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Center #: R6714-0A0
Contract#: AFOSR-89-0290

Project unit: AE
Project director(s):
  ZINN B T
  HEGDE U G
  DANIEL B R

Sponsor/division names: AIR FORCE
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Award period: 890301 to 900228 (performance) 900430 (reports)

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Funded 101,917.00

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Title: INVESTIGATION OF FLAME-ACOUSTIC WAVE INTERACTION AXIAL SOLID ROCKET...

PROJECT ADMINISTRATION DATA

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Security class (U,C,S,T5) : U
Defense priority rating : N/A
Equipment title vests with: Sponsor X GIT
ALL PERM. EQUIP.< $5000 VESTS W/ GIT AS WELL AS ALL M&S AND EXPENDABLE EQUIP.

Administrative comments -
PROJECT INITIATION FOR ONE YEAR BUT ADD'L YEAR CAN BE GRANTED (SEE PAGE GRANT); CURRENT FUNDS IAO $101,917, CURRENT TERM DATE IS 2/28/90;
GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF CONTRACT ADMINISTRATION

NOTICE OF PROJECT CLOSEOUT

Closeout Notice Date 05/17/91

Project No. E-16-663
Center No. R6714-0A0

Project Director ZINN B T
School/Lab AERO ENGR

Sponsor AIR FORCE/BOLLING AFB, DC

Contract/Grant No. AFOSR-89-0290
Contract Entity GTRC

Prime Contract No.

Title INVESTIGATION OF FLAME-ACOUSTIC WAVE INTERACTION AXIAL SOLID ROCKET

Effective Completion Date 910228 (Performance) 910430 (Reports)

Closeout Actions Required:  

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Comments

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Note: Final Patent Questionnaire sent to PDPI.
9. How can the information developed under the proposed studies be used to reduce the occurrence of instabilities in solid propellant rocket motors?

In the next section, accomplishments of this program under the current program are briefly described. This section is followed by the Proposed Research Section where the proposed research tasks are discussed. This is followed by sections describing the Personnel, Facilities and Proposed Budget. The proposal closes with appendices detailing professional interactions made possible by this program to date and resumes of the principal investigators.

II. PROGRESS TO DATE

II-A. Introductory Comments.

This section briefly describes the results obtained under this program during the past two years. These studies investigated the driving of axial acoustic fields by premixed and diffusion flames stabilized on the side wall of the duct and the damping provided by flow turning. The results of these studies are described in the following publications:


Since the accomplishments of this program during the past two years are described in detail in the above publications, only their main findings and conclusions are summarized in this section. All of the discussed experimental studies were carried out in the experimental setup shown in Fig. 4 which is described in detail in our previous AFOSR proposal and the above publications. In what follows, the results of these studies will be described according to the type of utilized burner and/or objective of the study.

II-B. Premixed Flame Investigations.

Initial efforts under this program investigated the interactions between a premixed flame stabilized on the side wall of the duct and axial acoustic fields (see papers Nos. 1, 3 and 6 above). A premixed flame was investigated initially in order to eliminate the effects of diffusion processes and determine the effects of other phenomena, such as oscillatory flame reaction rates, upon the driving of axial instabilities. These experimental and theoretical studies revealed that the interaction of a premixed flame stabilized at the side wall of a duct with an axial acoustic field produces the following effects:

1. A periodic flame reaction rate which oscillates with the frequency of the acoustic field and produces flame oscillations in a direction normal to the side burner surface (or propellant surface).

2. A complex flame structure consisting of driving and damping flame regions. The net effect of the flame depends upon the relative magnitudes of these driving/damping regions.

3. An oscillatory radial velocity component downstream of the flame which can drive or damp the axial acoustic field. The driving/damping provided by this velocity component is equivalent to that provided by an oscillating piston which periodically compresses the core flow.

4. The flame driving strongly depends upon the acoustic admittance of the burner surface.

In addition, the measured data were in good agreement with a developed theoretical model which describes the behavior of the investigated flames.
Fig. 4. Schematic of the Proposed Experimental Set-Up to Investigate the Interactions between Gas Phase Diffusion Flames and Longitudinal Acoustic Fields.
II-C. Diffusion Flames Investigations.

Large Scale Flames. Upon completion of the premixed flame studies, the interaction of a diffusion flame stabilized on the side wall of the duct with various axial acoustic fields was investigated. These studies were conducted with the burner which is shown in Figs. 4 and 5 and their results are described in papers Nos.3,4,5, 7,8 and 9.

The utilized diffusion flame burner, see Fig. 5, used propane and air to simulate the flow rates of binder pyrolysis products and oxidizer particles combustion products to the gas phase flame of a solid propellant. This type of burner has two advantages. First, the resulting diffusion flame configuration is geometrically simple and, therefore, amenable to theoretical analysis. Secondly, the developed diffusion flames are similar to the sandwich type of propellant flames with alternating oxidizer and binder sections which have been utilized in the past to study solid propellant combustion\(^{16}\). Its main disadvantage is that the scales of the resulting diffusion flames are much larger than the scales of diffusions flames which are encountered in actual solid propellant rocket motors. These scale effect have been considered in subsequent studies which are described in the last part of this section. The main results obtained with the burner configurations shown in Figs. 4 and 5 are summarized below.

1. A theoretical model which describes the interaction of a diffusion flame with an axial acoustic field has been developed (see paper No. 3). This model indicates that some regions of the diffusion flame drive the waves while the remaining regions damp the waves; the net flame driving increases as the fuel heating value increases; the flame driving is higher at low frequencies and at certain other frequencies which depend upon the flame properties; and the flame driving strongly depends upon the acoustic admittance of the burner wall.

2. Flame visualizations (see papers Nos. 5 and 7) revealed that the interactions of the diffusion flames with the axial acoustic field produced axial and transverse flame oscillations with the frequency of the imposed acoustic field.

3. The interaction of the flame with the acoustic field produced an oscillatory heat release rate, see papers Nos. 5 and 7. This heat release was in phase with the local pressure oscillations in some regions of the flame and out of phase in the remaining regions. Thus, the "in-phase" and "out-of-phase" regions of the flame drove and damped the waves, respectively.

4. LDV measurements revealed that the oscillatory flame reaction rate produces an oscillatory velocity component downstream of the flame whose direction is normal to the axis of the duct. This velocity component drives the
Developed Diffusion Flame Burner.

Fig. 5.
axial acoustic field when its oscillations are in phase with the local pressure oscillations (see paper Nos. 7-9)

5. Radiation and velocity measurements revealed that often one side of the flame drove the oscillations while the other side damped the oscillations. It has been conjectured that these opposite driving tendencies of opposite sides of the flame were caused by acoustic velocity oscillations which produced periodic variations of the oxidizer/fuel ratios within the flame.

**Small Scale Flames.** In the second part of the diffusion flames study, currently in progress, a new burner which produces a diffusion flame which better simulates a solid propellant flame was developed, see Fig. 6. This burner consists of approximately 1500 hypodermic needles which supply oxidizer jets into the gas flame. Fuel is supplied through the spaces around the hypodermic needles. Since the resulting flame consists of a multitude of diffusion flames, it will be denoted as the Multiple Diffusion Flame Burner (MDFB). In this burner, the oxidizer flow simulates the flow of the combustion products generated by the decomposition of the small ammonium perchlorate particles which are embedded in the fuel binder matrix of a composite solid propellant, and the fuel flow simulates the pyrolysis products of the binder.

The MDFB produced a flame which appeared to better simulate a gas phase solid propellant flame. In contrast to the relatively long diffusion flames produced by the burner in Fig. 5, the flame produced by the MDFB was very thin and it could be stabilized at various stand off distances from the burner surface. The resulting flame was blue and similar in shape to the previously investigated premixed flame. These observations strongly suggest that when the scales of the oxidizer and fuel streams which enter the gas phase flame are small, these streams rapidly mix and produce a quasi-premixed flame. Since the scales of the fuel and oxidizer streams which leave the surface of a solid propellant are expected to be very small, it is highly probable that solid propellant gas phase flames also behave as premixed flames. This is an important conjecture which should be further investigated.

Experiments with the MDFB also showed that for certain flame positions (relative to the standing acoustic wave in the experimental setup) and for certain wave frequencies the flame becomes wavy. Since flame "waviness" was most likely caused by wave propagation along the flame surface, it is also possible that the flame area and, consequently, the heat release rate of a wavy flame also vary with time. If the time dependence of the heat release rate contains a component with the frequency of the acoustic field, and if the heat release at this frequency is in phase with the local pressure oscillations, then the observed flame distortion may contribute significantly to the driving of the acoustic field. Since similar flame distortions could occur in actual solid propellant flames, the
Figure 6. A Schematic of the Multi-Diffusion Flame Burner
mechanisms responsible for the formation of wavy flames and the
driving provided by such flames will be investigated under the
proposed research program.

