AFOSR FINAL SCIENTIFIC REPORT

ROCKET RESEARCH AT GEORGIA TECH

Co-Principal Investigators

E. W. Price
W. C. Strahle
B. T. Zinn
J. E. Hubbartt
R. K. Sigman
B. R. Daniel

Prepared for

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH
AEROSPACE SCIENCES DIRECTORATE
BOLLING AIR FORCE BASE, D. C.

Under
Contract No. F49620-82-C-0013

November 1983

GEORGIA INSTITUTE OF TECHNOLOGY
A UNIT OF THE UNIVERSITY SYSTEM OF GEORGIA
SCHOOL OF AEROSPACE ENGINEERING
ATLANTA, GEORGIA 30332
**REPORT DOCUMENTATION PAGE**

1. **REPORT NUMBER**
   AFSR-TR-83

2. **GOVT ACCESSION NO.**

3. **RECIPIENT'S CATALOG NUMBER**

4. **TITLE (and Subtitle)**
   ROCKET RESEARCH AT GEORGIA TECH

5. **TYPE OF REPORT & PERIOD COVERED**
   Final
   27 Nov, 81 - 30 Sept, 83

6. **PERFORMING ORG. REPORT NUMBER**

7. **AUTHOR(s)**

8. **CONTRACT OR GRANT NUMBER(s)**
   AFSR F49620-82-C-0013

9. **PERFORMING ORGANIZATION NAME AND ADDRESS**
   GEORGIA INSTITUTE OF TECHNOLOGY
   SCHOOL OF AEROSPACE ENGINEERING
   ATLANTA, GA 30332

10. **PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS**

11. **CONTROLLING OFFICE NAME AND ADDRESS**
    AIR FORCE OFFICE OF SCIENTIFIC RESEARCH/NA
    BLDG. 410
    BOLLING AIR FORCE BASE, D.C. 20332

12. **REPORT DATE**
    Nov. 1983

13. **NUMBER OF PAGES**

14. **MONITORING AGENCY NAME & ADDRESS (IF different from Controlling Office)**

15. **SECURITY CLASS. (OF THIS REPORT)**
    UNCLASSIFIED

15a. **DECLASSIFICATION/DATE OF REPORT**

15b. **DECLASSIFICATION/DOWNGRADING SCHEDULE**

16. **DISTRIBUTION STATEMENT (OF THIS REPORT)**
   Approved for public release; distribution unlimited

17. **DISTRIBUTION STATEMENT (OF THE ABSTRACT ENTERED IN BLOCK 20, IF DIFFERENT FROM REPORT)**

18. **SUPPLEMENTARY NOTES**

19. **KEY WORDS (CONTINUE ON REVERSE SIDE IF NECESSARY AND IDENTIFY BY BLOCK NUMBER)**
   SOLID PROPELLANT COMBUSTION, ALUMINUM AGGLOMERATION, ALUMINUM COMBUSTION, COMBUSTION INSTABILITY, COMBUSTION OSCILLATION, RESPONSE FUNCTION, IMPEDANCE TUBE, TURBULENCE NOISE, TURBULENT COMBUSTION, EXTERNAL BURNING PROPULSION, BASE FLOW, SUPersonic FLOW.

20. **ABSTRACT (CONTINUE ON REVERSE SIDE IF NECESSARY AND IDENTIFY BY BLOCK NUMBER)**
   TASK I: Several investigations of phenomena related to the driving of combustible instabilities in solid propellant rocket motors were performed. A modified impedance tube setup for determining velocity coupled response functions of solid propellants was developed. Tests conducted in this facility showed that the characteristics of velocity coupled response functions depend upon the location of the propellant within the combustor. This
finding contradicts currently accepted notions that the velocity coupled response function is a propellant property. In addition, cold flow experimental studies of oscillatory flows in tubes with inflows through porous side walls and theoretical modelling of gas phase solid propellant flames were performed.

**TASK II:** Measurement and analysis of a simulated solid fueled ramjet combustion stabilization region. Flow field measurements were made using hot film, x-film, pitot-static and laser velocimeter instrumentation. Analysis was carried out using a $k-\varepsilon$ methods.

**TASK III:** Studies continued on behavior of aluminum in the combustion zone of AP-hydrocarbon binder propellants, with further consolidation of past work into a qualitative theory, and conduct of a combustion study on specially formulated propellants designed as a test of the theory. Proposed studies of aluminum behavior in other propellant systems have not been conducted because of difficulties in obtaining samples. Studies were initiated on behavior of other nonvolatile ingredients that are used in propellants either as burning rate modifiers or combustion instability suppressants. Preliminary results are presented on combustion of ZrC in propellant flames, and several burning rate modifiers in AP-hydrocarbon binder sandwiches.

**TASK IV:** Turbulence generated noise in the interior flow of rocket motors. Measurements were made in cold flow simulators of solid rocket combustors, and results were compared with predictions based on adaptation of the Bernoulli enthalpy theory of aeroacoustics.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Contents</th>
<th>i</th>
</tr>
</thead>
<tbody>
<tr>
<td>General Introduction</td>
<td>1</td>
</tr>
<tr>
<td><strong>Task I</strong> Driving of Combustion Instabilities by Solid Propellants</td>
<td>I-1</td>
</tr>
<tr>
<td>Research Objectives</td>
<td>I-2</td>
</tr>
<tr>
<td>Results and Discussion</td>
<td>I-2</td>
</tr>
<tr>
<td>Publications</td>
<td>I-5</td>
</tr>
<tr>
<td>Personnel</td>
<td>I-6</td>
</tr>
<tr>
<td>Professional Activities</td>
<td>I-7</td>
</tr>
<tr>
<td>Appendix A</td>
<td>I-9</td>
</tr>
<tr>
<td><strong>Task II</strong> Heterogeneous Diffusion Flame Stabilization</td>
<td>II-1</td>
</tr>
<tr>
<td>Research Objectives</td>
<td>II-2</td>
</tr>
<tr>
<td>Results and Discussion</td>
<td>II-2</td>
</tr>
<tr>
<td>Publications</td>
<td>II-3</td>
</tr>
<tr>
<td>Personnel</td>
<td>II-3</td>
</tr>
<tr>
<td>Professional Activities</td>
<td>II-3</td>
</tr>
<tr>
<td>Experiments and Computation on Two-Dimensional Turbulent Flow over a Backward Facing Step</td>
<td>II-4</td>
</tr>
<tr>
<td><strong>Task III</strong> Behavior of Aluminum in Solid Propellant Combustion</td>
<td>III-1</td>
</tr>
<tr>
<td>Research Objectives</td>
<td>III-1</td>
</tr>
<tr>
<td>Status of Research</td>
<td>III-2</td>
</tr>
<tr>
<td>References</td>
<td>III-10</td>
</tr>
<tr>
<td>Publications and Presentations</td>
<td>III-11</td>
</tr>
<tr>
<td>Personnel</td>
<td>III-12</td>
</tr>
<tr>
<td>Professional Activities</td>
<td>III-12</td>
</tr>
<tr>
<td><strong>Task IV</strong> Rocket Motor Aeroacoustics</td>
<td>IV-2</td>
</tr>
<tr>
<td>Research Objectives</td>
<td>IV-2</td>
</tr>
<tr>
<td>Results and Discussion</td>
<td>IV-2</td>
</tr>
<tr>
<td>Publications</td>
<td>IV-2</td>
</tr>
<tr>
<td>Personnel</td>
<td>IV-2</td>
</tr>
<tr>
<td>Professional Activities</td>
<td>IV-2</td>
</tr>
<tr>
<td>Sound Generation by Turbulence in Simulated Rocket Motor Cavities</td>
<td>IV-3</td>
</tr>
</tbody>
</table>
GENERAL INTRODUCTION

Activities and progress are summarized for the 2 years of AFOSR Contract No. F49620-82-C-0013, with emphasis primarily on progress not reported in the previous annual reports and publications. The project consists of four interrelated tasks that are reported individually in this report and described below.

Task I has been concerned with the determination of the mechanisms of driving by solid propellants in unstable rocket motors. Special attention was paid to the determination of the response of the solid propellant burn rate to velocity oscillations parallel to the propellant surface. This phenomenon has been investigated in an impedance tube which has been developed for this purpose. Tests conducted to date showed that the driving provided by the solid propellant section depends upon its location within the combustor; an observation which contradicts current practices which assume that the velocity coupled response function is a propellant property. Additional studies conducted under this program included hot wire measurements of oscillatory velocity fields in tubes with porous walls and mass addition through these walls, and theoretical modelling of oscillatory solid propellant gas phase flames.

Task II is concerned with experiments and analysis on a reacting flow configuration which models the flame stabilization region of a solid-fueled ramjet. Current experiments have completed the facility development and cold flow testing. Conventional intrusive diagnostics have been used and initial results have been obtained with LDV, non-intrusively. The k-ε
method of turbulence analysis has been used with excellent results in the cold flow case. Primary interest is in the predictability of this complex flow field.

Task III is concerned with combustion behavior of relatively nonvolatile particulate ingredients in solid propellants, including metal fuel powders, burning rate modifiers, and combustion stabilizers. Behavior of such ingredients does not conform to usual combustion models, and is resistant to experimental observation because of the nonsteady, and microscopic scale of the relevant processes. A family of combustion experiments have been developed that permitted the combustion behavior of aluminum powder to be clarified and controlled. These methods are now being applied to other metals of interest (e.g., boron), to ballister modifiers (e.g., Fe$_2$O$_3$), and to combustion stabilizers (e.g., 2rc).

Task IV is concerned with prediction of turbulence-induced pressure fluctuations in rocket motors, particularly as they pertain to production of vibrations in motor structure. The approach combines analytical developments and cold flow experiments, using each as a means to validate or guide improvement of the others. The analysis indicates what measurements and data analysis are necessary to interpret the experiments, and the measurements indicate the ability of the theory to predict the real behavior. The experiments involved development of a suitable cold flow simulators for combustion chamber flow, and measurement of velocity and pressure fluctuations at appropriate locations.
TASK I

Driving of Combustion Instabilities by Solid Propellants

B. T. Zinn
B. R. Daniel
L. L. Narayanaswami
Y. P. Kwon
F. Chen
TASK I

Driving of Combustion Instabilities by Solid Propellants


A. Research Objectives

1. Develop an understanding of the mechanisms responsible for the driving of axial instabilities in rocket motors with tubular solid propellants. Specifically, identify the processes in the immediate vicinity of the burning propellant surface which exert the greatest influence upon the propellant driving.

2. Use a modified version of the impedance tube setup to determine the validity of using the so-called velocity response function in determining the stability of solid propellant rocket motors.

B. Results and Discussion

The following were accomplished under this research program.

1. Development of a modified impedance tube setup for the determination of the velocity coupled response functions of solid propellants under different oscillatory flow conditions was completed.
2. Linearized versions of the conservation equations were utilized to investigate the characteristics of the oscillatory flow inside the impedance tube and to develop a data reduction procedure for the developed impedance tube. The latter used measured standing wave data (i.e., amplitudes and phases) to determine the test propellant samples velocity coupled response functions.

3. The developed impedance tube setup and data reduction procedures were used to determine the velocity coupled response functions of a number of different propellants when they were placed at different locations relative to the standing wave minima. The measured data showed that one cannot assume, as has been done to date, that the velocity coupled response function is a propellant property which is space independent.

4. A cold flow experimental setup consisting of a tube with porous walls was designed and developed for the investigation of characteristics of oscillatory flows next to walls with mass addition. The objective of this study is to gain insight into the characteristics of the flow next to the propellant surface in unstable rocket motors.

5. The cold flow setup described under Item 4 above was used to determine the variation of the oscillatory flow component with distance from the porous wall. Variables investigated in this study included the porosity of the wall, the magnitude of the steady flow through the wall and the frequency and amplitude of the imposed flow.
oscillations. This study pointed out serious limitations which are associated with the use of hot wires in oscillatory velocity measurements.

6. The modelling of the gas phase flame of a solid propellant burning in an unsteady rocket motor was initiated. The fundamental conservation equations are used as a starting point and viscous and heat conduction terms are retained in the model. A simpler version of the developed model is currently being programmed for the development of predictions which could be compared with cold flow experimental data.

7. The planning of a combustion experiment capable of simulating the oscillatory gas phase burning of solid propellants in unstable rocket motors was completed.

Some of the results obtained in the above described study are discussed in the following publications.


Papers 1, 3 and 4 are provided in Appendix A for further reference.

C. Publications


D. Personnel

Principal Investigators: Ben T. Zinn and Brady R. Daniel
Post Doctoral Fellow: Y. P. Kwon
Visiting Research Scholar: F. Chen
Graduate Research Assistant: L. L. Narayanaswami
E. Professional Activities/Interactions


8. Zinn, B. T., Associate Editor for Combustion and Aeroacoustics; AIAA Journal.


APPENDIX A

Some of the Publications
Prepared under Task I
INVESTIGATION OF THE CHARACTERISTICS OF THE VELLOCITY-COUPLED RESPONSE FUNCTIONS OF SOLID PROPELLANTS


School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332

* Graduate Research Assistant
** Regents' Professor
*** Senior Research Engineer

Subject Matter

Solid Propellant Combustion Instability
Abstract

This paper describes the results of an experimental investigation which has been concerned with the validity of the state of the art approaches which use the so-called velocity-coupled response functions to determine the stability of solid propellant rocket motors. These approaches are based upon the fundamental assumption that the propellant velocity-coupled response function is a propellant property which is independent of location within the combustor. The validity of this assumption was investigated in a modified impedance tube setup specifically developed for this study. It consisted of a "driver" propellant sample at the upstream end of the tube and "test" propellant samples on the side walls at a desired location downstream of the "driver" propellant. An acoustic driver located at the downstream end of the tube was used to simulate conditions in an unstable motor by exciting a standing acoustic wave of desired properties in the tube. During a test, the driver was turned on, all the propellant samples were ignited and a series of pressure transducers attached to the tube wall were used to measure the resulting standing wave structure. These data together with a specially developed data reduction program were used to determine the velocity-coupled response functions of the "test" propellant samples. Tests were conducted with the "test" propellant samples at different locations along the impedance tube standing wave. The results of these tests clearly showed that the velocity-coupled response function is strongly dependent upon the propellant sample location relative to the impedance tube standing wave. These results also indicate that predictions...
of existing solid propellant rocket motor stability programs are most likely invalid because they are based upon the erroneous assumption that the velocity-coupled response functions of the solid propellants are constant throughout the combustor. The paper points out the need for a reevaluation of current approaches for predicting the stability of solid propellant rocket motors.
Introduction

This paper describes the application of the impedance tube technique in the investigation of the validity of current practices which use the velocity-coupled response function to determine the stability of solid propellant rocket motors. Combustion instabilities occur when energy supplied by the combustion process excites one or more of the natural acoustic modes of the combustor. This driving by the combustion process depends upon the characteristics of the space dependent flow oscillations in the vicinity of the propellant surface. For example, in a rocket motor experiencing an instability of its fundamental, longitudinal acoustic mode, the propellant sections at the two ends of the motor experience primarily pressure oscillations, the propellant section at the center of the motor experiences primarily velocity oscillations parallel to its surface and the remainder of the propellant grain experiences both velocity and pressure oscillations of varying amplitude and phase relationships. To determine the contribution of the entire propellant grain to the instability, the driving or damping provided by various sections of the propellant must be known. To date, this requirement has been interpreted as the need to know the propellant burn rate response to both pressure and velocity oscillations. These responses are generally referred to as the pressure- and velocity-coupled responses, respectively, and they are related to the pressure and velocity fluctuations through constant proportionality factors called the pressure- and velocity-coupled response functions. While the existence of a pressure-coupled response function had been recognized earlier and it
has been the subject of a considerable number of investigations,\textsuperscript{4,5,6} the higher complexity of the physico-chemical processes associated with the velocity-coupled response function has limited the number of investigations which have been concerned with the elucidation of its fundamental properties and applicability. The latter is the subject of this investigation.

It is well known that the steady burning rate of a solid propellant depends upon the properties of the flow next to its surface, which under certain conditions causes erosive burning.\textsuperscript{7,8} Developing predictive capabilities of the steady burn rate and associated erosive burning of solid propellants would require detailed analysis of the complex, multidimensional mixing, heat transfer and chemical processes which occur next to the propellant surface. While such approaches have been pursued by several investigators\textsuperscript{9-12}, who have provided much insight into the causes of erosive burning, the complexity of the problem has, thus far, prevented the development of rigorous erosive burning models. In their absence, empirical models of erosive burning have been used to predict the performance of solid propellant rocket motors.

Experimental investigations of steady propellant burning indicate that there exists a threshold velocity $u_t$ below which the burn rate is unaffected by the parallel flow\textsuperscript{13-15} and above which the propellant burn rate is given by\textsuperscript{16}

$$\frac{m_b}{m_{bo}} = 1 + k(u - u_t)$$

\hspace{8cm} (1)
where $k$ and $u_t$ are determined experimentally. While models of this type are useful in design, they do not provide fundamental understanding of the phenomenon at hand.

By now, there is ample evidence that solid propellant combustion processes are sensitive to the presence of velocity oscillations parallel to the propellant surface.\textsuperscript{17-20} This sensitivity, which can be considered as the unsteady analog of steady state erosive burning, is termed velocity-coupling and the resulting burn rate oscillation, the velocity-coupled response. Considering the physics of the problem, one would expect that an oscillatory flow would produce oscillatory mixing, heat transfer and chemical processes next to the propellant surface which would result in an oscillatory propellant burn rate. When the proper phase relationship between the oscillatory propellant burn rate and the local pressure oscillations is established, driving of the flow oscillations occurs. Consequently, it is of utmost importance to develop dependable analytical capabilities which can predict such propellant combustion process-flow interactions. Again, as in the steady state case, the complexity of the problem has prevented the development of reliable solid propellant response models which are based upon fundamental principles and, consequently, all existing response models are heuristic in nature and they are merely extensions of the empirical steady state erosion model (see Eq. (1)) and concepts developed in pressure-coupling studies.

