Mechanisms of Time-Dependent Crack Growth at Elevated Temperature

Final Project Report

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1. INTRODUCTION AND SUMMARY

The prediction of design life of components subjected to elevated temperatures in energy conversion machinery require accurate methods for predicting crack growth behavior in the presence of time-dependent creep strains. Significant progress has been made in extending the linear-elastic and elastic fracture mechanics concepts into the creep regime. Thus, the use of fracture mechanics for predicting the crack growth life of elevated temperature components is a viable approach [1,2]. However, serious deficiencies exist in the understanding of the damage mechanisms that lead to time-dependent crack growth. Specifically, the interactions between creep and fatigue damage mechanisms for slow frequency cyclic loading are not well understood. Therefore, cumulative damage laws for creep-fatigue loading based on physical damage mechanisms are non-existent.

The objective of this three year study was to conduct creep and creep-fatigue crack growth experiments and to characterize the crack tip damage mechanisms in a model material (1% Sb-Cu) which is known to cavitate at the grain boundaries under creep deformation. These data and observations were to be used to develop mechanistic models for predicting crack tip damage and the resultant crack growth rates under complex loading conditions at elevated temperatures. The results of this program were to be integrated into Electric Power Research Institute (EPRI) sponsored studies at Georgia Tech. for characterizing the creep-fatigue crack growth behavior of Cr-Mo-V steels. Various new and established
techniques for detecting crack tip cavitation damage including electron metallography, synchrotron x-ray tomography, digital microradiography and small angle neutron scattering (SANS) were also evaluated. The study led to the following important conclusions.

- In the presence of large scale cavitation damage and crack branching, time rate of creep crack growth \((\frac{da}{dt})\) does not correlate with \(C_t\) or with \(C^*\). On the other hand, when the cavitation damage is constrained, \(\frac{da}{dt}\) is uniquely characterized by \(C_t\).

- Area fraction of grain boundary cavitated, \(A_c\), appears to be the single damage parameter for characterizing the extent of cavitation damage ahead of crack tips. The distribution of \(A_c\) ahead of the crack tip is a complex function of loading wave form and cycle time.

- The use of \(C_t\)-parameter was extended for characterizing the creep-fatigue crack growth behavior. It was shown that \(C_t\) is suitable for characterizing crack growth rate in the presence of different types of deformation transients.

- It was observed that in materials which are prone to rapid cavity nucleation, the creep cracks grow faster initially and then reach a steady-state during which their growth rate is uniquely characterized by \(C_t\).

- The percent creep life exhausted was shown to correlate with the average cavity diameter and also with fraction
of grain boundary area occupied by cavities.

- Synchrotron x-ray tomographic microscopy (XTM) was successfully demonstrated for imaging individual cavities in the 1 percent Sb-Cu material.
- The above concepts have been incorporated in methodologies for remaining life prediction of elevated temperature power-plant components. Specifically, \((C_t)_{\text{avg}}\) has been used to correlate creep-fatigue crack growth behavior in Cr-Mo and Cr-Mo-V steel and in weldments.

In Section 2 of this report, the important results from the program are presented. Section 3 of the report contains a list of the publications and presentations from the work performed under the grant and the status of the various students funded under this grant.

2. HIGHLIGHTS OF ACCOMPLISHMENTS

The creep and creep-fatigue crack growth experiments were conducted on a copper alloy containing 1 weight-percent antimony. The segregation of antimony to the grain boundaries embrittles the boundaries. Therefore, the room temperature fracture in this material is one hundred percent intergranular revealing the cavitation damage on the grain boundary facets. Thus, this alloy was ideally suited for studying the time-dependent damage accumulation at crack tips subjected to creep and creep-fatigue loading. The model alloy was specially produced at the Oak Ridge
National Laboratory and was supplied to Georgia Tech in the form of an extruded rectangular bar which was 2-3/4 in x 2 in cross-section. The chemical composition and the tensile and creep behavior of the material is summarized in reference [3] attached in the Appendix. The highlights of the various aspects of the work are summarized in the following sections.

2.1 Mechanisms of Creep Crack Growth

The creep crack growth data developed in air as a part of this study clearly showed that, in the presence of large scale cavitation damage ahead of the crack tip and crack branching, da/dt does not correlate with $C_t$ (Fig.1). On the other hand, when the damage in the crack tip region is not extensive (or constrained), da/dt is uniquely characterized by $C_t$ after steady-state damage conditions are reached (Fig. 2).

The extensive quantitative damage analysis of the specimens revealed that continuous nucleation and growth of cavities occurs near the crack tip. The fraction of grain boundary area cavitated, $A_c$, was found to best represent the state of damage at a point ahead of the crack tip. A steep gradient in the extent of cavitation damage as a function of distance from the crack tip was obtained in all creep crack growth specimens which showed constrained cavitation, Fig. 3. A governing equation for damage evolution was proposed (See Ref.3).

In order to promote constrained cavitation during subsequent creep crack growth testing, all specimens were sidegrooved. Also,
to protect the fracture surface from oxidizing, the subsequent testing was performed in ultra-high-purity nitrogen. Fig. 4 shows the creep crack growth rate behavior as a function of $C_t$ from these tests. Transient crack growth rates characterized by non-unique $da/dt$ versus $C_t$ relationship were observed for a substantial portion of the tests. The crack growth rates were high initially and they decreased with subsequent crack extension. Similar transient crack growth behavior has been observed previously [4] in ductile steels but the one observed here is peculiar in two ways. First, the crack growth rates in the transient regime are higher than those at equivalent $C_t$ values in the steady-state crack growth regime. Second, the duration of the crack growth transients in terms of the crack extension required to achieve steady-state was significantly higher in antimony-copper than observed in ductile steels. The transient crack growth rate behavior was examined in great depth through further testing. It was concluded that the transient behavior occurs due to (1) damage done during precracking prior to creep crack growth testing and (2) damage done during initial loading when the crack tip stresses are the highest. A very detailed discussion of this phenomena is contained in a forthcoming M.S. thesis of Mr. Richard Norris. The completed thesis will be mailed to DOE when available.

2.2 Damage and Life Fraction Correlations in Creep Rupture Specimens

Although studying the creep rupture behavior of antimony-copper was not the main objectives of this study, some rupture
testing was necessary to develop the creep constants for crack
growth data analysis. Also, the tested specimens were needed to
evaluate the nondestructive damage characterization techniques used
in this project.

Large gradients in the volume fraction of cavities $V_v$ were
observed as a function of distance from the rupture surface of
crept specimens (Fig. 5). Optical microscopy and synchrotron
microradiography both showed that lower stresses lead to
considerably higher damage levels and to more extensive zones of
enhanced damage (e.g. much higher than the uniform damage levels
elsewhere in the specimens). Each of the curves in Fig. 6
approaches the same asymptotic value of $V_v$ at large distances from
the rupture surface, which suggest that a critical level of damage
in creep of Cu-1% Sb must be attained to initiate necking leading
to fracture.

Both $A_c$ and $D$, the average cavity diameter, were determined
as a function of lifetime expended for two applied loads (Fig. 6).

Both quantities are linear functions of expended life, and
there appears to be little dependence on applied stress.
Apparently, failure by ductile rupture is imminent when about 0.45
of the grain boundaries are covered by cavities, regardless of the
stress level. This conclusion is consistent with the observation
of $V_v$ values far from the rupture surface.
2.3 Crack Growth Mechanisms During Creep-Fatigue Loading

The mechanism of crack advance in Cu-1% Sb is a function of the crack-tip stresses. Below a critical stress, crack growth occurs by coalescence of creep cavities with the crack front. Above this same critical stress, crack growth occurs by a combination of cavity coalescence and grain boundary peeling. The dendritic growth of the crack front, or peeling process occurs when the normal stresses on the boundaries exceed a critical value, and is thus sensitive to grain orientation with respect to the remotely applied stress.

The process of crack growth by cavity coalescence is controlled by the smaller crack-tip cavitation. The size and density distribution of these cavities are functions of loading waveform and cycle time. Any grain boundary area not previously failed by the growth of the large, remote cavities fails by the coalescence of the smaller voids. The widespread, larger cavities have only a secondary effect on the growth rates.

The average rate of crack growth during hold times in trapezoidal loading waveforms, \( \dot{a}_{avg} \) can be uniquely correlated with the average, measured value of the \( C_t \) parameter, \( (C_t)_{avg} \) (Fig. 7). The \( \dot{a}_{avg} \) vs. \( (C_t)_{avg} \) relationship for hold times of \( \frac{1}{2} \) hour and longer is identical to that obtained for creep crack growth conditions. This implies, that for hold times larger than \( \frac{1}{2} \) hour, creep-fatigue crack growth rates are correlated uniquely by \( (C_t)_{avg} \). The creep-fatigue interaction during elevated temperature crack growth may be the result of a change in crack growth mechanism.
during the hold times. In the initial portion of each hold time, peeling may occur, and as the stresses relax, crack extension may revert back to a process by cavity coalescence. Thus, shorter hold times maintain higher average crack-tip stresses and have a higher probability for the peeling mechanism to operate. Since peeling occurs at higher stress levels, it is expected to result in higher crack growth rates, and shorter hold times would be expected to have unexpectedly higher crack growth rates.

2.4 Development and Assessment of Advanced Damage Characterization Techniques

Electron radiography and small angle scattering were examined during the very early stages of this three-year study as possible alternatives to optical and scanning electron metallography for damage assessment. Neither was found to be suitable for this study as discussed in earlier progress reports [5]. On the other hand, industrial computed tomography (CT) and high-resolution variants termed x-ray tomographic microscopy (XTM) were found to be very useful for damage characterization; digital microradiography and synchrotron microradiography were also applied successfully to damage characterization in Cu-1%Sb. These novel techniques and results obtained with them as described below.

2.4.1 Microradiography

Synchrotron microradiographs were recorded at the Stanford Synchrotron Radiation Laboratory (SSRL) using white radiation. The nearly parallel, high flux beam makes synchrotron radiation an
ideal source for contact radiography. Optical densitometry tracings of the microradiographs were recorded along a direction normal to the rupture surface and yielded curves such as that shown in Figure 10. Standards of the appropriate thickness and containing no cavitation damage were not run, however, and absolute magnitudes of damage cannot be easily calculated.

Comparison of the relative optical densities with the relative cavity volume fractions for both 3.5 and 6.0 ksi samples (tested to failure) should show quantitative agreement; this is not observed. We suspect that local film saturation occurs at positions where distinct cavity images are seen. This is being checked with further densitometry. During the next experimental period we will also repeat the synchrotron microradiography. We will use undamaged and damaged samples of the same thickness.

Digital microradiography eliminates the photographic emulsion and directly measures the transmitted intensity with electronic x-ray detectors. High resolution can be obtained by using a multiple detector array with very small elements or by using a pinhole collimator and scanning the sample across the beam. The variation of x-ray absorption around a creep crack in the Cu-Sb material was quantified using the latter approach, Ag Kα radiation and a 15 μm diameter beam. The sample studied was CT5 which was crept at 400°C and 1.5 ksi(in.)^{0.5}. The sample was mechanically polished to about 0.3 mm thickness, and this was adequate to transmit nearly 30% of the incident beam in the vicinity of the crack. A square array of points with spacing of 0.5 mm was used to cover a 3 mm x 5 mm area.
around the crack. Counting times were 10 sec per point, and a minimum of $3 \times 10^5$ counts were obtained at each point, well under one percent standard error in counting statistics. We note that these measurements were performed at London Hospital Medical College in collaboration with Dr. J.C. Elliott's group in the Dept. of Child Dental Health.

Figure 11 shows the two dimensional distribution of transmitted intensity around the creep crack. Note that absorption is reduced as far as 1 mm from the crack. The decrease in absorption could be due to preferential thinning near the crack, i.e. near the center of the sample, and this unlikely source of increased transmissivity is presently being checked by precision metrology. Conversion of the data into volume fraction of cavities will follow shortly.

2.4.2 Industrial Computed Tomography

Industrial computed tomography offers considerable promise for characterizing damage in samples with macroscopic dimensions— as long as high spatial resolution (< 60 or 70µm) is not required. The compact tension samples crept and fatigued as part of this study are a good example of where use of industrial CT would be very advantageous. Large, long-range gradients in cavitation damage have been observed around the crack (see Sections 2.1 and 2.2) in sample CT2 (tested at $1.5 \text{ ksi(in.)}^{0.5}$ for 1250 hrs. and at $1.65 \text{ ksi(in.)}^{0.5}$ for times greater than 1250 hrs.). We have not yet reproduced the slices from our imaging terminal.
The adjacent slices surrounding the nominal crack plane for sample CT2 were obtained in collaboration with Dr. R. Isaacs of General Electric Corporation and had 62.5 μm pixels and 0.25 mm slice thicknesses. In these first experiments, the entire sample cross-sectional area (approximately 0.6 in. x 1.5 in.) was studied. Considerable beam hardening is evident which precludes quantitative analysis of the data until it is corrected. This correction is currently underway using a wedge-shaped reference sample from undamaged Cu-1% Sb.

The series of slices viewed on our terminal shows that the crack wandered from a single plane and that the crack extension was greater in the middle of the sample than at its edges. The crack tip structure is clearly evident even without correcting for beam hardening. Progress thus far indicates that prospects are excellent for quantitative measurements of the density variation ahead of creep cracks.

2.4.3. X-ray Tomographic Microscopy

A preliminary XTM scan was run of a small, rectangular prism of material taken from a sample which failed under creep loading at 400C. The XTM experiments were performed at SSRL in collaboration with Dr. J.H. Kinney's group at Lawrence Livermore National Laboratory. The creep test was run at 8 ksi, and one end of the centimeter-long prism was from the exposed fracture surface. Approximately 90 slices were recorded using 35 keV x-rays, and every ninth slice from the fracture surface is shown in Fig. 12.
The minimum dimension of this sample was 0.4 mm. Figure 13 shows enlargements of two of the slices. A few small cavities are visible in the interior of one of the slices, and their diameters of 10-20 μm are consistent with those measured from other volumes of the same sample by conventional techniques. Unfortunately, there were significant penetration problems at 35 keV, and higher energies could not available at SSRL. We intend to use the PEP ring at Stanford with higher x-ray energies to continue this investigation; we should be scheduled during the next PEP run. We have also approached CHESS (Cornell synchrotron) about obtaining time for these experiments. While the XTM sample clearly pushed the limits of what was then available, these results are very exciting: enough so that we will continue this aspect of the research even though the project has concluded.

3. LIST OF PAPERS PUBLISHED, CONFERENCE PRESENTATIONS AND STUDENTS SUPPORTED

In this Section, the published papers and the names of students and Research Associates who received support from the grant are listed.

3.1 Papers Published Acknowledging DOE Support


2. A. Saxena and B. Gieseke, "Transients in Elevated Temperature Crack Growth", in High Temperature Fracture


3.2 Manuscripts in Preparation


3.3 Conference Presentations


5. B. Gieseke and A. Saxena, "Mechanisms of Creep-Fatigue Crack Growth Interactions in 1wt % Sb-copper", TMS-AIME


3.4 Students and Research Associates Acknowledging DOE Support

Students


2. Brian Gieseke, Ph.D March 1990, Metals and Ceramics Division, Oak Ridge National Laboratory Oak Ridge, TN.


4. Abbas Guvenilir, Ph.D (expected December 1991)

Research Associates

1. Dr. K. Banerji (Jan 1987-Dec 1988) currently employed with Motorola Co., W. Palm Beach, Fla.
2. Dr. Y.H. Lee, Visiting Scientist from Korea Dec 86- Dec 87.

3. Dr. C.P. Leung (Jan 1989 - June 1989) currently employed at Southwest Research Institute, San Antonio, Texas.

4. REFERENCES


Fig. 4 (Top) Unconstrained cavitation at the tip of growing creep crack leading to extensive crack branching; (Bottom) the lack of correlation between $\frac{da}{dt}$ versus $C_t$ when data from specimens which exhibited this behavior were included in the correlation (Staley and Saxena, 1989).
Fig. 2 (Top) Constrained cavitation damage at the tip of a growing creep crack in 1 wt percent antimony-copper; (bottom) the creep crack growth rate as a function of $C_t$ in specimens which exhibited constrained cavitation (Staley and Saxena, 1989).
Fig. 3 - Fraction of grain boundary area cavitated as a function of distance from the crack tip for specimens CT-2 and CT-5.
Fig. 4 - Creep crack growth rate of 1 wt percent Sb - Copper in ultra-high purity N₂.
Fig. 5 - Volume fraction $V_v$ of cavities as a function of distance from the rupture surface of samples of Cu - 1 wt. % Sb tested at 400 C. The measurements were made with optical microscopy on a polished section of each sample.
Fig. 6 - a. Fraction of grain boundary area $A_c$ and b. Average cavity diameter $D$ as functions of fraction of elapsed lifetime $t/t_f$. The solid and open circles are for samples tested under 6.0 and 3.5 ksi loads, respectively.
Fig. 7 - The average $da/dt$ during hold time as a function of the $(C_t)_{avg}$ parameter and comparison with creep crack growth rate for a one percent antimony-alloy at 400°C (Gieseke and Saxena, 1989).
Figure 8. Volume fraction of cavities as a function of distance from the fracture surface of specimens deformed in creep.
Figure 9. Area fraction $A_c$ and average cavity diameter $D$ for samples tested at 3.5 ksi (open circles) and 6 ksi (closed circles).
Figure 10. Densitometry trace showing gradient of damage in synchrotron microradiographs of sample 3C.
Figure 11. Two-dimensional distribution of transmitted intensity around a creep crack. The vertical axis is transmitted intensity.
Figure 13. Slices recorded with synchrotron XTM parallel to the rupture surface of a Cu-1 wt. % Sb sample. The sample failed under creep, and the irregularities of the slices shown the jagged rupture surface. The small, dark images, indicated by arrows, appear to be grain boundary cavities.
APPENDIX

Copies of papers published on the Grant.
Creep Crack Growth under Transient Conditions*

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Abstract

The recent advances in time-dependent fracture mechanics concepts for characterizing creep crack growth rate under transient conditions are reviewed. At elevated temperatures, transients in the crack tip stress fields occur owing to stress relaxation during constrained creep due to primary and tertiary creep deformation, cyclic loading and crack growth. A unified crack tip parameter $C_*$, which is formulated to account for all types of transient is described and evaluated. The currently available creep crack growth data support the validity of $C_*$. Under steady state conditions, $C_*$ reduces to the familiar $C^*$ integral.

1. Introduction

The field of time-dependent fracture mechanics for characterizing crack growth at elevated temperatures has progressed considerably in the recent years. It is now generally accepted that the $C^*$ integral is a good correlating parameter for creep crack growth behavior under steady state conditions in creeping materials [1-7]. Steady state conditions exist in a cracked body when widespread creep deformation characterized by power law creep occurs. Under these conditions, $C^*$ becomes path independent, it equals the stress power (or energy rate) release rate and it also represents the strength of the crack tip stress singularity [1, 3, 4, 7].

