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Abstract

Most systems that support the visual interaction with 3D models use shape representations based on triangle meshes. The size of these representations imposes limits on applications, where complex 3D models must be accessed remotely. Techniques for simplifying and compressing 3D models reduce the transmission time. Multi-resolution formats provide quick access to a crude model and then refine it progressively. Unfortunately, compared to the best non-progressive compression methods, previously proposed progressive refinement techniques impose a significant overhead when the full resolution model must be downloaded. The CPM (Compressed Progressive Meshes) approach proposed here eliminates this overhead. It uses a new "patching" technique, which refines the topology of the mesh in batches, which each increase the number of vertices by up to 50%. Less than 4 bits per triangle encode where and how the topological refinements should be applied. We estimate the position of new vertices from the positions of their topological neighbors in the less refined mesh using a new estimator that leads to representations of vertex coordinates that are 50% more compact than previously reported progressive geometry compression techniques.

1. Introduction

Although many representations have been proposed for 3D models [Ros94], polyhedra (or more precisely triangular meshes) are the de facto standard for exchanging and viewing 3D data sets. This trend is reinforced by the wide spread of 3D graphic libraries (OpenGL [NDW93], VRML [CBM97]), and of 3D graphics adapters for PCs that have been optimized for triangles. Therefore, triangle count is a suitable measure of a model's complexity. Common representations of triangulated meshes usually store the coordinates of each vertex as 3 floating-point numbers per vertex and the incidence relation between triangles and vertices as 3 integer vertex-references per triangle. Such a simple representation requires 18 bytes per triangle (there are roughly twice more triangles then vertices in a typical model), not counting color and texture information.

The complexity of 3D models in CAD, AEC, GIS, and medical applications has been rising steadily, fueled by the improvements in interactive 3D design tools, in data acquisition technologies, and in the storage, processing and graphics capabilities of personal workstations. Early designers and scientist were deliberately limiting the accuracy of their data sets to what could be stored and manipulated on their workstations. Today, the more complex models used by the automotive, aerospace, construction, petroleum, and architecture industries contain millions or even hundreds of million triangles. Their complexity will continue to increase rapidly in response to a need for higher accuracy during analysis, planning, and inspection.

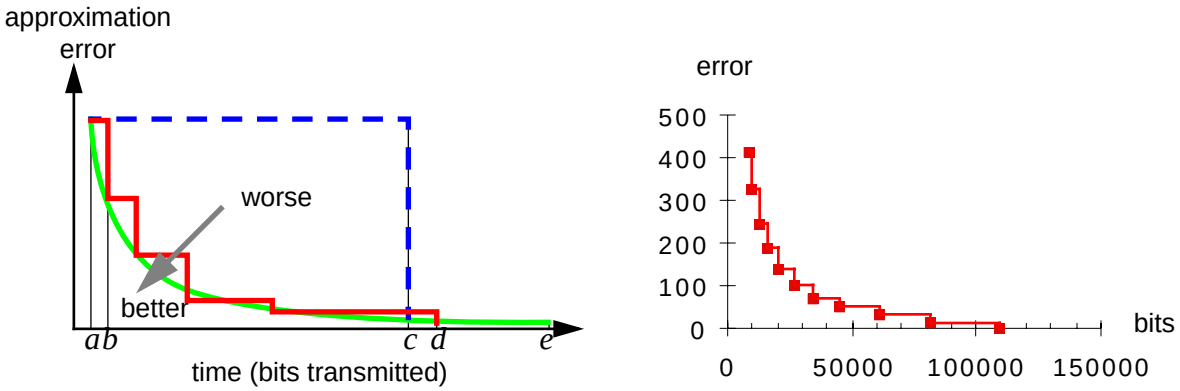
The internet and the intranet provide a convenient medium for posting 3D databases on-line for general or restricted access. However, users who need to access these databases are often equipped with personal computers and standard telephone line connections. They do not have enough storage to locally cache all the models they wish to interact with and lack automatic consistency mainte-

nance procedures for keeping such local copies updated. Consequently, most PC users must download the 3D models each time they wish to inspect or use them. A transmission cost of 18 bytes per triangle over a 56kbauds line implies a transmission rate of 400 triangles per second, or equivalently only 1.5M triangles per hour.

Because the exact representation of the visible geometry is not always required to produce an image of sufficient accuracy for navigation or inspection, geometric simplification and compression techniques may be invoked to reduce the transmission and rendering time. Geometric compression reduces the number of bits used to encode a geometric model. Simplification techniques reduce the number of triangles in an object's representation, and may be viewed as a form of lossy compression. Progressive transmission permits to first send a coarse model and then send information sufficient to refine the representation of the entire model or of specific features of the model.

This paper introduces several novel techniques, which significantly improve upon previously reported compression and progressive transmission approaches [Hop96, TGHL98, LK98]. The comparison of progressive transmission solutions often involves subjective (visual) criteria and may not be safely reduced to the comparison of a single scalar measure. We suggest the use of error/time curves (Figure 1), which express how the accuracy of the model increases with time, or more generally with the total number of transmitted bits. For simplicity, we assume that it takes *a* bits to transmit the crude model in all cases shown in Figure 1 left.

FIGURE 1. Accuracy vs. time in progressive transmission



Non-progressive approaches are often unacceptable, because they require the user to wait a long time before the entire model is decoded. A simple alternative would be to precompute a crude model, send it first, and then send the full resolution model independently, not taking advantage of the information received as part of the crude model. The user may start navigating the crude model immediately, but will have to wait more than the full transmission time of the complete model to see a more accurate resolution. For reference, this approach is shown by the dashed blue curve in Figure 1 left, the crude model is used until a full resolution model is received at time *c*.

Fine-grain compression techniques, which refine the model by inserting one vertex at a time may offer increasingly better accuracy early on, but require significantly longer to reproduce the full resolution model, and thus tie up the communication channel for longer than needed. The green curve in Figure 1 left shows an ideal fine-grain progressive transmission, which immediately starts improving the crude model, but takes much longer (*e* bits) than a non-progressive technique to download the entire model. This penalty results from the overhead of encoding the refinement steps individually.

Techniques that group refinements into batches strike an optimal compromise. Although the accuracy remains constant while the next batch of upgrades is received and decoded, the overall waiting time is reduced, because batched upgrades may be compressed more efficiently than individual upgrades. In Figure 1 left, the red staircase curve illustrates the approach presented in this paper, which groups the refinements in batches, and hence achieves a better compression. This technique may at times show higher error values than the fine-grain one, and take slightly longer to download the full model than a non-progressive approach. Still it offers the best compromise.