In modeling the velocity-coupled response it was argued that the combustion process is only responsive to the magnitude of the flow velocity and independent of the flow direction.\textsuperscript{17,21,22} Then, following the steady
state erosion model, it was assumed that (1) there is a threshold velocity below which the propellant response is zero\textsuperscript{1,23,24} and (2) when the magnitude of the total velocity (i.e., the vector sum of the steady and fluctuating velocities) is greater than the threshold velocity, the propellant response is proportional to the difference. Thus, the mass flux fluctuation due to a velocity oscillation parallel to the propellant surface was given by the following relationship

\[
\frac{\Delta m}{m_b} = \frac{R_v}{a} \left[ \left( |\bar{u} + u'| - u_L \right) - (\bar{u} - u_L) \right], \quad |\bar{u} + u'| > u_L, \quad \bar{u} > u_L
\]

where \( R_v \) is the velocity-coupled response function. Implicit in Eq. (2) is the assumption that \( R_v \) is a propellant property and thus independent of the propellant location within the combustor. While the validity of Eq. (2) has never been verified experimentally or theoretically, it has nevertheless served as a basis for a number of experimental investigations which were concerned with the determination of \( R_v \textsuperscript{2,3,25-27} \) as a propellant property.

The manner in which a given propellant section responds to flow oscillations depends upon the structure of the steady state combustion zone next to its surface, as different steady state combustion zones may respond differently to oscillatory excitation. Since the local, steady state combustion zone is expected to depend upon the characteristics of the steady flow, it follows from the above discussion that both the steady and oscillatory components of the flow next to the propellant surface should affect the propellant response. Since the flow conditions within the combustor are space dependent, the effect of the flow upon the propellant
response is likely to be different at different combustor locations. Consequently, one would expect that the velocity-coupled response function $R_v$ (see Eq. (2)), which is supposed to account for steady state flow and combustion effects, will also be space dependent;$^{28}$ a conjecture which contradicts the underlying assumption of Eq. (2) that $R_v$ is strictly a propellant property. Since the clarification of this contradiction is of crucial importance to the development of analytical capabilities for predicting the stability of solid propellant rocket motors, both critical experimental and theoretical studies aimed at the resolution of this problem must be performed. This paper describes the results of an experimental investigation in which the impedance tube technique was used to explore this issue.

In what follows, a modification of the impedance tube technique, specifically developed for this study, is briefly discussed. This is followed by a description of the application of the modified impedance tube technique in the investigation of the velocity-coupled responses of solid propellants. The paper closes with a discussion of the measured data and their implications for solid propellant rocket motors stability analyses.

The Modified Impedance Tube Technique

This section describes the measurement technique developed for this study. The impedance tube technique was initially developed by acousticians who utilized it to measure the sound absorbing characteristics of various materials.$^{29,30}$ In these applications, the impedance tube consisted of a rigid wall tube with the tested material placed across one end and an
acoustic driver at the other. During a test, the driver was used to generate a 
train of acoustic waves of a desired frequency that propagated toward the 
tested material. These waves reflected off the tested sample with modified 
amplitude and phase and then combined with the incident wave train to form 
a standing wave in the tube. A traversing microphone was used to measure 
the structure of the resulting standing wave and this data together with 
appropriate analytical solutions were utilized to determine the amplitude 
and phase changes occurring at the tested sample surface; data which 
determine the sound absorbing characteristics of the tested material. 
Subsequently, this technique was modified to determine the admittances of 
choked nozzles,\textsuperscript{31} acoustic liners,\textsuperscript{32} and pressure coupled responses of 
burning solid propellants.\textsuperscript{33,34} In this investigation, the impedance tube 
technique was further modified to investigate the velocity-coupled responses 
of solid propellants.

A schematic of the modified impedance tube setup used in this study 
is shown in Fig. 1. The objective of the experiment is to measure the 
velocity coupled response function of the "test" propellant samples. The 
"driver" propellant sample provides a stream of hot combustion products 
that flows past the "test" propellant samples in an attempt to simulate 
actual rocket motor flow conditions. In this configuration, the driver 
propellant experiences only pressure oscillations while the "test" propellant 
samples are subjected to both pressure and velocity oscillations. The 
experimental setup permits moving the "driver" propellant to different 
locations upstream of the "test" samples; a capability that enables the
investigation of the response of the "test" propellant samples at different acoustical environments along the standing wave.

In an experiment to determine the velocity-coupled response function, the acoustic driver is first used to setup a standing wave of a desired frequency in the impedance tube. Next, the propellant samples are ignited and a series of transducers mounted at preselected locations along the impedance tube walls are used to measure the continuously varying (due to the presence of ignition, quasi steady state and extinguishment periods) wave structure in the impedance tube. These pressure transducers are used in this setup because the short duration of a test (between one and three seconds) precludes the use of a traversing microphone for measuring the wave structure. While the test is in progress, a stepping motor is utilized to keep the "test" propellant surfaces flush with the adjacent impedance tube walls. The measured acoustic pressure data are used to determine the unknown velocity-coupled response function by utilizing a data reduction procedure specifically developed for this purpose.$^{35}$

The issues raised in the previous section were investigated in this study by measuring the value of $R_v$ in tests in which the "test" propellant samples were placed at different locations along the standing wave structure. If the same values (or close) for the response function $R_v$ were obtained in all of these tests, then this would support the argument that $R_v$ is a propellant property which is independent of location, and vice versa.
Theoretical Considerations

These theoretical studies were undertaken with the objective of developing an analytical methodology for determining the velocity-coupled response function from measured impedance tube pressure data. The system of conservation equations which is used to determine the axial, linear stability limits of solid propellant rocket motors also serves as the starting point for this study. It consists of the linearized, one-dimensional mass, momentum and energy conservation equations. Neglecting terms of $O(u^2)$, and assuming periodic time dependence of the solutions, these wave equations can be expressed in the following form:\(^{34,35}\)

### Continuity:

\[ i\omega p' + \frac{d}{dx}(\rho u' + \bar{u}_p') = \frac{b}{A} m'_b \] (3)

### Momentum:

\[ i\omega \bar{u}' + \rho \frac{d\bar{u}}{dx} + \rho \frac{d\bar{u}}{dx} u' + \frac{dp'}{dx} + \frac{b}{A} m_b u' + Gu' = 0 \] (4)

### Energy:

\[ i\omega p' + \bar{u} \frac{dp'}{dx} + \frac{d\bar{u}}{dx} u' + \gamma p \frac{d\bar{u}}{dx} + \gamma \frac{d\bar{u}}{dx} p' = \frac{b}{A} m_b E \left( \frac{E'}{E} + \frac{m'_b}{m_b} \right) \] (5)

where

\[ E = \gamma RT_F + \frac{R}{2C_v} (u^2 + u_b^2) \] (6)

describes energy addition (and driving) at the propellant surface.

Examination of the above equations shows that the momentum and energy equations are decoupled from the continuity equation, since they contain only the dependent variables $p'$ and $u'$. These equations can be solved provided the expression, $\frac{b}{A} m_b E \left( \frac{E'}{E} + \frac{m'_b}{m_b} \right)$ on the right hand side of the
energy equation is known. This expression describes the response of the propellant to disturbances in the flow field. To date, this term has been related heuristically to the acoustic pressure and velocity by the following expression

$$\frac{m_b'}{m_b} + E' = (R_p + \theta_p) \frac{p'}{P} + (R_v + \theta_v) \frac{u'}{a}$$

(7)

where

$$R_p + \theta_p = \frac{m_b'/m_b + E'/E}{p'/\bar{P}}$$

and

$$R_v + \theta_v = \frac{m_v'/m_v + E'/E}{u'/\bar{a}}$$

(8)

where $R_p$ and $R_v$ are the pressure- and velocity-coupled response functions, respectively, and $\theta_p$ and $\theta_v$ relate $E'/E$, see Eq. (6), to $p'$ and $u'$. It should be pointed out that in using Eq. (2) to obtain Eq. (7) it has been assumed that $u_t = 0$ and that $\bar{u} > |u'|$; a restriction which must be satisfied by the developed experiment. Furthermore, the distinction between $R_v$ and $R_p$ and $\theta_v$ and $\theta_p$ is purely academic, since currently there is no known way for distinguishing between these two sets of response functions. Consequently, it is customary to consider the combination $(R_p + \theta_p)$ and $(R_v + \theta_v)$ as the pressure-and velocity-coupled response functions respectively; a practice which will be also followed in the remainder of this paper.

It is important to note that driving of oscillations in a combustor by velocity-coupling will occur if the latter has a component in phase with the local pressure oscillation. According to Eq. (7) the driving due to velocity oscillations is given by
Noting that the acoustic velocity oscillation $u'$ is $90^\circ$ out of phase with the local pressure oscillation, it is clear from Eq. (9) that driving will occur only if the response function, $(R_v + \theta_v)$ will introduce a $90^\circ$ phase change that will result in the propellant response having a component in phase with the local pressure oscillation. Consequently, it is the imaginary part of the velocity-coupled response function which determines the contribution of velocity-coupling to rocket motor stability.$^{24}$

When $R_p + \theta_p$ and $R_v + \theta_v$ are known, Eqs. (3) through (7) could be used to predict the characteristics of the standing wave inside the impedance tube or the stability of solid propellant rocket motors. Alternately, these equations provide a starting point for the determination of $R_v + \theta_v$ from measured impedance tube data. To optimize the planned impedance tube experiments, Eqs. (3) through (7) were used in a parametric study$^{35}$ which investigated if changes in the velocity-coupled response function $R_v + \theta_v$ produced measurable changes in the impedance tube wave structure. This study revealed that the largest changes in the impedance tube wave structure, due to changes in $R_v + \theta_v$, occur in the vicinity of the acoustic pressure minima. Consequently, accurate measurements of the wave structure near these minima would be required to accurately determine $R_v + \theta_v$. 

\[
\frac{m_b'}{m_b} + \frac{E'}{E} = (R_v + \theta_v) \frac{u'}{a}
\]  (9)
The next step was the development of a suitable data reduction procedure to determine the velocity-coupled response function from the measured acoustic pressure data. Such a procedure was developed and it is discussed in detail in reference 35. It is based on the method of quasilinearization and it determines the value of \((R_v + \Theta_v)\) which provides the "best" fit between the measured data and the solutions to the impedance tube wave equations. It should be noted, however, that the developed data reduction procedure presumes knowledge of the pressure-coupled response function. Consequently, the pressure-coupled response function has to be determined in a separate experiment or by the use of a reliable theory.

Experimental Efforts

The modified impedance tube developed for the investigation of the velocity-coupled response of solid propellants is shown in Fig. 2. It is approximately 6' long with a 4" x 1" rectangular cross-section and it has provisions for mounting 4" long "test" propellants samples on the sidewalls. Special adaptors for holding the Sunstrand pressure transducers, model No. 211B-5, which were used in this study are available at 0.3 inch intervals along the upper wall. An electro-pneumatic Ling EPT-94B acoustic driver is mounted a short distance upstream of the exhaust end and a spectral dynamic oscillator, model SD104A-5, is used to control the frequency and the amplitude of the oscillations generated by the driver. During an experiment, the entire setup is placed inside a high pressure tank to simulate actual rocket motor conditions.
The data acquisition system consists of a minicomputer equipped with a disc drive and an analog-to-digital converter. The analog data from the transducers are digitized and stored for post test analysis by this system. The test duration is divided into a series of data acquisition periods, separated from each other by periods of data transfer. Each data acquisition period, called a block, can be programmed to acquire data over a period whose duration is a multiple of 12 periods of the driven oscillations. Thus, data were collected over discrete periods of time (i.e., blocks) as no data was acquired during data-transfer periods. After the test, the stored data were numerically Fourier-analyzed to obtain the amplitudes and phases of the measured data at the test frequency. A study of the measured data (e.g., see Fig. (3)) showed the existence of ignition and extinguishment transients with a quasi-steady period in between. Data obtained during the quasi-steady period are input into the developed data reduction procedure to determine the propellant velocity-coupled response function.

Results and Discussion

Several velocity-coupled response function determination tests which differed from one another by the location of the "test" propellant samples along the standing wave structure were performed. The same non-aluminized propellant was used in all of these experiments and the measured data are presented and discussed in this section.

As stated earlier, a reliable determination of the velocity-coupled response function requires a careful measurement of the standing wave
structure in the vicinity of the minimum point. Therefore, many of the pressure measurements had to be performed near the pressure node where the desired signal was masked by noise from other sources and signal averaging was required to reduce the effect of this noise. Since the available memory of the mini-computer limited the amount of data which could be recorded in a given block, an estimate of the minimum averaging time needed to sufficiently enhance the signal-to-noise ratio was required. This problem was investigated by comparing data averaged over different numbers of cycles of the test signal. The spatial amplitude and phase distributions obtained by averaging data measured during the quasi-steady burning period over 18, 36 and 72 cycles are compared in Figs. 4 and 5, respectively. Examination of these figures shows that the amplitude and phase data obtained as an "18 cycle average" exhibit considerable scatter about the "72 cycle average" while the "36 cycle average" data is consistent with the "72 cycle average" data. This indicates that averaging data over 36 cycles provides sufficient enhancement of the signal-to-noise ratio.

In the experiments to determine the velocity-coupled response functions, the spatial amplitude and phase distributions were obtained by averaging the measured signals over 36 cycles. The needed pressure-coupled response functions \( R_p + \theta_p \) were measured by performing separate experiments with the same propellant in the impedance tube using a previously developed data reduction procedure.\(^{34}\)

In the first of the velocity-coupled tests, the "test" samples were located upstream of the first pressure minimum and acoustic pressure data
measured around this minimum were used to determine $R_v + \theta_v$. In the second experiment, the "test" samples were located downstream of the first minimum and the acoustic pressure data measured around the second acoustic pressure minimum were used to determine $R_v + \theta_v$. The standing wave structures measured in these experiments are shown in Figs. 6 and 7. Also shown in these figures are the determined wave structures which provided the "best" agreement with the experimental data. The determined optimum values of $\text{Im} \ (R_v + \theta_v)$ which provided the "best" agreement are also presented in the figures.

An examination of Figs. 6 and 7 shows that there is satisfactory agreement between the measured and computed wave structures. However, the value of $\text{Im} \ (R_v + \theta_v)$ determined in the first experiment is considerably different from that obtained in the second; that is, the data indicate that $\text{Im} \ (R_v + \theta_v)$ of the propellant sample upstream of the pressure minimum is -3 and it equals 30 when the propellant sample is located downstream of the pressure minimum. Considerations of the physics of the problem together with the measured data indicate that in both tests the velocity coupled responses of the tested propellant samples attenuated the oscillations in the impedance tube. Qualitative support for this argument was provided by the observation that in both tests the amplitudes of the oscillation in the impedance tube decreased after propellant samples ignition.

The above reported data clearly show that the same propellant samples exhibit different velocity responses when positioned at different locations along a standing acoustic wave. Consequently, it must be
concluded that the velocity-coupled response function cannot be regarded as a propellant property and stability analyses which are based upon this notion are bound to yield erroneous rocket motor stability limits. Furthermore, while the results of this study clearly indicate that the propellant burn rate indeed responds to velocity oscillations parallel to the propellant surface, the nature of this response is currently not understood. Considering the importance of this type of response in solid propellant rocket motors stability analyses, it is strongly recommended that new endeavors aimed at the understanding of this type of propellant response be undertaken.

The results reported herein point out another misconception in current considerations of the contribution of velocity-coupling to motor stability. It has been argued that velocity-coupling does not contribute to the linear stability of the fundamental axial mode because the standing pressure wave undergoes a 180 degrees phase change between the fore and aft ends of the rocket motor. Consequently, if the assumption that the velocity-coupled response function is independent of location in the rocket motor holds, one half of the propellant will drive the oscillation and the remaining half will attenuate the oscillation with no net contribution to the driving of the combustor oscillation from velocity-coupling. The results of this investigation indicate that the velocity-coupled response function varies with location in the combustor and consequently velocity-coupling may contribute to the linear stability of the fundamental axial mode in a solid propellant rocket motor.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>speed of sound, m/s</td>
</tr>
<tr>
<td>A</td>
<td>cross-sectional area, m²</td>
</tr>
<tr>
<td>b</td>
<td>perimeter of the sidewall propellant samples, m</td>
</tr>
<tr>
<td>C</td>
<td>heat transfer parameter, m</td>
</tr>
<tr>
<td>C_v</td>
<td>specific heat at constant volume, J/kg-K</td>
</tr>
<tr>
<td>G</td>
<td>gas phase bulk loss coefficient, N-s/m⁴</td>
</tr>
<tr>
<td>Im( )</td>
<td>imaginary part of ( )</td>
</tr>
<tr>
<td>m</td>
<td>mass burn rate per unit area, kg/m²-s</td>
</tr>
<tr>
<td>p</td>
<td>pressure, N/m²</td>
</tr>
<tr>
<td>R</td>
<td>specific gas constant, J/kg-K</td>
</tr>
<tr>
<td>R_p</td>
<td>pressure-coupled response function, nondimensional</td>
</tr>
<tr>
<td>R_v</td>
<td>velocity-coupled response function, nondimensional</td>
</tr>
<tr>
<td>t</td>
<td>time, s</td>
</tr>
<tr>
<td>T</td>
<td>temperature, K</td>
</tr>
<tr>
<td>u</td>
<td>velocity, m/s</td>
</tr>
<tr>
<td>uₜ</td>
<td>threshold velocity, m/s</td>
</tr>
<tr>
<td>x</td>
<td>distance, m</td>
</tr>
<tr>
<td>γ</td>
<td>ratio of specific heats</td>
</tr>
<tr>
<td>ρ</td>
<td>density kg/m³</td>
</tr>
<tr>
<td>θ_p, θ_v</td>
<td>defined in Equation (8)</td>
</tr>
<tr>
<td>ω</td>
<td>radian frequency, 1/s</td>
</tr>
</tbody>
</table>
Subscripts
b \quad \text{properties at the sidewall propellant surface}
F \quad \text{flame property}

Superscripts
' \quad \text{fluctuating quantity}
- \quad \text{mean quantity}
References


Figure Captions

Fig. 1. A Schematic of the Impedance Tube Modified for Velocity-Coupled Response Measurements.

Fig. 2. An Isometric View of the Developed Impedance Tube Setup.