Despite the strong mechanics basis, the application of $C^*$ is limited because of the restriction of steady state creep. Most elevated temperature components are designed to resist widespread creep deformation. As a result, the creep strains are expected to be in the primary creep regime for a significant portion of the life. Because of the stress and temperature gradients and periodic start-up and shut-downs in thick section components, there is also a good likelihood of the presence of the small-scale creep conditions during most of the service life. The condition of small-scale creep has been analyzed by several researchers [7-13]. As will be discussed later in the paper, the presence of crack growth also promotes transient stress conditions near the crack tip [12, 13] and so does the occurrence of tertiary creep [14]. Thus, in several practical situations, the consideration of transient stress field is important and, for these applications, $C^*$ is no longer a valid crack tip parameter.

This paper focuses on a crack tip parameter $C_*$ [8] which is general enough to correlate creep crack growth behavior under all types of transient conditions and it reduces to $C^*$ when steady state conditions prevail. Provided that the correct material constitutive laws are used, $C_*$ can be calculated by finite element analysis for components [15, 16]. It can be accurately measured at the loading pins of a specimen [8]. Also, closed-form solutions for estimating $C_*$ are currently available for materials which deform by elastic power law creep [8, 9, 14]. The use of $C_*$ under cyclic loading conditions will also be briefly discussed [17].

2. Formulation of the $C_*$ parameter

The initial formulation of the $C_*$ parameter is made with the assumption that the crack is stationary. Following Saxena [8], let us consider several identical pairs of cracked specimens. For each pair, one specimen has a crack length $a$ and the other has a crack length of $a + \Delta a$. The specimens of each pair are loaded to various constant load levels $P_1, P_2, ..., P_n$ at elevated temperatures. As creep deformation progresses in the various specimens, additional deflections $\delta_i$ accumulate at the load lines owing to growth of the creep zones, as shown in Fig. 1(a). This deflection is
attributed to primary and secondary creep in the crack tip region and also to change in elastic strain due to stress relaxation within the body. Under small-scale creep it has been shown [8, 15] that the load-line deflection rate \( \dot{V}_c \) is directly proportional to the rate \( r_c \) of expansion of the creep zone size.

In Fig. 1(b) the instantaneous value of \( \dot{V}_c \) is plotted at time \( t \) as a function of load \( P \) for specimens with crack length \( a \) and for specimens with crack length \( a + \Delta a \). For other times, similar plots of \( P \) vs. \( \dot{V}_c \) can be constructed. The area between the curves corresponding to crack lengths \( a \) and \( a + \Delta a \) is designated \( \Delta U^* \). The subscript \( t \) denotes the value for a fixed time. \( C_t \) is then defined as

\[
C_t = \lim_{\Delta u \to 0} \left( \frac{1}{B} \frac{\Delta U^*}{\Delta a} \right) = -\frac{1}{B} \frac{\Delta U^*}{\Delta a} \tag{1}
\]

where \( B \) is the specimen thickness. Under extensive creep conditions where power law creep dominates, \( C = C^* \) by definition because \( \Delta U^* \) assumes a steady state value \( \Delta U^* \). Under extensive creep conditions where primary creep effects are still significant, it follows from Reidel's [14] definition of \( C_0^* \) that \( C = C_0^*/[1 + p]. \)

From the relationship between \( \dot{V}_c \) and \( C_0^* \), the following expression can be derived which is convenient for measuring \( C_t \) at the loading pins [8]:

\[
C(t) = \frac{K^2}{E(n+1)t} + C^* \tag{2}
\]

Equation (2) is valid for very short to long times [15, 16, 20]. It is important to point out that under small-scale creep conditions \((C)_{SSC} \neq C(t)\). In contrast, \((C)_{SSC}\) is related uniquely to the rate of expansion of the creep zone size by the following equation [8, 15]:

\[
(C)_{SSC} = 2(1 - \nu) \beta \left( \frac{F'}{F} \right) \frac{K^2 t_c}{E W} \tag{3}
\]

where the \( K \) calibration factor \( F = (K/P)BW \) and \( F' = dF/d(a/W) \). In eqns. (3) and (4), no assumptions have been made about the constitutive behavior of the material. Hence these equations hold for any constitutive law.* A most general expression for \( C_t \) which is valid for primary, secondary and tertiary creep as well as for extensive conditions ranging from small-scale creep to extensive creep is given by the following equation:

\[
(C)_{SSC} = \frac{PV_c F'}{B W F} \tag{4}
\]

where the \( K \) calibration factor \( F = (K/P)BW^{1/2} \) and \( F' = dF/d(a/W) \). In eqns. (3) and (4), no assumptions have been made about the constitutive behavior of the material. Hence these equations hold for any constitutive law. A most general expression for \( C_t \) which is valid for primary, secondary and tertiary creep as well as for extensive conditions ranging from small-scale creep to extensive creep is given by the following equation:

\[
C_t = \lim_{\Delta u \to 0} \left( \frac{1}{B} \frac{\Delta U^*}{\Delta a} \right) = -\frac{1}{B} \frac{\Delta U^*}{\Delta a} \tag{1}
\]
\[ C_i = (C_{i,ssc} + C_i^* (1 + p) t^{1 - p}) \]  

If we make the assumption of elastic power law creep, an analytical expression for estimating \( C_i \) is derived and given in earlier papers [8, 15].

Figure 2 shows the creep crack growth behavior of Cr-Mo and Cr-Mo-V steels [8, 21], demonstrating the excellent correlation between creep crack growth rates \( \frac{da}{dt} \) and \( C_i \), obtained over a wide range of growth rates. In these correlations, specimens tested at different load levels are involved and, to a limited extent, specimens of different geometries are also included. In Fig. 3 the average \( \frac{da}{dt} \) during the hold period of a fatigue cycle with trapezoidal waveform is also correlated with \( C_i \). The hold times in this study [17] ranged from 50 s to 24 h and the data appear to correlate with \( C_i \). However, in comparison with the \( \frac{da}{dt} \) values under static loading, the crack growth rates during the hold time are much lower. This may be due to the influence of reversed plasticity during unloading in the previous fatigue cycle. For details of these correlations, readers are referred to earlier papers [17].

In these correlations, the presence of cyclic loading promotes transient conditions at the crack tip; hence, the correlation between \( \frac{da}{dt} \) and \( C_i \) under these conditions is important in establishing \( C_i \) as a unifying crack tip parameter which can account for different types of transient.

3. Considerations due to crack growth

Under extensive creep conditions, the region of influence of crack growth on the crack tip stress fields is small in comparison with the zone of HRR field dominance [22]. In contrast, under small-scale creep conditions the presence of crack growth will significantly alter the crack tip stress fields [12, 13]. The extent of this effect depends on the crack growth rate and the rate at which creep is spreading in the component or specimen. Thus, it is important to discuss the influence of crack growth and its implications on the validity of \( C_i \) under small-scale creep conditions.

The consideration of crack growth can be divided into two regimes. The first regime is one of slow crack growth in which the crack tip stress fields are perturbed owing to elastic unloading associated with crack extension near the crack tip. Hence, \( r_c \) for a growing crack is different from the \( r_c \) estimated for a stationary crack. We define slow crack growth as a condition under which \( r_c > 0 \); so the crack tip always extends beyond the creep zone boundary. Under such conditions, \( C_i \) obtained from eqn. (3) with \( r_c \) correctly calculated is expected to represent the crack driving force. Also, if \( C_i \) is estimated from a measured deflection rate substituted in eqn. (4), the effects on the transient stress field associated with crack growth are included in the estimate of \( C_i \).

The other condition is that of a high crack growth rate under which \( r_c < 0 \). This condition
will only occur if another cracking mechanism such as one dominated by environment or stable crack growth due to ductile tearing is involved. Under such conditions, the characterizing crack tip parameter is expected to change gradually from $C_* \text{ to } J$ (or $K$) [23–25]. In between, there can be a regime where neither $C_*$ nor $J$ will characterize the crack growth rate. However, this regime is expected to occur only at very high crack growth rates ($\frac{da}{dt} > 0.1 \text{ mm h}^{-1}$) and perhaps does not contribute significantly to life.

A very special situation arises when $r_c = 0$. This condition implies that the creep zone size remains constant. The mechanism of crack growth will still be creep with the crack tip and the creep zone boundary moving at equal rates. This will lead to a time-independent stress field with respect to a moving crack tip and is therefore a case of steady state. $\frac{da}{dt}$ under such circumstances will correlate with $J$ (or $K$). The creep-brittled materials such as ceramics or embrittled ferritic steels may exhibit such trends. In general, $r_c < 0$ should lead to a $K$-controlled crack growth rate and $r_c > 0$ should lead to a $C_*$-controlled crack growth rate. In an experiment, the condition of a $C_*$- or $K$-controlled crack growth rate can be identified through the use of deflection rate partitioning analysis [24, 25]. However, much work remains to be done in the area of how transition from $C_*$-controlled to $J$-controlled (or $K$-controlled) crack growth occurs in creeping materials after ductile tearing begins.

4. Summary and conclusions

Significant progress has occurred in the development of concepts for characterizing crack growth behavior at elevated temperatures where time-dependent creep deformation occurs. The current state of these developments is summarized as follows:

1. The $C_*$ integral is widely accepted as being the correct crack tip parameter for characterizing creep crack growth behavior under steady state conditions.

2. Under transient crack tip stress conditions, $C_*$ appears to be promising for characterizing the creep crack growth behavior. There are no inherent limitations in the use of $C_*$ for transients resulting from stress relaxation, primary and tertiary creep deformation, cyclic loading and crack growth.

3. Expressions for measuring $C_*$ for various specimen geometries are currently available. The validity of these expressions includes situations involving all types of transient deformation. Much work remains to be done in developing accurate expressions to estimating $C_*$ in components in the presence of primary creep deformation and crack growth. Analytical expressions for calculating $C_*$ in components which deform by elastic power law creep are currently available.

Acknowledgments

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References


Transients in Elevated Temperature Crack Growth


ABSTRACT Transients which affect the crack growth behaviour of metals at elevated temperature under cyclic and static loading are classified as: (1) deformation transients which are present due to the relaxing crack tip stress fields during small scale creep, and (2) damage or crack growth transients which are present during the period in which a steady-state damage state evolves at the crack tip starting from an initially undamaged state.

It is shown using extensive creep and creep-fatigue crack growth data on 1Cr-1Mo-0.25V steel that deformation transients under static as well as cyclic loading are completely normalized by the $C_r$ parameter. It is also shown that the other crack tip parameters such as $K$ and $C(t)$ are unable to normalize the influence of deformation transients. However, $C_r$ fails to uniquely characterize the crack growth rate during the damage or crack growth transient.

Introduction and background

Recently, considerable progress has been made in the understanding of micro-mechanics of damage development in the vicinity of crack tips subjected to creep deformation (1)-(5). Also, experimental studies have been conducted to evaluate crack tip parameters for characterizing creep crack growth behaviour under steady-state and under transient conditions (6)-(7). The objective of this paper is to evaluate the suitability of the different crack tip parameters for characterizing the crack growth behaviour under two primary types of transient conditions. These include: (1) deformation transients due to the relaxing stress fields under small-scale creep (SSC) resulting in an expanding crack tip creep zone, and (2) crack growth transients associated with the initial portions of the test during which a steady-state damage pattern develops.

Considerable crack growth data are assembled from previous experimental studies to document the presence of the above transients and to also identify some possible approaches to predict crack growth behaviour in their presence. The nature of the various transients is first discussed in detail. We will be restricting our primary discussion to sharp, Mode I (Fig. 1) cracks in materials which deform elastically and by power-law creep. The uniaxial version of the material constitutive law is as follows

$$\varepsilon = \frac{\sigma}{E} + A\sigma^n$$

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where, $\dot{\varepsilon}$ is strain rate, $E =$ elastic modulus, $\sigma =$ applied stress, and $A$ and $n$ are the power-law creep constants.

**Deformation transients**

The plane strain Mode I stationary crack subject to a constant load has been investigated by Ohji *et al.* [8], by Riedel and Rice [9], and by Bassani and McClintock [10]. The instantaneous response is elastic and the crack tip stress field ($r \rightarrow 0$) is completely specified by the stress intensity parameter, $K$. As time elapses, a time-dependent crack tip stress field which resembles the HRR fields in elastic–plastic fracture mechanics [11][12] evolves. The crack tip stress is then given by the following equation

$$\sigma_{ij} = \left( \frac{C(t)}{Al_{n}r} \right)^{1/(n+1)} \tilde{\sigma}_{ij}(\theta, n)$$  \hspace{1cm} (2)

where, $\tilde{\sigma}_{ij}(\theta, n)$ and $I_{n}$ are the HRR field quantities listed in tabular form in reference [13]. $C(t)$ is an amplitude factor which is given by the following equation (14)

$$C(t) = \frac{1 - v^{2}}{(n + 1)Et} + C^{*}$$  \hspace{1cm} (3)

where, $v =$ Poisson's ratio, $t =$ elapsed time, and $C^{*}$ is a path-independent integral defined in several papers [15]. $C^{*}$ also represents the steady-state value of $C(t)$ when power-law creep dominates the entire specimen (or component). At short times, $C(t)$ is dominated by the first term on the right hand side of
equation (3) and small-scale-creep (SSC) conditions prevail. Figure 2 schematically shows the three regimes, namely, the SSC, the transition creep (TC) regime, and the steady-state (SS) regime of extensive creep. The creep zone size \( r_c(\theta, t) \) under SSC conditions is given by (9) equation (4)

\[
    r_c(\theta, t) = \frac{1}{2\pi} K^2 \left\{ \frac{(E\alpha t)}{2\pi(1 - \nu)} \right\}^{2/(n - 1)} \bar{r}_c(\theta)
\]

where \( \bar{r}_c(\theta) \) is an angular function and is described in reference (9).

Under SSC and TC conditions, the crack tip creep zone size as well has the crack tip stress field are continuously changing with time giving rise to a transient condition, labelled deformation transient. If slow crack growth is occurring such that the rate of crack growth, \( \dot{a} \), is much smaller than the rate of expansion of the creep zone size, \( \dot{r}_c \), it can be argued that the crack tip driving force decreases with time for a constant, \( K \). It is then interesting to
determine if \( a \) correlates uniquely with any crack tip parameter under such transient conditions. A candidate crack tip driving force parameter can be \( C(t) \) which is a measure of the instantaneous amplitude of the crack tip stress field. Further, the parameter \( C(t) \) is a local crack tip parameter which cannot be measured at the loading pins in a specimen; its value can be calculated from equation (3) only.

Saxena (7) has proposed an alternate crack tip parameter, \( C_s \), which is based on the stress-power release rate and can be measured at the loading pins. This parameter converges to \( C^* \) by definition when steady-state conditions prevail much like \( C(t) \). \( C_s \) is defined as follows

\[
C_s = \frac{-1}{B} \frac{\partial U^*}{\partial a}
\]

where, \( U^* \) is the instantaneous stress-power release rate and \( B = \text{thickness of the body.} \) The following expression can be used to estimate \( C_s \) from the measured load, \( P \), and the load-line deflection rate due to creep, \( \dot{V}_c \).

\[
C_s = \frac{P\dot{V}_c}{BW} F'/F - C^* \left( \frac{F'/F}{\eta} - 1 \right)
\]

where, \( W = \text{width of the body,} \ F \text{ is the } K \text{-calibration factor given by} F = (K/P) BW^{1/2}, F' = dF/d(a/W), \text{and } \eta \text{ is a geometry and crack size dependent parameter used in the calculation of } C^* \text{(16)}

\[
C^* = \frac{P\dot{V}_s}{BW} \eta
\]

where, \( \dot{V}_s = \text{steady-state deflection rate.} \) For details of \( C_s \) estimation, the readers are referred to earlier papers (7)(17). \( C_s \) has recently shown (17)(18) to be uniquely related to the rate of expansion of the creep zone under SSC conditions by the following relationship

\[
(C_s)_{\text{exp}} = 2(1 - v^2) \frac{K^2}{E \pi} F'/F \cdot \beta \dot{V}_c
\]

where, \( \beta = \text{scaling factor which can be determined numerically.} \) Recent finite element calculations (17)(18) for compact type and centre crack tension geometries have confirmed the relationship between \( (C_s)_{\text{exp}} \) and \( \dot{V}_c \) beyond reasonable doubt. This relationship makes \( C_s \) a unique parameter which can be both measured at the loading pins and also be related to the rate of expansion of the crack tip creep zone size.

Combining equations (4) and (8), an analytical expression for estimating \( C_s \) can be derived for elastic, power-law creep as follows

\[
C_s = \frac{4\pi \beta \dot{V}_s (\beta)}{E(n - 1)} (1 - v^2) K^* F' \left( \frac{F'}{W} \right) (E \pi)^{2/(n - 1)} - 2^{(n - 3)(n - 1)} + C^*
\]

where, \( n = \text{strain hardening exponen...} \)
In summary, there are two candidate crack tip parameters, $C(t)$ and $C$, for correlating creep crack growth behaviour in the presence of deformation transients. $C(t)$ is based on the instantaneous amplitude of the crack tip stress field and $C$, relates to the rate of expansion of the creep zone. Both parameters collapse to $C^*$ when steady-state conditions are reached. $C$, can be measured at the loading pins while $C(t)$ can only be calculated. In a later section, both parameters will be evaluated using the same set of creep crack growth data.

**Damage or crack growth transients**

Several analytical studies have modelled creep crack growth as a process governed by nucleation, growth, and coalescence of grain boundary cavities (1)(2)(4). These studies have focused mostly in the steady-state creep regime where crack growth rate has been shown to be governed by $C^*$. The experimental evidence supports such a mechanism for slow crack growth (5). In this discussion, we will first focus on steady-state conditions with regard to deformation transients by assuming that the extensive creep conditions dominated by $C^*$ prevail. Under these conditions it is intuitively appealing to think that a unique relationship between $a$ and $C^*$ can only be achieved if the state of creep damage in the form of cavitation within the process zone ahead of the crack tip is a characteristic of the applied $C^*$ level. In other words, at any given time, the size, spacing, and the number of cavities ahead of the crack tip are only a function of $C^*$, and as the crack tip advances, the state of damage moves with it in a self-similar manner. However, it takes a certain amount of crack extension in order to achieve a steady-state level of crack tip damage, as explained below.

When the crack tip is first subjected to creep deformation at the beginning of the test, the cavity closest to the crack tip grows from its nucleation size to the coalescence size. This situation is compared with one following a substantial amount of crack extension in which the cavities grow significantly past their nucleation size before reaching the position of the cavity nearest to the crack tip. Hence, the time needed to grow this cavity to the coalescence size is expected to be smaller than the time required to grow the former. Thus, during the transient period of crack extension, a steady-state relationship between $a$ and $C^*$ is not expected. The existing experimental data tend to show this behaviour which is schematically illustrated in Fig. 3. In this figure, it is shown that tests begun at different $C^*$ values have a non-unique $a$ vs $C^*$ relationship until the crack extends a certain distance. After some crack extension has occurred, the results from the three tests fall on the same trend. Actual data supporting such trends is shown later in the paper.

