STRESS RELAXATION OF POST-TENSIONED STAINLESS STEEL RODS FOR BRIDGE PIER CAP SHEAR STRENGTHENING

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Carlyn N. Krapf

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SUMMARY

This thesis examines the stress relaxation phenomenon that may occur in stainless steel rods that are used as part of an all-stainless steel externally post-tensioned rod system for strengthening reinforced concrete pier caps in shear. An overview of the stress relaxation phenomenon and its modeling is presented. Previous work on stress relaxation of stainless steel is reviewed and used to select an appropriate stress relaxation model to analyze the stress relaxation data obtained from experiments performed during this research. An experimental program utilizing six specimens of Type 304/304L stainless steel rods similar to those likely to be candidates for pier cap rehabilitation is presented, and the data are analyzed using the model identified from the review. This model reasonably captures the behavior of the resulting stress relaxation with time, and calculates stress values that are fairly close to those obtained experimentally. Using this model to predict long-term stress losses in a service environment, guidelines for design and installation of the post-tensioned stainless steel rods are proposed.
CHAPTER 1
INTRODUCTION

1.1 Background

Many older bridges in the State of Georgia are no longer considered adequate in terms of their load carrying capacity. In some cases, these bridges were designed for lower loads than those specified in the current *AASHTO LRFD Bridge Design Specifications* (2007) and may have suffered deterioration in the form of cracks that has resulted in perceived shear deficiencies. To keep these deficient bridges operational, they are posted. Approximately one-third of the 2,000 posted bridges in the Georgia Bridge Inventory are posted due to deficiencies in shear capacity of their reinforced concrete pier caps based on calculations according to the AASHTO *Manual for Bridge Evaluation* (2008).* These postings limit the maximum permitted truck loading and the required rerouting of goods and services results in economic losses to the community that they serve. An alternative to posting is replacement, repair, or strengthening.

Research is underway to determine cost-effective strategies for strengthening reinforced concrete pier caps. Three methods are currently being examined for in-place shear strengthening and repair of reinforced concrete pier caps:

1. An all-stainless steel externally post-tensioned rod system;
2. A carbon fiber reinforced polymer composite system; and
3. A stainless steel fiber reinforced polymer composite system.

A schematic of the external post-tensioning system in Method 1 is illustrated in Figure 1.1. The introduction of a compressive force into the concrete pier cap in the shear-critical region of the pier cap between the bridge girder carrying the superstructure and the column decreases the diagonal tensile stresses in the concrete, thereby, eliminating shear cracks. Effectively, the system acts as an external stirrup to increase the shear capacity of the concrete pier caps.

Figure 1.1: Current Post-Tensioned Rod System (Courtesy of GDOT)

The Georgia Department of Transportation (GDOT) recently implemented this shear strengthening concept and installed the externally post-tensioned rod system to one of the reinforced concrete pier caps of the I-675 Ramp over I-285 in Atlanta, Georgia (Figures 1.2 and 1.3). The system consisted of 1-inch ASTM A722 Grade 150 threaded
bars, ASTM A572 Grade 50 C6 x 10.5 channels, and ASTM A325 bolts to construct the post-tensioning system shown in Figures 1.2 and 1.3. A hot-dipped galvanized coating was applied to all structural steel components. The range of load applied resulted in an equivalent stress between 16.6 and 76.4 ksi in each threaded bar, and since the ultimate tensile strength (UTS) of Grade 150 threaded bars is 150 ksi and the minimum yield strength (YS) is 80% UTS (= 120 ksi), this corresponds to a range of stress from 14% YS to 64% YS and from 11% UTS to 51% UTS.

Figure 1.2: Bottom View of Post-Tensioning System (Photo Courtesy of GDOT)

Figure 1.3: Installation of Post-Tensioning System (Photo Courtesy of GDOT)
Experience with this initial application indicated that Method 1 is feasible. However, stainless steel behaves differently than other grades of traditional construction steel. Therefore, general guidelines for rehabilitation, retrofit, and field installation of an all-stainless steel external post-tensioned system must be developed that takes its unique behavioral characteristics into account.

1.2 Stainless Steel as a Structural Material

Stainless steel has advantages in an aggressive service environment. While its initial cost is higher, it is more resistant to corrosion than carbon or low-alloy steel and thus it is an attractive alternative when total life-cycle costs are considered.

Stainless steels are considered “stainless” because they contain chromium which forms a chromium oxide passive film and prevents corrosion. In order for a steel to be classified as a stainless steel, it must contain a minimum of approximately 11% chromium (Davis, 2001). There are five basic families of stainless steel, which are distinguished by their microstructure: ferritic, austenitic, duplex, martensitic, and precipitation-hardening. Some mechanical properties of stainless steel in the annealed condition as listed in ASTM Standard A276 (ASTM, 2008) are reproduced in Table 1.1.

<table>
<thead>
<tr>
<th>Steel Type</th>
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<th>Yield Strength (ksi)</th>
<th>% Elongation</th>
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<td>Austenitic: 304, 316</td>
<td>75</td>
<td>30</td>
<td>30</td>
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<td>Ferritic: 430</td>
<td>60</td>
<td>30</td>
<td>20</td>
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<tr>
<td>Duplex: S32205</td>
<td>95</td>
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Because ferritic stainless steel has a body-centered cubic (bcc) crystal structure, it has good strength and moderate ductility (see Table 1.1). It is magnetic and has good resistance to stress corrosion cracking (SCC). Its resistance to pitting and crevice corrosion is not as good as that of austenitic stainless steel. Austenitic stainless steel contains nickel, which is an austenite-stabilizing element. This produces a face-centered cubic (fcc) crystal structure. Austenitic stainless steel has good strength and high ductility (see Table 1.1). It can be cold-worked to attain a higher strength than ferritic stainless steel but at the cost of losing ductility. It is nonmagnetic and has poor resistance to SCC. Austenitic stainless steel is also susceptible to sensitization, which lowers resistance to pitting, crevice corrosion, and SCC. Duplex stainless steel has a combination of austenitic and ferritic microstructures. This produces a stainless steel with desirable mechanical and corrosion resistance properties that cannot be obtained with only a ferritic or austenitic phase present in the microstructure (Davis, 2001).

As structural engineering materials, the constitutive relations for stainless steel are different from those of carbon and low-alloy steels, with which bridge engineers and designers are usually familiar. Their stress-strain curves typically have no definite yield point and exhibit initial linear elastic behavior. Moreover, stainless steel is more likely to undergo stress relaxation over an extended service period than plain carbon steel. Over an extended period of service, this decrease in stress will affect the beneficial compression in the reinforced concrete pier cap that has been rehabilitated with the proposed post-tensioned stainless steel bracket system.
1.3 Research Objectives and Scope

This research examines the stress relaxation phenomenon that occurs in the stainless steel rods over an extended service period. These stainless steel rods will be used as part of an all-stainless steel externally post-tensioned rod system for concrete pier cap shear strengthening. Taking a combined experimental and analytical approach, this research will help develop guidelines for design and installation of the post-tensioned stainless steel rods. These guidelines are established based on mathematical models that predict long-term stress losses in a service environment from short-term stress relaxation experiments and will be supported by the results of a series of stress relaxation experiments.

The scope of this research is limited to the stress relaxation that occurs in austenitic stainless steel rods under uniaxial loads.

1.4 Organization of Thesis

The remainder of this thesis is divided into five chapters. Chapter 2 presents an overview of the stress relaxation phenomenon and its modeling. Chapter 3 presents previously published work on stress relaxation of stainless steel and the data are used to aide in the selection of an appropriate stress relaxation model. An experimental program utilizing stainless steel rods similar to those likely to be candidates for pier cap rehabilitation is presented in Chapter 4, and the data are analyzed using the selected model in Chapter 5. In Chapter 6, the conclusions are summarized, and design recommendations are made.
CHAPTER 2

STRESS RELAXATION OF STAINLESS STEEL

This chapter presents an extensive review of the literature on stress relaxation. An overview of viscoelastic materials and their stress relaxation behavior is presented. Specific analytical models that describe the stress relaxation phenomenon that occurs in linear viscoelastic, nonlinear viscoelastic, and nonlinear viscoplastic materials are also presented.

