Figure 2.22. E-W Autospectrum on 8th Floor and its Best Fit

Figure 2.23. N-S Autospectrum on 24th Floor and its Best Fit
Figure 2.24. N-S Autospectrum on 22nd Floor and its Best Fit

Figure 2.25. N-S Autospectrum on 18th Floor and its Best Fit
Figure 2.26. N-S Autospectrum on 13th Floor and its Best Fit

Figure 2.27. N-S Autospectrum on 8th Floor and its Best Fit
Table 2.6. Estimates for the N-S Directional Response

<table>
<thead>
<tr>
<th>Mode No. (Type)</th>
<th>Floor No.</th>
<th>8</th>
<th>13</th>
<th>18</th>
<th>22</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (Bending)</td>
<td>Freq.(Hz)</td>
<td>0.443</td>
<td>0.444</td>
<td>0.444</td>
<td>0.444</td>
<td>0.444</td>
</tr>
<tr>
<td></td>
<td>Damp. (%)</td>
<td>2.42</td>
<td>1.05</td>
<td>0.97</td>
<td>1.24</td>
<td>1.07</td>
</tr>
<tr>
<td>2 (Torsion)</td>
<td>Freq.(Hz)</td>
<td>0.709</td>
<td>0.706</td>
<td>0.706</td>
<td>0.707</td>
<td>0.706</td>
</tr>
<tr>
<td></td>
<td>Damp. (%)</td>
<td>2.19</td>
<td>1.90</td>
<td>1.88</td>
<td>1.88</td>
<td>1.78</td>
</tr>
<tr>
<td>3 (Bending)</td>
<td>Freq.(Hz)</td>
<td>1.393</td>
<td>1.393</td>
<td>1.394</td>
<td>1.392</td>
<td>1.394</td>
</tr>
<tr>
<td></td>
<td>Damp. (%)</td>
<td>1.29</td>
<td>1.24</td>
<td>1.31</td>
<td>1.30</td>
<td>1.29</td>
</tr>
<tr>
<td>4 (Bending)</td>
<td>Freq.(Hz)</td>
<td>2.073</td>
<td>2.073</td>
<td>2.070</td>
<td>2.074</td>
<td>2.075</td>
</tr>
<tr>
<td></td>
<td>Damp. (%)</td>
<td>2.08</td>
<td>2.09</td>
<td>2.16</td>
<td>2.10</td>
<td>2.01</td>
</tr>
<tr>
<td>5 (Bending)</td>
<td>Freq.(Hz)</td>
<td>2.530</td>
<td>-</td>
<td>2.545</td>
<td>-</td>
<td>2.556</td>
</tr>
<tr>
<td></td>
<td>Damp. (%)</td>
<td>1.67</td>
<td>-</td>
<td>1.41</td>
<td>-</td>
<td>1.58</td>
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</table>

Table 2.7. Estimates for the E-W Directional Response

<table>
<thead>
<tr>
<th>Mode No. (Type)</th>
<th>Floor No.</th>
<th>8</th>
<th>13</th>
<th>18</th>
<th>22</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (Bending)</td>
<td>Freq.(Hz)</td>
<td>0.335</td>
<td>0.335</td>
<td>0.335</td>
<td>0.335</td>
<td>0.335</td>
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<tr>
<td></td>
<td>Damp. (%)</td>
<td>1.62</td>
<td>1.89</td>
<td>2.32</td>
<td>2.44</td>
<td>1.90</td>
</tr>
<tr>
<td>2 (Torsion)</td>
<td>Freq.(Hz)</td>
<td>0.707</td>
<td>0.707</td>
<td>0.705</td>
<td>0.704</td>
<td>0.702</td>
</tr>
<tr>
<td></td>
<td>Damp. (%)</td>
<td>2.02</td>
<td>2.03</td>
<td>1.92</td>
<td>1.96</td>
<td>1.89</td>
</tr>
<tr>
<td>3 (Bending)</td>
<td>Freq.(Hz)</td>
<td>1.051</td>
<td>1.053</td>
<td>-</td>
<td>1.055</td>
<td>1.056</td>
</tr>
<tr>
<td></td>
<td>Damp. (%)</td>
<td>2.30</td>
<td>2.18</td>
<td>-</td>
<td>2.42</td>
<td>2.07</td>
</tr>
<tr>
<td>4 (Bending)</td>
<td>Freq.(Hz)</td>
<td>1.952</td>
<td>-</td>
<td>1.955</td>
<td>-</td>
<td>1.957</td>
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<tr>
<td></td>
<td>Damp. (%)</td>
<td>2.09</td>
<td>-</td>
<td>2.06</td>
<td>-</td>
<td>1.81</td>
</tr>
<tr>
<td>5 (Torsion)</td>
<td>Freq.(Hz)</td>
<td>2.092</td>
<td>2.077</td>
<td>2.068</td>
<td>2.081</td>
<td>2.077</td>
</tr>
<tr>
<td></td>
<td>Damp. (%)</td>
<td>2.45</td>
<td>2.17</td>
<td>1.90</td>
<td>2.24</td>
<td>2.08</td>
</tr>
</tbody>
</table>
greater variation in the E-W response (Table 2.7). This is attributable to the fact that this mode appears as one of the two closely spaced modes and, except in the 24th floor response, is not the dominant of the two. In fact, this mode is barely discernible in the 8th floor response. The estimates obtained from this response are considerably different from estimates for other floors (Figure 2.18 and Table 2.7). Damping values for the fundamental N-S bending and torsional modes are somewhat high for the 8th floor response because their modal peaks are relatively insignificant in this response (Figures 2.23 and Table 2.6).

Typical values computed for the approximate coefficients of variation and the correlation coefficients will be given next. Table 2.8 lists the values of $\varepsilon_r$ calculated from the diagonal elements of the parameter covariance matrix $V_0^*$ for the E-W response from the 24th floor. The correlation coefficients computed from the off-diagonal elements are shown in Table 2.9. As would normally be anticipated, the values of $\varepsilon_r$ are the least for the parameters of the dominant mode, which in this case is the second torsional mode. Relatively high values of $\varepsilon_r$ for the 3rd bending mode are mostly due to this mode being the lesser pronounced of the two closely spaced modes. But the highest $\varepsilon_r$ corresponds to the damping ratio of the lowest mode. This trend was observed in almost all the estimated spectra. The probable cause is the resolution bias error which is higher for the lower modal frequencies since, for the same value of $p$, a lower frequency mode is defined by fewer points than a higher frequency mode. The above trend is also confirmed by Table 2.7, in which the
Table 2.8. Approximate Coefficients of Variation for the Modal Parameters from the E-W Directional Roof Response

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Frequency (Hz)</th>
<th>Damping Ratio (%)</th>
<th>Participation Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.335</td>
<td>0.0009</td>
<td>1.90</td>
</tr>
<tr>
<td>2</td>
<td>0.702</td>
<td>0.0011</td>
<td>1.89</td>
</tr>
<tr>
<td>3</td>
<td>1.056</td>
<td>0.0011</td>
<td>2.07</td>
</tr>
<tr>
<td>4</td>
<td>1.957</td>
<td>0.0010</td>
<td>1.81</td>
</tr>
<tr>
<td>5</td>
<td>2.077</td>
<td>0.0004</td>
<td>2.08</td>
</tr>
</tbody>
</table>

(1) Not the actual or true values. The actual values are obtained using a multiplication factor that depends on the maximum magnitude of the spectrum and the gain of the amplifier.
Table 2.9. Correlation Coefficients for the Modal Parameters from the E-W Directional Roof Response