The novel techniques introduced here encode each batch of refinements more efficiently than previously reported solutions [TGHL98, LK98]. As a result, they provide a whole series of accuracy refinements with little or no effect on the overall transmission time for the full resolution model, when compared to previous single-resolution compression techniques [Dee95, TR98, GS98]. The actual error curve produced by the CPM method for the *Bunny* model of Table 1 is shown in Figure 1 on the right in red.

The following section reviews the relevant prior art in geometric compression and progressive refinement. Then we provide a short overview of the CMP technique in Section 3. Next follow detailed descriptions of the CPM format (Section 4), of the compression algorithm (Section 5), and of the decompression process (Section 6). We analyze the storage cost in Section 7, and discuss our experimental results in Section 8. Finally, the paper is concluded with a summary in Section 9.

2. Prior art

We distinguish between loss-less, lossy, and progressive compression. Previously reported loss-less compression techniques include:

- A bit-efficient encoding of the connectivity graph, which captures triangle-vertex incidence from which other incidence and adjacency relations may be derived. See for example [Dee95, TR98, GS98, TG98]. A more comprehensive survey may be found in [Ros98] and in [TR98b]. In practice, these approaches produce a compressed format with less than 2 bits per triangle for the connectivity information alone.
- A predictor-based compression of the vertex locations: These solutions encode the corrections between the actual and the estimated location of each vertex. If the predictions are accurate, the corrective vectors are short and their integer coordinates may be efficiently encoded using entropy coding [Huf52, CNW87]. In [TR98] each vertex is predicted using a linear combination of its ancestors in a vertex spanning tree. In [Hop96], if a vertex v is split into an edge, v is used as a predictor for the two ends of the new edge. The approach in [TG98] constructs a chamfered parallelogram to estimate the location of the free vertex of a new triangle that is adjacent to a known triangle.

Additional compression may be achieved through the following lossy approaches:

- Vertex locations may be quantized by expressing the vertex coordinates as k-bit integers in a normalized coordinate system derived from a minimum axis-aligned bounding box [Dee95, PH97, TR98]. The origin is placed at one vertex of the box and the units are selected so that the all coordinates span the range $[0..2^k]$. The choice of k is dictated by the absolute precision required by the application and by the size of the bounding box. Often k may be kept below 12 [Dee95, THLR98], which makes the entropy coding of the corrective vectors, discussed above, very effective, bringing the vertex location storage to about 12 bits per vertex for uniform tessellations of smooth surfaces [THLR98].

- The mesh may be simplified by coalescing vertices [RB93], by decimating them [SL96, SZL92], or by collapsing edges [HRD+93, RR96, GH97]. A more complete discussion may be found in [HG97]. Most of these techniques remove vertices one at a time in an order that attempts to maximize the accuracy of the approximating model at each stage. Saving intermediate results generates a series of approximations at several levels of detail. The differences between the various techniques lie principally in the error metric they use.

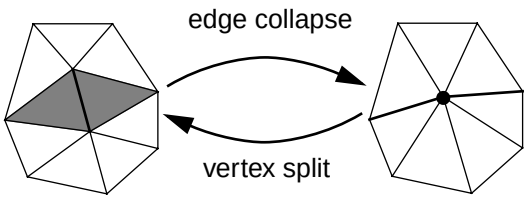
When the bandwidth precludes the transmission of the full resolution model, a crude model may be used initially, and then refined when necessary by downloading higher levels of detail [FS93] or by downloading upgrades which contain information sufficient to refine the current model. Refinements [Hop96, XEV97] which insert vertices one at a time, provide a fine-grain control of the accuracy and support view-dependent (non-uniform) refinements. However, this flexibility limits the compression ratio significantly and such progressive models require about 13 bits per vertex to encode the mesh connectivity. Grouping refinements into larger batches reduces the flexibility, but results in an economy of scale. For example, the *Progressive Forest Split* (PFS) technique [TGHL98], cuts the triangulated surface at a subset of its edges. The connected components of the cuts open up to become the boundaries of holes. The cuts and the internal triangulations of these holes may be encoded efficiently using the Topological Surgery compression [TR98]. The amortized connectivity encoding takes 10 bits per vertex. The geometry is encoded with about 30 bits per vertex.

Because simplified models are crude approximations of the original model, their vertices may be quantized with fewer bits without significantly increasing the geometric error. Thus, upgrades should combine mesh refinements and the encoding of the higher precision bits for vertex coordinates [LK98].

3. Overview

The CPM approach introduced here is based on the notion of a global upgrade. As in *Progressive Meshes* (PM) [Hop96], a crude model is transmitted first and then refined progressively through a series of vertex splits, which are the inverse of the edge collapse operations introduced in [HRD+93], see Figure 2 for an example.

FIGURE 2. Edge collapse and vertex split



The cost of encoding each vertex split operation in the Progressive Meshes approach [Hop96] has three components:

1. $\log_2(n)$ bits are needed to identify the vertex v to be split, where n is the number of previously recovered vertices.
2. $\log_2(x \cdot (x-1)/2)$ bits are used to identify two amongst all the x edges incident upon vertex v.
3. 31 to 50 bits are used to encode the displacements of the new vertices with respect to v.

The CPM approach, based on several original research contributions, improves on all three components:

1. We group the vertex splits into batches, which each split about 50% of the previously decoded vertices. We traverse the triangulated mesh, and specify split-vertices by marking vertices with one bit only instead of naming them explicitly as it was done in [Hop96]. The amortized cost of the marking is less than 3 bits per vertex for the transmission of the entire model – compared to more than 10 bits needed by the PM approach when compressing an average model. In [SvK97] a triangle-tree traversal is used to avoid expensive point location tests for incremental 2D Delaunay triangulations, whereas in CPM a vertex-tree traversal is used to reduce the storage space needed to specify progressive refinements in 3D meshes.
2. We encode the identifiers of the two cut-edges for each split-vertex as a choice of two out of d incident edges. This method requires less than 5 bits per split-vertex on average.
3. We use a novel prediction for the displacement of the new vertices. It reduces the average length of the corrective vectors and results in a compressed format of less than 20 bits per vertex location (for 10 bit coordinate quantization). Our vertex displacement prediction could be viewed as a reverse variant of the edge-split *Butterfly subdivision* scheme [DLG90, ZSS96].

