A_1} \]

\[ S_{LP21} = \text{U1/CMB} \]

\[ S_{LP22} = \text{U2/CMB} \]

\[ S_{LP31} = \frac{(U_1 + A_1)}{CMB} \]

\[ S_{LP33} = \frac{(U_3 + A_3)}{CMB} \]

\[ S_{LOP3} = \frac{(S_{LP31} + S_{LP33})}{2.0} \]

\[ S_{LP41} = \text{UP1/CMB} \]

\[ S_{LP44} = \text{UP4/CMB} \]

\[ S_{LOP4} = \frac{(S_{LP41} + S_{LP44})}{2.0} \]

\[ \text{GO TO 10} \]

\[ \text{14 WRITE(6,200) NTH,P1,P10LD} \]

\[ \text{15 CONTINUE} \]

\[ P_{UD}(2,1) = P_1 \]

\[ S_{UD}(2,1) = S_1 \]

\[ R_{UD}(2,1) = R_1 \]

\[ U_{UD}(2,1) = U_1 \]

\[ S_{SSPUD}(2,1) = A_1 \]

\[ R_{PUD}(2,1) = R_{P1} \]

\[ U_{PUD}(2,1) = U_{P1} \]

RETURN

END
SUBROUTINE TAYLOR(KS)
COMMON/VARG/R(200, 5), RP(200, 5), U(200, 5), UP(200, 5), S(200, 5)
1, P(200, 5), SSP(200, 5)
COMMON/FIXPR/MTOTA(5), GAM, GM1, GM12, GM126, G2, GGM1, DGK, DGKNL
COMMON/COMFX/ HAF(200, 5), SMDG(200, 5), SMDP(200, 5), XI, HAF, SMDGC
1, REACTN, SMDC
COMMON/TIMTRN/ DT, DT22, DXA(5), XI, SW(5), IDDA(5), ACMB(5), ASMB(5),
1 ASMC(5)
COMMON/SPCTRN/ CAPX(200, 5), XI(200, 5), SMX(200, 5),
1 RV2(200, 5), DLNA(200, 5)
COMMON/DDMG/ SMDG(200, 5)
C ** R IS RHO(GAS) 
C ** RP IS RHO(PARTICLES) 
C ** U IS VELOCITY(GAS) 
C ** UP IS VELOCITY(PARTICLES) 
C ** S IS ENTROPY(GAS) 
C *** CORRECTIONS HAVE BEEN MADE FOR NEW PARTICLE MOMENTUM EQUATION* 
C *** AS OF LEVINES REPORT --- 
C****** THIS SUBROUTINE VALID FOR GAS AND PARTICLE FLOWS*******
C****** INCLUDES NONLINEAR DRAG LAW*******

MF=MTOTA(KS)-1
DX=DXA(KS)
TWDX=2.00*DX
DX2=DX*DX
CMB=ACMB(KS)
R1=R(1,KS)
RP1=RP(1,KS)
U1=U(1,KS)
UP1=UP(1,KS)
S1=S(1,KS)
P1=P(1,KS)
H1=P1/R1
R2=RP(2,KS)
R21=RP(2,KS)

