A DIRECT METHOD FOR CALCULATING THE THRUST
AND TORQUE OF A HELICOPTER IN VERTICAL ASCENT

A THESIS

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A DIRECT METHOD FOR CALCULATING THE THRUST
AND TORQUE OF A HELICOPTER IN VERTICAL ASCENT

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Angles in Radians

\( \alpha \) = angle of attack of a blade element at radius, \( r \), measured between the zero lift chord line and the relative wind

\( \chi \) = angle between the induced component of the inflow velocity at radius, \( r \), and the normal to the rotor

\( \Theta_r \) = angle of incidence of a blade element at radius, \( r \), measured between the zero lift chord line and the plane of rotation

\( \phi \) = induced angle of attack of a blade element at radius, \( r \), measured between the relative wind and the plane of rotation

Areas

\( dA = 2\pi r dr \) = area of an elemental rotor annulus

\( A = \pi R^2 \) = rotor disk area

Forces

\( dL \) = lift on an elemental rotor annulus

\( dT \) = thrust on an elemental rotor annulus

\( T \) = rotor thrust

Lengths

\( Cr \) = chord of a blade element at radius, \( r \)

\( r \) = distance of a blade element from the axis of rotation

\( R \) = rotor radius
Miscellaneous

\( \alpha_o = \text{slope of the lift curve of a blade element} \)

\( = \frac{C_c}{\text{radian}} \)

\( dM' = \text{mass flow through an elemental annulus} \)

\( \Omega = \text{angular velocity of the rotor in radians/second} \)

\( \rho = \text{mass density of air} \)

Moments

\( dQ = \text{torque about the rotor axis of rotation of the air forces on an elemental annulus} \)

\( Q = \text{rotor torque about the axis of rotation of the air forces} \)

Nondimensional Coefficients

\( C_o = \text{rotor blade element drag coefficient} \)

\( C_{o0} = \text{profile drag coefficient of a blade element} \)

\( C_{oL} = \text{inflow drag coefficient of a blade element} \)

\( C_{L_e} = \text{lift coefficient at a blade element at radius, } r \)

\( dC_q = \text{rotor torque coefficient about the axis of rotation of air forces on an elemental annulus} \)

\( C_\Phi = \frac{Q}{\rho \pi \Omega^2 R^5} = \text{rotor torque coefficient} \)

\( dC_T = \text{rotor thrust coefficient of air forces on an elemental rotor annulus} \)

\( C_T = \frac{T}{\rho \pi \Omega^2 R^4} = \text{rotor thrust coefficient} \)
Nondimensional Parameters

\[ b = \text{number of rotor blades} \]

\[ k_m = \text{factor to account for the total head loss above the annulus due to the effects of viscosity} \]

\[ k_e = \text{factor to account for the total head loss below the annulus due to the effects of viscosity} \]

\[ k_i = \text{weighted mean value of } k_e \text{ across the rotor} \]

\[ \sigma_r = \frac{b C_r}{n r} = \text{rotor solidity at radius, } r \]

\[ \lambda_v = \frac{V_v}{V_0} = \text{ratio of the vertical ascent velocity to the induced velocity in hovering flight of an actuator disk with the given thrust} \]

\[ \lambda = \frac{V_v}{\Omega R} = \text{ratio of the vertical ascent velocity to the rotor blade tip velocity} \]

Pressure

\[ H = \text{total head} \]

\[ P_o = \text{atmospheric pressure} \]

\[ P_i = \text{static pressure just above the rotor annulus} \]

\[ P_2 = \text{static pressure just below the rotor annulus} \]

Velocities

\[ V_i = \text{induced velocity at a blade element at radius, } r \]

\[ V_{\infty} = \text{resultant induced velocity at a large distance below the rotor annulus where the streamlines are normal to the rotor plane} \]

\[ V_r = \text{velocity at which the rotor is ascending vertically} \]
\[ \Delta V = \text{total change in velocity along the line of thrust of the air in any elemental stream tube} \]

\[ V_0 = \sqrt{\frac{T}{ZrA}} = \text{value of the induced velocity for an actuator disk in hovering flight} \]
A Direct Method for Calculating the Thrust and Torque of a Helicopter in Vertical Ascent

I Summary

The object of this investigation is to find a more convenient and sufficiently exact method for calculating the thrust and torque of a helicopter rotor in vertical ascent.

Equations for the general case of rotors having blades with any twist and taper are derived by a combination of the momentum and blade element theories. A semi-empirical constant, \( \hat{A} \), which corrects for the effects of viscosity is introduced. Several simplifying approximations are made in order to obtain a direct solution.

The thrust and torque coefficients are calculated from the theory for a particular model rotor on which test data was available. The results are satisfactory for the special case of hovering flight and at low rates of vertical ascent, but for higher rates of vertical ascent the theoretical values of the thrust coefficients are progressively higher than the experimental values. The theoretical torque coefficient values are in good agreement with the experimental values over the whole range.
II Introduction

Very little work has been done on predicting the thrust and torque characteristics of a helicopter rotor in vertical ascent. Glauert\(^1\) derived a theory for the analysis of helicopter rotor performance in vertical ascent by the use of the vortex theory. This theory, with certain modifications, was used by Fagan\(^2\) for the performance calculations of a lifting airscrew in vertical ascent. The calculations are very involved and the results did not check well enough with the existing experimental data to warrant this application of the vortex theory in computing the thrust and torque characteristics.