In order to reduce the total cost of marking the split vertices, we seek to maximize the ratio of split-vertices at each batch. On the other hand, we must ensure a clear separation of the different cut-edges, so that each pair – belonging to a vertex split – may be encoded without ambiguities with a minimum number of bits. Our solution, detailed in Section 5.1, is the most effective compromise amongst all the variations that we have considered.

As a result, the CPM format takes 50% less storage than the PM format. In fact, our experimental results show that, thanks to the combination of the three new techniques mentioned above, the CPM format is competitive to previously reported *non-progressive* compressed formats [TR98, TG98, GS98, Dee95]. Thus the benefits of progressive refinements come at little or no additional cost.

The CPM method compares favorably with other progressive transmission schemes. For example, the PFS experiments in [TGHL98] report 10 bits per vertex for the connectivity and about 30 bits or more per vertex for the geometry, which correspond to respectively 25% and 50% larger storage or transmission costs than our CPM format. Similarly, [LK98] requires $\log_2(n+6)$ bits per vertex for connectivity, which is 50% more than our CPM format for average meshes.

4. CPM format

The CPM compressed format is organized as follows. The crude model, M_0, is stored using a single resolution compressed format [Ros98]. The vertex geometry in M_0 is stored at a reduced resolution optimized for M_0 using [LK98]. M_0 usually contains 5% to 10% of the number of triangles of the entire model. The second part of the CPM format contains the missing least significant bits of the vertex coordinates of M_0. (For simplicity, we chose not to implement the full progressive coordinate encoding scheme by [LK98].)

The third part of the CPM format contains the sequence of the refinement batches. These may be downloaded systematically, or only when needed, and create the sequence of increasingly precise approximations M_1, M_2, …, M_∞. For instance, if the model's screen representation remains small, only M_0 may be needed. Each batch $M_i \rightarrow M_{i+1}$ includes the split-vertex marking bits in M_i, the cut-edges encoding for every split-vertex, and the entropy encoded prediction correction vectors of the split-vector.

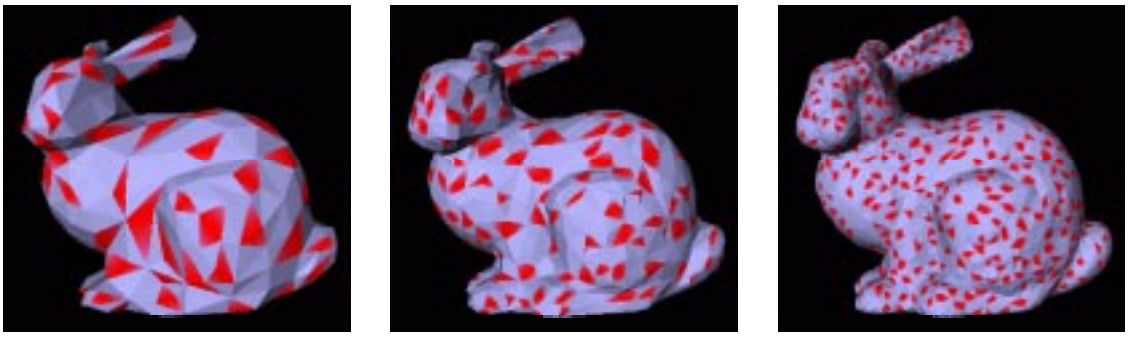
5. Compression algorithm

5.1 Batched simplification

The full resolution mesh, M_∞, is simplified in batches, creating a series of meshes M_∞, $M_{\infty-1}$, …, M_{i+1}, M_i, …, M_1, M_0 of decreasing accuracy. The simplification stops at a crude model M_0, when a given error threshold or mesh complexity is reached. This model M_0 is then used as the initial base mesh for reconstruction, and encoded using an efficient single-resolution mesh compression method, such as [Ros98, TG98, TR98]. In each simplification batch $M_{i+1} \rightarrow M_i$, the number of triangles is decreased by $3 \cdot \tau_e \cdot |M_{i+1}|$. The ratio τ_e denotes the fraction of edges of M_{i+1} that can be collapsed in the same batch.

Figure 3 shows three out of the eight different levels of detail produced by the CPM method, where the triangles inserted by the previous refinement batch are indicated in red. The batches are created by the CPM compression process by selecting, at each batch about 11% of the model's edges, by collapsing them, and by encoding the information necessary to reverse these steps.[1]

FIGURE 3. Batches of edge collapses



To optimize coding, CPM attempts to maximize the selection ratio τ_e of collapsed edges. However, to be able to uniquely identify independent vertex splits in the refined mesh M_i, the following two restrictions for collapsing edges in M_{i+1} must be fulfilled:

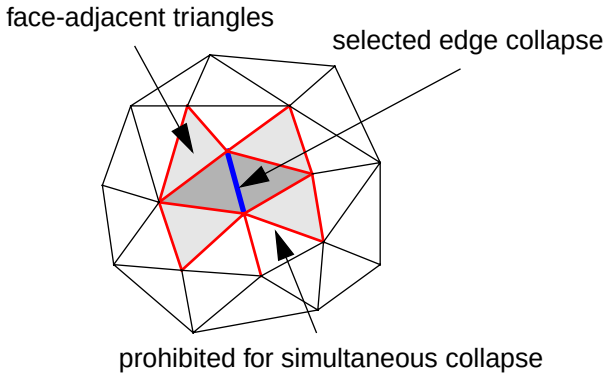
- at most two vertices may be collapsed into one
- at most two edges may be collapsed into one

These two restrictions are depicted in Figure 4. To prevent the simultaneous collapse of three vertices, no edge incident to one that will be collapsed can itself be collapsed in the same batch. Additionally, to avoid collapsing three edges into one, no face-adjacent triangles of the collapsed triangles may be part of another edge collapse in the same batch.

As a result, in the simplified mesh M_i a split-vertex will yield a single edge, and no cut-edge is used for more than one vertex split.

1. Collapsing 11% of the edges reduces the number of vertices by 33% because there are about three times more edges than vertices. A 33% reduction during simplification results in a 50% relative increase during refinement.

