Theory behind the Tool for Assessing Separation And Throughput (TASAT)

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Procedure Framework

- Low noise descent leg starts from an intermediate metering point
- Target separation suggested at metering point to assure uninterrupted operation at desired confidence
- Controller free to vector to establish target separation at metering point
- Preferably no controller intervention during low noise descent leg
  - Intervene only when separation violation is predicted, low probability

Diagram:
- Streaming/Sequencing
- Spacing
- Monitoring/Intervention
- Descent from Cruise
- Descent to Final
- Missed App.
- Height
- Intermediate metering point
- Target separation
- FAF
- Conventional
Trajectory Variation and Separation

Along Track Distance

- Separation at Metering Point
- Time
- Initial Position of Trailing AC
- Initial Position of Leading AC
- Leading AC
- Trailing AC
- Max. Final Separation
- Min. Final Separation
- Shaded area indicates trajectory variation
- Final separation will be a probability distribution
The Separation Problem

- Probability distribution of final separation
  - Target separation – assure final separation at desired confidence

![Diagram showing probability density vs separation with key points labeled: Separation Minimum, Final Separation Buffer $\beta_f$, Final Separation, Target Separation]
The Inverse Separation Problem

Along Track Distance

Separation at Metering Point in Time

Intermediate Metering Point

Runway Threshold

Max. Final Separation

Min. Final Separation

Initial Position of Trailing AC

Initial Position of Leading AC

Leading AC

Trailing AC

• Shaded area indicates trajectory variation
• Final separation will be a probability distribution
The Inverse Separation Problem

- Feasible separation for an aircraft pair
  - Trajectory: distance vs. time
    \[ D = f(t) \quad t = f^{-1}(D) \]
  - Distance at metering point
    \[ D_0 = f(0) \]
  - Time when leading aircraft is at runway threshold
    \[
    \begin{cases} 
      t_{l,f} = f_{l}^{-1}(0) \\
      t_{t,f} = f_{t}^{-1}(-S_f) 
    \end{cases}
    \]
  - Feasible separation
    \[ s = f_{l}(0) - f_{t}(t_{t,f} - t_{l,f}) = D_0 - f_{t}(t_{t,f} - t_{l,f}) \]
Conditional Probability Method

- Probability distribution of feasible separation
  - Distribution determined from large pool of aircraft trajectories

Feasible Separation, $p_1$
AC Type A – Type B

Feasible Separation, $p_2$
AC Type B – Type A

Target Separation $S_1$
Conditional Probability Method

- **Sequence-Independent Target Separation**
  - Conditional level of confidence: probability of no separation violations if separation at metering point exactly equal to the target separation
  - Given pdf $p_i$ of the feasible separations for aircraft sequence $i$, target separation $S_i$, the conditional level of confidence is

  $$P_{R_i} = \int_0^{S_i} p_i ds$$

  - Conditional level of confidence will be different for different aircraft sequences
  - Target separation: the minimum $S_i$ that every single $P_{R_i}$, or the average of $P_{R_i}$ is greater than or equal to the desired value
Conditional Probability Method

- Sequence-Specific Target Separations - Definition

- Probability Density

- Separation at Metering Point

Target Separation $S_{i1}$
Feasible Separation, $p_1$
AC Type A – Type B

Target Separation $S_i$

Target Separation $S_{i2}$
Feasible Separation, $p_2$
AC Type B – Type A
Conditional Probability Method

- **Sequence-Specific Target Separations**
  - Use different target separations for different aircraft sequences to achieve same conditional level of confidence for all sequences.
  - Given pdf $p_i$ of the feasible separations and target separation $S_{li}$ for aircraft sequence $i$, the conditional level of confidence is

  $$ P_{Ri} = \int_0^{S_{li}} p_i ds $$

  - Lower average target separation can be achieved, throughput benefits.
Conditional Probability Method

- Sequence-Specific Target Separations - Benefits

![Diagram showing probability density and cumulative probability for different separations between AC Type A and Type B at metering points, with feasible separations and cumulative probabilities indicated.]
Actual Separations Will Not Be Exact
- Depend on traffic condition and target separation

- Actual Separation in Arrival Stream

- Actual Traffic Unadjusted, $p_T$
- Actual Traffic Adjusted, $p_{Ta}$
- Normal MIT Restriction $S$
- Target Separation $S_i$

Separation at Metering Point
Actual Separation in Arrival Stream

- Mean Sequence in Unadjusted Arrival Stream
  \[ E(s_T) = \int_0^\infty p_T s ds \]