R51=0.0
R52=0.0
R53=0.0
R5T=0.0
R11=SMDG1*RW21-R1*U1*DLNA1
R21=SMDP1*RW21-RP1*UP1*DLNA1
UMUP1=U1-UP1
RUUP1=ABS(R1*UMUP1)
DKVR1=DGK*(1.0+DGKNL*RUUP1**0.66666667)
SMU1=SMDG1*U1
SMUP1=SMDP1*UP1
DKRUUP=DKVR1*RP1*UMUP1
R31=(DKRUUP+RW21*SMU1)/R1
R41=GGM1*(RW21*(SMDG1*(HAF1-H1)/GM1+(SMU1*U1
+SMUP1*UP1)/2.00)+DKRUUP*UMUP1)/P1
IF (SMDPC*GT*0.0) R51=(DKRUUP+RW21*SMUP1)/RP1
R2=RP(2,KS)
RP2=RP(2,KS)
U2 = U(2, KS)
UP2 = UP(2, KS)
S2 = S(2, KS)
P2 = P(2, KS)
H2 = P2/R2
RW22 = RW2(2, KS)
DLNA2 = DLNA(2, KS)
SMDG2 = SMDG(2, KS)
SMDP2 = SMDP(2, KS)
HAF2 = HAF(2, KS)
R12 = SMDG2*RW22 - R2*U2*DLNA2
R22 = SMDP2*RW22 - RP2*UP2*DLNA2
UMUP2 = U2 - UP2
RUUP2 = ABS(R2*UMUP2)
DGKV R2 = DGK*(1.0 + DGKNL*RUUP2**0.66666667)
SMU2 = SMDG2*U2
SMUP2 = SMDP2*UP2
DKRUUP = DGKV R2*RP2*UMUP2
R32 = -(DKRUUP + RW22*SMU2)/R2
R42 = GGM1*(RW22*(SMDG2*(HAF2-H2)/GM1 + (SMU2*U2
1 + SMUP2*UP2)/2.00) + DKRUUP*UMUP2)/P2
IF (SMDPC GT 0.0) R52 = (DKRUUP - RW22*SMUP2)/RP2

** THESE ARE TEMPORARY ASSIGNMENTS ONLY **
HAFD2 = 0.00

DO 100 M = 2, MF
SMDG2 = SMDG(M, KS)
SMDP2 = SMDP*SMDDG2
CU2 = U2/CMB
CUP2 = UP2/CMB
ETA2 = 1.0/CMB
M3 = M + 1
R3 = R(M3, KS)
RP3 = RP(M3, KS)
U3 = U(M3, KS)
UP3 = UP(M3, KS)
S3 = S(M3, KS)
P3 = P(M3, KS)
H3 = P3/R3
RW23 = RW2(M3, KS)
DLNA3 = DLNA(M3, KS)
SMDG3 = SMDG(M3, KS)
SMDP3 = SMDP(M3, KS)
HAF3 = HAF(M3, KS)
R13 = SMDG3*RW23 - R3*U3*DLNA3
R23 = SMDP3*RW23 - RP3*UP3*DLNA3
UMUP3 = U3 - UP3
RUUP3 = ABS(R3*UMUP3)
DGKV R3 = DGK*(1.0 + DGKNL*RUUP3**0.66666667)
SMU3 = SMDG3*U3
SMUP3 = SMDP3*UP3
DKRUUP = DGKV R3*RP3*UMUP3
R33 = -(DKRUUP + RW23*SMU3)/R3
R43 = GGM1*(RW23*(SMDG3*(HAF3-H3)/GM1 + (SMU3*U3
1 + SMUP3*UP3)/2.00) + DKRUUP*UMUP3)/P3