<table>
<thead>
<tr>
<th></th>
<th>$f_1$</th>
<th>$\rho_1$</th>
<th>$A_1$</th>
<th>$f_2$</th>
<th>$\rho_2$</th>
<th>$A_2$</th>
<th>$f_3$</th>
<th>$\rho_3$</th>
<th>$A_3$</th>
<th>$f_4$</th>
<th>$\rho_4$</th>
<th>$A_4$</th>
<th>$f_5$</th>
<th>$\rho_5$</th>
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<tbody>
<tr>
<td>$f_1$</td>
<td>1</td>
<td>-0.21</td>
<td>-0.53</td>
<td>0.92</td>
<td>-0.02</td>
<td>0.01</td>
<td>0.01</td>
<td>-0.21</td>
<td>0.06</td>
<td>0.06</td>
<td>-0.21</td>
<td>0.06</td>
<td>0.06</td>
<td>-0.21</td>
<td>0.06</td>
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<td>$\rho_1$</td>
<td>1</td>
<td>0.89</td>
<td>0.89</td>
<td>0.89</td>
<td>0.89</td>
<td>0.87</td>
<td>0.87</td>
<td>0.87</td>
<td>0.87</td>
<td>0.87</td>
<td>0.87</td>
<td>0.87</td>
<td>0.87</td>
<td>0.87</td>
<td>0.87</td>
</tr>
<tr>
<td>$A_1$</td>
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<td>-0.002</td>
<td>-0.16</td>
<td>-0.06</td>
<td>-0.09</td>
<td>-0.09</td>
<td>-0.16</td>
<td>-0.09</td>
<td>-0.09</td>
<td>-0.16</td>
<td>-0.09</td>
<td>-0.09</td>
<td>-0.16</td>
<td>-0.09</td>
</tr>
<tr>
<td>$f_2$</td>
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<td>-0.04</td>
<td>0.14</td>
<td>-0.06</td>
<td>0.07</td>
<td>-0.10</td>
<td>-0.10</td>
<td>0.12</td>
<td>-0.06</td>
<td>-0.06</td>
<td>0.12</td>
<td>-0.06</td>
<td>-0.06</td>
<td>0.12</td>
<td>-0.06</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>1</td>
<td>0.16</td>
<td>-0.06</td>
<td>0.07</td>
<td>-0.04</td>
<td>0.08</td>
<td>0.08</td>
<td>0.11</td>
<td>-0.06</td>
<td>-0.06</td>
<td>0.11</td>
<td>-0.06</td>
<td>-0.06</td>
<td>0.11</td>
<td>-0.06</td>
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<tr>
<td>$A_2$</td>
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<td>-0.007</td>
<td>-0.02</td>
<td>-0.02</td>
<td>0.07</td>
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<td>-0.03</td>
<td>0.03</td>
<td>-0.03</td>
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</tr>
<tr>
<td>$f_3$</td>
<td>1</td>
<td>-0.09</td>
<td>0.05</td>
<td>-0.09</td>
<td>-0.14</td>
<td>-0.17</td>
<td>-0.17</td>
<td>-0.08</td>
<td>-0.12</td>
<td>-0.12</td>
<td>-0.08</td>
<td>-0.12</td>
<td>-0.12</td>
<td>-0.08</td>
<td>-0.12</td>
</tr>
<tr>
<td>$\rho_3$</td>
<td>1</td>
<td>0.05</td>
<td>-0.16</td>
<td>-0.16</td>
<td>0.14</td>
<td>0.20</td>
<td>0.20</td>
<td>0.09</td>
<td>0.20</td>
<td>0.20</td>
<td>0.09</td>
<td>0.20</td>
<td>0.20</td>
<td>0.09</td>
<td>0.20</td>
</tr>
<tr>
<td>$A_3$</td>
<td>1</td>
<td>-0.02</td>
<td>0.086</td>
<td>0.20</td>
<td>-0.18</td>
<td>-0.18</td>
<td>-0.18</td>
<td>-0.086</td>
<td>-0.18</td>
<td>-0.18</td>
<td>-0.086</td>
<td>-0.18</td>
<td>-0.18</td>
<td>-0.086</td>
<td>-0.18</td>
</tr>
<tr>
<td>$f_4$</td>
<td>1</td>
<td>-0.09</td>
<td>-0.04</td>
<td>-0.04</td>
<td>-0.49</td>
<td>-0.60</td>
<td>-0.60</td>
<td>-0.49</td>
<td>-0.60</td>
<td>-0.60</td>
<td>-0.49</td>
<td>-0.60</td>
<td>-0.60</td>
<td>-0.49</td>
<td>-0.60</td>
</tr>
<tr>
<td>$\rho_4$</td>
<td>1</td>
<td>-0.04</td>
<td>-0.49</td>
<td>-0.49</td>
<td>-0.60</td>
<td>-0.60</td>
<td>-0.60</td>
<td>0.04</td>
<td>-0.49</td>
<td>-0.49</td>
<td>-0.60</td>
<td>-0.60</td>
<td>-0.60</td>
<td>0.04</td>
<td>-0.49</td>
</tr>
<tr>
<td>$A_4$</td>
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<td>-0.08</td>
<td>-0.64</td>
<td>-0.64</td>
<td>-0.02</td>
<td>-0.02</td>
<td>-0.02</td>
<td>0.08</td>
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<td>0.08</td>
<td>-0.64</td>
</tr>
<tr>
<td>$f_5$</td>
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<td>-0.10</td>
<td>-0.64</td>
<td>-0.64</td>
<td>-0.02</td>
<td>-0.02</td>
<td>-0.02</td>
<td>0.10</td>
<td>-0.64</td>
<td>-0.64</td>
<td>-0.02</td>
<td>-0.02</td>
<td>-0.02</td>
<td>0.10</td>
<td>-0.64</td>
</tr>
<tr>
<td>$\rho_5$</td>
<td>1</td>
<td>0.08</td>
<td>0.14</td>
<td>0.14</td>
<td>1.00</td>
<td>0.98</td>
<td>0.98</td>
<td>0.08</td>
<td>0.14</td>
<td>0.14</td>
<td>1.00</td>
<td>0.98</td>
<td>0.98</td>
<td>0.08</td>
<td>0.14</td>
</tr>
<tr>
<td>$A_5$</td>
<td>1</td>
<td>0.98</td>
<td>1.00</td>
<td>1.00</td>
<td>0.98</td>
<td>1.00</td>
<td>1.00</td>
<td>0.98</td>
<td>1.00</td>
<td>1.00</td>
<td>0.98</td>
<td>1.00</td>
<td>1.00</td>
<td>0.98</td>
<td>1.00</td>
</tr>
</tbody>
</table>
damping values for the lowest mode show the greatest variation.

Table 2.9 shows moderate values of $\tau$ for the parameters of the 4th and the 5th modes and high values between the damping ratio and the participation factor for each mode. The correlation values for the closely spaced modes are to be expected, whereas the high values between $\rho_1$ and $A_i$ are due mainly to the following reason.

Considering the case of a single degree-of-freedom system, the value of $|H(f)|$ at the peak frequency, using Equation (2.30), is given by

$$|H(f)|_f = f_1 = \frac{A_i}{2\rho_i}$$

Thus, including only those points that lie close to the peak will lead to significant correlation between $A_i$ and $\rho_i$. The high values of $\tau$ could possibly be reduced by using a weighted least squares approach in which the weights are chosen inversely proportional to the measured $\sqrt{G_{yy}(f)}$. This problem was not pursued further in this study.

The frequency and damping estimates found using three different methods for the N-S response from the 24th floor will now be compared. The methods employed here are the following:

1. Multidegree-of-freedom curve fitting
2. Single degree-of-freedom curve fitting for each mode
3. Direct method in which the peak frequencies are taken as the natural frequencies and the damping ratios are computed using the half-power points.

The 24th floor N-S response was chosen for comparison since it does not contain any closely spaced modes and the direct as well as the single
degree-of-freedom curve fitting methods can be applied to all the modes.

It has already been pointed out that the spectral estimates in the vicinity of the lower modes suffer from insufficient resolution. The damping ratios computed using the half-power bandwidth method will be acceptable only if the frequency resolution is such that there are at least 4 points between the half-power points \((\Delta f)\). This condition was not satisfied for the first 3 modes in the spectrum given in Figure 2.23. So, the time domain data was processed again with twice the frequency resolution as before \((\Delta f = 0.0078125 \text{ Hz})\) to obtain the parameters for the third mode and with four times the frequency resolution \((\Delta f = 0.00390625 \text{ Hz})\) to obtain the parameters for the first 2 modes. Figures 2.28 and 2.29 show the resulting high resolution autospectra. Owing to the smaller number of averages used in their computation, these spectra are subject to higher variance.

The second modal peak in the high resolution spectrum of Figure 2.29 exhibits behavior that is characteristic of nonlinear modes. At this point, it is not clearly understood what causes such behavior. Possible reasons other than nonlinearities include nonstationary response and peculiarities in the input spectrum. The half-power bandwidth method could not be applied to determine the damping ratio for this mode. In any event, the modal parameters obtained by applying the single and multidegree-of-freedom curve fitting procedures to the low resolution spectrum would be those for an equivalent linear system.

The estimates for the remaining parameters calculated using the three methods are given in Table 2.10. All the three methods give
Figure 2.28. High Resolution Autospectrum with $\Delta f = 0.0078125$ Hz

Figure 2.29. High Resolution Autospectrum with $\Delta f = 0.003906$ Hz
Table 2.10. Comparison of Modal Parameter Estimates from SDOF Fit, MDOF Fit and the Direct Method

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Frequency (Hz)</th>
<th>Damping Ratio (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Direct</td>
<td>SDOF</td>
</tr>
<tr>
<td>1</td>
<td>0.443</td>
<td>0.445</td>
</tr>
<tr>
<td>2</td>
<td>0.701</td>
<td>0.702</td>
</tr>
<tr>
<td>3</td>
<td>1.393</td>
<td>1.396</td>
</tr>
<tr>
<td>4</td>
<td>2.075</td>
<td>2.073</td>
</tr>
<tr>
<td>5</td>
<td>2.544</td>
<td>2.533</td>
</tr>
</tbody>
</table>
about the same values for the frequencies. The damping estimates for the second mode are considerably different for the single and multidegree-of-freedom curve fitting methods, which could be due to the underlying abnormal modal behavior. The values are comparable for all the other modes. The estimates for the fifth mode show excellent agreement. This could be the result of the fact that the spectral estimates in this region are subject to comparatively low bias errors.

The estimates of damping from the different procedures given above suggest the use of the following technique to obtain reasonable damping values from a given autospectrum with minimum effort.

(i) To start with, the half-power bandwidth method could be used.

(ii) If the frequency resolution is not high enough so that there is an insufficient number of points between the half-power points, a single degree-of-freedom fit could be carried out.

(iii) If the mode is not well separated from the neighboring modes, a multiple degree-of-freedom curve fitting procedure including all the modes in the neighborhood could be used.

2.8 Effect of Cladding

The discussion that follows, on the influence of cladding on the modal parameters, is restricted to frequencies and damping only. Mode shapes are not considered because not enough measurements could be made for sufficient characterization of each mode. Because of the long periods of time required for each measurement, the five available
accelerometers could not be moved around to measure the response from additional floors. Hence the response data could not be obtained for more than five floors on any given day. Also, the locations used for placing the accelerometers were different on different days and not the same five floors could be used on all days due to the inaccessibility of certain floors.

Attempts were made to keep track of the mass in the building on different test dates but were unsuccessful. Quantification of the amount of mass on various days would enable easier correlation of the changes in the modal parameters to structural modifications with construction, but was found infeasible. Nonetheless some important activities and events on the test dates were taken note of and these are included in Table 2.1. Despite the lack of estimates of mass in the structure, it can be assumed with reasonable confidence that, once the steel frame is erected and the floor slabs are in place, further construction activities would tend to increase the total mass of the structure due to the accumulation of materials for later use in construction.