FIGURE 4. CPM edge collapse restrictions



To achieve good approximations at each stage, the CPM method uses an error metric to evaluate the error introduced by every single edge collapse. However, the CPM approach is independent of the error metric which may be selected so as to satisfy the application requirements. The error metric used in our current implementation is discussed in Section 5.4. Based on that error metric, the CPM method selects a subset of the less expensive edges that do not violate the two constraints defined above. These will be collapsed in the next batch. Different selection strategies might be applied to achieve an optimum with respect to the approximation error introduced per batch. In fact, the batch-wise processing of simplifications in CPM cannot anymore guarantee the same order as proposed in [Hop96] or [GH97].

Without maintaining a dynamic heap of collapsible edges based on the approximation error, the current CPM implementation greedily selects edges in increasing error order, as long as they do not conflict with the topological restrictions mentioned above. Maintaining a heap is not necessary because all affected edges are incident to an edge collapse and cannot be collapsed within the same batch, see also Figure 4. Thus it is both necessary and sufficient to compute the approximation errors and sort the edges accordingly after every batch. One can also avoid the sorting by selecting edges iteratively with increasing threshold until no more can be selected due to topological restrictions. Choosing a good initial and incremental threshold will result in few iterations.

All selected edges are collapsed to their midpoint.

5.2 Connectivity coding

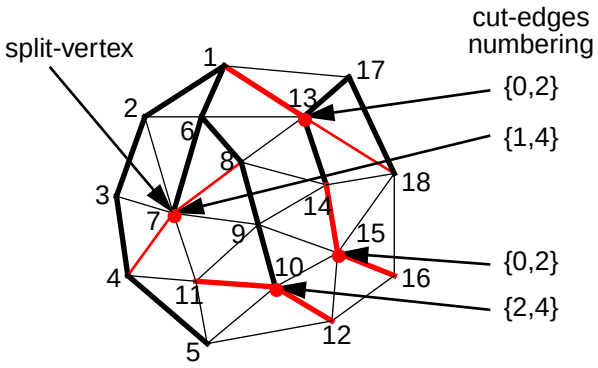
The encoding of the connectivity information needed to restore M_{i+1} from M_i can be summarized as follows:

- We construct and traverse a vertex spanning tree of M_i and mark vertices that are the results of an edge collapse. For every marked split-vertex *v*, we encode its cut-edges as follows:
  1. We compute the indices of the two cut-edges in the sorted list of the incident edges on *v*, clockwise, starting with the edge from *v* to its parent in the vertex spanning tree (Figure 5).
  2. Given the degree *d* of the split-vertex in mesh M_i, the two edge numbers are identified as one possible choice out of $\binom{d}{2}$ for selecting the cut-edges, we encode this choice using exactly $\left\lceil \log_2 \binom{d}{2} \right\rceil$ bits.

Since the degree *d* of a split-vertex in M_i is known by the encoder and the decoder, Step 2 can use a table look-up mechanism for the conversion between the two cut-edge numbers and the index out of $\binom{d}{2}$. The variable length coding of CPM uses exactly $\left\lceil \log_2 \binom{d}{2} \right\rceil$ bits to encode such a pair of cut-

edges in Step 3. Figure 5 illustrates a vertex spanning tree (thick black and red lines) and the corresponding vertex enumeration order. Four vertices are marked as split-vertices (7, 10, 13 and 15). The corresponding cut-edges are drawn in red. The two cut-edges of a split are identified by a pair of numbers as explained above. For example, 0 means that the first cut-edge is the edge to the parent in the tree.

FIGURE 5. CPM vertex split encoding

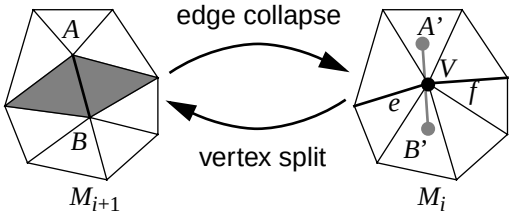


5.3 Geometry prediction and coding

To complete the compression we encode the geometry coordinates of the original vertices of the collapsed edges. In CPM we apply the prediction error coding model used for image compression [Kou95] to 3D geometry coordinates. The basic idea is to predict an unknown vector from known vertices and to encode the prediction error. The decompression process uses the same prediction and reconstructs the correct vector by applying the decoded correction.

1. We estimate the originally collapsed vertex locations by *A'* and *B'*, based on the split-vertex *V* and cut-edges *e* and *f* in mesh M_i.
2. We calculate the prediction correction vector $E = \overrightarrow{BA} - \overrightarrow{B'A'}$ between the estimated and actual vertex locations, see Figure 6.
3. We encode *E* using an entropy coding scheme.

FIGURE 6. Geometry prediction

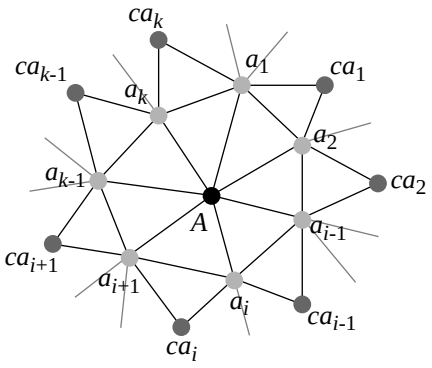


CPM uses a prediction method inspired by the *Butterfly subdivision* [DLG90, ZSS96] to estimate the original non-collapsed vertex locations. The basic idea of the method is that a vertex *A* can be approximated by a linear combination of its immediate neighbors a_i, with topological distance 1 on the triangulation graph, and the vertices ca_i at topological distance 2 as shown in Figure 7. The approximation *A'* of *A* is:

$$A' = \alpha \cdot \frac{\sum_{i=1}^{k} a_i}{k} + (1-\alpha) \cdot \frac{\sum_{i=1}^{k} ca_i}{k} \qquad \textbf{(EQ 1)}$$

Specifying a value of less than 1 for the parameter α denotes a weighted averaging between the two crowns, and a value of more than 1 expresses an extrapolation based on the difference between the two crowns for estimating *A*. The value of α is computed for each model and is consistently close to 1.15.