Thus, there is a need for a more accurate and less involved method for calculating the thrust and torque of a helicopter rotor in vertical ascent. In this paper the induced velocity at a given radius, \(r\), is set up as a function of the thrust at that radius and the rate of ascent. An empirical constant is introduced to account for the effects of viscosity.


The streamlines which pass through the circumference of a circle about the axis of symmetry of the flow form a surface which, for the flow under consideration, is a stream tube. The cross sectional area between two adjacent stream tubes at the plane of rotation of a rotor defines a rotor annulus of radius, \( r \), and width, \( dr \).

In writing Bernoulli's equation, \( H = P + \frac{\rho}{2} V^2 \), between any two points along a stream tube it is assumed that there is no loss of energy for a perfect fluid. However, in a real fluid there is a change in energy due to viscosity. The velocity distribution at the rotor is non-uniform, but at a great distance below the rotor the velocity distribution tends to become uniform, so there will be an exchange in energy between the adjacent stream tubes. A viscosity correction factor is introduced in Bernoulli's equation to account for the change in energy.

It is first assumed that the coordinates are fixed in space. Then as shown in Figure (1), at some point (1) a great distance above the rotor annulus, the pressure is the atmospheric value, \( P_0 \), and the velocity is zero. Just above the rotor at point (2) the pressure is \( P_1 \) and the velocity is \( V_i \), where \( P_1 \) = static pressure and \( V_i \) = induced velocity. At a point (3), just below the rotor, the static pressure is \( P_2 \) and the velocity is also \( V_i \). At point (4), a great distance below the rotor, the static pressure is again equal to the atmospheric pressure, \( P_0 \), and the velocity is \( V_\infty \) where \( V_\infty \) = resultant induced velocity at a large distance below the rotor annulus where the streamlines are normal to the rotor plane.

---

The pressure loss above the rotor is a function of the induced velocity, $f(v_i^2)$, and that below the rotor, $f(v_o^2)$. Putting in the factor to account for the change in energy between the adjacent stream tubes due to viscosity, the dynamic pressure terms above and below the rotor become, respectively, $\frac{1}{2} \kappa_m v_i^2$ and $\frac{1}{2} \kappa_f v_o^2$

where

$\kappa_m = \text{factor to account for the total head loss above the annulus due to the effects of viscosity}$

$\kappa_f = \text{factor to account for the total head loss below the annulus due to the effects of viscosity}$

Assuming now that the coordinates are moving with the rotor, the relative velocity between the rotor and the free stream is the rate of vertical ascent as can be seen from Figure (2). A point (1) is selected at a very great distance above the rotor where the static pressure is equal to the atmospheric pressure, $P_o$, and the velocity is $v_v$, the velocity at which the rotor is ascending vertically. At point (2), just above the rotor, the static pressure is $P$, and the velocity is $(v_v + v_i)$. Bernoulli's equation is written between points (1) and (2) as

$$P_o + \frac{1}{2} v_v^2 = P_o + \frac{1}{2} (v_v + v_i)^2$$  \hspace{1cm} (1)

or by expanding the dynamic pressure terms and introducing the viscosity correction factor, equation (1) becomes

$$P_o = P_o + \frac{1}{2} (v_v^2 + 2 v_v v_i + \kappa_m v_i^2) - \frac{1}{2} v_v^2$$  \hspace{1cm} (2)

Bernoulli's equation cannot be written between a point above the rotor and another point below the rotor, because there is a discontinuity on the pressure field at the rotor.
At a point (3), just below the rotor, the static pressure is $P_2$ and the velocity is $(V_v + V_i)$. At point (4), a great distance below the rotor, the static pressure is again equal to the atmospheric pressure, $P_0$ and the velocity is $(V_v + V_\infty)$. Applying Bernoulli's equation between these two points gives

$$P_2 + \frac{\rho}{2} (V_v + V_i)^2 = P_0 + \frac{\rho}{2} (V_v + V_\infty)^2$$  \hspace{1cm} (3)

and again by expanding the dynamic pressure terms and including the viscosity correction factor, equation (4) becomes

$$P_0 = P_2 + \frac{\rho}{2} (V_v^2 + 2V_vV_i + V_i^2) - \frac{\rho}{2} (V_v^2 + 2V_vV_\infty + \kappa_2 V_\infty^2)$$  \hspace{1cm} (4)

Combining equations (2) and (4) gives

$$P_1 + \frac{\rho}{2} V_r^2 + \rho V_r V_i + \frac{\rho}{2} \kappa_1 V_i^2 - \frac{\rho}{2} V_r^2 = P_2 + \frac{\rho}{2} V_r^2 + \rho V_r V_i$$

$$+ \frac{\rho}{2} V_i^2 - \frac{\rho}{2} V_r^2 - \rho V_r V_\infty - \frac{\rho}{2} \kappa_2 V_\infty^2$$  \hspace{1cm} (5)

or

$$P_2 - P_1 = \frac{\rho}{2} V_i^2 (\kappa_1 - 1) + \frac{\rho}{2} V_\infty^2 (2V_v + \kappa_2 V_\infty)$$  \hspace{1cm} (6)

According to the propeller momentum theory, approximately three-fourths of the dynamic pressure change takes place below the rotor for a perfect fluid with no viscosity or compressibility effects. The approximation is made that the total head loss due to viscosity above the rotor is small compared to the loss below the rotor. Then $\kappa_1 = 1$, and equation (6) can now be written as