Variations of frequencies and damping ratios with construction, for the period when the installation of cladding was completed, are given in Tables 2.11 and 2.12. It is evident from these tables that all the frequencies show a decreasing trend initially. This can be ascribed to an increase in the mass of the building. The largest drop, as seen in the values of the E-W bending frequencies, occurs between February 22 and March 7, indicating a large influx of mass into the building during this period. However, after March 7, the frequencies
Table 2.11. Variation of Frequencies with Construction

<table>
<thead>
<tr>
<th>Direction</th>
<th>Mode No.</th>
<th>1/16</th>
<th>2/8</th>
<th>2/15</th>
<th>2/22</th>
<th>3/7</th>
<th>4/24</th>
<th>5/16</th>
<th>5/29</th>
</tr>
</thead>
<tbody>
<tr>
<td>N-S</td>
<td>1</td>
<td>0.479</td>
<td>0.475</td>
<td>0.469</td>
<td>-</td>
<td>0.458</td>
<td>0.446</td>
<td>0.444</td>
<td>0.444</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.444</td>
<td>1.424</td>
<td>1.412</td>
<td>-</td>
<td>1.367</td>
<td>1.365</td>
<td>1.368</td>
<td>1.375</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>2.604</td>
<td>2.587</td>
<td>2.581</td>
<td>-</td>
<td>2.473</td>
<td>2.511</td>
<td>2.526</td>
<td>2.545</td>
</tr>
<tr>
<td>E-W</td>
<td>1</td>
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<td>-</td>
<td>-</td>
<td>0.320</td>
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<td>-</td>
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<td>3</td>
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<td>-</td>
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<tr>
<td>Torsion</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>0.702</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>1.960</td>
<td>2.008</td>
<td>2.024</td>
<td>2.038</td>
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Table 2.12. Variation of Damping With Construction

<table>
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<th>Direction</th>
<th>Mode No.</th>
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<th>2/8</th>
<th>2/15</th>
<th>2/22</th>
<th>3/7</th>
<th>4/24</th>
<th>5/16</th>
<th>5/29</th>
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</thead>
<tbody>
<tr>
<td>N-S</td>
<td>1</td>
<td>0.91</td>
<td>0.61</td>
<td>1.92</td>
<td>-</td>
<td>1.02</td>
<td>0.88</td>
<td>0.82</td>
<td>0.71</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.47</td>
<td>0.67</td>
<td>0.62</td>
<td>-</td>
<td>0.62</td>
<td>1.04</td>
<td>1.54</td>
<td>1.32</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>-</td>
<td>0.66</td>
<td>0.92</td>
<td>-</td>
<td>1.10</td>
<td>1.56</td>
<td>1.30</td>
<td>1.23</td>
</tr>
<tr>
<td>E-W</td>
<td>1</td>
<td>0.44</td>
<td>-</td>
<td>-</td>
<td>0.56</td>
<td>0.53</td>
<td>-</td>
<td>3.15</td>
<td>2.32</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.57</td>
<td>-</td>
<td>-</td>
<td>0.97</td>
<td>1.04</td>
<td>-</td>
<td>1.72</td>
<td>1.78</td>
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<td></td>
<td>3</td>
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<td>0.62</td>
<td>-</td>
<td>1.76</td>
<td>1.49</td>
</tr>
<tr>
<td>Torsion</td>
<td>1</td>
<td>1.15</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.32</td>
<td>1.33</td>
<td>1.56</td>
<td>1.56</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.71</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.79</td>
<td>1.11</td>
<td>1.56</td>
<td>1.47</td>
</tr>
</tbody>
</table>
of all the modes except the fundamental bending and torsional modes show an increasing trend in contrast to the decreasing trend that would generally be expected throughout.

Comparison of Tables 2.1 and 2.11 brings out an interesting feature. From Table 2.1, which lists the cladding levels on different days, it is observed that there is a considerable change in the levels on all four faces before and after March 7. This leads to the possible inference that the increase in frequencies after this date could be attributed, at least in part, to the continuing installation of exterior cladding. The frequencies of the fundamental modes in N-S bending and torsion remain relatively unaffected. This again is in conformity with the expected behavior of cladding, which could influence the higher modes more since they involve greater curvatures. The cladding levels and frequencies are plotted against time in days in Figures 2.30 and 2.31 respectively.

Although there are definite trends in the frequency variations, the magnitudes of the variations themselves are quite small. But one must realize that the building considered here is only a glass-clad structure, which, due to the lightweight nature of glass, perhaps is not the best candidate for studying cladding effects (This particular building was employed because it was the only one available at the time of the study.) On the other hand, it is also possible that the observed variations in the modal parameters were caused by other effects or components such as the interior partitions.

Table 2.12 shows increasing values for the damping of torsional modes in general. Except the 2nd mode in the E-W direction, the bending
Figure 2.30. Variation of Cladding Levels with Construction
Figure 2.31. Variation of Frequencies with Construction
Figure 2.31 (Cont'd.). Variation of Frequencies with Construction

(c) Torsional Frequencies
modes do not exhibit a definite trend. But the values of damping for all the modes tend to be high after March 7, when most of the cladding was in place.

The average modal parameters computed from the data taken on November 13, well after the cladding installation was complete, are given in Table 2.13. All the frequencies in E-W bending and torsion shown increases of considerable amounts but only the 2nd frequency in N-S bending exhibits an increase. The observed changes in this case are mostly due to other nonstructural elements such as the interior partition walls.

In conclusion, it can be stated that the possible influence of cladding on the modal parameters, based on the findings here, is to increase the frequencies of the higher modes slightly and, to a lesser degree, increase the damping in general.
Table 2.13. Frequency and Damping Estimates from Data Taken on 11/13/80

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>N-S Direction</th>
<th>E-W Direction</th>
<th>Torsion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Freq. (Hz)</td>
<td>Damp. (%)</td>
<td>Freq. (Hz)</td>
</tr>
<tr>
<td>1</td>
<td>0.444</td>
<td>1.12</td>
<td>0.335</td>
</tr>
<tr>
<td>2</td>
<td>1.393</td>
<td>1.28</td>
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</tr>
<tr>
<td>3</td>
<td>2.543</td>
<td>1.49</td>
<td>1.954</td>
</tr>
</tbody>
</table>
3.0 FORCED VIBRATION TESTING

3.1 Introduction

Forced vibration testing is the most widely employed technique in the experimental determination of the dynamic characteristics of structures. While, under certain conditions, it is possible to resort to special techniques like ambient testing that obviate the need for a prescribed external input, the measurement of the dynamic properties of a general structure requires one or more excitation sources. Force generating devices or shakers that provide well-defined controllable and measurable force input are usually employed as the excitation sources. Even for structures like highrise buildings where ambient testing is applicable, the use of prescribed external excitation eliminates the uncertainty regarding the input and thus improves the credibility of the results obtained. A survey of full scale testing techniques can be found in Reference 37.

There are many different ways in which a structure can be excited with a shaker. In the conventional method known as harmonic testing, the steady-state response of the structure at different frequencies is obtained by incrementally increasing the exciter frequency. The natural frequencies are taken as those corresponding to the peaks in the gain versus frequency plot. The damping ratio for each mode is computed by applying the half-power bandwidth method to
the modal peaks or the logarithmic decrement method to the decaying response acquired by turning the power off to the shaker at resonance. The mode shapes are determined from the values of the gain at modal frequencies, measured at various locations. Other analysis tools such as the vector or Nyquist plot method can also be used to extract the vibrational parameters (38). In this method, the parameters are derived from a curve obtained by plotting the in- phase response against the out-of-phase response.

A more accurate but laborious testing procedure would require using multiple shakers (3). The structure is made to response in one particular mode by controlling the forces provided by the shakers, which are distributed throughout the structure. This process is referred to as "modal tuning". It may not always be possible to apply this technique because, due to the inaccessibility of certain locations in the structure, it may be impracticable to position all the shakers in optimum spots to tune a specific mode.

When the modal properties for several modes are needed, the process of acquiring the steady-state response around each mode becomes lengthy and tedious. In such cases, it may be easier to make use of techniques that impart energy to the structure over the entire frequency range of interest in a short interval of time, and compute the frequency response function from the input-output measurements. Such techniques simplify the testing procedure at the cost of increased calculations in the post-test analysis of the data. With the rapid development and deployment of computers, these are increasingly being used in dynamic testing of many types of structures. However, the
methods that have been employed to date in vibration tests on buildings are almost exclusively of the harmonic type \((40-45)\). This can mostly be traced to the unavailability of an exciter that fills the needs for full scale testing of structures like buildings.

The three most commonly employed types of shakers are the electrodynamic, electrohydraulic and mechanical (rotating eccentric mass) models. The electrodynamic shakers produce force directly whereas the others produce motion. All types of shakers must possess certain desirable performance characteristics for their effective use. In addition to delivering the required force levels, they must have good frequency response over the frequency range of interest. This implies that for testing buildings whose natural frequencies vary from a fraction of a Hertz to several Hertz, the shakers employed must be capable of low frequency operation.

The mechanical exciters can usually generate only steady harmonic motion which renders them useless when an arbitrary waveform is desired for the forcing function. On the other hand, the electrodynamic exciters often have performance limitations owing to their size and construction. The force levels generated by practical-sized models are inadequate for testing civil engineering structures such as buildings. Besides, they suffer from poor low frequency response due to the inherent restrictions on the components used in their construction. As a consequence, these models have found little use in dynamic tests on buildings.
An electrohydraulic shaker that circumvents the above problems has recently been developed for use in full scale testing (46). This chapter describes the forced vibration tests conducted on a 25 story steel frame building employing this shaker. The results of these tests are compared to the ambient test results obtained by measuring the ambient response of the building. The forced vibration test results are also used in the estimation of stiffness parameters in the analytical model of the building, to be dealt with in the next chapter. Detailed discussions on the design, construction and operation of the shaker are given in Reference 46. A brief outline of some of its important aspects will now follow.

3.2 The Shaker

The electrohydraulic force generator in its basic form consists of a seismic mass that moves in a rectilinear manner under the action of a hydraulic actuator. The seismic mass is made-up of lead "bricks" (4" x 4" x 8") that weigh approximately 55 lbs. each. This mass is placed on a weight table which is supported on 4 trackless air bearings (Figure 3.1). The hydraulic actuator produces horizontal to-and-fro motion of the weight table. Supporting the table on air bearings enables easy and quick reorientation of the table so that the axis of the actuator can be aligned in any desired direction.