FIGURE 7. Vertex prediction



Using the vertex prediction model of Equation 1 and the notations of Figure 8, we can combine this prediction formula for the two original vertices *A* and *B* of an edge collapse and estimate them as:

$$A' = \alpha \cdot \frac{\sum_{i=1}^{k_a} a_i + v_1 + v_2 + B'}{k_a + 3} + (1-\alpha) \cdot \frac{\sum_{i=1}^{k_a+1} ca_i + b_1 + b_{k_b}}{k_a + 3} \quad \textbf{(EQ 2)}$$

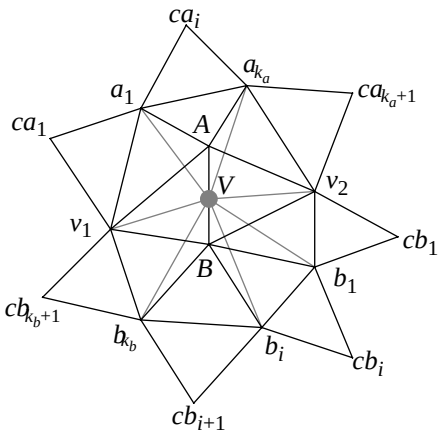
$$B' = \alpha \cdot \frac{\sum_{i=1}^{k_b} b_i + v_1 + v_2 + A'}{k_b + 3} + (1-\alpha) \cdot \frac{\sum_{i=1}^{k_b+1} cb_i + a_1 + a_{k_a}}{k_b + 3} \quad \textbf{(EQ 3)}$$

Based on the collapsed vertex $V = (A+B)/2$ and the split-vector $D = B - A$, we have $A = V - 0.5D$ and $B = V + 0.5D$. Therefore, we can express both Equations 2 and 3 in terms of an estimated split-vector *D*'. To simplify the expressions, we introduce $SA = (\sum_{i=1}^{k_a} a_i + v_1 + v_2)/(k_a + 3)$ and $CA = (\sum_{i=1}^{k_a+1} ca_i + b_1 + b_{k_b})/(k_a + 3)$ (for *SB* and *CB* respectively). Thus using *D*, Equations 2 and 3 can be rewritten as:

$$V - 0.5D' = \alpha \cdot \left(SA + \frac{V + 0.5D'}{k_a + 3}\right) + (1-\alpha) \cdot CA \Rightarrow -0.5 \cdot D'_A = \frac{(k_a+3)((1-\alpha)CA + \alpha SA) + (\alpha - k_a - 3)V}{k_a + 3 + \alpha} \quad \textbf{(EQ 4)}$$

$$V + 0.5D' = \alpha \cdot \left(SB + \frac{V - 0.5D'}{k_b + 3}\right) + (1-\alpha) \cdot CB \Rightarrow 0.5 \cdot D'_B = \frac{(k_b+3)((1-\alpha)CB + \alpha SB) + (\alpha - k_b - 3)V}{k_b + 3 + \alpha} \quad \textbf{(EQ 5)}$$

FIGURE 8. CPM Butterfly prediction



At decompression time everything is known but the split-vector *D*, for which we want to have a short encoding. Based on the estimates *A*' and *B*', Equations 4 and 5 provide two different predictions for *D*': D'_A and D'_B,. Therefore, we estimate $A' = V - 0.5D'_A$, $B' = V + 0.5D'_B$, and thus $D' = 0.5(D'_A + D'_B)$. Since *D*' is known at compression and decompression time, we can encode the prediction error $E = D - D'$ only. At decompression, *D* is reconstructed by adding the decoded correction vector *E* to the estimate *D*'.

One can observe that prediction errors have a probability distribution that decreases exponentially with the absolute value of the prediction error. We approximate the actual prediction error histogram with a *Laplace* probability distribution:

$$L(x) = \frac{1}{\sqrt{2\upsilon}} e^{-\sqrt{\frac{2}{\upsilon}}|x-\mu|}$$

For symmetric error distributions, the mean μ is 0, and the variance υ uniquely defines the Laplace distribution. For each batch of vertex splits the variance of the prediction errors, $\upsilon = \sum_i^{E_i \in \text{batch}} (E_i - \mu)^2 / |\text{batch}|$, is computed and encoded with the compressed batch. The mean prediction error μ is assumed to be 0.

Given this probability distribution, entropy coding methods compress the quantized coordinate prediction errors. Based on the variance υ and the probability distribution $L(x)$, we compute a Huffman code [Huf52] for each batch to compress the corresponding prediction errors. In contrast to other geometry compression approaches, CPM does not store the complete Huffman coding table for each batch, but only the variance value υ. This is sufficient for the decompression algorithm to reconstruct the required Huffman coding table.

The CPM experiments in Section 8 demonstrate that this prediction error coding model indeed produces short codings for the geometry coordinates of the tested models.

5.4 Error metric

Most attempts at estimating the error that results from using approximations to nominal shapes for graphics are either limited to view-independent geometric deviations [RB93, HRD+93, KT96, GH97] or to heuristic measures focused on preserving image characteristics, such as the location of silhouettes or highlights [Hop97, LE97]. In the current CPM implementation we chose to use a variation of the *Quadric Error Metrics* [GH97], enhanced by a normalization factor for every error quadric – the number of planes.

6. Decompression algorithm

The decompression algorithm performs the inverse operations of the compression process to reconstruct the sequence of meshes M_0, M_1, …, $M_{\infty-1}$, M_∞. Geomorphs [Hop96] may be used to eliminate the popping effect of each update. Therefore, at the beginning, prior to the actual CPM method, the base mesh M_0 is decompressed. Thereafter, the individual refinement batches $M_i \rightarrow M_{i+1}$ are decompressed from the CPM file as needed. In each batch, the number of triangles is increased by $\tau_v \cdot |M_i|$ on average. The decompression builds a vertex spanning tree of M_i and uses the same vertex traversal order as the compression algorithm to read and process the CPM vertices. Within each batch, the following steps are performed for every visited vertex in the vertex tree traversal of M_i:

- Read bit of CPM data. If 0, the vertex is not marked to split. If 1, the vertex has to be split, and we extract the additional vertex split information as follows:
  1. Read $\left\lceil \log_2\binom{d}{2} \right\rceil$ bits of CPM data, where d is the degree of the current marked vertex in M_i and use these bits to identify the corresponding two cut-edges in M_i.
  2. Decompress the prediction error vector E from the CPM data and use it to estimate the split vector D' in M_i using Equations 4 and 5. We use that vector to reconstruct the correct split vector as $D = D' + E$ for the current split-vertex.

Note that in Step 1 all cut-edges are numbered with respect to mesh M_i. Therefore, the actual mesh refinements of one batch have to be performed only after all cut-edges of that batch have been identified in M_i.

At the beginning of each batch, the variance υ is read from the CPM data, and the corresponding Huffman table is constructed in the same way as in the compression algorithm. Using that Huffman code, the prediction errors E can exactly be decompressed in Step 3.