The thrust acting on the annulus is equal to the difference in static pressures above and below the annulus multiplied by the annulus area or

$$dT = (P_2 - P_1)\,dA$$

where

\[ dT = \text{thrust on an elemental rotor annulus} \]
\[ dA = 2\pi r \,dr = \text{area of an elemental rotor annulus} \]

The mass flow through an elemental annulus can be written as

$$dM' = \rho (V_r + V_i \cos \chi)\,dA$$

where

\[ \rho = \text{mass density of air} \]
\[ \chi = \text{angle between the induced component of the inflow velocity at radius, } r, \text{ and the normal to the rotor} \]
\[ V_i \cos \chi = \text{component of the induced velocity normal to the rotor} \]

The thrust acting on an elemental annulus is equal to the rate of change of momentum along the line of thrust of the air passing through the annulus. Thus

$$dT = dM' \Delta V = \rho (V_r + V_i \cos \chi)\,dA \,V_\infty$$

where

\[ \Delta V = \text{total change in velocity along the line of thrust of the air in any elemental stream tube} \]
\[ = V_\infty \]

Combining equations (8) and (10) gives
\[(P_2 - P_1) \, dA = \rho \, (\nu \, + \, \nu_i \, \cos \, \chi) \, dA \, V_\infty \quad (11)\]

Substituting the value of \((P_2 - P_1)\) of equation (7) into equation (11) gives

\[
\frac{\rho}{2} (2 \nu \, + \, \nu_i \, V_\infty) \, dA = \rho \, (\nu \, + \, \nu_i \, \cos \, \chi) \, V_\infty \, dA \quad (12)
\]

and cancelling out \(\rho\), \(dA\), and \(V_\infty\) from both sides of the equation,

\[
\nu \, + \, \nu_i \, \cos \, \chi = \nu \, + \, \frac{\nu_i \, V_\infty}{2} \quad (13)
\]

or

\[
\nu_\infty = \frac{2 \nu_i \, \cos \, \chi}{\nu_i} \quad (14)
\]

Substituting the value of \(V_\infty\) from equation (14) into equation (10) gives

\[
dT = \frac{\rho}{2} (\nu \, + \, \nu_i \, \cos \, \chi) \, 2 \nu_i \, \cos \, \chi \, dA \quad (15)
\]

or since \(dA = 2\pi \, r \, dr\)

\[
dT = \frac{4\pi \, \rho \, \nu_i \, (\nu \, + \, \nu_i \, \cos \, \chi) \, \nu_i \, \cos \, \chi \, r \, dr}{\nu_i} \quad (16)
\]

The value of \(\nu_i\) will vary across the rotor. Defining \(\nu_i\) as the weighted mean value of \(\nu_i\), equation (16) becomes

\[
dT = \frac{4\pi \, \rho \, \nu_i \, (\nu \, + \, \nu_i \, \cos \, \chi) \, \nu_i \, \cos \, \chi \, r \, dr}{\nu_i} \quad (17)
\]

The total thrust acting on the blade element is the vector sum of the components of the lift and profile drag acting in the vertical direction. Since the component due to the profile drag is very small compared to the component of the lift (see Figure (3)) it may be neglected. The thrust equation is then written as

\[
dT = \frac{1}{2} \, b \, C_r \, V^2 \, \cos \, \phi \, r \, dr \quad (18)
\]
where

\( b \) = number of rotor blades
\( C_r \) = chord of a blade element at radius, \( r \)
\( C_{lr} \) = lift coefficient at a blade element at radius, \( r \)
\( \phi \) = induced angle of attack of a blade element at radius, \( r \), measured between the relative wind and the plane of rotation

\( V_i \cos \chi \) is small compared to \( V \) for large rates of vertical ascent as shown in Figure (3). For small rates of vertical ascent \( V_i \cos \chi \) is small compared to \( \Omega r \), over the outer portion of the blades where the forces are large. Therefore, the following approximations are made:

1. \( V = \sqrt{V_i^2 + \Omega^2 r^2} \)
2. \( \cos \phi = \frac{\Omega r}{V} = \frac{\Omega r}{\sqrt{V_i^2 + \Omega^2 r^2}} \)
3. \( \sin \phi = \frac{V_i + V_i \cos \chi}{V} = \frac{V_i + V_i \cos \chi}{\sqrt{V_i^2 + \Omega^2 r^2}} \)

where \( \Omega \) = angular velocity of the rotor in radians/second

Substituting the values of \( V \) and \( \cos \phi \) of approximations (1) and (2) into equation (18) gives

\[
\frac{dT}{\Omega} = \frac{1}{2} b C_r \left( V_i^2 + \Omega^2 r^2 \right) C_{lr} \frac{\Omega r}{\left( V_i^2 + \Omega^2 r^2 \right)^{1/2}} dr
\]

or

\[
\frac{dT}{\Omega} = \frac{1}{2} b C_r \left( V_i^2 + \Omega^2 r^2 \right) C_{lr} \frac{\Omega r}{\left( V_i^2 + \Omega^2 r^2 \right)^{1/2}} dr
\]
Equating the values of thrust, $dT$, of equations (17) and (20) gives

$$\frac{4\pi e (V_r + V_v \cos \chi) V_i \cos \chi \, r \, dr}{\rho_k} = \frac{1}{2} \rho C_r \left( V_r^2 + r^2 \right) \rho \omega r \, dr$$