2.1 Viscoelastic Materials and Stress Relaxation

The uniaxial stress ($\sigma$) in most solid materials, with a small strain ($\varepsilon$) applied, can be described by Hooke’s law:

$$\sigma = E\varepsilon$$

(2.1)

where $E$ is the modulus of elasticity. A Newtonian fluid under shear stress behaves according to,

$$\sigma = \eta \frac{d\varepsilon}{dt}$$

(2.2)

where $\eta$ is the viscosity of the material. Stainless steel is a material in which viscoelastic effects are significant, and the relationship between stress and strain depends on time. The stress-strain curve becomes steeper if the strain-rate is increased because for a given strain rate, both time and strain increase simultaneously (Lakes, 2009).

Stress relaxation is a time-dependent phenomenon that occurs in viscoelastic materials. When the material is subjected to a constant total strain (or deformation), stress in the material gradually decreases (or relaxes) with time. The phenomenon is
illustrated in Figure 2.1. When a step, or a constant, strain is applied to a material (Figure 2.1, left), the stress decreases with time (Figure 2.1, right). The rate of relaxation is determined by several mechanisms including grain boundary slipping, movement of edge dislocations, and the formation of vacancies.

![Figure 2.1: Stress Relaxation Behavior Caused by a Step Strain](image)

The total constant strain ($\varepsilon_T$) present in the material can be separated into two components: the elastic strain ($\varepsilon_e$) and the plastic strain ($\varepsilon_p$).

$$\varepsilon_T = \varepsilon_e + \varepsilon_p$$ (2.3)

The elastic strain varies linearly according to Hooke’s law:

$$\sigma = E\varepsilon_e$$ (2.4)

where $\sigma$ is the uniaxial stress and $E$ is the modulus of elasticity. As stress in the material decreases with time, the elastic strain also decreases. This means the plastic strain must increase. When the material is initially loaded into the elastic region ($\varepsilon_p = 0$ at $t = 0$), the resulting strain behavior with time is shown in Figure 2.2.
Figure 2.2: Strain Behavior with Time

From points 0 to 1 in Figure 2.2, $t = 0$ and a virtually instantaneous strain increment is applied to the material. At point 1, only elastic strain is present in the material, and the plastic strain is zero. As time increases from points 1 to 2, the elastic strain is replaced by plastic strain. As the plastic strain increases, the elastic strain decreases because the total strain remains constant (Dowling, 1993).

2.2 Analytical Modeling of Stress Relaxation

The viscoelastic effect in structural materials can be either linear or nonlinear. Linear viscoelastic materials obey linear differential equations or linear integral equations, and nonlinear viscoelastic materials obey nonlinear differential equations or nonlinear integral equations.

2.2.1 Linear Viscoelasticity

For linear viscoelastic materials, the modulus relaxation function, $E(t)$, is independent of strain, and the materials relax according to,

$$\sigma(t) = E(t)\epsilon_t$$

(2.5)

$^\dagger$ Since the total strain is constant, this is a simplified version of the more common integral representation.
where the stress, $\sigma(t)$, and the relaxation modulus of elasticity, $E(t)$, are functions of time. For a viscoelastic solid, $E(t)$ reaches a nonzero asymptotic value as $t \to \infty$. Many different relaxation functions for linear materials have been proposed, such as exponentials, power laws, and stretched exponentials functions. The constitutive equations for relaxation are determined by experimentation (Lakes, 2009).

Spring and dashpot models are helpful to visualize viscoelastic behavior, although real materials seldom can be reasonably modeled by a small number of springs and dashpots (Lakes, 2009). One of the simplest models of stress relaxation uses two elements: a linear spring element and a linear viscous dashpot element connected in series as shown in Figure 2.3. It is an elastic, steady-state creep model known as the Maxwell model (Dowling, 1993).

The spring represents the elastic strain and the dashpot represents the plastic strain. At $t = 0$, the strain is entirely elastic (represented by the spring), and the plastic strain is zero. As time increases, motion occurs in the dashpot that represents the plastic strain. The amount of plastic strain present in the system depends on the coefficient of tensile viscosity ($\eta$); a larger $\eta$ yields more plastic strain and a smaller $\eta$ yields less plastic strain.
The Maxwell model can be used to derive an overall equation that represents the stress at any time during relaxation. First, differentiating both sides of Eq. 2.3 yields
\[ \dot{\varepsilon}_T = \dot{\varepsilon}_e + \dot{\varepsilon}_p \] (2.6)
in which the dot indicates differentiation with respect to time. For a stress relaxation test, the total strain is held constant, so \( \dot{\varepsilon}_T = 0 \), and Eq. 2.6 reduces to
\[ 0 = \dot{\varepsilon}_e + \dot{\varepsilon}_p \] (2.7)

The elastic strain rate (\( \dot{\varepsilon}_e \)) can be found by taking the derivative of Eq. 2.4 with respect to time:
\[ \frac{d\sigma}{dt} = \sigma = E\dot{\varepsilon}_e \] (2.8)

The plastic strain rate (\( \dot{\varepsilon}_p \)) can be found by substituting only plastic strain into Eq. 2.2:
\[ \frac{d\varepsilon_p}{dt} = \varepsilon_p = \frac{\sigma}{\eta} \] (2.9)

Substituting the values for elastic strain rate and plastic strain rate from Eqs. 2.8 and 2.9, into Eq. 2.7 yields
\[ \frac{1}{E}\frac{d\sigma}{dt} + \frac{\sigma}{\eta} = 0 \] (2.10)

Finally, the stress at any time during relaxation can be determined by solving this linear homogeneous first-order differential equation. With the initial condition of \( \sigma(0) = E_0\varepsilon_T \) because the plastic strain is initially zero, one obtains
\[ \sigma(t) = E_0\varepsilon_T e^{-\frac{E_0\dot{\varepsilon}_T}{\eta}} \] (2.11)

where Eq. 2.11 is consistent with the formulation in Eq. 2.5 when
The Maxwell model yields a prediction of stress relaxation obtained from Eq. 2.11 that is a single exponential function, as illustrated in Figure 2.4.

\[
E(t) = E_0 e^{-\frac{E_d}{\eta} t}
\]  

\[\text{(2.12)}\]

While the Maxwell model is among the simplest of the stress relaxation models, it only has one degree of freedom (the coefficient of tensile viscosity, \(\eta\)) and only describes loading into the elastic strain range (\(\varepsilon_p = 0\) at \(t = 0\)). By increasing the number of elements in a model, it becomes better at describing real material behavior, but at the expense of requiring more experimental data to define the elements. A three-element model is the standard linear solid (SLS) model (Lakes, 2009). It consists of a spring in parallel with a spring and dashpot in series as seen in Figure 2.5. Using the same techniques as those used to solve for the Maxwell model, one obtains the relaxation function of the SLS model:

\[
\sigma(t) = E_2 \varepsilon_T + E_1 \varepsilon_T e^{-\frac{E_d}{\eta} t}
\]

\[\text{(2.13)}\]
As with the Maxwell model relaxation function, the relaxation function for the SLS model is also in the form of a single exponential. Although increasing the number of elements in the model from two to three may lead to a better description of time-dependent behavior, models built around a single exponential function undergo all of their relaxation relatively quickly. In contrast, real materials undergo relaxation over an extended amount of time (Lakes, 2009).

2.2.2 Nonlinear Viscoelasticity and Viscoplasticity

Several different constitutive models have been developed to describe the stress relaxation of nonlinear viscoelastic and viscoplastic materials. Two of these are reviewed in this section.

2.2.2.1 Liu and Krempl’s Model

Liu and Krempl (1979) developed a uniaxial viscoplastic model based on total strain and overstress in which viscoplasticity is represented using piecewise nonlinear viscoelastic functions. The model is linear in the stress-rates and the strain-rates but nonlinear in the Cauchy stress and the strain. It does not require a constant volume.
hypothesis or concept of a yield surface, and utilizes the concept of an equilibrium stress-strain curve. The model is represented by,

\[
\dot{\varepsilon} = \frac{\dot{\sigma}}{E} + \frac{\sigma - g(\varepsilon)}{Ek[\sigma - g(\varepsilon)]}
\]  

(2.14)

where square brackets denote a function of the enclosed quantity. Parameter \( \varepsilon \) is the total strain, \( \sigma \) is the Cauchy stress, and \( \dot{\varepsilon} \) and \( \dot{\sigma} \) are their time derivatives, respectively. 