Fig. 3. Typical Time Variations of Measured Pressure Amplitudes and Phases for Two Tube Locations. Note the Various Test Periods.

Fig. 4. Comparison of Spatial Amplitude Distributions Obtained by Averaging the Pressures Measured During the Quasi-Steady Burning Period Over 18, 36 and 72 Cycles.

Fig. 5. Comparison of Spatial Phase Distributions Obtained by Averaging the Pressures Measured During the Quasi-Steady Burning Period Over 18, 36 and 72 Cycles.

Fig. 6. A Comparison of the Experimental and Theoretically Determined Axial Variations of the Amplitude and Phase of the Impedance Tube Standing Wave when the "Test" Samples were Located Upstream of the First Pressure Minimum.

Fig. 7. A Comparison of the Experimental and Theoretically Determined Axial Variations of the Amplitude and Phase of the Impedance Tube Standing Wave with the "Test" Samples Located Downstream of the First Pressure Minimum.
ACOUSTIC DRIVER FOR GENERATING STANDING WAVE. EST! PROPELLANT SAMPLE

"DRIVER" PROPELLANT

"TEST" PROPELLANT SAMPLE

ACOUSTIC DRIVER FOR GENERATING STANDING WAVE

HOT COMBUSTION PRODUCTS

PRESSURE TRANSUDERS

EXH

DISTANCE

EXCITED STANDING WAVE PATTERN
Im($R_v + \Theta_v$) = +30°
Experimental Investigation of Pressure and Velocity Coupled Response Functions of Aluminized and non-Aluminized Solid Propellants
L.L. Narayanaswami, B.R. Daniel and B.T. Zinn, Georgia Institute of Technology, Atlanta, GA
EXPERIMENTAL INVESTIGATION OF PRESSURE AND VELOCITY COUPLED RESPONSE FUNCTIONS OF ALUMINIZED AND NON-ALUMINIZED SOLID PROPELLANTS

L. L. Narayanaswami*, B. R. Daniel** and B. T. Zinn***
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, Ga. 30332

Abstract

This paper describes investigations of the characteristics of the response functions of different aluminized and nonaluminized propellants. The results of an investigation of the dependence of propellant driving and damping characteristics upon its aluminum content indicate that aluminum addition increases both the propellant driving and the gas phase losses. This paper also discusses the status of an ongoing experimental investigation of the "so-called" velocity coupled response function of solid propellants. A modified impedance tube was developed for this study and comparison between measured data and corresponding theoretical predictions are presented and discussed.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>cross sectional area of the tube, (m^2(ft^2))</td>
</tr>
<tr>
<td>b</td>
<td>perimeter of the side wall propellant samples, (m(ft))</td>
</tr>
<tr>
<td>(C_p)</td>
<td>specific heat at constant volume, (3/kg-K(Btu/slug-°R))</td>
</tr>
<tr>
<td>F</td>
<td>drag force experienced by the flow per unit volume, (N/m^3(lb/ft^3))</td>
</tr>
<tr>
<td>G</td>
<td>gas phase bulk loss coefficient, (N-s/m^4(lb-s/ft^4))</td>
</tr>
<tr>
<td>(m_b)</td>
<td>mass injection rate per unit area, (kg/m^2-s(lug-ft^2-s))</td>
</tr>
<tr>
<td>p</td>
<td>pressure, (N/m^2(lb/ft^2))</td>
</tr>
<tr>
<td>(R_p)</td>
<td>pressure coupled response function</td>
</tr>
<tr>
<td>(R_V)</td>
<td>velocity coupled response function</td>
</tr>
<tr>
<td>T</td>
<td>temperature, (K(°R))</td>
</tr>
<tr>
<td>(ΔT)</td>
<td>non-isentropic temperature perturbation at the combustion zone, (K(°R))</td>
</tr>
<tr>
<td>u</td>
<td>velocity, (m/s(ft/s))</td>
</tr>
<tr>
<td>x</td>
<td>distance, (m(ft))</td>
</tr>
<tr>
<td>Y</td>
<td>propellant admittance</td>
</tr>
<tr>
<td>b</td>
<td>properties at the side wall propellant surface</td>
</tr>
<tr>
<td>F</td>
<td>properties at the flame</td>
</tr>
<tr>
<td>(I)</td>
<td>fluctuating quantity - mean quantity</td>
</tr>
<tr>
<td>(γ)</td>
<td>ratio of specific heats</td>
</tr>
<tr>
<td>(ρ)</td>
<td>density, (kg/m^3(lug-ft^3))</td>
</tr>
<tr>
<td>(ω)</td>
<td>radian frequency, (l/s)</td>
</tr>
</tbody>
</table>

Copyright © American Institute of Aeronautics and Astronautics, Inc., 1983. All rights reserved.

Response Functions and Damping of Aluminized Propellants

The first part of the paper describes the results of an investigation of the effect of aluminum addition upon the response functions and gas phase losses associated with aluminized propellants. This study had been undertaken because of indications that aluminum addition to a propellant may worsen the stability of a given rocket motor; a situation that may arise when the overall increase in propellant driving due to aluminum addition is larger than the corresponding overall increase in damping due to the presence of aluminum oxide particles in the gas phase. To investigate the possible occurrence of such a situation, it is necessary to determine the dependence of the propellant driving and its associated gas phase losses upon its aluminum contents. In this study, the previously developed impedance tube set-up \(^{1,2}\) has been utilized to investigate the dependence of the pressure coupled response function and associated gas phase losses of a given propellant formulation upon its aluminum content. The second part of the paper describes some recent results obtained in an experimental study that is concerned with the determination of the velocity coupled response functions of solid propellants in a modified impedance tube set-up specifically developed for this purpose \(^3\).

Response Functions and Damping of Aluminized Propellants

This section describes the results of the investigation of the effect of aluminum addition upon the pressure coupled response functions and gas phase losses of a specific solid propellant formulation. The pressure coupled response function, \(R_p\), of a solid propellant is defined as \(^3\)
where \( \mathbf{M}_a = \mathbf{u}/\mathbf{a} \) is the Mach number of the mean flow at the propellant surface and \( \gamma \) is the non-dimensional, complex, propellant admittance. The real part of the admittance, \( \gamma_r \), is a measure of the driving provided by the propellant. When the oscillations at the propellant surface are isentropic, Eq. (1) reduces to

\[
R = \frac{m^2/m}{p'p'} = \frac{u'p'/p'}{p'/p'} = \frac{Y + \rho_0/p}{p'/p'} \quad \forall \mathbf{M}_a \neq 0
\]

(1)

The three propellants were tested in the impedance tube over the frequency range of 400 to 1200 Hz and typical results are presented in Fig. 2 where the frequency dependence of the propellants' driving \((i.e., Y)\) and their gas phase damping \((i.e., G)\) are presented. Examination of this figure shows (1) the existence of maxima in the response function curves and that the frequency at which these occur changes with the aluminum content of the propellant; and (2) that the propellant driving and associated gas phase damping increase with an increase in the propellant aluminum content. Since increasing both the propellant driving and the gas phase damping would exert counteracting effects on the stability of a rocket motor, one cannot determine the effect of aluminum addition upon a given rocket stability without conducting an analysis capable of properly accounting for the above indicated effects. Finally, it remains to establish that the observed effects are due to the aluminum addition only and not due to the differences in the compositions of the nonaluminized fractions of the tested propellants.

**Velocity Coupled Response Measurements**

In this section, recent progress made in the experimental investigations of the so called velocity coupled response functions of solid propellants, is reported. The experimental configuration of the modified impedance tube facility utilized in this study is shown in Fig. 3, along with the impedance tube wave equations \(\text{Eq. (2)}\). The "driver" propellant sample provides a stream of hot combustion products that moves past the two test propellant samples in an attempt to simulate actual rocket flow conditions. In this configuration, the driver propellant "experiences" only pressure oscillations while the test propellant samples are subjected to both pressure and velocity oscillations. The experimental set-up permits moving the "driver" propellant to different locations upstream of the test propellant samples; a capability that provides an opportunity for investigating the response of the test propellants at different acoustical environments along the standing wave. During a test, a stepping motor is utilized to feed the test propellant samples inward at the propellant burn rate. This is done in order to maintain the burning test propellant surfaces flush with the adjacent impedance tube walls.

The wave equations presented in Fig. 3 are those utilized in this study to determine the unknown velocity coupled response function \(R_v\) and they represent an "accepted" formulation of the axial instability problem \(\text{Eq. (1)}\). It should be pointed out, however, that the definitions of \(R_v\) and \(R_p\) utilized herein differ from those used by other investigators. (See for example, Eq. 1). Furthermore, it appears that the introduction of these definitions was motivated by the need to simplify the mathematical formulation of the combustion instability problem and not by any sound theoretical and/or experimental justification.

A new data reduction program has been developed for the determination of the unknown velocity coupled response function \(R_v\) from the measured acoustic pressure data. The data reduction procedure is based on the method of quasi-linearization and has been discussed in detail in reference 1. The method requires that the acoustic pressure data used in the estimation of \(R_v\) be measured in that region of the impedance tube where the test propellant samples are located. It should also be noted that the data reduction program presumes a knowledge of the pressure coupled response function and it determines only the velocity coupled response function. Consequently, the pressure coupled response function needs to be determined in a separate experiment or by the use of a reliable theory.
During an experiment to determine the velocity coupled response function, the impedance tube is placed inside a pressurized tank for the reasons mentioned in the previous section. Next, the acoustic driver is turned on to set up a standing wave of a desired frequency inside the impedance tube and pressure transducers mounted at pre-selected locations along the impedance tube walls are used to measure the continuously varying wave structure in the impedance tube. As in the pressure coupled case, the acoustic pressure data are continuously fed, via an analog-to-digital converter, into a minicomputer-disc system for storage. The test duration is divided into a series of data acquisition periods, separated from each other by periods of data transfer. Each data acquisition period, called a block, can be programmed to acquire data over a period whose duration is a multiple of 12 periods of the test oscillation. It is to be noted that data taken during a test is discrete since no data is acquired during data-transfer periods. After the test, the stored data are Fourier-analyzed to obtain the amplitudes and phases of the measured data at the test frequency. A study of the analyzed data shows the existence of ignition and extinguishment transients with a quasi-steady burning period in between. Data obtained during this quasi-steady period is used, along with the solutions of the impedance tube wave equations, to evaluate the propellant response.

In what follows, some recent results obtained in the experimental determination of the velocity coupled response functions of solid propellants are presented. A non-aluminized propellant was used in this investigation and the presented results are for an excitation frequency of 750 Hz.

In the experiments conducted to date, the test propellant samples were located near a pressure node where the velocity oscillations are maximum. Since, as mentioned earlier, the pressure measurements need to be performed in the region of the impedance tube where the test propellant samples are located, some of the acoustic pressure measurements had to be performed near a pressure node. Consequently, the desired signal was often buried in noise from other sources. Signal averaging can be used to separate the signal from the noise. Since the available memory of the minicomputer limited the amount of data that could be recorded in a given block, an estimate of the non-purification of the test signal over which the data should be averaged to sufficiently enhance the signal-to-noise ratio was required. This problem was investigated by comparing data averaged over different numbers of cycles of the test signal.

The spatial amplitude and phase distributions, obtained before propellant ignition by averaging data over 12, 18 and 36 cycles, are compared in Figs. 4 and 5, respectively. Examination of these figures shows that while the amplitude distribution agrees in all cases, the "12 cycle average" phase data exhibits considerable scatter about the "36 cycle average" and the "18 cycle average" phase data exhibits almost no scatter. This indicates that, during the quasi-steady burning period, a "36 cycle average" would be required to increase sufficiently the signal to noise ratio.

Consequently, the spatial amplitude and phase distributions used to evaluate the velocity coupled response functions were obtained by averaging the measured signals over 36 cycles. The needed pressure coupled response functions were measured by the experimental procedure described in the previous section. A comparison between the computed standing wave pattern (i.e., a solution of the wave equations presented in Fig. 3) that provided the "best" agreement, and the experimental data is presented in Fig. 8. The determined optimum value of R, and the measured value of R, that were used to predict the standing wave pattern are also indicated in the figure.

Examination of Fig. 8 indicates a reasonable agreement between the predicted and measured amplitude distributions. In contrast, some disagreement is noted in the compared phase distributions. These discrepancies could be due to errors in measurements and/or the data reduction procedure or due to shortcomings in the wave equations that are currently utilized (See Fig. 3) to model the axial instability problem. Specifically, the model for the axial instability problem utilized in this study assumes that the driving due to velocity coupling is directly related to the acoustic velocity itself. However, other functional relationships may be more appropriate as, for example, has been noted by Cullick, who argued that the burn rate response to the velocity fluctuations should only depend upon the amplitudes of the velocity oscillation and not its direction. Also, there exists no proof that the utilized "one-dimensional" formulation is indeed capable of accounting for the multi-dimensional aspects of the problem where driving of the axial oscillations occurs on the side walls.

Summary

In this study, the impedance tube developed to measure the pressure coupled response functions of solid propellants was used to investigate the dependence of a propellant's driving and damping characteristics upon its aluminum content. The results of this study indicate that aluminum addition increases both the propellant "pressure-coupled" driving and the associated gas phase acoustic losses. Depending on which of these effects is dominant, the addition of aluminum may stabilize or destabilize a given rocket motor. The development of a modified impedance tube for the measurement of velocity coupled response function has also been described. Some recent results obtained in this investigation are presented and discussed.

References


Table I. Comparison of desired and actual compositions of tested propellants.

<table>
<thead>
<tr>
<th>INGREDIENT</th>
<th>ACTUAL</th>
<th>DESIRED</th>
<th>ACTUAL</th>
<th>DESIRED</th>
<th>ACTUAL</th>
<th>DESIRED</th>
</tr>
</thead>
<tbody>
<tr>
<td>R-45m</td>
<td>9.34</td>
<td>9.34</td>
<td>9.34</td>
<td>9.34</td>
<td>9.34</td>
<td>7.6588</td>
</tr>
<tr>
<td>INDOPOL</td>
<td>1.85</td>
<td>1.85</td>
<td>1.85</td>
<td>1.7575</td>
<td>1.85</td>
<td>1.517</td>
</tr>
<tr>
<td>TEPANOL</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
<td>0.1425</td>
<td>0.15</td>
<td>0.123</td>
</tr>
<tr>
<td>AP-200µ</td>
<td>51.00</td>
<td>51.00</td>
<td>51.00</td>
<td>48.45</td>
<td>35.00</td>
<td>41.82</td>
</tr>
<tr>
<td>AP-50µ</td>
<td>15.00</td>
<td>15.00</td>
<td>12.00</td>
<td>14.25</td>
<td>13.00</td>
<td>12.3</td>
</tr>
<tr>
<td>AP-8µ</td>
<td>22.00</td>
<td>22.00</td>
<td>20.00</td>
<td>20.90</td>
<td>19.00</td>
<td>18.04</td>
</tr>
<tr>
<td>ALUMINUM:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.5µ</td>
<td>0.00</td>
<td>0.00</td>
<td>5.00</td>
<td>5.00</td>
<td>18.00</td>
<td>18.00</td>
</tr>
<tr>
<td>IDPI</td>
<td>0.66</td>
<td>0.66</td>
<td>0.66</td>
<td>0.627</td>
<td>0.66</td>
<td>0.5642</td>
</tr>
</tbody>
</table>
Fig. 1. Schematic Diagram of the Impedance Tube Facility.

Fig. 2. Dependence of Propellant Driving, $Y_r$, and Associated Gas Phase Losses, $G$, upon Frequency and Propellant Aluminum Content.

Fig. 3. Set-up and Equations Utilized for the Determination of Velocity Coupled Response Functions.

Fig. 4. Comparison of Spatial Amplitude Distributions Obtained by Averaging the Pressure Measured Prior to Ignition over 12, 18 and 36 Cycles.
Fig. 5. Comparison of Spatial Phase Distributions Obtained by Averaging the Pressures Measured Prior to Ignition over 12, 18 and 36 Cycles.

Fig. 6. Comparison of Spatial Amplitude Distributions Obtained by Averaging the Pressures Measured during the Quasi-Steady Burning Period over 18, 36 and 72 Cycles.

Fig. 7. Comparison of Spatial Phase Distributions Obtained by Averaging the Pressures Measured during the Quasi-Steady Burning Period over 18, 36 and 72 Cycles.
Fig. 8. Axial Variation of Amplitude and Phase of the Standing Wave in the Impedance Tube; A Comparison Between Theory and Experiment.
INTRODUCTION

This paper describes the status of an investigation which has been concerned with the determination of the flow characteristics inside solid propellant rocket motors experiencing axial instabilities. When the instability drives one of the longitudinal modes of the combustor, different sections of the propellant are exposed to different oscillatory flow conditions; that is, portions of the propellant near pressure nodes are subjected primarily to axial oscillations parallel to their surfaces, propellant sections near velocity nodes experience primarily acoustic pressure oscillations and the remaining propellant sections experience both pressure and velocity oscillations. The interactions of these oscillations with the propellant combustion processes are responsible for the observed axial instabilities and they involve complex fluid mechanical, heat transfer and chemical mechanisms. In general, these interactions occur near the burning propellant surface and they produce velocity oscillations $v'$ at the propellant surface which are normal to the direction of the core flow oscillations. Consequently, both the steady state and oscillatory flow fields are multidimensional and their velocities change direction, from radial to axial, somewhere between the propellant surface and the combustor centerline. This observation also indicates that while axial instabilities are generally treated as one dimensional problems (e.g., see Refs. 1-3) these phenomena are actually multidimensional and they should be treated as such.

A propellant exhibits an unsteady burn rate during an instability because the presence of flow oscillations probably results in periodic mixing, diffusion, heat transfer and chemical processes next to its surface. The manner in which a given propellant section responds to flow oscillations depends upon the structure of the steady state combustion zone next to its surface, as "differences" in the steady state combustion zones may respond differently to oscillatory excitation. Since the structure of the local, steady state combustion zone is expected to depend upon the characteristics of the steady flow, it follows from the above comments that both the steady and oscillatory components of the flow next to the propellant surface affect the propellant response. In addition, one should keep in mind that (1) the characteristics of the oscillatory flow field are known to depend upon the steady flow properties; (2) the presence of oscillations may affect the characteristics of the steady flow by causing early transition to turbulent flow, modifying the turbulence structure and the introduction of acoustic streaming; and (3) the processes described under (2) above may be amplitude dependent.