Upon simplification equation (21) becomes

$$\frac{V_i^2 \cos^2 \chi + V_v V_i \cos \chi - \frac{k_i b C_r (V_r^2 + r^2)}{8 \pi}}{\rho \omega r} = 0$$

Substituting the value of $\sigma_r = \frac{b C_r}{\pi r}$ in equation (22) gives

$$V_i^2 \cos^2 \chi + V_v V_i \cos \chi - \frac{k_i \sigma_r C_r (V_r^2 + r^2)}{8} \rho \omega r = 0$$

where

$$\sigma_r = \text{rotor solidity at radius, } r$$

Solving equation (23) for $V_i \cos \chi$

$$V_i \cos \chi = -\frac{V_v}{2} \pm \sqrt{\left(\frac{V_v}{2}\right)^2 + \frac{k_i \sigma_r C_r (V_r^2 + r^2)}{8} \rho \omega r}$$

The sign in front of the radical is positive because the value of $V_i \cos \chi$ in hovering flight, when $V_v = 0$, is equal to $+$ $\sqrt{\frac{k_i \sigma_r C_r r^2}{8}}$

The torque loading in ft-lb/ft$^2$ of a helicopter rotor is relatively small compared to that of a propeller. The induced rotation of the air-stream is, therefore, small enough to be neglected. Thus the angle $\Phi$ is written with sufficient accuracy as

$$\Phi = \tan^{-1} \left( \frac{V_v + V_i \cos \chi}{\omega r} \right)$$

where

$$\Phi = \text{inflow angle of a blade element at radius, } r, \text{ measured between the relative wind and the plane of rotation}$$

Substituting the value of $V_i \cos \chi$ of equation (24) into equation (25) gives
\[ \Phi = \text{TAN}^{-1}\left[\frac{V_v}{2\pi R} + \frac{V_v}{2\pi R} + \sqrt{\left(\frac{V_v}{2\pi R}\right)^2 + \frac{C_{dr}}{8} \left[\left(\frac{V_v}{2\pi R}\right)^2 + 1\right]^{\frac{1}{2}}} \right] \]  

(26)

Letting \( \lambda = \frac{V_v}{2\pi R} \), equation (26) becomes

\[ \Phi = \text{TAN}^{-1}\left[\frac{\lambda R}{2r} + \sqrt{\left(\frac{\lambda R}{2r}\right)^2 + \frac{C_{dr}}{8} \left[\left(\frac{\lambda R}{r}\right)^2 + 1\right]^{\frac{1}{2}}} \right] \]  

(27)

From Figure (3) it can be seen that

\[ \Phi = \Theta_r - \alpha = \Theta_r - \frac{C_{ar}}{a_o} \]  

(28)

where

- \( \Theta_r \) = angle of incidence of a blade element at radius, \( r \), measured between the zero lift chord line and the plane of rotation
- \( \alpha \) = angle of attack of a blade element at radius, \( r \), measured between the zero lift chord line and the relative wind
- \( a_o \) = slope of the lift curve of a blade element

Equation (28) is rearranged so that

\[ \frac{C_{ar}}{a_o} = \Theta_r - \Phi \]  

(29)

Substituting the value of \( \Phi \) from equation (27) into equation (29) gives

\[ \frac{C_{ar}}{a_o} = \Theta_r - \text{TAN}^{-1}\left[\frac{\lambda R}{2r} + \sqrt{\left(\frac{\lambda R}{2r}\right)^2 + \frac{C_{dr}}{8} \left[\left(\frac{\lambda R}{r}\right)^2 + 1\right]^{\frac{1}{2}}} \right] \]  

(30)

For any given values of \( \Theta_r, a_o, \lambda, \frac{C_{dr}}{8} \), and \( R/r \), equation (30) can be solved for \( C_{ar} \).

From equation (20) the rotor thrust is written as

\[ T = \int_0^R bc_r \left( V_v + \frac{\lambda R}{2r} \right) \frac{1}{2} \cdot \frac{C_{dr}}{8} \cdot \frac{\lambda R}{r} \cdot dr \]  

(31)
In terms of $\sigma_r = \frac{b_c r}{\pi r}$ and $\lambda = \frac{V_r}{\sqrt{\Omega R}}$, equation (31) becomes

$$T = \int_0^R \frac{b_c \pi r}{\pi r} \left( \frac{V_r^2}{\Omega^2 r^2} + 1 \right) C_{d_r} \lambda^2 r^2 \, dr$$

or

$$T = \int_0^R \frac{\rho}{\pi^2} \pi \sigma_r^2 \left[ \left( \frac{\lambda R}{r} \right)^2 + 1 \right] C_{d_r} \lambda^3 r^3 \, dr$$

Both sides of equation (33) are divided by $\rho \pi \sigma_r^2 R^4$ so

$$\frac{T}{\rho \pi \sigma_r^2 R^4} = \frac{\rho \pi \sigma_r^2}{\pi^2} \int_0^R \sigma_r \left[ \left( \frac{\lambda R}{r} \right)^2 + 1 \right] C_{d_r} \lambda^3 r^3 \, dr$$

By definition $C_T = \frac{T}{\rho \pi \sigma_r^2 R^4}$ so equation (34) becomes

$$C_T = \int_0^R \frac{\sigma_r}{\pi^2} \left[ \left( \frac{\lambda R}{r} \right)^2 + 1 \right] C_{d_r} \lambda^3 r^3 \, dr$$

Expressing the elemental radius as a fraction of the rotor radius, i.e., $r/R$, the integral of equation (35) becomes

$$C_T = \int_0^1 \frac{\sigma_r}{\pi^2} \left[ \left( \frac{\lambda R}{r} \right)^2 + 1 \right] C_{d_r} \left( \frac{r}{R} \right)^3 \, d\left( \frac{r}{R} \right)$$

The integration in the above equation for the rotor thrust coefficient is most easily performed graphically.