344
IF (SMDPC > 0.0) R53 = (DKRUUP - RW23*SMUP3)/RP3
RX = (R3 - R1)/TWDX
RPX = (RP3 - RP1)/TWDX
UX = (U3 - U1)/TWDX
UPX = (UP3 - UP1)/TWDX
SX = (S3 - S1)/TWDX
PX = (P3 - P1)/TWDX
HX = (PX - H2*RX)/R2
EGR2 = ETA2/(GAM*R2)
RT = R12 - (CUP2*RPX + ETA2*RP2*UPX)
UT = R32 - (CUP2*UX + EGR2*PX)
ST = R42 - CU2*UX
UPT = R52 - CUP2*UPX
PT = P2*ST + GAM*H2*RT
HT = (PT - H2*RT)/R2
CUX = ETA2*UX
CUPX = ETA2*UPX
CUT = UT/CMB
CUPT = UPT/CMB
RXX = (R3 - 2.00*R2 + R1)/DX2
RPXX = (RP3 - 2.00*RP2 + RP1)/DX2
UXX = (U3 - 2.00*U2 + U1)/DX2
UPXX = (UP3 - 2.00*UP2 + UP1)/DX2
SXX = (S3 - 2.00*S2 + S1)/DX2
PXX = (P3 - 2.00*P2 + P1)/DX2
R1X = (R13 - R11)/TWDX
R2X = (R23 - R21)/TWDX
R3X = (R33 - R31)/TWDX
R4X = (R43 - R41)/TWDX
R5X = (R53 - R51)/TWDX
R7X = R1X - (CUX*RX + CU2*RX*ETA2*(RX*UX + R2*UX))
RPTX = R2X - (CUPX*RPX + CUP2*RPXX + ETA2*(RPX*UPX + RP2*UPXX))
UTX = R3X - (CUX*UX + CU2*UX*EGR2*(PXX*PX*RX/R2))
STX = R4X - (CUX*SX + CU2*SXX)
UPTX = R5X - (CUPX*UPX + CUP2*UPXX)
PTX = PX*ST + P2*STX + GAM*(HX*RT + H2*RTX)
R7T = RW22*SMDG2 - (RT*U2 + R2*UT)*DLNA2
R2T = RW22*SMDP2 - (RPT*UP2 + RP2*UPT)*DLNA2
UMUPT = UT - UPT
UTUMUP = RPT*UMU2
RUTMPT = RP2*UTMUPT
IF (SMDPC <= 0.0) GO TO 80
IF (UMUPT2) 50, 80*60
50 ATIV1UPT = -UTMUPT
GO TO 70
60 ATIV2UPT = UTMUPT
70 VARKT = 0.666666667*DGK*DGKNL*(RT*ABS(UMU22) + R2*AUMUPT)/
1 + RUUP2**0.333333333)
GO TO 90
80 VARKT = 0.0
90 R3T = -(R32*RT + DGKVR2*(RTUMUP + RUTMPT) + VARKT*RP2*UMUP2
1 + RW22*(UT*SMDG2 + U2*SMDG2))/R2
R4T = (GGM1 * (RW22 * (SMDDG2 * (HAF2 - H2) + SMDG2 * (HAFD2 - HT)) / GM1 + 1 (SMDDG2 * U2 + U2 + SMDDP2 * UP2 + UP2) / 2.00
2 + (SMU2 * UT + SMUP2 * UPT)) +
3 DGKV2 * UMUP2 * (RTUMUP + 2.00 * RUTMPT) + VARKT * RP2 * UMUP2
4 - R42 * PT) / P2
IF (SMDPC *LE* 0.0) GO TO 91
R5T = DGKV2 * UMUP2 + VARKT * UMUP2 - RW22 * (UPT * SMDG2 + UP2 * SMDG2
1 - SMUP2 * (RPT/ RP2)) / RP2
91 RTT = R1T - (CUT * RX + CU2 * RTX + ETA2 * (UX * RT + R2 * UTX))
RPTT = R2T - (CUPT * RPX + CUP2 * RPTX)
1 + ETA2 * (UPX * RPT + RP2 * UPTX)
UTT = RTT - (CUT * RX + CU2 * RTX + ETA2 * (UX * RX + RTE + R2 * UTX))
STT = R4T - (CUT * SX + CU2 * STX)
UPTT = R5T - (CUPT * RX + CUP2 * UPTX)
RMKS = R2 + RT * DT + RTT * DT2
R(M, KS) = RMKS
U(M, KS) = U2 + UT * DT + UTT * DT2
IF (SMDPC *LE* 0.0) GO TO 92
RP(M, KS) = RP2 + RPT * DT + RPTT * DT2
UP(M, KS) = UP2 + UPT * DT + UPTT * DT2
GO TO 94
92 RP(M, KS) = RP2
UP(M, KS) = UP2
94 SFMK S = S2 + ST * DT + STT * DT2
SMKS = SFMK S
PFMK S = EXP (SF MK S + GAM * ALOG (RMKS))
PM(K, KS) = PFMKS
SSP(M, KS) = SQRT (PFMK S / RMKS)
R1 = R2
RP1 = RP2
U1 = U2
UP1 = UP2
S1 = S2
P1 = P2
R2 = R3
RP2 = RP3
U2 = U3
UP2 = UP3
S2 = S3
P2 = P3
H2 = H3
SMDDG2 = SMDDG3
SMDDP2 = SMDDP3
HAF2 = HAF3
DLNA2 = DLNA3
RW22 = RW23
UMUP2 = UMUP3
RUUP2 = RUUP3
DGKV2 = DGKV3
SMU2 = SMU3
SMUP2 = SMUP3