II-D. Flow Turning Losses Investigations.

These studies are currently in progress and they consist of theoretical and
experimental efforts. The experimental efforts use the experimental setup shown
in Fig. 4 with the MDFB. The flow turning losses are being investigated by
evaluating the acoustic energy fluxes

\[ I = \langle p' v' \rangle = \frac{1}{T} \int_0^T p' V' \, dt \] (1)

which enter and leave small control volumes located within the region where the
flow leaving the burner changes direction (i.e., it "turns"). The investigated
region is shown on the bottom of Fig. 7 and a typical control volume is shown
on top. By evaluating the magnitudes and directions of the acoustic energy fluxes
along the boundaries of each control volume it was possible to determine
whether a net amount of acoustic energy is absorbed or generated within the
control volume. The former indicates that an acoustic energy was dissipated
within the control volume and the latter indicates that acoustic energy was
generated within the control volume.

A typical experiment is conducted with a stream of cold flow with velocity
U being supplied through the porous plate at the upstream end of the setup and a
secondary stream supplied through the burner, see Fig. 7. The flow entering
through the burner changes direction in the investigated region. The needed data
were obtained from measured velocities and acoustic pressures in the indicated
region. The former was obtained with an LDV system and the latter with an
acoustic pressure transducer attached to probe.

Both cold and "hot" experiments have been carried out to date. In the cold
experiments cold flow was supplied through the burner while in the "hot"
experiments a flame was stabilized close to the burner surface as shown in Fig.
7. Typical results obtained in experiments with a 500 Hz. acoustic waves are
presented in Fig. 8. The numbers inside the boxes indicate the net amount of
acoustic energy absorbed (negative) or generated (positive) within each control
volume. It has been estimated that when the magnitudes of these numbers are
less than one, their accuracy is open to question.

An analysis of the control volume on top of Fig. 8 shows that the
magnitudes of all of the measured "sources" and "sink" terms are smaller than
one. Furthermore, they appear to change signs randomly. These observations
suggest that if flow turning losses exist, their magnitudes under cold flow
conditions are very small. In contrast with the cold flow results, data obtained in
"hot" experiments, presented in the middle and bottom of Fig. 8, indicate that
significant acoustic sources and sinks may be possibly present in the investigated
Figure 7. Utilized Measurement Control Surfaces and Technique for Determining the Spatial Distribution of Acoustic Power Entering and Leaving the Boundaries of the Control Surfaces.
(a) cold flow case, measured at a pressure node.

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sub-control surface

(b) hot flow case, measured at a pressure antinode.

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main control surface

(c) hot flow case, measured at a pressure node.

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location of maximum temperature gradient

Figure 8. Percentage of Acoustic Energy Generated/Absorbed within Each Sub-Control Surface for a 500 Hz. Acoustic Wave.
region. The broken line which "divides" these control volume indicates the measured boundary between cold (above) and hot (bottom) regions of the control volume. The results presented in the middle of Fig. 8 indicate that when the flame was located near an acoustic pressure maximum acoustic energy was absorbed in the hot region near the burner and it was generated in the cold region in the upper left side of the control volume. On the other hand, the data on the bottom of Fig. 8 show that when the flame was located near an acoustic pressure minimum acoustic energy was absorbed throughout the control volume.

The results presented in Fig. 8 are perplexing and unexpected. Current efforts and those which will be carried out in the initial phase of the proposed program will investigate the processes which are responsible for the observed trends. These efforts will involve repetition of some of the experiments, the use of alternate measurement approach, and a review of the theoretical analysis of Culick and others which predicted the existence of flow turning losses.

III. PROPOSED RESEARCH

The proposed research program is divided into two tasks. Task I will complete the ongoing investigation of flow turning losses and Task II will investigate flame processes which can significantly contribute to the driving of axial instabilities in solid rockets.

**TASK I: INVESTIGATION OF FLOW TURNING LOSSES.**

**A - Task I Objectives:**

1. Determine whether flow turning losses are significant under conditions simulating those encountered in unstable solid propellant rocket motors.
2. Determine the mechanism responsible for the occurrence of flow turning losses in solid propellant rocket motors.
3. Evaluate the magnitude flow turning losses.
4. Determine regions within the rocket motor where flow turning losses are significant.
5. Evaluate whether sound refraction effects affect flow turning losses.
6. Evaluate the effect of near wall temperature gradients upon flow turning losses.
7. Obtain experimental data which could be used to guide the numerical and analytical efforts of Drs. Baum and Kassoy.

**Theoretical Efforts:**

The proposed theoretical efforts were recently initiated in an effort to interpret unexpected experimental results. As described in the previous section, the flow turning loss was measured by evaluating the integral of the acoustic energy flux \( I \) (see Eq. 1 in the previous section) around the boundaries of the...
AFOSR ANNUAL TECHNICAL REPORT

on

INVESTIGATION OF THE FLAME-ACOUSTIC WAVE INTERACTION DURING AXIAL SOLID ROCKET INSTABILITIES

Prepared for

Air Force Office of Scientific Research
Aerospace Sciences Directorate
Bolling Air Force Base

Co-Principle Investigators

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Approved for public release; distribution unlimited

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ABSTRACT

The primary objectives of this research program are: (1) to investigate the mechanisms responsible for the driving of axial instabilities by solid propellant flames, (2) to determine whether state-of-the-art theoretical models can predict the characteristics of the flame driving mechanisms and (3) to investigate the effect of flow turning upon axial instabilities in solid propellant rocket motors. To attain the program's objectives, the response of diffusion flames, stabilized on the side wall of a duct, to imposed axial acoustic waves has been investigated by using flame radiation measurements and laser Doppler velocimetry (LDV). The flame radiation measurements revealed that the presence of an acoustic field produced space dependent oscillatory heat release rates which depend upon the characteristics of the flame and the excited acoustic field. Measurements of the flame radiation and velocity field both showed that at a given instant some sections of the flame drive the acoustic field while others damp it. The net effect of the flame upon the acoustic field depends upon the relative magnitudes of these driving and damping regions. During the reporting period, the validity of a previously developed flame response model was also investigated by comparing measured and predicted oscillatory velocity components in the flame region. To utilize the developed model, data describing the steady state temperature distribution in the flame region, the flame shape and the acoustic admittance of the burner surface were needed as inputs. Using the measured input data, the developed model was used to predict oscillatory vertical velocity distributions in the flame region. These were then compared with velocity distributions measured with an LDV system, showing good qualitative agreement between the predicted and measured results. Towards the end of the reporting period, a new diffusion flame burner which better simulates the characteristics of gas phase solid propellant flames was developed. It consists of 1537 hypodermic tubes arranged in a 29X53 matrix. These tubes supply oxidizer to the flame and they simulate the decomposition products of the solid propellant oxidizer particles. The hypodermic needles are embedded in a fuel flow which simulates the pyrolysis products of the propellant fuel binder. The new burner is currently being used in an investigation of flow turning losses in solid propellant rocket motors.
INTRODUCTION

This research program is concerned with the development of an understanding of the mechanisms responsible for the driving of axial instabilities in solid propellant rocket motors. The onset of combustion instabilities in rocket motors depends upon the relative magnitudes of the gain (i.e., driving) and loss (i.e., damping) mechanisms within the combustor which add and remove energy from the oscillations, respectively. The elimination or reduction of the onset of combustion instabilities in solid propellant rocket motors requires identification of the processes which add or remove energy from the oscillations, and the development of an understanding of the mechanisms which control these processes. At present, it is generally recognized that the response of the solid propellant combustion process to the flow oscillations is responsible for providing the energy required for the initiation and maintenance of instabilities inside rocket motors. On the other hand, nozzle damping, viscous dissipation, flow turning and so on are processes which damp rocket instabilities. This research program has been concerned with developing an understanding of the gas phase flame processes which contribute to driving of the instabilities, and the damping provided by the flow turning process.