It follows from the above discussion that there is a need to develop an understanding of the flow fields in unstable rocket motors. Ideally, one would want to acquire such knowledge by performing specific diagnostics in unstable rocket motors. However, difficulties associated with performing the needed measurements inside rocket combustors and the high cost of firing actual rocket motors render such an approach impractical. Also, considerations of the complexities of the problem when chemical reactions occur in the combustor strongly suggest that as a first step towards the solution of the problem at hand one should investigate, both experimentally and theoretically, the characteristics of the flow in a "cold flow" rocket-like facility capable of simulating the flow conditions inside solid propellant rocket motors experiencing axial instabilities. One such possibility is to use a channel in which the propellant is simulated by mass addition through porous side walls.

Steady flows in channels with porous walls have been investigated experimentally by Yagodkin, Yamada et al., Taylor Heusman and Eckert, Dunlap et al and others. These studies show that the following occur in a closed end porous tube with mass injection through the wall: (a) the axial velocity distribution is self similar over substantial axial distances; (2) turbulence begins forming while the velocity profile remains self similar; (3) transition of the self similar laminar profile to a turbulent profile occurs at large distances from the closed end and (4) maximum turbulence intensity occurs in an annular region between the wall and the centerline and the downstream development of the turbulence profile is not similar.

Keeping the above flow characteristics in mind, one is faced with the following questions when considering the combustion instability problem: (1) does the presence of flow oscillations in the motor affect the above-mentioned steady flow characteristics and under what conditions? and (2) how do these steady flow characteristics affect the acoustic flow field and unsteady propellant response? With the exception of the recent study of Brown, it does not appear that these questions were addressed in studies of flows in channels with mass addition from the side walls. On the other hand, related questions were considered in studies of oscillatory flows in tubes with solid walls. These indicate that the presence of pulsations in the flow may (1) change the turbulence structure of the flow; (2) enhance
EXPERIMENTAL SETUP

The experimental setup is shown in Fig. 1. It consists of a 1.9 inch I.D. porous wall tube whose ends are connected to solid wall tubes. The tube on the right is connected to a large vacuum tank and its flow rate is controlled by a valve located upstream of the vacuum tank. An acoustic driver attached to the opposite end of the tube was used to excite a standing acoustic wave of desired amplitude and frequency in the tube. The hot wire probe was used to obtain radial distributions of the velocity at the indicated location and the microphone to measure the amplitude of the sound wave.

Tests were conducted with different steady state flow rates and different excitation conditions. In addition, in an effort to check the validity of the hot wire measurements under oscillatory flow conditions, the hot wire probe was used to measure centerline velocities at different locations of a solid wall tube in which a standing acoustic wave was driven by means of an acoustic driver in the absence of a steady flow.

RESULTS AND DISCUSSION

This section presents radial dependence of the velocity distributions measured with the hot wire probe shown in Fig. 1. Figure 2 presents a comparison of the steady state, velocity profiles in the presence and absence of a 291 Hz standing acoustic wave in the tube. In the absence of acoustic oscillations, the magnitude of the velocity varied between 2.5 m/sec near the centerline, where the velocity was axial, to 0.3 m/sec near the wall where the velocity was radial. Since the hot wire traverse was performed with the hot wire perpendicular to both the axial and radial components of the velocity, the hot wire probe responded to both the axial and radial velocity components and its signal was proportional to the magnitude of the velocity vector. Figure 2 shows that when sound is present, the hot wire probe indicates that the magnitude of the steady velocity increases near the wall and decreases near the centerline. These effects increase as the wave amplitude increases from 141 dB to 146 dB.

Before discussing the data presented in Fig. 2, it should be pointed out that an examination of the hot wire output on an oscilloscope as the radial traverse was carried out revealed that it remained nearly sinusoidal between the centerline of the tube and a short distance (of the order of a fraction of an inch) from the wall where the signal became partially rectified. The probe output became fully rectified when the probe traverse was completed at a distance of approximately .01 inches away from the wall, where the steady (radial) velocity is very low. To elucidate the causes of the observed increase in steady velocity near the wall in the presence of sound (see Fig. 2), the hot wire was used to measure the centerline velocity in a tube closed at both ends (with no steady flow) in which a standing acoustic wave was excited by means of an acoustic driver. As expected, the probe waveform output (on an oscilloscope) was fully rectified throughout the tube and its voltmeter output indicated the presence of both steady and oscillatory flow components.

The probe indications of the presence of a steady flow component in a situation where such a flow was absent is symptomatic of the difficulties that such probes experience when used in flow situations where the magnitude of the oscillatory component of the velocity is larger than the magnitude of steady component of the velocity. These problems are discussed in detail in Ref. 13 and they are caused by flow reversal and the inability of the probe to respond to the rapid changes in flow direction when the magnitude of the velocity is very low. In view of these comments, it is not clear whether the observed increase in the magnitude of the steady component of the velocity near the wall in the presence of an acoustic field is indeed due to a fluid mechanical phenomenon (such as nonlinear acoustic effect) or due to the peculiarities of the hot wire probe when used in an oscillating flow field, or both.

Next, the observed reduction (see Fig. 2) in the magnitude of the steady velocity component near the center line of the tube in the presence of sound is considered. Again, it could be a fluid mechanical phenomenon but most likely it is caused by the nonlinearity of the hot wire response whose output voltage is proportional to the square root of the magnitude of the velocity. For the data presented in Fig. 2, the magnitude of the steady component of the velocity is larger than the magnitude of the oscillatory velocity component outside the steady state velocity boundary layer. Consequently, one can argue (qualitatively) that in the presence of an acoustic field the hot wire output near the tube centerline is given by

$$
\varepsilon = \frac{1}{2} \left| \sqrt{\bar{u} + \left| u' \right|^2} + \sqrt{\bar{u} - \left| u' \right|^2} \right| < \sqrt{\bar{u}}
$$

(1)
Finally, it should be pointed out that the bands which are used in Fig. 2 to describe the steady velocities inside the tube represent the ranges of measured variations of the steady velocity components at the indicated locations. The test conditions described in Figure 3 differ from those considered in Figure 2 by the fact that they involve a lower steady state velocity; that is, the maximum centerline velocity of the data in Fig. 3 is approximately 1.5 m/sec while it is 2.5 m/sec in the case considered in Fig. 2. For the data presented in Fig. 3 the magnitude of the oscillatory component of the velocity was comparable to that of the steady velocity component along all the radius when the dB level was 146. As a result, the hot wire output was partially rectified near the tube centerline and fully rectified near the wall. Thus, using the arguments presented above, it is believed that this rectification effect was the cause of the observed increase in the magnitudes of the steady state velocity which 146 dB oscillations were present in the tube. In contrast, the steady velocity distribution measured with 141 dB oscillations in the tube resembles those presented in Fig. 2 and the reasons for its deviation from the steady velocity distribution measured in the absence of sound are probably due to the effects considered in the discussion of the data presented in Fig. 2.

Figure 4 contains a comparison of steady state velocity distributions measured when standing waves of different frequencies and the same amplitudes were excited inside the experimental setup. In each of the tests, the DC output of the hot wire probe were the same at the centerline of the tube. When this condition was satisfied, the measured steady state velocity distributions lie within the band shown in Fig. 6 for frequencies of 291, 392, 581, 600, 866 and 1470 Hertz. This result indicates that if the presence of an oscillatory velocity in the tube has any effect upon the steady velocity distribution, then this effect is independent of frequency at least for the conditions investigated under this study.

Figure 5 presents a comparison of the radial distributions of the AC components of the hot wire outputs in the presence and absence of a 291 Hz sound wave. The variation of the measured AC voltage at each radial location is described by the indicated band. An examination of Figure 5 reveals that both distributions have a maximum between the centerline and the wall at a distance of approximately .06 inches from the wall. When no sound is present in the tube, the AC voltage describes the turbulence level of the flow and its magnitude drops rapidly when one moves away from the location of maximum turbulence. Figure 5 also indicates that when 291 Hz sound is excited in the tube the AC output of the hot wire increases considerably throughout the tube cross sectional area. This increase in the AC signal output is due to the presence of sound in the tube. It is interesting to note that when the hot wire signal was observed on an oscilloscope during tests with sound excitation, it became difficult to "see" (on the oscilloscope) the oscillatory velocity signal as the region of maximum AC output was traversed. This was caused by the masking of the acoustic wave signal by the random, high intensity turbulence in this region.

The observed existence of a region of maximum fluctuating velocities (i.e., turbulence and acoustic velocity) between the centerline and the wall is consistent with the findings in related studies. Reference 11, which investigated the effect of sound upon convective heat transfer, referred to such a region as "abnormal turbulence" and it argues that this phenomenon is most likely responsible for the observed increase in convective heat transfer between the flow and the wall. If this hypothesis is indeed correct, then the hot wire supports that supports the hot wire (or the wire itself) in this condition was rectified near the wall. Thus, using the arguments presented above, it is believed that this rectification effect was the cause of the observed increase in the magnitudes of the steady state velocity when 146 dB oscillations were present in the tube. In contrast, the steady velocity distribution measured with 141 dB oscillations in the tube resembles those presented in Fig. 2 and the reasons for its deviation from the steady velocity distribution measured in the absence of sound are probably due to the effects considered in the discussion of the data presented in Fig. 2.

Before leaving this section a few comments regarding the hot wire measurements are in order. First, as indicated above, the hot wire data presented in this paper only describes magnitude variations of the velocity vector which changes direction from being radial at the wall to being axial near the tube centerline. To check for possible wall effects upon the hot wire measurements, the hot wire output was monitored as the hot wire probe was moved from the centerline towards the wall with no flow or sound present in the tube. In this case the probe output remained at zero level until it reached a distance of approximately .06 inches from the wall. At this point the DC output of the probe started increasing and it further increased as the probe was moved from this point towards the wall in spite of the fact that there was no sound or flow in the tube. It is believed that this voltage increase was caused by heat transfer from the hot wire to the wall. This phenomenon could affect the accuracy of the velocity measurements near the wall where the magnitudes of the measured velocities are small. Another phenomenon which can affect the accuracy of the hot wire measurements is free convection from the wall which was shown to be important when the magnitude of the measured velocity is less than, approximately, 25 cm/sec. Finally, Ref. 15 reports that the orientation of the hot wire supports relative to the steady component of the velocity can also affect the accuracy of the hot wire measurements. These studies indicate that while the shape of the measured velocity profiles are not affected by the orientation of the hot wire supports, their magnitudes can differ by as much as 25 percent with the highest and lowest accuracies obtained when the hot wire supports are aligned along or perpendicular to the direction of the steady flow, respectively. Unfortunately, the supports of the hot wire used in this study were perpendicular to direction of the mean flow which according to Ref. 15 may have resulted in some errors in the measured velocities. However, since the orientation of the wire supports does not affect the shapes of the measured velocity profiles, then the velocity distributions presented herein should at least provide a correct qualitative description of the measured velocities.
SUMMARY

This paper describes the status of an ongoing study in which a hot wire is used to investigate the effect of acoustic oscillations upon the radial distribution of the velocity in a tube simulating the flow conditions in solid propellant rocket motors. While the data indicates that the presence of oscillations may strongly influence the velocity field near the wall, where mass addition occurs, there are also indications that the measured hot wire velocities are in error in flow regions where the magnitudes of the steady velocity components are of the same order as the magnitudes of the oscillatory velocity components. Hence, utmost caution should be exercised in the interpretation of these data, especially in the wall region where the largest errors in the hot wire measurements most likely occur.

REFERENCES

Figure 1. A Schematic of the Experimental Setup.

Figure 2. A Comparison of the Steady State, Radial Distributions of Velocities Measured in the Presence and Absence of Sound Fields of the Same Frequency and Different Amplitudes.
Figure 3. A Comparison of the Steady State, Radial Distributions of Velocities Measured in the Presence and Absence of Sound Fields of the Same Frequency and Different Amplitudes.

Figure 4. Dependence of the Radial Distributions of the Steady State Velocities upon the Frequency of the Excited Acoustic Wave.
Figure 3. A Comparison of the Steady State, Radial Distributions of Velocities Measured in the Presence and Absence of Sound Fields of the Same Frequency and Different Amplitudes.

Figure 4. Dependence of the Radial Distributions of the Steady State Velocities upon the Frequency of the Excited Acoustic Wave.
Figure 3. A Comparison of the Steady State, Radial Distributions of Velocities Measured in the Presence and Absence of Sound Fields of the Same Frequency and Different Amplitudes.

Figure 4. Dependence of the Radial Distributions of the Steady State Velocities upon the Frequency of the Excited Acoustic Wave.
TASK II

Heterogeneous Diffusion Flame Stabilization

J. E. Hubbartt
J. I. Jagoda
W. C. Strahle
A. Research Objectives

This program utilizes a facility which models the flame anchoring region of a solid fueled ramjet. The major objective is to ensure predictability of this kind of a flow through computation which is supported by an adequate data base.

B. Results and Discussion

The following tasks were completed under this research program

1. Design, development, construction and checkout of a wind tunnel system which models the flame stabilization region of a solid fueled ramjet.

2. Acquisition and set-up of a two component laser velocimeter and construction of a three component actuation system for the velocimeter.

3. Cold flow tests using hot film, x-film and laser velocimetry together with pitot-static testing.

4. Set-up and development testing with laser Rayleigh scattering.

The results are described in AIAA Paper No. 84-0013 which will be given in Reno, Nevada at the 22nd AIAA Aerospace Sciences Meeting. That paper follows, since it properly describes the program.
C. Publications


D. Personnel

Principal Investigators: James E. Hubbartt, Warren C. Strahle and Jechiel I. Jagoda

Research Engineer: R. E. Walterick

Graduate Research Assistants: C. R. J. Richardson, W. A. deGroot, W. Grissom, W. I. McNicoll

E. Professional Activities/Interactions:


EXPERIMENTS AND COMPUTATION ON TWO-DIMENSIONAL TURBULENT FLOW OVER A BACKWARD FACING STEP

R. E. Walterick, J. J. Jagoda C. R. J. Richardson
W. A. De Groot, W. C. Strahle, and J. E. Hubbard
School of Aerospace Engineering, Georgia Institute of Technology
Atlanta, Georgia

Abstract

Measurements are made of two components of velocity and shear stress on a turbulent incompressible two-dimensional flow over a backward facing step in a confining channel. A $k-\varepsilon$ method of calculation was developed which produces good agreement with experiment for both time mean and turbulence quantities. Reattachment length is particularly well predicted. Sensitivity of this flow to geometrical details and initial conditions has been numerically investigated and found to be high, consequently comparison with other data sets is difficult. Fluctuations of the order of 2% were found in a region of the flow which should not have been vortical, and acoustic motions due to an unsteady recirculatory flow are suspected.

List of Symbols

- \( \rho \): density
- \( \sigma_k, \sigma_\varepsilon \): turbulent Prandtl numbers for diffusion of \( k \) and \( \varepsilon \)
- \( \gamma \): shear stress
- \( \theta \): dependent variable (\( U, V, k, \varepsilon \) etc.)
- \( \delta_{ij} \): Kronecker delta
- \( \delta_{Vol} \): elemental volume corresponding to a given cell
- \( a \): coefficient in the finite difference equation
- \( b, c \): coefficients in the linearized source term
- \( C_1, C_2 \): turbulence model constants
- \( E, F, G \): empirical constant in 'law of the wall' equation
- \( H \): shear force at the solid boundary
- \( k \): production term in the turbulent kinetic energy equation
- \( p \): step height
- \( P \): turbulent kinetic energy
- \( S \): 'source' term in the governing equation for \( \theta \)
- \( U, U_{inlet} \): mean axial velocity
- \( u^2 \): mean freestream axial velocity at the inlet
- \( u' \): turbulent fluctuating velocity in the axial direction
- \( \overline{u'v'} \): Reynolds normal stress in the axial direction
- \( v' \): turbulent fluctuating velocity in the transverse direction
- \( \overline{u'v'} \): turbulence shear stress
- \( x, y, \alpha \): transverse coordinate
- \( \beta \): empirical constant in the velocity - pressure gradient correlation model
- \( e \): effective turbulent exchange coefficient for \( \theta \)
- \( \varepsilon \): dissipation rate of turbulence energy
- \( \chi \): von Karman constant
- \( \mu \): molecular (laminar) viscosity
- \( \mu_e \): effective viscosity
- \( \mu_t \): turbulent viscosity
- \( \nu \): laminar kinematic viscosity

Subscripts

- \( i, j \): tensor notation
- \( p \): node at which calculations take place
- \( E, N, S, W \): nodes surrounding the \( p \) node
- \( t \): turbulent value
- \( o \): values pertaining to the dependent variable \( \theta \)
- \( s \): value at solid boundary
- \( \theta_{rms} \): root mean square

Introduction

Solid-fueled ramjets operate by ingestion of air and subsequent combustion with a solid fuel. The low velocity of flow near the head end of the fuel grain is imperative for flame stabilization. This may be achieved by creating a backward facing step at the flow boundary thereby forcing a recirculation zone behind the step. In practice the flow field is highly turbulent, and, even without combustion, is highly complex.

A goal at this laboratory is to be able to compute the solid-fueled ramjet flowfield to an accuracy commensurate with providing scaling laws for quantities such as the flame blowoff limit and fuel regression rate. To this end a facility is under development to simulate a solid fueled ramjet flame stabilization region. The first phase of this development has yielded the cold flow facility of Fig. 1. The first phase tests of a backward facing step in a cold flow have been completed and are the subject of this paper.