The maximum force generated by the shaker at low frequencies (f < 1 Hz) is limited by the maximum stroke of the actuator. In the intermediate frequency range (1 < f < 3.5 Hz), the constraining
Figure 3.1. Rectilinear Electrohydraulic Shaker
factor is the hydraulic flow rate and at high frequencies \( f > 3.5 \, \text{Hz} \),
the performance is restricted by the maximum permissible hydraulic
pressure and the actuator piston area. For a given seismic mass, the
force produced is thus proportional to the square of the frequency up
to 1 Hz and to the frequency itself from 1 to 3.5 Hz. Above 3.5 Hz,
the force remains a constant. The performance curve for the shaker
is reproduced from Reference 46 in Figure 3.2.

The most important advantage in using this shaker is its
ability to produce motion of the weight table according to any arbitrary
prescribed waveform. In the present study, the approach adopted was
to compute the desired waveform at discrete time intervals in a desktop
computer and transform the resulting digital signal to the analog
form by means of a digital-to-analog converter (DAC). The output
waveform from the DAC is lowpass filtered to remove the jaggedness in
the signal caused by the "sample and hold" mode of operation of the
DAC. The final analog waveform is fed to the servocontroller unit
which controls the actuator to generate corresponding table motion.
The block diagram in Figure 3.3 illustrates this procedure.

3.3 The Forcing Function

The types of waveforms that permit the excitation of the structure
over a range of frequencies can be grouped into the following three
categories (47):

(a) Bandlimited white noise or random input

(b) Swept-sine input

(c) Impact or impulse input
Figure 3.2 Performance Curve of the Shaker
Figure 3.3. Block Diagram for the Shaker Input
All of the above inputs give rise to time waveforms that possess approximately flat spectral density functions. Since impact techniques are not suitable for use in buildings, they will not be considered here. White noise input involves synthesizing the time waveform by inverse Fourier transforming the desired linear spectrum in the frequency domain. A comparatively easy method of generating a time function that has approximately uniform distribution of power between two frequency limits is to use a sinusoid whose frequency is varied between these limits in a predetermined manner. This technique of varying the frequency between the lower and upper limits is known as "sweeping" and the input thus generated is usually referred to as "swept-sine wave" or "chirp".

For the case where the frequency is varied linearly, the swept-sine wave is defined by

\[ x(t) = \sin \left[ \frac{\pi(f_b - f_a)t^2}{T} + 2\pi f_a t \right] \] (3.1)

where

- \( x(t) = \) swept-sine input
- \( f_a = \) lower frequency limit
- \( f_b = \) upper frequency limit
- \( T = \) sweep time

The function \( x(t) \) has a linear spectrum whose magnitude between \( f_a \) and \( f_b \) is a constant with a ripple superimposed on it (48). Figures 3.4a and b show a typical linearly swept sine wave and its spectrum. All the forcing functions employed here are swept-sine waves computed on the basis of Equation (3.1).
Figure 3.4. Linearly Swept Sine Wave; $f_1 = 0.05$ Hz, $f_2 = 0.75$ Hz, $T = 102.4$ sec
In creating waveforms to drive the shaker, some pertinent aspects of its performance must be taken into account. Equation (3.1) can be assumed to be the force produced from which the required displacement signal to control the shaker can be derived. However, since higher force levels can be obtained with increasing frequencies for a given stroke, the shaker can be operated more efficiently by using Equation (3.1) for displacement rather than force. But, due to the physical limitations of the exciter, the displacement signal must be tapered in such a fashion that the amplitude of the generated force follows a curve similar to the operating curve given in Figure 3.2. Specifically, it must be attenuated so that the force varies linearly between 1 and 3.5 Hz and remains constant thereafter. To provide some safety margin, the tapering of all the signals utilized here was started at frequencies 10 percent below these values (0.9 and 3.15 Hz). The complete waveform is determined using

\[
x(t) = \tilde{A} \sin \left[ \frac{\pi(f_b - f_a)t^2}{T} + 2\pi f_a t \right]
\]  

(3.2)

where

\[
\tilde{A} = A, \quad f \leq 0.9
\]

\[
= \left(\frac{0.9}{f}\right)A, \quad 0.9 < f \leq 3.15
\]

\[
= \left(\frac{0.9}{f}\right)\left(\frac{3.15}{f}\right)A, \quad f > 3.15
\]

\[
f = \frac{(f_b - f_a)t}{T} + f_a', \text{ the instantaneous frequency}
\]

and \(A\) is the amplitude initially at \(f = f_a\). Figures 3.5a and b show a tapered waveform and its spectrum.
Figure 3.5. A Tapered Sine Wave Input; $f_1 = 0.1$ Hz, $f_2 = 4.9$ Hz, $T = 51.2$ sec
3.4 Structure

The structure chosen for forced vibration and optimum stiffness estimation studies is a 25-story steel frame office tower. It consists of a central steel core surrounded by a lightweight exterior steel frame which supports a highly contoured precast concrete panel curtain wall. The building core is constructed with braced framing in one direction and rigid framing in the other (Figure 3.6). The exterior frame is supported by a reinforced concrete, rigid frame pedestal. Further aspects of the construction of this building can be found in Reference 49.

3.5 Measurement

3.5.1 Ambient Tests

Prior to conducting full scale forced vibration tests, the ambient response of the structure was measured on November 20 (1980). The objectives of this measurement were first to determine the modal frequencies for later use in forced vibration tests, and second to provide a basis from which to make comparisons of the ambient and forced vibration testing methods. The equipment used was the same as that employed in the ambient tests described in the previous chapter. Five accelerometers were placed on the 2nd, 9th, 17th and 21st floors and the roof. The accelerometers were situated near the center of the building and about 3 hours of data were recorded in each of the two bending directions. No torsional response was measured.

3.5.2 Forced Vibration Tests

Full scale dynamic tests using the shaker were carried out
(a) Structural Framing

(b) Cladding Panel

Figure 3.6. Structure Used in Forced Vibration Tests
starting June 26 (1981). The instrumentation used consisted of the response measuring equipment utilized in ambient measurements and the shaker along with its peripheral devices. The latter included a hydraulic pump to provide the hydraulic power for the actuator, an air compressor to provide the air supply for the bearings and a specially constructed vibration control system. The control system was based around an HP 9825A calculator which was used to generate the desired waveforms and to synchronize the operation with other equipment. The control system also included a DAC to convert the digital waveforms to analog form and an MTS 406 servocontroller to control the hydraulic valve.

Ideally, it would be desirable to locate the shaker near the building top so as to avoid placing it near the node points for any of the modes of interest. This was not possible because adequate floor space and power connections were not available in this area. Rather, the shaker was mounted on the 15th floor, which is the mechanical floor in the building. The shaker assembly was left in place between tests and was dismantled only after all tests were completed. The rest of the equipment was set up and removed when required.

Since only five accelerometers were employed to measure the response, the testing process was carried out in several stages to get sufficient definition of the mode shapes. The accelerometers were first placed on the roof and four lower floors and a set of measurements was made. Next, the accelerometers on the lower floors were moved to four different floors and the measurements were repeated. The response from the roof was used as the reference in computing the mode shapes,
which were thus defined by a total of nine coordinate points. Along with the roof, the 23rd, 21st, 19th, 17th, 15th, 12th, 9th and 6th floors were chosen for response measurement. The accelerometers were located in a stairwell close to the center of the building for measuring the bending response and near the edges of the building for torsional response. The shaker was located at the midpoint of the north edge on the 15th floor. Since access to the four sides of the building was limited, torsional measurements were made only on the roof, 15th and 10th floors. Consequently, proper definition of the torsional mode shapes could not be obtained.

The orientation of the accelerometers and the line-of-action of the shaker were selected depending upon the type of response desired. For measuring bending response in either the braced or the rigid frame direction, the shaker and the accelerometers were aligned in the braced or rigid directions respectively. Torsional data was obtained by locating the accelerometers along the east and west edges oriented in the rigid frame direction, with the shaker aligned in the braced frame direction (Figure 3.7).

The HP 9825A calculator employed to synthesize the waveforms was programmed so that, for given values of $f_a$, $f_b$ and $T$, a swept-sine data block containing 2048 points is calculated according to Equation (3.2). To avoid large inertial forces that might occur if the shaker is started or stopped abruptly, two additional harmonic waveforms, one with frequency $f_a$ and the other with frequency $f_b$, were synthesized and used together with the swept-sine waveform. The calculator was instructed to repeatedly output the lower harmonic
Figure 3.7. Accelerometer and Shaker Layout
cycle initially and the upper harmonic cycle after every sweep.
The actual testing procedure consists of the following steps.

(i) With the calculator transmitting the lower harmonic
cycle, increase the stroke of the shaker from zero
to the required level.

(ii) Issue a command to the calculator to start the sweep.

(iii) After the sweep is over, the stroke of the shaker,
which is now being driven by a harmonic signal of
frequency $f_b$, is reduced to zero.

(iv) Issue a second command to the calculator to switch to
the lower harmonic signal.

(v) Go to step (i) and repeat as many times as needed.

All data blocks were computed with a constant time interval $\Delta t$
between two successive points, given by

$$\Delta t = \frac{T}{2048}$$

An external clock, which is a part of the integral DAC unit, was
used to pace the transmission of data points from the calculator to
the DAC at $\Delta t$ intervals.

The input measurement consisted of a displacement signal from
an LVDT attached to the actuator. This signal is proportional to
the displacement of the weight table. The table acceleration can
be computed from this signal if necessary, and used as a measure of
the force produced.
To facilitate post-processing of the measurements using signal analyzers, a trigger signal was created in DAC with the aid of the calculator and recorded. This signal was such that it assumed a constant dc value for the duration of the sweep and zero otherwise. By employing this signal to control the triggering, the start of data acquisition in the analyzer can be synchronized with the start of the recorded sweep.

The seismic mass used on the weight table comprised 32 lead bricks for a combined mass of about 1760 lbs. The stroke length was adjusted for a maximum displacement of about 8 inches in the low frequency range.