- Mean Sequence in Adjusted Arrival Stream
  \[ E(s_{Ta}) = \int_0^\infty p_{Ta} s ds \]

- Traffic Throughput
  \[ C = 360\varnothing E(T) \]
Total Probability under Unadjusted Traffic

- Suitability of procedure without using a higher target separation

Feasible Separation, $p_1$
AC Type A – Type B

Feasible Separation, $p_2$
AC Type B – Type A

Actual Traffic Unadjusted, $p_T$

A small slice of traffic at separation $s$
Total Probability Method

- **Total Probability under Unadjusted Traffic**
  - Probability for a small slice of traffic at $s$ for aircraft sequence $i$
    \[ dP_{T_i,s} = p_T ds \int_0^s p_i dx \]
  - Total probability for aircraft sequence $i$
    \[ P_{T_i} = \int_0^\infty \left( \int_0^s p_i dx \right) p_T ds \]
  - Overall total probability
    \[ P_T = \sum_i P_i P_{T_i} = \sum_i P_i \int_0^\infty \left( \int_0^s p_i dx \right) p_T ds \]
Total Probability Method

- **Total Probability under Adjusted Traffic**
  - Total level of confidence for target separation

![Diagram]

- Feasible Separation, $p_1$
- AC Type A – Type B

- Feasible Separation, $p_2$
- AC Type B – Type A

- Actual Traffic Unadjusted, $p_T$

- Actual Traffic Adjusted, $p_{Ta}$

- Target Separation $S_1$

- A small slice of traffic at separation $s$
Total Probability Method

- **Sequence-Independent Target Separation**
  - Total probability for aircraft sequence $i$
    \[
P_{Tai} = \int_0^\infty \left( \int_0^s p_i dx \right) P_{Ta} ds
    \]
  - Overall total probability
    \[
P_{Ta} = \sum_i P_i \int_0^\infty \left( \int_0^s p_i dx \right) P_{Ta} ds
    \]
  - The pdf of the adjusted traffic separations is to the right of the pdf for the unadjusted traffic, resulting in a higher total level of confidence
Sequence-Specific Target Separations
- Generic model of separation in adjusted traffic needed

- Target Separation $S_{l1}$
- Feasible Separation, $p_1$
- AC Type A – Type B

- Target Separation $S_l$

- Target Separation $S_{l2}$
- Feasible Separation, $p_2$
- AC Type B – Type A

- Actual Traffic Adjusted, $p_{Ta1}$
- Unadjusted, $p_T$

- Actual Traffic Adjusted, $p_{Ta2}$

Separation at Metering Point
Sequence-Specific Target Separations

- Probability for a small slice of traffic at $s$ for aircraft sequence $i$
  \[ dP_{Tai,s} = p_{Tai} \int_0^s p_i \, dx \]

- Total probability for aircraft sequence $i$
  \[ P_{Tai} = \int dP_{Tai,s} = \int_0^\infty \left( \int_0^s p_i \, dx \right) b_{Tai} \, ds \]

- Overall total probability
  \[ P_{Ta} = \sum_i P_i P_{Tai} = \sum_i P_i \int_0^\infty \left( \int_0^s p_i \, dx \right) b_{Tai} \, ds \]
Numeric Examples

- Traffic Separation at SACKO – Metering Point for KSDF CDA

Graph showing probability density against separation in nm.
Numeric Examples

- **Modeling Traffic Separation**
  - Erlang distribution model and parameter estimation

\[ p(s) = \begin{cases} \frac{\lambda^k s^{k-1} e^{-\lambda s}}{(k-1)!} & k = 1, 2, \ldots; s \geq 0 \\ 0 & \text{otherwise} \end{cases} \]

- Model parameters for traffic separation at SACKO

<table>
<thead>
<tr>
<th>Traffic Condition</th>
<th>Traffic Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E(s)</td>
<td>(\sigma_s)</td>
</tr>
<tr>
<td>Unadjusted</td>
<td>15.79 nm</td>
<td>5.88 nm</td>
</tr>
<tr>
<td>Adjusted</td>
<td>19.05 nm</td>
<td>4.28 nm</td>
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</table>
CDA to Runway 17R, Target Separation = 15 nm

Initial separation, conditional level of confidence, and traffic throughput.