346
SUBROUTINE NUTEMP(KST, NOPT, NT)
  COMMON/TIMTRAN/ DT, DT22, DXA(5), XI SW(5), IDDA(5), ACMB(5), ASMB(5),
                   ASMC(5)
  COMMON/SPCTRAN/ CAPX(200,5), XI(200,5), SMX(200,5),
                   RW2(200,5), DLNA(200,5)
  COMMON/VARG/R(200,5), RP(200,5), U(200,5), UP(200,5), S(200,5)
  COMMON/MSFSLD/ SMXBS(100), APLOC(100), FSMDG(100), DXSBS, NBS, APC(100)
  COMMON/BRNCON/ DY A(3), NT Y(3), ALFTA(3), TYLOC(50), Y O, Z I, Z 2P, WCON,
  COMMON/EORG, EORS, ETF, QMC, BRAY, JTOT, JTO TM1, ARSTR, RHRSTR, Z IT, TS
  APLOC(1) = P(1, 1)
  IF(KST .GT. 1) GO TO 30
  C   KST=1, HENCE ONLY ONE REGION INTERPOLATION NEEDED
  8   ML=2
      DO 20 IBS=2, NBS
        SMXBSI = SMXBS(IBS)
      9   IF(SMXBSI .LE. SMX(ML,1)) GO TO 10
            ML=ML+1
            GO TO 9
      10  ML1=ML-1
          SMX1 = SMX(ML1,1)
          P1 = P(ML1, 1)
          APLOC(IBS) = ((P(ML1) - P1)/(SMX(ML,1) - SMX1)) * (SMXBSI - SMX1) + P1
      20  CONTINUE
            GO TO 71
  C   KST .GT. 1 MULTIPLE REGION INTERPOLATION NEEDED
     30   ML=2
          KSL=1
          CMBL = ACMB(1)
          SMBL = ASMB(1)
          XKSEND = CMBL + SMBL
          DO 70 IBS=2, NBS
            SMXBSI = SMXBS(IBS)
            38  IF(SMXBSI .GT. XKSEND) GO TO 50
            39  IF(SMXBSI .LE. SMX(ML, KSL)) GO TO 40
                ML = ML+1
                GO TO 39
40  ML1=ML-1
    SMX1=SMX(ML1,KSL)
    P1=P(ML1,KSL)
    APLOC(IBS)=((P(ML,KSL)-P1)/(SMX(ML,KSL)-SMX1))*(SMXBSI-SMX) + P1
    GO TO 70
50  ML=2
    KSL=KSL+1
    CMBL=ACMB(KSL)
    SMBL=ASMB(KSL)
    XKSSEND = CMBL + SMBL
    GO TO 38
70  CONTINUE
71  IF (NT EQ 0) GO TO 100
75  IF (NOPT*EQ.2) GO TO 80
    DO 78 IBS = 1,NBS
        PDEL = APLOC(IBS) - APC(IBS)
        CALL LINTHW(IBS,PDEL)
    78 CONTINUE
    GO TO 100
80  DO 90 IBS = 1,NBS
    CALL THRMWV(IBS,APLOC(IBS),TFLM,NT)
90  CONTINUE
100 CONTINUE
RETURN
END