Different ranges of sweeps were used by choosing different values for $f_a$ and $f_b$. All the modal parameters for the higher modes were estimated from sweep data with $f_a = 0.1$ Hz and $f_b = 4.9$ Hz. Although this range includes the frequencies of the fundamental modes, the force generated at low frequencies ($f < 0.5$ Hz) is so small that the estimates of the frequency response function in this range are very poor. For this reason, a separate sweep with $f_a = 0.05$ Hz and $f_b = 0.85$ Hz was employed for measuring the fundamental mode response. The sweep time $T$ was selected as 51.2 seconds for the $0.1 - 4.9$ Hz sweep and 102.4 seconds for the $0.05 - 0.85$ Hz sweep. These figures were chosen to comply with the time taken for a block of data to be accumulated in the signal analyzer employed to compute $H(f)$.

Each sweep was repeated 5 to 10 times and the data recorded each time so that ensemble averaging could be used to estimate the spectral density functions from which the frequency response function is obtained.
3.6 Frequency Response and Spectral Estimation

Procedures similar to those given in the previous chapter were used to compute the autospectra and the cross-spectra from the ambient data. The autospectra measured on the roof in the braced and rigid frame directions are shown in Figures 3.8 and 3.9 respectively.

In the case of forced vibration test data, the frequency response functions are estimated as

\[
\hat{H}(f) = \frac{\hat{G}_{xy}(f)}{\hat{G}_{xx}(f)} \tag{3.3}
\]

where \(\hat{H}(f)\) is the estimate of \(H(f)\), \(\hat{G}_{xx}(f)\) is the estimate of the input power spectral density and \(\hat{G}_{xy}(f)\) is the estimate of the cross-spectral density between the input and the output. The coherence function between the input and the output, computed according to Equation (2.15), can be utilized as a measure of the accuracy of the estimate \(\hat{H}(f)\).

The bias and variance errors involved in estimating the frequency response function are discussed in Reference 28. The variance of the estimate decreases as the coherence approaches unity. The variance is also reduced by increasing the number of averages used to calculate \(\hat{H}(f)\). The bias in the estimate depends on many factors. An important fact worth noting here is that the presence of other sources of excitation does not cause any bias in \(\hat{H}(f)\) if the input due to these sources is not correlated with the externally applied input. Therefore, the modal parameter estimates will not be influenced by these sources. In contrast, the harmonic or steady-state testing method employs the response directly to
Figure 3.8. Ambient Results; Autospectrum on Roof in the Rigid Frame Direction

Figure 3.9. Ambient Results; Autospectrum on Roof in the Braced Frame Direction
determine the modal properties. Thus, if the external input is not large enough so that other inputs can be considered negligibly small, the parameter estimates will be significantly affected. This is an added advantage in using the frequency response function to determine the modal parameters.

The computation of $H(f)$ was carried out in an HP 5451B mini-computer based Fourier analyzer. The Hanning window was employed to reduce leakage and smooth the spectra. The trigger signal was used to identify the start of each sweep and trigger the analog-to-digital conversion of the recorded shaker input and the response at each point. Time records of 1024 points were employed to compute 512-point frequency domain functions. Number of averages used range from 5 to 10 depending on the number of measurements made. Values of the coherence function estimates were close to one near all the modal frequencies except the fundamental modes. Since the force levels near the fundamental frequencies were quite low, the values of coherence in this region were only about 0.5 - 0.6.

Figures 3.10 - 3.18 show the frequency response functions computed using 0.1 - 4.9 Hz sweep data in the braced frame direction. The torsional frequency response functions measured on the roof are given in Figures 3.19 - 3.20. It can be seen from Figures 3.19 and 3.20 that the $H(f)$ measured along the east and the west sides show a 180° phase shift, confirming that the response measured in either of these locations was indeed predominately torsion. A typical coherence function obtained is given in Figure 3.21. A frequency response function measured in the 0.05 - 0.85 Hz range and its associated coherence are illustrated in Figures 3.22a and b.
Figure 3.10. Transfer Function Measured on the Roof

Figure 3.11. Transfer Function Measured on the 23rd Floor
Figure 3.12. Transfer Function Measured on the 21st Floor

Figure 3.13. Transfer Function Measured on the 19th Floor
Figure 3.14. Transfer Function Measured on the 17th Floor

Figure 3.15. Transfer Function Measured on the 15th Floor
Figure 3.16. Transfer Function Measured on the 12th Floor

Figure 3.17. Transfer Function Measured on the 9th Floor
Figure 3.18. Transfer Function Measured on the 6th Floor
Figure 3.19. Transfer Function For Torsional Response (North)
Figure 3.20. Transfer Function for Torsional Response (South)
Figure 3.21. A Typical Coherence Function
Figure 3.22. A Transfer Function in the Low Frequency Range (0.05 - 0.85 Hz); Real and Imaginary Parts
Figure 3.22 (Cont'd.)

(b) Coherence
3.7 Modal Parameter Estimation

The autospectra from the ambient data were curve fitted by the procedure detailed in the preceding chapter. The average values of frequencies and damping ratios for the first three bending modes in the braced and rigid frame directions are tabulated in Table 3.1.

The frequency response functions from the forced vibration tests were curve fitted using a commercially available software system, the HP Modal Analysis Package (50). This system fits a transfer function model to the measured $H(f)$ for the estimation of frequencies, damping and complex mode shapes. The complex mode shapes, which are characterized by a magnitude and phase for each mode shape coefficient, will be obtained if the damping matrix $[C]$ cannot be uncoupled using the modal matrix $[\phi]$ (51). When this occurs, all the points in the structure do not move exactly in or out of phase with each other and no normal modes exist. For the present application, the real parts of the complex modes were extracted and normalized to obtain the approximate natural modes.

Careful examination of all the frequency response functions revealed that only the lowest four modes in bending and torsion could be identified. High modal density in the upper frequency region, which included modes other than the primary bending and torsional modes, and a lack of sufficient number of response measurements complicated the identification of the higher modes with any degree of certainty. The parameters for the first four modes were obtained using a combination of single and multidegree-of-
Table 3.1. Ambient Test Results

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Braced Frame Direction</th>
<th>Rigid Frame Direction</th>
<th>Torsion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Freq. (Hz)</td>
<td>Damp. (%)</td>
<td>Freq. (Hz)</td>
</tr>
<tr>
<td>1</td>
<td>0.43</td>
<td>3.8</td>
<td>0.33</td>
</tr>
<tr>
<td>2</td>
<td>1.34</td>
<td>3.4</td>
<td>1.01</td>
</tr>
<tr>
<td>3</td>
<td>2.41</td>
<td>4.8</td>
<td>1.73</td>
</tr>
</tbody>
</table>

Table 3.2. Forced Vibration Test Results

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Braced Frame Direction</th>
<th>Rigid Frame Direction</th>
<th>Torsion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Freq. (Hz)</td>
<td>Damp. (%)</td>
<td>Freq. (Hz)</td>
</tr>
<tr>
<td>1</td>
<td>0.41</td>
<td>2.7</td>
<td>0.32</td>
</tr>
<tr>
<td>2</td>
<td>1.30</td>
<td>1.4</td>
<td>0.96</td>
</tr>
<tr>
<td>3</td>
<td>2.35</td>
<td>3.0</td>
<td>1.66</td>
</tr>
<tr>
<td>4</td>
<td>3.37</td>
<td>2.3</td>
<td>2.32</td>
</tr>
</tbody>
</table>
freedom curve fitting procedures. Single degree-of-freedom fits were used for all the fundamental modes and the fourth bending mode in the braced frame direction. All the other modes were fitted using multidegree-of-freedom functions. An example of the type of fit obtained using the software system is given in Figure 3.23 for a frequency response function measured on the roof.

The final estimates for the frequencies and the damping ratios obtained by curve fitting are listed in Table 3.2. The mode shapes for bending modes are plotted in Figures 3.24 and 3.25. Torsional mode shapes are omitted here since only 3 floors were used for torsional response measurement.

Comparison of Tables 3.1 and 3.2 shows that the ambient test frequencies are higher than the forced vibration test frequencies. This is in agreement with the findings of previous investigations on highrise buildings (14, 52). The damping values in the ambient results are also consistently higher. But in this case, the signal-to-noise ratio in the ambient measurements used to compute the autospectra was rather low. Therefore, no definite conclusions can be drawn from the above results. Only the modal parameter values from the forced vibration tests were employed in the identification of an optimum stiffness matrix for the analytical model of the building. This identification procedure, carried out to determine the possible contribution of the exterior curtain wall to the total stiffness, is described in the next chapter.
Figure 3.23. A Transfer Function and its Fit
Figure 3.24. Braced Frame Bending Mode Shapes

Figure 3.25. Rigid Frame Bending Mode Shapes
4.0 ESTIMATION OF STIFFNESS

4.1 Introduction

One of the primary reasons for structural dynamic testing is to validate, modify, update or construct the analytical or finite-element model of the structure. In most cases an a priori model of the structure exists. This model, for many complex structures, usually consists of a set of second order linear differential equations in terms of the mass, stiffness and damping parameters (Equation 2.16). But the dynamic behavior of the structure is often better interpreted in terms of the modal parameters. However, for several applications such as the direct integration of the equations of motion, the system matrices containing the mass, stiffness and damping constants are required to be known explicitly.

A number of procedures have been employed in the past to construct or alter the system matrices utilizing experimental data. The fundamental motivation for such procedures has been the need for an analytical model that is capable of simulating the experimentally observed behavior as closely as possible. Some of these procedures use the modal parameters while others employ the time domain response directly.

Flannelly and others (53) and Thoren (54) describe schemes to construct the system matrices from measured modal data without
making use of an a priori theoretical model. These are applicable only when the model is restricted to have as many degrees of freedom as the number of measured modes, which in turn must be equal to the number of measurement points. Their usefulness is thus limited for structures such as highrise buildings whose analytical models are generally required to possess many more degrees of freedom than the number of measured modes. Berman and Flannelly (55) used a procedure that overcomes this problem to some extent. In this procedure, the analytical model is derived from data for fewer modes than the number of measurement points, using an initial estimate of the mass matrix. The resulting "incomplete" model simulates the structural response in a specific frequency range when all the modes within this range are included in model construction.