<table>
<thead>
<tr>
<th>Aircraft Type/Sequence</th>
<th>Ideal Case</th>
<th>$S_i = 15$ nm</th>
<th>$P_{Ri} = 68.2%$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$C_i$</td>
<td>$E(S_i)$</td>
<td>$P_{Ri}$</td>
</tr>
<tr>
<td>B757 – B757</td>
<td>32.02</td>
<td>14.21</td>
<td>71.8%</td>
</tr>
<tr>
<td>B757 – B767</td>
<td>36.89</td>
<td>11.64</td>
<td>100.0%</td>
</tr>
<tr>
<td>B767 – B757</td>
<td>25.82</td>
<td>17.82</td>
<td>1.5%</td>
</tr>
<tr>
<td>B767 – B767</td>
<td>35.18</td>
<td>12.22</td>
<td>99.6%</td>
</tr>
<tr>
<td>Average</td>
<td>31.88</td>
<td>13.97</td>
<td>68.2%</td>
</tr>
</tbody>
</table>

Total levels of confidence assuming 50-50 traffic mix of B757 and B767.

<table>
<thead>
<tr>
<th>Sequence</th>
<th>$P_T$</th>
<th>$P_{Ta}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B757–B757</td>
<td>55.6%</td>
<td>87.5%</td>
</tr>
<tr>
<td>B757–B767</td>
<td>74.2%</td>
<td>97.1%</td>
</tr>
<tr>
<td>B767–B757</td>
<td>33.6%</td>
<td>59.7%</td>
</tr>
<tr>
<td>B767–B767</td>
<td>70.3%</td>
<td>95.7%</td>
</tr>
<tr>
<td>Overall</td>
<td>58.7%</td>
<td>85.0%</td>
</tr>
</tbody>
</table>
The Effect of Wind

Nominal Wind Profiles

- Tail Wind
- Zero Wind
- Head Wind

Feasible Separation, nm

- B757-B757
- B757-B767
- B767-B757
- B767-B767
Numeric Examples

- The Effect of the Location of the Metering Point

![Bar chart showing the effect of different locations on the metering point.](image-url)
The Effect of the Location of the Metering Point

<table>
<thead>
<tr>
<th>Metering Point</th>
<th>$S_f^a$</th>
<th>SACKO</th>
<th>CHERI</th>
<th>-34 nm</th>
<th>-25 nm</th>
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</thead>
<tbody>
<tr>
<td>B757–B757</td>
<td>15</td>
<td>13.69</td>
<td>10.65</td>
<td>9.41</td>
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<tr>
<td>$\beta_f^a$</td>
<td>0.27</td>
<td>0.21</td>
<td>0.19</td>
<td>0.26</td>
<td></td>
</tr>
<tr>
<td>$C$</td>
<td>30.40</td>
<td>30.70</td>
<td>31.00</td>
<td>30.73</td>
<td></td>
</tr>
<tr>
<td>B757–B767</td>
<td>1.25</td>
<td>0.96</td>
<td>0.58</td>
<td>0.34</td>
<td></td>
</tr>
<tr>
<td>$C$</td>
<td>28.88</td>
<td>28.55</td>
<td>29.25</td>
<td>30.73</td>
<td></td>
</tr>
<tr>
<td>B767–B757</td>
<td>-0.94</td>
<td>-0.66</td>
<td>-0.46</td>
<td>-0.46</td>
<td></td>
</tr>
<tr>
<td>$C$</td>
<td>30.40</td>
<td>30.70</td>
<td>31.00</td>
<td>30.73</td>
<td></td>
</tr>
<tr>
<td>B767–B767</td>
<td>1.02</td>
<td>1.10</td>
<td>0.95</td>
<td>0.62</td>
<td></td>
</tr>
<tr>
<td>$C$</td>
<td>28.88</td>
<td>28.55</td>
<td>29.25</td>
<td>30.73</td>
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<tr>
<td>Average</td>
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<td>0.31</td>
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<td>29.60</td>
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<td>30.73</td>
<td></td>
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$^a$ Target separations and final separation buffer are in nm.
Future Directions

- Develop Generic Traffic Separation Model
  - Radar data of traffic flow under different miles-in-trail restrictions
  - Controller-in-the-loop simulation

- Traffic coordination for merging arrival routes
  - Target separation can extended to a traffic coordination problem
  - Wind profile different on different routes
  - Aircraft pairs to be treated as sequence/procedure combinations
  - Use distances to metering points instead of target separation
  - Time of arrival to each procedure’s respective metering point