The torque of the drag forces on the blade elements lying within the annulus is

$$dQ = \frac{\rho}{\pi} b_c r V^2 C_o r \, dr$$

where $dQ = \text{torque about the rotor axis of rotation of air forces on an elemental annulus}$

$C_o = \text{rotor blade element drag coefficient}$

The drag is broken up into components as can be seen in Figure (3) so that
equation (37) is written

$$dQ = \frac{\rho}{2} b c_r \frac{V^2}{r} \left( C_{D0} \cos \Phi + C_D \sin \Phi \right) r \, dr$$

or

$$dQ = \frac{\rho}{2} b c_r \frac{V^2}{r} C_{D0} \cos \Phi \, dr + \frac{\rho}{2} b c_r V^2 C_D \sin \Phi \, dr \tag{39}$$

where

$$C_{D0} = \text{profile drag coefficient of a blade element operating at a lift coefficient, } C_{AR} \,$$

Substituting the values of $V$, $\cos \Phi$, and $\sin \Phi$ of approximations (1), (2) and (3) into equation (39) gives

$$dQ = \frac{\rho}{2} b c_r \left( V^2 + r^2 \right) C_{D0} \frac{\sqrt{2 \bar{r}}}{\left( V^2 + r^2 \right)^{1/2}} \, dr \tag{40}$$

The value of $V \cos \theta$ of equation (24) is substituted in equation (40) which then reduces to

$$dQ = \frac{\rho}{2} b c_r \left( V^2 + r^2 \right) C_{D0} \sqrt{2 \bar{r}} \, dr \tag{41}$$

or

$$dQ = \frac{\rho}{2} b c_r \left( V^2 + r^2 \right) C_{D0} \sqrt{2 \bar{r}} \, dr + \frac{\rho}{2} V \frac{\rho}{2} b c_r \left( V^2 + r^2 \right)^{1/2} \, dr \tag{42}$$
Equation (42) can be expressed in terms of \( \sigma_r = \frac{b C_r}{\pi r} \) and \( \lambda = \frac{V_r}{\pi^2 R} \) as

\[
dQ = \frac{\pi r}{4} \rho \frac{b C_r}{\pi r} \left[ \frac{V_r^2 R^2 + 1}{R^2 + 1} \right] \pi^2 C_{D_0} r^3 dr
\]

\[
+ \frac{\pi r}{4} \rho \frac{b C_r}{\pi r} V_r \frac{\pi R}{\pi^2 R} \left[ \frac{V_r^2 R^2 + 1}{R^2 + 1} \right] \pi^2 C_{f_r} r^2 dr
\]

\[
+ \frac{\pi r}{4} \rho \frac{b C_r}{\pi r} \left[ \frac{V_r^2 R^2 + 1}{R^2 + 1} \right] \lambda R C_{f_r} r^3 dr
\]

or

\[
dQ = \frac{\pi}{2} \pi r^2 \sigma_r \left[ \frac{\lambda R}{r} + 1 \right] C_{D_0} r^4 dr
\]

\[
+ \frac{\pi}{4} \pi R^2 \sigma_r \left[ \frac{\lambda R}{r} + 1 \right] \lambda R C_{f_r} r^3 dr
\]

\[
+ \frac{\pi}{2} \pi r^2 \sigma_r \left[ \frac{\lambda R}{r} + 1 \right] C_{f_r} \left[ \lambda R \frac{1}{2} + \frac{\lambda R}{r} C_{f_r} \left[ \frac{\lambda R}{r} + 1 \right] \right] \pi^2 r^4 dr
\]

The equation for the rotor torque about the axis of rotation due to the air forces is written as

\[
Q = \frac{\pi}{2} \pi R^2 \int_0^R \sigma_r \left[ \frac{\lambda R}{r} + 1 \right] C_{D_0} r^4 dr
\]

\[
+ \frac{\pi}{4} \pi R^2 \int_0^R \sigma_r \left[ \frac{\lambda R}{r} + 1 \right] C_{f_r} r^3 dr
\]

\[
+ \frac{\pi}{2} \pi R^2 \int_0^R \sigma_r \left[ \frac{\lambda R}{r} + 1 \right] C_{f_r} \left[ \lambda R \frac{1}{2} + \frac{\lambda R}{r} C_{f_r} \left[ \frac{\lambda R}{r} + 1 \right] \right] r^4 dr
\]

By definition \( C_Q = \frac{Q}{\rho \pi R^2 R^5} \), so dividing both sides of equation (45) by \( \rho \pi R^2 R^5 \) gives
Expressing the elemental radius as a fraction of the rotor radius, i.e., \( r/R \), the integrals of equation (46) become

\[
C_q = \frac{1}{2} \int_0^R \sigma r \left[ \left( \frac{AR}{r} \right)^2 + 1 \right] C_{c0} \left( \frac{r}{R} \right)^4 \frac{r}{R^5} \, dr + \frac{\Delta}{4} \int_0^R \sigma r \left[ \left( \frac{AR}{r} \right)^2 + 1 \right] C_{c0} \left( \frac{r}{R} \right)^3 \frac{r}{R^4} \, dr \\
+ \frac{1}{2} \int_0^R \sigma r \left[ \left( \frac{AR}{r} \right)^2 + 1 \right] C_{r} \left( \frac{r}{R} \right)^{12} \frac{r}{R^4} \, dr
\]

These integrals are easily integrated by graphical integration in calculating the rotor torque coefficients.
IV Procedure

In order to make the thrust and torque coefficient calculations less laborious, a plot of equation (30) is shown on Figure (4) for the positive values of the lift coefficients at radius, \( r \), and on Figure (5) for the negative lift coefficients. These plots afford a graphical trial and error solution of equation (30).