**Transients during cyclic loading**

Both deformation and crack growth transients can exist during cyclic loading at elevated temperature. For simplicity, a trapezoidal loading waveform with a
fast loading/unloading segment and a hold time in between will be considered. This is equivalent to a situation in which a static load is periodically interrupted by an unloading event. The unloading and reloading is accomplished in a time much smaller than the time between successive interruptions, also called the hold time, $t_h$. Also, $t_h$ is much smaller than the transition time, $t_T$, from SSC to SS conditions given by the following relationship:

$$t_T = \frac{K^2}{E(n + 1)C^*}$$  \hspace{1cm} (10)

Thus, we will be restricting the discussion here to SSC conditions and also to situations where no creep deformation is occurring during the loading/unloading portions of the cycle.

Figure 4(a) schematically shows the various deformation zones in the vicinity of the crack tip, and Fig. 4(b) shows the expected stress–strain behaviour within the various deformation zones. If both small-scale-yielding (SSY) as well as small-scale-creep (SSC) are assumed, the stress–strain behaviour in Region 1 is linear-elastic. In Region 2 (the monotonic plastic zone), monotonic plasticity occurs during the first loading cycle, but no reversed plastic deformation occurs. In Region 3 (cyclic plastic zone), reversed plasticity occurs but no substantial creep deformation during the hold period occurs. In Region 4 (creep zone), substantial creep deformation occurs during the hold time, which is reversed during the unloading portion of the cycle via plastic deformation. The extents of creep, cyclic, and monotonic plastic zones depend on the applied $K$ level, the rate of creep deformation, and the hold time. It is by no means implied in Fig. 4 that the creep zone is always contained in the cyclic zone, which in turn is contained in the monotonic zone. Region 4 can be larger than Region 2 for sufficiently large hold times. However, for short hold times
the creep zone will be contained within the cyclic plastic zone, which will always be within the monotonic plastic zone. During the period in which the creep zone (Region 4) grows through the cyclic plastic zone (Region 3), a creep–fatigue interaction can be expected to occur. This will give rise to crack growth as well as deformation transients, because the fatigue loading will influence the creep deformation rates as well as the rate of damage accumulation.

Analysis of crack growth data

All data analyzed in this paper was obtained on a single heat of 1Cr–1Mo–0.25V steel manufactured in accordance with the ASTM Specification for Vacuum-Treated Carbon and Alloy Steel Forgings for Turbine Rotors and Shafts (A 470, Class 8). The tensile and steady state creep properties have been extensively characterized at 427, 482, and 538°C in previous papers (7)–
The pertinent creep constants, equation (1), were taken as \(5.18 \times 10^{-31} \text{ (MPa)}^{-n} \text{ hr}^{-1}, \quad n = 10.5\), with Young's modulus taken as \(1.62 \times 10^5 \text{ MPa}\). For additional data, the reader is referred to the earlier papers.

**Analysis of creep crack growth test data**

Creep crack growth tests were conducted at 538°C on eight, standard 25.4 mm (1 in) thick compact-type (1T-CT) specimens subjected to dead weight loading in lever-type creep machines. The load-line deflection was recorded periodically during all the tests. In two of the tests, the crack length was monitored using the electric potential drop method (20). The crack growth rates during these tests were calculated using the secant method. In other tests, the cracks were grown only a short distance and the crack extension was measured directly from the fracture surfaces. The crack growth rates, \(da/dt\) were calculated by dividing the crack extension by the test time. The \(da/dt\) data were correlated with crack tip parameters, \(K, C^*, C(t),\) and \(C_0\). The results of these correlations will be discussed in the next section. In the following discussion, the procedure used for calculating the crack tip parameters are described.

\(K\) was calculated using the standard expression given by Srawiev (21).

For calculating \(C^*\) using equation (7), a steady-state deflection rate, \(\dot{\varphi}_{ss}\), is required but it cannot be measured unless the test itself is under steady-state conditions. Since none of our tests were entirely under steady-state conditions, an expression was used to calculate \(C^*\) in which the steady-state deflection rate was analytically estimated. That expression is given as follows for CT specimens (6)(22)

\[
C^* = \frac{AW}{(1 - a/W)^n} h_1 \left( \frac{P}{1.455 a_1 B} \right)^{n+1}
\]  

(9)

In this equation, \(A\) and \(n\) represent the steady-state creep constants. \(W\) = the specimen width, \(B\) = the specimen thickness, and \(P\) = the applied load. The factor \(a_1\) is given by the equation

\[
a_1 = \left[ \left(\frac{2a/(W-a)}{2} + 4a/(W-a) + 2 \right)^{1/2} - 2a/(W-a) + 1 \right]\)

(10)

and \(h_1 (a/W, n)\) is a dimensionless calibration function given in (22).

The crack tip parameter \(C(t)\) was calculated using equation (3). To obtain an average value of \(C(t)\) during a time interval, equation (3) was integrated with respect to time and divided by the time interval. The values of \(K\) and \(C^*\) were considered as constant within the small crack length interval and were calculated at the average crack length for the interval. Thus, the average \(C(t)\) during the interval \(t_i \leq t \leq t_{i+1}\) is given by the following equation

\[
C(t) = \frac{1}{t_{i+1} - t_i} \frac{K^2}{E(n+1)} \ln \left( \frac{t_{i+1}}{t_i} \right) + C^*
\]

(11)

In cases where \(t_i\) was zero it was assumed to be 1 second to avoid the singularity.
The average value of $C_i$ over an interval $t_i \leq t \leq t_{i+1}$ was calculated using equation (6). $F'/F$, $C^*$, and $\eta$ were calculated for the average $a/W$ over the interval. The deflection rates were calculated as follows:

$$V_i = \frac{(V_{i+1} - V_i)/(t_{i+1} - t_i)}{(12)}$$

The deflection rate due to creep was estimated by using the expression (23)(24)

$$\dot{V}_c = \dot{v} - \frac{\Delta B}{P}(2K^2/E)$$

\[13\]

\[14\]

Analysis of fatigue crack growth data with hold times

Fatigue crack growth rate behaviour of the Cr–Mo–V steel was characterized at 538°C for loading waveforms which included hold times of 0 s, 50 s, 0.5 h, and 24 h (25)(26).

The tests with 0 and 50 second hold times were conducted on 1T-CT specimens in servo-hydraulic test machines. The crack length was monitored by the electric potential drop method. The 0.5 and 24 hour hold time tests were conducted on creep machines in which the tests were periodically cycled by unloading followed by reloading. The unloading and reloading times during all the hold time testing was on the order of 0.5 seconds. The 0.5 hour hold time test was conducted on a 1T-CT specimen. The twenty-four hour hold time tests were conducted on multiple-edge-crack (MEC) specimens. The MEC specimens contained five shallow edge cracks of different sizes. These specimens were cycled constant load amplitude for periods of six months to a year and then the specimens were broken open at the various crack locations to determine the crack extension. Thus, five crack growth rate data points were obtained from each test. The overall cyclic crack growth rate $da/dN$, was correlated with $\Delta K$ in earlier studies (25)(26). The time rate of crack growth during the hold time were correlated with the average value of the $C_i$ parameter during the hold time as a part of this study.

The above cyclic tests were conducted in a dominantly small-scale creep regime. The longest hold time of 24 hours was much smaller than the smallest transition time $t_T$ (equation (10)) which was at least 585 hours. Hence, the transient term in equation (9) will dominate the magnitude of $C_i$ and the $C^*$ term can be neglected. Thus, the average $C_i$ is given by the following equation

$$C_{i_{avg}} = \frac{1}{t_{h}} \int_{t_h}^{t_{h}} C_i \, dt$$

(15)

In order to estimate $C_i$, it was necessary to determine the values of $F'/F$ for the MEC specimens. Since the cracks were small, the $F'/F$ values for single-edge-notch (SEN) specimens were used. Finite element analysis was performed on the specimens (27) to assure that virtually no interaction occurred between the various cracks. Thus, the use of SEN $K$-calibration expressions is justified.
The average crack growth rate during the hold time was calculated as follows:

\[ \frac{da}{dt}_{avg} = \frac{\left( \frac{da}{dN} - \frac{da}{dN_0} \right)}{t_n} \times \frac{1}{n} \]

where \( \frac{da}{dt}_{avg} \) is the average \( \frac{da}{dt} \) value during the hold time and \( \frac{da}{dN_0} \) is the crack growth rate per cycle for zero hold time. Therefore, the fatigue contribution to the overall cyclic crack growth rate was subtracted in order to estimate the average \( \frac{da}{dt} \) during the hold time.

**Results and discussion**

In this section, the results of the crack growth rate correlations with the various crack tip parameters are presented and subsequently discussed in light of the three types of transients described in Section 1. First, the creep crack growth rate data are discussed and then the results of the fatigue with hold time tests will be discussed.

**Creep crack growth data**

Correlation between \( \frac{da}{dt} \) and \( K \)

The correlation between creep crack growth rate, \( \frac{da}{dt} \), and \( K \) is presented in Fig. 5. In general, \( \frac{da}{dt} \) increases with \( K \) for a given specimen, but the correlation does not appear to be unique for two reasons. The first is the possible violation of the small-scale-creep condition in the test specimens. This can be examined further by comparing the test time with the transition time, \( t_T \). Table 1 lists some typical test times along with the transition times based on the average crack length during the various tests. From these comparisons, it appears that significant creep deformation could have occurred during the tests and may have caused the lack of correlation between \( \frac{da}{dt} \) and \( K \). However, the above transition time calculation is valid only for a stationary crack, and when moving crack considerations are taken into account, the extent of creep deformation can be significantly less in the test specimen than predicted by equation (10). The second reason for lack of correlation between \( \frac{da}{dt} \) and \( K \) is due to the non-uniqueness of the relationship between \( K \) and

<table>
<thead>
<tr>
<th>Test time (hrs)</th>
<th>( t_T ) (hrs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2035</td>
<td>64.298</td>
</tr>
<tr>
<td>760</td>
<td>499</td>
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<tr>
<td>220</td>
<td>373</td>
</tr>
</tbody>
</table>
TRANSIENTS IN ELEVATED TEMPERATURE CRACK GROWTH

Fig. 5. Correlation between creep crack growth rate and the stress intensity parameter, $K$

the crack tip stress, even under SSC conditions. This is evident upon examination of equations (2) and (3). Under SSC conditions, the crack tip stress field characterized by $C(t)$ is clearly a function of $K$ and time, $t$, and not just $K$. The creep zone size and its rate of expansion are similarly dependent on $K$ and time under SSC conditions, equation (4). However, equations (2)–(4) are for stationary cracks and the conditions for moving cracks are different. A general and complete crack tip stress field solution for a growing crack under creep conditions is not available. Some early work of Hui and Riedel (28), recent work of Hawk and Bassani (29), and by Hui (30) may be used to make qualitative arguments on when a unique relationship between $da/dt$ and $K$ can be expected. This is further explained below.

We consider a moving Cartesian coordinate system attached to the crack tip with its $x$ axis located along the crack plane and perpendicular to the crack front. Under SSC conditions, $K$ controlled crack growth can occur only if $\partial\sigma(x, t)/\partial t = 0$, where, $\sigma(x, t)$ represents the crack tip stress. This is possible only if the creep zone is non-existent or its size remains constant with time. In other words, $r_e = 0$. The numerical results of Hawk and Bassani (29) in Mode III (Fig. 6) show that such a condition is theoretically possible (at least in an approximate way). In Fig. 6, the creep zone size is normalized by a reference
creep zone size, $r_c$, and is plotted as a function of time normalized by a reference time $t_R$. The creep zone size for a stationary crack is also shown for comparison. Up to a $t/t_R$ value of 2.0, $r_c$ is non-zero, therefore steady-state relationship between $da/dt$ and $K$ is not expected. For $t/t_R > 2.0$, the creep zone size appears to be approaching a constant size and a unique $da/dt$ vs $K$ relationship may be obtained. The amount of crack extension prior to attainment of steady-state conditions is dependent on $da/dt$ and the creep properties. It is expected that $K$-dominated conditions will be promoted for extremely creep brittle materials and at high $da/dt$ values. More experimental work is needed in order to determine a quantitative criterion for $K$-controlled creep crack growth.

Another result which can be derived from Fig. 5 is that the creep zone size ahead of a moving crack is smaller than predicted from stationary crack considerations. Therefore, for growing cracks, the conditions of SSC prevail for longer periods than predicted by transition time, $t_T$, equation (10).

Correlation between $da/dt$ and $C^*$

Figure 7 shows that the correlation between $da/dt$ and $C^*$ is not good. Since a significant portion of the various tests were in the SSC and TC regions where $C^*$ is not valid as a crack tip parameter, this result is not surprising.

Correlation between $da/dt$ and $C(t)$

The correlation between $da/dt$ and $C(t)$ is shown in Fig. 8. A unique correlation appears to be lacking. The crack tip parameter $C(t)$ accounts for the stress relaxation at the crack tip due to creep in the SSC and the TC regions.
and characterizes the amplitude of the HRR fields for stationary cracks. However, when growing crack effects become significant over the creep zone, \( C(t) \) loses its significance because, as shown by the finite element analysis of Hawk and Bassani (29), HRR type stress fields are annihilated. The new singularity which develops at the crack tip is not of HRR type (28). This explains the lack of correlation between \( \frac{da}{dt} \) and \( C(t) \).

**Correlation between \( \frac{da}{dt} \) and \( C_t \)**

Figure 9 shows the same creep crack growth data which was plotted in Figs 5, 7, and 8 but correlated with the \( C_t \) parameter. There is an excellent correlation between \( \frac{da}{dt} \) and \( C_t \). The two slashed points represent data from the very early portion of the test. The reason for why they do not fall in the general trend will be discussed later.

The \( C_t \) parameter is uniquely related to the rate of expansion of the creep zone size, equation (8), and the stress power release rate, equation (5) in the SSC regime. Neither of these definitions of \( C_t \) are affected by crack growth. In estimating \( C_t \), the creep zone expansion rate should be determined from an appropriate analysis. When deflection rates are measured and used to estimate
In the correlation between da/dt and C, shown in Fig. 9, there are data which were obtained under a variety of conditions, ranging from SSC to extensive creep or SS conditions. The following parameter, \( \tau \), determines the extent to which transient conditions exist in the specimen. \( \tau \) is defined by

\[
\tau = \frac{C_*}{C_i}
\]

For SSC conditions, \( \tau \ll 1 \). For extensive creep conditions \( C_i = C_* \) therefore, \( \tau = 1 \). The values of \( \tau \) ranged from \( 10^{-2} \) to essentially 1 for the various points included in Fig. 9. It is clear that the data span a wide variety of conditions ranging from SSC to SS. Recently, several other experimental studies have shown that creep crack growth rate over a wide range of conditions correlate
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Fig. 9. Correlation between creep crack growth rate and the \( C_t \) parameter: slashed points represent data from the very early portion of the test.

extremely well with \( C_t \) (32). Hence, it can be stated that \( C_t \) is able to account for the influence of deformation transients adequately.

The two slashed points from the very early portions of the test were in the region where crack growth transients operate. A rough estimate of this region is 0.25 mm. The analytical calculations show that the crack should advance by fifteen intercavity distances before steady-state crack growth rates can be established (4). If intercavity distance is on the order of 10 \( \mu \)m, the amount of crack extension required prior to establishment of steady-state is 0.15 mm. This compares favourably with the experimental findings.

**Creep–fatigue crack growth data**

Figure 10 shows the average time rate of crack growth (equation 15) during the hold time for hold times of 50 s, 30 minutes and 24 h. The data for each hold time are plotted with a different symbol and are correlated with the average \( C_t \) parameter estimated from equation (14). For comparison, the creep crack growth data are plotted in the same figure for comparison. There appears to be an excellent correlation between the creep–fatigue crack growth...
Fig. 10. Creep-fatigue crack growth rates as correlated with the $C_i$ parameter

rates for various hold times when correlated with $C_i$. However, there is a significant difference between the crack growth rates under creep conditions and those under creep-fatigue conditions. This difference can be attributed to the crack growth transients introduced by unloading. Correlating the data with $C_i$ does not account for these transients. On the other hand, the deformation transients from the expanding creep zone appear to be completely normalized when the crack growth rates are correlated with the $C_i$ parameter.

The excellent correlation between $da/dt$ and $C_i$ during fatigue with hold time has several interesting implications. By inspecting equations (18) and (9) it is evident that, even for a given loading waveform and cycle time (constant $t_0$), the average $C_i$ is not uniquely determined by $K$ because of the additional crack size and geometry dependent term $F'/F$. However, in the past, fatigue crack growth data at elevated temperature have been routinely correlated with $\Delta K$ for constant loading waveforms and cycle times (25)(26)(33). These correlations have been obtained with a simple specimen geometry, mostly CT specimens for which $F'/F$ does not vary significantly over a wide range of crack length interval. Also, if the cycle times are small, the contribution of the time-dependent crack growth is small and the overall crack growth per cycle ($da/dN$) can be dominated by the cycle-dependent portion which correlates with $K$. Thus, it may be that the correlation between $da/dN$ and $\Delta K$ observed in the past have been fortuitous. To the knowledge of the authors, there is no
elevated temperature crack growth data on the same material from two very different specimen geometries for long cycle times.

On the other hand, if \( \frac{da}{dt} \) were correlated with \( C(t) \) for creep–fatigue crack growth, unique relationship between \( \frac{da}{dN} \) and \( \Delta K \) for a constant cycle time and loading waveform is in fact implied. Figure 11 shows the same data correlated with the average \( C(t) \) parameter, according to equation (9) with \( t_{i+1} \) taken as \( t_h \) and \( t_i \) taken as 1 s. The lack of correlation between the data for various hold times is clearly evident. Therefore \( C(t) \) is unable to normalize the influence of deformation transients.

A significant difference between the \( \frac{da}{dt} \) values for creep and creep–fatigue crack growth exists for hold times of 24 h. If the hold time is increased further, this difference should gradually vanish. From these data it appears that the time needed for creep–fatigue interactions to vanish is much larger than 24 h at the load levels used during testing.

In correlating \( da/dt \) with \( C \), for creep–fatigue crack growth tests, the analytical expression, equation (9), was used to estimate \( C \). For reducing the creep crack growth data the experimentally measured \( V_c \) values were utilized. A possibility exists that some difference between creep crack growth and creep–fatigue crack growth maybe due to differences in expression for estimating \( C \).
Specifically, the value of $\beta r_f(\theta)$ was chosen to be one because for CT specimens this value has been shown to give estimates of $C$, (32) which agreed with experiments on 1.25Cr–0.5Mo steels. On the other hand, the finite element calculations yield a value of 0.13 for $\beta r_f(\theta)$ and the difference was thought to be due to the presence of primary creep. The extent of primary creep may be different for 1Cr–1Mo–0.25V steel used in this study as compared to the other steel. Therefore, the value of $\beta r_f(\theta)$ of one is not completely justified for this material and there is some uncertainty in the calculation of $C$, for the creep-fatigue tests. However, the uncertainty factor is a constant; therefore, it does not influence the conclusions regarding the general trends. It is also unlikely that the discrepancy, if any, in the estimation of $C$, is two orders of magnitude which is what it will take to collapse the creep-fatigue and creep crack growth into a single trend. Therefore, we feel that creep-fatigue interaction effects are strong. There is a need for developing models which will predict the extent of creep-fatigue interaction effects for making accurate life time predictions of elevated temperature components.

