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- STRAHLE W C  
- JAGODA J I

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Title: FRACTAL IMAGE COMPRESSION OF RAYLEIGH, RAMAN, LIF AND LW DATA IN TURBULENT....

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INITIATION OF AFOSR PROJECT E-16-693. SEE AFOSR GRANTS MANUAL DATED 11/85.
FOR INFORMATION ON ADMINISTRATION OF GRANT.

INITIATION OF AFOSR PROJECT E-16-693. SEE AFOSR GRANTS MANUAL DATED 11/85.
NOTICE OF PROJECT CLOSEOUT

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Project No. E-16-693
Center No. R6418-0A0
Project Director STRAHLE W C
Center No. R6418-0A0
School/Lab AERO ENGR
Sponsor AIR FORCE/BOLLING AFB, DC
Contract/Grant No. AFOSR-88-0001
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Title FRACTAL IMAGE COMPRESSION OF RAYLEIGH, RAMAN, LIF AND LDV DATA IN TURBULENT
Effective Completion Date 900930 (Performance) 901130 (Reports)

Closeout Actions Required: 

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Comments

Subproject Under Main Project No. 

Continues Project No. 

Distribution Required:

- Project Director: Y
- Administrative Network Representative: Y
- GTRI Accounting/Grants and Contracts: Y
- Procurement/Supply Services: Y
- Research Property Management: Y
- Research Security Services: N
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NOTE: Final Patent Questionnaire sent to PDPI.
This interim report covers progress in experiment and analysis on turbulent reacting flows, with experimental emphasis on data obtained in a two dimensional windtunnel with a backward facing step, hydrogen injection and combustion. Analytical modeling of the flow, Raman spectroscopy measurements and fractal geometry applications in data analysis are discussed.
Experiments and analysis were carried out for data base generation, comparison of theory and experiment and new fractal-based analysis methods on a complex turbulent reacting flow. The experimental configuration was a two-dimensional backward facing step windtunnel with hydrogen injection from the floor behind the step. Efforts during the reporting period included Raman spectroscopy measurements, completion of a finite rate kinetics calculation of the turbulent flow field and initial analysis of turbulent reacting flow data by methods based on fractal geometry.

During the present reporting period the Spontaneous Raman system was brought on line. The system was tested by measuring the concentration of nitrogen in a variety of cold jets of different nitrogen concentrations. The experimentally determined mean nitrogen concentrations agreed to within better than 1% with the known levels of nitrogen in the jets. In fact, the Raman system is now being "calibrated" in this fashion before every run. However, the fluctuations in the Raman output were unacceptably high (> 15%) even when species concentrations in the jets were kept steady. Since the RMS value of the species concentration and the concentration-velocity correlations in the flow are required to fully describe the turbulent mixing it was essential that the noise contribution to the RMS be reduced. Tests showed that the noise was essentially caused by statistical limitations related to
the number of Raman shifted photons that could be collected and the efficiency of their conversions to an electrical signal. After considerable effort to increase the laser output power and to improve the alignment and the collection optics the noise level was reduced to about 4%.

Preliminary Raman measurements have been carried out in the hot flow. The results have shown excellent reproducibility. As expected, the amount of nitrogen decreases as one approaches the plate. This is, in part, due to an increase in temperature (decrease in density) and, in part, due to the presence of other species (bleed gases, combustion products, etc.). These two effects will become separable once the local temperatures have been measured. Measurements of mean and RMS concentrations at other stations are currently under way. These will be followed by the determination of local temperatures and of velocity-concentration correlations.

Computational modeling, began during a prior program, finished a major phase by incorporation of a finite rate kinetics method into the turbulent reacting flow calculation. The conclusion was that, at least for the \( \text{H}_2 - \text{air} \) case in this configuration, the drop in computed temperature would not amount to more than 50°C compared with the assumption of local equilibrium. Future computational efforts will center on the theoretical-experimental discrepancy in reattachment length which was indicated in the final report on the preceding program.

Fractal geometry concepts were applied to some representative data in Rayleigh scattering from a hot flow and hot film data in a cold flow. Areas being concentrated upon are a) recreation of data in sparse data rate situations as are encountered, for example, in pulsed Raman spectroscopy, b) new visual representations of data by a new development in the creation of a
multifractal probability density function and c) use of the fractal character of the data to improve signal to noise ratio in some situations.

The multifractal pdf concept has already resulted in the creation of a new type of digital filter, whereby certain noise is removed if it does not have fractal content known to be characteristic of turbulence. A second demonstration made during the past year is that the ordinary pdf of turbulence data may be markedly improved over that obtained from sparse data through use of the fractal characteristics of the data.

Current efforts are being directed toward the use of fractal geometry in analysis of joint, correlated data. Again, the purpose is to improve correlation estimates for heat, mass and momentum transport in sparse data situations.
This interim report covers progress in experiment and analysis on turbulent reacting flows, with experimental emphasis on data obtained in a two dimensional windtunnel with a backward facing step, hydrogen injection and combustion. Analytical modeling of the flow, Raman spectroscopy measurements and fractal geometry applications in data analysis are discussed.
Experiments and analysis were carried out for data base generation, comparison of theory and experiment, and new fractal-based methods of data reduction on a complex, turbulent reacting flow. The experimental configuration was a two-dimensional backward facing step windtunnel which simulates the flame stabilization region of a solid fuel ramjet. Hydrogen along with several diluents were injected from the floor behind the step and burned with the air entering over the step. Efforts during the reporting period included Raman spectroscopy measurements with several diluents in the fuel, generation of a new computational scheme to include the diluent effect, and development of new data reduction techniques based upon fractal geometry.

During the present reporting period, work on the spontaneous Raman measurements was continued. In this technique the local concentration of nitrogen is determined by measuring the intensity of the nitrogen Stokes line.

The system was calibrated against a platinum - platinum rhodium thermocouple using the combustion products of a hydrogen air flame mixed with considerable quantities of excess air. This produces a hot stream of gas having essentially the same nitrogen concentration as air. In this case, the density of nitrogen may be considered to be inversely proportional to the temperature of the stream. The temperatures measured using Raman exceeded those determined with the thermocouple by about 50 degrees. This discrepancy was to be expected since the thermocouple results were not corrected for radiative losses.
Next, a pane of quartz was inserted between the Raman system and the calibration flame in order to simulate the window of the backward facing step facility. With the window in place, the apparent temperature measured using the Raman system decreased by some 300 degrees. Subsequent recalibration of the Raman optics using gas streams carrying known quantities of nitrogen revealed that the window caused a significant amount of stray light to enter the detection optics. This excess light is added to the nitrogen Stokes line intensity from the combustion products as well as that from the cold flow. This reduces the ratio between these two nitrogen Stokes lines which is used to calculate the local temperature is proportional.

The amount of stray light caused by the window was determined by comparing the Raman measured nitrogen concentration of a cold air stream with the known composition of air. When the thus determined stray light level was subtracted from the Raman scattered intensities of the cold and hot air streams, before their ratio was formed, the correct temperature was, once again, obtained. In addition, the level of stray light scattered by the window was directly measured by observing the intensity of light measured by the detector at the wavelength of the Raman Stokes line of nitrogen when the calibration stream was replaced by pure argon. The level of stray light determined by both techniques were found to be in good agreement.

The procedure for carrying out Raman measurements in the tunnel was, therefore, modified. At each location, the intensity of light scattered by the window is determined by introducing a stream of pure argon into the probe volume. This background noise is then subtracted from the Stokes signals with and without combustion before the local concentration of nitrogen molecules is determined from their ratios.
Raman Stokes line intensities of nitrogen without and with combustion were measured at various downstream locations at small intervals of height above the bleed plate. Since the local ratios of these intensities are a measure of the relative concentration of nitrogen molecules in the hot flow it was necessary to determine to what degree the observed fluctuations in nitrogen levels are due to fluctuations in temperature and to what extent they are caused by mixing of the inlet air with the fuel and its diluent which was argon in these tests. For this purpose, the argon diluent was replaced by nitrogen. A comparison of the measured Stokes signals obtained with the two diluents revealed no measurable changes in nitrogen concentration at the locations, hereto, investigated. Thus, it seems that increases in temperature rather than dilution by the gas flow through the bleed plate are responsible for the observed decrease in nitrogen concentration. The Stokes line intensities can, therefore, be used to determine the local temperature fluctuations directly using an equation of state.