For a particular rate of vertical ascent, \( V_v \), and a given value of \( C_t \), the corresponding value of \( \lambda_v \) is obtained as follows:

\[
\lambda_v = \frac{V_v}{V_o}
\]

by definition

where

\[
V_o = \sqrt{\frac{T}{2 \rho A}}
\]

\[
\lambda_v = \frac{V_v}{\sqrt{\frac{T}{2 \rho A}}} = \frac{V_v}{\sqrt{\frac{\pi \rho \pi R^2 R^4 C_t}{2 \rho \pi R^2}}} = \frac{V_v}{\sqrt{\frac{\pi R^2 C_t}{\pi R^2}}}
\]

where

\[
T = \rho \pi R^2 R^4 C_t \quad \text{and} \quad A = \pi R^2
\]

Then

\[
\lambda_v = \frac{V_v}{\lambda R \sqrt{\frac{C_t}{Z}}}
\]

or finally

\[
\lambda_v = \frac{\lambda}{\sqrt{\frac{C_t}{Z}}}
\]

(49)

The value of \( \lambda_v \), corresponding to the calculated value of \( \lambda_v \), is obtained from Figure (6).
Glauert developed a semi-empirical curve relating the inflow velocity to the rate of descent for the case of a propeller in the vortex ring state. This relation accounted for the effects of viscosity. According to Glauert's curve, the viscosity correction factor becomes zero for relatively small rates of vertical ascent.

For this paper the approximation is made that the values of $\delta$, of Figure (6) are the reciprocal of the contraction ratio. The value for $\delta$, of approximately $\sqrt{2}$ for hovering flight has previously been substantiated by experimental data.

Knowing $\theta, a, \lambda, \text{ and } \kappa$, equation (30) can be solved for $C_r$, the lift coefficient at any radius, $r$. For the calculation of $C_r$ and $C_d$ it is assumed that the value of the slope of the lift curve, $a$, is $2 \pi$/radian. For a cascade of thin two dimensional airfoils, perfect fluid theory predicts a slope slightly greater than $2 \pi$/radian. The effects of viscosity reduce this value to about $5.75$/radian for the case of a thin two dimensional airfoil in a real fluid. As the effects of viscosity are corrected for in this paper by a factor, $\kappa$, it is felt that the use of a slope of $2 \pi$/radian is reasonable.

The graph on Figure (4) is divided into four quadrants. To use this graph for the solution of equation (30), an arbitrary positive value of $C_r / \delta$ is selected. The lower left quadrant is a plot of $C_r / \delta$ vs. $\delta, a_r, C_r / \delta$ for different values of $\kappa, \sigma$. At any radius, $r$, the value of $\sigma = \frac{bC_r}{\pi r}$ is calculated and knowing $\kappa$, the value of $\kappa, \sigma, C_r / \delta$

---

5 Durand, op. cit., p. 350, Vol. IV
6 Ibid., p. 230
can be obtained from the graph.

The upper left quadrant is a plot of $k_{i, r} C_{i r} / g$ vs. $\phi$ for different values of $\lambda R / r$. From the value of $k_{i, r} C_{i r} / g$ determined in the lower quadrant and the known value of vertical ascent, $\lambda$, the value of $\phi$ can be found.

The upper right quadrant contains a plot of $\phi$ vs. $C_{i r} / a_o$ for different values of $\theta_r$. Using the value found for $\phi$ and an assumed value of $\theta_r$, the value of $C_{i r} / a_o$ is determined.

The lower right quadrant is a plot of $C_{i r} / a_o$ vs. $C_{i r} / g$ with the slanting lines joining equal values of $C_{i r}$.

The method of solution is to select positive values of $C_{i r} / g$ and trace through the graph as explained until the value of $C_{i r} / g$ at the end of the trial corresponds to the assumed value at the beginning of the trial. This procedure will yield the value of $C_{i r}$ at any particular radius, $r$, at which the elemental lift coefficient is positive.

At some inboard radius along the rotor blade, the lift coefficient may become negative. Coming inboard along the radius from the rotor blade tip the value of $C_{i r}$ becomes increasingly small and at some radius becomes zero. Beyond this radius the positive graph (Figure (4)) yields no solution so the negative graph (Figure (5)) is used with the same procedure as for the positive graph to yield the negative values of $C_{i r}$.

Values of $C_{i r}$ must be determined at a sufficient number of stations along the rotor blade in order that the thrust and torque coefficients can be calculated.

At some radius along the rotor blade near the axis of rotation, the radical term in equation (30) may become imaginary, thus no solution from
Figure (4) or (5) can be obtained. This indicates that there is a vortex ring in the flow at that point and the equations derived in this paper are not valid. The thrust and torque at the axis of rotation are zero so the approximation is made that the curves of $d C_T$ and $d C_p$ vs. $(r/R)$ are faired in from a point on the curves where equation (30) becomes imaginary to zero at the hub or axis of rotation.