SUBROUTINE THRMWV(IBS,PLOC,TFLM,NT)
COMMON/TEMWAV/CT(101,20),CNU(101,20),SW(101,20),SV(101,20),RGR(20)
COMMON/MSFSLD/SMXBS(100),APLOC(100),FSDG(100),DKSBS,NBS,APC(100)
COMMON/BRNCON/DYA(3),NY(3),ALFTA(3),TYLOC(50),YO,ZZP,WCON
1EORG=EORS,ETF,QMC,BRAY,JTOT,JTOTM,ARSTR,RHRSTR,Z1,T1,TS
COMMON/TIMTRN/DT,DT22,DXA(5),XI,SW(5),IDDA(5),ACME(5),ASMB(5),
1ASM(5)
DIMENSION SAVE(8)
250 FORMAT(2X,**STEP=**,I5,**IBS=**,I2,**ITSUB,D,N,C IN THRMWV FOR
1PLOC=**,E14.8,**CTS=**,E14.8,**CAPF=**,E14.8)
    IF (PLOC.LT.10.00) PLOC = 10.00
    WSTR=WCON*PLOC*TFLM*ETF
    Z2=Z2P*WSTR
    R=RGR(IBS)
    ROT=R/2.00
    SQRT=SQRT(ROT*ROT + 1.00/DT)
    XI1=SQRT*ROT
    XI2=SQRT*ROT
    X1D=XI2-XI1
    CNU(J,IBS)=0.00
    DO 20 J=2,JTOT
    Y=TYLOC(J)*YO
    EXID=EXP(X1D*Y)
    CNUJB=(1.00-EXID)/(XI1*EXID-XI2)
    IF (ABS(CNUJB-CNU(J,1,IBS))*LT.1.0E-07) GO TO 22
    CNU(J,IBS)=CNUJB
20 CONTINUE
GO TO 24

22 CNU(J, IBS) = CNUJB
   JST = J + 1
   Y = TLOC(JST) + Y
   EXID = EXP(XID*Y)
   CNUFX = (1.00 - EXID) / (XI1*EXID - XI2)
   DO 23 J = JST, JTOT
      CNU(J, IBS) = CNUFX
   23 CONTINUE

24 CONTINUE

   SW(1, IBS) = 1.00
   CNUJ = 0.00
   CTJ = CT(1, IBS)
   SWJ = 1.00
   NF = 0
   DO 31 I = 1, 3
      NI = NF + 1
      NF = NF + NTY(I)
      ALFT = ALFTA(I)
      DO 30 J = NI, NF
         JP1 = J + 1
         CNUJP1 = CNU(JP1, IBS)
         CTJP1 = CT(JP1, IBS)
         SWJP1 = SW*(1.00 - ALFT*CNUJ) + ALFT*(CNUJP1*CTJP1 + CNUJ*CTJ)/1.00 + ALFT*CNUJP1
         SW(JP1, IBS) = SWJP1
         SWJ = SWJP1
         CTJ = CTJP1
         CNUJ = CNUJP1
      30 CONTINUE
   31 CONTINUE

   CNU = CNU(JTOT, IBS)
   SWL = SW(JTOT, IBS)
   CTS = CT(JTOT, IBS)
   SAVE(1) = 1.00
   SAVE(2) = 0.010*CTS

35 EETS = EXP(EORS/CTS)
   GH = ((Z1 + Z1T*(CTS-TS))/EETS) + Z2*EETS
   CAPF = CTS + CNU*GH + SWL
   CALL ITSUB(CAPF, CTS, SAVE, 0.00001, 50)
   KBR = SAVE(1)
   GO TO (35A, 35B, 35C, 35D, 40), KBR

40 WRITE (*, 250) NT, IBS, PLOC, CTS, CAPF
41 EETS = EXP(EORS/CTS)
   R = BRAY/EETS
   ROT = R/2.00
   GH = ((Z1 + Z1T*(CTS-TS))/EETS) + Z2*EETS
   SV(JTOT, IBS) = GH
   SWJ = SWL
   CNUJ = CNUL
   CTJ = CT(JTOT, IBS)
   SVJ = GH
   NF = 0
DO 51 II=1,3
   I=4-II
   NI=NF+1
   NF=NF+NTY(I)
   ALFT=ALFTA(I)
   BETN=DYA(I)*ROT
   DO 50 N=NI,NF
      JM1=JTOT - N
      SWJM1=SW(JM1,IBS)
      CNUJM1=CNUC(JM1,IBS)
      CTJM1=CT(JM1,IBS)
      SVJM1=(SVJ*(1.00+BETN+ALFT*CNU(J)) - ALFT*(SVJ+SWJM1-CTJ-CTJM1))
      1/(1.00+BETN+ALFT*CNUJM1)
      SV(JM1,IBS)=SVJM1
      SVJ=SVJM1
      SWJ=SWJM1
      CNUJ=CNUJM1
      CTJ=CTJM1
   50 CONTINUE
   51 CONTINUE
   DO 60 J=1,JTOT
      CT(J,IBS)=CNUC(J,IBS)*SV(J,IBS) + SW(J,IBS)
   60 CONTINUE
   RGR(IBS)=R
   FSMDG(IBS)=QMC*R
   RETURN
END