Summary and conclusions

In this paper, an effort was made to correlate creep crack growth rate data developed under conditions ranging from small scale to extensive creep on a 1Cr–1Mo–0.25V turbine rotor steel with several crack tip parameters. The data clearly show a lack of correlation with $K$, $C^*$, and the $C(t)$ parameters, but good correlation was obtained when the data were plotted with $C_i$. The creep-fatigue crack growth rates during hold times ranging from 50 s to 24 h and under small scale creep were also shown to correlate with $C_i$ for the same material. The crack growth rate vs $C_i$ relationship is significantly different for creep as compared with creep-fatigue conditions.

It was also shown that the transient crack growth characterized by non-unique $da/dt$ vs $C$, behaviour is observed early during creep crack growth tests. In reporting creep crack growth behaviour, it is recommended that the data developed over the first 0.25–0.5 mm of crack extension should be discarded to avoid the influence of such transients on the creep crack growth behaviour reported. Since such transient behaviour is important for component life times, it should be studied in greater depth.

Acknowledgements

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INTRODUCTION

Critical gas and steam turbine, power-plant boiler, and petrochemical reactor components that operate at elevated temperature tend to develop cracks during the early stages of service life. Some components contain crack-like defects even as they enter into service. These defects can grow and cause failures during service. Some examples of major failures in the power industry involving elevated temperature components where creep was a major contributing factor are described in Ref 1 to 3. These failures resulted in millions of lost dollars in downtime and repair costs and, in some cases, also loss of human lives. Thus, crack growth under creep conditions is a major industrial problem.

Further impetus for studying creep crack growth comes from the need to assess the remaining life of components that have been in service and are approaching their originally predicted design life. More and more equipment operators such as the Air Force, Navy, utility companies, and petrochemical companies are turning to a retirement-for-cause (RFC) philosophy rather than rely on life predictions made several years ago that were based on concepts that are now outdated.

Failures due to creep can be classified either as resulting from widespread bulk damage, or resulting from localized damage. Structural components are vulnerable to bulk damage are subjected to uniform loading and uniform temperature distribution during service, for example, thin-walled pressure vessels. The life of such a component can be estimated from creep-rupture data. On the other hand, components that are subjected to stress and temperature gradients (typical of thick section components) will not fail by creep rupture. It is more likely that, at the end of the predicted creep-rupture life, a crack develops at a critical location, which propagates and ultimately causes failure. Figure 1 shows cracks in the interior of a steam header that had been in service for 24 years. The cracks emanating from holes in the interior surface of the Mississippi Power and Light header used as test material in this study. (Courtesy of Babcock and Wilcox Co.)

In this paper, the concepts of time-dependent fracture mechanics (TDFM) for characterizing creep crack growth behavior are reviewed. We start by describing the stresses in front of crack tips in creeping solids. This includes a discussion of the crack tip parameters for characterizing creep crack growth. The microscopic aspects of creep crack growth and the metallurgical variables affecting the creep crack growth behavior are discussed in the subsequent sections of this paper.
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difference between two identically loaded bodies having incrementally differing crack lengths:

$$C^{*} = - \frac{1}{B} \frac{dU^{*}}{d\alpha}$$  \hspace{1cm} (4)$$

where $U^{*}$ is the power or energy rate defined for a load $P$ and an associated load-line displacement rate, $V$, as shown in Fig. 3(b); $B$ is the thickness of the cracked body. The other property of $C^{*}$ is its ability to uniquely characterize the stress distribution at the crack tip. For a power law creep material, the crack tip stress is given by: \cite{11}

$$\sigma_{\theta} = \frac{C^{*}}{2A_{r}} \left[ \frac{1}{1 + n} \sigma_{\theta}(\theta, n) \right]$$  \hspace{1cm} (5)$$

where $\sigma_{\theta}(\theta, n)$ is an angular function and $I_{n}$ is a nondimensional constant with a value ranging between 3.8 and 6.3 for a range of $n$ values. The numerical values of both are listed elsewhere. \cite{12} Here, $r$ is the distance from the crack tip.

Figure 4 shows a plot of creep crack growth rate, $da/dt$, as a function of the $C^{*}$-integral in 304 stainless steel obtained from specimens tested at 594 °C (1100 °F). \cite{13} There is good agreement between data obtained from compact-tension (CT) and the center-crack tension (CCT) specimens. There is now considerable experimental evidence to show that under steady-state conditions $C^{*}$ is able to characterize the creep crack growth rates.

In addition to calculating the value of the contour integral (Eq 2) or measuring it using Eq 4, there are two other methods of determining $C^{*}$. The first of these methods is suitable for determining $C^{*}$ in test specimens where the load, the load line deflection rates, and the crack size measurements are available. A number of investigators \cite{14, 16} have used this
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A crack tip parameter for characterizing creep crack growth is defined by setting the value of \( C(t) \) from Eq 9 equal to \( C^3 \). In the SSC regime, the creep zone is defined as the region around the crack tip where creep strains exceed the elastic strains. The definition of \( C(t) \) is as follows:

\[
C(t) = C^3 \quad (15)
\]

For long times under constant applied load, when extensive creep occurs everywhere in the specimen, \( C \) approaches a constant, steady-state value \( C^3 \), which was discussed earlier. The time \( t_1 \) between small-scale creep (SSC) and extensive creep is approximated by setting the value of \( C(t) \) from Eq 9 equal to \( C^3 \):

\[
t_1 = (1 - r^2) K_{t} (n + 1) E^2 C^3
\]

The growth of the creep zone and, therefore, \( V_r \), is intimately connected with the elastic strain rates that arise due to crack tip stress relaxation, as well as the creep strain rates.

To define the partitioning of load-line displacements in small-scale creep (SSC), imagine a specimen with a crack of length \( a \), subjected to load \( P \). The total load-line deflection is expressed as:

\[
V = V_r + V_c (17)
\]

where \( V_r \) is the total deflection of an identical specimen that undergoes only the instantaneous elastic deformation, and \( V_c \) is the remaining deflection that accumulates with time for the actual specimen undergoing both elastic and creep deformation. The latter elastic deformation is caused by the redistribution of stresses at the crack tip. It is important to note that \( V_r \) and \( V_c \) are not compatible with the elastic and creep strains, respectively, of the specimen. In fact, because in SSC \( V_r \) is due to the growth of the creep zone, it is intimately connected with the crack tip stress relaxation and the associated elastic strain rates. It may be recalled that in the time-independent elastic-plastic case, Edmonds and Willis have demonstrated that this partitioning is asymptotically exact (see Hutchinson for an analogous discussion based on the Dugdale solution).

Note that for a stationary crack under constant load, \( V = V_r \).

\( C_t \) is defined as the instantaneous rate at which stress power is dissipated. Consider several identical pairs of cracked specimens. For each pair, one specimen has a crack length \( a \) and the other has a crack length \( a + \Delta a \). The specimens of each pair are loaded to various load levels \( P_1, P_2, P_3, \ldots, P_n \), etc., at elevated temperature. It is assumed that no crack extension occurs in any of the specimens and that the instantaneous response is linear-elastic.
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elds can be written in the HRR format as:

$$
\sigma_n(r, \theta, t) = \frac{C_n}{A_1(1 + ptL\ln r)^{1 + \frac{1}{1 + p}}} \sigma_n(\theta, n_1)
$$

(31)

where all terms have been defined previously. It should be noted that by the correspondence principle, the stationary stress states for time-hardening primary creep (first term in Eq 24) and secondary creep are the same.

When a steady-state creep zone is growing out of the primary creep zone such that both terms on the right side of Eq 29 are comparable in magnitude somewhere in the cracked body, it cannot be shown rigorously that the C'-integral in Eq 2 is path independent. This regime also corresponds to conditions of extensive creep. It is understood in discussion of primary creep that the C'-integral in Eq 2 becomes time dependent, whereas the more narrow definition of Ref 6 pertains to steady-state secondary creep conditions only. However, recent finite-element numerical calculations have shown that C' in this regime is in fact path independent to a good degree of approximation. It thus follows that C, is approximately equal to C' in this regime and that the HRR amplitude factor C(t) is also approximately equal to C'. Hence, C = C(t) in this regime of extensive creep conditions involving both primary and secondary creep behavior. We can thus make use of the recent interpolation formula derived by Riedel and Detampel for estimating C(t) in this regime to also estimate C:

$$
C(t) = C(t) = (\frac{t_1}{t_1})^{1 + \frac{1}{p-1}}
$$

(32)

where C' represents the steady-state value at time t → ∞ when secondary creep conditions dominate, and t1 is a transition time obtained by the following expression:

$$
t_1 = \left(\frac{C_n}{(1 + ptL)}\right)^{\frac{1}{1 + \frac{1}{1 + p}}} A_1
$$

(33)

C' can be obtained for various cracked geometries by substituting A(1 + ptL)^{-1 + p} for A and n, for n in Eq 9, which was originally derived for calculating C'. Thus, from the knowledge of the primary and steady-state creep constants, geometry, crack size, and applied stress (or load), C(t) can be completely calculated from handbook-type solutions.

Next, we look at inclusion of primary creep when small-scale creep conditions exist. For small-scale primary creep alone, the HRR amplitude factor, C, and the creep zone size have been estimated by Riedel. Furthermore, Riedel and Detampel have recently modified Eq 32 to include only primary creep in the small-scale creep regime (Eq 34):

$$
C(t) = [t_1/t + (t_2/t)^{1 + \frac{1}{1 + p}} + 1]C'
$$

(34)

where

$$
t_1 = \left(\frac{K(t_1 - t_2)}{E(1 + n_1 + pA_1)}\right)^{1 + \frac{1}{p}} \frac{1}{1 + n_1}
$$

(35)

The value of C' in Eq 34 corresponds to the steady-state value of the C'-integral. The above equations also assume that, in the presence of significant primary creep, the transition to extensive creep conditions is dominated by the primary creep strain, as opposed to steady state creep as in Eq 15. This assumption is realistic because the initial primary creep rates are significantly higher than the secondary creep rates. If primary creep is not relevant, p = 0, and the magnitude of t1 = t. Therefore, the expression for C(t) becomes consistent with the earlier expression for C(t) for elastic materials deforming by elastic power-law creep (Eq 15).

In the definition of C, for small-scale creep, there is a provision to include primary creep deformation. For example, Eq 22 is general and is valid for any type of creep deformation behavior. However, the expression for C, must include the correct deformation law. Assuming that when primary creep is present the transition from SSC to extensive creep will be dominated by the spread of the primary creep deformation, the following expression for $\dot{r}$ can be derived from Riedel's previous work:

$$
\dot{r} = \frac{K(t_1)\sigma_n(\theta)}{2\pi E(1 - \nu^2)} \left[2 + \frac{1}{n_1 - 1} \left(1 + \frac{n_1}{n_1 - 1}\right)\right]
$$

(36)

where all terms in Eq 36 have been defined before. Thus, substituting Eq 36 into Eq 22 gives the expression for C, for the presence of primary creep.

The wide range expression for C, in the presence of primary creep can also be derived as:

$$
C_1 = (C_1)_\infty + [t_2/t]^{1 + \frac{1}{1 + p}} + 1]C'
$$

(37)

where $(C_1)_\infty$ is derived as mentioned previously. Considerable numerical work has been performed recently to verify the validity of Eq 37. For test specimens under creep conditions ranging from small-scale to extensive
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correlate data. A brief discussion of these parameters is included below. Under extensive creep conditions dominated by steady-rate creep, the crack-tip opening displacement rate and \( C^* \) are related to each other

\[ \dot{\delta}_t \propto a_0 C^* = a_0 C_t \]  

(40)

Here \( a_0 \) is a constant on the order of the yield strength and the constant proportionality is on the order of one. Thus, for extensive creep conditions, and \( C^* \) (or \( C_t \)) can be considered as equivalent parameters.

Under small-scale creep conditions, we can show (see Appendix for derivation) that:

\[ (C_t)_{ss} = \left( \frac{2\pi}{\beta} \right)^{1/2} \left( \frac{F'F}{W} \right)^{1/2} K \frac{1}{\epsilon} \]  

(41)

and

\[ (C_t)_{ss} = \frac{n+1}{n+1} \frac{K}{\beta} \left( \frac{2\pi}{\beta} \right)^{1/2} \]  

(41a)

Whereas Eq 41 is valid for any general constitutive law, Eq 41(a) is only valid for elastic power-law creep behavior. From these equations, it is observed that neither \( (C_t)_{ss} \) and \( \dot{\delta}_t \), nor \( (C_t)_{ss} \) and \( \dot{\delta}_t \), are related to each other by simple constants. However, within the realm of SSC only, the relationships between \( C_t \) and \( \dot{\delta}_t \), and also \( C(t) \) and \( \dot{\delta}_t \), are single valued.

As we had argued previously when we examined the relationship between \( C_t \) and \( C(t) \): Eq 27, \( \dot{\delta}_t \), and \( C(t) \) cannot correlate with creep crack growth rate data over a wide range of creep conditions, because their interrelationships are uniquely if either SSC or extensive creep conditions are assumed. Also, if one of these parameters correlates with creep crack growth behavior, it necessarily implies that others cannot. A further difficulty with \( \dot{\delta}_t \) is that measurement is not practical. For this reason, there are not many experimental studies in which an attempt to correlate \( \dot{\delta}_t \) and \( \delta_{dd} \) has been made. The \( C_{C_{exp}} \) parameter of Webster et al.\(^{14,42} \) is determined as follows:\(^{14} \)

\[ C_{C_{exp}} = \frac{PV}{BW} \frac{\eta(u/W,n)}{\eta(u/W,n)} \]  

(42)

where \( \eta \) in Eq 42 is the same as in Eq 6. Thus, when extensive steady-state creep conditions prevail, \( \dot{V}_t = \dot{V}_m \) (see Eq 6 for the definition of \( \dot{V}_m \)), and it is easy to see that \( \dot{C}_t = C_{C_{exp}} \). Under extensive primary creep, the \( \eta \) function changes slightly because a different value of \( n \) applies instead of the one describing the steady-state creep response of the material. However, this dependence is small and is not of much consequence when treating experimental data. It thus follows that \( C_{C_{exp}} \) and \( C_t \) are approximately equal in the extensive creep regime.

In the small scale creep range, \( C_{C_{exp}} \) is not a viable parameter. Experimentally, it can still be determined by Eq 42 for test specimens. However, it must be remembered that \( \eta \) is derived from analogy to a limit-load type analysis for fully plastic structures and thus assumes that the specimen is under extensive creep.\(^{14} \) Therefore, the use of \( \eta \) derived from such analyses under small scale creep is not justified. An interesting point is noted when we compare Eq 42 for determining \( C_{C_{exp}} \) to Eq 38 for determining \( C_t \). We find that for geometries for which \( F'/F = \eta \), \( C_{C_{exp}} = C_t \).

In a previous study,\(^{16} \) the author has compared the values of \( \eta \) and \( F'/F \) for the CT and CCT specimen geometries over a wide range of \( n \) and \( a/W \) values. These results are reproduced in Table 1. For CT specimens, \( \eta \) is within 15% of \( F'/F \) over a wide range of \( a/W \) and \( n \) values. Thus, for these specimens, \( C_{C_{exp}} = C_t \). Much of the success of \( C_{C_{exp}} \) in correlating creep crack growth data may be due to the fact that a large amount of the data are obtained on CT specimen geometry for which it is an approximation for \( C_t \). It may also be mentioned that the \( C_t \) parameter of Jaske\(^{44} \) and an old version of \( C_t \)\(^{38} \) are identical to \( C_{C_{exp}} \).
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experimentally shown to correlate with CCGR behavior over a wide range of creep conditions, it implies that a unique relationship between CCGR and the other two parameters cannot exist for the same conditions.

In this section, creep crack growth data in a recently concluded ASTM standard on a Cr-Mo-V steel at 594 °C (1100 °F) are analyzed using different crack tip parameters. (Fig. 8 to 11). These data were developed using impact-type specimens that were 50.8 mm (2 in.) wide. The steady-state creep deformation behavior in this material is shown in Fig. 12. No further consideration of $\delta$ is possible, because no systematic studies where it has been measured are available.

Figures 8 to 11 show plots of the creep crack growth rate, $\frac{da}{dt}$, as a function of $K_{eff}$, which is a plasticity-modified stress-intensity parameter, $C'$, and $C_0$, respectively. Specimens 19 through 22 were 6.25 mm (0.25 in.) thick and specimens 11 and 12 were nominally 25.4 mm (1 in.) thick with 25% side groove.

As expected from the previous discussion, little correlation exists between the data from different specimens when plotted with $K_{eff}$. Figures 9 shows similar lack of correlation with $C'$, indicating that transient conditions prevailed during these tests. $C'$ was calculated using Eq 9. A parameter $\tau$, defined to determine whether the test is in the transient or steady-state regime:

$$\tau = \frac{C'}{C_0} \quad (43)$$

When steady-state conditions are reached, $C_0 = C'$ and $\tau$ approaches 1. When highly transient conditions exist, $\tau < 1$. The values of $\tau$ at the beginning and end of each test are listed in Table 2. Also listed are the transition time $t_r$ and the test duration, $t_d$. In several of the tests, the value of $\tau$ is considerably less than 1, thus confirming that the specimens were under transient conditions at least for some time during each test.

A similar lack of correlation is also observed when $\frac{da}{dt}$ values are plotted with $C(t)$ as determined from Eq 15. However, when the same data are plotted with $C_0$, they correlate very well with each other. Some distinction can be made between data from the 25.4 mm-thick side-grooved specimens and the 6.25 mm-thick non-side-grooved specimens. This may be attributed to the state-of-stress effects. The side-grooved specimens are expected to be under plane-strain conditions, whereas the others are closer to plane-stress conditions. Riedel and Detampl52 have also made a similar evaluation of $C'$, $C(t)$, and $C_0$ on a 2.25Cr-Mo material tested at 540 °C and have reached the conclusion that better correlations are obtained when creep crack growth rates are plotted with $C_0$.