As mentioned in last year's annual report, Shot noise in the photomultiplier resulted in a non-negligible contribution to the RMS of the Raman signal even when the level of nitrogen was kept constant. Since this RMS is a measure of the local turbulent mixing a technique had to be developed to remove the Shot noise contributions from the signal. A calibration curve of Shot noise versus mean intensity of light reaching the detector was, therefore, generated. The data acquired in the tunnel are now corrected by calculating the mean Stokes line intensity at that location, reading the shot noise RMS off this curve and subtracting it from the measured RMS. The validity of this technique was verified by generating a simulated Raman signal having statistical properties equal to those of a typical true Raman signal. The simulated signal was then "contaminated" point by point using a randomly generated pdf with an RMS
value obtained from the RMS vs. mean intensity curve. The RMS of the new, "contaminated" simulated Raman signal was then determined. When the RMS of the noise was subtracted from the that of the "contaminated" pdf the original RMS was obtained to within better than 1% for a wide variety of cases.

Two important results have been gained by computational efforts. First, a bifurcation in the flame position has been found. That is, there are computationally two stable flame positions found numerically, separated by a critical fuel flow rate. In the experiment both flames are seen, one on the floor of the tunnel and one in the shear layer. In practice, the second is of most practical interest. The second important finding was guided by the experimental result that the reattachment length of the recirculation zone increased with burning while the computation was shortening it. Here again, there are two computationally allowed solutions, one with a short reattachment length and one which is long. The crucial step in the computation is to start out the iteration for convergence by assuming a long recirculation zone, so that, as in the experiment, all fuel is injected into the recirculation zone rather than escaping from it. The latter is what happens in the experiment.

Efforts continued on the development of fractal geometry uses in data reduction of turbulence data. Incredible accuracy enhancement of decimated, but correlated, data has been achieved. Efforts were expended to theoretically explain why these results were achieved, and the reason lies in the field of chaotic dynamics. What was happening during data augmentation by fractal interpolation was the creation of a chaotic dynamics representation of the data. It has been found in other fields that such a representation gives excellent statistical properties of data. Appended is AIAA Paper No. 90-759 which describes efforts on this program in fractal geometry as applied to turbulent combustion data analysis.
Hidden variable fractal interpolation was investigated as a means for analysis of joint properties of correlated data. This technique will be put to use when the experiment yields joint Raman-LDV data.
This paper investigates several types of data analysis, based upon fractal geometry concepts, using time series generated in turbulent combustion research. The techniques are quite general and may be used for other turbulent flows. The data selected for this paper is generated in constant density turbulence, such as in Refs. (1)-(3), it is reasonable that it should find even greater use in description of turbulent reacting flows.

Fractal geometry has, in fact, found initial introduction into analysis of turbulent premixed combustion and turbulent diffusion flames. Several workers (too numerous to cite here and not particularly relevant to the points which are to be made in this paper) have used the calculation of fractal dimension in description of experimental results. As a descriptor of the "wiggliness" of some picture, the fractal dimension (to be quantified in more detail below) has become a common quantity to present, in addition to other detail such as spectra and probability density functions (pdf).

Turbulence is a chaotic motion of a fluid for which fractal geometry becomes a natural tool of analysis. In fact, the trend is toward analysis of turbulence as not quite so chaotic (random) as previously thought, but with more determinism in analysis. This has led the author toward application of fractal geometry to time series analysis and filter applications in Refs. (7)-(8). Those works were preliminary in nature and will be expanded upon in this paper.

This paper is concerned with several techniques of fractal geometry application to time series analysis. Turbulent combustion is the vehicle used for generation of the time series.

INTRODUCTION

Time series or instantaneous space distributions obtained from turbulent combustion data analysis are rich in complexity. There are simply more physical phenomena occurring in such flow fields than in those of constant density, fixed composition fields as in incompressible turbulence. Since fractal geometry has found its uses in describing fluid quantization in constant density turbulence, such as in Refs. (1)-(3), it is reasonable that it should find even greater use in description of turbulent reacting flows.

Fractal geometry has, in fact, found initial introduction into analysis of turbulent premixed combustion and turbulent diffusion flames. Several workers (too numerous to cite here and not particularly relevant to the points which are to be made in this paper) have used the calculation of fractal dimension in description of experimental results. As a descriptor of the "wiggliness" of some picture, the fractal dimension (to be quantified in more detail below) has become a common quantity to present, in addition to other detail such as spectra and probability density functions (pdf).

The data presented are "wiggly". They are chaotic with broad band power spectra and pdf which are smooth and broad (except for the rectangular pulse train which has two near-delta functions at the upper and lower mean positions). Actually, the pdf for both the Rayleigh scattering trace and the rectangular pulse train show bi-modal properties reminiscent of, say, a temperature trace which would be seen in a premixed flame of BML structure. The Raman trace shows near Gaussian structure as does the hot film trace, more closely resembling a pdf in cold flow turbulence. This information is only presented for the curious reader. From now on, only the fractal geometry of the traces and the uses of fractal geometry in data treatment are of interest.

ABSTRACT

This paper investigates several types of data analysis, based upon fractal geometry concepts, using time series generated in turbulent combustion research. The techniques are quite general and may be used for other turbulent flows. Analysis of the data selected for this paper is generated in constant density turbulence, such as in Refs. (1)-(3), it is reasonable that it should find even greater use in description of turbulent reacting flows.

Fractal geometry has, in fact, found initial introduction into analysis of turbulent premixed combustion and turbulent diffusion flames. Several workers (too numerous to cite here and not particularly relevant to the points which are to be made in this paper) have used the calculation of fractal dimension in description of experimental results. As a descriptor of the "wiggliness" of some picture, the fractal dimension (to be quantified in more detail below) has become a common quantity to present, in addition to other detail such as spectra and probability density functions (pdf).

Turbulence is a chaotic motion of a fluid for which fractal geometry becomes a natural tool of analysis. In fact, the trend is toward analysis of turbulence as not quite so chaotic (random) as previously thought, but with more determinism in analysis. This has led the author toward application of fractal geometry to time series analysis and filter applications in Refs. (7)-(8). Those works were preliminary in nature and will be expanded upon in this paper.

This paper is concerned with several techniques of fractal geometry application to time series analysis. Turbulent combustion is the vehicle used for generation of the time series.

DATA SETS

Four sets of data are used for analysis, three real and one contrived. Shown in Fig. 1 are these time series, two taken from a turbulent premixed flame, one from a subsonic ramjet combustor simulator and the last generated by a random number generator. The apparatus used for the first two, the Rayleigh scattering trace and the hot film anemometer trace, is described in Ref. (8). The apparatus used for the Raman scattering trace is described in Ref. (10). The last trace is a train of rectangular pulses with random arrival times with the tops and bottoms contaminated with high frequency noise of a uniformly distributed pdf with a peak to peak of 10% of the mean peak amplitude.

The most important point concerning these data sets is that the neighboring points of Figs. 1a, 1b and 1d are close enough to be correlated with one another, whereas the data points of Fig. 1c (the Raman trace) are not. The time series of Fig. 1c were taken by a pulsed laser with a repetition time of 0.2 sec, which was far too long for significant correlation between data points (the correlation coefficient between points was .017 for 1024 points used in the calculation). For viewability Fig. 1 has all data points connected by straight lines. The actual points were from digitized data.