\( E \) is the elastic modulus. Function \( g \) is a function of the total strain and represents the stress-strain curve for very slow rates of loading (denoted as the equilibrium stress-strain curve). The overstress, \( \sigma - g(\varepsilon) \), is the difference between the Cauchy stress and the corresponding equilibrium stress obtained from the equilibrium stress-strain curve at the same strain value. The model is only valid for positive values of overstress. Function \( k \) is a material function of the overstress (Liu, et al., 1979). In previous studies, modified power laws and single and double exponentials with suitable constants have been used for the \( k \)-function (Krempl, 2001). One common form of the \( k \)-function is the double exponential expression,

\[
k = Ae^{Be^{C[\sigma - g(\varepsilon)]}} \]  

(2.15)

where \( A, B, C, \) and \( D \) are constants for one particular type of material. Eq. 2.14 is similar to the form of Eq 2.6, with

\[
\dot{\varepsilon}_e = \frac{\dot{\sigma}}{E}
\]

(2.16)

and

\[
\dot{\varepsilon}_p = \frac{\sigma - g(\varepsilon)}{Ek[\sigma - g(\varepsilon)]}
\]

(2.17)
Using the chain rule to rearrange Eq. 2.14 yields

$$\dot{\varepsilon}\left(1 - \frac{d\sigma}{d\varepsilon E}\right) = \frac{\sigma - g(\varepsilon)}{Ek[\sigma - g(\varepsilon)]}$$

(2.18)

In the plastic region, $\frac{d\sigma}{d\varepsilon E} << 1$; therefore, the initial plastic strain rate is approximately equal to the loading strain-rate preceding the stress relaxation test.

During a stress relaxation test, the specimen is held at a constant total strain after initial loading, so the total strain rate is constant and zero ($\dot{\varepsilon} = 0$). Eq. 2.14 simplifies to

$$\sigma = \frac{g(\varepsilon_0) - \sigma}{k[\sigma - g(\varepsilon_0)]}$$

(2.19)

where $\varepsilon_0$ is the total constant strain at $t = t_0$. Applying a separation of variables technique, Eq. 2.19, is integrated to yield,

$$t - t_0 = \frac{\sigma - g(\varepsilon_0)}{\sigma_0 - g(\varepsilon_0)} \int k[\sigma - g(\varepsilon_0)] \frac{d(\sigma - g(\varepsilon_0))}{\sigma - g(\varepsilon_0)}$$

(2.20)

where $\sigma_0$ is the stress at the start of relaxation. In the plastic strain range, the value of $\sigma_0 - g(\varepsilon_0)$ is constant for a given loading strain rate regardless of the initial strain value because in this range, the stress-strain curve for a given loading strain rate will be parallel to the equilibrium stress-strain curve. Eq. 2.20 represents an equation for time as a function of over stress, enabling the stress vs time relaxation curve to be plotted and used to predict long-term stress relaxation losses based on short-term stress relaxation tests.

To use Eqs. 2.19 and 2.20 to model stress relaxation, the constant $g(\varepsilon_0)$ and the material $k$-function must be found. After performing multiple stress relaxation tests on the same material, log stress vs log strain rate curves can be extrapolated to a strain rate
of 1.00×10^{-12} \text{s}^{-1} \text{ (strain rate used by Liu, et al. (1979) to establish the equilibrium stress-strain curve). The resulting stress values at a strain rate of 1.00×10^{-12} \text{s}^{-1} for each initial strain should fall on the equilibrium stress-strain curve for that material. Using this equilibrium stress-strain curve, } g(\varepsilon_0) \text{ from Eq. 2.19 can be found for each relaxation test. Then, Eq. 2.19 can be used to calculate the experimental values for } k[\sigma - g(\varepsilon_0)] \text{ and the data can be plotted as } \log k[\sigma - g(\varepsilon_0)] \text{ vs } \sigma - g(\varepsilon_0). \text{ The theoretical } k\text{-function can be determined as the resulting curve fit.}

When appropriate constants and material functions are chosen, Eq. 2.14 can model not only stress relaxation behavior (} \dot{\varepsilon} = 0 \text{, but also creep behavior (} \dot{\sigma} = 0 \text{) and the preceding stress-strain curve for a given material (Liu, et al., 1979). Also, once the equilibrium stress-strain curve and } k\text{-function are determined for a given material, they can be used to simulate a stress relaxation test at any initial strain.}

2.2.2.2 Gupta and Li’s Model

A second model reviewed in this study is the model developed by Gupta and Li (1970). This model is a combination of three equations. First, the plastic strain rate is given by the Orowan equation presented in Gupta and Li (1970),

$$\dot{\varepsilon}_p = \phi \rho b \overline{\nu}$$ \hspace{1cm} (2.21)

where } \phi \text{ is a geometric factor, } \rho \text{ is the density of mobile dislocations, } b \text{ is the Burgers vector of the mobile dislocations, and } \overline{\nu} \text{ is the average velocity of the mobile dislocations. The second equation relates the plastic and elastic strain rates,}

$$\dot{\varepsilon}_p = -\dot{\varepsilon}_e = -\frac{1}{E^*} \frac{d\sigma}{dt}$$ \hspace{1cm} (2.22)
where \( E^* \) is the combined elastic modulus of the testing machine and test specimen. The third and final equation needed to formulate the model is a relation between stress and average dislocation velocity. The Johnston-Gilman relation presented in Gupta and Li (1970) is used to define this average velocity,

\[
\bar{v} = B(\sigma - \sigma_i)^{m^*}
\]

(2.23)

where \( B \) is the average velocity at unit effective stress, \( \sigma_i \) is the internal stress, and \( m^* \) is the dislocation velocity-stress exponent. When Eqs. 2.21, 2.22, and 2.23 are combined, one obtains,

\[
\bar{\sigma} = -K'(\sigma - \sigma_i)^{m^*}
\]

(2.24)

where \( K' = E^* \phi \rho b B \). Integration of Eq. 2.24 yields the equation for stress relaxation,

\[
\sigma - \sigma_i = K(t + a)^{-n}
\]

(2.25)

where \( n = 1/(m^* - 1) \), \( K = (K'(m^* - 1))^{-n} \), and \( a \) is a constant of integration. Eq. 2.25 represents an equation for stress as a function of time. With appropriate constants based on short-term stress relaxation tests, this equation can be used to predict long-term stress relaxation losses.

To use Eqs. 2.24 and 2.25 to model stress relaxation, the constants \( K' \), \( \sigma_i \), \( a \), and \( m^* \) must be found. After performing a stress relaxation test, \( a \) and \( m^* \) can be found by making a plot of log stress rate vs log time. The initial nonlinear portion of this curve is due to the constant \( a \); therefore \( a \) can be determined from this curve. After determining the slope of this graph, without including the data points from initial nonlinear portion,

\[
m^* = \frac{\text{SLOPE}}{\text{SLOPE} + 1}
\]

(2.26)
\( \sigma_i \) can be determined from any two points on the stress relaxation curve using the following equation:

\[
\frac{\sigma_i - \sigma_i}{\sigma_2 - \sigma_i} = \left( \frac{t_1 + a}{t_2 + a} \right)^{-n}
\] (2.27)

where \( \sigma_i \) and \( \sigma_2 \) are the stresses at times \( t_1 \) and \( t_2 \), respectively. \( K' \) can be found by plotting stress rate vs \( (\sigma - \sigma_i) \) and finding the resulting constant according to Eq. 2.24.

These unknown constants must be determined for each stress relaxation test; therefore, this model cannot simulate a stress relaxation test for an arbitrary initial strain.
CHAPTER 3

PREVIOUS STRESS RELAXATION STUDIES

In this chapter, the previous experimental work on stress relaxation of stainless steel is presented. Several sources of data are examined and analyzed to determine an appropriate model for the stress relaxation of stainless steel.