Baruch and others (56, 57) developed a method to determine an optimum stiffness matrix when the mass matrix is known. This method requires that the measured mode shapes satisfy the orthogonality conditions. Caravani and Thomson (58) considered the estimation of damping assuming that the mass and stiffness matrices are known. This problem was pursued further by Thomson and others (59). Beliveau (60) describes a procedure to obtain the mass, stiffness and damping matrices using the Bayesian estimation technique. Experimentally measured frequencies, damping and complex mode shapes are used in the estimation scheme. Ross (61) applied the least squares method to find an optimum mass matrix.
Collins and others (62) discuss an estimation procedure to alter the mass and stiffness matrices. Ibanez (63) outlines a perturbation method to compute the necessary changes in the mass and stiffness matrices. Torkamani and Hart (64) split the total stiffness of the structure into those due to different components.

\[
[K] = \sum_{i=1}^{NC} \theta_i [K_i]
\]  

(4.1)

where \([K_i]\) is the stiffness due to the \(i\)th component, \(\theta_i\) is a scalar parameter and NC is the number of components. The values of \(\theta_i\) (which are all equal to one in the a priori model) are adjusted to produce the best possible match with experimental results. Recently, Chen and Garba (65) proposed another method to identify the mass and stiffness matrix coefficients.

Numerous other investigators have studied the dynamic behavior of structural systems under various conditions by applying system identification and parameter estimation techniques. An extensive survey of the work done in this area can be found in the articles by Young and On (66), Collins and others (67), Hart and Yao (68) and Ibanez (69).

The purpose of the present chapter is to describe the procedures employed to estimate the stiffness due to the exterior cladding for the 25-story building with heavyweight cladding which was used in forced vibration tests. An a priori finite-element model of this
building has recently been developed and used in analytical cladding-structure interaction studies (10, 49). It was found in these studies that the analytical frequencies increased by substantial amounts, particularly for torsional modes, when the cladding stiffness was added. The influence of cladding is explored further here by utilizing both the a priori model and the experimental results from forced vibration tests. Starting from the initial model, an improved model is arrived at so that the modal parameters of the improved model are closer to the experimental values than the parameters of the original model. The contribution of cladding to the total stiffness is determined in the process.

Since the major objective was to investigate the stiffness effects of cladding as a lateral force resisting system, the procedures described below do not make any attempt to alter the mass matrix of the original model. Furthermore, the damping effects are not considered. Only the frequencies and the normal mode shapes measured in experiments are employed to modify the a priori stiffness matrix.

The process of revising the stiffness matrix, to make the model conform with the experimental results, can be carried out using any of the several techniques discussed in the references given above. For highrise building models with many degrees of freedom, methods that alter the individual elements of the stiffness matrix directly may involve large, and at times prohibitive, amount of computational effort. The technique that may be preferable in such cases is to decompose the structural stiffness matrix into
components due to various structural subsystems. A structural or stiffness parameter is associated with each of the component stiffness matrices. The total stiffness is obtained using Equation (4.1). This particular approach of manipulating the stiffness matrix by decomposition and the subsequent estimation of the stiffness parameters offers the advantage of being able to handle large systems with relative ease. However, this technique is less powerful than others that modify the coefficients of the stiffness matrix directly, which, for this very reason, can be expected to give better results and produce closer matches.

In the current study, the above technique has a distinct advantage since the role of cladding can be determined by estimating the parameter $\theta_4$ associated with the approximate cladding stiffness matrix developed in the a priori model. Therefore, this technique is adopted and the stiffness matrix is decomposed to explicitly represent the effects of various components including the curtain wall.

4.2 The A Priori Model

The construction details and features of the prior analytical model for the building employed are given in Reference 49. The mass of the structure was lumped at each floor level giving rise to a diagonal mass matrix. The stiffness matrix was assembled from independently developed stiffnesses for three different parts, namely the primary core, the exterior frame that supports the cladding panels and the cladding. The model consists of three degrees of freedom per floor (bending in the braced and rigid frame
directions and torsion). The cladding stiffness matrix was developed in terms of an interstory shear stiffness parameter that quantifies the stiffness effects of cladding and its connection elements on each face between floors. The initial value for this parameter was determined by the least squares method on a trial and error basis, employing preliminary ambient test results. This value was found to be 625 Kips/inch.

4.3 The Estimation Methods

The stiffness parameters for the three components, viz. the core, the frame and the cladding, are estimated using a weighted least squares approach. The weighting matrix is selected by considering the uncertainties in the measurements as well as the prior values of the parameters. The estimates are also obtained by the usual ordinary least squares, maximum likelihood and maximum posterior density methods. The results from these three procedures, are compared with the results of the weighted least squares method. A concise review of the three standard estimation procedures will now be presented followed by a description of the weighted least squares method used.

4.3.1 Ordinary Least Squares (OLS) Estimation

The cost function for the OLS method is given by Equation (2.32). Experimental values \( \{y_1\} \) correspond to the measured frequencies and mode shape coefficients and \( \{F_1(\theta)\} \) correspond to the analytical eigenparameters obtained by solving the eigenvalue problem for given
values of the parameters \{\theta\}. The summation index \(i\) in this case refers to the different dependent variables rather than different experiments as for the single equation least squares problem. The OLS procedure does not take into account any information regarding the distribution of the data or the confidence in the prior values of \{\theta\}; the measurement and the modeling errors are completely ignored.

### 4.3.2 Maximum Likelihood (ML) Estimation

The ML estimates are obtained by maximizing the likelihood function \(L(\theta)\). If \(P(Y/\theta)\) denotes the probability density function of the observations for given \{\theta\},

\[
L(\theta) = P(Y/\theta) \tag{4.2}
\]

where \{\theta\} is considered variable. When the errors in the measurement are normally distributed with zero mean and a covariance matrix \([V_y]\), the logarithm of the likelihood function can be expressed as (34)

\[
\ln[L(\theta)] = -\frac{1}{2} r [\ln(2\pi)] - \frac{1}{2} \ln(|V_y|)
- \frac{1}{2} e^T [V_y]^{-1} e \tag{4.3}
\]

where \(r\) is the number of dependent variables, \(|V_y|\) is the determinant of \([V_y]\) and \(e\) is the \(r \times 1\) error vector. Maximizing \(L(\theta)\) reduces to minimizing \(\psi\), where
\[
\psi = \{e\}^T [V_y]^{-1}\{e\}
\]  

(4.4)

when \([V_y]\) is completely known. Thus, the maximum likelihood method with the above assumptions is equivalent to weighted least squares estimation where the weighting matrix is taken as the inverse of the covariance matrix. If \([V_y]\) is not completely known, an objective function that is different from Equation (4.4) can be used. The form of this function will depend on the amount of knowledge of the covariance (34); that is, it depends on the degree to which \([V_y]\) is known.

The ML procedure considers the measurement errors in estimation but does not treat the parameters to be estimated as random variables. Although the ML estimates do not possess optimal statistical properties, they are usually satisfactory.

4.3.3 Maximum Posterior Density (MPD) or Bayesian Estimation

If a prior density function \(P_b(\theta)\) can be assigned to the parameters, the posterior density function \(P_a(\theta/Y)\) can be written, using Bayes' theorem, as

\[
P_a(\theta/Y) = \frac{P(Y/\theta) P_b(\theta)}{P(Y)} = \frac{L(\theta) P_b(\theta)}{P(Y)}
\]

(4.5)

where

\[
P(Y) = \int P(Y/\theta) P_b(\theta) \, d\theta
\]
The process in which the parameters are estimated by maximizing the posterior density function is known as the maximum posterior density method. If both \( P(Y|\theta) \) and \( P_b(\theta) \) are normally distributed, maximizing \( P_a(\theta|Y) \) is the same as minimizing the following function (34):

\[
\psi = \{e\}^T [V_y]^{-1}\{e\} + \{\theta - \mu\}^T[V_\theta]^{-1}\{\theta - \mu\}
\]

(4.6)

where \( \{\mu\} \) and \( [V_\theta] \) are the mean and covariance of the prior distribution of \( \{\theta\} \).

The MPD method includes the effects of uncertainty in the data and the model. It is even applicable to some cases where it is not feasible to use least squares or maximum likelihood method.

### 4.3.4 Weighted Least Squares (WLS) Estimation

In WLS estimation, the objective function \( \psi \) takes the form

\[
\psi = \{e\}^T [W] \{e\}
\]

(4.7)

where \( [W] \) is the weighting matrix. The OLS method is a special case of the WLS procedure with \( [W] \) equal to the identity matrix. Also, ML estimation with normally distributed errors and known covariance is equivalent to assuming \( [W] = [V_y]^{-1} \). But these are by no means the only types of weighting possible. In the present study, the weighting matrix is taken as the inverse of the error covariance \( [V_e] \) computed by treating both \( \{Y\} \) and \( \{F(\theta)\} \) as random. Under the assumption that the experimental values \( Y_i \) and the analytical values...
$F_1(\theta)$ are uncorrelated, it is easy to show that

$$\text{cov}[\mathbf{e}_i \mathbf{e}_j] = \text{cov}[\mathbf{Y}_i \mathbf{Y}_j] + \text{cov}[F_1(\theta) F_j(\theta)] \tag{4.8}$$

in which $\text{cov}[x_i x_j]$ denotes the covariance of two random variables $x_i$ and $x_j$. In matrix form,

$$[\mathbf{V}_e] = [\mathbf{V}_y] + [\mathbf{V}_F] \tag{4.9}$$

where

$$[\mathbf{V}_e] = \begin{bmatrix}
\text{Var}(e_1) & \text{Cov}[e_1, e_2] & \cdots & \text{Cov}[e_1, e_r] \\
\text{Cov}[e_2, e_1] & \text{Var}(e_2) & \cdots & \\
& \ddots & \ddots & \\
\text{Cov}[e_r, e_1] & \cdots & \text{Var}(e_r)
\end{bmatrix}$$

and

$$[\mathbf{V}_F] = \begin{bmatrix}
\text{Var}(F_1(\theta)) & \text{Cov}[F_1(\theta) F_2(\theta)] & \cdots & \text{Cov}[F_1(\theta) F_r(\theta)] \\
\text{Cov}[F_2(\theta) F_1(\theta)] & \text{Var}(F_2(\theta)) & \cdots & \\
& \ddots & \ddots & \\
\text{Cov}[F_r(\theta) F_1(\theta)] & \cdots & \text{Var}(F_r(\theta))
\end{bmatrix}$$