Actual solid propellant gas phase flames are believed to consist of complex combinations of premixed and diffusion flames. Specifically, it has been argued that premixed flames occur above the oxidizer particles (e.g., ammonium perchlorate) and that the diffusion flames are stabilized above the interfaces between the oxidizer particles and the fuel binder. Since premixed and diffusion flames are expected to respond differently to acoustic excitation, this investigation has been divided into two parts. In the first part of this study, completed earlier under this program, the interaction of premixed flames with axial acoustic fields was investigated. The second part, described herein, investigated the response of diffusion flames to excited axial acoustic fields which simulate axial instabilities in solid propellant rocket motors.
Since actual solid propellant flames cannot be used in these studies, because of their extremely small dimensions, their smoky nature, and rapid burnout, other flames which simulate, in some respects, solid propellant flames have been used in such studies. In the present investigation, the response of diffusion flames, stabilized on the bottom wall of a long duct, to the excitation of axial acoustic fields has been studied. As with the previously conducted premixed flame investigations, the diffusion flame studies consist of parallel experimental and theoretical investigations of the diffusion flame response. Specifically, this research program has been investigating:

(1) the effects of diffusion processes on the acoustic driving/damping characteristics of gas phase flames stabilized on the side wall of a duct,

(2) the validity of state-of-the-art solid propellant combustion response models, and

(3) the effect of the flow turning upon axial instabilities in solid propellant rocket motors.

During the report period, the experimental study of the interaction of diffusion flames with axial acoustic fields has been completed. In addition, the validity of the previously developed theoretical model of a diffusion flame response to axial acoustic fields was checked by comparing its predictions with experimental data. These efforts were followed by the development of a new diffusion flame burner which is currently used in the investigation of flow turning losses. These studies are briefly described in the remainder of this report.
RESEARCH ACCOMPLISHMENTS

(A) Experimental Studies.

The experimental phase of this program was conducted in the experimental setup shown in Fig. 1 which was developed earlier under this program. The interaction of three diffusion flames, stabilized on the bottom wall of a duct, with excited axial acoustic fields were investigated. The response of the diffusion flames to excited acoustic fields was studied by measuring steady and unsteady velocity components and C-H flame radiation distributions using laser Doppler velocimetry and photomultipliers. In addition, pressure transducers were used to characterize the excited acoustic field.

In a rocket motor, the interactions between unsteady solid propellant flames and the local flow oscillations involve complex fluid mechanical, heat transfer and chemical processes. These interactions occur near the lateral boundaries of the motor cavity and they produce vertical velocity oscillations \( v'(y) \) at and near the propellant surface in a direction normal to the direction of the axial oscillations in the core flow. These normal velocity oscillations act as lateral pistons which periodically compress the core flow, thus providing the energy required for initiating and maintaining the core flow oscillations. Specifically, if

\[
I_v(y_i) = \int_{L} \int_{T} p' \text{ Real}(v'(y_i)) \, dt \, dx > 0
\]

where \( p' \), \( L \), \( T \) and \( y_i \) are the local pressure oscillation, a relevant axial integration distance, the period of the oscillation, and the height above the flame, respectively, then these normal velocity oscillations pump acoustic energy into the core flow oscillations. Thus, if \( I_v \) increases with \( y \) in a given flame region, then the flame tends to drive the acoustic oscillation in this region, and vice versa.

The velocity measurements were conducted by using an existing LDV system, which had been also used for the premixed flame study. A typical, measured time dependence of the normal velocity component at a
The variations of $I_v$ in the vertical direction for two different driving frequencies measured at a pressure antinode are presented in Figs. 3 and 4. Figure 3 shows that $I_v$ increases continually up to $0.55y_f$ ($y_f$ is the flame height) and that it decreases subsequently. Based upon the above discussion, this result indicates that when the flame is subjected to a 300 Hz. acoustic oscillation the lower (where $I_v$ increases) and upper (where $I_v$ decreases) sections of the flame drive and damp the oscillation, respectively. The overall driving/damping of the flame depends upon the net effect of these two regions. Since $I_v(y_f) > I_v(y=0)$, the net effect of the flame is to drive the acoustic field. In contrast to the driving and damping regions which exist in the flame when it is subjected to a 300 Hz. wave, Fig. 4 shows that $I_v$ increases continually along the whole length of the flame when the flame interacts with a 400 Hz. acoustic field, indicating that driving occurs throughout the flame region.

Flame radiation measurements are of interest because earlier studies\textsuperscript{3} have shown that the radiation intensities from radicals such as C-H, C-C and O-H are proportional to the reaction and heat release rates in the flame. Such radiation measurements were used in this study to determine the effect of the acoustic field upon the magnitude of the heat release rate and the phase relationship between the heat release and local pressure oscillations. The latter is important because, as stated by Rayleigh's criterion\textsuperscript{1}, the phase difference between the oscillatory heat release rate and pressure oscillations determines whether the flame adds or removes energy from the acoustic waves. Expressed mathematically\textsuperscript{2}, a heat source, such as a flame, adds energy to the acoustic waves when the following inequality is satisfied:

$$\int_v |S_{p'q'}| \cos \phi_{p'q'} dV > 0$$

where $|S_{p'q'}|$ and $\phi_{p'q'}$ are the magnitude of the cross-spectrum between pressure ($p'$) and oscillatory heat release rate ($q'$) oscillations, and the phase difference between $p'$ and $q'$, respectively. The above integration is performed over the whole space where driving or damping by the flame occurs. In a flame region where the integrand is positive, driving of the acoustic waves occurs. This integrand is positive where $p'$ and $q'$ are in
phase; that is, where $-90^\circ < \phi p'q' < 90^\circ$. On the other hand, damping of the acoustic field by the flame occurs when $p'$ and $q'$ are out of phase; that is, when $-90^\circ > \phi p'q' > 90^\circ$.

In the present study, C-H radiation measurements were carried out at different wave frequencies, different sections of the flame region and with the flame located at different portions of the standing acoustic field; that is, at a pressure node and a pressure antinode. The radiation emission from the oscillatory flame was collected by the setup shown in Fig. 5. Paper shields containing slots at desired locations were placed on the optical windows of the test section in order to permit radiation measurements from specific regions of the flame. An appropriate bandpass filter (431 nm for C-H) was placed between the collecting lens and the photomultiplier to permit passage of only the wavelength of interest. The local pressure oscillations were measured with a transducer mounted on the wall above the flame. These studies revealed that:

1. The autospectrum of the flame radiation exhibits a peak at the same frequency as the measured pressure autospectrum, and the amplitude of this peak is proportional to the amplitude of the excited pressure oscillation, see Figs. 6 and 7. These results indicate that the presence of an acoustic field results in periodic heat release rate having the same frequency as the imposed acoustic field. Also, the magnitude of the oscillatory heat release process is proportional to the amplitude of the acoustic waves.

2. Contrary to the results obtained with premixed flames which produced periodic radiation only when they were located away from an acoustic pressure node, the investigated diffusion flames produced oscillatory radiation when they were located next to acoustic pressure nodes and antinodes, see Fig. 8. This result indicates that the mechanisms which control the responses of diffusion and premixed flames to acoustic oscillations are different. This result also suggests that the mechanism which controls the response of diffusion flames to acoustic excitation is responsive to both pressure and velocity oscillations.

3. The phase difference between the total flame radiation and pressure oscillations depends upon both the frequency and the flame
location relative to the standing acoustic wave, see Fig. 9. This figure shows that the flame adds energy to the acoustic waves at low frequencies. However, a transition from flame driving to damping occurs as the frequency increases beyond approximately 700 Hz. No oscillatory flame radiation was observed at frequencies higher than 800 Hz., suggesting that the investigated flames do not respond to these high frequencies, or that their response was too weak to be detected by the photomultiplier.

(4) An investigation of the variation of the flame driving $I_q$ (i.e., $I_q = \int \mid S_{p'q'} \cos(\phi_{p'q'}) \mid dV$) within the flame region along a distance normal to the burner surface showed that for a 300 Hz. acoustic wave the driving occurred at the lower part of the flame while damping occurred at its upper part, see Fig. 10. When the flame was exposed to a 400 Hz. wave, driving occurred along the whole length of the flame, as shown in Fig. 11. In these studies, the driving was determined by setting $dV=dy$ in the $I_q$ integral, and using the radiation which passed through slits parallel to the $x$ axis to evaluate the integrand of the $I_q$ integral. A comparison of these two results with those obtained using LDV velocity measurements (see Figs. 3 and 4, respectively), shows excellent qualitative agreement between the findings of the velocity and radiation measurements.

(5) The investigation of variation of the flame driving with axial distance showed that, for a 200 Hz. acoustic wave, the left and right sides of the flame drove and damped the acoustic waves, respectively, see Fig. 12. In this case, the integral describing $I_q$ was determined by setting $dV=dx$ and using radiation which passed through vertical slits.

