Algorithms for Economically and Environmentally Efficient Terminal Area Transition Metering

John-Paul Clarke, Liling Ren, Clayton Tino (Georgia Tech)
Marcus Lowther (was Georgia Tech now Metron Aviation)

28 July 2009
Achieving the Desired Spacing
Objective Function: Minimizing Fuel Over Given Distance

\[
\min Z = \sum_{n=1}^{N} f_{M_d_i} \left( T_i - \Delta t_i \right)
\]
Inclusion of \( dt \) in objective function is a simple addition, but necessitates additional constraints [1].

Must now approximate bilinear term with a grid.

Further constraints at right limit selection of grid points to four adjacent planes.

Objective Function:
Minimize Fuel Burn Over Given Distance

\[
\begin{aligned}
r_i &= f\big|_{M_i} \Delta t_i \\ 
\lambda_{M_i} &= F_k \Delta T_i \\
\end{aligned}
\]

Fuel burn value equal to sum of fuel grid points

\[
\sum_k \sum_l F_k \lambda_{M_i} = \text{sum of fuel grid points}
\]

Combined bilinear term equal to sum of grid points

\[
\Delta t_i = \sum_k \sum_l \Delta T_i \lambda_{M_i}
\]

\[
r_i = \sum_k \sum_l (F_k \Delta T_i) \lambda_{M_i}
\]

Sum of grid points must be equal to a complete term

\[
\sum_k \sum_l \lambda_{M_i} = 1
\]

\[
\mu_{M_i} = \sum_l \lambda_{M_i}
\]

Use SOS variables to limit number of adjacent planes

\[
\eta_{M_i} = \sum_k \lambda_{M_i}
\]

\[
\lambda_{M_i} \geq 0
\]

\[
\mu_{M_i} \in \text{SOS2}
\]

\[
\eta_{M_i} \in \text{SOS2}
\]

Can have a positive or negative term (speed increase or decrease)

\[
r_i \rightarrow \text{free}
\]
Constraints: Sequence and Spacing

- Necessary for aircraft to rearrange scheduled arrival times
- Allows algorithm to examine all possible arrival sequences

Separation constraints for a pair of aircraft

Example for three aircraft, variable sequence constraints will create additional constraints

\[ T_2 - T_1 + \alpha_{1,2} \leq Pz_1 \]
\[ 2\alpha_{2,1} - (T_2 - T_1 + \alpha_{2,1}) \leq P(1 - z_1) \]

\[ T_3 - T_1 + \alpha_{1,3} \leq Pz_2 \]
\[ 2\alpha_{3,1} - (T_3 - T_1 + \alpha_{3,1}) \leq P(1 - z_2) \]

\[ T_3 - T_2 + \alpha_{2,3} \leq Pz_3 \]
\[ 2\alpha_{3,2} - (T_3 - T_2 + \alpha_{3,2}) \leq P(1 - z_3) \]
Maximum one speed change per aircraft

Limit number of aircraft able to make a change

Mach-Time Derivation

Calculation of decision Mach and final ETA

Bounds on decision variables

\[
\delta_i \geq \frac{\Delta t_i}{M_i} \\
\delta_i \leq M_i |\Delta t_i| \\
\delta_1, \delta_2, \ldots, \delta_n \text{ binary} \\

\sum_{j=1}^{j} \delta_i \leq j \\

\Delta t_i = T_i \frac{\Delta M_i}{M_i} \\

M_{d_i} = M_i + \Delta M_i \\

T_{f_i} = t_i - \Delta t_i \\

\text{Bounds} \\
-\infty \leq \Delta t_i \leq \infty \\
-0.02 \leq dM_i \leq 0.02
\]
Constraints: Fairness

\[ P_i = \frac{\dot{f}_{i|M_j}}{\dot{f}_{\text{min}}} \cdot 100 \]

\[ P_{f_i} - P_{f_{i+1}} \leq |\text{tolerance}|. \]