3.1 Experimental Data from Previous Studies

In many of the studies reviewed, successive stress relaxation tests were run on a single specimen at increasing levels of initial strain. After one stress relaxation test was complete, the same specimen would be reloaded to a higher initial strain and a new stress relaxation test would proceed from this new initial strain.

The data from previous studies are commonly reported as log stress vs log strain rate. The strain rate value associated with the initial stress value is commonly taken as the strain rate during initial loading. The data are also commonly reported as a “master curve.” A master curve is a compilation of many different stress relaxation tests on one type of material represented on one graph. It represents a wider range of strain rate values than what is obtained during one stress relaxation test. The data is usually scaled to the lowest initial strain. To construct a master curve, an appropriate scaling relation \((\Delta \log \sigma, \Delta \log \dot{\varepsilon})\) is used to transform the stress and strain rate data onto a single curve. The scaling relation is obtained by plotting log stress vs log strain rate values from each test having the same slope. The resulting line represents the scaling relation. If a scaling relation exists, the overlapping segments of each log stress vs log strain rate curve should match, within experimental error, on the constructed master curve (Yamada, et al., 1973).
3.1.1 Yamada and Li (1973)

Yamada and Li conducted room temperature stress relaxation tests using three different types of stainless steel: Type 304 stainless steel (304SS), heat-treated either at 1100°C for 30 min or at 850°C for 30 min, and Type 316 stainless steel (316SS) heat-treated at 1100°C for 30 min. Each type of stainless steel was air cooled after its respective heat treatment. Three specimens, one of each type of stainless steel, were used for the stress relaxation tests. The specimens had a gage length of 2 in and diameter of 0.10 in. The yield strength of these specimens was not reported. The tests were conducted on a screw-driven Instron machine with an initial loading rate of 0.02 in/min. This is equal to a strain rate of approximately $1.67 \times 10^{-4} \text{ s}^{-1}$ for the reported specimen gage length.

The 304SS specimen heat-treated at 1100°C, Specimen #1, was tested at four initial strains: 0.58%, 1.55%, 2.50%, and 4.45%. The 304SS specimen heat-treated at 850°C, Specimen #2, was tested at four initial strains: 0.58%, 1.51%, 2.46%, and 4.36. The 316SS specimen heat-treated at 1100°C, Specimen #3, was tested at four initial strains: 0.54%, 2.47%, 4.39%, and 9.23%.

The log stress vs log strain rate curves for Specimen #1 (Figure 3.1) exhibited a decrease in slope with increasing initial plastic strain. These curves were concave upwards. Yamada and Li found that this was similar to the shape of the curves for bcc metals, although austenitic stainless steel has an fcc crystal structure. Yamada and Li reported that in other stress relaxation tests, the log stress vs log strain rate curves for fcc metals were concave downwards.
A master curve was generated for Specimens #1, #2, and #3 (Figures 3.2, 3.3, and 3.4 respectively). The slope of the scaling relations used to generate the master curves was 0.101, 0.130, and 0.093, respectively. The slopes of the scaling relations were different for the two types 304 SS because their two different heat treatments produced two different microstructures.
Figure 3.2: Log Stress vs Log Strain Rate Master Curve for Specimen #1

Figure 3.3: Log Stress vs Log Strain Rate Master Curve for Specimen #2
3.1.2 Thomas and Yagee (1975)

Thomas and Yagee performed stress relaxation tests on 316SS at room temperature (23°C). Two types of finishes were utilized to investigate how microstructure affects stress relaxation. The test specimens were prepared from a single rectangular 316SS bar. The bar was reduced to a 0.030 in sheet by alternate cold rolling and solution annealing. There were four specimens in total. Half of the samples were 20 pct cold-worked, and the other half were solution-annealed at 1025°C for 1 hr and water quenched. The yield strength of these specimens was not reported. The relaxation tests were performed on a screw-driven Instron machine at an extension rate of 0.08 in/min (strain rate of approximately $1 \times 10^{-3} \text{ s}^{-1}$). The duration of each test was approximately $10^4 \text{ s}$. 

Figure 3.4: Log Stress vs Log Strain Rate Master Curve for Specimen #3
The first solution-annealed specimen (SA-1) was tested at five initial strains: 2.8%, 8.3%, 13.4%, 18.1%, and 22.6%. The second solution-annealed specimen (SA-2) was tested at nine initial strains between 3.0% and 21.6%. For both solution-annealed specimens, the log stress vs log strain rate curves were concave upwards and showed a decrease in average slope (stress sensitivity) with increasing initial plastic strain. A master curve depicting log stress vs log strain rate was prepared for SA-1 (Figure 3.5) by using an appropriate scaling relation ($\Delta \log \sigma, \Delta \log \dot{\varepsilon}$). The slope of the scaling relation used to construct the master curve was 0.14. This is comparable to the scaling relation of 0.09 used by Yamada and Li to construct the master curve for their solution-annealed specimens.

Figure 3.5: Log Stress vs Log Strain Rate Master Curve for SA-1
The first cold-worked specimen (CW-1) was tested at three initial strains: 3.3%, 9.1%, and 14.7%. The second cold-worked specimen (CW-2) was tested at five initial strains between 3.2% and 14.9%. The log stress vs log strain-rate curves for the cold-worked specimens represent a higher initial stress than was observed in the solution-annealed specimens. Even the lowest initial strains for the cold-worked specimens have a higher initial stress than the highest initial strains for the solution-annealed specimens.

The log stress vs log strain rate curves for CW-1 (Figure 3.6) were concave upwards for strain rates larger than \(5 \times 10^{-6}\) \(s^{-1}\). The curvature varied at lower strain rates except for the test with an initial strain of 3.3%. For intermediate values of initial strain, such as 9.1%, a dip occurs at strain rates near \(1 \times 10^{-5}\) \(s^{-1}\).

![Figure 3.6: Log Stress vs Log Strain Rate Curves for CW-1](image-url)
The log stress vs log strain rate curves for CW-1 did not translate well onto the master curve for SA-1 over the entire strain rate range (Figure 3.7). The slope of the scaling relation used to construct the master curve for CW-1 was 0.60, which is much larger than the slope of the scaling relation (0.14) used to construct the master curve for SA-1. This suggests that the slope increases as the initial plastic strain increases. The high strain rate region (\(>5 \times 10^{-6} \text{ s}^{-1}\)) of CW-1 translated well onto the master curve for SA-1, but the strain rates lower than \(5 \times 10^{-6} \text{ s}^{-1}\) lay above the master curve (due to a lower average slope). This is a different finding than what Hart and Solomon (1973) found for high-purity aluminum, which also has a face-centered cubic microstructure. They showed that a swaged specimen (400% plastic strain) fit onto a master curve determined from a solution-annealed specimen. Cook (1973) also observed a lower average slope of his cold-worked specimens versus his solution-annealed specimens. He suggested that this may be due to the nonuniform deformations introduced by cold rolling, observing that differences in microstructure of cold-worked and solution-annealed specimens may contribute to the observed difference in their log stress vs log strain rate curves.
3.1.3 Anciaux (1981)

Anciaux conducted stress relaxation tests at 290°C on three different types of austenitic stainless steels: 304SS, XM-19, and Aquamet-22 (a warm-worked version of XM-19). The reported room temperature yield strengths of these steels were 29.0, 49.3, and 89.9 ksi, respectively, but there was no indication of how these values were obtained. Specimens with a diameter of 0.635 in and a gage length of 1.0 in were used. The specimens were first allowed to equilibrate at the testing temperature and then strained at a rate of $1.67 \times 10^{-4} \text{s}^{-1}$ until the desired initial strain was reached. Most of the test durations were between 1 and 3 days, but some tests were as short as 4 hours or as long as 10 days.
Four 304SS specimens from the same heat were tested at various strains. Specimen 304T5 was tested at five initial strains: 0.175%, 0.32%, 0.75%, 2.0%, and 3.0%. Specimen 304R10 was tested at two initial strains: 0.75% and 2.0%. Specimen 304R8 was tested at one initial strain of 2.6%. Specimen 304R9 was tested at two initial strains: 0.05% and 2.0%. The resulting stress vs log time curves are shown in Figure 3.8.