In an effort to better understand the processes which control the driving and damping by the flame, the phase differences between the pressure and radiation oscillations in different regions of the flame were investigated. Figure 13 shows that when the flame is located at a pressure node of the acoustic wave the phase differences between the radiation and pressure oscillations on the right side of the flame differ from those on the left side of the flame by almost 180° for all of the investigated frequencies. On the other hand, Fig. 14 shows that when the flame is located at a pressure antinode, the phase differences between the radiation and pressure
signals on the left and right sides of the flame are in the 120°-180° range. These results are consistent with the driving data presented in Fig. 12 and they indicate that, in most instances, when one side of the flame tends to drive the acoustic field, the other side of the flame tends to damp the acoustic oscillations. It is the relative magnitudes of the driving and damping provided by these two regions that determine whether the net effect of the flame is to drive or damp the acoustic motions.

The differences in the driving and damping tendencies of different regions of the investigated diffusion flames, which is discussed above, could have been caused by the variations in the oscillatory velocity components in the flame region. In a steady, laminar, diffusion flame the flame location and reaction rate are primarily controlled by rates of convection and diffusion of the oxidizer, fuel and other species into the flame sheets. The presence of oscillatory flow velocities in the flame region is expected to modify the steady flame behavior. For example, when the flame is located at a pressure node, the oscillatory axial velocity vectors on both sides of the fuel sheet are observed to be in phase, see Fig. 15. Consequently, they may increase and decrease the gas (i.e., primarily oxidizer) convection into the left and right sides of the flame, respectively. These, in turn, may increase and decrease the reaction rates of the left and right sides of the flame with respect to their mean reaction rates, respectively. Such behavior is expected to result in a 180° phase difference between the rates of heat addition of the left and right sides of the flame, which is consistent with the experimental findings of this study.

(B) Investigation of the Validity of the Flame Response Model

An important objective of this research program was to examine the validity of the investigated flame response model which had been developed during the previous reporting period. This linear flame model requires the steady temperature distribution in the flame region, the flame shape and acoustic admittance of the burner surface as inputs. Using these input data, the model has been used to predict the distribution of the oscillatory velocity component normal to the burner. During the reporting period, the required input data were determined experimentally and the model was
used to predict the velocity distributions within the investigated flames. These predictions were compared with corresponding experimental data. These efforts are briefly described in the remainder of this section.

The steady temperature distributions within the flame were performed by using a "Rayleigh scattering" technique. This technique has been widely used in flame temperature measurements because it provides a non-intrusive way to measure the temperature with high spatial and temporal resolutions. A schematic of the utilized Rayleigh scattering temperature measurement setup is shown in Fig. 16. A 5 watt argon-ion laser was used as the light source. Scattered light from molecules was collected by a receiving system. A Hamamatsu R-268 photomultiplier was used to measure the intensity of the scattered light which contains the temperature information. The relationship between the temperature and the measured scattered light for the flame configuration of this study can be expressed as

\[ T = T_{\text{ref}} \times I_{\text{ref}} / I \]

where \( T_{\text{ref}} \) is a reference temperature (e.g., at room temperature) and \( I_{\text{ref}} \) is the corresponding intensity of the scattered light. Thus, by measuring the Rayleigh scattering intensity, \( I \), the temperature at any desired location can be obtained.

An important input parameter of the model is the Peclet number (\( Pe \)). In the present study this parameter was determined from a comparison of theoretically and experimentally determined flame heights. The experimental flame height was measured by a cathetometer and the theoretical flame height was obtained from a solution of a modified Burke-Schumann model.

The acoustic admittance of the side wall burner assembly is another input required by the theoretical model. The admittance of the burner system was measured by the impedance tube technique. The experimental setup used to determine the acoustic admittance of the burner is shown in Fig. 17. An acoustic standing wave of desired frequency was excited inside the duct by acoustic drivers. The interaction between the incident and reflected waves results in the formation of a standing wave pattern having
successive maxima and minima along the duct, see Fig. 17. Using a transversing microphone to measure the amplitudes of the maxima and minima of the standing wave and the location of the first acoustic pressure minimum, it is possible to determine the required acoustic admittance.

Using the above described inputs, the model was used to predict the oscillatory vertical velocity distribution for different acoustic wave frequencies and amplitudes. Comparisons between predicted and measured distributions of the real part of the oscillatory vertical velocity, \( \text{Re}(v') \), are presented in Figs. 18-20. Both the theoretical predictions and measured data show that \( \text{Re}(v') \) is space dependent and that it maximizes in the vicinity of the steady flame sheet location and minimizes away from the flame region. The sharp peak in \( \text{Re}(v') \), which is predicted by the model, is a result of the utilized flame sheet assumption. Figures 18 and 19 present comparisons of measured and predicted distribution of \( \text{Re}(v') \) for a 444 Hz. and a 300 Hz. waves, respectively, for the same amplitude of acoustic excitations. These results indicate that \( \text{Re}(v') \) is frequency dependent and that the predicted and measured data are in qualitative agreement. Figure 20 presents a comparison of predicted and measured distribution of \( \text{Re}(v') \) for a 300 Hz. wave with an amplitude larger than that used in Fig. 19. These results show that a higher excitation amplitude causes a larger magnitude of \( \text{Re}(v') \) as expected. The model also predicts the same increase in the amplitude of \( \text{Re}(v') \) as observed experimentally.

\( \text{(C) Flow Turning Studies.} \)

In the latter part of the report period, an investigation of the characteristics of flow turning losses in solid propellant rocket motors was initiated. The investigation was initiated with the development of a new diffusion flame burner which better simulates the characteristics of gas phase solid propellant flames and is, thus, more suitable for the proposed flow turning studies. These studies are currently in progress, and this section briefly describes the developed burner.

In an effort to better simulate the interaction of a gas phase solid propellant flame with axial acoustic fields, it was decided to develop a new
burner, see Fig. 21, for the flow turning studies. This burner consists of 1537 hypodermic tubes (0.762 mm inner diameter) arranged in a 29X53 matrix. The oxidizer is supplied to the flame through these tubes, which simulate the oxidizer rich flow generated, for example, by the decomposition of the small ammonium perchlorate particles which are embedded in the fuel binder matrix of a composite solid propellant. The fuel is supplied to the flame through the spaces surrounding the oxidizer supply tubes, and it simulates the flow of the pyrolysis products leaving the surface of the binder during the combustion of a composite solid propellant. By using this arrangement, a diffusion flame which consists of a multitude of small scale diffusion flamelets, could be stabilized just above the burner surface.

The current investigation of the flow turning process utilizes an "acoustic intensity" approach which has been described in Ref. 8. This analysis requires the determinations of the spatial and temporal dependences of the pressures and velocities near the flame region. The velocity measurements are conducted by using the existing LDV system, and the pressures are measured with a long water cooled probe which has a microphone inserted at the end of probe away from the flame region to protect the microphone from the hostile environment. The construction of this water cooled probe has been completed.

Detailed measurements are in progress which will determine the effect of the following parameters upon the flow turning process:

(1) The location of the developed flame relative to the excited acoustic wave; that is, the flame will be positioned at pressure maxima, minima, or in between.
(2) The relative amounts of the injected fuel and oxidizer.
(3) The composition of the fuel and oxidizer flows.
(4) The fuel and oxidizer supply velocities at the burner surface.
(5) The axial mean flow velocity.
(6) The frequency of the excited acoustic field.
(7) The amplitude of the acoustic excitation.
Detailed results of these studies will be discussed in the next progress report.
SUMMARY

During the reporting period, theoretical and experimental investigations of the driving of axial acoustic fields by diffusion flames stabilized on the side wall of an acoustically excited duct were completed. In addition, an investigation of the effect of flow turning upon axial combustion instabilities in solid rockets was initiated.

Flame radiation measurements conducted during this study show that the excitation of acoustic waves produces space dependent oscillatory reaction and heat release rates within the investigated diffusion flames. At a given instant, some parts of the flame release energy in phase with the waves and, thus, drive the waves while other parts of the flame release energy out of phase with the pressure oscillations and, thus, damp the waves. This observation is consistent with LDV velocity data also measured during this study. The overall effect of the flame upon the acoustic wave depends upon the relative magnitudes of these "driving" and "damping" regions of the flame which add and remove energy from the acoustic waves. The results obtained in this study also suggest that both the pressure and velocity oscillations play an important role in the mechanism which controls the response of the diffusion flame to acoustic oscillations.

The velocity field measurements conducted during this year show that the interaction between the diffusion flames and the axial acoustic field produces an oscillatory transverse velocity component whose characteristics depend upon the spatial location, frequency and amplitude of the acoustic wave. The predicted and measured distributions of Re(v') are in good qualitative agreement.