- Equate percentage increase in fuel burn for every group of aircraft belonging to an individual airline.
### Sample Scenario

<table>
<thead>
<tr>
<th>Flight Number</th>
<th>Aircraft Type</th>
<th>Initial Mach</th>
<th>Flight Departure Time</th>
<th>Initial ETA</th>
<th>Required Sep. (s)</th>
<th>Initial Sep. (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>940</td>
<td>752</td>
<td>0.78</td>
<td>3:35 AM</td>
<td>9:05 AM</td>
<td>131.1</td>
<td>240</td>
</tr>
<tr>
<td>788</td>
<td>763</td>
<td>0.785</td>
<td>3:39 AM</td>
<td>9:09 AM</td>
<td>107.2</td>
<td>720</td>
</tr>
<tr>
<td>780</td>
<td>763</td>
<td>0.785</td>
<td>3:51 AM</td>
<td>9:21 AM</td>
<td>115</td>
<td>240</td>
</tr>
<tr>
<td>1002</td>
<td>752</td>
<td>0.78</td>
<td>3:55 AM</td>
<td>9:25 AM</td>
<td>135</td>
<td>60</td>
</tr>
<tr>
<td>752</td>
<td>752</td>
<td>0.78</td>
<td>3:56 AM</td>
<td>9:26 AM</td>
<td>131.1</td>
<td>1080</td>
</tr>
<tr>
<td>1478</td>
<td>763</td>
<td>0.785</td>
<td>4:14 AM</td>
<td>9:44 AM</td>
<td>115</td>
<td>180</td>
</tr>
<tr>
<td>716</td>
<td>752</td>
<td>0.78</td>
<td>4:17 AM</td>
<td>9:47 AM</td>
<td>135</td>
<td>0</td>
</tr>
<tr>
<td>1076</td>
<td>752</td>
<td>0.775</td>
<td>4:17 AM</td>
<td>9:47 AM</td>
<td>107.2</td>
<td>300</td>
</tr>
<tr>
<td>1282</td>
<td>764</td>
<td>0.79</td>
<td>4:22 AM</td>
<td>9:52 AM</td>
<td>107.2</td>
<td>160</td>
</tr>
<tr>
<td>480</td>
<td>763</td>
<td>0.785</td>
<td>4:23 AM</td>
<td>9:53 AM</td>
<td>115</td>
<td>180</td>
</tr>
<tr>
<td>1642</td>
<td>752</td>
<td>0.78</td>
<td>4:26 AM</td>
<td>9:56 AM</td>
<td>135</td>
<td>2400</td>
</tr>
<tr>
<td>714</td>
<td>752</td>
<td>0.78</td>
<td>6:06 AM</td>
<td>10:36 AM</td>
<td>131.1</td>
<td>780</td>
</tr>
<tr>
<td>806</td>
<td>763</td>
<td>0.78</td>
<td>6:19 AM</td>
<td>10:49 AM</td>
<td>115</td>
<td>540</td>
</tr>
<tr>
<td>898</td>
<td>752</td>
<td>0.775</td>
<td>6:28 AM</td>
<td>10:58 AM</td>
<td>135</td>
<td>1020</td>
</tr>
<tr>
<td>816</td>
<td>752</td>
<td>0.78</td>
<td>6:45 AM</td>
<td>11:15 AM</td>
<td>135</td>
<td>1500</td>
</tr>
<tr>
<td>636</td>
<td>752</td>
<td>0.78</td>
<td>7:10 AM</td>
<td>11:40 AM</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Flights in RED would be unable to fly the CDA as initially spaced. Aside from the obvious spacing conflicts, there are clusters of aircraft that would be affected by isolated Mach change decisions.
Results Without Fairness:
Initial and Final ETA Separation

Unable to fly CDA with initial spacing
Results Without Fairness: Fuel Burn Change
Results With Fairness:
Initial and Final ETA Separation

Unable to fly CDA with initial spacing
Results With Fairness:
Fuel Burn Change