The matrix $[\mathbf{V}_F]$ is evaluated using the prior covariance matrix of the parameters $[\mathbf{V}_\theta]$. 
The procedures for computing the elements $\text{cov}[F_i(\theta)F_j(\theta)]$ using $[V_0]$ are well established (70-72). These procedures are based on the small perturbation assumption that the analytical values can be expressed as

$$F_i(\theta) = F_i(\bar{\theta}) + \sum_{k=1}^{P} \frac{\partial F_i(\theta)}{\partial \theta_k} |\{\theta\} = \{\bar{\theta}\} (\theta_k - \bar{\theta}_k) \quad (4.10)$$

where $\{\bar{\theta}\}$ is the mean of the distribution of $\{\theta\}$. From the above equation,

$$E[F_i(\theta)] = F_i(\bar{\theta})$$

and

$$\text{Cov} [F_i(\theta)F_j(\theta)] = \sum_{k=1}^{P} \sum_{l=1}^{P} \frac{\partial F_i(\theta)}{\partial \theta_k} \frac{\partial F_j(\theta)}{\partial \theta_l} |\{\theta\} = \{\bar{\theta}\} \text{Cov}(\theta_k, \theta_l)$$

where $E[\cdot]$ denotes expectation. Cast in the matrix form,

$$\{E[F(\theta)]\} = \{\bar{F}(\theta)\} = \{F(\bar{\theta})\} \quad (4.11a)$$

and

$$[V_F] = \left[ \frac{\partial F}{\partial \theta} \right]^T [V_0] \left[ \frac{\partial F}{\partial \bar{\theta}} \right] \quad (4.11b)$$
where

\[
\frac{\partial f}{\partial \theta} = \begin{bmatrix}
\frac{\partial f_1}{\partial \theta} & \frac{\partial f_2}{\partial \theta} & \cdots & \frac{\partial f_r}{\partial \theta} \\
\frac{\partial f_1}{\partial \theta} & \frac{\partial f_2}{\partial \theta} & \cdots & \frac{\partial f_r}{\partial \theta} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial f_1}{\partial \theta} & \frac{\partial f_2}{\partial \theta} & \cdots & \frac{\partial f_r}{\partial \theta}
\end{bmatrix}
\]

in which the dependence on \(\{\theta\}\) is omitted for convenience.

The matrix \(\frac{\partial f}{\partial \theta}\) consists of the derivatives of the modal parameters with respect to the stiffness parameters. In the case of frequencies, the derivative \(\frac{\partial f_i}{\partial \theta_j}\) is given by

\[
\frac{\partial f_i}{\partial \theta_j} = \left(\frac{df_i}{d\lambda_i}\right)\left(\frac{\partial \lambda_i}{\partial \theta_j}\right) = \frac{1}{4\pi \sqrt{\lambda_i}} \frac{\partial \lambda_i}{\partial \theta_j}
\]

(4.12)

where \(\lambda_i\) is the analytical eigenvalue, expressed as

\[
\lambda_i = \omega_i^2 = (2\pi f_i)^2
\]

Therefore, in order to compute \(\frac{\partial \lambda_i}{\partial \theta_i}\), the eigenvalue derivative \(\frac{\partial \lambda_i}{\partial \theta_j}\) and the eigenvector derivatives \(\frac{\partial \phi_i}{\partial \theta_j}\) must be evaluated. Several investigators have dealt with the problem of obtaining the eigenparameter derivatives (72-76). These derivatives are acquired as follows.

The eigenvalue problem to be solved can be written as

\[
[K] \{\phi_i\} = \lambda_i [M] \{\phi_i\}
\]

(4.13)
Differentiating Equation (4.13) with respect to $\theta_j,$

$$[K, j]\{\phi_i\} + [K]\{\phi_i, j\} = \lambda_{i, j}[M]\{\phi_i\} + \lambda_{i, [M, j]}\{\phi_i\}$$

$$+ \lambda_{i, [M]}\{\phi_i, j\}$$

(4.14)

where $[K, j]$ represents the matrix of the derivatives of the stiffness elements with respect to $\theta_j$ and so on. Multiplying Equation (4.14) by $\{\phi_i\}^T$ and recognizing that $[M]$ and $[K]$ are symmetric, the above equation reduces to

$$\{\phi_i\}^T[K, j]\{\phi_i\} = \lambda_{i, j}\{\phi_i\}^T[M]\{\phi_i\} + \lambda_{i, [M], j}\{\phi_i\}$$

where Equation (4.13) has been used. Therefore

$$\lambda_{i, j} = \frac{\{\phi_i\}^T[K, j]\{\phi_i\}}{\{\phi_i\}^T[M]\{\phi_i\}}$$

(4.15)

Rearranging Equation (4.14)

$$[K] - \lambda_{i, [M]}[\phi_i, j] = [\lambda_{i, j}[M] + \lambda_{i, [M], j} - [K, j]]\{\phi_i\}$$

(4.16)

Since the matrix $[K] - \lambda_{i, [M]}$ is of rank (n-1), Equation (4.16) cannot be directly solved for $\{\phi_i, j\}$. If one of the elements of $\{\phi_i, j\}$ is fixed, the other elements can be determined by deleting the corresponding equation from Equation (4.16). It is obvious from
Equations (4.15) and (4.16) that to evaluate the derivatives of an

eigenpair, only that pair is needed and Equation (4.13) need not be

solved completely for all the eigenvalues and eigenvectors.

Another method to determine the eigenvector derivatives

utilizes the representation of \{\phi_{i,j}\} as a linear combination of the

n independent eigenvectors which span the n-dimensional space. This

method requires all the eigenvectors though it is possible to

truncate the sum of the eigenvector contributions in some cases and

obtain an approximation for \{\phi_{i,j}\}. However, it does not involve

the solution of a set of simultaneous equations. Details of this

procedure can be found in the references cited above. In the current

procedure, Equations (4.15) and (4.16) are employed to obtain the

required derivatives.

Once the eigenparameter derivatives are known, \[V_F\] can be

evaluated using Equation (4.11). The weighting matrix \[W\] is computed

as

\[
[W] = [V_e]^{-1} = [V_y + V_F]^{-1}
\]

(4.17)

The amount of computational effort involved can be greatly reduced

if the off-diagonal terms in \[W\] are neglected. For this case,

\[
[W] = \left[ \frac{1}{\text{Var}(Y_1) + \text{Var}(F_1)} \right]
\]

(4.18)

Equation (4.18) defines the weighting matrix used here. This
procedure can be viewed as one in which the weights are chosen based on the confidence in the measured as well as the analytical values of the dependent variables.

4.4 The Minimization Algorithm

The objective functions in all four estimation procedures described in the previous section were minimized by the inverse rank one correction (IROC) method. This method involves replacing the Hessian \([S]\) with an approximation that is updated every iteration. A discussion of this method and a derivation are given in Appendix A. The improved estimates for the parameters \(\{\theta^{k+1}\}\) are calculated from the old values \(\{\theta^k\}\) using Equation (A.11) in the Appendix. The approximation \([A^k]\) to the Hessian is updated by adding a correction \([\Delta A^k]\) computed according to Equation (A.10). The series of matrices \([A^k]\) thus generated converges to the Hessian evaluated at the minimum as \(\{\theta\} \) converges to \(\{\theta^*\}\). The Hessian at the minimum is therefore generated along with the minimum itself.

Each iteration in the algorithm involves solving an eigenvalue problem of order \(n\). Only the first few eigenvalues and eigenvectors corresponding to the experimentally measured frequencies and mode shapes are needed to evaluate the objective function and its derivatives with respect to \(\{\theta\}\). In addition, the computation of the eigenvector derivative employing Equation (4.16) requires the solution of a set of simultaneous equations which, if it is assumed that one of the equations has been deleted, is of order \((n-1)\). In the present case,
the eigenvectors and the measured mode shapes were normalized so that the coefficient of the degree of freedom corresponding to the roof was set equal to one. Hence, the mode shape derivatives were calculated with respect to the roof response by excluding the equation for the roof coordinate. The actual solution was carried out by replacing the off-diagonal elements of the row and the column corresponding to the roof with zeros and solving the system of n equations.

The weighted least squares method requires the evaluation of the analytical covariance $[V_F]$. But for this additional computation in WLS estimation, all four methods require approximately the same amount of computations with a major part of the effort being spent on the function and the gradient evaluations. Therefore, in the discussion to follow, these methods will be compared on the basis of the number of function and gradient evaluations necessary for convergence and not the number of iterations.

The termination criterion adopted to stop the algorithm after the minimum has been approximately located is that given in Equation (2.45d). The computations were terminated if the values of the objective function in two successive iterations differed by less than 0.5 percent by choosing $\varepsilon_5 = 0.005$.

4.5 Application and Results

The prior analytical model of the subject highrise building consists of a coupled 63 x 63 stiffness matrix assembled using independent component stiffness matrices belonging to the core, the
exterior frame and the cladding. The stiffness matrix for each of these components is assembled, in turn, using three stiffness matrices corresponding to the response in the three directions, namely bending in braced and rigid frame directions and torsion.

In the following procedure, it is assumed that the response of the structure in any direction is independent of the response in the other directions and the coupling effects in the stiffness matrix are ignored. Uncoupling the stiffness matrix allows decomposition of the original problem into three smaller problems, one for each of the three directions. The order of the system to be solved is now reduced from 63 to 21, but the complete solution is obtained by solving three 21 x 21 systems (The 21 coordinates correspond to the topmost 21 floors. The lower floors were considered laterally supported at the floor levels due to the stiff concrete pedestal at the base of the building.) Dealing with the reduced systems instead of the original coupled system offers the following advantages.