Finally, a new diffusion flame burner which better simulates the characteristics of gas phase solid propellant flames was developed. This burner is currently used in an investigation of the effect of flow turning upon axial instabilities in solid propellant rocket motors. Specifically, the effects of frequency, flame position relative to the acoustic field, flow velocity and amplitude of the acoustic wave upon the flow turning process will be studied.
REFERENCES

PROFESSIONAL INTERACTIONS

A. Professional Personnel:
   Dr. Ben T. Zinn, Regents' Professor
   Mr. Brady R. Daniel, Senior Research Engineer
   Dr. Uday G. Hegde, Research Engineer
   Mr. Tzengyuan Chen, Ph. D. Student

B. Publications:

C. Presentations:
Figure 1. A Schematic of the Experimental setup
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**TITLE (Include Security Classification)**
INVESTIGATION OF THE FLAME-ACOUSTIC WAVE INTERACTION DURING AXIAL SOLID ROCKET INSTABILITIES

**PERSONAL AUTHOR(S)**
B. T. ZINN, B. R. DANIEL, U. G. HEGDE.

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The primary objectives of this research program are: (1) to investigate the mechanisms responsible for the driving of axial instabilities by solid propellant flames, (2) to determine whether state-of-the-art theoretical models can predict the characteristics of the flame driving mechanisms, and (3) to investigate the effect of flow turning upon axial instabilities in solid propellant rocket motors. To attain the program's objectives, the response of diffusion flames, stabilized on the side wall of a duct, to imposed acoustic waves has been investigated by using flame radiation measurements and laser Doppler velocimetry (LDV). The flame radiation measurements revealed that the presence of an acoustic field produced space dependent oscillatory heat release rates which depend upon the characteristics of the flame and the excited acoustic field. Measurements of the flame radiation and velocity field both showed that at a given instant some sections of the flame drive the acoustic field while others damp it. The net effect of the flame upon the acoustic field depends on the relative magnitude of these driving and damping regions. The validity of a previously developed flame response model was investigated by comparing measured and predicted oscillatory velocity components in the flame region. Using measured values of the acoustic admittance of the burner, the flame shape, and the steady state temperature distribution in the flame region, the model was used to predict oscillatory vertical velocity distributions in the flame region. These were then compared to velocity distributions measured with an LDV system, showing good qualitative agreement. An experimental investigation was initiated into the effect of flow turning on an excited acoustic field. Failure of the experiments to show any effect larger than the inherent experimental errors of the measurement system prompted a theoretical investigation into the nature of the flow turning loss. This theoretical study revealed that because the flow turning loss is of first order in the mean flow Mach number, neglecting energy convection terms of the first order in mean Mach number in the original experimental procedure resulted in errors on the order of the flow turning loss. An improved experimental study into the flow turning losses in solid propellant rocket motors which includes the effects of terms proportional to the mean Mach number has been initiated.
ABSTRACT

The primary objectives of this research program are: (1) to investigate the mechanisms responsible for the driving of axial instabilities by solid propellant flames, (2) to determine whether state-of-the-art theoretical models can predict the characteristics of the flame driving mechanisms, and (3) to investigate the effect of flow turning upon axial instabilities in solid propellant rocket motors. To attain the program's objectives, the response of diffusion flames, stabilized on the side wall of a duct, to imposed acoustic waves has been investigated by using flame radiation measurements and laser Doppler velocimetry (LDV). The flame radiation measurements revealed that the presence of an acoustic field produced space dependent oscillatory heat release rates which depend upon the characteristics of the flame and the excited acoustic field. Measurements of the flame radiation and velocity field both showed that at a given instant some sections of the flame drive the acoustic field while others damp it. The net effect of the flame upon the acoustic field depends on the relative magnitude of these driving and damping regions. The validity of a previously developed flame response model was investigated by comparing measured and predicted oscillatory velocity components in the flame region. Using measured values of the acoustic admittance of the burner, the flame shape, and the steady state temperature distribution in the flame region, the model was used to predict oscillatory vertical velocity distributions in the flame region. These were then compared to velocity distributions measured with an LDV system, showing good qualitative agreement. An experimental investigation was initiated into the effect of flow turning on an excited acoustic field. Failure of the experiments to show any effect larger than the inherent experimental errors of the measurement system prompted a theoretical investigation into the nature of the flow turning loss. This theoretical study revealed that because the flow turning loss is of first order in the mean flow Mach number, neglecting energy convection terms of the first order in mean Mach number in the original experimental procedure resulted in errors on the order of the flow turning loss. An improved experimental study into the flow turning losses in solid propellant rocket motors which includes the effects of terms proportional to the mean Mach number has been initiated.
INTRODUCTION

This research program is concerned with the development of an understanding of the mechanisms responsible for the driving of axial instabilities in solid propellant rocket motors. The onset of combustion instabilities in rocket motors depends upon the relative magnitudes of the gain (i.e., driving) and loss (i.e., damping) mechanisms within the combustor which add and remove energy from the oscillations, respectively. The reduction or elimination of the onset of combustion instabilities in solid propellant rocket motors requires identification of the processes which add or remove energy from the oscillations, and the development of an understanding of the mechanisms which control these processes. At present, it is generally recognized that the response of the solid propellant combustion process to the flow oscillations is responsible for providing the energy required for the initiation and maintenance of instabilities inside rocket motors. On the other hand, nozzle damping, viscous dissipation, flow turning, among others are processes which damp rocket instabilities. This research program has been concerned with developing an understanding of the gas phase flame processes which contribute to driving the instabilities, and the damping provided by the flow turning process.

Actual solid propellant gas phase flames are believed to consist of complex combinations of premixed and diffusion flames. Specifically, it has been argued that premixed flames occur above the oxidizer particles (e.g., ammonium perchlorate) and that diffusion flames are stabilized above the interfaces between the oxidizer particles and the fuel binder. Since premixed and diffusion flames are expected to respond differently to acoustic excitation, this investigation has been divided into two parts. In the first part of the study, completed earlier under this program, the interaction of premixed flames with axial acoustic fields was investigated. The parts of the study described herein investigated the response of diffusion flames to excited axial acoustic fields which simulate axial instabilities in solid propellant rocket motors, and began an investigation into the acoustic damping provided by the flow turning process.

Since actual solid propellant flames cannot be used in these studies because of their extremely small dimensions, their smoky nature, and rapid burnout, other
flames which simulate, in some respects, solid propellant flames have been used. In the present investigation, the response of diffusion flames, stabilized on the bottom wall of a long duct, to the excitation of axial acoustic fields has been studied. As with the previously conducted premixed flame investigations, the diffusion flame studies consist of parallel experimental and theoretical investigations of the diffusion flame response. Specifically, this research program has been investigating:

(1) the effects of diffusion processes on the acoustic driving/damping characteristics of the gas phase flame stabilized on the side wall of a duct,

(2) the validity of state-of-the-art solid propellant combustion response models,

and

(3) the effect of the flow turning upon axial instabilities in solid propellant rocket motors.

During the investigation, the experimental study of the interaction of diffusion flames with axial acoustic fields has been completed. In addition, the validity of the previously developed theoretical model of a diffusion flame response to axial acoustic fields was checked by comparing its predictions with experimental data. Preliminary results from the flow turning study revealed a need for a better theoretical understanding of the process, stimulating a theoretical investigation of the problem. This investigation has suggested a new experimental approach for investigating the effects of the flow turning upon axial acoustic fields, leading to the initiation of a new experimental study of the problem. Also, a simple computational model has been constructed that can be utilized to predict the approximate magnitudes of the various effects that are important to this type of flow field. This model will help in the analysis of future experimental data.
RESEARCH ACCOMPLISHMENTS

(A) Experimental diffusion flame studies:

The experimental phase of this program was conducted in the experimental setup shown in Fig. 1 which was developed earlier under this program. The interaction of three diffusion flames, stabilized on the bottom wall of a duct, with excited axial acoustic fields were investigated. The response of the diffusion flames to excited axial acoustic fields was studied by measuring steady and unsteady velocity components and distributions of C-H radiation from the flame using laser Doppler velocimetry and photomultipliers. In addition, pressure transducers were used to characterize the excited acoustic fields.

In a rocket motor, the interactions between unsteady solid propellant flames and the local flow oscillations involve complex fluid mechanical, heat transfer, and chemical processes. These interactions occur near the lateral boundaries of the motor cavity and they produce vertical velocity oscillations \( v'(y) \) at and near the propellant surface in a direction normal to the direction of the axial oscillations in the core flow. These normal velocity oscillations act as lateral pistons which periodically compress the core flow, thus providing the energy required for initiating and maintaining the core flow oscillations. Specifically, if

\[
I_v(y) = \int_0^L \int_0^T p' \text{Real}(v'(y)) \, dt \, dx > 0
\]

where \( p' \), \( L \), \( T \), and \( y_i \) are the local pressure oscillation, a relevant axial integration distance, the period of the oscillation, and the height above the flame, respectively, then these normal velocity oscillations pump acoustic energy into the core flow oscillations. Thus, if \( I_v \) increases with \( y \) in a given flame region, then the flame tends to drive the acoustic oscillation in this region, and vice versa.