Table 2. Values of $\tau$ and Transition Time During Various Creep Crack Growth Tests

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$a_0 / W$</th>
<th>$\tau$</th>
<th>Test duration, h</th>
<th>$t_r$, h</th>
<th>$C_0 / t_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CT 19</td>
<td>0.557</td>
<td>0.300</td>
<td>26.3</td>
<td>26.6</td>
<td>10.1</td>
</tr>
<tr>
<td>CT 20</td>
<td>0.561</td>
<td>0.125</td>
<td>16.2</td>
<td>108.3</td>
<td>6.7</td>
</tr>
<tr>
<td>CT 21</td>
<td>0.590</td>
<td>0.044</td>
<td>7.69</td>
<td>15.3</td>
<td>14.1</td>
</tr>
<tr>
<td>CT 22</td>
<td>0.516</td>
<td>0.00646</td>
<td>1.34</td>
<td>139.6</td>
<td>12.2</td>
</tr>
<tr>
<td>SG 11</td>
<td>0.500</td>
<td>0.0136</td>
<td>1.40</td>
<td>53.6</td>
<td>60.5</td>
</tr>
<tr>
<td>SG 12</td>
<td>0.500</td>
<td>0.296</td>
<td>1.40</td>
<td>323.3</td>
<td>13.9</td>
</tr>
</tbody>
</table>

The data shown in Fig. 11 and from other studies clearly show that $C_0$ should be the crack tip parameter of choice for correlating creep crack growth rate data. However, it was surprising that the correlation between $\frac{da}{dt}$ and $C(t)$ is not better than the correlation with $C'$. From examining the ratios $\frac{a}{t}$, given in Table 2, it is expected that all tests should lie in the extensive creep regime, and the data should correlate just as well with $C'$ as with $C_0$ and $C_0$, because all three are the same in this regime. This point merits further discussion.

The measured values of $C_0$ are significantly higher than the values of $C'$, indicating that transient conditions still existed at times significantly larger than $t_r$. It may be argued that the $t_r$ estimate of transition time in Eq 14 is for stationary cracks, and it may not be an accurate representation of the transition time for growing cracks. However, these differences are too large for such a simplistic explanation. At this point, there is no rigorous explanation for this behavior. However, Bassani46 has recently given an explanation that has considerable appeal and is discussed below.

In the remaining ligament of CT specimens, there is a point (neutral axis) where the stress fields change from tension to compression. The region along this point is subjected to low stresses. As crack advances in a creep crack growth test and the remaining ligament shrinks, the neutral axis also shifts. Thus, a previously undeformed region now undergoes creep deformation, which possibly increases the transition period. Such an effect will
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3. CREEP CRACK GROWTH TRENDS IN ELEVATED TEMPERATURE MATERIALS

In this section, the creep crack growth behavior of austenitic and ferritic steels used in elevated temperature applications is considered. Creep cracks in these materials grow by grain boundary cavitation, and the environmental effects are only secondary. In that sense, these materials are quite distinct.
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Figure 14 shows the influence of temperature on the CCGR behavior of 1Cr-1Mo-0.25V steel. Again, the behavior at 482 °C (900 °F) and 538 °C (1000 °F) is very comparable, even though the creep deformation rates at these temperatures for the material are very different (Fig. 12). The lack of strong material composition and temperature dependence of CCGR behavior requires further explanation.

Consider two identical cracked specimens of the same material loaded with identical loads, except that one is subjected to a temperature $T_1$ and the other to a temperature $T_2 > T_1$. To simplify matters, assume that both specimens are under steady-state conditions and that the creep crack growth rate correlates with $C'$. To estimate $C'$, either Eq 6 or 9 can be used. Because the applied load (or stress) is the same in the two specimens and $n$ does not change significantly with temperature, the value of $C'$ from Eq 9 depends strongly on the value of $A$ or on the value of $V$, if Eq 9 is used. In either case, a higher value of $C'$ is predicted for the test at temperature $T_2$ compared to $T_1$. Thus, even if the $\frac{da}{dt}$ versus $C'$ relationships at both temperatures were the same, the crack growth rate in the higher temperature specimen is expected to be much faster.

From the above argument, it appears that correlating creep crack growth rate with $C'$ (or $C_1$) provides a first-order normalization with respect to changes in temperature. The same argument can be applied to rationalize the similarity in the $\frac{da}{dt}$ versus $C$ relationship for materials with different compositions. However, these trends are expected to hold true only when there are no substantial changes in rupture ductilities due to changes in temperature, composition, or thermal history. When significant changes in creep ductilities occur, the $\frac{da}{dt}$ versus $C$ relationship in fact changes, as discussed below.

### 3.2 Influence of In-Service Aging

Figures 15 and 16 show the CCGR and the secondary-stage creep deformation rate behavior, respectively, of a 1.25Cr-0.5Mo steel from a component in service for 24 years. The region marked “hot” was subjected to a temperature of 538 °C (1000 °F) and the region marked “cold” was subjected to a temperature of approximately 510 °C (950 °F). Both the CCGR and deformation rates in the hot region were significantly higher than the cold region. The corresponding microstructures of the regions at low and high magnification are shown in Fig. 17 and 18. The grain size and the microstructure at low magnification appear to be virtually indistinguishable between the two regions. There was also no evidence of creep cavitation at the grain boundaries in either material. However, at the high magnification, the difference in the grain boundary carbide morphology becomes evident. This was further confirmed by transmission electron...
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Fig. 18 (a) High-magnification micrograph of the cold end materials (Nital etch).

a). South (Cold)

Fig. 18 (b) High-magnification micrograph of the hot end materials (Nital etch).

b). North (Hot)
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temperature and higher levels of impurity (S,P,As) content results in the worst CCGR behavior. These rates are as much as 50 times higher than the worst case behavior in the HAZ/FL data of Fig. 21.

The influence of impurities on the creep crack initiation and growth behavior in 2.25Cr-1Mo steel was also investigated by Lewandowski et al. Their data also show higher crack growth rates in materials with higher levels of impurities. Among the Konosu and Maeda and Lewandowski et al. studies, there is controversy over the role of sulfur in causing the embrittlement. The former investigators did not find any segregation of sulfur on the fracture surface of the material that had a high amount of sulfur, whereas the latter investigators report considerable amounts of sulfur segregation. Despite the disagreement, both studies are quite revealing on the role of impurities in determining creep crack growth behavior. This appears to be an interesting area for further investigation.

4. MICROSCOPIC ASPECTS OF CREEP CRACK GROWTH

Figure 23 shows the cavitation damage ahead of a 1.25Cr-0.5Mo creep crack growth rate specimen. The tested specimen was sectioned at mid-thickness and the area directly ahead of the creep crack growth was...
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In explicit, only a one-dimensional array of cavities was considered, and the singular dependence of the stress field (which is a factor on the order of one) is dropped, because only qualitative trends are presently being considered. The crack is supposed to advance by a distance 2b when the cavity nearest the crack tip grows to a critical size, \( p_c \). If this occurs over a time interval \( \Delta t \), the crack growth rate is:

\[
\frac{da}{dt} = \frac{2b}{\Delta t} \tag{45}
\]

During the time interval \( \Delta t \), the cavity with radius \( \rho_2 \) grows to a radius \( \rho_1 \), and a new cavity of radius \( \rho_N \) nucleates at a distance \( 2b \) from the crack. In this manner, a steady-state crack growth process can be established, which leads to the following set of integral equations:

\[
\Delta t = \int_{\rho_0}^{\rho_1} \frac{dp}{\dot{\rho}} \quad m = 0, 1, 2, \ldots N - 1 \tag{46}
\]

where \( \rho_0 = \rho_c \). The solution of these integral equations yields:

\[
\frac{da}{dt} = \frac{(2b)^{n+1}}{\psi_0 - \psi_N} \left( \frac{1}{N} \sum_{m=1}^{N} \frac{C'}{I_{m}A} \right)^{\frac{1}{n+1}} \tag{47}
\]

where

\[
\psi = \int \frac{dp}{\phi(\rho,b)} \tag{48}
\]

For constrained cavity growth, the function \( \phi \) can be derived from previous work by Rice\(^{49} \) and subsequently used by Jani and Saxena:\(^{49} \)

\[
\psi = \frac{1}{d} \left( \frac{2b}{\rho} \right)^{3} - A \tag{49}
\]

where \( d \) is the grain diameter, and \( A \) is the power law creep coefficient. Therefore:

\[
\psi = \int \frac{dp}{\phi(\rho,b)} = \frac{2.5}{3(2b)^2} \left( \rho_c - \rho_N \right) \tag{50}
\]

and

\[
\psi_0 - \psi_N = \frac{2.5}{3(2b)^2} \left( \rho_c - \rho_N \right) \tag{51}
\]

Substituting Eq 18 into Eq 14 yields:

\[
\frac{da}{dt} = \frac{(2b)^{n+1}}{2b} \cdot \frac{A}{3d} \cdot \frac{C'}{I_{m}A} \left( \frac{C'}{I_{n}A} \right)^{\frac{1}{n+1}} \tag{52}
\]

The following interpretations can be drawn from the model presented in Eq 52:

- The crack growth rate is not very sensitive to \( A \) for a constant value of \( n \). For this reason, the crack growth rates are expected to be only weakly dependent on temperature, which has been observed for Cr-Mo steels.\(^{44} \)

- Creep crack growth rate will be sensitive to the intercavity spacing \( 2b \), the critical cavity size for coalescence \( p_c \), and the cavity nucleation size \( p_N \). If cavities nucleate at grain boundary carbides, then the carbide size can be considered to be \( p_N \). Therefore, a coarsened grain boundary carbide structure will result in higher crack growth rates.

Although the model does qualitatively predict the trend observed in this study, which is encouraging, it is important to note its limitations. The model assumes an idealized cavitation behavior on a grain boundary along the plane of the crack. It also assumes that cavity growth occurs under steady-state stress fields characterized by the HRR singularity, which limits its application to steady-state conditions. Also, the critical cavity radius for coalescence is a constant value, which may be simplification. In reality, \( p_c \) must be stress dependent over a wide range of crack growth rates, and such dependence must be incorporated into the model. Also, under the more realistic transient creep conditions (SSC and TC), the stress fields ahead of the crack tip are relaxing with time, and the model must be modified to account for that and the stress fields must be modified to account for damage. The constant cavity spacing precludes simultaneous nucleation and growth of cavities. This is a major shortcoming of the model.

For the Cr-Mo material used in the study mentioned earlier,\(^{48} \) it was observed that the largest cavities ahead of a crack tip were on the order of 70 \( \mu \)m, whereas the average carbide size \( p_N \) was on the order of a few microns. If the largest cavities can be taken as \( p_c \), then the term \( \rho_c - \rho_N \) in the model is relatively insensitive to small changes in \( p_N \), and the crack growth rate will be controlled by \( p_c \). The possibility of embrittling species such as tin, antimony, sulfur, and phosphorus affecting the critical radius for cavity coalescence becomes relevant and needs to be addressed in the future.
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MECHANISMS OF CREEP CRACK GROWTH IN 1 wt% ANTIMONY–COPPER: IMPLICATIONS FOR FRACTURE PARAMETERS

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Abstract—A copper alloy with 1% (by weight) antimony was used as a model material and tested at 400°C to study the mechanisms of creep crack growth. At this temperature, the creep deformation in this material was dominated by secondary and tertiary creep. The expression for estimating $C_t$ (a crack tip parameter for creep conditions) in a compact type specimen used for testing was modified to include tertiary creep deformation. Extensive damage characterization was conducted on tested creep crack growth specimens using optical metallography and scanning electron microscopy. The following observations were derived from the test results. The creep crack growth rates correlated with $C_t$ only when intense cavitation damage was restricted to a region approximately 1.3 mm in size near the crack tip and no crack branching occurred. It was observed that the average diameter, areal density, and percent of grain boundary area cavitated decreased as function of distance from the crack tip. From these results it is argued that simultaneous nucleation and growth of cavities occur on grain boundary facets during the creep crack growth process. Percent grain boundary area cavitated is proposed as the most reasonable measure of creep damage. The critical amount of damage for crack extension appears to depend on the magnitude of the $C_t$ parameter.

Résumé—

Zusammenfassung—

BACKGROUND

The development and use of materials for elevated temperature applications is of critical importance for many industries. Within the power generation and jet engine industry, there is a need for optimum utilization of elevated temperature components which is being accomplished by the implementation of retirement-for-cause criteria [1–4]. Components are no longer routinely retired at the end of their originally estimated design life. Thus, there is a need for developing accurate methods for predicting the
remaining service life of these components for establishing appropriate inspection intervals and criteria.

At high temperatures, a prominent damage mechanism for metals and alloys which can ultimately cause rupture is intergranular cavitation which occurs in response to creep deformation [5]. However, several high temperature structures consist of thick sections which are designed to resist widespread creep deformation. These components do not completely fail by which are designed to resist widespread creep deformation. These components are spent in crack propagation. Cracks in components can often be introduced during fabrication such as in welded structures. In these components, the entire life of the component can be spent in crack propagation.

The phenomenon of crack extension due to creep deformation and damage in the crack tip region in response to sustained (or slowly varying) load is known as creep crack growth. Creep cracks often grow by nucleation, growth and coalescence of grain boundary cavities [6]. However, much more work is needed for developing a better understanding of the mechanisms of creep crack growth.

In this paper, the mechanisms of creep crack growth at 400°C (~0.5 Tm) in a Cu–1 wt% Sb alloy are investigated. Unlike most structural alloys, the simple copper system contains minimal secondary phases and artifacts and is known to cavitate on grain boundaries when subjected to creep deformation [7, 8]. By adding 1 wt% Sb to the Cu, it is possible to reveal cavitated grain boundaries by intergranular fracture at room temperature. Recent creep crack growth results on the same material by Nix et al. [8] showed that the size of the grain boundary cavities ahead of the crack tip was dependent on the distance from the crack tip. Therefore, this material was considered ideal for studying evolution of creep damage in specimens subjected to different load histories.

The objectives of this study were to (i) quantify grain boundary cavitation damage in this model material as function of the magnitude of the crack tip parameter, C, [9], (ii) to compare crack tip damage in specimens subjected to the same levels of C, but undergoing different levels of creep deformation via a vs small-scale vs extensive creep and (iii) characterize the influence of the level of creep damage (constrained vs unconstrained) on the validity of the crack tip field parameters which are deformation based and do not explicitly account for cavitation damage. This work is a part of an ongoing study which in the near future will also consider crack tip damage under creep-fatigue conditions as affected by loading variables such as cycle time and waveform.

**EXPERIMENTAL**

**Material preparation and characterization**

The Cu–1 wt% Sb alloy used in this study was produced by Oak Ridge National Laboratory (ORNL). 99.99% pure OFHC copper and 99.999% pure antimony were induction melted in a ZrO, crucible at a vacuum of 10⁻⁵ torr and cast in the form of 126 mm (5 in.) diameter ingot. The ingot was homogenized at 950°C for 5 h and extruded into a rectangular bar of 38.1 x 75 mm (1.5" x 3") cross section at 700°C. Considerable antimony segregation to the grain boundary occurred during the extrusion process itself. This was confirmed by examining fracture surfaces of small charpy type specimens taken from the as-extruded material which showed 100% intergranular fracture at room temperature. Fig. 1. A piece of the extruded material was also given a further segregation treatment for 24 h at 675°C. The Auger analysis of this material compared well with that of the extruded material [10] implying that the antimony segregation was nearly complete in the as-extruded material itself. This also provided assurance that no further antimony segregation will occur during the creep tests which were conducted at 400°C.

The material had an average grain diameter of 57 μm (Fig. 2) and was uniform on samples taken from all faces of the rectangular bar. Based on these results the as-extruded material was used for our studies. All bars were ultrasonically tested for defects. The defective material near the end of the bars was discarded to avoid taking any samples from those areas.

The chemical composition of the material is given in Table 1. Table 2 lists the short term tensile properties of the test material at 400°C which was the test temperature for all our subsequent work. The following equation describes the plastic strain (εp) vs applied stress, σ, behavior

\[
ε_p = 5.39 \times 10^{-4} (σ/σ_{YS})^{1.84}
\]

where, σ_{YS} = 0.2% yield strength given in Table 2.

**Creep deformation tests**

Cylindrical specimens of 12.6 mm (0.505") diameter were tested at 400°C in air under constant load using SATEC lever type, dead weight creep machines.
Testing standards and procedures were in accordance with American Society for Testing and Materials (ASTM) specification E139-79 [11]. Four specimens were subjected to stress levels ranging from 24.1 to 55.1 MPa (3.5 to 8ksi). Continuous measurement of strain in the gage length was made to characterize the full creep curve. Figure 3 shows typical creep curves from specimens tested at various stress levels. The creep curves exhibited mostly secondary and tertiary creep deformation. The primary creep region seemed to be absent in this material. The data were fit to the following equation

\[ \varepsilon = A_0 \sigma^n + A_1 \sigma^m (e - A_0 \sigma^n)^p \]  

(2)

where, \( A_0, n, A_1, m, \) and \( p \) are regression constants derived from the creep data. The values of these constants are listed in Table 3 for 400°C. Figure 3 shows the comparison between the measured creep rates and those predicted from equation (2) at various stress levels. Equation (2) appears to describe the creep deformation accurately. The steady-state creep rate as a function of stress is compared with other data in the literature [7, 12] for Cu and antimony-copper. Fig. 4. Creep rates generally appear to decrease with increase in the grain size of the material.

Creep crack growth tests

Creep crack growth testing was conducted at 400°C in air under constant load also using lever type, dead weight creep machines. Five standard compact type (CT) specimens [13] which were 50.8 mm (2 in.) wide and nominally 15.24 mm (0.6 in.) thick were fatigue-precracked at room temperature prior to testing. A MTS servo-hydraulic test system under load control conditions was used. Subsequently, these specimens were mounted on a creep machine. The load-line deflection as a function of time was measured using a LVDT system and the crack size was continuously monitored using an electric potential system. The details of the test method are described elsewhere [14, 15]. Each test was terminated prior to fracture to allow crack tip metallography to be performed for characterizing damage in the tested specimens.

Damage characterization in crack growth specimens

All creep crack growth specimens were sectioned through mid-thickness and the crack tip region of one-half of the specimen was mounted for metallography. Thus, a complete two-dimensional view of the creep cavitation surrounding the crack tip could be observed. The second half of the compact type specimens CT-2 and CT-5 were fractured to reveal the creep crack fracture surface and also the cavitated grain boundary facets ahead of the crack tip. These fracture surfaces were examined at a high magnification in a scanning electron microscope (SEM).

The SEM micrographs obtained at various distances (0.2, 0.6, 1.3, 5.5 and 8.3 mm) from the crack tip for the two specimens were analyzed for cavity sizes and distribution using quantitative metallography procedures [16] briefly described below. The magnifications of the pictures analyzed were 330 to 400 x.

A system of grids with a known total length was placed on the pictures. The total length sampled in a single field of view (FOV) was 5.091 mm. In all eight fields of view were analyzed at each of the various distances from the crack tip. The total numbers of cavities which intersected the grid lines were counted and a linear density of cavities, \( N_l \), per mm of length was determined. The total number of cavities in all fields of view were determined and divided by the area to determine the areal density of cavities, \( N_A \). The average diameter is then computed from the following relationship [16]

\[ D_{avg} = 2 \left( \frac{2N_c}{\pi N_A} \right) \]  

(3)

The fraction of projected grain boundary cavitated, \( A_c \), is then approximately obtained by [16]

\[ A_c = \frac{\pi}{4} N_A D_{avg}^2 \]  

(4)

The \( D_{avg}, N_A \) and \( A_c \) were related to the distance ahead of the crack tip.