Incompressible two dimensional turbulent flow over a backward facing step has received considerable work from several investigators and a recent review article summarizes this work. This flow was labeled as a standard flow with a number 0420 assigned to it in Ref. 3. Consequently, the work here adds to the data base of such flows. It should be noted that this flow is highly complex, there are discrepancies in the results of several investigators, and truly satisfactory calculability of the flow has not been demonstrated. A calculation method is developed here which gives somewhat better predictability of the flow than has been heretofore demonstrated, especially with regard to reattachment length. Comparisons are given between calculations and the experimental results of several workers together with the experimental work at this laboratory.
**Tunnel**

The tests described in this paper were carried out in a specifically designed suction type open tunnel. In this facility, shown in Fig. 1, air is drawn in directly from the room through a bellmouth (3.5 cm radius of curvature) into a rectangular boundary layer development section, 7 cm high, 41.9 cm wide and 61.6 cm long. At the end of this section the flow passes over a 3.47 cm high backward facing step into the test section which is 43.2 cm long, 10.47 cm high and 41.9 cm wide. A ratio of step height to width of 12 was selected to eliminate possible wall effects near the axis of the test section, Ref. 7. This section is followed by a transition section consisting of a 30 cm long constant area duct joined to a 61 cm long small angle diffuser of expansion ratio 21, which dumps into a large plenum chamber. The diffuser and plenum chamber are separated by a thin, fine mesh screen.

A 60 HP centrifugal blower draws the air from the other end of the plenum through a 1 cm honeycomb structure followed by a screen. The plenum chamber, honeycomb and screens serve to prevent flow disturbances created in the blower from passing back up into the test section. The exhaust from the blower is ducted through a throttling orifice to a port in an external wall of the building through which the flow is discharged.

The side walls of the test section consist of broadband anti-reflection coated, optical quality glass plates which permit optical diagnostics of the flow to be carried out. The tunnel is fitted with a row of static pressure taps in the streamwise direction, displaced 1.27 cm from the center line, for determining local static pressures as well as the axial static pressure distribution. Seven and six such ports, equally spaced, are fitted into the floor of the boundary layer development section and the constant area connecting duct, respectively. Both the ceiling and the floor of the test section are instrumented with 23 static pressure taps. The separation between these ports in the test section is 2.5 cm, except near the reattachment point of the flow, where this spacing has been halved.

Lateral rows of five static pressure taps were also incorporated into the tunnel at selected positions in the boundary layer development section, the step itself, the test section, and beyond, in order to monitor the two dimensionality of the flow. All static pressures were measured by a Scanivalve system using variable-capacitance, precision pressure transducers. The Mach number in the test section is about 0.2 and the flow was observed to be almost perfectly two dimensional. Furthermore, smoke and tufts attached to all tunnel walls were used to verify the quality of the flow.

A second row of access ports was incorporated in the test section ceiling, parallel to the static pressure ports on the opposite side of the tunnel center line, in order to permit diagnostic probes to be introduced into the flow. An actuator fitted to the top of the tunnel facilitates a remotely controlled vertical traversing of the probes, while the streamwise translation in discrete steps corresponding to the position of the probe access ports was carried out manually. The stagnation temperature of the flow was determined with a thermocouple placed just downstream of the test section.

**Diagnostics**

Probe measurements of velocities were carried out using single hot-films (TSI 1210-20), x-films (TSI 1210-T 1.5) and pitot probes. Some preliminary laser Doppler velocimeter (LDV) results will also be reported here.

The hot film and x-film probes were calibrated in a uniform nonturbulent flow using 15 discrete velocities over a range from 45 to 260 ft/sec. At each velocity the probe output signal was periodically sampled, digitized and a mean output voltage determined. These mean voltages for the fifteen sampled velocities were fitted to King's law using the least square technique in order to establish the calibration coefficients. These calibrations were carried out before and after each run. In general, it was found that the discrepancy between the calibration carried out before and after three complete vertical traverses was less than 1.5%. Single and x-film measurements were carried out at each measurement station in the tunnel. The x-films were placed in the vertical plane, normal to the tunnel floor with each film at an angle of 45° to it. The single film was positioned normal to the plane of the x-films. Mean velocities in the streamwise and the vertical directions were determined from the mean voltages. The mean squares of the fluctuating components of the signal of each of the three films were used to determine the mean squares of the fluctuating components of velocity as well as the Reynolds shear stresses. All values were normalized with respect to the steady component of the streamwise velocity upstream of the step. The effects of the normal and binormal velocity components on the films were taken to be equal while the effects of velocities tangential to the films were neglected.

Mean stagnation pressure distributions throughout the flow in the test section were determined using a traversable pitot probe connected to a 100 mm Hg pressure transducer. These stagnation pressures were referenced to the static pressures as determined at the tunnel ceiling for the relevant downstream locations in order to determine the mean streamwise velocity distributions.

Both hot wire and pitot probe techniques have problems in areas of high turbulence superimposed on low velocities as well as in reverse flow regions of the flow field. Hot wire probes, which generally permit the determination of mean velocities and turbulent intensities, are unable to differentiate between forward and reverse flow. This makes them unsuitable in locations where the peak velocity fluctuations exceed the mean flow velocity. Furthermore, the linearization techniques used become inaccurate at local turbulence intensities larger than about 30%. The pitot probes and pressure transducers exhibit too slow a response for turbulence intensity measurements. The probes further suffer for having to be reversed for backward flows. They are, therefore, again inaccurate in regions where the velocity fluctuations exceed the mean flow velocity.

In order to overcome these problems the position of flow reversal (the recirculation region zero velocity line) as obtained using laser Doppler velocimetry is also being reported here. The velocimeter used is a two-component system (TSI 9100-7) fitted with a 4 watt argon ion laser (Spectra Physics 165-08) and operates in the back-scatter mode using two counter type signal processors (TSI 1990 A), Bragg cell frequency shifters (TSI 9108) in one beam of each of the two beam pairs permit a detection of reverse flow. The frequency shift introduced by the Bragg cells is adjusted to local flow conditions to permit acceptance of the entire velocity related signal with maximum accuracy. Filter bandwidths are adjusted simultaneously to yield a maximum signal to noise ratio without loss in extreme velocity fluctuations. The system is mounted on an actuator which is remotely controlled in three degrees of linear translation and manually adjustable in tilt. The
digital counter outputs are analysed in the form of velocity probability density functions which permits the determination of mean velocities and turbulence intensities.

Analysis

Governing Equations

A two-equation turbulence model (k-ε) is used for the prediction and analysis of the two-dimensional steady, turbulent flow over a backward facing step. In this model, transport equations for the turbulent kinetic energy and its dissipation rate are employed along with the conservation equations for mass and momentum and an equation for the turbulent viscosity.

For a two-dimensional, steady, turbulent flow, the governing equations can be written in the general form

\[ \frac{\partial}{\partial x_j} \left( \rho u_j u_k \right) = \frac{\partial}{\partial x_j} \left( \rho \frac{\partial u_k}{\partial x_j} \right) - \frac{\partial}{\partial x_j} \left( \rho u_k \frac{\partial u_j}{\partial x_j} \right) + \frac{\partial}{\partial x_j} \left( \Gamma \frac{\partial u_k}{\partial x_j} \right) + S_k \]

(1)

where \( \delta \) represents the dependent variable (U, V, k, ε etc.) being considered (\( \delta = 1 \) for the continuity equation), \( \Gamma_\delta \) is the appropriate effective exchange coefficient for the turbulent flow and \( S_k \) is the "source" term. This model assumes isotropic diffusion with the diffusion coefficient \( (F) \) given as

\[ F = \frac{1}{3} \right) \rho k \]

The definitions of \( \Gamma_\delta \) and \( S_\delta \) for the various dependent variables (\( \delta \)) are listed in Table 1. These definitions are similar to those given in Ref. (7) except those for the turbulent kinetic energy equation.

The turbulent kinetic energy equation for a steady flow can be written as

\[ \frac{\partial}{\partial x_j} \left( \rho u_j k \right) = \frac{\partial}{\partial x_j} \left( \rho \frac{\partial k}{\partial x_j} \right) - \frac{\partial}{\partial x_j} \left( \rho u_j \frac{\partial u_j}{\partial x_j} \right) + \frac{\partial}{\partial x_j} \left( \Gamma_k \frac{\partial k}{\partial x_j} \right) + \frac{\partial}{\partial x_j} \left( \Gamma_{\epsilon} \frac{\partial \epsilon}{\partial x_j} \right) + S_k \]

(2)

Usually the velocity-pressure gradient correlation term, which is the last term on the right hand side of Eq. (2), is lumped into the diffusion term, the third term on the right hand side of Eq. (2). Based on independent work done at the Georgia Institute of Technology, it was decided to include this correlation term explicitly in the turbulent kinetic energy equations. This velocity-pressure gradient correlation term is modelled as

\[ \frac{\partial u_k}{\partial x_j} = \frac{2}{3} \rho k \delta_{ij} - \mu_{\epsilon} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \]

(3)

where \( \alpha \) is an empirical constant. This correlation term is added to the source term in the turbulent kinetic energy equation (see Table 1).

The following equation for the Reynolds stresses has been used in this analysis,

\[ \gamma \frac{\partial u_i u_j}{\partial x_j} = \frac{2}{3} \rho k \delta_{ij} - \mu_{\epsilon} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \]

(4)

Wall functions which link the flow variables at solid boundaries to those in the near-wall region are employed.

Solution Technique

A numerical code called TEACH, originally developed at Imperial College, is used to solve the elliptic partial differential equations given by Eq. (1) and Table 1 along with the appropriate boundary conditions. This code was modified to include the velocity-pressure gradient correlation term and the modified version of the numerical code is referred to as TEACH-GT in the present work.

In the numerical code, the flow domain is overlaid with a rectangular grid. The grid is arranged such that the flow boundaries lie along the control volume boundaries. All the dependent variables except the velocities are calculated at the intersection points or "nodes" of the grid. The velocities are calculated at locations midway between these nodes.

A finite difference counterpart of Eq. (1) is derived by supposing that each variable is enclosed in its own control volume or "cell" formed by the grid liner. Equation (1) is integrated (microintegration) over the control volume corresponding to each node. A hybrid differencing scheme is used. The source term is linearized as

\[ \int S_\delta \, dVol = b \frac{\partial u_p}{\partial x} + c \]

Volume

where the subscript 'p' denotes the node at which calculations take place.

The continuity and momentum equation are combined to derive an equation which relates the change in the pressure field to those in the mean velocity field. The pressure changes obtained from this equation are used to adjust the velocity field thus satisfying the continuity equation.

The finite difference equations are assembled to yield the general equation

\[ (a_p - b) \phi_p = a_N \phi_N + a_S \phi_S + a_E \phi_E + a_W \phi_W + C \]

(5)

where \( N, S, E \) and \( W \) denote the nodes surrounding the node 'p' and

\[ a_p = a_N + a_S + a_E + a_W \]

(6)

These equations are solved with appropriate modifications made to account for the boundary conditions.

Boundary Conditions

The conditions at the outlet are seldom known. The practice employed here is to locate the outlet boundary in a region where the flow is strongly outwards directed and hence insensitive to downstream conditions. The upwind differencing scheme used here helps in this respect.

The variables at the solid boundaries (denoted by the subscript o) are linked to those at the first grid node near the wall (denoted by the subscript '1') by algebraic relations which are consistent with the logarithmic "law of the wall" (wall functions). Use of these wall functions minimizes computer storage and run times.
### Table 1. Definitions of Terms

<table>
<thead>
<tr>
<th>Conserved property</th>
<th>$\bar{s}$</th>
<th>$\Gamma_{\bar{s}}$</th>
<th>$S_{\bar{s}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>x momentum</td>
<td>$U$</td>
<td>$\mu_{\text{eff}}$</td>
<td>$\frac{\partial}{\partial x} \left[ \frac{\mu_{\text{eff}}}{\partial_{\text{eff}}} \frac{\partial U}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \frac{\mu_{\text{eff}}}{\partial_{\text{eff}}} \frac{\partial V}{\partial y} \right] - \frac{\partial P'_{\text{eff}}}{\partial x}$</td>
</tr>
<tr>
<td>y momentum</td>
<td>$V$</td>
<td>$\mu_{\text{eff}}$</td>
<td>$\frac{\partial}{\partial x} \left[ \frac{\mu_{\text{eff}}}{\partial_{\text{eff}}} \frac{\partial U}{\partial y} \right] + \frac{\partial}{\partial y} \left[ \frac{\mu_{\text{eff}}}{\partial_{\text{eff}}} \frac{\partial V}{\partial y} \right] - \frac{\partial P'_{\text{eff}}}{\partial y}$</td>
</tr>
<tr>
<td>Turbulent energy</td>
<td>$k$</td>
<td>$\frac{\mu_{\text{eff}}}{\sigma_k}$</td>
<td>$\frac{\varepsilon}{k} \left( c_1 \sigma - c_2 \rho \varepsilon \right)$</td>
</tr>
<tr>
<td>Dissipation rate</td>
<td>$\varepsilon$</td>
<td>$\frac{\mu_{\text{eff}}}{\sigma_\varepsilon}$</td>
<td>$\frac{\varepsilon}{k} \left( c_1 \sigma - c_2 \rho \varepsilon \right)$</td>
</tr>
</tbody>
</table>

where

$$ G = -\rho \frac{\partial}{\partial x} \left[ \frac{\mu_{\text{eff}}}{\partial_{\text{eff}}} \frac{\partial U}{\partial x} \right] $$

$$ \mu_{\text{eff}} = \mu_t + \mu $$

$$ \mu_t = C_\mu \rho \frac{k^2}{\varepsilon} $$

and

$$ C_\mu = 1.44, \quad C_1 = 1.92, \quad C_2 = 1.0, \quad \sigma_\varepsilon = 1.217, \quad \sigma = 0.067 $$

The wall function for the momentum equation is,

$$ U_1^+ = \frac{1}{N} \ln(Ey_1^+) \quad (7) $$

where

$$ U_1^+ = \frac{U_1 C_L k_1^{-3/2} \varepsilon_{\text{w}}}{\sqrt{\tau \rho_0}} \quad (8) $$

The quantity $k_1$ is calculated from the regular balance equation for the cell corresponding to the near-wall node. While calculating $k_1$, an average value is assigned to the dissipation term in the turbulent kinetic energy equation which is deduced from the assumption.

$$ \int_0^{\varepsilon_1} \varepsilon \, dy = \frac{C_3}{k_1^{3/2}} \tau \ln(Ey_1^+) $$

The production term in the turbulent kinetic energy equation is approximated as

$$ G \approx \frac{\tau (U_1 - U_o)}{\varepsilon_1} $$

The boundary condition along the solid boundary for the momentum equation is applied in the form of a shear force ($F_o$) given as

$$ F_o = \tau_0 \frac{C_L k_1^{-3/2} \varepsilon_{\text{w}} (U_1 - U_o)}{U_1^+ \delta Vol} \quad (12) $$

$E = 9.79$ and $\chi = 0.4187$
where $k_{pw} = \frac{1}{2} (k_p + k_w)$ for the near wall node. The $\tau_0$ in Eq. (11) is deduced by inverting the logarithmic law of the wall given by Eq. (7). The dissipation rate at the near wall node is calculated directly from the expression.

\[
e^{-1} = \frac{u}{\kappa_{1}} \left( \frac{2}{H} \right)^{3/4} \frac{2}{2 \sqrt{2}}
\]

(Eq. 13)

Predictions

Grid dependency tests were carried out and a 31 x 41 grid was chosen for the analysis. A grid spacing which is expanding in the axial direction has been used. The grid spacing is constant in the transverse direction. The outlet boundary is located at a distance of twenty four stepheights from the inlet (step).

Preliminary calculations with plug flow conditions at the inlet and $\alpha = 0$ (Eq. (3)) yielded a reattachment length of 5.86 stepheights. This is lower than the experimentally observed reattachment length of $7 \pm 1$ stepheights.

The reattachment length measured at the experimental facility at Georgia Institute of Technology was 7.33 stepheights. Inclusion of the velocity-pressure gradient correlation term given by Eq. (3) improved the prediction of reattachment length. With $\alpha = 0.067$ and plug flow conditions at the inlet, the reattachment length was predicted to be 6.22 stepheights.

Inlet conditions are important in this analysis. The inlet location coincides with the stepface in this analysis. Since the calculations in the TEACH-GT code start at the grid just downstream of the stepface, it is important to have accurate inlet conditions. Specifically, the mean axial velocity profile at the inlet must be accurate. Calculations were carried out with different mean velocity profiles at the inlet. The most accurate among these profiles was the one measured at the experimental facility mentioned in the present work using hot-wire anemometry. With this profile at the inlet and $\alpha = 0.067$, the reattachment length was predicted to be 7.26 stepheights which compares very well with the experimentally measured reattachment length. The computer execution time for a typical calculation was about 1200 CPU seconds on a CYBER 835 computer.

Use of Eq. (6) in the analysis implies that the normal turbulent stresses are nearly equal which is inaccurate, especially in the high turbulence regions. With this equation, the streamwise turbulence intensities are underpredicted and the transverse turbulence intensities are overpredicted. Any comparison of the turbulence intensities evaluated from Eq. (6) with experimental data must therefore be inaccurate. For this reason the following empirical relations for plane shear layers were used to calculate the turbulence intensities after obtaining the converged solution using the numerical code TEACH-GT.

\[
\frac{v^2}{k} = \frac{2}{3} = 0.3
\]

\[
\frac{v^2}{k} = \frac{2}{3} = 0.18
\]

(Eq. 14)

Results

The experimental results presented here are primarily from hot-film anemometry. Some pitot probe data and preliminary laser velocimeter data are included for the purpose of comparison. In all cases the probe measurements in the vertical plane are terminated above $y/H = 2.0$ due to physical restrictions between the probes and the test facility.

Figure 2 shows the development downstream of the step of the longitudinal component of the mean velocity, the longitudinal and vertical components of the turbulence intensity and the shear stress. The freestream velocity, which nominally is 74.0 meters per second, is used to normalize the results. Pitot probe derived mean velocities are included for comparison with hot-film data at six $x/H$ locations that bracket the shear layer mean reattachment point. The mean reattachment point is shown from the pitot data to occur at an $x/H$ of 7.33, which is in excellent agreement with the TEACH-GT prediction of $x/H = 7.26$. The characteristic reverse flow in the recirculation region is seen at locations $x/H = 6.4$ and $x/H = 5.13$. Pitot data in the reverse flow of the recirculation region was obtained with the pitot probe reversed. Pitot measurements upstream and downstream of the six presented here are available, but only those that coincide exactly with the hot-film $x/H$ locations are shown. Note that the hot-film measurement presentation is terminated prior to entry into the recirculation region. The longitudinal turbulence intensity based on local mean velocity approaches 50 percent at the termination point. Inaccurate probe readings result beyond this point because the signal is rectified. In the freestream above the shear layer the comparison of pitot and hot-film data yields acceptable agreement; however, disagreement between the pitot and hot-film data at $x/H = 7.33$ and $x/H = 8.06$ as the test section floor is approached can be attributed to the above mentioned signal rectifying problems and truncation of the voltage-velocity relation. Finally, a freestream flow acceleration of approximately 2.5 percent is seen at $x/H = 1.1$ and $x/H = 2.56$. This fact is confirmed by pitot measurements.