Figure 3.8: Stress vs Log Time for 304SS at 290°C

One XM-19 specimen, Specimen 19BR1, from heat 376564 was tested at two initial strains: 0.50% and 1.20%. Three XM-19 specimens from heat 686408 were tested
at various strains: Specimen 19AR1 was tested at two initial strains (0.10% and 0.80%), Specimen 19AR3 was tested at two initial strains (0.20% and 1.20%), while Specimen 19AR4 was tested at one initial strain (0.50%). The resulting stress vs log time curves are shown in Figure 3.9.

![Figure 3.9: Stress vs Log Time for XM-19 at 290°C](image)

One Aquamet-22 specimen, Specimen 22BR1, from heat 376564 was tested at two initial strains: 0.30% and 1.20%. There were three Aquamet-22 specimens from heat 686408 tested at various strains. Specimen 22AR1 was tested at three initial strains:
0.15%, 0.20%, and 0.80%. Specimen 22AR2 was tested at two initial strains: 0.30% and 1.20%. Specimen 22AR3 was tested at two initial strains: 0.50% and 2.00%. The resulting stress vs log time curves are shown in Figure 3.10.

Each of the three stainless steels tested in these experiments showed an approximate linear decrease in stress with log time as seen in Figures 3.8 through 3.10. At initial stresses below yield, stress relaxation occurred in both 304SS and XM-19 stainless steels, but not in the Aquamet-22 stainless steel. Anciaux found that the stress...
loss over the six decades of time (in seconds) that data was taken was approximately 10%. An independent analysis of Anciaux’s data conducted in the course of this research found that for 304SS, between 6% and 12% stress loss occurred in 24 hours. The specimens with a lower initial strain had smaller stress loss, and the specimens with a higher initial strain had a larger stress loss; therefore the percentage stress loss for both low and high initial strain was approximately equal. This means a specimen with a higher initial load will maintain a higher load.

A log stress vs log strain rate graph was constructed for 304SS (Figure 3.11). As seen in Figure 3.11, most of the log stress vs log strain rate curves were concave upwards, which is consistent with previous literature. However, there were two tests that had a variation in curvature at lower strain rates. This is similar to the behavior of the cold-worked specimens tested by Thomas and Yagee (1975). The log stress vs log strain rate curves did not “readily translate onto a master stress-strain trajectory” (Anciaux, 1981).
Stress relaxation tests were performed at room temperature on duplex and superduplex stainless steel specimens. The test specimens were fabricated from two plates of commercially available duplex and superduplex stainless steel with thicknesses of 0.43 in and 0.59 in, respectively. For each type of stainless steel, specimens were cut both longitudinally and transversely to the rolling direction. The room temperature yield strengths of the duplex steels that were cut longitudinally to the rolling direction and transversely to the rolling direction were 74.4 ksi and 82.0 ksi, respectively. The ultimate tensile strengths of these stainless steels were 114.6 ksi and 120.0 ksi, respectively. The room temperature yield strengths of the superduplex steels that were cut longitudinally to the rolling direction and transversely to the rolling direction were 71.2 ksi and 87.8 ksi, respectively. The ultimate tensile strengths of these stainless steels were 121.0 ksi and
127.9 ksi, respectively. The test specimens had a gage length of 0.98 in and a gage diameter of 0.32 in. The specimens were initially loaded to a predetermined stress and strain value at a crosshead rate of approximately 0.020 in/min. This is equal to a strain rate of approximately $3.33 \times 10^{-4} \text{s}^{-1}$ for the reported specimen gage length. The load decrease and strain data were recorded for 30 minutes.

The duplex stainless steel specimen that was cut longitudinally to the rolling direction (D-L) was tested at three initial strains: 1.21%, 1.55%, and 1.84%. This corresponds to initial stress values of 73%, 76%, and 78% of the ultimate tensile strength, respectively. A log stress vs log strain rate graph was constructed for the D-L specimens (Figure 3.12).

![Figure 3.12: Log Stress vs Log Strain Rate Curves for D-L](image-url)
The duplex stainless steel specimen that was cut transversely to the rolling direction (D-T) was tested at four initial strains: 0.99%, 1.42%, 1.94%, and 2.09%. This corresponds to initial stress values of 72%, 75%, 78%, and 81% of the ultimate tensile strength, respectively. A log stress vs log strain rate graph was constructed for the D-T specimens (Figure 3.13).

![Figure 3.13: Log Stress vs Log Strain Rate Curves for D-T](image)

The superduplex stainless steel specimen that was cut longitudinally to the rolling direction (SD-L) was tested at four initial strains: 0.90%, 1.37%, 1.80%, and 2.42%. This corresponds to initial stress values of 74%, 76%, 79%, and 80% of the ultimate tensile strength, respectively. A log stress vs log strain rate graph was constructed for the SD-L specimens (Figure 3.14).
The superduplex stainless steel specimen that was cut transversely to the rolling direction (SD-T) was tested at four initial strains: 0.77%, 1.11%, 1.72%, and 2.20%. This corresponds to initial stress values of 68%, 74%, 75%, and 77% of the ultimate tensile strength, respectively. A log stress vs log strain rate graph was constructed for the SD-T specimens (Figure 3.15).
The shape of the log stress vs log strain rate curves for all test specimens in this study could be represented using straight lines, as seen in Figures 3.12 to 3.15. The log stress vs log strain rate curves for duplex and superduplex stainless steel did not exhibit a decrease in slope with increasing initial plastic strain as was observed in the log stress vs log strain rate curves for austenitic stainless steel (Figure 3.1).

3.2 Analytical Modeling of Stress Relaxation Data from Previous Studies

In this section, Liu and Krempl’s model and Gupta and Li’s model are used to model the stress relaxation data of stainless steel from selected previous studies. Based on the results of this analysis, an appropriate model is chosen to analyze the stress relaxation data from the experiments conducted within this research.
3.2.1 Yamada and Li (1973)

Liu and Krempl (1979) performed an analysis using their model with the data obtained from Yamada and Li’s Specimen #1 tested at an initial strain of 1.55%. They plotted the estimated loading stress-strain curve and equilibrium stress-strain curve in their Figure 1. From this figure, the initial stress value was 39.2 ksi and the equilibrium stress value was 32.1 ksi. This corresponds to an initial overstress value of 7.1 ksi. Their theoretical $k$-function was in the form of Eq. 2.15 with $A = 2.296 \times 10^{-4}$ s, $B = 21.28$, $C = 8.453$ ksi, and $D = 1$.

Using the $k$-function and initial overstress value from Liu and Krempl’s analysis (Liu, et al., 1979), the stress relaxation behavior of Specimen #1 from Yamada and Li (1973) at the remaining initial strain values (0.55%, 2.50%, and 4.45%) can be modeled because the $k$-function is material-dependent and the initial overstress value is dependent upon the initial loading strain rate. The initial stress values are obtained from Figure 1 in Yamada and Li (1973) and are 33.9 ksi, 43.5 ksi, and 50.7 ksi, respectively. Using the initial overstress from Liu and Krempl’s analysis (Liu, et al., 1979) of 7.1 ksi, the equilibrium stress for Specimen #1 at the remaining initial strain values can be back-calculated since this initial overstress value should be constant for all initial strains in the plastic region as described in Section 2.2.2.1. The calculated equilibrium stress values are 26.8 ksi, 36.4 ksi, and 43.6 ksi, respectively. Since the data reported by Yamada and Li was log stress vs log strain rate, Eq. 2.19 is rewritten in terms of strain rate:

$$\dot{\epsilon} = \frac{g(\varepsilon_0) - \sigma}{E[k(\sigma - g(\varepsilon_0))]}, \quad (3.1)$$

The value for $E$ was not reported, so an assumed value of $E = 28,000$ ksi is used. The resulting log stress vs log strain rate curves using Eq. 3.1 are plotted in Figure 3.16.
Figure 3.16: Comparison of the Experimental Log Stress vs Log Strain Rate Curves from Specimen #1 in Yamada and Li (1973) with the Log Stress vs Log Strain Rate Curves Obtained using the Liu and Krempl (LK) Model