---

Table 1. Chemical composition of the test material (wt%)

<table>
<thead>
<tr>
<th>Element</th>
<th>Ag</th>
<th>Si</th>
<th>S</th>
<th>Sb</th>
<th>Cr</th>
<th>Cu</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.01</td>
<td>0.06</td>
<td>0.01</td>
<td>0.98</td>
<td>0.02</td>
<td>Balance</td>
</tr>
</tbody>
</table>

---

Table 2. Tensile properties of the test material (400°C)

<table>
<thead>
<tr>
<th>Property</th>
<th>( E ) (GPa)</th>
<th>( \sigma_{YS} (0.2%) ) (MPa)</th>
<th>( \sigma_{UTS} ) (MPa)</th>
<th>25.4 mm gage length</th>
<th>( D_{avg} ) (MPa)</th>
<th>( m^{abc} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>103.4 ( \times 10^4 )</td>
<td>64.1</td>
<td>102.7</td>
<td>7</td>
<td>2 ( \times 10^{-7} )</td>
<td>4.76</td>
</tr>
</tbody>
</table>

\[ n_s = D_l (\sigma/\sigma_{YS}) \]
Fitted Vs. Experimental Data

THE C, PARAMETER

The definition of \( C_i \) is as follows [9]. Consider several identical pairs of cracked specimens. Within each pair, one specimen has a crack length, \( a \), and the other has an incrementally differing crack length, \( a + \Delta a \). The specimens of each pair are loaded to various load levels \( P_1, P_2, P_3, \ldots \), etc., at elevated temperatures, and the load–line deflection as a function of time is recorded. Fig. 5(a). The load–line deflection due to creep is \( V_c \) and its time rate is given by \( \dot{V}_c \). It is assumed that no crack extension occurs in any of the specimens, and the instantaneous response is linear-elastic. Consider the behavior at a fixed time, \( t \), in the transient region where \( \dot{V}_c \) is continuously changing due to the presence of small-scale creep conditions or due to transient creep resulting from primary and/or tertiary creep strains. The load vs creep deflection rate behavior is plotted for all specimens. A schematic of the expected behavior is shown in Fig. 5(b). Several such plots can be generated from the above tests by varying time.

The area between the \( P - V_c \) curves for specimens of crack lengths, \( a \) and \( a + \Delta a \), is called \( \Delta U_c^* \). Physically, \( \Delta U_c^* \) (the subscript denotes that this value is at a fixed time \( t \)) represents the difference in the energy rates (or power) supplied for deforming the two crack bodies with identical creep deformation histories as they are loaded to different load or deflection-rate levels. The \( C_i \) parameter is given by the following equation

\[
C_i = \lim_{\Delta a \to 0} \left( \frac{\Delta U_c^*}{B \Delta a} \right) = \left( \frac{\partial U_c^*}{\partial \Delta a} \right)
\]

where \( B = \) specimen thickness. Under small-scale-creep (SSC) conditions, \( C_i \) uniquely characterizes the rate of expansion of the creep zone size [18, 19] irrespective of the type of creep deformation, i.e., primary, secondary, or tertiary creep. Under extensive creep conditions, \( C_i \) uniquely characterizes the HRR crack tip stress fields if either primary or secondary or tertiary creep dominate the deformation behavior.

If a combination of more than one of the three creep deformation stages exist under extensive creep conditions, \( C_i \) still approximately characterizes the HRR field [17]. Under the special condition of extensive secondary-stage creep deformation, \( C_i \) by definition is equal to the \( C^* \)-integral of Landes and Begley [22].

A general expression for estimating \( C_i \) in test specimens has been derived [2, 9, 17]

\[
C_i = \frac{PF}{BW \dot{V}_c} = C^*(t) \left[ \frac{F'P'}{F} \right] \left[ \frac{F''}{F'} \right] \left[ \frac{P''}{P'} \right]
\]

where

- \( F = K \)-calibration factor given by \( F = (K/P)BW^{1/2} \)
- \( P = \) applied load
- \( W = \) width of a specimen
- \( K = \) applied stress intensity parameter at the time of the loading
where, $C_\infty$ can be obtained from a simple modification of equation (8) which involves substituting $[A_3(1-\rho_2)]^{1/\eta}$ for $A$ and $n_3(1-\rho_2)$ for $n$ [23].

$V_\nu$ in equation (6) can be estimated from the measured deflection rate, $V$, by the following equation [26]

$$V_\nu = V - \frac{\Delta B}{P} \left( \frac{2K^3}{E} \right)$$

where, $E$ = elastic modulus. For slowly growing cracks, the second term in equation (11) is negligible in comparison to $V$ and $V_\nu \approx V$. Since the definition of $C_\nu$ is based on stationary cracks, it is important that the condition of slowly growing cracks be met. This can be checked by assuring that the second term on the right hand side of equation (11) is less than 10% of the measured value of $V$.

Creep crack growth rate, $\frac{da}{dt}$ at any given point in the test was estimated by the secant method in which an increment in crack extension, $\Delta a$, was divided by the corresponding increment in time, $\Delta t$. Thus, $\frac{da}{dt}$ and the corresponding $C_\nu$ values were determined. These results are discussed in the next section.

RESULTS AND DISCUSSION

$\frac{da}{dt}$ vs $C_\nu$ correlation

Figure 6 shows the creep crack growth rate, $\frac{da}{dt}$, plotted as a function of $C_\nu$ at 400°C for all five compact specimens. The data from the various tests do not correlate with $C_\nu$, which was, at first, a surprising result. However, a possible explanation of this trend is as follows. Since this material cavitates extensively with large cavity sizes, it is suspected that the lack of correlation may be due to the development of large scale cavitation damage at the crack tip. $C_\nu$.
is a deformation based global parameter much like the J-integral [27]. Therefore, the relationship between $C$ and the creep zone expansion rate in small scale creep regime [18] as well as the relationship between $C$ and the crack tip stress in extensive creep regime [17, 19] are based on the assumption that the damaged region containing cavities or micro-cracks is small in comparison to the crack size and the remaining ligament size of the specimen (or component). This explanation was further explored by characterizing the extent of damage ahead of the crack tip in the various creep crack growth specimens using metallography and scanning electron microscopy (SEM).

Figures 7 to 10 show photo-micrographs from the crack tip regions of specimens CT-2 through CT-5, respectively. Specimens CT-2 and CT-5 show isolated grain boundary cavities while as the micrographs from specimens CT-3 and CT-4 show extensive microcracking and crack branching at their respective crack tips. In fact, the damage in these latter specimens was present over a large region ahead of the main crack. In specimen CT-4, crack branching occurs almost from the tip of the deformed precrack indicating that very little, if any, of the crack growth occurred as growth of a single dominant flaw. Since fracture mechanics concepts are limited to the characterization of the growth of single flaws, the lack of correlation between $\frac{da}{dt}$ and $C$, is understandable.

In view of the above findings, the creep crack growth rate data obtained on specimens CT-1, CT-2 and CT-5 (the behavior of CT-1 was similar to CT-2 and CT-5) are plotted on a separate plot shown in Fig. 11. A good correlation between $\frac{da}{dt}$ and $C$, in these set of data was in fact obtained which further supports the argument that the lack of correlation between creep crack growth rate and $C$, in Fig. 6 was
due to development of large scale damage in some of the specimens tested. The specimens in which the problem of large scale damage was encountered were the ones in which the initially applied $K$ levels were higher (factor of 2 higher than the specimens from which consistent data were obtained). The high initial $K$ levels may have led to nucleation of large amounts of cavities and subsequently a high rate of growth over a substantial portion of the ligament. The cavities can then coalesce to form microcracks and branched cracks. At lower initial $K$ levels, cavitation may have been isolated as well as restricted to the near crack tip process zone with less opportunity to form microcracks. This facilitated the growth of a single dominant crack in these specimens.

Since the onset of tertiary creep is often associated with significant cavitation, it is interesting to see if crack branching occurs in specimens in which significant tertiary creep deformation occurs also. One measure of the extent of tertiary creep in specimens is obtained when the test time is compared with $t_r$, the transition time for significant tertiary creep to develop from extensive secondary creep conditions. Thus, the transition time, $t_r$, was calculated for the various test specimens for plane strain and plane stress conditions using equation (10) and are listed in Table 4. Also reported in Table 4 is the transition time, $t_m$, for extensive steady-state creep conditions to develop from small-scale creep conditions. $t_r$ is given by the following equation [28, 29].

$$ t_r = \frac{[K^2(1-v^2)]}{E(n+1)C_n^d} \quad (12) $$

All terms in equation (12) have been defined previously. It is observed that $t_r$ for all tests are considerably smaller than the corresponding test durations. Thus, for practical purposes, all tests can be considered to have been in extensive creep conditions for the entire duration. We now turn our attention to the transition from extensive secondary creep to extensive tertiary creep given by $t_r$. Since plane stress estimates of $t_r$ are considerably shorter than the plane strain estimates, the former are used for comparison with the test duration. The appropriateness of this assumption is further discussed later in this section.

For specimens CT-2 and CT-5 which exhibited singular crack growth, $t_r$ is approximately two orders of magnitude higher than the operating time. Hence, tertiary creep conditions in these specimens are expected to be restricted only to a small region ahead of the crack tip. This is also in agreement with the earlier observations from crack tip metallography. On the other hand the test duration is 16 and 40% of $t_r$ for specimens CT-3 and CT-4, respectively. These specimens did not exhibit singular crack growth and the creep crack growth rate data obtained from these specimens did not correlate with $C_n$. Therefore, in order to maintain constrained cavity
growth in the crack tip region, it appears necessary to restrict test time to a small fraction of $t_r$.

We now return to the question of the appropriateness of the plane stress vs plane strain assumption. Plane strain conditions are expected to prevail in the near crack tip region in test specimens. Therefore, when the specimen is in small scale creep where the creep deformation is restricted to the crack tip region, plane strain conditions are the appropriate condition. This will be the case in the beginning of the test. On the other hand, as the test progresses and the creep deformation becomes more widespread, plane stress conditions which exist away from the crack tip region will contribute significantly to the overall deformation behavior of the specimen. Therefore, under extensive creep conditions, perhaps the plane stress estimates are more appropriate.

Another aspect of the above discussion deals with side grooving of specimens which promotes plane strain conditions. From the above discussion it follows that side grooving will have the effect of increasing $t_r$ and forcing constrained cavitation in a regime where unconstrained cavity growth is expected without the side grooves. Therefore, side grooves may help in extending the regime over which valid crack growth rate measurements can be obtained.

**Damage zone growth**

From the previous discussion it is evident that creep cracks in antimony-copper grow due to cavitation damage ahead of the crack tip. As discussed earlier, $C_r$ is a deformation based parameter and can only correlate creep crack growth data if the damage zone is completely constrained by a significantly larger deformation zone in which the crack tip fields apply. It is thus interesting to explore the relationship between the characteristics of cavitation with the damage zone and the applied $C_r$ levels.

The terminal $C_r$ levels for specimens CT-2 and CT-5 were 1.63 and 3.29 J/m$^2$, respectively. The corresponding average crack growth rates were $2.36 \times 10^{-4}$ and $1.86 \times 10^{-4}$ mm/h, respectively.

Figures 12 and 13 show the cavitation damage ahead of the crack tip in the mid-thickness region of specimens CT-2 and CT-5, respectively. The cavity sizes and numbers as a function of distance ahead of the crack tip were quantitatively measured in these specimens following the procedure described in an earlier section. Figures 14a and 14b show the linear density and Fig. 14(c) and (d) show the areal density respectively, of cavities as a function of distance ahead of the crack tip in specimens CT-2 and CT-5. The average cavity diameter (calculated using equation (3)) as a function of distance from the crack tip is plotted in Fig. 15 for the two specimens. The percent grain boundary area cavitated (calculated using equation (4)) as a function of distance from the crack tip is also plotted, Fig. 16. There are several interesting implications of these results which merit further discussion.

It is observed that the areal density, diameter and the fraction of grain boundary area cavitated ($A_c$) increase significantly as the crack tip is approached. Cavities were found to exist as far as 8 mm ahead of the crack tip, however, the diameter, density and $A_c$ decreased at a rapid rate up to a distance of 1.3 mm and changed only marginally for larger distances. Therefore, it can be argued that the zone of intense damage extends up to approximately 1.3 mm from the crack tip. The remaining ligament size was about 34 mm which was more than 25 times of the damage zone. Therefore, the condition that the damage be contained in a small region near the crack tip for $C_r$ to be applicable appears to have been met in these specimens.
Fig. 13. Same as Fig. 12 except for specimen CT-5.

The C level of specimen CT-5 was twice that of CT-2. The cavity diameter as a function of distance from the crack tip shows an opposite trend. In other words, the cavity diameters in specimen CT-2 appear to be consistently larger than in specimen CT-5 (Fig. 15). However, the areal density of cavities was higher in specimen CT-5 (Fig. 14) and the percent grain boundary area cavitated also appears to be somewhat higher in specimen CT-2, Fig. 16. From these observations, the following interpretations can be made regarding the damage accumulation during the creep crack growth process.

The smallest stable cavities which were detected in the crack tip region were approximately 3 μm in diameter and the largest were approximately 15 μm in diameter. Hence, considerable cavity growth must occur due to creep deformation in the crack tip region as a fixed point in the material ahead of the crack tip is being approached by the growing crack. The cavity growth can occur by a Hull-Rimmer type atomic diffusion process which in all likelihood in the crack tip region is constrained by deformation in the surrounding grains. Simultaneously, cavity growth can also occur by a process in which the larger cavities grow by consuming the smaller ones in its neighborhood. This will generally lead to a reduction in areal density of cavities. It was observed [31] that in uniformly and uniaxially loaded creep specimens of this material which were interrupted at various life fractions that the cavity density (number of cavities/unit area) decreased with increasing life fraction (or creep strain). Therefore, the latter mechanism of cavity growth at the crack tip is in fact supported by experimental observations.

The data also clearly show that fresh cavities must continuously nucleate at fixed points as they are approached by the moving crack tip. This is apparent from Fig. 14(b) which shows an increasing density of cavities with decreasing distance from the crack tip. This is not surprising in view of the high stresses (or strains) encountered as the distance to the crack tip decreases thus causing new cavities to nucleate. Some of these cavities coalesce with the larger ones and contribute to their growth as discussed in the preceding paragraph while others grow to larger sizes by themselves. It appears that the nucleation rate of cavities increases with C [Fig. 14(c, d)].

The accumulated creep damage in the crack tip region is a complex function of the rate at which cavities nucleate and subsequently grow by one or more of the operative mechanisms. A suitable measure of overall creep damage is $A_c$ as shown in Fig. 16. This measure of damage includes the contributions from the different cavity nucleation and growth processes operating in the crack tip region. It appears that the damage levels are consistently higher for specimens with the lower crack growth rate. This trend is discussed later in this section.

In view of the experimental results, it is worthwhile to examine the various criteria for crack extension due to damage for cavitating materials. Previous researchers [32, 33] have used the attainment of a critical cavity size as a condition for coalescence with the crack tip. When this condition is met, the crack is postulated to extend by a distance equal to the average cavity spacing. For cavity growth mechanisms which are constrained by deformation, this is equivalent to a critical strain criterion. This criterion appears to be reasonable if cavity nucleation is not continuous. In other words, once cavity nucleation has taken place at a certain distance in front of the crack tip, no new cavities form at that location as it approaches the moving crack tip. The criterion applies well to materials in which cavity nucleation occurs only at second phase particles [6] where the areal density of cavities remains constant with respect to distance from the crack tip. For this condition, the fraction of grain boundary area cavitated, $A_c$, 


depends uniquely on the cavity size (equation (4)). Hence, there is no difference between using critical $A_c$ or critical cavity diameter as a condition for coalescence. On the other hand, in the presence of continuous nucleation, $A_c$ continues to be relevant while the critical cavity diameter no longer remains a valid crack extension criterion.

The other commonly used crack extension criterion is the Kachanov–Rabotnov $\omega$ which is an internal variable with a value of zero in undamaged condition and one in the critically damaged condition [34, 35]. In variations of this approach, the critical value of $\omega$, $\omega_1$, is assumed to be a constant which is less than one. A direct relationship between $\omega$ and $A_c$ has also been suggested. Thus, the use of $A_c$ also appears to be consistent with this approach.

In the current approaches [32–34], $\omega_c$ and the critical cavity sizes are treated as constants independent of stress. However, our data from the two specimens examined show that the value of $A_c$ in the immediate vicinity of the crack tip varies inversely...
with crack growth rate or with $C_\alpha$. These values of $A_\alpha$ represent the critical values of $A_\alpha$ for crack extension in these specimens. This trend will be confirmed by additional testing.

Another point of interest from the data of Fig. 16 is the distribution of damage as a function of distance from the crack tip and its relationship with the crack tip parameters such as $C_\alpha$ and $C_\beta$. Specifically, it is of interest to determine if the extent of the region of intense damage varies significantly and uniquely with the magnitude of $C_\alpha$. Before discussing these trends, it is necessary to examine some governing equations for evolution of damage in front of the crack tip. For simplicity, we will consider damage distribution along the crack plane only.

Consider a crack moving at a velocity $a$ ($da/dt$) and a fixed rectangular coordinate system with the $x$-axis located along the crack plane and the origin located at the original crack position. The crack growth rate varies with time in a transient problem. Damage at a point along the $x$-axis is modelled as a function of both position and time. Therefore, the critical damage $D(x,t)$, the rate of damage accumulation, $dD/dt$, will be given by (recognizing that $da/dt = -a$)

$$\frac{dD}{dt} = \frac{dA_\alpha}{dt} = \left[ \frac{\partial A_\alpha}{\partial A} \right] - a \left[ \frac{\partial A_\alpha}{\partial x} \right].$$

(13)

During the initial incubation period following loading, $a$ is nearly zero and $dD/dt$ is expected to be dominated by the $(\partial A_\alpha/\partial x)$ term. However, $(\partial A_\alpha/\partial x)$ will decrease with time and will nearly become zero when the crack tip stresses are completely relaxed and attain a stationary stress state. The rate of damage accumulation at this point will be given completely by the $(\partial A_\alpha/\partial x)$ term. With respect to a coordinate system which moves the crack tip, $dD/dt$ will then be zero. In order to determine the damage function $D(x,t)$, it is necessary to integrate equation (13) with knowledge of the functional forms of $A(x)$ and $A_\alpha(x)$. At present, sufficient information is not available to derive the complete damage distribution function. However, it seems reasonable to expect from equation (13) that a high value of $a$ will lead to a lower accumulated damage ahead of the crack tip.