The velocity measurements were conducted by using an existing LDV system, which had been also used for the premixed flame study. A typical measured time dependence of the normal velocity component at a point in the flame is presented in Fig. 2. The variations of \( I_v \) in the vertical direction for two different driving frequencies measured at a pressure antinode are presented in Figs. 3 and 4. Figure 3
shows that $I_v$ increases continually up to $0.55y_f$ ($y_f$ is the flame height) and that it decreases subsequently. Based upon the above discussion, this result indicates that when the flame is subject to a 300 Hz. acoustic oscillation, the lower (where $I_v$ increases) and upper (where $I_v$ decreases) sections of the flame drive and damp the oscillation, respectively. The overall driving/damping of the flame depends upon the net effect of these two regions. Since $I_v(y_f) > I_v(y=0)$, the net effect of the flame is to drive the acoustic field. In contrast to the driving and damping regions which exist in the flame when it is subjected to a 300 Hz. wave, Fig. 4 shows that $I_v$ increases continually along the whole length of the flame when the flame interacts with a 400 Hz. acoustic field, indicating that driving occurs throughout the flame region.

Flame radiation measurements are of interest because earlier studies\(^1\) have shown that the radiation intensities from radicals such as C-H, C-C, and O-H are proportional to the reaction and heat release rates in the flame. Such radiation measurements were used in this study to determine the effect of the acoustic field upon the magnitude of the heat release rate and the phase relationship between the heat release and the local pressure oscillation. The latter is important because, as stated by Rayleigh's criterion\(^2\), the phase difference between the oscillatory heat release rate and pressure oscillations determines whether the flame adds or removes energy from the acoustic waves. Expressed mathematically\(^3\), a heat source, such as a flame, adds energy to the acoustic waves when the following inequality is satisfied:

$$\int |S_{p'q'}| \cos(\phi_{p'q'}) \, dV > 0$$  \hspace{1cm} (2)

where $|S_{p'q'}|$ and $\phi_{p'q'}$ are the magnitudes of the cross-spectrum between the pressure ($p'$) and the oscillatory heat release rate ($q'$), and the phase difference between $p'$ and $q'$, respectively. The above integration is performed over the whole space where driving or damping by the flame occurs. In a flame region where the integrand is positive, driving of the acoustic waves occurs. This integrand is positive where $p'$ and $q'$ are in phase; that is, where $-90^\circ < \phi_{p'q'} < 90^\circ$. On the other hand, damping of the acoustic field by the flame occurs when $p'$ and $q'$ are out of phase; that is, when $-90^\circ > \phi_{p'q'} > 90^\circ$. 

5
In the present study, C-H radiation measurements were carried out at different wave frequencies, different sections of the flame region, and with the flame located at different portions of the standing acoustic field; that is, at a pressure node and a pressure antinode. The radiation emission from the oscillatory flame was collected by the setup shown in Fig. 5. Paper shields containing slots at desired locations were placed on the optical windows of the test section in order to permit radiation measurements from specific regions of the flame. An appropriate bandpass filter (431 nm for C-H) was placed between the collecting lens and the photomultiplier to permit passage of only the wavelength of interest. The local pressure oscillations were measured with a transducer mounted on the wall above the flame. The studies revealed that:

(1) The autospectrum of the flame radiation exhibits a peak at the same frequency as the measured pressure autospectrum, and the amplitude of this peak is proportional to the amplitude of the excited pressure oscillation, see Figs. 6 and 7. These results indicate that the presence of an acoustic field results in a periodic heat release rate having the same frequency as the imposed acoustic field. Also, the magnitude of the oscillatory heat release process is proportional to the amplitude of the acoustic waves.

(2) Contrary to the results obtained with premixed flames which produced periodic radiation only when they were located away from an acoustic pressure node, the investigated diffusion flames produced oscillatory radiation when they were located next to acoustic pressure nodes and antinodes, see Fig. 8. This result indicates that the mechanisms which control the responses of the diffusion and premixed flames to acoustic oscillation are different. This result also suggests that the mechanism which controls the response of diffusion flames is responsive to both pressure and velocity oscillations.

(3) The phase difference between the total flame radiation and pressure oscillations depends upon both the frequency and the flame location relative to the standing acoustic wave, see Fig. 9. This figure shows that the flame adds energy to the acoustic waves at low frequencies. However, a transition from flame driving to damping occurs as the frequency increases beyond approximately 700 Hz. No oscillatory flame radiation was observed at frequencies higher than 800 Hz., suggesting that the investigated flames do not respond to these high frequencies, or that their response was too weak to be detected by the photomultiplier.
(4) An investigation of the variation of the flame to driving $I_q$ (i.e., $I_q = \int \left| S_{p'q'} \right| \cos(\phi_{p'q'}) \, dV$) within the flame region along a distance normal to the burner surface showed that for a 300 Hz. acoustic wave the driving occurred at the lower part of the flame while damping occurred at its upper part, see Fig. 10. When the flame was exposed to a 400 Hz. wave, driving occurred along the entire length of the flame, as shown in Fig. 11. In these studies, the driving was determined by setting $dV = dy$ in the $I_q$ integral, and using the radiation which passed through slits parallel to the $x$-axis to evaluate the integrand of the $I_q$ integral. A comparison of these two results with those obtained using LDV velocity measurements (see Figs. 3 and 4), shows excellent qualitative agreement between the findings of the velocity and radiation measurements.

(5) The investigation of variation of the flame driving with axial distance showed that, for a 200 Hz. acoustic wave, the left and right sides of the flame drove and damped the acoustic waves, respectively, see Fig. 12. In this case, the integral describing $I_q$ was determined by setting $dV = dx$ and using radiation which passed through vertical slits.

In an effort to better understand the processes which control the driving and damping by the flame, the phase differences between the pressure and radiation oscillations in different regions of the flame were investigated. Figure 13 shows that when the flame is located at a pressure node of the acoustic wave the phase differences between the radiation and pressure oscillations on the right side of the flame differ from those on the left side of the flame by almost $180^\circ$ for all the investigated frequencies. On the other hand, Fig. 14 shows that when the flame is located at a pressure antinode, the phase differences between the radiation and pressure signals on the left and right sides of the flame are in the $120^\circ-180^\circ$ range. These results are consistent with the driving data presented in Fig. 12 and they indicate that, in most instances, when one side of the flame tends to drive the acoustic field, the other side of the flame tends to damp the acoustic oscillations. It is the relative magnitudes of the driving and damping provided by these two regions that determines whether the net effect of the flame is to drive or damp the acoustic motion.

The differences in the driving and damping tendencies of different regions of the investigated diffusion flames, which is discussed above, could have been caused
by the variations in the oscillatory velocity components in the flame region. In a steady, laminar, diffusion flame the flame location and reaction rate are primarily controlled by rates of convection and diffusion of the oxidizer, fuel, and other species into the flame sheets\(^1\). The presence of oscillatory flow velocities in the flame region is expected to modify the steady flame behavior. For example, when the flame is located at a pressure node, the oscillatory axial velocity vectors on both sides of the flame sheet are observed to be in phase, see Fig. 15. Consequently, they may increase and decrease the gas (i.e., primarily oxidizer) convection into the left and right sides of the flame, respectively. These, in turn, may increase and decrease the reaction rates of the left and right sides of the flame with respect to their mean reaction rates, respectively. Such behavior is expected to result in a 180° phase difference between the rates of heat addition of the left and right sides of the flame, which is consistent with the experimental findings of this study.

(B) Investigation of the validity of the flame response model:

An important objective of this research program was to examine the validity of the investigated flame response model which had been developed during the previous reporting period. This linear flame model requires the steady temperature distribution in the flame region, the flame shape, and the acoustic admittance of the burner surface as inputs. Using these input data, the model has been used to predict the distribution of the oscillatory velocity component normal to the burner. During the reporting period, the required input data were determined experimentally and the model was used to predict the velocity distributions within the investigated flames. These predictions were compared with corresponding experimental data. These efforts are briefly described in the remainder of this section.