The data from Specimen #1 in Yamada and Li (1973) are also analyzed using Gupta and Li’s model. The internal stress values reported in Yamada and Li (1973) were used. The values were 23.2 ksi, 28.2 ksi, 32.1 ksi, and 38.0 ksi for the initial strains of 0.58%, 1.55%, 2.50%, and 4.45%, respectively. When the data are plotted as stress rate vs \((\sigma - \sigma_i)\) the constants \(m^*\) and \(K'\) can be determined according to Eq. 2.24. The values for \(m^*\) are 14.32, 15.31, 15.25, and 26.17, and the values for \(K'\) are \(5.04 \times 10^{-15} ksi^{-13.32} s^{-1}\), \(4.44 \times 10^{-16} ksi^{-14.31} s^{-1}\), \(3.29 \times 10^{-16} ksi^{-14.25} s^{-1}\), and
$3.25 \times 10^{-33} \text{ksi}^{-25.17} \text{s}^{-1}$ for the initial strains of 0.58%, 1.55%, 2.50%, and 4.45%, respectively. Since the data reported by Yamada and Li was log stress vs log strain rate, Eq. 2.24 is rewritten in terms of strain rate:

$$\dot{\varepsilon} = \frac{-K'}{E} (\sigma - \sigma_i)^m$$

(3.2)

The resulting log stress vs log strain rate curves using Eq. 3.2 are plotted in Figure 3.17.

![Figure 3.17: Comparison of the Experimental Log Stress vs Log Strain Rate Curves from Specimen #1 in Yamada and Li (1973) with the Log Stress vs Log Strain Rate Curves Obtained using the Gupta and Li (GL) Model](image-url)
The Gupta and Li model presented in Figure 3.17 more closely fits the data reported by Yamada and Li (1973) than the Liu and Krempl model presented in Figure 3.16, but the unknown constants must be determined for each stress relaxation test. Therefore, the Gupta and Li model gives a better prediction of stress relaxation behavior than the Liu and Krempl model if a stress relaxation test is performed at the actual initial design strain. Although the Liu and Krempl model is not as good at representing experimental stress relaxation behavior as the Gupta and Li model, once the material constants are determined, it can be used to predict the stress relaxation behavior at any arbitrary initial design strain. For this reason, Liu and Krempl’s model is chosen for subsequent analysis in this paper.
CHAPTER 4
CURRENT EXPERIMENTAL PROGRAM AND RESULTS

4.1 Research Plan

Building upon the stress relaxation models and data summarized in Chapters 2 and 3, a combined experimental and analytical approach was employed in this study to shed further light on the likely performance of stainless steel rods used as post-tensioning in strengthening reinforced concrete bridge pier caps for shear, and to make recommendations for rehabilitating pier caps to enhance their shear capacity. First, tensile tests were performed to obtain the mechanical properties of the smooth austenitic stainless steel rods used in this study. Then, stress relaxation tests were performed on these specimens in accordance with ASTM Standard E328-02. The results from these stress relaxation tests will be interpreted in the subsequent chapter using Liu and Krempl’s model (Liu, et al., 1979) as described in Section 2.2.2.1. Finally, this analysis will be used to develop guidelines for design and installation of the post-tensioned stainless steel rod system.

4.2 Design of Experiments

4.2.1 Test Specimens, Experimental Setup, and Instrumentation

Smooth Type 304/304L stainless steel (SS304/304L) rods were used for the stress relaxation experiments. Smooth rods were chosen instead of threaded rods so that the local stress concentrations due to threading were not a factor and the calculation of stress area could be simplified. For a smooth rod, the area is calculated as
\[ A_s = \frac{\pi D^2}{4} \]  

(4.1)

where \( D \) is the diameter of the smooth rod.

The chemical composition of the smooth SS304/304L rods was provided in a Metallurgical Test Report (MTR) and is presented in Table 4.1.

<p>| | | | | | | | | | |</p>
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<td>S</td>
<td>Si</td>
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<td>0.0020</td>
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</tr>
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</table>

The MTR also listed a 0.2% offset yield strength of 83.72 ksi, an ultimate tensile strength of 99.29 ksi, and an elongation of 55.8%. To verify these mechanical properties, tensile tests were performed in accordance with ASTM Standard A370 to obtain an overall stress-strain curve. The tests were performed on an MTS 810 Material Testing System with Hydraulic Wedge Grips. The measured diameter of the test specimens was 0.365 in. For each test, the total specimen length was 16 in, and the initial distance between crossheads varied between 9.7 in and 9.9 in. A clip-on MTS extensometer with a gage length of 1 in was positioned at the approximate midpoint of the test specimen and was used to measure strain. The test setup is shown in Figure 4.1.
A typical stress-strain curve obtained for the material is presented in Figure 4.2, where both the strain obtained from the extensometer and the strain calculated via crosshead displacement are presented. This tensile test was conducted at a crosshead displacement of 0.20 in/min and the initial distance between crossheads was 9.7 in. This corresponds to calculated a strain rate of $3.44 \times 10^{-4} \text{s}^{-1}$. This closely corresponds to the strain rate obtained using data from the extensometer of $3.49 \times 10^{-4} \text{s}^{-1}$. The minor
difference in these values might be due to the slip that can occur within the grips and the compliance of the testing machine. The similar appearance of the two stress-strain curves presented in Figure 4.2 and the close correspondence of the strain rate calculated for both curves validates the use of the crosshead displacement to calculate strain when the strain limit of the extensometer is reached. When the extensometer strain limit of 20% was reached, the test was briefly paused to remove the extensometer and prevent any damage to it. Then, the test was resumed and continued to an approximate ultimate elongation of 32% and an ultimate stress of 120 ksi. The modulus of elasticity, \( E \), was found by adding a trendline to the stress-strain curve between the origin and a strain of 0.0012 in/in. This resulted in an elastic modulus of 27,462 ksi. Using this elastic modulus, the 0.2% offset yield strength was found to be 97.1 ksi.

![Figure 4.2: Stress vs Strain for SS304/304L](image-url)
4.2.2 Experimental Procedure for Stress Relaxation Tests

During the preliminary design stage, an alternative experimental setup was considered, as summarized in Appendix A. This approach was discarded early during the experimental design stage due to the unreliability of the instrumentation readings and poor temperature regulation. Therefore, the same servo-hydraulic testing machine described in Section 4.2.1 was used to perform the stress relaxation tests. The same specimens, test setup, and loading procedure as used for the tensile tests were used for the stress relaxation tests. The only difference for the stress relaxation tests was that each specimen was loaded using a crosshead displacement of 0.20 in/min to a specified strain level as shown in Table 4.2, and then the crosshead displacement was held constant throughout the duration of the test. During initial loading and throughout the duration of the relaxation test, the load, displacement, strain, and time were recorded every ½ s.

Tests #2, #4, and #6 were intended to replicate Tests #1, #3, and #5 to assess the repeatability of results.

Table 4.2: Stress Relaxation Experimental Program using $f_u = 120$ ksi

<table>
<thead>
<tr>
<th>Test Number</th>
<th>Initial Strain (in/in)</th>
<th>Initial Stress (ksi)</th>
<th>Initial Distance between Crossheads (in)</th>
<th>Test Time (hr)</th>
</tr>
</thead>
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<td>0.0367</td>
<td>0.95f_u 113.84</td>
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<td>9.8125</td>
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</tr>
<tr>
<td>5</td>
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<td>0.96f_u 114.74</td>
<td>9.6875</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
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<td>0.96f_u 115.51</td>
<td>9.6875</td>
<td>2</td>
</tr>
</tbody>
</table>
4.3 Experimental Results

The stress vs strain curves obtained during loading are displayed in Figure 4.3, and the values obtained from these curves are presented in Table 4.3.