SUBROUTINE LINTHW(IBS,PDEL)
COMMON/TEMWAV/CT(101,20),CNUC(101,20),SW(101,20),SV(101,20),RGR(20)
COMMON/MSFSLD/SMB(100),APLOC(100),FSMDG(100),DXTSB,NBS,APC(100)
COMMON/BRCNCON/DYA(3),NTY(3),ALFTA(3),TYLOC(50),YO,ZI,ZEP,WCON,
IDORG,EDRST,ETF,QMC,BRAY,JTOT,JBTOBM,ARSTR,RHSTR,1T5,TS
COMMON/TIMTRN/DT,DT22,DX(5),XI,SW(5),IDD(5),ACMB(5),ASM(5),
1ASM(5)
COMMON/SYVAL/RBAR(20),CTBAR(101,20),CTYBAR(101,20),
1Z2RP(20),ETGH(20),ERTS(20)
203 FORMAT (2X,*ITERATION ON R FAILED TO CONVERGE*,5X,
1*ROLD=*,E14.8**,RNEW=*,E14.8)

C
ROT = RBAR(IBS)/2.0
SQR=SQRT(ROT*ROT + 1.00/DT)
CNUC1,IBS)=0.00
DO 20 J=2,JTOT
   Y = TYLOC(J) + YO
   ALPHA = SQR * Y
   SINHA = (EXP(ALPHA) - EXP(-ALPHA))/2.0
   COSHA = (EXP(ALPHA) + EXP(-ALPHA))/2.0
   CNUJB = SINHA/(SQR*COSHA + ROT*SINHA)
   IF(ABS(CNUJB-CNUC(J-1,IBS)) LT 1.0E-07) GO TO 22
   CNU(J,IBS)=CNUJB
   20 CONTINUE
GO TO 24
22 CNU(J,I BS) = CNUJB
   JST = J + 1
   Y = T YLOC(JST) + YO
   ALPHA = SQ R * Y
   SINHA = ( EXP(ALPHA) - EXP(-ALPHA))/2.0
   COSHA = ( EXP(ALPHA) + EXP(-ALPHA))/2.0
   CNUFX = SINHA/( SQR*C OSHA + ROT*SINHA)
   DO 23 J = JST, JTOT
      CNU(J, I BS) = CNUFX
   23 CONTINUE
24 CONTINUE

C

   NSW = 0
   SW(J, I BS) = 0.0
   CNUJ = 0.00
   RDEL = RGR(I BS) - RBAR(I BS)
25 ROL D = RDEL
   CTDEL = CT(J, I BS) - CTBAR(J, I BS)
   CHJ = CTDEL - RDEL*CTYBAR(J, I BS)*DT
   SWJ = 0.0
   DO 31 J=1,3
      NI=NF+1
      NF=NF+NTY(I)
      ALFT = ALFTA(I)
      DO 30 J=NI,NF
         JP = J+1
         CNUJP1 = CNU(JP, I BS)
         CTDEL = CT(JP, I BS) - CTBAR(JP, I BS)
         CHJP1 = CTDEL - RDEL*CTYBAR(JP, I BS)*DT
         SWJP1 = SWJ*(1.00 - ALFT*CNUJ) + ALFT*( CNUJP1*CHJP1 + CNUJ*CHJ)
      1/(1.00 + ALFT*CNUJP1)
         SW(JP, I BS) = SWJP1
         SWJ = SWJP1
         CHJ = CHJP1
         CNUJ = CNUJP1
      30 CONTINUE
   31 CONTINUE