The steady temperature distributions within the flame were performed by using a "Rayleigh scattering" technique. This technique has been widely used in flame temperature measurements\(^4\) because it provides a non-intrusive way to measure the temperature with high spatial and temporal resolutions. A schematic of the utilized Rayleigh scattering temperature measurement setup is shown in Fig. 16. A 5 watt argon-ion laser was used as the light source. Scattered light from molecules was collected by a receiving system. A Hammamatsu R-268 photomultiplier was used to measure the intensity of the scattered light which
contains the temperature information. The relationship between the temperature and the measured scattered light for the flame configuration of this study can be expressed as:

\[ T = T_{\text{ref}} \times I_{\text{ref}} / I \]  

(3)

where \( T_{\text{ref}} \) is a reference temperature (e.g., room temperature) and \( I_{\text{ref}} \) is the corresponding intensity of the scattered light. Thus, by measuring the Rayleigh scattering intensity, \( I \), the temperature at any desired location can be obtained.

An important input parameter of the model is the Peclet number (Pe). In the present study this parameter was determined from a comparison of theoretically and experimentally determined flame heights. The experimental flame height was measured by cathetometer and the theoretical flame height was obtained from a solution of a modified Burke-Schumann model.

The acoustic admittance of the side wall burner assembly is another input required by the theoretical model. The admittance of the burner system was measured by the impedance tube technique. The experimental setup used to determine the acoustic admittance of the burner is shown in Fig. 17. An acoustic standing wave of desired frequency was excited inside the duct by acoustic drivers. The interaction between the incident and reflected waves results in the formation of a standing wave pattern having successive maxima and minima along the duct, see Fig. 17. Using a traversing microphone to measure the amplitudes of the maxima and minima of the standing wave and the location of the first acoustic pressure minimum, it is possible to determine the acoustic admittance of the test sample.

Using the above described inputs, the model was used to predict the oscillatory vertical velocity distribution for different acoustic wave frequencies and amplitudes. Comparisons between predicted and measured distributions of the real part of the oscillatory vertical velocity, \( \text{Re}(v') \), are presented in Figs. 18-20. Both the theoretical predictions and measured data show that \( \text{Re}(v') \) is space dependent and that it maximizes away from the flame region. The sharp peak in \( \text{Re}(v') \) predicted by the model is a result of the utilized flame sheet assumption. Figures 18 and 19 present comparisons of measured and predicted distributions of \( \text{Re}(v') \) for a 444 Hz. and a 300 Hz. acoustic wave, respectively, for the same amplitude of acoustic excitation. These results indicate that \( \text{Re}(v') \) is frequency dependent and that the
predicted and measured data are in qualitative agreement. Figure 20 presents a comparison of predicted and measured distribution of $\text{Re}(v')$ for a 300 Hz. wave with an amplitude larger than that used in Fig. 19. These results show that a higher excitation amplitude causes a larger magnitude of $\text{Re}(v')$ as expected. The model also predicts the same increase in the amplitude of $\text{Re}(v')$ as observed experimentally.

(C) Flow turning studies:

An experimental investigation was begun in order to develop a greater understanding of the damping of combustion instabilities in solid rocket combustors due to 'flow turning'. A schematic of the experimental set-up is shown in Fig. 21. The steady state velocity field measured above the surface of the burner assembly is shown in Fig. 22. This figure shows that the region in which the added gases are turned in the direction of the core flow extends several centimeters above the burner surface. This is the region where 'flow turning' losses are expected to occur. Initial attempts, based upon a state-of-the-art theoretical approach, to measure the flow turning loss in cold flow failed to show any measurable effect. The classical acoustic energy flux, $\text{Rea}(p'u')$, was measured and integrated over the surface of a control volume. Since no other sources or sinks of acoustic energy were present within the investigated control volume, any measured net loss of acoustic energy would represent the flow turning loss. This procedure was unable to measure any acoustic energy loss larger than experimental errors.

It was concluded that the inability to measure the flow turning loss may be due to one or more of the following:

1. The experiment failed to account for the acoustic fluxes convected by the mean flow.
2. The flow turning effect may be improperly predicted by Culick.
3. The flow turning loss may be on the order of terms neglected in the experimental analysis.

To determine which of these factors could explain the experimental results, it was decided to rederive the expression for the flow turning loss by using a different theoretical approach from that used by Culick. A one-dimensional acoustic stability
equation was developed from an energy balance approach, similar to that used by Cantrell and Hart\textsuperscript{10}.

The analysis was based on explicit considerations of mass, momentum, and acoustic energy fluxes at the control volume boundaries. It involved considerations of the first and second order perturbations of the acoustic quantities. The consideration of second order quantities is necessary for proper evaluation of terms of the order of the acoustic energy. Basically, the energy equation was expanded to second order in acoustic perturbations and the perturbed forms of the mass and momentum conservation equations were used to modify the energy equation. The original Cantrell and Hart analysis has been modified to include flow rotation effects. An expression for the acoustic growth rate $\alpha$ was derived which by careful manipulation was shown to be independent of any terms involving second order perturbations. The resulting stability equation is similar in form to Culick's 1-D result. This similarity to Culick's form of the acoustic stability equation is necessary for comparison of the results, and does not simplify interpretation of the terms. The resulting equation for a 2-D system is

$$-2\alpha <E^2> = <[p' u']_0^L> + <[\frac{p'^2 u_0}{a^2}]_0^L> + <[\rho_o u_o u'^2]_0^L>$$

$$+ \int_0^L \frac{\rho_o \ u'}{H} [v' u_0]_0^H \ dx > + \int_0^L \frac{\rho_o \ u'}{H} [u' v_0]_0^H \ dx > +$$

$$+ \int_0^L \frac{p'}{H} [\frac{m_b'}{\rho_o}]_0^H \ dx > - \int_0^L \frac{u'^2}{H} [m_{bo}]_0^H \ dx >$$

(4)

where $\alpha$ is the acoustic growth rate constant, $E^2$ is a term with units of energy, $u$ and $v$ represent velocity components in the axial and transverse directions, respectively, $m_b$ is the mass flux out of the burning surface, and $L$ and $H$ are the length and height of the control volume. Values not evaluated at the walls are cross-duct averages. The equation is valid for constant height control volumes with no large variation in mean density $\rho_o$, no residual burning and no particles. It must be noted that the term $E^2$ does not represent the total acoustic energy of the system, but as a function of terms on the order of the perturbation squared, the term has a
growth rate of $2\alpha$. The lack of a simple physical interpretation of the $E^2$ term is an unfortunate but necessary consequence of the cancellation of any terms involving second order perturbations.

The last term on the right hand side of Eq. 4 corresponds to Culick's flow turning term. This 1-dimensional acoustic stability equation is more general than Culick's result in that it does not require the acoustic oscillations to be near a resonance of the system. Comparison shows that this result contains terms that are not found in Culick's result. The absence of these terms in Culick's result may be due to the inappropriate application of approximate boundary conditions. This work also confirms the results of Van Moorhem$^{11}$ which suggested that the flow turning effect is inherent to the 3-dimensional problem, and should not be added to the 3-dimensional stability equation as per Culick$^{12}$.

A numerical investigation was initiated to predict the nature and the relative magnitudes of the terms in the 1-dimensional acoustic stability equation (Eq. 4) derived in the above procedure. For this investigation the 2-dimensional acoustic equations including mean flow terms were averaged across a constant area duct of height $H$. The mean pressure $p_o$ and mean density $\rho_o$ were considered constant throughout the duct to the first order in Mach number. The isentropic value of the speed of sound was used. The acoustic perturbations were assumed to be harmonic in time at a given frequency $\omega$, which represents a system that is neutrally stable. The equations were then integrated in a duct of length $L$. A schematic of the duct is shown in Fig. 23. The problem was simplified by setting the acoustic admittance of the injectors to 0, which allowed any terms proportional to the fluctuating part of the vertical velocity component $v'$ to be neglected. Note that this is not a realistic assumption for walls made up of solid propellant, but is tolerable in this study because it helps to isolate the effect of the flow turning loss. The axial velocity component at the walls was set to zero, and because the inviscid equations were used, an infinitely thin boundary layer resulted. The results of the integration were used to evaluate each of the terms on the right hand side of Eq. 4. Terms involving integration along the axis were evaluated from 0 to $x$ for each axial location. Neutral stability requires that the sum of these terms be constant along the duct when there is no net gain or loss of acoustic energy in the volume.