![Stress vs Strain Curve](image)

**Figure 4.3: Stress vs Strain Curves Prior to Relaxation Tests**

Table 4.3: Values Obtained from Stress vs Strain Curves

<table>
<thead>
<tr>
<th>Test Number</th>
<th>Strain Rate during Loading (s(^{-1}))</th>
<th>Initial Elastic Modulus (ksi)</th>
<th>0.2% Offset Yield Strength (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.44×10(^{-4})</td>
<td>26,404</td>
<td>98.8</td>
</tr>
<tr>
<td>2</td>
<td>3.36×10(^{-4})</td>
<td>26,964</td>
<td>97.1</td>
</tr>
<tr>
<td>3</td>
<td>3.39×10(^{-4})</td>
<td>27,292</td>
<td>97.6</td>
</tr>
<tr>
<td>4</td>
<td>3.38×10(^{-4})</td>
<td>27,290</td>
<td>91.9</td>
</tr>
<tr>
<td>5</td>
<td>3.53×10(^{-4})</td>
<td>26,525</td>
<td>95.3</td>
</tr>
<tr>
<td>6</td>
<td>3.51×10(^{-4})</td>
<td>26,582</td>
<td>96.3</td>
</tr>
</tbody>
</table>
The resulting stress vs time curves obtained from the six relaxation tests are shown in Figure 4.4. The results are plotted on a log time scale in Figure 4.5.

Figure 4.4: Stress Relaxation Results
The stress relaxation data are plotted as log stress vs log strain rate in Figure 4.6. As discussed in Chapter 3, the strain rate value for the initial stress value is assumed to be equal to the initial loading strain rate prior to the onset of stress relaxation. The log stress vs log strain rate curves are concave upwards, which is consistent with the results of the previous research surveyed in Chapter 3.
Figure 4.6: Log Stress vs Log Strain Rate for SS304/304L
CHAPTER 5

INTERPRETATION OF RESULTS

In this chapter, the stress relaxation test results presented in Chapter 4 are interpreted using the Maxwell model (Dowling, 1993), the SLS model (Lakes, 2009), and the Liu and Krempl model (Liu, et al., 1979). First, the experimental results from Tests #2 and #6 are used to show that the stress relaxation phenomenon is more complex than the Maxwell or standard linear solid models would suggest. Then, based on the appraisal of existing stress relaxation models in Chapter 3, the Liu and Krempl model is used to analyze the stress relaxation behavior of Tests #1 through #6.

5.1 Interpretation of the Experimental Data using the Maxwell and the Standard Linear Solid Models

According to Eq. 2.11, the constants needed to use the Maxwell model are $E_0$, $\varepsilon_T$, and $\eta$. $E_0$ is the initial elastic modulus for each test as listed in Table 4.3. For Tests #2 and #6, $E_0$ is 26,964 and 26,582 ksi, respectively. The boundary conditions required to obtain Eq. 2.11 from Eq. 2.10 are that $\varepsilon_T = 0$ and that the plastic strain is equal to zero at $t = 0$; therefore, $\varepsilon_T$ is taken as the initial elastic portion of the total strain. The initial stress values, $\sigma(0)$, from Table 4.2 for Tests #2 and #6 are 112.70 and 115.51 ksi, respectively. To satisfy the boundary conditions listed above, then, $\varepsilon_T$ must equal 0.00418 and 0.00435 in/in for Tests #2 and #6, respectively. The coefficient of viscosity, $\eta$, is obtained using Eq. 2.9. The $\sigma$ and $\dot{\varepsilon}_p$ values needed to use Eq. 2.9 are represented graphically in Figure 4.6. According to the Maxwell model, $\eta$ should be
constant. However, when $\eta$ is calculated for Tests #2 and #6, its value changes with each data point; therefore, it will be taken as an average value. This is the first indication that the Maxwell model is not able to represent the stress relaxation behavior of stainless steel accurately. For Tests #2 and #6, $\eta$ is $4.50 \times 10^9 \text{ ksi} \cdot \text{s}$ and $7.01 \times 10^9 \text{ ksi} \cdot \text{s}$, respectively. The resulting stress vs time curves are plotted as solid lines in Figure 5.1.

According to Eq. 2.13, the constants needed to use the SLS model are $E_1$, $E_2$, $\varepsilon_T$, and $\eta$. For Tests #2 and #6, the values of $\varepsilon_T$ and $\eta$ are assumed to be the same values as used for the Maxwell model. The boundary conditions used to obtain Eq. 2.13 were that $\sigma(0) = E_1 \varepsilon_T + E_2 \varepsilon_T$ and that (like the Maxwell model) the plastic strain is equal to zero at $t = 0$; therefore, $E_0 = E_1 + E_2$. Then, the values for $E_1$ and $E_2$ are chosen to satisfy the conditions listed above and to obtain a curve that best fits the data. For Tests #2 and #6, the values for $E_1$ are both 20,000 ksi while the values for $E_2$ are 6,964 and 6,582 ksi, respectively. The resulting stress vs time curves are plotted as dotted lines in Figure 5.1.
Since the calculated values for $\eta$ are very high, the Maxwell and SLS models both appear to be linear within the time interval shown in Figure 5.1, even though the functions are exponential in nature. It is clear that neither model captures the shape of the stress relaxation curve when the necessary model constants are calculated using Eqs. 2.7 to 2.13 and appropriate boundary conditions.

**5.2 Interpretation of the Experimental Data using the Liu and Krempl Model**

To use the Liu and Krempl model, an equilibrium stress-strain curve is first established. Using Figure 4.6, stress values are extrapolated to a strain rate of
1.00×10^{-12} \text{ s}^{-1} (strain rate used by Liu, et al. (1979) to establish the equilibrium stress-strain curve). Extrapolated values from all tests except Test #4 lie near the equilibrium stress-strain curve (Figure 5.2) that is generated for analysis purposes.

![Figure 5.2: Comparison of the Equilibrium and Overall Stress-Strain Curve](image)

Using the equilibrium stress-strain curve in Figure 5.2, \( g(\varepsilon_0) \) from Eq. 2.19 is found for each relaxation test, as summarized in Table 5.1. A plot of \( \log k[\sigma - g(\varepsilon_0)] \), as determined from Eq. 2.19, vs \( \sigma - g(\varepsilon_0) \) is then constructed, as shown in Figure 5.3. Test #4 cannot be plotted on the graph because at \( t = 48 \text{ s} \), the overstress value becomes negative. A theoretical \( k \)-function in the form of Eq. 2.15 is fit to the relaxation data in Figure 5.3 using \( A = 0.065 \text{ s} \), \( B = 30.4 \), \( C = 2.74 \text{ ksi} \), and \( D = 0.57 \).
Using Eq. 2.20, the time as a function of overstress is found. The initial stress is added to this overstress value, and the resulting stress vs time relationship is shown in Figure 5.4.
As seen in Figure 5.4, the Liu and Kreml model captures the shape of the stress relaxation curve and appears to reasonably describe the stress relaxation of the austenitic stainless steel used in this study. In particular, the model fits the data for Tests #2, #3, and #6 (Figure 5.4) very well. For the initial stress values tested, the model predicts an approximate stress loss of 10% in 50 years, with 80% of this stress loss occurring in the first hour of constant straining.
CHAPTER 6
CONCLUSIONS AND DESIGN RECOMMENDATIONS

6.1 Summary of Major Conclusions

This research examined the stress relaxation phenomenon that may occur in stainless steel rods that are used as part of an all-stainless steel externally post-tensioned rod system for reinforced concrete pier cap shear strengthening. A review of previously reported experimental data on the stress relaxation of stainless steel was accompanied by an experimental program utilizing six specimens of Type 304/304L stainless steel stainless steel rods similar to those likely to be candidates for pier cap rehabilitation. It was found that stress relaxation of stainless steel over an extended period is predictable based on short-term stress relaxation experiments. Using these results, guidelines for design and installation the post-tensioned stainless steel rods were developed.

The experimental results from Tests #2 and #6 show that the stress relaxation phenomenon in stainless steel is more complex than the commonly used Maxwell or SLS models would suggest. The stress relaxation of stainless steel can be modeled using either Gupta and Li’s model or Liu and Krempf’s model. The Gupta and Li model predicts stress relaxation behavior more accurately for design than the Liu and Krempf model if a stress relaxation test is performed at the actual initial design strain. Although the Liu and Krempf model is not as good as the Gupta and Li model at representing experimental stress relaxation behavior, once the material constants are determined, it can be used to predict the stress relaxation behavior at any arbitrary initial design strain.
The stress relaxation behavior of the Type 304/304L stainless steel that was used in this study can be represented using Liu and Krempl’s model for stress relaxation. When the appropriate material constants were used, the Liu and Krempl model accurately captured the shape of the stress relaxation curve and calculated theoretical stress values that were fairly close to those obtained experimentally. For the initial stress values tested, the model predicts an approximate stress loss of 10% in 50 years, with 80% of this stress loss occurring in the first hour after loading.