C

   CNUL = CNU(JTOT, I BS)
   SWL = SW(JTOT, I BS)
   SVL = (Z2RP(I BS)*PDEL - ETGH(I BS)*SWL)/(1.0 + CNUL*ETGH(I BS))
   SU(JTOT, I BS) = SVL
  CTSDEL = CNUL*SVL + SWL
   RDEL = ERTS(I BS)*CTSDEL
   IF (ABS(RDEL - ROLD) * LT * 1.0E-06) GO TO 40
   IF (NSW *GT* 15) GO TO 39
   NSW = NSW + 1
   GO TO 25
39 WRITE (6,203) ROLD, RDEL
40 CONTINUE

C
SUBROUTINE COMBNT(XND, KS, KST, HAFL, SMDGL, SMDPL)
COMMON/TIMTRN/DT, DT22, DXA(5), XI SW(5), I DDA(5), ACMB(5), ASMB(5),
1 ASMC(5)
COMMON/NOZDAT/ EL STR, RCHSTR, UT
COMMON/COMFIX/ HAF(200,5), SMDG(200,5), SMDP(200,5), XI FC, HAFC, SMDGC
1 , REACTN, SMDPC
COMMON/MFSFLD/ S B X S(100), A LOC(100), FSMDG(100), DX SBS, NBS, APC(100)
400 FORMAT(* / COMBNT ERROR - 3 POSSIBLE CASE FAILURE - --- , /, 5X,
1*K S*K 12, KST**, I2, **(XST,XSMN D, XMEND, XMBND, X J1, X J2)**, 6E13, 7)
401 FORMAT(* / IN COMBNT - A ONE POINT INTERPOLATION HAS BEEN FORC
2MND, XMBND)**, 215, 2E14, 8, /, 15X, 6E14, 8)
402 FORMAT(* / IN COMBNT - A ONE POINT INTERPOLATION HAS BEEN FORC
2MND, XMBND)**, 215, 2E14, 8, /, 15X, 6E14, 8)
XST=XND+ELSTR
IF(XST.LE.0.0) GO TO 10
SMDGL=0.00
GO TO 200
10 XSM=ELSTR + XST
XSMND=XSM/DXBS
J1=1 + XSMND
XJ1=(J1-1)
XJ2=J1
J2=J1 + 1
IF(KST.GT.1) GO TO 20
15 FSMGD1=FSMDG(J1)
SMDGL=(FSMDG(J2)-FSMDG1)*(XSMND-XJ1) + FSMDG1
GO TO 200
20 CMB=ACMB(KS)
SMB=ASMB(KS)
SMXE = CMB + SMB
SMXB = SMB
XMEND=(SMXE*ELSTR + ELSTR)/DXBS
XMEND = (SMXB*ELSTR + ELSTR)/DXBS
IF(XJ2.LT.XMEND.AND.XJ1.GE.XMEND) GO TO 15
IF(XJ2.GT.XMEND) GO TO 30
IF(XJ1.LT.XMEND) GO TO 40
WRITE(6,400) K, KST, XST, XSMND, XMEND, XMEND, XJ1, XJ2
STOP
30 J0 = J1 + 1
XJO = J0 - 1
IF (XJO.LT. XMEND) GO TO 35
FSMDG1 = FSMDG(J1)
SMDGL = (FSMDG1 - FSMDG(J0)) * (XSMND - XJ1) + FSMDG1
GO TO 200
35 SMDGL=FSMDG(J1)
WRITE(6,401) K, KST, J1, J0, XJ1, XJO, XST, XSMND, XMEND, XMEND
GO TO 200
40 J3 = J2 + 1
IF (J3.GT. NBS) GO TO 45
XJ3 = J2
IF (XJ3.GT. XMEND) GO TO 45
FSMDG2 = FSMDG(J2)
SMDGL = (FSMDG(J3)-FSMDG2)*(XSMND-XJ2) + FSMDG2
GO TO 200
45 SMDGL = FSMDG(J2)
WRITE(6,402) K, KST, J2, J3, XJ2, XJ3, XST, XSMND, XMEND, XMEND
200 HAFL=HAFC
SMDPL = SMDPC*SMDGL
RETURN
END
SUBROUTINE MOCI NY (Y1, Y2, Y3, DXN, Y0)
C1 = Y1 - Y2
C3 = Y3 - Y2
A = 0.500 * (C1 + C3)
B = -0.500 * (3.00*C1 + C3)
Y0 = A*DXN*DXN + B*DXN*Y1
RETURN
END