Results from two test cases are shown in Figs. 24-27. For these test cases the flow was assumed to be at room temperature. Figure 24 shows the acoustic pressure
amplitude and the phase of the pressure at a given axial location with respect to the phase of acoustic pressure at the head end of the duct for the case in which there is side injection at a local maximum of the pressure amplitude and no mean flow upstream of the side injection. The length of the side injector and the height of the duct are both 0.1 m. The injection is seen to have no discernable effect on the magnitude of the pressure and a relatively small effect on the phase of the pressure which is due to the mean axial velocity. The lack of any noticeable effect on the mode of the acoustic disturbance indicates that the effect of the flow turning, as well as the effects of the other terms of the acoustic stability equation, are relatively small. Figure 25 shows five terms relevant to the acoustic stability of the model and a graph of these terms and their sum as functions of axial location. Comparison of the graphed terms to the terms of Eq. 4 reveals that the stability of the system can be determined for an arbitrary section of the duct by subtracting the sum of the five terms at the left boundary of the volume from the sum of the five terms at the right boundary; i.e., if the growth rate $\alpha$ is zero throughout the control volume, the value of 'term 6' must be constant along the length of the duct. This is expected because the oscillations in the duct were assumed neutrally stable in the computational model. In Figs. 26 and 27, the injection occurs at a local minima of the acoustic pressure amplitude. Comparison of Figs. 25 and 27 reveals that the magnitude of the flow turning loss depends on the axial location of the injection; e.g., the flow turning loss is relatively higher when the injection occurs in the vicinity of an axial acoustic velocity maxima, and is relatively smaller when the injection occurs in the vicinity of an axial acoustic velocity minima. In the original experimental procedure, an attempt was made to measure the magnitude of the flow turning loss indirectly by measuring the deficit in the acoustic flux through the surface of the control volume. However, the convection of acoustic exergy by the mean flow was neglected as small with respect to the mean flow independent energy flux. The relative magnitudes of the terms in the acoustic stability equation reveal that the flow turning loss is of the order of these convective terms and therefore of the order of terms neglected in the original experimental procedure. The behavior of the terms predicted by the numerical model will be used to aid in the interpretation of data measured in an improved experimental procedure currently being initiated. This investigation will involve a more detailed study of the flow turning region
and the results will be analyzed using the acoustic stability equation derived above. The magnitude of the flow turning loss will be directly calculated from measured data, and then checked by indirect techniques. This new experimental procedure should ensure a more accurate determination of the magnitude of the flow turning loss and its importance to the stability of solid rocket motors.
SUMMARY

In the reported phase of this ongoing research program, theoretical and experimental investigations of the driving of axial acoustic fields by diffusion flames stabilized on the side wall of an acoustically excited duct were completed. In addition, initial results from an experimental study of the effects of flow turning upon axial acoustic instability failed to show expected results, which prompted a theoretical investigation into the nature of the flow turning loss.

Flame radiation measurements conducted during this study show that the excitation of acoustic waves produces space dependent oscillatory reaction and heat release rates within the investigated diffusion flames. At a given instant, some parts of the flame release energy in phase with the waves and, thus, drive the waves while other parts of the flame release energy out of phase with the pressure oscillations and, thus, damp the waves. This observation is consistent with LDV velocity data also measured during this study. The overall effect of the flame upon the acoustic wave depends upon the relative magnitudes of these "driving" and "damping" regions of the flame which add and remove energy from the acoustic wave. The results obtained in this study also suggest that both the pressure and velocity oscillations play an important role in the mechanism which controls the response of the diffusion flame to acoustic oscillations.

The velocity field measurements conducted during the diffusion flame study show that the interaction between the diffusion flames and the axial acoustic field produces an oscillatory transverse velocity component whose characteristics depend upon the spatial location, frequency, and amplitude of the acoustic wave. The predicted and measured distributions of Re(v') are in good qualitative agreement.

Finally, preliminary results from an experimental investigation of the effect of flow turning upon axial combustion instabilities in solid rockets revealed the need for a more complete theoretical development of the flow turning losses. A theoretical investigation and a simple numerical model revealed that the original experimental procedure was inadequate for measuring the flow turning loss. A more detailed experimental procedure has been initiated using an acoustic stability equation derived during this theoretical investigation.
REFERENCES

PROFESSIONAL INTERACTIONS

A. Professional Personnel:

Dr. Ben T. Zinn, Regent's Professor
Mr. Brady R. Daniel, Senior Research Engineer
Dr. Uday G. Hegde, Research Engineer
Dr. Tzengyuan Chen, Research Assistant
Mr. Lawrence M. Matta, Ph. D. Student

B. Publications:


C. Presentations:

Figure 1. A Schematic of the Experimental setup
Figure 2. Typical Time Trace of the Vertical Velocity of a 400Hz. Oscillation at a Pressure Antinode.
Figure 3. Measured Spatial Dependence of $\int \text{Re}(v') dx$ along a Distance Normal to the Burner Surface of a Flame at a Pressure Antinode of a 300 Hz Acoustic Wave.
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Figure 9. Phase Differences between the Radiation and Pressure Oscillations Measured with the Flame at a Pressure Antinode and a Pressure Node.
Figure 10. Variation of the Flame Driving/Damping along a Distance Normal to the Burner Surface of a Flame at a Pressure Antinode of a 300 Hz. Wave.
Figure 11. Variation of the Flame Driving/Damping along a Distance Normal to the Burner Surface of a Flame at a Pressure Antinode of a 400 Hz. Wave.
Figure 12. Axial Dependence of the Driving/Damping for a Flame Located at a Pressure Antinode of a 200 Hz. Wave.
Figure 13. Frequency Dependence of the Phase Differences between the Radiation and Pressure Oscillations in Different Flame Regions of a Flame Located at a Pressure Node.
Figure 14. Frequency Dependence of the Phase Differences between the Radiation and Pressure Oscillations in Different Flame Regions of a Flame Located at a Pressure Antinode.
Figure 15. Time Trace of Axial Velocity on Both Oxidizer Sides at Three Different Flame Locations of a 200 Hz Oscillation at a Pressure Node.
Figure 16. A Schematic of Experimental Setup for Rayleigh Scattering Temperature Measurement
Figure 17. A Schematic of the Setup for Acoustic Amdittance Measurement.
Figure 18. Measured and predicted Spatial Dependence of Real($v'$) at $Y=Y_f$ of a 444 Hz. Acoustic Wave at a Pressure Antinode.
Figure 19. Measured and Predicted Spatial Dependence of Real($v'$) at $Y=Y_\text{f}$ of a 300 Hz. Acoustic Wave at a Pressure Antinode.
Figure 20. Measured and Predicted Spatial Dependence of Real($v'$) at $Y=Y_F$ of a 300 Hz. Acoustic Wave at a Pressure Antinode. (Larger amplitude of acoustic Excitation)
Figure 21. A schematic of the experimental setup.
Figure 22. Vector diagram of the steady state velocity field above the burner in the experimental set-up. This example is for hot flow with a flame stabilized just above the burner surface.
Figure 23. Schematic of the test section used in the numerical investigation of the magnitudes of the terms in the 1-dimensional acoustic stability equation. Walls and injectors are acoustically 'hard'. 
Figure 24. Numerically predicted amplitude and phase of the pressure oscillation in a duct with side injection at a local pressure maxima. The phase is measured with respect to the pressure signal at $x = 0$. The injection occurs between $x = 1.0$ and $x = 1.1$. 
1 = \langle p' u' \rangle \\
2 = \frac{p'^2 u_0}{\rho_0 a^2} \\
3 = \langle \rho_0 u_0 u'^2 \rangle \\
4 = - \left\langle \int_0^x \frac{u'^2}{H} \left[ m_{bo} \right]_0^H dx \right\rangle \\
5 = \left\langle \int_0^x \frac{p'}{H} \left[ \frac{m_{bo}'}{\rho_0} \right]_0^H dx \right\rangle \\
6 = \text{The sum of above 5 terms.}

Figure 25. Numerical prediction of the relevant terms on the right hand side of the 1-dimensional acoustic stability equation for a duct with side injection at 1.0m, in this case a pressure maxima. Term (4) is identified with the flow turning loss.
Figure 26. Numerically predicted amplitude and phase of the pressure oscillation in a duct with side injection at a local pressure minima. The phase is measured with respect to the pressure signal at $x = 0$. The injection occurs between $x = 0.625$ and $x = 0.725$. 
Figure 27. Numerical prediction of the relevant terms on the right hand side of the 1-dimensional acoustic stability equation for a duct with side injection at 0.625m, in this case a pressure minima. Term (4) is identified with the flow turning loss.

\[ 1 = < p' u' > \]
\[ 2 = < \frac{p'^2 u_0}{\rho_0 a^2} > \]
\[ 3 = < \rho_0 u_0 u'^2 > \]
\[ 4 = - \int_0^x \frac{u'^2}{H} \left[ m_{b_0} \right]_0^H dx \]
\[ 5 = \int_0^x \frac{p'}{H} \left[ \frac{m_{b_1}}{\rho_0} \right]_0^H dx \]
\[ 6 = \text{The sum of above 5 terms.} \]