### 6.2 Design Recommendations

A *Guide Specification for Design and Installation of Externally Post-Tensioned Rods for Enhancing the Shear Capacity of Reinforced Concrete Pier Caps* was developed from the research reported herein. This *Guide Specification* can be found in Appendix B. The model predicts an approximate stress loss of 10% in 50 years for the initial plastic strains used in this study that are associated with stress values above $0.9f_u$; therefore, a limiting design strength of $0.6f_u$ is sufficient so that stress relaxation will not be a problem for the intended purpose. In general, as long as stress relaxation is accounted for in the initial design calculations, it will not be a problem for the intended purpose.

### 6.3 Recommendations for Further Study

This research only considered strains in the plastic region of the stress-strain curve; therefore, future work should examine elastic strains. Also, the experimental program only examined the stress relaxation that occurs in austenitic stainless steel. In the future, the stress relaxation that occurs of other types of stainless steel, such as ferritic or duplex, should be explored.
APPENDIX A

DEVELOPMENT OF STRESS RELAXATION TEST PROCEDURE

A.1 Preliminary Test Fixture Design and Instrumentation

A test fixture was designed in order to apply a constant displacement on the test specimens. Special consideration was made to ensure that only the test specimen would strain and not the entire testing frame. The testing frame consisted of two 10”x10”x1” steel plates separated by four 12-inch steel angles (L3x3x1/2). The relative stiffness of the testing frame to the test specimen was 14.01. Calculations were performed to ensure that flexural buckling, flexural-torsional buckling, and local buckling would not occur. To measure the load, a homemade thru-hole load cell with a capacity of 200 kip was securely fastened to the end of the threaded rod using a nut and washer. A dial gage was attached at the center of the bar to measure strain. The test setup is shown in Figures A.1 and A.2.
Figure A.1: Stress Relaxation Test Setup Elevation View
A.2 Preliminary Experimental Procedure

A hydraulic jack was used to apply the load at a maximum strain rate of 0.02 in/min until the desired initial stress level was reached. The setup is illustrated in Figure A.3. After the desired initial stress level was reached, the hexagonal nut labeled “Point A” in Figure A.3 was tightened down and the hydraulic jack and loading table were removed. The tests were run in a climate-controlled environment.

A.3 Preliminary Findings

Through preliminary testing, it was found that the temperature could not be well-regulated in the testing room. Also, since the load cell was homemade, the readings frequently drifted.
Figure A.3: Loading Setup
APPENDIX B

GUIDE SPECIFICATION FOR DESIGN AND INSTALLATION OF EXTERNALLY POST-TENSIONED RODS FOR ENHANCING THE SHEAR CAPACITY OF REINFORCED CONCRETE BRIDGE PIER CAPS

B.1 General Requirements

B.1.1 Scope

This Guide Specification is intended for the design and installation of stainless steel rods required for use in a post-tensioned stainless steel rod system for in-place shear strengthening and repair of reinforced concrete pier caps.

B.1.2 Referenced Standards and Specifications

The following standards and documents are referred to in this Guide Specification:


B.2 Material Requirements

The materials shall meet the requirements prescribed in this specification. To reduce the risk of galvanic corrosion, only compatible grades of materials as defined in MIL-STD-889B shall be joined together. If incompatible materials are in contact with one another, proper electrical insulation is required.

B.2.1 Post-Tensioning Rods

Stainless steel tensioning rods shall meet the requirements set forth in ASTM Standard A276. Steel rods that have been damaged or that indicate the presence of cracking or corrosion shall be rejected.

B.2.2 Brackets

Brackets shall meet the requirements of ASTM Standard A276 and shall be structural shapes designed to withstand all forces imposed by the post-tensioning rods.

B.2.3 Nuts and Washers

Nuts and washers shall meet the requirements of ASTM Standard F593.

B.3 Design

The external post-tensioning system shall be designed to provide the required capacity to withstand the shear deficiency in the reinforced concrete pier cap. The required shear capacity of the reinforced concrete pier cap, \( V_u \), shall be calculated using an HL-93 loading and the STRENGTH I limit state as described in Chapter 3 of the *AASHTO LRFD Bridge Design Specifications Fourth Edition (2007)* [hereinafter denoted as the *AASHTO Specifications*]. The nominal shear strength of the reinforced concrete pier cap, \( V_n \), shall be calculated in accordance with Article 5.8.3.3 of the *AASHTO Specifications*. 
Specifications. The tensile force in each rod, $T$, shall not exceed $0.6 f_u \times A_g$, where $f_u$ is the tensile strength of the stainless steel rod as specified in ASTM Standard A276, and $A_g$ is the gross area of the stainless steel rod. The number of stainless steel rods needed for in-place shear strengthening is calculated as,

$$\text{# of rods} = \frac{V_u - \phi V_n}{T}$$  \hspace{1cm} (B.1)

where the quantity, $V_u - \phi V_n$, is the shear deficiency in the reinforced concrete pier cap and $\phi$ is the resistance factor specified in the AASHTO Specifications, Article 5.5.4.2.

The maximum spacing between the post-tensioned stainless steel rods shall be calculated using the equations in Article 5.8.2.7 of the AASHTO Specifications, and each potential diagonal shear crack shall be crossed by at least one rod.

**B.4 Installation**

The Contractor shall construct the post-tensioned stainless steel bracket system as shown in the Plans.

**B.4.1 Calibration**

Each jack and its gage shall be calibrated and shall be accompanied by a certified calibration chart. The Contractor shall provide a copy of this chart to the Engineer. Jacks and gauges shall be recalibrated and recertified annually and any time there are indications that the jack calibration is in error.

**B.4.2 Inspection and Surface Preparation**

The concrete surface of the pier cap shall be inspected prior to the installation of the post-tensioning system. Any irregularities in the concrete surface that would prevent
uniform contact between the bracket and the pier cap shall be removed prior to post-tensioning the rods. The slope of the surfaces of parts in contact with the nut and washer shall not exceed 1:20 with respect to a plane that is normal to the rod axis. The use of grout is permitted if these requirements are not satisfied.

**B.4.3 Stressing Requirements**

The load applied to the stainless steel rods shall be measured by the Contractor with jacking gages. The Engineer shall use pressure cells or other appropriate methods to check jacks, gauges and calibration charts before and during tensioning. The Contractor's jacking pressure measurements shall make appropriate allowance for the loss of prestress in accordance with Article 5.9.5 of the *AASHTO Specifications*. In all stressing operations, the Contractor shall keep stressing forces symmetrical about the member's vertical axis and shall take appropriate safety precautions to prevent accidents.

**B.5 Inspection and Quality Assurance**

**B.5.1 Inspection and Testing**

The Contract Documents shall stipulate specific testing requirements, sampling frequency, acceptance criteria and tolerances. The Engineer shall develop a checklist to assist the Inspector in assessing conformance to all installation requirements in the Contract Documents and to ensure proper record-keeping.

**B.5.2 Quality Assurance**

The Contractor shall have in place a Quality Assurance Program (QAP) designed to ensure that all quality and engineering requirements have been recognized and that
consistent and uniform control of these requirements is established and maintained during the conduct of the work. The QAP shall be incorporated into the Contract Documents.

**B.5.3 In-service Inspection of Post-tensioned Stainless Steel Rods**

In-service inspection of externally post-tensioned stainless steel rods used to strengthen reinforced concrete pier caps in shear shall be performed no later than five (5) years following original installation. Tension forces in the rods shall be measured by approved means. Any necessary re-tensioning shall be performed in accordance with Article B.4.3. Loss of tensioning and corrective action shall be noted in the bridge maintenance record.

Rods that have been re-tensioned shall be inspected no later than five (5) years following re-tensioning. Rods found to have maintained their post-tension force during the first five-year interval need not be inspected further.
REFERENCES