SUBROUTINE SHDETL(U1, U2, U3, U4, SHDET, NYES)
NYES = 0
V31 = U3 - 2.00*U2 + U1
V41 = U4 - 2.00*U3 + U2
U22 = U2*U2
U32 = U3*U3
U42 = U4*U4
U12 = U1*U1
V32 = U32 - 2.00*U22 + U12
V42 = U42 - 2.00*U32 + U22
U33 = U32*U3
U33 = U32*U3
U23 = U22*U2
V33 = U33 - 2.00*U23 + U12*U1
V43 = U42*U4 - 2.00*U33 + U23
A = V31*V42 - V32*V41
B = V33*V41 - V31*V43
C = V32*V43 - V33*V42
DSHDET = B*B - 3.00*A*C
IF (DSHDET LT 0.000005) GO TO 20
IF (ABS(V31) LT 0.1E-06) GO TO 20
IF (ABS(V41) LT 0.1E-06) GO TO 20
SD = SQRT(DSHDET)
UR1 = (-B + SD)/(3.00*A)
UR2 = (-B - SD)/(3.00*A)
IF (UR1 .GT. U1 .AND. UR1 .LT. U4) NYES = 1
IF (UR2 .GT. U1 .AND. UR2 .LT. U4) NYES = 1
GO TO 20
15 CONTINUE
IF (UR1 .LT. U1 .AND. UR1 .GT. U4) NYES = 1
IF (UR2 .LT. U1 .AND. UR2 .GT. U4) NYES = 1
GO TO 20
20 SHDET = DSHDET
RETURN
END
SUBROUTINE ITSUB(FOFY, Y, SAVE, CONV, NTIMES)
DIMENSION SAVE(8)
N1=SAVE(3) + 1
FOFXCK=SAVE(8)
FOFX=FOFY
X=Y
IF(ABS(FOFX)-CONV.LE.0.) GO TO 110
TIME=SAVE(1) + 1
GO TO (10, 30, 50, 70), TIME
10 N1=1
TIME=2
FOFXCK=FOFX
SAVE(6)=FOFXCK
IF(FOFX*LT.0.) GO TO 50
30 IF(FOFX*LT.0.) GO TO 70
IF(FOFXCK*GE.0.) GO TO 35
SAVE(2)=-1.*SAVE(2)
X=X-2.*SAVE(2)
GO TO 90
35 SAVE(4)=X
SAVE(5)=FOFX
X=X-SAVE(2)
GO TO 90
50 TIME=3
IF(FOFX*GT.0.) GO TO 70
IF(FOFXCK*LE.0.) GO TO 55
SAVE(2)=-1.*SAVE(2)
X=X+2.*SAVE(2)
GO TO 90
55 SAVE(6)=X
SAVE(7)=FOFX
X=X+SAVE(2)
GO TO 90
70 TIME=4
N1=SAVE(3)
IF(FOFX*LT.0.) GO TO 75
SAVE(4)=X
SAVE(5)=FOFX
GO TO 80
75 SAVE(6)=X
SAVE(7)=FOFX
80 X=SAVE(4)-SAVE(5)*((SAVE(6)-SAVE(4))/(SAVE(7)-SAVE(5)))
90 IF(N1*GE. NTIMES) GO TO 100
N1=N1+1
SAVE(3)=N1
GO TO 120
100 TIME=6
GO TO 120
110 TIME=5
SAVE(4)=X
SAVE(5)=FOFX
SAVE(6)=X
SAVE(7)=FOFX
120 SAVE(1)=FLOAT(TIME)+0.1
Y=X
RETURN
END