The data presented are "wiggly". They are chaotic with broad band power spectra and pdf which are smooth and broad (except for the rectangular pulse train which has two near-delta functions at the upper and lower mean positions). Actually, the pdf for both the Rayleigh scattering trace and the rectangular pulse train show bi-modal properties reminiscent of, say, a temperature trace which would be seen in a premixed flame of BML structure. The Raman trace shows near Gaussian structure as does the hot film trace, more closely resembling a pdf in cold flow turbulence. This information is only presented for the curious reader. From now on, only the fractal geometry of the traces and the uses of fractal geometry in data treatment are of interest.
Although it is also irrelevant for the main purposes of this paper, the interested reader may wish to know the sampling rate of the data in Figs. la-ld. Fig. la was obtained at 100,000 data points per second, Fig. lb was sampled at 30,000 per second, Fig. lc, because of a pulsed laser, was gained at 5 per second, and Fig. ld was corrupted, with the “hash” at the first peak using 1/50 of the distance between the first two peaks and the same “time” separation for all “hash” on all peaks and troughs thereafter.

The lines on the piece of paper for Fig. 1 have topological dimension of unity, i.e. $y(t)$ with the ordinate and t the abscissa is a one dimensional graph. If one filled in the entire graph with ink to make a black area, the topological dimension would be two, an area. The entire idea of what follows is that the traces are so “wiggly” that they do not have a real dimension of unity, but that their apparent dimension lies between one and two.

**ANALYSIS**

**Generalized Dimension**

Consider a digitized time series so that at each time $t_i$, $i=0,1,\ldots,N$ there corresponds a value of the ordinate $y_i$. This is shown by example in Fig. 2a. Consider also that the entire graph is covered by boxes of width and height $\epsilon$ (although not necessary, one may presume that both coordinates have maximum amplitude of unity by appropriate normalization). Consider further throwing away all boxes not containing a data point, which yields $M$ boxes containing data, and label each box by an integer $m$. For each of the $M$ boxes one may define a probability $p_m$ that the mth box contains a point, $p_m = M/m$. The generalized dimension, $D_q$, is then defined by

$$D_q = \lim_{\epsilon \to 0} \frac{\log(p_m^q)}{(q-1)\log(\epsilon)}$$

In practice a finite data record is available, so the limit $N=\infty$ cannot be taken. Moreover, it is clear that $\epsilon \to 0$ is meaningless in practice since below some limit each of the $M$ boxes would contain one point and the numerator of Eq. (1) would no longer change with a change in $\epsilon$. However, if the derivative of the numerator with respect to $\log(\epsilon)$ is taken there results

$$(q-1)D_q = \log(p_m^q)/\log(\epsilon)$$

That is, a straight line with slope $(q-1)D_q$ should result on a log-log plot if the generalized dimension exists over a range of $\epsilon$.

What often results in turbulent combustion time series is that over a range of $\epsilon$, termed the inner and outer cutoffs, a straight line does occur. This is schematically depicted in Fig. 2b for $q=0$. This very important $q$ value yields $D_0$ and is denoted the fractal, capacity or similarity dimension. For $q=0$, $p_m^0$ is unity and the sum in the numerator is merely the number of covering boxes, $M$. In practice, in order to avoid voids between boxes, the data points may be connected by straight lines and the now continuous curve covered by boxes which always touch each other. This only works for $D_0$, however.

An alternative computational procedure, which has been found to yield answers very close to a box counting procedure, is to first pick an $\epsilon$ along the time axis. Second, search the data string between $t_i$ and $t_i+\epsilon$, finding the maximum and minimum $y$ values in this interval. Approximately (usually very close), the number of boxes, $n_1(\epsilon)$, in this interval is

$$n_1(\epsilon) = (y_{max} - y_{min})/\epsilon$$

and the total number of boxes is obtained by summing over all $\epsilon$-intervals. The error at each $i$ is a fraction of unity in the integer number of boxes which would be exactly calculated.

There are several other definitions and ways of computing dimensions, as may be found, for example, in Ref. (13). For some of the calculations the error bounds have been estimated in Ref. (14). However, in this paper, only the methods described above have been used.

**Fractal Dimension and the Multifractal pdf**

The fractal dimension of the curves in Figs. la and lb has previously been reported. The fractal dimension of the curves in Figs. lc and ld have been computed by the above method and the results are compiled here.

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These dimensions reveal the space-filling nature of the curves. Although the first and second traces look quite different, and represent different physical phenomena, the blackening of a piece of paper are about the same. The third curve is not as space-filling as the first two or the fourth. The Raman curve, containing mostly uncorrelated points, has a high fractal dimension, suggesting the obvious. Presentation of a single number such as the fractal dimension is not particularly revealing, just as presentation of simply the turbulence intensity leaves out a lot of detail. Most research workers might be more interested in the pdf of turbulence velocities; analogously the computation of the pdf of fractal dimensions turns out to be very revealing. This pdf will be called the multi-fractal pdf and measures the distribution of local fractal dimensions. This calculation is described in Ref. 8, but the method is repeated here.

What one is trying to do is to calculate the local slope of $\log(\epsilon)$ with respect to $\log(\epsilon)$. First calculate $n_1(\epsilon)$ from $t_i$ to $t_i+\epsilon$. Do it again from $t_{i+2}$ to $t_{i+2}$, and $t_{i+1}$ to $t_{i+2}$. Call the first two $n_1$ and $n_2$ and the last one $n_3$. The local fractal dimension is

$$D_{local} = \log(n_3) - \log(n_1^n_2))/\log(2)$$
Then construct a pdf of these local results from the total time series. Results of this calculation from 4096 points of the Rayleigh scattering trace are shown in Fig. 3 for various values of \( \epsilon \). At low values of \( \epsilon \) the pdf bunches about unity, whereas at high values of \( \epsilon \) the pdf gathers about two. But in an intermediate range of \( \epsilon \) the pdf is quite stable; it is in this range that the curve has fractal-like behavior with a fractal dimension, on average, of about 1.55.

A quite different picture is presented in Fig. 4 for the Rayleigh scattering trace with an average fractal dimension of about 1.7. This occurs because neighboring points on the Rayleigh scattering trace are uncorrelated with one another. The original time series is largely a space-filling curve.

Aside from giving a new pictorial representation of the complexity of the time series there is a technological use for the local fractal dimension. A local calculation of this dimension is shown for the short segment of the Rayleigh scattering trace in Fig. 5. Notice that in regions of large moves in the time series (the spikes in the trace) the fractal dimension is low, whereas in the noisy parts of the trace (the troughs) the fractal dimension is large. If one were to take a Fourier spectrum of this time series, both the spikes and the noise in the troughs would contribute to the high frequency portion of the spectrum. The local fractal dimension, however, distinguishes between the two types of events. It is known from the physics of the situation that the spikes are real, wanted events (they are cold spots in the turbulent flame), whereas the noise in the troughs is flash noise from the photomultiplier and is unwanted contamination of the data. A natural conclusion, therefore, is that the local fractal dimension may be used as an indicator for signal filtration between different types of high frequency events.

Fractal Filtration

A common technique for filtering out noise from a time series, in addition to analog filtration of the original signal, is to use some form of digital filtration. But such filtration is not discriminatory with different types of high (or low) frequency events. Monitoring of the local fractal dimension provides such discrimination, as has been pointed out in Ref. (7).

The use of a discriminator along with a linear digital filter creates a nonlinear filter. Any digital filter may be used, but here a simple first order lag recursive filter will be used as a demonstration. Consider that the trace, here chosen as Fig. 1d, is to be filtered by

\[
y_{i, \text{filtered}} = \alpha y_{i}(t) + (1-\alpha) y_{i-1}, \text{filtered}
\]

where the filter parameter is chosen by the user. Using \( \alpha = 0.05 \) this filter is applied to Fig. 1d, and the results are shown in Fig. 6a. A well known problem results in that, while it is true that the noise has been removed from the peaks and troughs, the vertical character of the rectangular transitions is lost. Moreover, the narrow troughs are completely lost and in general there is a phase lag introduced with this filter. The filter has shown no discrimination between two types of high frequency events.