Turbulence intensities are shown for $x/H = 1.47$ to $x/H = 10.99$ in the second portion of Fig. 2. For this case values of $u_{rms}$ have been determined from single film measurements and values of $v_{rms}$ from x-film measurements of $\left( \frac{v^2}{2} \right)$ with single film data of $\frac{v^2}{2}$ subtracted to determine $\frac{u^2}{2}$. Hence $v_{rms}$ data are presented at only six $x/H$ locations where single film and x-film measurements are coincident. Freestream turbulence intensity at the step location has been measured at 1.3 percent. This value is higher than normally found in a potential flow core. As noted in this investigation and as reported by other investigators there appears to be a low frequency flow oscillation indicated by the reattachment point wandering about a mean location by $\pm$ one stepheight. This oscillation could be seen as contributing to the velocity fluctuations. In the freestream $u_{rms}$ levels are the same, as is to be expected. Within the shear layer $v_{rms}$ is lower than $u_{rms}$ and shows more scatter in the data. Again measurements are not presented in the recirculation region. Finally, the characteristic broadening of the shear layer accompanied by a drop in maximum $u_{rms}$ values as the flow proceeds downstream is evident. The lower portion of Fig. 2 presents shear stress levels in the flow. Values are essentially zero in the freestream and grow as the shear layer is entered. A maximum is reached prior to reattachment followed by a gradual broadening and drop in levels. The return to values of zero at the wall for $x/H = 7.33$ is expected since it is the mean reattachment point. However, proceeding downstream at the wall one would expect the levels to increase toward those typical of a turbulent boundary layer. No such increase occurs with the levels remaining at zero. However, in this region the lower mean velocities coupled with high local turbulence intensity causes the x-film results to be inaccurate.

Figure 3 provides a mapping of the extent of the recirculation region. Shown is a comparison of pitot probe
data with the preliminary laser velocimeter data and the computation of TEACH-GT of the zero velocity streamline. The pitot derived zero velocity line was determined from extrapolation of pressure data from the pitot probe. Due to probe inaccuracies in low mean velocity, high turbulence regions (i.e., high flow angularity at the measurement point) accurate pitot data was not obtained in a band centered on the actual zero velocity point at each station measured. Hence, the extrapolation of the data outside of this band was required. The agreement of the pitot data with the computation is quite good from \( x/H = 2.2 \) downstream to the reattachment point. As stated previously the mean reattachment point is accurately predicted for this case. The inaccurate behavior of TEACH-GT in the vicinity of the step is due to the large grid spacing in the computational scheme. The pitot data indicates an initial drop in the recirculation region height in the region from \( x/H = 0.0 \) to \( x/H = 2.2 \) followed by a plateau between \( x/H = 2.2 \) and \( x/H = 5.5 \) and a final drop until \( x/H = 7.33 \). The preliminary laser data indicates a higher recirculation region between \( x/H = 2.2 \) and \( x/H = 3.66 \) and between \( x/H = 6.59 \) and \( x/H = 7.33 \). Further testing is required to verify this trend.

The recirculation region flow details as computed by TEACH-GT are shown in Fig. 4. Each arrow indicates the velocity magnitude and direction at a point. The magnitude and direction of \( u^* \) is shown for reference. The vector plot indicates the existence of one large recirculation region terminating at reattachment. There is no evidence of a second smaller opposite rotation recirculation region at the step test section floor juncture as reported in some investigations. This is probably due to the grid spacing being too large to pick up this small scale recirculation zone.

A more detailed comparison between several experiments and the TEACH-GT computation for local mean velocity is shown in Fig. 5 for an \( x/H \) of 7.33 (i.e., at reattachment). The pitot data agrees well with the computation except in the freestream where TEACH-GT overpredicts by four to five percent. In this case hot film data are included that confirm the pitot measurements except in the low mean velocity, high turbulence region near the wall where the hot-film data is inaccurate. The data of Kim et al. differs significantly with TEACH-GT in the shear layer with better agreement in the freestream. The data of Eaton & Johnston are obtained with a pulsed wire anemometer and are in excellent agreement with the hot film results.

Profiles of \( u_{rms} \) are presented in Fig. 6 for the downstream location \( x/H = 7.33 \) (i.e., reattachment) and \( x/H = 10.99 \). At reattachment both sets of data display higher freestream values than those computed by TEACH-GT. This is a reflection of the higher overall freestream \( u_{rms} \) values first shown in the initial conditions at the step. Above \( y/H = 1.0 \) the experimental data are in close agreement and indicate a thicker shear layer than TEACH-GT, as also indicated in Fig. 5. It should be noted that the TEACH-GT computations have been modified in this case to account for anisotropic turbulence conditions. Below \( y/H = 1.0 \) the data bracket the computation indicating good prediction of the maximum value and location of \( u_{rms} \).

In the second portion of Fig. 6 (\( x/H = 10.99 \)) freestream values of \( u_{rms} \) are again higher than those computed and the shear layer is thicker. The data profiles are similar except that the single film data indicate a shear layer displaced slightly further from the floor than that of Kim et al. Maximum values of \( u_{rms} \) are predicted well with just a slight discrepancy in the \( y/H \) location of the maximum. Here, TEACH-GT incorrectly shows \( u_{rms} \) values going to zero at higher \( y/H \) values than found for turbulent boundary layers. The plot has been terminated at \( x/H = 0.0385 \), the last computed point at the floor. The single film data more accurately shows the actual trend of higher levels closer to the floor.

A plot of the maximum value of \( \frac{\overline{u^2}}{\overline{v^2}} \) for each measurement station is shown in Fig. 7. Here again the results of TEACH-GT have been adjusted for anisotropic turbulence conditions. The single film data are slightly underpredicted but the agreement is quite acceptable. Despite some scatter in the single film data it too brackets the prediction and agrees with the single film data. All the data sets except those of Kim et al. predict a maximum of the maximum values of \( \overline{u^2} / \overline{v^2} \) slightly downstream of \( x/H = 5.0 \). Kim et al. indicates maximum values nearer the reattachment point. Values computed by TEACH-GT have been eliminated in the vicinity of the step due to grid generated inaccuracies.

A similar presentation for the maximum values of \( \frac{\overline{u^2}}{\overline{v^2}} \) is given in Fig. 8. TEACH-GT values are again adjusted for anisotropic turbulence conditions. Agreement between experiment and theory is fair. Kim's data levels are in better agreement with the theory except that the maximum is again predicted to occur at values closer to reattachment. x-film data were not obtained upstream of \( x/H = 4.4 \) nor downstream of \( x/H = 8.06 \). Therefore, it is not known if at least the maximum point is correctly predicted by the x-film. The Bremmer et al. data show large scatter and disagreement with theory.

Reynolds shear stress profiles at two different \( x/H \) locations are shown in Fig. 9, one at reattachment and one further downstream at the last x-film measurement location. Both Kim's data and the x-film data indicate a thicker shear layer at these locations than predicted by TEACH-GT. For \( y/H \) greater than about 1.0 the data are comparable but below \( y/H = 1.0 \) the x-film data are below those of Kim. At the reattachment location the data of Kim et al. show a maximum value which agrees well with that predicted by TEACH-GT. However, the \( y/H \) location of this maximum is higher than predicted. The x-film data, though lower in magnitude more nearly defines the maximum \( y/H \) location according to the theory. The stresses at reattachment as determined by TEACH-GT and the hot-film go to zero at the floor as they should. At \( x/H = 11 \) TEACH-GT and Kim's data are in good agreement as to magnitudes and maximum location. The x-film data maximum location is also in agreement. As previously stated the stresses should not go to zero at the floor so rapidly for this downstream location.

Figure 10 presents the streamwise development of the maximum value of Reynolds shear stress. The TEACH-GT computation is included as well as a curve computed for \( \overline{u'v'} = 0.3 \) based on the work of Ref. (12). The data of Bremmer et al., though sparse, are in good agreement with the unadjusted TEACH-GT computation. For \( x/H > 7 \) levels of Kim et al. are in excellent agreement with the three-tenths turbulent kinetic energy approximation. However, the maximum value is again measured at reattachment whereas the x-film data from this experiment provides a better correspondence with the results from TEACH-GT as to location of the maximum.

Note that the data from Kim et al. and Eaton and Johnston have been faired through for purposes of clarity.
In the preceding presentation of results from several different investigations there exist discrepancies among the various data sets and the TEACH-GT computations. In an effort to determine causes for these differences it was decided to investigate the influence of two factors using results from TEACH-GT. One of these factors was the effect of varying initial conditions in longitudinal mean velocity profile and longitudinal turbulence intensity at the step (i.e., x/H = 0.0). The other was a variation of test section height to step height (see Fig. 1). The sensitivity to initial flow conditions is shown in Fig. 11 which presents u, profiles at reattachment. Looking first at the two results with a uniform velocity profile (plug flow), the effect of varying initial u, levels is negligible except in the freestream region. Even in the freestream the differences are nearly damped out. Changing to the single wire measured initial velocity profile as input together with a turbulence intensity of one percent, there is a growth of the shear layer in height and maximum values that more closely approaches the experimental single film values at reattachment. Calculations of reattachment length give x/H = 6.3 for the uniform flow case and x/H = 7.26 for the experimental case. Therefore the recirculation region has experienced an overall enlargement. Apparently any differences amongst investigations as to initial conditions would contribute to further differences throughout the flow. It should be noted that all TEACH-GT computations presented to this point use the experimentally generated velocity profile as input for initial conditions. The effect of varying the test section height to step height is the second influential factor checked and is presented in Fig. 12 with area ratio as a parameter. Again a u, profile at x/H = 7.33 is depicted. The area ratio is determined by keeping the test section height constant and varying the height of the step. Several important trends become evident. First, with increasing area ratio the shear layer becomes thicker. This is a consequence of pressure rise and an overall increase in the extent of the recirculation region. At an area ratio of 1.33 the computed reattachment length is 5.8 step heights and at an area ratio of 2.0 the reattachment has grown to 7.94 step heights. Secondly, along with the growth in the recirculation region there occurs a movement away from the test section floor of the maximum u, values, again a consequence of an enlarging recirculation region. Finally, after reattachment u, values typically decline rapidly so that as the reattachment length decreases the u, values at x/H = 7.33 should decrease also. Differences of area ratio between investigators is another source of discrepancies in data sets.

Discussions

The contributions of this work are to a) add to the data base of two dimensional turbulent flow over a backward facing step and b) put forth a computational method based upon k-ε technique which will adequately predict the results. By adjustment of the usual method of treatment of the velocity-pressure gradient correlation in the turbulent kinetic energy equation, it has been found possible to obtain overall good agreement with experiment. This agreement comes about in both time mean and turbulence quantities. The primary deficiency in the calculation appears to be an underprediction of the width of the shear layer.

Neglect of the parallel velocity effect on the x-film has resulted in a somewhat low measurement of the magnitude of v, and u, values. This deficiency will be corrected with the future laser velocimetry measurements. However, the data are in general agreement with the body of data collected from other workers. The calculations have suggested that such comparisons must be made with care since the flow field is sensitive to initial conditions and the exact geometry.

A primary finding is that the freestream is turbulent in regions where no vorticity should be present. This can only be an unsteady low frequency potential motion set up by the unsteady, confined flow. Such unsteadiness has been reported by other workers.

Acknowledgements

This work was supported by the Air Force Office of Scientific Research under Contract No. F49620-82-C-0013.

References


Figure 1. Test Section Details.

Figure 2. Mean Velocity, Turbulence Intensity and Shear Stress Profiles.

Figure 3. Recirculation Region Zero Velocity Line.

Figure 4. Recirculation Region Velocity Vector Diagram.

Figure 5. Comparison of Longitudinal Mean Velocity.

Figure 6. Comparison of Longitudinal Turbulence Intensity Profiles.
Figure 7. Maximum of Longitudinal Turbulence Intensity Squared.

Figure 8. Maximum of Vertical Turbulence Intensity Squared.

Figure 9. Comparison of Shear Stress Profiles.

Figure 10. Maximum Reynolds Shear Stress.

Figure 11. Sensitivity at Reattachment to Initial Flow Conditions.

Figure 12. Sensitivity to Area Ratio.
Strahle, W. C.: Associate Editor for Combustion and Aeroacoustics, *AIAA Journal*


Jagoda, J. I.: Member of the AIAA Propellants and Combustion T. C.
A. Research Objectives

The objectives of this task are to gain understanding and improved control of combustion of the aluminum ingredient in solid propellant, and of the effect of aluminum on overall propellant combustion. In practical terms, this relates to attainment and assurance of desired burning rate, combustion efficiency, combustor stability and resistance to detonation, while striving for high propellant density and high specific impulse.

During the present year, the above objective has been extended to study of the behavior of other particulate propellant ingredients that, like aluminum, have relatively low volatility and hence tend to concentrate on the burning surface. Such ingredients include other metals, additives to modify burning rate, and additives to control combustion stability.

Specifically, the objectives for FY 1983 were: 1) to clarify the detailed behavior of aluminum in AP/HC binder/Al propellants; 2) to assemble the detailed behavior of aluminum into a systematic qualitative theory; 3) to develop an experiment for controlled perturbation of the burning surface of aluminized propellants by flow disturbances; 4) to carry out a screening study of aluminum behavior with propellants other than AP/HC/Al formulations; and 5) to start studies on other low volatility ingredients.
B. Status of Research

Detailed Behavior of Aluminum and Qualitative Theory

The primary emphasis for the year has been on consolidation of accumulated results into a qualitative theory for aluminum behavior and completing tests to evaluate critical features of the theory. Most effort has been on ammonium perchlorate-hydrocarbon binder-aluminum (AP/HC/Al) systems, with attention directed to the behavior of aluminum prior to and during detachment from the burning surface. This includes the ignition-agglomeration step.

Specific progress includes:

a) Exploration of a method for study of the response of aluminum particles during heating, using a scanning electron microscope. Single and interacting particles are located on an electrically heated element in the SEM and observed at high magnification during heating. The method holds promise for understanding (and modifying) the breakdown of the oxide skin that leads to sintering and/or coalescence of hot particles.

b) Completion of a qualitative theory for aluminum behavior in combustion of AP/HC/Al systems. This consists of a "scenario" for the complex sequence of processes that control behavior. That portion of the theory covering events on the propellant burning surface (up to and including agglomeration and ignition) is summarized in Ref. 1.

c) Because of a diversity of views regarding the conditions that precipitate ignition of accumulative aluminum on the burning surface (and because of the critical importance of ignition to overall behavior), the accumulated evidence regarding ignition was assembled into a single report (Ref. 2). As contended in the qualitative theory, ignition of agglomerate-forming aluminum is
a propagative inflammation of sintered particle arrays, precipitated locally by exposure to oxidizer-binder flamelets. This behavior cannot be described accurately without consideration of the multidimensional features of the combustion zone.

d) The qualitative theory was tested by evaluation of its predictive capability. A special series of propellants was prepared, and tendency towards aluminum agglomeration was measured in combustion plume quench tests. The test series was designed to verify a theoretically predicted transition in agglomeration behavior as a function of pressure and oxidizer particle size. Preliminary results were presented in last year's report (Ref. 3). Further quench tests and combustion photography have confirmed and extended the results, which clearly show the predicted transitions in agglomeration behavior (Fig. 1). Details are reported in Ref. 1, 4-7. It should be stressed that the experiment is a test of several aspects of the theory, including the criterion for ignition of aluminum noted above, and the requirement that this criterion be applied in terms of a multidimensional view of the combustion zone microstructure.

Experiment for Perturbation of the Burning Surface

The objective of this experiment is to learn more about the response of aluminum behavior to the gas flow environment, with particular emphasis on response of accumulation-agglomeration-ignition-detachment processes. In particular, the objective is to determine the response to transient and periodic disturbances. Such response is considered to be critical to understanding combustion instability of aluminized propellants, but has thus far received only minimal study. Our improved understanding of aluminum behavior in general should greatly facilitate interpretation of results of perturbation response tests.
Experimental verification of aluminum accumulation-ignition-agglomeration theory using propellants with bimodal oxidizer particle size and three different sizes for fine component.

Region I: Aluminum concentration continues until ignited by AP-binder flamelets of coarse oxidizer.
Region II: Transition region.
Region III: Aluminum concentration is limited by presence of AP-binder flamelets on all AP particles.
Preliminary design was made of a pulsating jet adaptation to the combustion window bomb, which would permit time-resolved observation of burning surface behavior during jet impingement. This phase of the work was not otherwise pursued during this fiscal year.

Aluminum Behavior with "Other" Propellants

Combined experience of various laboratories has shown that aluminum behavior is significantly different with propellants other than the AP/HC/Al system, and the theory clearly indicates why this should be. However, studies of other systems have been much less systematic, and some of the newer propellant systems have received almost no attention relative to aluminum behavior. In view of improved understanding of the detailed mechanisms of aluminum behavior, it is now timely to examine other propellant systems for application, extension and exploitation of the theory.

The intent expressed in the proposal for the present studies was to obtain propellant samples from ongoing programs in other laboratories and carry out screening tests by combustion photography and plume quenching. A number of efforts were made to obtain samples, but government restrictions on release and transport of samples has thus far frustrated these efforts. In view of the very small amount of propellant required, it does not seem that these difficulties should be insurmountable, but it seems likely that some initiative from AFOSR will be required.