7. Amortized storage cost analysis for connectivity

We want to express the number of bits used to encode the connectivity of a triangle mesh as a function of the size of the final mesh. For this, we first consider only one batch of refinement steps that increase the number of triangles from $|M_i|$ in the coarser model to $|M_{i+1}|$ in the finer model. Assume that, in each batch, we can select $\tau_v \cdot |M_i|$ vertices to be split, thus we have $|M_{i+1}| = (1+\tau_v)|M_i|$. Furthermore, to encode such a refinement we need $B \cdot |M_i|$ bits (B bits per triangle in M_i), which means $B/(1+\tau_v)$ bits per triangle in M_{i+1}.

Now we can examine a sequence of refinement batches, each of which increases the number of triangles by a factor of $1+\tau_v$. Expressing the overall cost based on the finest mesh M_∞ we derive the following recursive cost function:

$$C(|M_\infty|) = B \cdot |M_{\infty-1}| + C(|M_{\infty-1}|) = B\frac{|M_\infty|}{1+\tau_v} + C\left(\frac{|M_\infty|}{1+\tau_v}\right).$$

This recursive cost function can be rewritten as a geometric sum with $\delta = 1/(1+\tau_v)$, as:

$$C(|M_\infty|) = B\delta|M_\infty| + B\delta^2|M_\infty| + \ldots = B\delta|M_\infty|(1+\delta+\delta^2+\ldots) = B \cdot \delta|M_\infty|\frac{\delta^k-1}{\delta-1}.$$

When the number k of refinement batches is large and because $\delta < 1$, this cost can be bounded as follows:

$$C(|M_\infty|) \le B \cdot \frac{\delta}{1-\delta}|M_\infty| = B \cdot \frac{1}{\tau_v}|M_\infty| \qquad \textbf{(EQ 6)}$$

Therefore, the overall cost to transmit a series of refinement batches can be expressed as B/τ_v bits per triangle. Thus the coding scheme depends strongly on the fraction τ_v of split-vertices that are selected in every batch, and on the encoding of one single batch expressed as B bits per triangle of the batch's input mesh.

The relationship of split-vertices in the coarse mesh M_i to the corresponding edge-collapses in the refined mesh M_{i+1} can be expressed as $\tau_v = 3\tau_e/(1-3\tau_e)$ and $\tau_e = \tau_v/(3-3\tau_v)$. For example:

$$\tau_e = 1/15 \Leftrightarrow \tau_v = 1/4\,,$$

$$\tau_e = 1/12 \Leftrightarrow \tau_v = 1/3\,,$$

$$\tau_e = 1/9 \Leftrightarrow \tau_v = 1/2\,.$$

Considering the simplified mesh M_i and the corresponding vertex split operations, one can observe that an independent set of vertices in M_i is always a valid result of a set of independent edge collapses in M_{i+1} (see also Figure 4). Therefore, the 4-coloring theorem of planar graphs provides a lower bound for the ratio $\tau_v \geq 1/4$ of simultaneous vertex splits in M_i, and thus also for the ratio $\tau_e \geq 1/15$ of edge collapses in M_{i+1}.

Given the vertex split selection ratio τ_v the overall cost for transmitting the connectivity information of one batch is one bit per vertex, and $\left\lceil \log_2\binom{d}{2} \right\rceil$ bits for every vertex split, thus $B = 1/2 \cdot (1 + \tau_v \cdot \left\lceil \log_2\binom{d}{2} \right\rceil)$ bits per triangle in M_i. Using Equation 6 to calculate the amortized cost, and expressing the cost as bits per triangle in the full resolution mesh M_∞, CPM achieves the following connectivity encoding cost per triangle:

$$\frac{1}{2} \cdot \left(\frac{1}{\tau_v} + \left\lceil \log_2\binom{d}{2} \right\rceil \right)$$

For practical situations our experiments have shown that $\left\lceil \log_2\binom{d}{2} \right\rceil$ is less than 5 bits on average, and that split ratios of $\tau_v \geq 1/3$ are achievable. Overall, the experiments show that CPM can encode the connectivity of the complete mesh M_∞, including all intermediate incremental meshes M_0, …, M_i, …, M_∞ in about 3.5 bits per triangle of the original mesh M_∞.

8. Experimental results

In all the experiments presented in this section the base mesh M_0 is encoded using the Edgebreaker coding method [Ros98]. Therefore, we use 2 bits per triangle to encode the connectivity, and 3 times the number of quantization bits per vertex for the coordinates without using any geometry compression. In the tables, we use the notation C / Δ and G / Δ to denote the number of bits needed per triangle for connectivity and geometry. The different meshes that result from incrementally applying the CPM refinement batches, starting with the base mesh M_0, are also called levels of detail (LODs).

Examples for a selection of different models and LODs, produced by the CPM method, are presented in Figures 9 and 10.

Table 1 shows the detailed coding and compression results of the CPM method, applied to a Bunny model with 4833 vertices, quantized to 10 bits per coordinate. The CPM simplification was stopped at a base mesh M_0 containing approximately 5% of the number of input vertices. With selection ratio of $\tau_v > 1/3$ on average, the CPM method generated a sequence of 10 refinement batches. The rows $M_i \rightarrow M_{i+1}$ show the number of new vertices per batch, the connectivity and geometry bits per batch, and also per triangle. Row M_{10} presents the cumulative costs of the CPM method (including all batches and M_0). The CPM representation requires 3.6 bits per triangle for the bunny and between 3.5 to 3.7 bits per triangle for other models for encoding the connectivity. The vertex coordi-

nates are encoded using about 5 bits each which is equivalent to 7.7 bits per triangle. The timings reported here include file I/O for the data and for the intermediate results. The error, i.e. the *Hausdorf* distance, between the original model and the different LODs produced by the CPM method were estimated using the *Metro* tool [CRS96]. The error graph is plotted in Figure 1 on the right.