Now consider that 32 points to the left of \( t_i \) and 32 points to the right are used to compute the local fractal dimension. The following formula (a function linear in \( D_0 \) is then applied to pick a local value of the filter parameter, \( \alpha \):

\[
\alpha = (D_0 - 1.45)*0.73 + 0.05 \quad \text{if } D_0 \leq 1.45
\]

\[
\alpha = (D_0^2 - 1)*0.11 \quad \text{if } D_0 > 1.45
\]

This allows no filtration if \( D_0 \leq 1 \) and no pass condition if \( D_0 \geq 2 \). The results are shown in Fig. 6b. In this case, it is seen both the sharp moves are retained, there is no phase lags introduced and the noise on the peaks and troughs are largely removed.

There are other nonlinear filters designed to cope with the rapid changes of the large moves, e.g., the median filter. If one goes to a given number of points both before and after the central point of issue (in this case five) and chooses the median value of the \( y \)'s obtained as the filtered point, one can catch the large stepwise moves, as shown in Fig. 6c. However, the median filter does not do as well as the adaptive fractal filter in removing the noise in the tops and troughs.

In the examples given, the points chosen for both nonlinear filters were chosen equidistant from the central point of interest. This presumes that at real time only data points before the one of interest can be used for processing. This will delay the onset of recognition of a coming low (or high) fractal dimension event (or, in the case of the median filter, a large step). In the case of the fractal filter, this would allow some "hash" to appear right after the large step change. As with any filter, the practitioner must adjust the filter parameters and the number of data points used in the discrimination process to achieve the best results. At this time, it is reiterated that the fractal discrimination can be used with any digital filter, and this is also at the choice of the practitioner.

While there appear several uses for the use of fractal analysis in filtration, perhaps also in the cleaning up of images produced by fractal image compression, it currently appears that it is most useful in discrimination between wanted and unwanted high frequency events. It has been employed for precisely this problem for the Rayleigh trace of Fig. 1b in Ref. (7). This trace bears a strong resemblance to the model trace of Fig. 1b in frequency characteristics. Further work is warranted to find applications for such an adaptive filter.

Multifractal Spectrum

Just as the multifractal pdf gives information about the distribution of \( D_0 \) along a time series, this development was preceded by a concept of the multifractal spectrum by Halsey et al. This has been applied to the analysis of the dissipation field in incompressible turbulence in Ref. (3). The basic idea is that along a time series there is a distribution of scaling laws whereby the \( p_m \) of Eq. (1) behave as \( p_m \alpha \) and that the \( \alpha \) behave with a probability distribution.
proportional to $e^{-f(a)}$. The function $f$ then acts as a fractal dimension corresponding to a value $\alpha$ found along the time series. Insofar as the author can see, this is a postulate to be tested for real time series. It is stated in Ref. (18) that it is hoped "to encourage experiments along these lines". In any event, the technique has been explored for the traces of Figs. 1a and 1b to test the utility of the method.

For a full development of the theory the reader is referred to Ref. (18). However, $f(a)$ and $\alpha$ are related to the generalized dimension, $q$, introduced above. If one can compute $D_q$ from Eq. (1) the following two formulas allow construction of the $f(a)$ vs. a curve:

$$D_q = \frac{q(a) - f(a(q))}{(q-1)} \quad (2)$$

$$\alpha = \frac{q}{d} \cdot (q-1) \quad (3)$$

However, in the authors view, there are two critical assumptions made in the derivation. First, it is assumed in a method of steepest descent that $q=\alpha$ is "sufficiently" small while the probability distribution for $a$ contains no "holes" (finite values). Secondly, it is assumed that $f$ is everywhere concave when viewed from low $a$. No such presumptions are made in the multifractal pdf introduced above. It is a consequence of the theory that the maximum of the $f-a$ curve gives $f = 0$, so that there will nowhere appear a local fractal dimension greater than $D_0$. It will be noted that the computation of Eq. (1) must proceed by calculation of probabilities and none of the approximations used in calculation of $D_0$ will work. For $q=0$, small $p_m$ will yield large contributions to the sum, and, moreover, small numbers of points in a box yield a great statistical uncertainty in the calculation of $\alpha$. That is, on rarefied portions of the time series are made the largest contributions to the generalized dimension for negative $q$. The opposite is true for positive $q$. These facts pose a computational difficulty for a single valued digitized data time series as $a$ is varied. It matters not whether the data are digitized at fixed time intervals or at random time intervals (if the randomness is not tied to the physics of interest). There will come a point for negative $a$ at small enough $\epsilon$ that the data cannot be believed, and finally, at small enough $\epsilon$, $p_m$ will be zero, which will not work for $q>0$.

An opposite problem occurs for large positive $q$. The dominant contribution to $D_q$ for positive $q$ is on regions of the curve densely populated by points. This causes problems at large $\epsilon$, where an increase in $\epsilon$ by a factor of two, say, tends to increase $p_m$ by a factor of four for a given box because the whole graph is dense. On the other hand, at low $\epsilon$ the best answers are obtained because only a few boxes are densely populated and contribute to the sum of Eq. (1). Consequently, one has to look for the generalized dimension at high negative $q$ at relatively high $\epsilon$ and for high positive $q$ at relatively low $\epsilon$. The $D_0$ is still obtained from straight lines on a log-log plot.

This is illustrated in Fig. 7 for 16384 points of the Rayleigh trace of Fig. 1b (where only 4096 points are shown). Clearly, some judgement must be applied.

This analysis has also been applied to the data of Fig. 1a. Applying Eqs. (2) and (3) to the obtained $D_q$ values, the $f-a$ plots of Fig. 8 result. For both curves, within experimental error, the maxima indeed correspond to $D_0$. The intercepts with the $a$-axis would correspond to $D_0$ on the left and $D_{-\infty}$ on the right. That is, in both rarefied and compact regions regions of the time series, the implication is that the fractal dimension is zero. This is counter to the revelations given above by the multifractal pdf. Moreover, there is no real structural difference in the two $f-a$ curves to give an indication of a visual difference between the two data traces. This is believed to arise because some of the above mentioned assumptions concerning the derivation of the method. While the Rayleigh trace seems to be richer in high $\alpha$ content, this may be an artifact of some numerical error [note that a numerical derivative must be taken in Eq. (3)]. Finally, for a real fractal on a two dimensional piece of paper (a Henon map or a Poincare section of a multidimensional motion, for example), the separation between points in both vertical and horizontal directions has a connection with the real fractal properties of the figure. In the case of a time series digitized at fixed time intervals or at intervals not connected with the physics the local fractals has nothing to do with the fractal-like properties of the figure. Consequently, the $a$-scaling is distorted in one of the dimensions.

The conclusion of the author is that while the multifractal spectrum may give some insight into true fractals (recall here that the time series are only fractal-like between an upper and a lower time scale), there is little to be gained with this analysis in turbulent combustion data series. What is more, the computation involved is time consuming. It is acknowledged that this conclusion may be somewhat controversial.

**Fractal Interpolation**

A serious question concerning statistical accuracy arises in many experiments which provide a sparse number of data points along a time series. This situation may arise, for example, because of expense or hazard in long run times, or in slow repetition rate of, say, a pulsed laser. Two problems are involved: there is a problem of obtaining correlation of neighboring points and, secondly, one of obtaining a reliable, statistically accurate pdf. Addressing the second point first, the question is asked, is it possible to augment the obtained data by an appropriate interpolation scheme, which retains physics, and yields more accurate pdf information such that moments of the pdf can be believed?