The values of $C_{1r}$, determined at various stations between the axis of rotation and the rotor blade tip, are substituted into equation (50) which is the differential form of equation (36).

$$d C_T = \frac{\rho v^2}{2} \left[ \left( \frac{\alpha R}{R} \right)^2 + 1 \right] C_{1r} \left( \frac{L}{R} \right) d \left( \frac{L}{R} \right)$$  \hspace{1cm} (50)

where $d C_T$ = rotor thrust coefficient of air forces on an elemental rotor annulus.

A curve of $d C_T$ vs. $(r/R)$ is plotted and upon graphical integration of this curve the value of the thrust coefficient, $C_T$, is determined.

Several different values of $\Theta_r$ at the blade root are assumed and the same procedure of calculating $C_T$ is followed through. The correct values of $\Theta_r$ are the values at which the known thrust coefficient is equal to the integrated thrust coefficient. A cross plot is made of $\Theta_r$ at the root and $C_T$ from which $\Theta_r$ is determined at the known value of $C_T$.

The procedure for the determination of the torque coefficient is similar to that of the thrust coefficient, except that in this case $\Theta_r$ is known from the thrust solution. Equation (51) is used to calculate the values of $d C_p$ which are plotted against $(r/R)$. This equation is the differential form of equation (47).
\[
d C_q = \frac{d}{2} \left[ \left( \frac{\lambda R}{r} \right)^2 + 1 \right] C_{D_o} \left( \frac{r}{R} \right) \frac{v_z^4}{d \left( \frac{r}{R} \right)} + \frac{1}{4} \frac{d}{2} \left[ \left( \frac{\lambda R}{r} \right)^2 + 1 \right] \left( \frac{r}{R} \right) \frac{v_z^4}{\left( \frac{r}{R} \right)} + \frac{\lambda}{2} \frac{d}{2} \left[ \left( \frac{\lambda R}{r} \right)^2 + 1 \right] \left( \frac{r}{R} \right) \frac{v_z^4}{d \left( \frac{r}{R} \right)}
\]

(51)

where

\[
d C_q = \text{rotor torque coefficient about the axis of rotation of air forces on an elemental annulus}
\]

The first term of equation (51) contains a profile drag coefficient, \(C_{D_o}\). Values of \(C_{D_o}\) for the corresponding values of \(C_{D_r}\) at a given value of \((r/R)\) can be obtained from the rotor blade airfoil profile drag polar. The profile drag polar used in this paper was obtained from experimental data\(^7\) (see Figure (7)). The profile drag coefficient is usually based on the value of the Reynold's Number at the 3/4 radius which is a weighted average value for the blade.

Graphical integration of the curve of \(d C_q\) vs. \((r/R)\) yields the torque coefficient.

\(^7\)Montgomery Knight and Ralph A. Heffner, Static Thrust Analysis of the Lifting Airscrew, (U. S. National Advisory Committee for Aeronautics, Technical Note No. 626, 1937), pp. 37-38
Results and Discussion

From a report by Fagan\textsuperscript{8} experimental data was obtained for a three-bladed rotor having a five foot diameter. The blades were untwisted and had a two inch constant chord with an N. A. C. A. 0015 airfoil section. The tests were conducted in a nine foot open jet wind tunnel.

The thrust and torque coefficients, the rates of vertical ascent, and the blade angles of the experimental results are shown in Tables (I, II, III, IV and V).

Calculations were made for the theoretical values of the thrust and torque coefficients using equations (50) and (51). The values, listed in Tables (VI, VII, VIII, IX and X), were plotted vs. the blade angles as shown in Figures (8) and (9) and against each other as shown in Figure (10). On these curves the experimental values are shown as points. A table of sample calculations for the theoretical values of \( C_T \) and \( C_Q \) of the rotor model is shown in the appendix.

For low rates of vertical ascent the theoretical thrust and torque coefficients are in close agreement with the experimental values. The plot of \( C_T \) vs. \( C_Q \) for both the theoretical and experimental values indicates that for a given value of \( C_T \) the theory predicts too small a value of \( C_Q \).

For greater rates of vertical ascent and blade angles the theoretical thrust coefficients are progressively higher while the theoretical torque coefficients are about the same value as the experimental for the whole range.

\textsuperscript{8}Fagan, op. cit.
VI Conclusions

(1) This general theory predicts the thrust and torque on rotors having constant chord untwisted blades with good accuracy for hovering flight and small rates of vertical ascent. Further study should be made of agreement of this theory with experimental data for the cases of rotors having twisted and tapered blades.

(2) Better correlation of the experimental and theoretical thrust and torque coefficients can be obtained by the selection of another viscosity correction factor curve. The values of $k$, should be increasingly greater for the larger values of $\lambda \nu$.

(3) An investigation should be made concerning the validity of assuming that the thrust and torque coefficient curves can be faired through the vortex ring area.


APPENDIX

As an example of the calculations for the theoretical thrust and torque coefficient values, the condition of the untwisted constant chord rotor in Fagan's paper with a blade angle, $\theta_r = 4^\circ$ and a rate of vertical ascent, $\lambda = .03$ was selected. As shown in Table II the experimental values of $C_r$ and $C_q$ are .000636 and .00013, respectively, for the above condition. The procedure for the thrust and torque coefficient calculations is shown in Table XI.