Behavior of Other Nonvolatile Ingredients

In addition to aluminum, there are a variety of other particulate ingredients used in propellants that accumulate on the burning surface without
gassification. These include burning rate modifiers such as iron, chromium and copper oxides, and instability suppressants such as zirconium carbide. The mechanisms by which these ingredients act are matters of speculation, and the search for new ballistic modifiers is largely empirical. However, the experimental methods and background of observational data now available on behavior of low volatility ingredients offer improved opportunity to clarify the combustion mechanisms that make these ingredients useful. In the present program, two studies have been started on additives, one on the combustion of ZrC particles in propellant combustion, and the other on combustion of AP/HC binder sandwiches with ballistic modifiers in the binder or oxidizer laminae.

  a) Combustion of ZrC was studied by hot stage microscopy, and by combustion photography and plume quench tests on propellant samples. In the hot stage microscope, particles exhibited no change during heating to 1500°C in CO or Ar atmospheres. In an O₂ atmosphere, the particles formed a white oxide coating at about 1350°C.

Combustion photography of propellant samples with 1% ZrC showed particles burning vigorously above the burning surface. No agglomeration on the burning surface was evident, and the small particles burned very rapidly. Quench tests showed that the particles burned in a very complex fashion, with oxide accumulation on the surface and burning in fissures. This is consistent with the high melting point of ZrC and the high boiling point of ZrO₂. The presence of CO as a reaction product appears to play a role in preventing ZrO₂ from blocking diffusion of oxidizing species to the particle surface. Judging from the apparent fissure-burning, the interplay of accumulating ZrO₂, escaping CO, and inward-diffusing oxidizing species is a very complex process. At this point it is too soon to tell how ZrC might stabilize combustion, and the state of the oxide product
droplets after burnout has not yet been determined. Future tests will address this latter question, and tests will be concentrated on ZrC samples with demonstrated effect on oscillatory combustion at known frequency (so that the possible role of particulate damping by ZrO$_2$ can be evaluated).

b) Combustion of AP-polymer sandwiches has been pursued in great detail on a companion project, with the goal of relating observable features with the structural details of the flame complex. The results of this research provide a framework of experimental methods, observational results, and mechanistic interpretation that offer the opportunity of elucidating the mechanisms by which burning rate modifiers act in the combustion zone.

A series of quench tests were conducted on AP-PBAN sandwiches with 10% ballistic modifier (by weight) in the PBAN lamina. For those initial tests, a burning pressure of 3.45 MPa was chosen. PBAN lamina thickness was 50 μm. Ballistic modifiers were ZrC, Al$_2$O$_3$, Fe$_2$O$_3$, B$_4$C, Cu$_2$Cr$_2$O$_4$, CuO, Iron Blue and Ferrocene. In this series of tests, two classes of results were obtained, illustrated by the two quenched samples in Fig. 2. In part a of the figure, the sample looks very much like the ones with pure PBAN laminae. Such samples resulted with Al$_2$O$_3$, ZrC, and B$_4$C additives. Part b of the figure is typical of samples with Cu$_2$Cr$_2$O$_4$, Fe$_2$O$_3$, Iron Blue and Ferrocene. In all these cases the additive accumulated on the surface of the binder lamina. The burning rate was enhanced. The surface profile of the quenched sample showed characteristic changes that have been interpreted as resulting from approach of the kinetically limited leading edge of the oxidizer-binder flame closer to the surface. It seems likely that this is a result of catalytic "cracking" of large fuel vapor molecules in the filigree of accumulated additive above the binder. Concentration of catalytically active additives on the binder surface appears to be necessary to produce burning rate
Fig. 2 Scanning electron microscope pictures of quenched sandwiches.

a) Example of surface with non-accumulating additive (Al₂O₃) that did not modify burning.

b) Example of surface with ballistically active, accumulating additive (Cr₂O₃) (dull appearance of surface compared to part a is due to imaging technique).
Fig. 2c  Explanation of Figure 2a.

A) Area of AP self-detlagration, unaffected by AP-binder flame.
B) Burning-rate controlling regions, where AP deflagration and AP-binder flames in combination yield maximum surface heating (high rate results in a "Vee"-shaped overall surface profile as in 2b).
C) "Smooth band," where heat flow into the endothermic binder retards AP regression and AP surface decomposition is shifted to dissociative sublimation.
D) Binder lamina (recessed under these test conditions unless additive accumulates).
change. These issues will be more fully elucidated when tests are made with other binders and at other pressures.

References


C. Publications and Presentations

The following were published or presented during 1 October 1982 to 30 September 1983.


D. Personnel

Principal Investigators -- E. W. Price, Professor, and R. K. Sigman, Senior Research Engineer.

Post Doctoral Fellow — J. K. Sambamurthi.

Graduate Research Assistant — B. B. Lieber.

E. Professional Activities

1. Participant, 19th JANNAF Combustion Meeting, presentation of Ref. 7 above, October 1982.

2. Participant, AFOSR/AFRPL Rocket Propulsion Research Meeting, presentation of Ref. 5 above, March 1983.


5. Member, AIAA National Publications Committee, Editorial Board of AIAA Educational Book Series.
TASK IV

Rocket Motor Aeroacoustics

W. C. Strahle
TASK IV

ROCKET MOTOR AEROACoustics

A. Research Objective

The fundamental objective was to demonstrate that the fluctuating pressure field in a rocket motor may be calculated if the state of the turbulence of the interior flow is known.

B. Results and Discussion

The overall results of this program are documented in AIAA Paper No.84-0287 which will be presented at the 22nd Aerospace Sciences Meeting. This paper follows as a proper summary of the program.

C. Publications


D. Personnel

Principal Investigator: Warren C. Strahle
Graduate Research Assistant: Uday G. Hegde

E. Professional Activities/Interaction

Abstract

The present investigation is motivated by vibration problems in solid propellant rocket motors. A class of interior flows modelled to simulate flow conditions inside rocket motor cavities is considered. The turbulence generated pressure fluctuation consists of two components—acoustic and hydrodynamic. The Bernoulli enthalpy theory of aeroacoustics is employed to extract acoustic pressure spectra from experimentally obtained turbulence data and acoustic impedance values at flow boundaries. The effects of turbulence intensities, sidewall acoustic impedance, length to diameter ratio of the cavity and different mass flux on the acoustic pressure level are investigated in experimental configurations. Typical pressure levels inside rocket motor environments are calculated utilizing the A-B representation for propellant response.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>tube radius</td>
</tr>
<tr>
<td>A</td>
<td>tube cross section area, parameter in Eq. (18)</td>
</tr>
<tr>
<td>B</td>
<td>parameter in Eq. (18)</td>
</tr>
<tr>
<td>c</td>
<td>isentropic speed of sound</td>
</tr>
<tr>
<td>f</td>
<td>quantity defined by Eq. (5)</td>
</tr>
<tr>
<td>F(w)</td>
<td>frequency correction factor</td>
</tr>
<tr>
<td>g</td>
<td>Green's function for the Bernoulli enthalpy problem</td>
</tr>
<tr>
<td>g_T</td>
<td>Green's function for the acoustics problem</td>
</tr>
<tr>
<td>g_w</td>
<td>inverse Fourier transform of k^2 g_w</td>
</tr>
<tr>
<td>h</td>
<td>specific enthalpy</td>
</tr>
<tr>
<td>H</td>
<td>Bernoulli enthalpy</td>
</tr>
<tr>
<td>H_o</td>
<td>cross section average of Bernoulli enthalpy fluctuations</td>
</tr>
<tr>
<td>k</td>
<td>wave number</td>
</tr>
<tr>
<td>k_oo</td>
<td>modified wave number</td>
</tr>
<tr>
<td>L</td>
<td>duct length</td>
</tr>
<tr>
<td>L_corA</td>
<td>correlation length in axial direction</td>
</tr>
<tr>
<td>M_e</td>
<td>mean Mach number at nozzle entrance plane</td>
</tr>
<tr>
<td>M_0</td>
<td>mean Mach number at head end</td>
</tr>
<tr>
<td>M_r</td>
<td>mean radial Mach number at propellant surface</td>
</tr>
<tr>
<td>n</td>
<td>index in burning rate law</td>
</tr>
<tr>
<td>p</td>
<td>pressure</td>
</tr>
<tr>
<td>P_H</td>
<td>hydrodynamic pressure fluctuation</td>
</tr>
<tr>
<td>P_a</td>
<td>acoustic pressure fluctuation</td>
</tr>
<tr>
<td>Q(x)</td>
<td>factor in Eq. (15)</td>
</tr>
<tr>
<td>r</td>
<td>radial coordinate, propellant burning rate</td>
</tr>
<tr>
<td>( \tilde{r} )</td>
<td>two dimensional polar coordinate</td>
</tr>
<tr>
<td>R</td>
<td>defined by Eq. (20)</td>
</tr>
<tr>
<td>R_T</td>
<td>inverse Fourier transform of R</td>
</tr>
<tr>
<td>s</td>
<td>specific entropy</td>
</tr>
<tr>
<td>S_oo</td>
<td>defined by Eq. (13)</td>
</tr>
<tr>
<td>S_a</td>
<td>acoustic pressure auto spectrum</td>
</tr>
<tr>
<td>S_{ij}</td>
<td>cross power spectrum between signal i and signal j</td>
</tr>
<tr>
<td>( \tilde{S} )</td>
<td>defined by Eq. (12)</td>
</tr>
<tr>
<td>( \tilde{U} )</td>
<td>incompressible field velocity vector</td>
</tr>
<tr>
<td>( \tilde{U}_1 )</td>
<td>mean axial velocity</td>
</tr>
<tr>
<td>( \tilde{U}_{lc} )</td>
<td>cross section averaged axial velocity</td>
</tr>
<tr>
<td>( \tilde{V} )</td>
<td>total velocity vector</td>
</tr>
<tr>
<td>( \tilde{V}_a )</td>
<td>acoustic velocity vector</td>
</tr>
<tr>
<td>( \tilde{V}_{x,x_o} )</td>
<td>tube volume</td>
</tr>
<tr>
<td>z</td>
<td>axial coordinate</td>
</tr>
<tr>
<td>( z_s )</td>
<td>axial separation coordinate</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>thermal diffusivity of propellant</td>
</tr>
<tr>
<td>( \gamma_s )</td>
<td>side wall/propellant specific acoustic admittance</td>
</tr>
<tr>
<td>( \beta_s )</td>
<td>critical ( \beta ) for stability analysis</td>
</tr>
<tr>
<td>( G )</td>
<td>Green's function, azimuthal coordinate in Fig. 6</td>
</tr>
<tr>
<td>( \phi )</td>
<td>defined by Eq. (18)</td>
</tr>
<tr>
<td>( \rho )</td>
<td>mean density of gaseous products from combustion</td>
</tr>
<tr>
<td>( \rho_s )</td>
<td>mean density of solid propellant</td>
</tr>
<tr>
<td>( \rho_{po} )</td>
<td>specific acoustic impedance at nozzle entrance plane</td>
</tr>
<tr>
<td>( \rho_{po}^* )</td>
<td>specific acoustic impedance at head end</td>
</tr>
<tr>
<td>( \omega )</td>
<td>angular frequency</td>
</tr>
<tr>
<td>( \omega_0 )</td>
<td>modified acoustic wavelength</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>acoustic wavelength</td>
</tr>
<tr>
<td>( \lambda_oo )</td>
<td>specific acoustic wavelength</td>
</tr>
<tr>
<td>( \zeta_e )</td>
<td>specific acoustic impedance at nozzle entrance plane</td>
</tr>
<tr>
<td>( \zeta_o )</td>
<td>specific acoustic impedance at head end</td>
</tr>
<tr>
<td>( \tau )</td>
<td>mean or ensemble or cross section average</td>
</tr>
<tr>
<td>( \bar{\tau} )</td>
<td>fluctuation</td>
</tr>
<tr>
<td>( \bar{\tau}^* )</td>
<td>complex conjugate</td>
</tr>
<tr>
<td>( \bar{\tau} \rightarrow )</td>
<td>vector quantity</td>
</tr>
</tbody>
</table>

Introduction

The issue of pressure fluctuations in interior flows has attracted the attention of the propulsion community due to its relevance to vibration and instability problems in solid propellant rocket motors. Under normal operating conditions these fluctuations can be of the order of 1-2% of the mean chamber pressure, and usually these fluctuations are random. However, fluctuations near the acoustic resonant frequencies of the chamber may develop a nearly stationary phase relationship and aggravate the vibration problem.
One source of pressure fluctuations inside rocket motors is the turbulence present within the chamber. Pressure fluctuations associated with the turbulent eddies give rise to balance the local, unsteady acceleration. A small part of the turbulence energy can, however, escape in the form of acoustic radiation (Fig. 2). Modification of the acoustic radiation takes place due to reflections from the nozzle and at the propellant surface, and a standing wave may be set up in the chamber. The prediction of the pressure levels associated with this standing wave, given sufficient data about the turbulence field and the acoustic properties at the flow boundaries, is the objective of this paper.

The Bernoulli enthalpy approach of Yates (2) will be used. The fluctuating pressure field is specified by two scalar fields - the Bernoulli enthalpy and the acoustic potential. The resulting equations resemble the formalism proposed by Ribner (13), who distinguished between the fluctuating pressure field (labelled pseudosound) associated with the incompressible motion of eddies and the pressure field (acoustic) associated with the compressible motion of the fluid. Investigation of the acoustic field in cold flow simulations of rocket motors will be described here.

**Analysis**

The configurations of Figs. 3 and 4 are considered. Figure 3(a) depicts a pipe flow terminated by a choked nozzle and Fig. 3(b) shows a simulation of a rocket cavity by means of a porous tube with the entire mass injected from the side walls. Two other versions of the porous tube are shown in Fig. 4. Figure 4(a) shows a modified version of the porous tube that has acoustically stiffer walls due to sections of the tube being taped. Figure 4(b) shows a shortened version of the porous tube. The majority of the pipe (Fig. 3(a)) is filled with fully developed turbulent flow. The pressure drop across the walls of the porous tube is, on the average, about 4500 N/m² and is much lower than the pressure drop across the nozzles under choked conditions. This ensures that the net mass flux through the nozzle is determined purely by its area ratio. In all the cases considered the exit Mach number was about 0.1. A version of the shortened porous tube having an average exit Mach number of 0.07 was also investigated.

Due to the low Mach numbers considered, the primary flow in the configurations may be taken to be incompressible. Following Yates (2), the fluid velocity is split into two components, one associated with the incompressible field and the other being the acoustic velocity.

\[ \vec{v} = \vec{u} + \vec{v}_a, \quad \nabla \cdot \vec{u} = 0 \]  
(1)

\[ \vec{v}_a = \vec{v}_b \quad \text{and} \quad \vec{w} = \nabla \times \vec{u} \]

The governing equations are

\[ \frac{1}{c^2} \frac{D^2 \phi}{Dt^2} - \nabla^2 \phi = \frac{1}{c^2} \frac{DH}{Dt} \]  
(2)

\[ \nabla \cdot \vec{H} = - \frac{D\vec{u}}{Dt} \]  
(3)

where the Bernoulli enthalpy, \( H \), is given by

\[ H = h + \frac{D\phi}{Dt} + \frac{|\vec{v}_a|^2}{2} \]

and the operator

\[ \frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{u} \cdot \nabla \]

is the substantial derivative following the non acoustic motion of the fluid.

Entropy variations have been neglected because it is the larger energy containing eddies that are responsible for sound generation and these are inviscid to a good approximation. Acoustic velocities have been neglected compared to the velocities associated with the incompressible field. Magnitude estimates show this to be an excellent approximation. Moreover, Eq. (2) has been linearized with respect to the acoustic potential \( \phi \).

Fluctuations in the pressure field are obtained from the isentropic equation of state

\[ \frac{dh}{\rho} = \frac{d\rho}{\rho} \]

In terms of \( H \) and \( \phi \), this becomes

\[ p' = \rho_o (\frac{dH}{\rho} - \frac{D\phi}{Dt}) \]

where under the linear approximation the density is replaced by the reference state density \( \rho_o \). The pressure fluctuation therefore consists of two parts, one being associated with the unsteady, compressible motion of the fluid, and therefore the acoustic pressure

\[ p_\phi = -\rho_o \frac{D\phi}{Dt} \]  
(4)

The other is associated with \( H \) and the incompressible motion, and hence is the hydrodynamic pressure fluctuation

\[ p_H = \rho_o H' \]

It will be the acoustic pressure \( p_\phi \) that is of interest in the following development.

The nonhomogeneous wave equation, Eq. (2), is to be solved. This requires a knowledge of the \( H \) field. The solution for \( H \) is obtained by taking the divergence of Eq. (3) and solving the resultant Poisson's equation

\[ \nabla^2 \vec{H} = - \frac{\partial^2 \vec{v}_a}{\partial \vec{x} \cdot \partial \vec{x}} = - f(x, t) \]  
(5)

The boundary condition \( \nabla \cdot \vec{H} \cdot \vec{n} \) is specified from Eq. (3) at the head end, the side walls and the nozzle entrance plane.

Only plane wave acoustic motion will be considered since it is readily shown that the turbulence energy is negligible at transverse mode frequencies. Thus, the cross section average of Eq. (2) representing plane wave motion is investigated with \( H \) being replaced by its cross section
The solution for $H_0$ may be written in terms of the Green's function $g_0$ for the equation

$$\frac{d^2 g_0}{dx^2} = -\frac{1}{\alpha} \delta(x-x_0)$$

with $g_0 = 0$ at the head end

$$\frac{dg_0}{dx} = 0$$

at the nozzle entrance plane.

The solution is

$$g_0(x,x_0) = \begin{cases} \frac{x}{\alpha} & x \geq x_0 \\ \frac{x_0}{\alpha} & x \leq x_0 \end{cases}$$

and $H_0$ is given in terms of $g_0$ by

$$H_0(x) = \frac{1}{\alpha} \int (\tilde{u}_1 \tilde{u}_1') dA$$

where the integral is over the local cross section area.