Typically in turbulence one is interested in no more than a fourth moment of a pdf. But even at the second moment (variance or covariance), much less at a fourth moment (kurtosis or fourth order correlation) tails on the pdf may have a zero value of the probability. Statistical uncertainty is large and noise may be dominant. At high moments, however, these tails contribute significantly to
the answer. It would be nice if a method of at least decreasing the statistical uncertainty were available by increasing the number of available data points near these tails. Fortunately, there does appear a method, based upon fractal geometry, to accomplish this goal.

Consider the situation of Fig. 9 where certain data points are known, but they are too sparsely spaced to clearly define the true experimental curve. If it is known that the curve is fractal-like over some range of time scales, it appears that an interpolation between points by a fractal would be superior to, for example, connecting the data points by straight lines. Fortunately, there is now a technique which allows a rational interpolation by fractals which, by the way, contains straight line interpolation as a special case (it is known as the "condensation set" of the map).\(^{(19,20)}\)

There are many ways to implement the procedure, and the one to be presented is only one way. It was the method found by the author on the first try, but other methods may be invented by other workers. The strategy is shown on Fig. 10. Given a time series, divide it up into \(N\) (even) intervals, the interval number being denoted by \(n\). To each endpoint of the interval there is a data point (assumes equidistant time spacing of data points) through which it is demanded that the fractal curve fit will pass. At the midpoint of each interval there is also assumed a data point through which the curve will pass, but these points are called target points. The target points will determine some parameters of the fitting procedure.

Assume, for the moment, that the algorithm below will converge to a line on a piece of paper that passes through all data points. Say that one is in interval \(n\) and a random number generator has picked a new \(n\) with uniform probability over the interval \(0\leq n< N\). Associated with the old \(n\) is \(y_0\) and \(t_0\); a new \(y\) and new \(t\) are generated by the following affine (linear) relation:

\[
y_n = b_n (t_n - t_0) + C_n (y_n - y_0) + y_0
\]

where

\[
b_n = [y_n (y_n - 1 - C_n (y_n - y_0))] / (y_n x_0)
\]

The \(N\) values of the constants \(C_n\) are free parameters which are to be chosen. Provided \((19,20)\)

\[R[C_n] > 1\] for all \(n\) \(|C_n| < 1\) (the transformation, or map, is contractive), the procedure will generate a continuous (but not necessarily differentiable), unique fractal curve passing through all \(y, t\) points, where these have been chosen as data points at the interval end points. This procedure still does not guarantee passage through the target points; a choice of the \(C_n\) is required to do this.

The \(t\)-transformation of Eqs. (4) takes an old \(t\) and moves \(1\) to interval \(n\) in the same fraction of distance from the left endpoint of interval \(n\) to the right point of interval \(n\) as \(t\) was from \(n = 0\) to \(n = N\). Consequently, if the central data point is designated as special, as it is on Fig. 10, it will always map to the central point of any chosen interval \(n\). Then choose the \(C_n\) so that the \(y\) map of Eqs. (4) always maps \(y_n/2\) into each target point for each \(n\). One then has a fractal curve (called an attractor of the map) which hits all data points. The fractal dimension of the curve may be estimated by the implicit formula\(^{(19)}\)

\[R[C_n] (C_n - t_n - 1)^{1/2} = 1\]

Under this procedure the formula for determination of the \(C_n\) is

\[
y_n = (1/2) (y_n x y_n - 1)
\]

If the map is contractive, the fractal dimension may then be determined by Eq. (5). A problem is that the \(C_n\)'s may not all have magnitude less than unity with this procedure. They will if the \(y\) values of the endpoints \(n=0\) and \(n=N\) are high (low) enough while the midpoint at \(n=N/2\) is low (high) enough so that the denominator of Eq. (6) has a magnitude which is sufficiently high. This requires careful selection of the data trace by the practitioner, and the data may be sufficiently sparse that this freedom is not available.

The above procedure has been applied to the data trace of Fig. 1b. First, 4096 data points were decimated to 512 data points, because good definition of the curve still resulted. More precisely, even with this decimation neighboring points are highly correlated. This 512 point trace was then taken as the data trace "standard" against which all approximations are to be tested. Shown in Table 1 on the first line are the various statistical moments of this actual trace. In this case \(p\) is the autocorrelation coefficient at zero time delay which is obviously unity. In other cases to be investigated \(p\) is the cross correlation coefficient at zero time delay of the approximate curve against the standard, exact curve. The nearness to unity is a measure of the accuracy of the data fit. The nearness of the various statistical moments of the approximations compared with those of the exact curve are of more engineering use. That is, while it would be nice to hit the standard curve exactly, there are reasons why this cannot be expected, but the useful information (the moments) can be reproduced.

<table>
<thead>
<tr>
<th>Table 1 Fractal Analysis of Rayleigh Data</th>
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<tbody>
<tr>
<td>Full Data</td>
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<tr>
<td>Truncated Data</td>
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<tr>
<td>Fractal Fit</td>
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<tr>
<td>Straight Line</td>
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<td>Fractal Data</td>
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<td>Fractal Fit</td>
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<td>Fractal Data</td>
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<td>Fractal Fit</td>
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The results of Table 1 are rather remarkable. If one only had one-fourth of the real data, all of the statistical moments are badly estimated by
The estimates become even worse through two further decimations by a factor of two. The fractal fit for 128 data points is shown in the upper curve of Fig. 11. The fractal dimension computed by Eq. (5) is 1.45, quite close to the dimension calculated from many more points. From Table 1 it is seen that a remarkable improvement in the moments is achieved, compared with the raw truncated data. Also shown in Table 1 is the result obtained using a straight line fit between each of the 128 data points (this may be generated from the condensation set by setting all C's equal to zero or by direct straight line fit). The straight line method is highly inferior to the fractal fit. There is improvement of the statistical moments with the fractal fit at all decimations, as compared with the truncated data, but the comparison with the exact moments becomes worse as the decimation increases.

The worth of the method depends highly on \( p \). Unless \( p \) can be kept in the high .9's, highly accurate results will not be obtained, although any improvement can still be gained by fractal fitting - if neighboring data points have some correlation with each other. The same procedure does not work with the Raman data of Fig. 1c precisely, as mentioned above, because adjacent data points are not correlated with each other. For the original data of Fig. 1b, the autocorrelation coefficient, shown in Fig. 12, goes through the first zero crossing at a data point separation of 140 points, and there is statistically significant correlation to about three times that value. For the 512 point data, therefore, the first zero crossing would take place at about 420 points, and for the first truncation of Table 1 at about 11 points. Even for the 32 point data the zero crossing would take 3 points, with significant correlation out to 9 points.

The interpolation fractal produced is a self-affine curve which reproduces itself (with rotation and contraction) on smaller and smaller scales. This can be seen upon magnification on the bottom half of Fig. 11. On the upper trace notice the nearly straight line segments which form a "V" on the left third of the trace. This can be seen as a (rotated, smaller) nearly inverted "V" on the magnified portion, but the curve would repeat upon further magnification since the curve produced is really a self-affine fractal. The actual data curve is only fractal-like between an upper and lower time scale. But the picking of the data points assures that the large scales are reproduced, the fractal interpolation ensures that the intermediate scales behave with proper fractal-like behavior, and the smaller scales are of little importance in the computation of statistical moments. The reason the method works is that it reproduces a roughness to the trace which increases the number of delay equations (and, therefore statistical confidence) of points near the tail ends (and in the middle, for that matter) of the pdf.

It may be shown with standard statistical analysis techniques that if the fitting curve, \( y_{fit} \), consists of the true curve, \( y_{true} \), plus a noise, \( v \), where the noise is uncorrelated with the true curve, there is a bias introduced into an estimate of the variance of the fitted curve. That is

\[
\sigma^2_{fit} = \sigma^2_{true} (1 - \epsilon^2); \quad \epsilon^2 = (1 - p^2)/p^2
\]

This bias is highly sensitive to \( p \). Notice in Table 1 that as declination is increased and \( p \) drops, the estimated variance increases over the true variance for the fractal fit.