As a first approximation, it is assumed that the theoretical value of $C_T$ is equal to the experimental value so that a value of $\lambda_v$ can be obtained. It was later found that the resulting value of $C_T$ was greater than that first assumed, so another value of $C_T$ between the previous two values was chosen and a new value of $C_T$ calculated. In calculating the theoretical values for this paper it was found that although second and third approximate values of $C_T$ to insert in equation (49) were assumed, the first integrated value was very near the integrated values for the second or third approximation.

From equation (49) the value of $\lambda_v$ is calculated as

$$\lambda_v = \frac{\lambda}{\sqrt{\frac{C_r}{2}}} = \frac{.03}{\sqrt{.000636}} = 1.685$$

and

$k_1 = 1.1$ from Figure (6).

An explanation of Table XI is as follows:

Column (1): Stations of $(r/R)$ are selected between the axis of rotation and the tip of the rotor blade.

Column (2): The rotor solidity at each station is calculated from the
equation

\[ \sigma_r = \frac{bC_r}{\pi r} = \frac{3(2/12)}{\pi r} = \frac{1}{2\pi r} \]

Column (3): The values of \( \sigma_r \) for each station are multiplied by the factor \( \mathcal{A}_r = 1.1 \).

Column (4): \( \lambda \) is multiplied by the reciprocal values of column (1).

Column (5): The values of \( C_{1r} / a_0 \) are obtained from Figures (4) and (5) which are plots of equation (30) for positive and negative values of \( C_{1r} \) respectively. For each station a value of \( C_{1r} / B \) is assumed. With the corresponding value of \( \mathcal{A}_r \), \( \sigma_r \) from column (3) the value of \( \mathcal{A}_r C_{1r} / B \) is determined from either Figure (4) or (5). The value of \( \lambda R / r \) in column (4) is used to determine the value of \( \Theta \). With \( \Theta \) and the known value of \( \Theta_r \), \( C_{1r} / a_0 \) is obtained. The slanting lines in the lower right quadrant of Figures (4) and (5) connect equal values of \( C_{1r} \). This value of \( C_{1r} \) should correspond to the assumed value. Different values of \( C_{1r} / B \) for each station are assumed and the same procedure is followed until the final value of \( C_{1r} \), as determined from the graph, is equal to that value assumed. This is a graphical trial and error solution of equation (30). No solution of equation (30) is obtained from the graphs for stations \( r/R \) = 0.1 and 0.2 which indicates that the radical term of the equation is imaginary and also the presence of a vortex ring in the flow starting somewhere between stations \( r/R = 0.2 \) and 0.3.

Column (6): The values of \( C_{1r} / a_0 \) in column (5) are multiplied by the slope of the lift curve, \( a_0 \), which is \( 2 \pi / \text{radian} \) as stated on page (16) of this paper.

Column (7): The drag coefficients are obtained from the plot of \( C_{D_{1r}} \) vs.
shown in Figure (7).

Column (8): \( dC_r \) is calculated from equation (50) for each station.

Column (9): \( dC_\phi \) is calculated from equation (51) for each station.

The values of \( dC_r \) and \( dC_\phi \) are plotted vs. \( (r/R) \) in Figures (11) and (12) respectively. The curves are fairied from the values of \( dC_r \) and \( dC_\phi \) at station \( (r/R) = 0.3 \) to zero at \( (r/R) = 0. \)

Graphical integration of these curves yields the theoretical values of \( C_r \) and \( C_\phi \).
### Table I

Experimental Thrust and Torque Coefficient Values

<table>
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<tr>
<th>( \theta )</th>
<th>( C_T )</th>
<th>( C_Q )</th>
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### Table II

Experimental Thrust and Torque Coefficient Values

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<th>( C_Q )</th>
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### Table III

Experimental Thrust and Torque Coefficient Values

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### Table IV

Experimental Thrust and Torque Coefficient Values

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### Table V

Experimental Thrust and Torque Coefficient Values

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### Table VI

Theoretical Thrust and Torque

Coefficient Values

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### Table VII

Theoretical Thrust and Torque

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### Table VIII

Theoretical Thrust and Torque

Coefficient Values

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### Table IX

Theoretical Thrust and Torque

Coefficient Values

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### Table X

Theoretical Thrust and Torque

Coefficient Values

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### Table XI

**Calculation of** $C_r$ and $C_0$

$\theta = 4^\circ$, $\lambda = .03$

#### Three-bladed Rotor

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<th>$r/R$</th>
<th>$\sigma_r$</th>
<th>$A, \sigma_r$</th>
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<th>$C_{jr}/a_0$</th>
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</tr>
</tbody>
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FIGURE 1

VELOCITIES WITH RESPECT TO FIXED COORDINATES
FIGURE 2

VELOCITIES WITH RESPECT TO THE ROTOR
FIGURE 3

VELOCITIES, ANGLES, AND FORCES
AT A BLADE ELEMENT
FIGURE 5  PLOT OF EQUATION (30) FOR NEGATIVE VALUES OF $C_\phi$. 
FIGURE 4

PLOT OF EQUATION (30)

FOR POSITIVE VALUES OF C

\[ A + C \]
Figure 7

Values of $C_{D,0}$ vs. $C_{L,0}$

for an N.A.C.A. 0015 airfoil
Comparison of theoretical and experimental values of $C_T$ vs. $\beta$.
 Comparison of theoretical and experimental values of $C_T$ vs $\theta$.