TABLE 1. 10bit quantized Bunny model

meshes	error	vertices	C bits	C/Δ	G bits	G/Δ	C+G/Δ	decode time	encode time
M_0	413	243	972	2	7290	15	17		
$M_0 \rightarrow M_1$	324	56	516	4.6	1344	12	16.6	0.05 s	0.06 s
$M_1 \rightarrow M_2$	245	88	712	4.1	1888	10.7	14.8	0.06 s	0.06 s
$M_2 \rightarrow M_3$	185	115	943	4.1	2320	10.1	14.2	0.06 s	0.11 s
$M_3 \rightarrow M_4$	138	163	1279	3.9	3120	9.6	13.5	0.07 s	0.12 s
$M_4 \rightarrow M_5$	100	234	1752	3.8	4248	9.1	12.9	0.10 s	0.16 s
$M_5 \rightarrow M_6$	70	331	2467	3.7	5584	8.4	12.1	0.11 s	0.24 s
$M_6 \rightarrow M_7$	48	469	3416	3.6	7392	7.9	11.5	0.12 s	0.33 s
$M_7 \rightarrow M_8$	29	692	4976	3.6	10472	7.6	11.2	0.16 s	0.46 s
$M_8 \rightarrow M_9$	15	998	7081	3.6	13504	6.8	10.4	0.23 s	0.68 s
$M_9 \rightarrow M_{10}$	0	1444	10316	3.6	17744	6.1	9.7	0.33 s	1.12 s
M_{10}		4833	34430	3.6	74906	7.7	11.3		

TABLE 2. 10bit quantized models

meshes	vertices	C bits	C / Δ	G bits	G / Δ	C+G / Δ	LODs
fohe	3620	25545	3.5	73332	10.1	13.7	7
fandisk	6475	48289	3.7	99626	7.7	11.4	9

The CPM connectivity coding is 25% more efficient than the PSF method [TGHL98] which requires about 5 bits per triangle on average, whereas CPM uses about 3.5 (Tables 1, 2 and 3). The PSF experiments use only 6 bits for coordinate quantization, and apply a sophisticated smoothing operation to avoid visual artifacts. This makes it difficult to directly compare the geometry compression results. Still, even using a much finer quantization of 10 instead of only 6 bits per coordinate, the CPM method outperforms the PSF method in terms of geometry compression. CPM only needs 7 to 10 geometry bits per triangle, whereas PSF requires about 15 to 20 bits per triangle.

A comparison to state-of-the-art single-resolution mesh compression methods is given in Table 3 for two 8-bit quantized models. The column C+G denotes the overall data size in bytes, and the column LODs shows how many different meshes, LODs, are created with the CPM method. The numbers of the two comparing compression methods are replicated from the tables in [TG98]. One can observe that the CPM method mainly suffers in terms of connectivity encoding compared to the single-resolution methods. In some cases, the geometry compression is even better than the proposed method in [TR98]. However, in contrast to the single-resolution methods, the CPM algorithm pro-

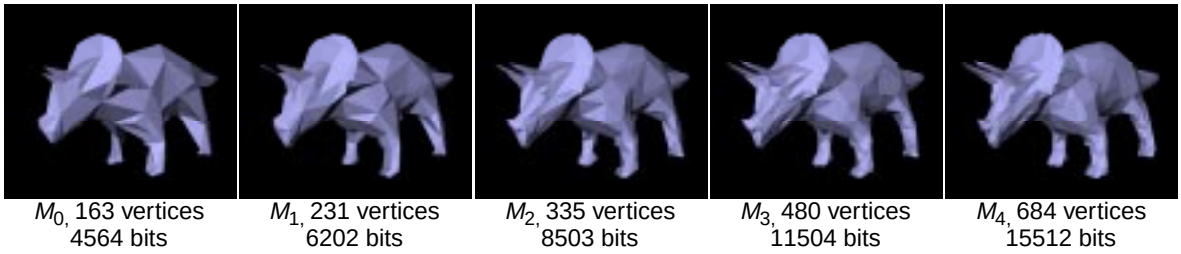
vides different LODs, and produces good approximations of the final shape of the model at early stages during decompression.

TABLE 3. 8bit quantized model comparison

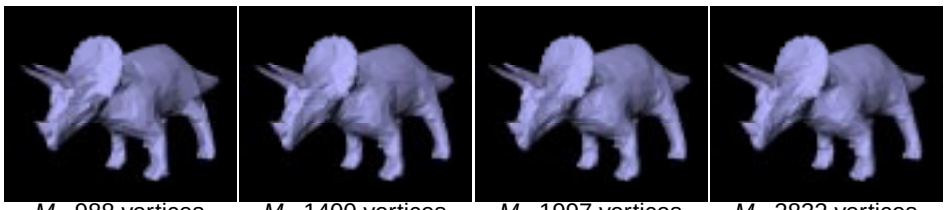
		Topological Surgery [TR98]			Touma & Gotsman [TG98]			CPM			
model	**vertices**	**C/Δ**	**G/Δ**	**C+G**	**C/Δ**	**G/Δ**	**C+G**	**C/Δ**	**G/Δ**	**C+G**	**LOD**
triceratops	2832	2.2	5.2	5196	1.1	4.1	3701	3.5	5.8	6527	9
shape	2562	1.1	7.1	5291	0.1	4.7	3038	3.5	6.9	6637	10

In Figure 9, a complete sequence of meshes demonstrates the progressive mesh refinements achieved by the CPM method. A vertex split ratio of $\tau_v \approx 0.43$ could be achieved, resulting in 9 different LODs M_0, …, M_8. The bits indicated for M_i denote the cumulative bits for transmitting all batches up to M_i, including all intermediate meshes $M_{j<i}$.

FIGURE 9. CPM sequence of meshes



M_0, 163 vertices, 4564 bits | M_1, 231 vertices, 6202 bits | M_2, 335 vertices, 8503 bits | M_3, 480 vertices, 11504 bits | M_4, 684 vertices, 15512 bits



M_5, 988 vertices, 21045 bits | M_6, 1400 vertices, 28356 bits | M_7, 1997 vertices, 38441 bits | M_8, 2832 vertices, 52215 bits

Figure 10 provides on overview of CPM compression results using a representative set selected from our test set of 3D models. The rightmost column shows the original quantized models, the number of vertices, and the number of bits needed to store it using a standard binary encoding (three vertex indices per face and three times the number of quantization bits per vertex). We show also an estimated timing for transmitting the meshes over a 56Kbit per second communication line. The other three columns show the base mesh M_0, an intermediate mesh, and the full resolution mesh. The ratio of the number of vertices to the original model is given with each image. The number of bits shown is the accumulated cost needed by the CPM method, thus the bits for mesh M_i include all the meshes $M_{j<i}$ too. Furthermore, in the full-resolution models of the CPM method (column 3 of Figure 10), the triangles selected by the first batch of the simplification are shown. All edge collapses and the corresponding two triangles are highlighted in red. Note that the CPM method not only saves time and space to transmit or store the complete model compared to a standard binary representation but also provides good approximations already at small fractions of time, or number of bits, used for the full-resolution model.