It may also be shown, under the same assumption as above that an estimate of the variance of the variance may be calculated by

\[
\text{Var}[\sigma^2_{fit}] = \sum_{k=0}^{3} \text{Var} \left[ \sigma^2_{true}(k) \right] R^2_{fit}(k) / N
\]

where \( R \) is the autocorrelation coefficient at a time delay of \( k \) data points and \( N \) is the number of fit points. What one is trading off is a larger number of data points against some noise. Note, however, that when \( k \) is equal to the separation between data or data and target points the noise is zero by construction, and this helps in accuracy.

**Chaotic Dynamics**

There is probably a deeper reason why the above procedure appears to work well. All of the data traces presented are really fairly simple insofar as chaos theory is concerned. That is, they are single valued, aperiodic traces with a fairly low fractal dimension. According to Ref. (22), one should be able to model these traces with four to five delay equations in the form of iterative maps. That is, if a time delay, \( r \), between points is appropriately chosen, it should be possible to reconstruct the statistical properties of the original curve by a set of equations of the form

\[
\begin{align*}
  v_1 &= y_i = f_1(v_2, v_3, \ldots, v_z) \\
  v_2 &= y_{i-1} = f_2(v_2, v_3, \ldots, v_z) \\
  & \vdots \\
  v_z &= y_{i-(z-1)} = f_z(v_2, v_3, \ldots, v_z)
\end{align*}
\]

with a clever choice of (nonlinear) \( f \)'s. This view is reinforced when applying the procedure of Ref. (24) as a method of finding the minimum number of delay equations needed. Shown in Fig. (13) is a calculation of the eigenvalues of the covariance matrix of the data in Fig. 1b using ten values of the time delay, \( r \). Here \( r \) is chosen as 50, although a better estimate of an appropriate time delay may have been obtained by the laborious computations indicated in Ref. (23). The number of eigenvalues off of the noise floor is an indicator of the number of delay equations, \( z \), needed to produce an adequate chaotic dynamic for the data. Here the number indicated is five. While there may be some limited forecasting ability of the generated analytical curve, sensitivity to initial conditions will eventually destroy exact agreement. Nevertheless, the statistical properties of the experimental curve should be produced for all future times. The creation of the necessary \( f \)'s is almost an art form and has not been attempted here. Nevertheless, in the cited references there is ample evidence that a procedure could be implemented to produce the desired result.
Hidden Variable Fractal Interpolation

There was a problem above with the fractal interpolation that all data sets would not produce a contractive mapping, contractivity being necessary for the attractor to be useful (and even exist). Moreover, the two dimensional fractal graph is self-affine and upon magnification produces pictures at small t scales which are clearly not consistent with physics. There is a way out of these two problems (the first being the most important) by using hidden variable fractal interpolation \(^{(27)}\).

The method consists of adding a third dimension, \(u(t_1)\), and placing any (although some may be better that others) curve in this plane. Then using the following affine maps for \(t, y\) and \(u\) and the same random jumping between intervals \(n_a\) curve in the \(y, t\) plane may be generated:

\[
\begin{bmatrix}
y \\
u \\
n \end{bmatrix}_{\text{new}} = \begin{bmatrix}
\alpha_n & 0 & 0 \\
\beta_n & 0 & 0 \\
\gamma_n & 0 & 0 \\
\end{bmatrix} \begin{bmatrix}
y \\
u \\
n \end{bmatrix}_{\text{old}} + \begin{bmatrix}
\delta_n \\
\epsilon_n \\
\zeta_n \\
\end{bmatrix}
\]

All \(n\)-subscripted numbers in the 3x3 matrix must be chosen so that the map is contractive, and there are restrictions on all coefficients which are analogous to those used in the above \(y, t\) map (Eqs. (4)). However, there are more free coefficients to be chosen by the practitioner to produce any fractal dimension desired in the \(y, t\) plane and (more importantly) b) make any data set with a contractive map, still going through all data and target points. That is, the problem above of some \(n\) having magnitude greater that unity can be avoided by choosing an appropriate background function, \(u\). The resulting \(y, t\) plot need not be fractal and need not be self-affine \(^{(27)}\).

There is another potential use for the hidden variable technique in the construction of cross correlation between two physical quantities of interest, say, turbulent shear stress and heat transfer. Also say that one curve is well defined (as the Rayleigh trace) and the other is not well sampled (as the Raman trace). Yet presume there is a cross correlation between the two. If the well sampled curve is placed in the background, \(u\), and the sparse points of the poorly sampled curve is placed in the foreground, \(y\), there is the possibility of creating a better cross correlation, from a statistical sense, than from a sparse set of points, \(y\). Work on the hidden variable technique will be reported upon in the future.

CONCLUSIONS

1. Many high Reynolds number turbulent combustion flows have physical quantity measurement time series with strong fractal-like properties. Quantification of the fractal properties yields new visual insight into the data and can indicate some new physical insight.

2. The distribution of fractal dimension in time along a time series can be used as a discriminator for nonlinear adaptive filtration, to remove unwanted noise from a signal while retaining much information usually removed by ordinary filtration even when adaptive and nonlinear.

3. The multifractal pdf introduced here appears to have superior properties in discrimination between different types of data traces as compared with another type of multifractal analysis called the multifractal spectrum. This conclusion is not general, but only applies to the type of time series investigated here with their digitization properties.

4. Fractal interpolation has been shown to be a valuable tool in increasing statistical confidence of moments of a data pdf when the data are sparse or the data record is short. The caveat is that neighboring data points must have some meaningful correlation with each other.

5. There appears a natural connection between fractal interpolation and chaotic dynamics whereby data may be reconstructed and extrapolated by nonlinear maps. Future work in this direction is warranted.

6. With the drawback of algebraic complexity, there appear advantages in pursuing multiple variable fractal interpolation for both increasing flexibility in matching data and in constructing joint properties of data.

ACKNOWLEDGEMENTS

Dr. Michael F. Barnsley was my primary teacher in fractal geometry and many of the ideas in this paper were a direct outgrowth of multiple discussions with him; his help (and patience) are gratefully acknowledged. Discussions with Professors Ronald W. Schafer and Russell M. Mersereau were very helpful in developing the fractal filter. Discussions with Professor John Elton on the multifractal spectrum were enlightening. Continuing discussions with Professor Jechiel I. Jagoda have been highly useful on all aspects of the problem. The illustrations were prepared by Mr. Ronald E. Walterick. This work was sponsored by the Air Force Office of Scientific Research under Contract No. AFOSR-88-0001. Dr. Mitat Birkan is the contract monitor.

REFERENCES


Figure 1. Four data traces used for analysis. Fig. la is from a hot film measurement in a relatively cold portion of a turbulent premixed flame. Fig. lb is a Rayleigh scattering measurement in the hot portion of the same flame. Fig. lc is a Raman spectroscopy measurement in a turbulent diffusion flame in a windtunnel and Fig. ld is a contrived waveform using a random number generator.

Figure 2. The upper curve (a) illustrates the box counting procedure for calculating probabilities needed for the generalized dimension. The lower curve (b) shows a schematic of an expected result for the dimension of order zero (fractal dimension).
Figure 3. Multifractal pdf of the Rayleigh trace for various values of the box size. A comparison curve is given at the bottom for a given box size for the hot film trace.
**Figure 4.** Multifractal pdf for the Raman trace.

**Figure 5.** Two segments of the Rayleigh time series showing radically different local fractal dimensions.

**Figure 6.** Three types of digital filtration applied to the Rayleigh time series; (a) used a first order lag recursive filter, (b) used a fractal filter as a discriminant to the use of filter (a), and (c) used a median filter.
Figure 7. Computations required for calculation of the generalized dimension of order $q$. $\chi$ is the probability sum of Eq. (1). The straight lines are drawn in the regions of highest confidence for dimension determination for the various $q$. The solid curved lines, show the asymptotic values obtained for negative and zero $q$ if boxes are omitted when no points are found in a box located between the lower and upper bounds of $y$ for a given $t_i$ and $\varepsilon$. The data points to the left of the "point insertion" arrow indicate the results for negative $q$ when points are added to a null box by taking points from another box.