**SUMMARY AND CONCLUSIONS**

Tensile, creep deformation and creep crack growth tests were conducted on a model 1% antimony–copper alloy at 400°C. The mechanical tests were followed by extensive characterization of grain boundary cavitation damage in the creep crack growth specimens by optical metallography and scanning electron microscopy (SEM). The following observations and conclusions can be made from the results of these experiments.

1. Creep deformation behavior in this model alloy at 400°C is dominated by secondary and tertiary creep deformation. It is, therefore, important to account for tertiary creep in estimating the magnitude of $C_\alpha$, a crack tip parameter for creep conditions.
2. The creep crack growth rate correlates with $C_\alpha$ provided the cavitation damage is contained and therefore restricted to a small region near the crack tip. Further, no crack branching should also occur.
3. It is shown that creep cracks propagate by simultaneous nucleation, growth and coalescence of grain boundary cavities. The cavity diameter, area, cavity density and percent fraction of grain boundary area cavitated decrease with distance from the crack tip.
4. The most suitable index of crack tip damage appears to be the percent grain boundary area cavitated. The extent of damage is a complex function of position and time. The critical damage for crack extension appears to vary inversely with the applied value of $C_\alpha$. Further tests are recommended for completely mapping the complex damage function and determining whether a unique relationship exists between the distribution of damage and the applied value of $C_\alpha$.

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Recent Advances in Elevated Temperature Crack Growth and Models for Life Prediction

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ABSTRACT
This paper summarizes the recent developments in the field of time-dependent fracture mechanics over the past few years. The state of understanding in this area is now at the same level as elastic-plastic fracture mechanics. The important developments and their applications are described in this paper.

KEYWORDS
Creep, Cracks, J-Integral, C^r-Integral, C^s, Fatigue

INTRODUCTION
Critical gas and steam turbine, power-plant boiler and petro-chemical reactor components which operate at elevated temperature tend to develop cracks during the service life. Some components have crack like defects even at the time they go into service which can grow and cause failure. Some examples of major failures in the power industry involving elevated temperature components where creep was a major contributing factor include turbine rotors (Kramer and Randolph, 1978) and steam pipes. These failures resulted in millions of lost dollars in down time and repair costs and, in some cases, also loss of human lives. Thus, crack growth under elevated temperature creep conditions is a major industrial problem.

Further impetus for studying elevated temperature crack growth comes from the need to assess the remaining life of components which have been in service and are approaching their originally predicted design life. More and more operators of equipment are turning to a retirement-for-cause (RFC) philosophy rather than to rely on life predictions made several years ago with concepts which are now out-dated (Harris et al., 1980; Saxena et al., 1988).
Failures due to creep can be classified either as resulting from widespread or bulk damage, or resulting from localized damage. The structural components which are vulnerable to bulk damage are subjected to uniform loading and uniform temperature distribution during service, for example, thin wall pipes. The thin life of such a component can be estimated from creep rupture data. On the other hand, components which are subjected to stress and temperature gradients (typical of thick section components) will not fail by creep rupture. It is more likely that at the end of the predicted creep rupture life, a crack develops at the critical location which propagates and ultimately causes failure. Thus, crack growth is an important part of a component's overall life.

In this paper, the concepts of time dependent fracture mechanics (TDFM) for characterizing elevated temperature crack growth behavior and their role in life prediction are briefly reviewed. No attempt is made to summarize developments in the methods for predicting creep rupture which is broad enough by itself to warrant a separate review.

ADVANCES IN TIME-DEPENDENT FRACTURE MECHANICS (TDFM) CONCEPTS

Crack Tip Stress Analysis

The difference between the crack tip stress and strain fields of bodies loaded in the creep and subcreep temperature regime occurs due to the presence of time dependent strains in the creep regime. The accumulated strain in front of a stationary crack tip changes continuously and the crack tip stress can also vary with time depending on whether or not steady-state conditions have been reached.

The levels of creep deformation under which creep crack growth can occur include the small-scale-creep (SSC) region, the transition creep region and the extensive creep region. Under SSC, the creep zone is small in comparison to the size of the body and the crack size and its growth is constrained by the surrounding elastic material. Under extensive creep conditions, the creep zone engulfs the entire ligament. The transition creep region is the intermediate condition. These regions are analogous to the small-scale-yielding, the elastic-plastic and fully plastic conditions, respectively, encountered in the sub-creep temperature regimes. In addition to the above, the picture at elevated temperature is further complicated when primary and tertiary creep deformations occur either by themselves or in conjunction with elastic and/or secondary creep deformations. Fortunately, following the pioneering work of Landes and Begley (1976) which led to the discovery of $C^*$, the crack tip stress fields for a variety of creep deformation laws have been worked out (Riedel and Rice, 1980; Riedel, 1981; Riedel and Detemple, 1987; Ehlers and Riedel, 1981; Ohji et al., 1979).

In general, we can express the amplitude of the Hutchinson, Rice and Rosengren (HRR) (Hutchinson, 1968; Rice and Rosengren, 1968) crack tip stress fields by the following equation.
Here $B$ and $r$ are treated as generic material constants whose values depend on the dominant mechanism of creep deformation operating in the crack tip region. For example, for power-law creep $B=A$ and $r=n$; for primary creep $B=A/(1+p)$ and $r=n_1 A_1 A_0 p A_1 n_1$ are material constants in the respective creep deformation equations given in Table 1. $L$ is a nondimensional constant which depends on $n$ and ranges between 3.8 and 6.3 for a range of $r$ values (Shih, 1983). $\sigma_{ij}(\theta,r)$ is an angular function and $r$-distance from the crack tip. The expressions for estimating $C(t)$ for several creep deformation laws are given in Table 1. Note that for extensive power-law creep $C(t)=C'$. There is an abundance of experimental data which show that under these conditions $C'$ characterizes the creep crack growth behavior (Saxena, 1980).

Since $C(t)$ uniquely relates to the amplitude of the crack tip stress fields for a variety of creep deformation laws, it is an attractive candidate crack tip parameter for characterizing creep crack growth for other than just the conditions for which it is equal to $C'$. However, there are two major shortcomings of this approach. First, $C(t)$ cannot be measured at the loading pins of the test specimens except for the two very special conditions of extensive power-law creep and the extensive primary creep shown in Table 1 (Saxena, 1980). Under these conditions, $C(t)$ is also equal to the stress-power dissipation rate ($U'$) in the cracked body. For extensive power-law creep, this relationship is given by (Landes and Begley, 1978)

$$C(t) = C' = - \frac{1}{B} \frac{dU'}{da}$$

where $B$ = thickness of the specimen. However, under small-scale and transition creep conditions $C(t) \neq -1/B (dU'/da)$.

The second shortcoming results from the question about the dominance of the HRR fields for growing cracks. It has been experimentally (Saxena et al., 1984) and numerically (Hawk and Bassani, 1986) shown that this limitation is not important in the extensive creep regime when crack velocities are low and the HRR fields essentially dominate over large distances ahead of the crack tip. Recent numerical results (Hawk and Bassani, 1986), reproduced in Fig. 1, have shown that in SSC, the region of influence of growing cracks (known as the Hui-Riedel (or HR) field (Hui and Riedel, 1981; Hui, 1983) can in fact be significant in comparison to the HRR fields. Under these circumstances the use of $C(t)$ as a crack tip parameter is invalid. Other approaches must be considered.

$C_t$ Parameter

The $C_t$ parameter (Saxena, 1986) is an extension of the stress-power dissipation rate interpretation of $C'$ into the transient regime. It also
Table 1. Summary of major crack tip stress analysis results in creeping bodies.

<table>
<thead>
<tr>
<th>Deformation Law</th>
<th>Expression</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power-law creep</td>
<td>$C$</td>
<td>Launay and Berger (1975)</td>
</tr>
<tr>
<td></td>
<td>$t + A t^b$</td>
<td>Goldman and Dutchak (1976)</td>
</tr>
<tr>
<td>Elastic - power-law creep</td>
<td>$1 - \kappa \frac{x^2}{r^2}$</td>
<td>Riedel and Rice (1980)</td>
</tr>
<tr>
<td></td>
<td>$\frac{1}{(1+\kappa)^2}$</td>
<td>Shih, Gupta and Kube (1978)</td>
</tr>
<tr>
<td>Primary creep</td>
<td>$\frac{C_0}{A_0 + B_0 t}$</td>
<td>Riedel (1980)</td>
</tr>
<tr>
<td>Elastic - primary creep</td>
<td>$x = \frac{C_0}{A_0 + B_0 t}$</td>
<td>Riedel (1981)</td>
</tr>
<tr>
<td></td>
<td>$x = \frac{C_0}{A_0 + B_0 t}$</td>
<td>Riedel (1981)</td>
</tr>
<tr>
<td>Elastic - primary - secondary creep</td>
<td>$x = \frac{C_0}{A_0 + B_0 t}$</td>
<td>Riedel and Oktawski (1987)</td>
</tr>
</tbody>
</table>

Fig. 1. Normalized creep zone size as a function of normalized time for a growing and stationary crack in Mode III. Hawk and Bassani (1986). 
alleviates the two primary shortcomings of the $C(t)$ approach as will be explained in this section. $C_t$ is defined as the instantaneous stress-power dissipation rate as follows:

$$C_t = \frac{1}{B} \frac{\partial U_t^2}{\partial a}$$

(3)

where, $U_t$ represents an instantaneous value at time, $t$, after application of the load. The parameter is defined for a stationary crack and the implications for the growing crack will be considered later in this section. Under the conditions of extensive steady-state creep it is obvious from the respective definitions that $C_t = C'$. Therefore, under these conditions and for extensive primary creep conditions, $C_t = C(t)$ and hence it also characterizes the amplitude of the HRR field. Under conditions of small-scale and transition creep $C_t \neq C(t)$ as shown clearly in the numerical results (Bassani et al., 1988) and reproduced in Fig. 2.

In small-scale-creep regime, a general expression for estimating $C_t$ is derived as follows (Saxena, 1986)

$$C_t = \frac{P \nu_c}{BW} \frac{F'}{F}$$

(4)

where, $W$ = width of the specimen, $P$ = applied load, $F = (K/P)B^{1/2}$, $K$ - calibration factor, $F' = dF/d(a/W)$, $\nu_c$ = rate of deflection at the load-line due to creep deformation. Equation (3) is equally valid for power-law or primary creep law provided small-scale-creep conditions are met. In a test specimen, if $\nu_c$ is measured, the magnitude of $C_t$ is readily obtained irrespective of the prevailing creep deformation law. In components, where $\nu_c$ cannot be measured, it can be estimated under small-scale-creep conditions by (Bassani et al., 1988).
where \( r_c \) = rate of expansion of the creep zone size and \( \beta \) is a scaling factor and is approximately equal to 1/3 as determined from finite element analyses (Bassani et al., 1988). The value of \( r_c \) is dependent on the prevailing creep deformation law. For power-law creep and primary creep it can be obtained from the following relationship.

For power-law creep (Riedel and Rice, 1980):

\[
\dot{r}_c = \frac{1}{2} \left( \frac{K}{E} \right)^2 \left[ \frac{(n+1)/n}{2\pi(1-\nu^2)} \right]^{2/3} \frac{1}{r_c^{(n-3)/(n-1)}} \cdot \frac{r_c}{r_c^{(n-1)/(n-1)}}
\]

For primary creep (Riedel, 1981):

\[
\dot{r}_c = \left[ \frac{2}{2\pi(1-\nu^2)} \right] \left[ \frac{1}{r_c} \right]^{2/3} \left[ \frac{(n+1)/(1+p)A_1}{(n+1)} \right]^{1/3} \left[ \frac{1}{r_c^{(n-1)/(n-1)}} \right]^{1/3} \frac{1}{r_c^{(n-1)/(n-1)}}
\]

It should be pointed out that both Eqs (6) and (7) are strictly valid for stationary cracks only. However, they may be used for slowly growing cracks defined by the condition \( \Delta \approx r_c \). Expressions for estimating \( r_c \) which account for growing crack effects are currently not available. Hence, even though there is no fundamental difficulty in the use of \( C \), for situations where crack growth effects significantly influence \( r_c \), there are practical limitations due to lack of adequate analyses. However, this is not a problem in test specimens where \( V_c \) is easily obtained from the measured deflection rates following the deflection rate partitioning method (Saxena et al., 1984). By combining Eqs 6 and 5, the relationship between \( C \) and the crack tip creep zone size is easily derived. This relationship is unique for a fixed applied value of \( K \). Thus, \( C \) can relate the load and deflection rate measurements made at the load-point which is remote from the crack tip to the crack tip creep zone expansion rate. The following equations relates \( C \) to \( C(t) \) under the small-scale-creep condition (Saxena, 1981).

\[
(C \big|_{ssc}) = \beta(1-\nu^2)\left( \frac{F'}{F} \right) \frac{r_c}{\bar{r}} (C(t))_{ssc}
\]

Since \( r_c \) is a function of applied \( K \) and time, \( t \), the relationship between \( (C)_{ssc} \) and \( (C(t))_{ssc} \) is not unique. Often, creep crack growth data are correlated with crack tip opening displacement (CTOD) rate, \( \delta_t \). The following relationships can be derived between \( C \) and \( C(t) \) and \( \delta_t \) in the small-scale-creep regime (Saxena, 1988):
Neither of the two parameters are uniquely related to $\delta_t$ in the small-scale-creep region. In the extensive creep region, $\delta_t$ and $C'$ are uniquely related.

Over a wide range of creep deformation conditions ranging from small-scale to extensive creep, it was shown in the above discussion that neither parameters, $C(t)$, $C_o$ or $\delta$, are uniquely related to each other. However, in the extensive creep region the relationship is indeed unique. In correlating wide range creep deformation data a good correlation with one of these parameters necessarily implies no unique correlation with the others. Figures 3 and 4 show the creep crack growth data correlations with $C_t$ and $C(t)$. From these results it is concluded that $C_t$ parameter is most appropriate for correlating wide range creep crack growth data.

**Fig. 3.** Creep crack growth rate as a function of $C_t$ in a Cr-Mo-V steel at 694°C.
APPLICATION OF TDFM IN LIFE PREDICTION

As mentioned earlier in this paper, there are several potential applications of TDFM concepts in life prediction analyses of elevated temperature components. In this section a methodology for predicting creep crack growth life is described. Subsequently, the methods for predicting creep-fatigue crack growth life are also briefly discussed. Some areas which need further development to achieve greater accuracy in life prediction are outlined at the end of this section.

Prediction of Creep Crack Growth Life

Figure 5 shows a general methodology for predicting the remaining creep crack growth life of an elevated temperature component (Saxena and Liaw, 1985). The top of the figure shows the type of specimens used in material testing, and the data obtained from these tests. Material data needed are fracture toughness (to establish the crack size which will cause rupture), creep deformation properties, the tensile stress-strain properties and the creep crack growth rate behavior. Crack growth rates in structural components are predicted using the calculated value of C_t. These rates are integrated to develop the crack size versus time curve or the remaining life.
versus crack size curve. The procedure outlined above is similar to the procedure used for predicting fatigue crack growth life except, fatigue cycles are replaced with time and $\Delta K$ (cyclic stress intensity parameter) is replaced by $C_r$.

Most components such as steam pipes or gas turbine disks are periodically shut down. This has a very significant influence on the estimated value of $C_r$. As time elapses the relative contribution of the transient term in estimating $C_r$ decreases as steady-state conditions dominated by $C_t$ are approached. When load interruptions occur, the stress relaxation process is also interrupted. Upon re-start the stress relaxation process is reinitiated independent of the relaxation in the previous cycle if small-scale-creep conditions dominate. This is schematically illustrated in Fig. 6 where a step increase in $C_t$ value is shown following each start-up.

Since the methods of TDFM are complex and the remaining life is affected by so many factors, it is desirable to conduct the analyses with the help of a computer. Several personal computer based (Saxena, 1987; Wells, 1986) computer programs are now available for predicting creep crack growth life in specialized components. Figure 7 shows results from example calculations of creep crack growth life of a high temperature steam pipe (solid lines) containing surface defects in the radial-axial plane for several operating pressures. The dotted lines show the corresponding maximum allowable half crack lengths for assuring a leak-before-break situation. These calculations were performed using the PCPIPE computer code.
Crack Growth Due to Creep-Fatigue

The approach which has been most widely used to characterize creep-fatigue crack growth is to sum the cycle-dependent \( \frac{da}{dN} \) and time-dependent contributions to crack growth to obtain an overall crack growth rate \( \frac{da}{dN} \) for the cycle (Saxena et al., 1981). The governing equation for such an approach is:

\[
\frac{da}{dN} = \left( \frac{da}{dN} \right)_c + \int_0^{t_c} \left( \frac{da}{dt} \right) dt
\]

where \( t_c \) = cycle time and \( \frac{da}{dt} \) is the average time dependent crack growth rate. Within this general approach there are some variations between researchers. Nikbin and Webster (1988) estimate \( \frac{da}{dt} \) from creep crack growth tests to calculate the time-dependent contribution. This implies that there is no creep-fatigue interaction which influences the time-
dependent crack growth behavior. Our approach (Saxena and Gieseke, 1987) obtains da/dt from creep-fatigue tests which implies that creep-fatigue interactions cannot be ruled out as a general rule. For materials which do not show creep-fatigue interactions, both approaches are identical.

There are also apparent variations in the crack driving force used to characterize da/dt among the two approaches. Webster and co-workers use $C'$ to characterize da/dt in a cyclic situation where small-scale-creep is expected to dominate during one cycle. Our approach uses the $(C_t)_{avg}$ parameter which is the average value of $C_t$ during the cycle to characterize da/dt. Figure 8 shows the correlation between da/dt and $(C_t)_{avg}$ in a 1Cr-Mo-0.25V steel (Banerji and Saxena, 1988) at 427°C and 538°C for a trapezoidal loading waveform with hold times ranging from a few seconds to twenty-four hours. A single plot is obtained for different hold times and also for different temperature. Such collapsing of data into a single trend for widely varying conditions is valuable for life prediction schemes.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig8.png}
\caption{Creep-fatigue crack growth rate data in Cr-Mo-V steel for trapezoidal waveforms with hold times ranging from 50 seconds to 24 hours and at temperatures of 427°C (800°F) and 538°C (1000°F).}
\end{figure}

Other approaches used for characterizing creep-fatigue crack growth (Ohji and Kubo, 1988) sum driving forces according to cycle and time-dependent parts. This gives rise to a combined $\Delta J_c$ parameter which is the sum of $\Delta J_c$, the time-independent cyclic $J$-integral and the integral of $C'$ over the cycle period. The relationship between $\Delta J_c$ and the crack tip stress, strain or a related quantity is not clear. For a detailed discussion of these
Recommendations for Future Work

Several problems of cracking in elevated temperature components result from thermal-mechanical stresses. As yet, there are no accepted approaches for predicting crack growth due to thermal-mechanical loading. Little work has been done in the area of transition from slow creep crack growth to fast fracture which may hold the key to accurate predictions of leak-before-break conditions. Additional work in the area of predicting time-dependent crack growth for long cycle times from laboratory tests conducted over short cycle periods is needed. On the analytical side, accurate methods are still needed for estimating \( C_t \) for complex material constitutive laws which properly account for effects due to primary creep, cyclic loading and crack growth. The solutions for estimating \( C' \) are limited to only a few geometries and almost exclusively to 2-dimensional crack cases. This area also needs attention in the future.