9. Summary of research contributions

The CPM method introduced here transmits 3D models through a series of progressive refinements. The total transfer time is comparable to – and sometimes even better than – the time required to transfer only the original model when using the best non-progressive 3D techniques reported so far. However, instead of having to wait until the entire transmission is over, a crude but often sufficient approximation of the model is already available after the initial 5 to 10% of this period. Furthermore, that crude approximation is refined incrementally during the transfer through a series of 7 to 15 steps, and its accuracy drastically increased by the first few refinements, often reducing the need to ever transfer the final batches of refinements.

The significant improvements in compression ratios offered by CPM over previously reported compression and progressive transmission techniques result from a new approach which combines three new ideas that were introduced in this paper:

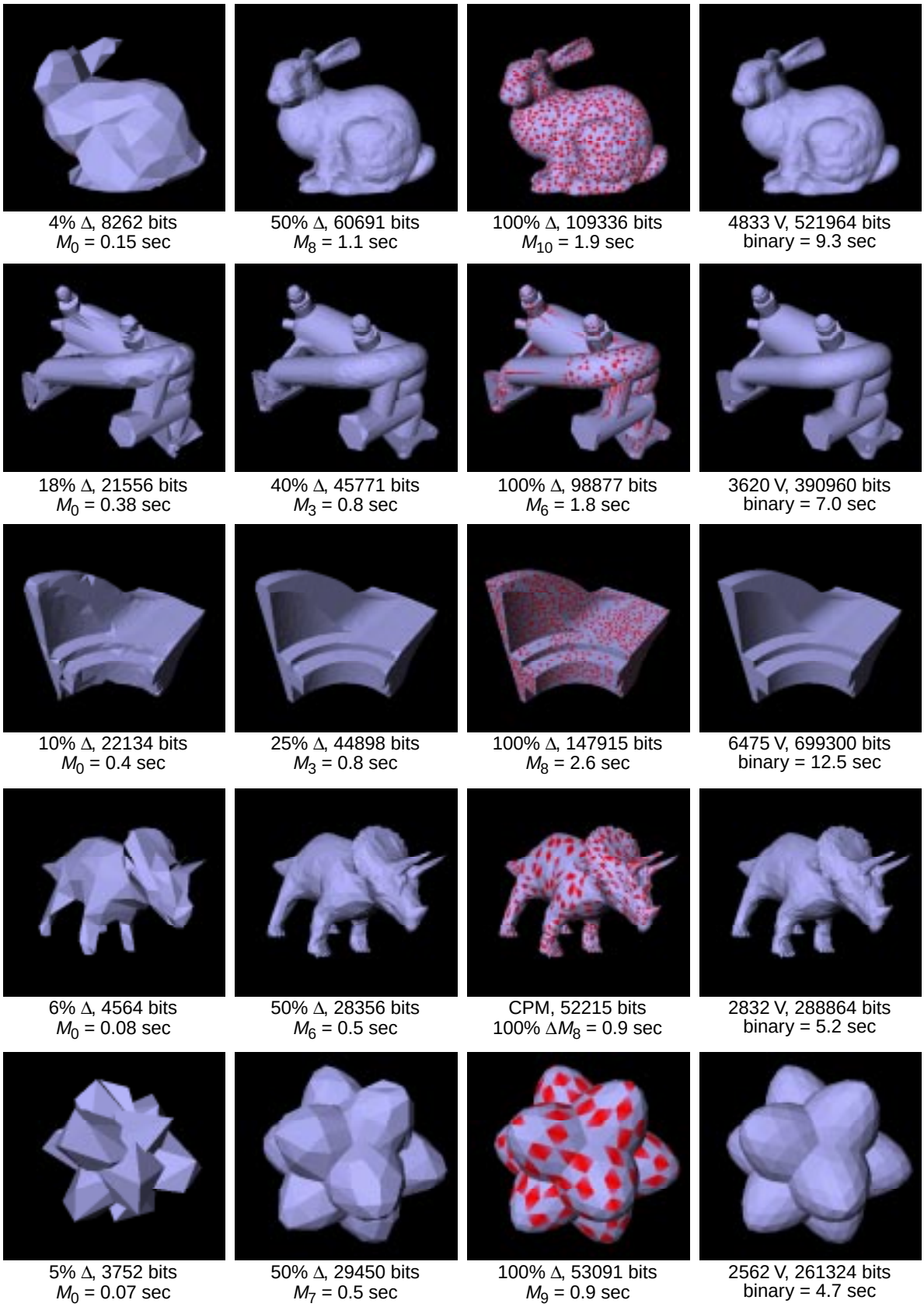
1. Split vertices are identified using a single bit per vertex, rather than $\log_2 n$ bits, as needed by the original PM solution [Hop96].
2. The two cut-edges amongst the d edges incident upon a given split-vertex are identified using an optimal code and a look-up table. The table is defined by the value of d, which is known to both the compression and decompression algorithms, for each split-vertex.
3. The location of the pair of vertices produced by each vertex split is predicted unprecedented accuracy by our new *Butterfly estimator*, which takes into account the vertices in the topological vicinity of the split-vertex.

We have derived simple validity conditions, which govern the simplification steps of the CPM compression algorithm. These conditions guarantee that our 1-bit-per-vertex marking method is unambiguous and still offer sufficient flexibility for our greedy and simple algorithm to achieve a 25% or more vertex reduction for each batch. Furthermore, because at each batch CPM first marks all the edges that must be collapsed and then collapses them all at once, there is no need, nor benefit, from maintaining a priority queue of the edge candidates. The compression algorithm is thus significantly simpler and more efficient.

The CPM algorithm and format are independent of the chosen complexity and of the particular technique used to compress the initial crude approximation and of the particular error metric used to select the best edge candidates for simplification at each batch. These may be selected based on the particular application needs.

Our experimental results conducted on a variety of models exhibit a 50% improvement over progressive compression ratios reported elsewhere [Hop96, TGHL98]. The average cost per triangle for transmitting the entire mesh using our progressive method is 3.6 bits for the connectivity and 7.7 for the vertex location (for the bunny model). For the same model, the PM approach would require 8 bits for the connectivity and between 15 and 25 for the geometry. According to the experiments reported in [TGHL98] the PFS method would require 5 bits for connectivity and about 20 bits for geometry.

FIGURE 10. CPM experimental results



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