Figure 8. The multifractal spectrum $f-\alpha$ curve for the hot film and Rayleigh data.

Figure 9. Schematic of fractal reconstruction of data, given sparsely separated data points.

Figure 10. Description of how the fractal interpolation scheme works and the strategy used to determine the unknown coefficients in the affine maps.
Figure 11. Fractal interpolation results for the Rayleigh data decimated by a factor of four. The lower curve shows a magnified sector of the upper curve, illustrating the self-affine nature of the transformations.

Figure 12. Autocorrelation function of the Rayleigh data.

Figure 13. Eigenvalues, in increasing size (by number encountered), of the covariance matrix of the time delayed Rayleigh trace.
Research Summary and Forecast Report (U)

Warren C. Strahle and J. I. Jagoda

Progress is reported on fractal image compression studies with laser based data using a backward facing step turbulent reacting flow facility. Work covers the first six months of this new contract and a forecast is made for the next six months of work.
Dr. Mitat Birkhan  
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Bolling Air Force Base  
Washington, DC 20332-6448

Subject: Research Summary and Forecast Report for AFOSR 88-0001

Dear Dr. Birkhan:

Request: This is a Research Summary and Forecast Report for our work on AFOSR 88-0001. Our request is for funding for our first option year, commencing on October 1, 1988.

Research Summary: Fractal analysis techniques have been applied to some Rayleigh scattering and cold flow velocity data as a preliminary step toward treatment of joint properties of data. Fractal dimension, multiscale pdfs and fractal interpolation techniques have been applied to the data. The results show in a graphic way the differences and similarities in the data. A paper on these results has been accepted for and will be presented at the 22nd International Combustion Symposium.

The data used in the development of the fractal data interpolation technique are being obtained in a turbulent reacting flow over a backward facing step with blowing of the fuel from the porous floor behind the step. The properties of the flow and concentration fields for the cold flow in this facility were obtained under a previous contract. Also obtained were preliminary data on the distribution of the axial velocities in the reacting flow.

Since the velocity data obtained in the reacting flow did not agree with the predictions made simultaneously using a modified k - ε model, the velocity measurements were continued during the first one-half year of this contract. All the axial velocity measurements were repeated and additional measurements were carried out at new axial stations. In addition, axial velocities very close to the porous floor were measured along the centerline of the tunnel floor. For these latter measurements the optical axis of the LDV was placed parallel to the tunnel which, while blocking one of the laser beams required for vertical velocity measurements, resulted in a much improved signal to noise ratio in the determination of the axial velocity.
component. All these measurements indicated that counter to the predictions by the k-ε model, the heat release lengthens the recirculation zone.

Currently, the Raman system is being tested in a non-reacting flow. The linear relationship between the intensity of the stokes line and the nitrogen concentration in the test region is being confirmed and the level of noise in the signal is being determined. Various adjustments are being made in order to improve the signal to noise ratio.

Research Forecast: During the remaining six months of the current contract year it is anticipated that the nitrogen concentration in the turbulent reacting, recirculatory flow behind the backward facing step will be mapped out. For these tests the fuel will be diluted with argon rather than nitrogen. The measured local nitrogen concentrations will then be representative of the local concentration of the oxidizer which entered the flowfield over the step. Next, the local concentration and velocity will be measured simultaneously using the combined LDV-Raman scattering system. Finally, work will commence on the measurement of the local temperatures by simultaneously determining the stokes and antistokes line intensities in the reacting flow.

During the next year of the contract simultaneously determined velocities, concentrations and temperatures will be correlated for the reacting flow behind the step. Furthermore, preparations will begin for Laser Induced Fluorescence (LIF) measurements which will help establish the degree of deviation from local equilibrium in the turbulent reacting flow.

By the end of the current contract year various data sampling techniques will have been investigated to see if an optimal technique for fractal image compression may be obtained. Also, a decision will be made between two conflicting methods in the literature for multiscale pdf construction. Joint fractal properties of simultaneous two channel data from the windtunnel will have been investigated in a preliminary manner with the goal of compressing the data taking process for joint data.

The second year of the contract will be devoted to joint Raman and LV data and the limits of fractal image compression will be defined.

Sincerely,

[Signature]

Warren C. Strahle
Co-Principal Investigator

[Signature]

Jagoda
Co-Principal Investigator

WCS/ps
Fractal Image Compression of Rayleigh, Raman, LIF and LDV Data in Turbulent Reacting Flows (U)

W. C. Strahle and J. I. Jagoda

Fractal geometry analysis methods, analytical modeling and several experimental diagnostics were applied to an experimental reacting flow in a two-dimensional subsonic windtunnel with a backward facing step and provision for injection of inerts and combustibles through the porous floor behind the step. Experiments used laser velocimetry and Raman spectroscopy measurements for two components of the mean and fluctuating velocity and the local nitrogen concentration, which was found in this flow to be an accurate temperature sensor. The fuel used was hydrogen diluted with argon or nitrogen.

Fractal geometry yielded a new nonlinear adaptive filter and an interpolation method for improving the statistics of sparsely separated data points. Analytical modeling was improved over that from a prior program, and a two equation turbulent model now reproduces many of the observed features of the reacting flow.
AFOSR FINAL REPORT

FRACTAL IMAGE COMPRESSION OF RAYLEIGH, RAMAN, LIF AND LDV DATA IN TURBULENT REACTING FLOWS

Co-Principal Investigators
Warren C. Strahle
Jechiel I. Jagoda

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AEROSPACE SCIENCES DIRECTORATE
BOLLING AIR FORCE BASE, DC

Under Contract No. AFOSR-88-0001

November, 1990

GEORGIA INSTITUTE OF TECHNOLOGY
School of Aerospace Engineering
Atlanta, Georgia 30332
SUMMARY

Analytical modelling, new fractal-based methods of data analysis and several experimental diagnostics were applied to an experimental flow in a two-dimensional windtunnel with a backward facing step and provision for the injection of inerts and combustibles through the porous floor behind the step. Laser-based diagnostics for velocity, species concentration and temperature measurements included LDV and Raman spectroscopy, augmented by some intrusive temperature measurements for validation of results. Two fractal based methods of data analysis were developed, one to improve signal to noise ratio and the other to provide better certainty in the experimental data when confronted by temporally sparse data points. Analytical modelling techniques were based upon a two equation model of turbulence, Favre averaging being incorporated to account for variable density effects. The hot flows used hydrogen-argon and hydrogen-nitrogen mixtures as the fuel injectant.

Major findings were a) the invention of a fractal filter, b) the development of a fractal interpolation technique for data augmentation, c) the discovery of extreme analysis sensitivity of the shear layer flame location to the fuel flow, d) the failure of the analysis method to yield as accurate results for the hot flow as had been found in cold flow (nevertheless yielding the gross features of the hot flow), e) the discovery that in this flow the Raman signal is primarily a temperature sensor, and f) the development of a unique method of data reduction for the Raman signal in the face of severe glare in the windtunnel. An additional year on the program was requested to complete joint measurements of temperature and velocity and further turbulence modelling with fractal based methods of data reduction as well as to complete the LIF measurements on OH concentration.

RESEARCH OBJECTIVES

The primary objective was to determine the limits of scientific understanding and predictability of a particular complex turbulent reacting flow. This flow field models that in the flame stabilization region of a solid fueled ramjet. Secondary objectives included a) the development of several laser diagnostic methods operating under particularly severe conditions of signal to noise ratio, b) the development of fractal based methods of data analysis to aid in item a) and c) the determination of necessary modifications to the turbulence
model to affect agreement between theory and experiment.