SUMMARY AND CONCLUSIONS

This paper summarizes the recent developments in time-dependent fracture mechanics (TDFM) over the past few years. The applications of TDFM concepts have also kept pace with the new developments. The following conclusions can be drawn at this time about the status of the field.

1. It is now widely accepted that under the conditions of extensive steady-state creep, the creep crack growth rate is characterized by \( C' \).

2. Significant progress has occurred in the understanding of creep crack growth behavior under transient conditions to include the effects due to small-scale-creep, primary creep and cyclic loading. \( C_t \) appears to be a promising candidate parameter for unifying the crack growth during the transient conditions with those during the steady-state conditions.

3. Applications of TDFM have kept pace with the concept developments largely because of the timely appearance of user-friendly computer codes.

4. Areas needing further development include crack growth due to thermal-fatigue, extension of the available \( C' \) and \( C_t \) solutions and accurate models for predicting crack growth due to creep-fatigue.

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Correlation of Creep-fatigue Crack Growth Rates Using Crack-tip Parameters

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ABSTRACT

Several approaches for correlating creep-fatigue crack growth (CFGG) rates are reviewed which incorporate time-dependent fracture parameters. The parameters \( C(t) \), \( C^* \), and \( C_t \) have been evaluated for correlating CFCG rates by partitioning the overall growth rates into cycle- and time-dependent contributions. It is shown that the use of \( C_t \) is the most appropriate for describing the time-dependent contribution. Further, an additional crack length and geometry dependent factor is introduced by the presence of creep at the crack tip. This factor is not included in correlations with \( AK \). Considerable creep-fatigue data from various sources are used to support these conclusions.

KEYWORDS

Creep; Fatigue; Creep-fatigue; Crack Growth: \( C_t \); \( C^* \)

INTRODUCTION AND BACKGROUND

Since the early work of James (1972), high temperature fatigue crack growth rates have routinely been correlated using \( AK \) with full recognition of the fact that creep deformation occurs at the crack tip below certain frequencies. It is now well established that the crack-tip stress fields are affected by creep (Riedel, 1983; Saxena et al., 1986), and a fresh look at the validity of using \( AK \) for characterizing creep-fatigue crack growth (CFCG) rates is needed. In the past few years, the concepts in time-dependent fracture mechanics have evolved yielding several parameters such as \( C^* \), \( C(t) \), and \( C_t \) which may be used to correlate crack growth under static loading, in other words, creep crack growth (Landes et al., 1976; Riedel et al., 1980; Saxena, 1986). Recently, attempts have also been made to use these parameters for correlating CFCG rates in combination with \( AK \) or \( AK \) (Saxena, 1980; Okazaki et al., 1983; Saxena et al., 1987; Dimopoulos et al., 1988; Nikbin et al., 1988; Ohji et al., 1988). Of the various fracture mechanics parameters, \( C_t \) appears to show the most promise (Saxena et al., 1987). The various crack-tip parameters which can
account for creep deformation are first briefly described with emphasis on their attributes and limitations in regard to correlation of creep-fatigue crack growth behavior. General methodologies for modeling creep-fatigue crack growth behavior are then described. Subsequently, data correlations are presented with the objective of evaluating the various parameters.

Candidate Time-Dependent Fracture Mechanics Parameters

Under static loading, the stress fields near a crack tip in a material undergoing elastic, power-law creep deformation are of the Hutchinson, Rice and Rosenblatt (HRR) type, the magnitude of which is denoted by \( C(t) \) (Riedel et al., 1980; Bassani et al., 1981). \( C(t) \) is approximately given by (Ehlers et al., 1981):

\[
C(t) = \frac{\sigma^2(1,\nu^2) + C'}{E(n+1)t}
\]

where \( E \) is Young's modulus, \( n \) is the creep exponent in Norton's creep law, \( t \) is time, \( \nu \) is Poisson's ratio and \( C' \) is a path-independent integral first introduced by Landes et al. (1976). For small scale creep conditions (SSC), which are analogous to small scale yielding, the first term in Eq. (1) dominates (Riedel et al., 1980). When extensive creep (ES) conditions exist, \( C(t) \approx C' \) (Goldman et al., 1975). A useful method for determining when extensive creep conditions exist is to compare the Riedel-Rice transition time, \( t_1 \) (the time at which the first term in Eq. (1) equals \( C' \)), with the cycle time, \( n_0 \) (Riedel, 1983; Saxena, 1988). SSC conditions exist when \( t_1 << n_0 \) and extensive creep conditions exist when \( t_1 >> n_0 \). \( C' \), in Eq. (1), has also been interpreted as the stress-power dissipation rate in cracked bodies (Landes et al., 1976). This interpretation (or definition) can be used to measure the value of \( C' \) at the loading pins for bodies undergoing dominantly steady-state creep deformation.

The \( C_t \) parameter is an extension of the stress-power dissipation rate definition of \( C' \) into the SSC regime (Saxena, 1986). It has been proposed for correlating creep crack growth for a wide range of conditions ranging from small-scale to extensive creep. \( C_t \) is also uniquely related to the rate of expansion of the crack tip creep zone under SSC conditions (Bassani et al., 1986; Leung et al., 1988), and \( C_t = C' \) when extensive creep conditions exist. \( C_t \) can be measured at the loading pins for all deformation conditions including those involving primary creep (Leung et al., 1988). There is also considerable evidence that \( C_t \) can correlate creep crack growth data for conditions ranging from small-scale to extensive creep (Saxena, 1986).

The crack-tip stress fields in creep-fatigue are more complicated than in creep. However, Riedel (1983) has numerically investigated the crack-tip stress fields for a few simple loading waveforms and his results are of interest here. He has shown that if rapid load variations occur within otherwise slow cycles, crack-tip stress peaks follow the peaks of the load variations. On the other hand, for waveforms with hold times and a rise time of approximately one half \( t_1 \), he has shown that the stress peak can be as much as 75% higher than the steady-state value. For waveforms with hold times, the stresses decay with time according to Eq. (1) and \( C' \) does not characterize the stress fields except for very long hold times \( (t_1 >> t_1) \). Hence, \( C' \) is not a valid parameter for characterizing the crack growth rates for short hold times. On the other hand, both \( C(t) \) and \( C_t \) are valid in the transient regime and can be considered as candidate parameters.
Methodologies for Modeling Creep-Fatigue Crack Growth

Creep-fatigue crack growth rates as a function of frequency, for a fixed ΔK range and waveform, show three regimes of behavior. At high frequencies, a cycle-dependent region exists where crack growth is controlled by fatigue processes. At very low frequencies, crack growth is entirely controlled by time-dependent processes. The crack growth rates in each regime are characterized by a different crack driving force. In the intermediate frequency regime, where the creep-fatigue interactions occur, two approaches have emerged to model crack growth rates.

Partitioning of Crack Growth Rates. Time-dependent fracture mechanics parameters are typically incorporated into creep-fatigue analysis by decomposing the CFCG rates into cycle-dependent and time-dependent contributions and using superposition to determine the overall rate, i.e. (Saxena, 1980; Okazaki et al., 1983; Saxena et al., 1984; Saxena et al., 1987; Dimopulos et al., 1988; Nikbin et al., 1988; Ohji et al., 1988):

\[
\frac{da}{dN} = (\frac{da}{dN})_f + (\frac{da}{dN})_t
\]  

(2)

where \((\frac{da}{dN})_f\) and \((\frac{da}{dN})_t\) are the fatigue and time-dependent components, respectively. \((\frac{da}{dN})_f\) is uniquely characterized by either \(\Delta K_{eff}\) (Dimopulos et al., 1988) or \(\Delta J\) (Ohji et al., 1988) depending upon the amount of instantaneous plasticity. Here \(\Delta K_{eff}\) is the effective stress intensity factor range and \(\Delta J\) is the cyclic J-integral of Dowling et al. (1976). Thus \((\frac{da}{dN})_t\) is given by:

\[
(\frac{da}{dN})_t = C(\Delta K_{eff})^m
\]  

(3a)

or by:

\[
(\frac{da}{dN})_t = C_0(\Delta J)^m
\]  

(3b)

where \(C, C_0, m\) and \(m_0\) are material constants of which only two are independent. For example, \(m = 2m_0\). For estimating \((\frac{da}{dN})_t\), the time rate of crack growth is first estimated using:

\[
\frac{da}{dt} = D(A)^b
\]  

(4)

where \(D\) is a time-dependent fracture parameter. This has been chosen as \(D(C)\) by some researchers (Saxena, 1980; Saxena et al., 1981; Saxena et al., 1984; Saxena, 1988), as \(C\) by others (Dimopulos et al., 1988; Nikbin et al., 1988; Ohji et al., 1988), and finally as \(C\) by Saxena et al. (1987). The time rate of crack growth can be converted to a growth rate per cycle by a simple integration over the cycle time.

Partitioning of Crack Driving Forces. An alternate approach to that of partitioning the crack growth rates has been to partition the crack driving force into cycle-dependent and time-dependent contributions (Taira et al., 1979; Ohji et al., 1988). For elastic-plastic-creep conditions, this has resulted in the so-called "total J-integral", \(\Delta J_T\), which is defined as (Taira et al., 1979):

\[
\Delta J_T = \Delta J_f + \Delta J_c
\]  

(5)

\(\Delta J_f\) applies to fatigue only and \(\Delta J_c\) is the time-integral of \(C\) over one stress cycle. Crack growth rates are then correlated using Eq. (3b) with the substitution of \(\Delta J\) for \(\Delta J_c\). The combined \(\Delta J\) parameter does not currently have a mechanics interpretation because it is not related to crack-tip stress, strain, or any other related crack-tip quantity.
DATA CORRELATIONS

Correlations Using $C^*$ and $C_t$.

In the time-dependent regime, under cyclic loading, Dimopulos et al. (1988) and Nikbin et al. (1988) have shown that CFCC rates fall on the same trend as creep crack growth data when converted to time rates of crack growth, $da/dt$, and plotted versus an experimentally measured value of $C^*$. They have observed this behavior in a nickel-base superalloy, a creep-ductile steel, and creep-brittle steel using a square loading waveform. An example of the correlation for a 2.25 Cr-Mo steel is shown in Fig. 1a. Their version of Eq. (2) is as follows (Dimopulos et al., 1988):

$$\frac{da}{dN} = C\Delta K_{eff}^2 + \frac{D(C^*)}{(3600\tau)}$$  \hspace{1cm} (6)

where $C^*$ is experimentally determined at the maximum load in the trapezoidal cycle. Figure 1b shows an example of the experimental and predicted CFCC rates determined via Eq. (6) for one material (Dimopulos et al., 1988).

![Fig. 1. (a) Dependence of crack growth/cycle on frequency. (b) Low frequency crack growth rates as a function of $C^*$. (Dimopulos et al., 1988)](image)

There is in fact a good correlation which has led Dimopulos et al. (1988) to conclude that the crack growth rates for creep-fatigue loading can be determined from tests carried out under static loading and high frequency fatigue conditions to determine the time- and cycle-dependent components of crack growth. It should be noted that the Riedel-Rice transition times in many of these tests were on the order of one hour (Nikbin et al., 1988). Since the longest reported cycle times were only 1000 seconds, SS conditions could not have developed during each cycle. The use of $C^*$ can then be questioned. However, experimentally measured values of $C^*$ for CT specimens are nearly equal to the values of $C_t$ for a wide range of crack sizes (Saxena.)
Therefore, these data support the use of \( C_i \) as the time-dependent crack-tip parameter instead of \( C' \). The data correlations with \( C_i \) will be discussed later.

Creep-fatigue crack growth in 304 stainless was approached differently by Ohji et al. (1988). In their method, the term \((da/dN)_{c}\) in Eq. (2) is given by:

\[
(da/dN)_{c} = AC'/\nu - A\Delta J_c
\]

where \( A \) is a material constant. In these studies also, \( C' \) also was experimentally measured. The overall fatigue crack growth rate was given by:

\[
\frac{da}{dN} = A(\Delta J_c + B\Delta J_f)
\]

where \( A \) and \( B \) are experimentally determined constants. Using this relationship, they have obtained good correlation of fatigue data on 304 stainless tested using sinusoidal waveforms. Equation (8) is in fact a special case of Eq. (2) with constants \( m \) and \( \phi \) in Eqs. (3b) and (4) being one.

Okazaki et al. (1983) have used the total J-integral approach in correlating data generated using balanced and unbalanced triangular strain-controlled waveforms on 304 stainless at 600 and 700°C. Fatigue data plotted versus \( \Delta J_c \) collapsed into a band that lies between cyclic- and time-dependent data. However, a unique relationship between \( da/dN \) and \( \Delta J_c \) was not observed for all waveforms. Because of this non-unique relationship between \( da/dN \) and \( \Delta J_c \), these researchers have suggested that \( \Delta J_c \) and \( \Delta J_f \) cannot be added linearly. As a result they have proposed a somewhat modified form of the \( \Delta J_c \) approach which does differentiate somewhat between waveforms. However, discussion of this technique is beyond the scope of this short paper.

Correlations Using \( C_i \) and \( C(t) \)

Saxena and co-authors (Saxena, 1980; Saxena et al., 1981; Saxena et al., 1984; Saxena, 1988) have used \( C(t) \) to correlate the time-dependent contribution to CFCG rates. Initially Saxena (1980) noted that the stress and strain fields at the crack tip, and therefore the crack growth rates, were dependent upon \( K^2/t \), which is proportional to \( C(t) \), for SSC. Thus in Eq. (2), the time-dependent crack growth could be described by:

\[
\frac{da}{dt} = b(K^2/t)^{m} - b_{1}(C(t))^{m}
\]

Equation (9) can be integrated over the fatigue cycle and the result can be expressed in the form which relates \((da/dN)_{c}\) to \( \Delta K \) by a power-law relationship including terms which relate to cyclic frequency. Reasonable correlations were obtained by this approach.

Recently Saxena et al. (1987) evaluated whether time-dependent contributions to CFCG rates can be better correlated with \( C_i \) instead of \( C(t) \) in Eq. (9). Creep-fatigue data from hold time tests on a 1.0Cr-1.0Mo-.25V rotor steel were re-evaluated using the \( C_i \) and \( C(t) \) parameters. In these tests, the hold times were much shorter than the smallest calculated transition time, \( t_1 \), which was 585 hours. Thus these tests were conducted in a dominantly small scale creep regime.

The term \((da/dN)_{c}\), from Eq. (2), was taken to be the additional crack growth per cycle due to the hold time. This was converted to an average crack growth rate during the hold time, \((da/dt)_{h} \), as follows:
\[
(da/dt)_{\text{avg}} = \frac{(da/dN) - (da/dN)_c}{t_h}
\]  
(10)

where \(da/dN\) is the overall crack growth rate, \(t_h\) is the hold time, and \((da/dN)_c\) is the crack growth rate per cycle associated with tests having zero hold time.

The average \(C_e\) parameter was calculated by integrating the analytical expression for estimating \(C_e\) and dividing it by the hold time. The expression for \(C_e\) in the small-scale-creep regime assumes elastic, power-law creep deformation and SSC conditions and is given by (Bassani et al., 1988):

\[
(C_e)_{\text{avg}} = \frac{\Delta \sigma \mu K_{\text{avg}}}{E(n-1)W F'}
\]  
(11)

In Eq. (11), \(E\) is Young's modulus, \(F\) is the K-calibration function for the cracked body of interest, \(F' = dF/d(a/W)\) where \(W\) is the body width, \(a\) is the crack length, and the other symbols are described elsewhere (Saxana, 1986).

\((da/dt)_{\text{avg}}\) was plotted versus \((C(t))_{\text{avg}}\) in Fig. 2a and versus \((C_e)_{\text{avg}}\) in Fig. 2b. The correlation with \((C(t))_{\text{avg}}\) is considerably better than the correlation with \((C_e)_{\text{avg}}\). In Fig. 2b, creep crack growth data are also plotted for comparison. It is noted that the periodic loading/unloading events of fatigue have reduced the crack growth rates during the hold time. Excellent correlation between \((da/dt)_{\text{avg}}\) and \((C_e)_{\text{avg}}\) has also been observed in a 1.25Cr-0.5Mo steel waveforms with hold times of 10, 98, and 600 seconds preceded by an initial 100 percent overload (Yoon et al., 1988). In this case, however, \((da/dt)_{\text{avg}}\) vs. \((C_e)_{\text{avg}}\) followed a trend similar to the creep crack growth data.

The correlation between \((da/dt)\) and \((C_e)_{\text{avg}}\) during the fatigue hold time in the approach described above has several interesting implications. First, it is evident from Eq. (11) that for a given waveform and cycle time (constant \(t_h\)).
the average \( C_t \) is not uniquely determined by \( K \) because of the additional crack size and geometry dependent term \( F'/F \). However, in the past, fatigue crack growth data have been routinely correlated with \( \Delta K \) for constant loading waveforms and cycle times (James, 1972; Saxena, 1980; Saxena et al., 1981; Saxena et al., 1984). This represents an apparent contradiction which is resolved as follows. It is noted that these correlations were obtained usually with a single specimen geometry, mostly CT specimens, for which \( F'/F \) does not vary significantly over a wide range of crack sizes (Saxena, 1986). In this case, \( (C_t)_m \) is approximately related to \( K \) for a constant waveshape and frequency. Also for short cycle times, the contribution of the time-dependent crack growth is small and the overall crack growth per cycle \((da/dN)\) can be dominated by the cycle-dependent portion which correlates uniquely with \( \Delta K \). Hence the observed correlations with \( \Delta K \) are not surprising, but at the same time should be viewed with caution especially for geometries other than the compact type.

Correlations of crack growth rates for waveforms other than trapezoidal have not yet been satisfactorily addressed. This is an area of future study.

**SUMMARY AND CONCLUSIONS**

Several approaches for correlating creep-fatigue crack growth rates have been reviewed in this paper. Time-dependent fracture parameters have been incorporated into the correlation of CFGC rates under elastic-creep and elastic-plastic-creep conditions with reasonable success. The presence of time-dependent creep deformation at the crack tip during cyclic loading at elevated temperatures introduces an additional crack length and geometry dependent factor which is not included in crack growth rate correlations with \( \Delta K \) even for constant loading frequency and waveform. The implication is that a non-unique relationship is expected between \( da/dN \) and \( \Delta K \) at elevated temperatures at loading frequencies where creep occurs. It is shown that the use of \( C_t \) is most appropriate parameter for describing the time-dependent contribution to the overall crack growth rates. Considerable creep-fatigue crack growth data from several sources are used to support these arguments.

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