Velocity fluctuations in the experimental configurations are on order of magnitude below the mean velocity. This enables the linearization of Eq. (7)

$$H_0(x) = -\frac{2}{\alpha} \int \tilde{u}_1 \tilde{u}_1' dA$$

Now $\tilde{U}_1 = \tilde{U}_1 + (\tilde{U}_1 - \tilde{U}_1)$ where $\tilde{U}_1$ is constant over the cross section. Substituting into Eq. (7), the term $\int \tilde{U}_1 \tilde{u}_1' dA = \tilde{U}_1 \int \tilde{u}_1' dV = 0$ because $\tilde{v}_1 = 0$. Hence,

$$H_0(x) = -\frac{2}{\alpha} \int \left(\tilde{U}_1 - \tilde{U}_1\right) \tilde{u}_1' dA$$

The acoustic field is obtained from Eq. (2). As mentioned previously, $H$ is replaced by $H_0$ on the right hand side of Eq. (2). A further simplification occurs by noting that the axial length scale of $H_0$ is that of $g_0$ and is $\ell$, the length of the duct, while its time scale is $\alpha/\tilde{U}_1$, the duct radius, is a typical vortical length scale. Accordingly, the partial time derivative of $H_0$

dominates the convective acceleration term in $DH_0/\alpha t$ for large $\ell/\alpha$ as in the cases under consideration. Therefore $DH_0/\alpha t$ is approximated by simply $\alpha H_0/\alpha t$.

Then, for low Mach numbers, the cross section averaged Fourier transform of Eq. (2) becomes

$$\frac{d^2 \tilde{\phi}_w}{dx^2} + k_\infty^2 \tilde{\phi}_w = \frac{i\omega}{c^2} g_\infty$$

where the modified wave numbers $k_\infty$ is given by

$$k_\infty^2 = k^2 - i \frac{2k_\alpha}{\alpha}$$

and may be approximated by

$$k_\infty = k - i \pi x$$

with $k = \omega/c$.

The boundary conditions on $\tilde{\phi}_w$ are

$$\frac{d\tilde{\phi}_w}{dx} \bigg|_{x=0} = 0$$

$$\frac{d\tilde{\phi}_w}{dx} \bigg|_{x=\ell} = 0$$

The solution is obtained by using the Green's function $g_\infty$ satisfying

$$\frac{d^2 g_\infty}{dx^2} + k_\infty^2 g_\infty = -\delta(x-x_0)$$

which has the same boundary conditions as $g_\infty$.

The acoustic potential $\tilde{\phi}_w$ is then given by

$$\tilde{\phi}_w(x) = -\frac{2i\omega}{Ac} \int \tilde{u}_1 \tilde{u}_1' g_\infty(x,x_0) dV_0$$
and the acoustic pressure transform by

\[ p_{\phi}(x) = \frac{2 p_0 k^2}{A} \int \left( \left( 1 - \frac{r}{r_c} \right) u_{\frac{1}{2}} s_{\phi}(x, x_0) \right) dV_0 \]

The autospectrum, \( S_{\phi \phi}(x) \), of the acoustic pressure is obtained by multiplying \( p_{\phi} \) by its complex conjugate and taking an ensemble average

\[ S_{\phi \phi}(x) = \frac{2 p_0^2 k^4}{A} \int \left( \left( 1 - \frac{r}{r_c} \right) u_{\frac{1}{2}} \right) dV_0 dV_2 \]

where

\[ T_w(\vec{z}) = 2 \left( \left( 1 - \frac{r}{r_c} \right) u_{\frac{1}{2}} \right) \]

To estimate \( S_{\phi \phi} \), the axial correlation length is introduced

\[ l_{\text{corA}} = \frac{1}{S_{11}} \int \frac{S_{12} dz}{S_{12}} \]

where \( S_{11} \) is the autospectrum of \( T_w \) and \( S_{12} \) is given by

\[ S_{12} = \text{Real} \left[ T_w^*(x_0, \vec{z}) \ T_w(x_0 + \vec{z}, \vec{r}) \right] \]

and is the real part of the cross spectrum of \( T_w \) with respect to axial separation \( z \), the acoustic wavelength. For frequencies under consideration, \( \lambda \gg l_{\text{corA}} \), so that \( S_{12} \) varies slowly over the distance scale where the axial correlation of \( T_w \) falls to zero. Introducing

\[ S_{\infty}(x) = \left( \frac{p_0^2}{A} \right)^2 \int dA(\vec{T}_1) \int dA(\vec{T}_2) \ T_w^*(x_1, \vec{T}_1) \ T_w(x_2, \vec{T}_2) \]

the expression for \( S_{\phi \phi} \) is approximated by

\[ S_{\phi \phi}(x) \approx k \left[ \int_0^L S_{\infty}(x_0) \left| s_w(x, x_0) \right|^2 dx_0 \right] l_{\text{corA}} \]

For purposes of estimation, it is assumed that

\[ S_{\phi \phi}(x) = S_{\infty}(x) Q(x) F(w) \]

For pipe flow, since the majority of the pipe is filled with fully developed turbulent flow both \( Q(x) \) and \( F(w) \) are taken to be unity. For the porous tube cases, \( Q(x) \) is taken to be a second degree polynomial and is constructed by noting that \( S_{\infty}(0) = 0 \) and that \( Q(x) = 1 \). The frequency correction \( F(w) \) that has been used here is obtained by considering the autospectrum, \( S_{11} \), of the fluctuating axial velocity at several axial stations at half radius,

\[ F(w) = \frac{1}{n} \sum_{i=1}^{n} S_{11}(x_i) \]

Typically, \( n = 2 \) or \( 3 \) has been used with the autospectra being measured near the head end and at half the length of the tube.

Finally, \( S_{\phi \phi}(\vec{z}) \) has to be estimated. Using the coordinate system of Fig. 6, and following Ref. 6, the approximation is used

\[ S_{\phi \phi}(\vec{z}) = \frac{2 p_0^2 \pi}{A} \left[ \int_0^{a/2} S_{12}(\vec{z}) d\theta + \int_0^{3a/2} S_{12}(\vec{z}) d\theta \right] \]

This requires fixing an anemometer at half the radius and carrying out a traverse of another anemometer along the diameter defined by the reference probe and the tube center (Fig. 6).

**Experimental**

The pipe in Fig. 3(a) is 6m. long and 5 cm. in diameter. The tubes in Figs. 3(b) and 6 are constructed on porous sleeves made of sintered steel and are also 5 cm. in diameter. The setups are instrumented with flush mounted microphones and hot film anemometers. The wall microphones are used to measure wall pressure spectra and cross spectra. The hot films are used to obtain necessary axial velocity correlations.

Wall pressure spectra and cross spectra associated with the modified porous tubes are shown in Fig. 7. At the head end, where there is no turbulence, the spectrum shows peaks at the resonant acoustic frequencies of the tube. In the nozzle end spectrum, the acoustic peaks are lost in the broadband background level. The head to nozzle end cross spectrum reveals only the acoustic part. A comparison of the head end spectrum with the head to nozzle end cross spectrum shows both to be of the same level. This strengthens the belief that the head end spectrum is mainly acoustic whereas the nozzle end spectrum is dominated by flow noise (pseudo sound) which is local in nature. Investigations of flow noise for the case of fully developed turbulent pipe flow are reported in Ref. 7.

The real part of the axial velocity cross spectra at the nozzle entrance plane at a frequency of 100 Hz referred to a point at half radius is plotted in Fig. 8 for the pipe and porous tube configurations having an average Mach number of 0.1 at the nozzle entrance plane. The value for zero separation corresponds to the autospectrum at 100 Hz. Clearly, turbulence levels in the porous tube configurations are higher as compared to those in the pipe. Also, the turbulence levels are highest for the modified porous tube configuration.

Acoustic impedance measurements were carried out using the classical impedance tube technique. Figure 9 shows the measured impedance at the bellmouth inlet for the pipe and its side wall loss factor. Figure 10 plots the wall loss factor for the porous tube configurations (\( R_e = \))
The wall loss factors for the porous tubes are an order of magnitude higher than those for the pipe indicating that the side walls of the tube act as acoustic dampers. Among the porous tube configurations, the loss factor for the modified porous tube is lowest since those sections of the tube that were taped tended to acoustically stiffen the side walls. It should be noted that it is the quantity $\gamma$ that has been plotted in the figures.

The specific acoustic impedance, $z_s$, at the nozzle entrance plane was theoretically obtained using the short nozzle approximation. The expression for $z_s$ for low Mach numbers is

$$z_s = \frac{2}{y_e(y-1)}$$

Comparisons of the predicted acoustic spectra with wall measured pressure spectra are shown in Fig. 11 through Fig. 15. For the pipe flow, the comparison is at the nozzle end where the wall measured spectrum contains both hydrodynamic and acoustic components. However, the resonant peaks are clear and definite and a valid comparison can be made. For the porous tube cases, the comparison is at the head end where the wall measured spectrum is mainly acoustic.

It is seen in all cases the resonant frequencies are well predicted and the magnitude of the spectra is predicted to within a factor of three. Since only limited turbulence data was used in the prediction this factor is acceptable. The most serious discrepancy is with regard to the relative magnitude of the first mode as compared to the other modes for the porous tube cases. The experimental spectra show the level at the first mode to be much higher than the levels at the other modes. The predicted spectra for the porous tubes does not show this behavior to the degree present in the experimental spectra. Two reasons for this discrepancy may be advanced. First, by using the impedance tube technique, wall loss factors can be measured only near to and above the second natural frequency. Values at lower frequencies were obtained by linear extrapolation and may have been in error to some extent. Secondly, the frequency correction factor $F(\omega)$ may not be sufficient. In particular, a better estimate of $s_2(\omega)$ may be obtained by measuring cross spectra of velocity fluctuations with respect to radial separation for more than one representative point. In other words, the limited amount of turbulence data used could also be responsible for the discrepancy. Considering the fact, however, that it is certainly not feasible to map the entire turbulence field in the setups, the predicted spectra are in good agreement, overall, with the measured spectra.

An understanding of the effects of different aspects of the flow configuration on the acoustic pressure level may be obtained from the spectra in the porous tubes. From Fig. 8 and Fig. 10, it is clear that turbulence intensities are higher and wall loss factors lower for the modified porous tube as compared to the basic porous tube. This is reflected in the higher pressure levels in the modified porous tube. Considering the basic and the shortened porous tube, both have approximately the same wall loss factors (Fig. 10), but at the common resonant modes, the levels in the shortened porous tube are higher (Fig. 12 and Fig. 14). When the side walls tend to damp the acoustic motion, a lower $h/d$ ratio results in higher pressure levels. When the side walls drive the acoustic motion, as is generally true in rocket motors, the effect is reversed. As far as exit Mach number is concerned higher pressure levels are associated with higher exit Mach numbers (see Figs. 14 and 15). The velocity terms in Eq. (11) provide a $M^2$ scaling for the pressure fluctuation while the denominator of the Green's function, $g_{pp}$, contains $c_e$ which varies inversely as the Mach number (Eq. (17)). The net effect, then, is a scaling of the pressure fluctuation with the exit Mach number $M_e$ and therefore of the pressure level with $M_e^{2}$. This scaling is also evident in both the measured and predicted spectra for the two versions of the shortened porous tube.

### Application to Rocket Motors

The present theory may be extended to center perforated rocket motors by identifying $\beta$, the side wall specific acoustic admittance with the propellant admittance. The propellant admittance has been modeled here by the A-B representation of propellant response (8).

For simplicity, it is assumed that fluctuations at the burning surface of the propellant are isentropic. In terms of the A-B model

$$\beta = -\frac{\rho}{\gamma (A/A) - (1 + A) + A^2 - 1}$$

where

$$\Lambda = \frac{1}{2} + \frac{1}{\gamma} - \frac{\epsilon}{2}$$

and $\epsilon$ is the burning rate of the solid propellant.

The criterion for linear instability of a rocket motor is obtained by setting the denominator of the Green's function, $g_{pp}$, to zero. For small $M_e$, terms of the order of $M_e^2$ and higher may be neglected. Then, using Eqs. (10) and (17) the criterion for instability becomes

$$k_{oo} \sin k_{oo} r - \frac{\tan (\gamma - 1) \beta}{2} \cos k_{oo} r - 0$$

The effect of $\beta$ is felt through the modified wave number $k_{oo}$ according to Eq. (9).

For a given rocket geometry and exit Mach number the value of $\beta$, denoted by $\beta_{cri}$, may be obtained that makes the rocket unstable. It is then possible to specify the parameters $A$ and $B$ that yield $\beta_{cri}$ given the thermal diffusivity of the propellant and the burning rate law. An example of the stability curve in terms of the parameters $A$ and $B$ is shown in Fig. 16. The calculation assumes the following values

$$L/a = 30, \quad n = 0.65, \quad \epsilon = 1 c m/sec, \gamma = 1.2,$n = 0.65, \quad \epsilon = 1 \text{ cm/sec}, \gamma = 1.2, \quad \beta_{oo} = 0.2, \quad \frac{c_e}{ca} = 7.2 \times 10^{-7} \text{ and } \frac{\beta_{oo}}{f_g} = 150$$

As the parameters $A$ and $B$ are varied in a suitable manner (e.g., in the direction of the arrow in Fig. 16), the propellant becomes more driving and the acoustic pressure level in the chamber increases. Typical variations in the pressure level may be estimated by scaling the turbulence quantities obtained experimentally.

The scaling is done by assuming that $A^2 s_{oo}(x)$ varies as $c_o U_{ic}$ and evaluating the density $c_o$ in terms of the mean pressure and temperature in the rocket chamber...
via the perfect gas equation. Further, it is assumed that \( l_{\text{cor}} \) remains constant.

A sample calculation is shown in Fig. 17. The mean pressure and temperature inside the rocket motor were taken as 13.8 x 10^6 N/m^2 and 2500 K. The radius of the motor was taken as 0.05 m. Turbulence levels were scaled from the shortened porous tube (M, = 0.1) configuration. The rocket motor was assumed to have \( M_0 = 0.2 \) and \( \alpha = 20 \). It is seen that the pressure level becomes significant only very near to a stability limit. Away from the stability limit the rise in pressure level is not steep but gradual as the propellant becomes more driving. But very near to the stability limit there is a steep increase in the pressure level.

Another interesting result is obtained by considering the nature of the Green's function \( g_w \). The acoustic pressure transform (Eq. (11)), under the assumption leading to Eq. (14), may be approximated as

\[
P_{\text{aw}}(\mathbf{x}, t) \approx -\frac{2\rho_0}{A} \sum_{n=0}^{N} k^2 g_w(x_a x_n) R(\omega x_n)
\]

where \( x_n = n l_{\text{cor}} \), \( x_n \approx l \) and

\[
R(\omega x_n) = \int_{A(x_n)} (\mathbf{B}_1 - \mathbf{A}_1) u_{1w} \, dA
\]

The inverse Fourier transform of Eq. (20) is

\[
p_{\text{aw}}(x, t) \approx -\frac{2\rho_0}{A} \sum_{n=0}^{N} g_T * R_T
\]

where the * symbol denotes the convolution operation.

As a simple exercise one term of Eq. (21) corresponding to \( x = 0 \) and \( x_n = l \) was chosen. The function \( g_w(\mathbf{0}, t) \) for a 0.3% pressure level (Fig. 17) was calculated as a function of frequency. The signal from a white noise generator simulated \( R_T \). The inverse Fourier transform of \( k^2 g_w(\mathbf{0}, t) \) was convoluted with a sample \( R_T \) over a time of 0.25 seconds. The result is shown in Fig. 18. The signal \( R_T \) is the source strength for the pressure fluctuations and is a random signal. But when it is operated upon by \( g_T \) a periodic signal of nearly constant amplitude results. This occurs because \( g_T \) acts like a bandpass filter such that signals at or near the resonant frequencies of the motor are passed unhindered whereas fluctuations at other frequencies are suppressed. In this numerical experiment, the amplitude of the resulting signal will depend upon the content of the white noise sample chosen. For an actual rocket motor, the pressure fluctuation according to Eq. (21) is given by a sum of convolutions which will depend on the frequency content of the turbulence over the entire volume of the rocket chamber. For statistically stationary turbulence, the net frequency content of the turbulence over the entire chamber should not vary appreciably with time. Hence, in this case also the pressure fluctuation will be nearly periodic and of nearly constant amplitude.

Conclusion

The generation of sound by turbulence in simulated rocket motor interior flows has been investigated analytically and experimentally. The entire volume of turbulence within the motor cavity is responsible for the generation of acoustic pressure fluctuations. However, it is possible to estimate the pressure spectra from limited data on the turbulence field and from knowledge of the acoustic impedance at flow boundaries. Higher turbulence levels increase the pressure level. If the energy in the turbulence spectrum is high in frequency bands containing resonant frequencies of the motor, the problem may become severe. Increase in pressure level with increase in propellant driving characteristics is very gradual away from a stability limit. Significant pressure levels are generated only extremely close to a stability limit where for all practical purposes the motor may be deemed unstable. However, an interesting result is that while the driving turbulence is band limited noise the resulting pressure oscillation is nearly periodic in character.

Acknowledgements

This work was supported by the Air Force Office of Scientific Research under Contract No. F49620-78-C-0003.

References

Figure 1. Fluctuating pressure components inside a rocket motor.

Figure 2. Turbulence generated pressure fluctuations.

Figure 3. Schematic diagram of the experimental configurations (I).

Figure 4. Schematic diagram of the experimental configurations (II).
Figure 5. Axial correlation lengths as a function of frequency ($M_e = 0.1$).

Figure 6. Coordinate system for cross correlation integrals.

Figure 7. Wall pressure spectra and cross spectra in modified porous tube.

Figure 8. Cross spectra of axial velocity fluctuations at 100 Hz with respect to radial separation ($M_e = 0.1$).
Figure 9. Measured acoustic impedances for the pipe.

Figure 10. Measured wall loss factors for porous tube configurations ($M_e = 0.1$).

Figure 11. Pressure spectra at nozzle entrance plane for pipe flow.
Figure 13. Pressure spectra at head end for modified porous tube.

Figure 14. Pressure spectra at head end for shortened porous tube ($M_e = 0.1$).

Figure 15. Pressure spectra at head end for shortened porous tube ($M_e = 0.07$).
Figure 16. Typical stability limit in terms of the parameters A and B.

Figure 17. Typical pressure levels as a function of (B/\beta_{exit}).

Figure 18. Action of the Green's function g_{\omega} as a filter.