ACCOMPLISHMENTS

Facility

The facility used in this study is described in Ref. A, in the REFERENCES AND PUBLICATIONS section of this report. The combustion windtunnel is a two-dimensional, backward facing step facility with the provision for injection of inerts and combustibles through a porous floor behind the step. Injectants used were mixtures of hydrogen-argon and hydrogen-nitrogen. The facility was developed during the forerunner of this program under Contract No. AFOSR-83-0356, entitled "Heterogeneous Diffusion Flame Stabilization". The facility simulates the flame stabilization region in a solid fueled ramjet. For scientific purposes, however, it was used as an experimental device to investigate a highly complex, turbulent, recirculatory reacting flow with mass addition and combustion.

Experimental Effort

Following publication of the final cold flow, foreign gas injection results in Ref. B, full effort was turned to the investigation of the flow field with combustion. Velocity distributions were measured using laser Doppler velocimetry and temperature distributions were determined using Raman spectroscopy. Hydrogen was used exclusively for the fuel so that soot radiation and other species would not contaminate the Raman measurement, which concentrated on the species nitrogen. The fuel injection rate is relatively low, 1.82x10^{-3} and 5.4x10^{-3} lb/sec of H\textsubscript{2} and diluent, respectively, in accord with solid fuel ramjet fuel pyrolysis rates.

The horizontal and vertical components of velocity were mapped by LDV for both the cold and hot flows, showing a significant difference between the two, both in the mean and fluctuating components. The correlation between the vertical velocity fluctuation and the Raman measurement would show the mass and heat transfer in the vertical direction, to be compared with analysis. However, time ran out on this program before these simultaneous measurements could be made. Reference C documents the results of the Raman and LDV measurements.

The development and use of the Raman system in this configuration represented a non-trivial task. The windtunnel has glare-producing walls and the beam must be directed through quartz windows, which cause some window fluorescence. These
problems add severe noise to the measurement as compared to measurements made by other investigators on open flames. The ultimate goal was to develop the system to the point where simultaneous Raman-vertical velocity measurements (both mean and fluctuating) could be made. This would provide species concentration, temperature, and turbulent heat and mass transport measurements to be compared to the analytical modelling results.

The Raman system consists of a linear flashlamp pumped pulsed dye laser capable of ten shots per second. This produces data points separated far enough that they are uncorrelated with one another. That is, they are sparse data points requiring long run times to obtain statistically significant pdf's for data analysis. Knowing this in advance was one reason for entering the fractal image compression methods, discussed below.

Following system development, the Raman signal was mapped throughout the flow field. In order to determine the effect of the bleed gas upon the N₂ concentration, both Argon and Nitrogen were used as fuel diluents. Little difference was found between the two cases, indicating that the bleed gas addition was too small to affect the Nitrogen concentration in the test section. This permits the Nitrogen concentration measurements to be used to determine the local temperature, except very near the wall. This is, of course, only true if local chemical equilibrium is obtained (fast kinetics) and nearly unity turbulent Lewis number occurs for the main species in the flow. Thermocouple traverses showed equality with the temperature (density) deduced from the Raman measurements, so that at once concentration and temperature are being measured.

Reduction of the data proved nontrivial because of glare and window fluorescence. A novel method was developed to remove the resulting noise from the Raman signal. The flow was turned off and the upstream and downstream sections of the windtunnel were blocked off. The test section was then loaded with Argon by bleeding the gas through the porous floor. A reading of the Raman signal (theoretically zero, but nonzero because of noise) was made throughout the tunnel. This local noise was then subtracted from the hot flow signal at every point in order to deduce the true N₂ concentration. This is the first known time that such a calibration has been demonstrated.
Analytical Effort

a. Fractal Geometry

References D-F document the progress in using fractal geometry in data analysis during the course of this program. First of all, a fractal filter was developed. This nonlinear adaptive filter monitors the local fractal dimension of a time series and will adjust its strength in accordance with the prevailing fractal dimension. This invention was a byproduct of the program and is applicable to time series analysis in general. It should be of general interest to many practitioners of turbulent combustion data analysis. It was applied to previously obtained Rayleigh scattering data, which were contaminated by photomultiplier shot noise of a higher fractal dimension than the true signal. The fractal filter was used to clean these data.

Also of direct interest to this program was the development of fractal interpolation techniques to increase the confidence in the statistics of the pdf's generated. It was found for widely separated (but still correlated) data points a fractal interpolant could generate data points in between the sparse points that could be justifiably considered "real" data. The technique was originally intended to be used on the Raman data. However, because of the low laser repetition rate neighboring data points were uncorrelated and simple interpolation was, therefore, not possible. However, it is likely that another technique, hidden variable fractal interpolation, which is currently being developed will be applicable to the analysis of the joint Raman-LDV data once they have been obtained. This is a three-dimensional interpolation method which may work if one of the time series is well represented (LDV) while the other (Raman) is sparse. A crucial assumption in the application of this method is that the two traces being worked with have approximately the same fractal dimension. It is the experience of the PI's that this is often true in turbulence at high Reynolds number, but it must be independently checked. Preliminary work on this technique was completed but must await the joint data for application.

b) Analytical Modelling

References G-I were published during this effort, but document the results obtained during the prior contract. At the beginning of this contract, the two equation turbulence modelling had progressed to the hot flow case including the effects of temperature fluctuations and finite rate reactions. However, the
bleed section did not include the effects of diluent, nor did it include the
effects of an inert injection section downstream of the fuel injection region
used in the experimental setup for tunnel wall cooling. In the absence of the
diluent flow the model predicted a flame lying on the floor of the tunnel, which
was not what was experimentally observed. Moreover, the original analysis showed
a shortening of the reattachment zone with combustion while the experiment showed a
lengthening.

During this program the above deficiencies were corrected. The grid in the
vicinity of reattachment was tightened. As a result the lengthening of the
reattachment zone with combustion is now predicted. A more detailed modeling of
the diluent and cooling gas flows moved the flame off the floor into the shear
layer. Nevertheless, the flame position is extremely sensitive to the fuel flow
rate. While gross features of the flow are now well predicted by the model the
mixing rates are still over-predicted so that the flame is still too close to the
floor. The last deficiency was being investigated at the end of the program and
will be addressed during the one-year follow-on. Suspicions are that either the
temperature fluctuation effect (not used in the most recent calculations) may be
responsible or that a new model is needed for the pressure-velocity correlation
in the turbulent kinetic energy equation.

REFERENCES AND PUBLICATIONS

test facility and optical instrumentation for a complex turbulent reacting

B. De Groot, W.A., Walterick, R.E., and Jagoda, J.I., "Combined LDV and Rayleigh
measurements in a complex turbulent flow," AIAA J., 27, No. 1, pp. 108-110,
1989

C. Wu, M.Z., Walterick, R.E., De Groot, W.A., Jagoda, J.I. and Strahle,
W.C.,"Turbulent diffusion flame properties behind a step," AIAA Paper
#91-0079, 1991

D. Strahle, W.C. and Jagoda, J.I., "Fractal geometry applications in turbulent
combustion data analysis," 22nd Symposium (International) on Combustion, The
Combustion Institute, Pittsburg, pp. 561-568, 1989

F. Strahle, W.C., "Turbulent combustion data analysis using fractals," AIAA Paper #90-0279, 1990; accepted for publication in *AIAA J.*


**OTHER INTERACTIONS AND PRESENTATIONS**

A. Presentations at three AFOSR Contractors' Meetings, 1988-1990
B. Strahle, W.C., "Application of fractal geometry in turbulent combustion," Invited seminar at University of Arizona, March, 1989

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Mr. Donald Kenzakowski, M.S., September, 1989
Mr. John Szillat, M.S., September, 1990

AWARDS

No new awards.

AFOSR PROGRAM MANAGER INFORMATION

The program was given one more year to finish the work delineated above. The fractal filter attracted international attention; no patent was deemed required since it was a computer program and published in the public domain.