(1) The treatment of three 21 x 21 systems is more economical in terms of computer time and storage than the treatment of the coupled 63 x 63 system.

(ii) By introducing three parameters for the core, the frame, and the cladding in each direction, a total of nine parameters can be estimated. The match between the experimental and the analytical frequencies and mode shapes using these 9 parameters is likely to be better.
than the match that would be obtained with only 3 parameters associated with the coupled component stiffness matrices.

Whereas the frame and the cladding have the same stiffness in both the bending directions, the core does not. This is due to the fact that the core is not symmetric and consists of both rigid and braced frame components.

The uncoupled stiffness matrices in the three directions can be written as

\[ \kappa_i = \sum_{j=1}^{3} \theta_{ij} [K]_{ij}, \quad i=1,2,3 \] (4.19)

where

- \( j = 1 \) corresponds to the core
- \( j = 2 \) corresponds to the exterior frame
- \( j = 3 \) corresponds to the cladding
- \( i = 1 \) corresponds to bending in braced frame direction
- \( i = 2 \) corresponds to bending in rigid frame direction
- \( i = 3 \) corresponds to torsion

\( [K]_i = \) total stiffness in the \( i \)th direction

\( [K]_{ij} = \) stiffness of the \( j \)th component in the \( i \)th direction

and

\( \theta_{ij} = \) parameter associated with \( [K]_{ij} \)

The equations of motion to be solved are

\[ [\ddot{\mathbf{x}}]_i + [\ddot{\kappa}]_i [\dot{x}]_i = 0, \quad i=1,2,3 \] (4.20)

where \( [\ddot{\mathbf{x}}]_i \) = the response vector in the \( i \)th direction and \( [\ddot{\kappa}]_i \) = the diagonal 21 x 21 mass matrix for the \( i \)th directional response.
For estimating the parameters in the two bending directions, the first four frequencies and the first three mode shapes measured in forced vibration tests were considered. The fourth bending mode shapes were not deemed accurate enough to be included here because the convergence of the curve fitting program employed to fit the measured transfer functions was relatively poor in this region. In torsion, only the first four frequencies were used and no mode shapes were considered due to the lack of a sufficient number of response locations to define these adequately.

The analytical values for the modal parameters as predicted by the uncoupled a priori model, which were virtually the same as those predicted by the original coupled model, are compared with the experimental values in Tables 4.1 and 4.2. The a priori model is the one in which all $\theta_{ij}$ equal unity. Table 4.1 lists the computed and the measured frequencies. The mode shape coefficients are tabulated in Table 4.2 and the analytical shapes are compared with the experimental mode shapes in Figures 4.1 and 4.2. Also listed in Tables 4.1 and 4.2 are the values of the sum of the squares of the errors (SSQ), which is a measure of the deviation of the predicted values from the observed values and which forms the objective function in OLS estimation.

From Table 4.1 it is observed that the frequencies in the braced frame direction are the closest to the experimental values (SSQ = 0.0108) while the frequencies in torsion show the greatest deviation (SSQ = 0.0322). The analytical frequencies in braced frame
Table 4.1. Comparison of Experimental and A Priori Analytical Frequencies

<table>
<thead>
<tr>
<th>Dir. Mode No.</th>
<th>Braced Frame</th>
<th>Rigid Frame</th>
<th>Torsion</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.41</td>
<td>0.402</td>
<td>0.32</td>
</tr>
<tr>
<td>2</td>
<td>1.30</td>
<td>1.263</td>
<td>0.96</td>
</tr>
<tr>
<td>3</td>
<td>2.35</td>
<td>2.349</td>
<td>1.66</td>
</tr>
<tr>
<td>4</td>
<td>3.37</td>
<td>3.273</td>
<td>2.32</td>
</tr>
<tr>
<td>SSQ</td>
<td>0.108 X 10</td>
<td>0.179 X 10</td>
<td>-1</td>
</tr>
</tbody>
</table>
Table 4.2. Comparison of Experimental and A Priori Analytical Mode Shapes

<table>
<thead>
<tr>
<th>Floor No.</th>
<th>Braced Frame Direction</th>
<th>Rigid Frame Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mode No.</td>
<td>1</td>
</tr>
<tr>
<td>Roof</td>
<td>1.00 1.00</td>
<td>1.00 1.00</td>
</tr>
<tr>
<td>23</td>
<td>0.94 0.93</td>
<td>0.68 0.69</td>
</tr>
<tr>
<td>21</td>
<td>0.87 0.85</td>
<td>0.33 0.30</td>
</tr>
<tr>
<td>19</td>
<td>0.75 0.76</td>
<td>-0.06 -0.11</td>
</tr>
<tr>
<td>17</td>
<td>0.66 0.67</td>
<td>-0.41 -0.47</td>
</tr>
<tr>
<td>15</td>
<td>-</td>
<td>0.56</td>
</tr>
<tr>
<td>12</td>
<td>-</td>
<td>0.45</td>
</tr>
<tr>
<td>9</td>
<td>0.31 0.28</td>
<td>-0.68 -0.69</td>
</tr>
<tr>
<td>6</td>
<td>-0.16 -0.13</td>
<td>-0.45 -0.37</td>
</tr>
</tbody>
</table>

SSQ (4 Freq. +3 Mode Shapes) 0.197 0.590
Figure 4.1. Comparison of Experimental and Analytical Mode Shapes; Braced Frame Direction

Figure 4.2. Comparison of Experimental and Analytical Mode Shapes; Rigid Frame Direction
bending and torsion are lower than the measured values. On the other hand, the analytical rigid frame bending frequencies are higher than the experimental frequencies. When the mode shapes are included, it is again seen that the parameters in the braced frame direction are closer (SSQ = 0.197) than in the rigid frame direction (SSQ = 0.590). In both the directions, the deflections in the experimentally determined fundamental mode shape are higher than in the analytical shape in general. For higher modes, the measured shapes possess nodal points that are closer to the base.

In most cases, the experimental frequencies are much more accurate than the experimental mode shapes. Accordingly, the coefficients of variation for all the frequencies were taken as 0.002 (0.2% standard deviation) while the values for the mode shapes were taken as 0.05, 0.10 and 0.15 for the first, second and third modes respectively (5, 10 and 15% standard deviation). These values were chosen after examining the modal parameter estimates from different response locations in the building. The diagonal measurement covariance matrix \([V_y]\) was constructed using the above values of standard deviation for the modal parameters.

The matrix \([V_F]\) employed in WLS estimation will be a true representation of the analytical covariance of the modal parameters only for small values of standard deviations for \(\theta_{ij}\). This is due to the first order approximations of the eigenparameters employed in deriving expressions for the covariance. The values assumed here in both WLS and MPD estimation are the following:
\( \sigma(\theta_{i1}) \), the standard deviation for the core parameter = 0.03

\( \sigma(\theta_{i2}) \), the standard deviation for the exterior frame parameter = 0.04

\( \sigma(\theta_{i3}) \), the standard deviation for the cladding parameter = 0.05

The diagonal prior covariance \([V_0]\) was computed using these values.

The approximate posterior covariance \([V_\theta^*]\) for the optimum parameters \(\{\theta^*\}\) can be computed by making use of the general expression given in Reference 34.

\[
[V_\theta^*] = [S^*]^{-1} \left[ \frac{\partial^2 \psi}{\partial \theta \partial Y} \right] [V_y] \left[ \frac{\partial^2 \psi}{\partial \partial Y} \right]^T [S^*]^{-1}
\]

(4.21)

where

\[
\left( \frac{\partial^2 \psi}{\partial \theta \partial Y} \right)_{ij} = \frac{\partial^2 \psi}{\partial \theta_i \partial Y_j}
\]

and \([S^*]\) is the Hessian evaluated at the optimum estimate \(\{\theta^*\}\). For objective functions of the form considered here,

\[
\left[ \frac{\partial^2 \psi}{\partial \theta \partial Y} \right] = -2 \left[ \frac{\partial F}{\partial \theta} \right] [W]
\]

In OLS estimation, \([W] = [I]\), the identity matrix and

\[
[V_\theta^*] = 4 [S^*]^{-1} \left[ \frac{\partial F}{\partial \theta} \right]^* [V_y] \left[ \frac{\partial F}{\partial \theta} \right]^* [S^*]^{-1}
\]

(4.22)
For ML estimation, $W = [V_y]^{-1}$ and Equation (4.21) reduces to

$$[V_{\theta^*}] = 4[S^*]^{-1} \left[ \left( \frac{\partial F}{\partial \theta} \right)^* \right]^T [V_y]^{-1} \left[ \left( \frac{\partial F}{\partial \theta} \right)^* \right] [S^*]^{-1}$$

Equation (4.23) also holds good for MPD estimation. For the WLS procedure, $W = [V_e]^{-1}$ and

$$[V_{\theta^*}] = 4[S^*]^{-1} \left[ \left( \frac{\partial F}{\partial \theta} \right)^* \right]^T [V_e]^{-1} [V_y] [V_e]^{-1} \left[ \left( \frac{\partial F}{\partial \theta} \right)^* \right]^T [S^*]^{-1}$$

Equations (4.22), (4.23), (4.24) and (4.25) can be used to compute the covariance in the four different procedures. When the IROC method is employed to minimize the objective function, the final updated matrix $[A^*]$, which is an approximation to the Hessian at $\{\theta^*\}$ can be substituted for $[S^*]$ in the above equations. However, in this study, no attempts are made to evaluate the performance of the methods using the posterior covariance. Consequently, the values of $[V_{\theta^*}]$ are excluded from the results reported below.

The IROC minimization algorithm was first applied to a simulated case in which the measured modal parameters were taken as the values corresponding to $\theta_{11} = 0.85$, $\theta_{12} = 0.95$ and $\theta_{13} = 0.90$. The parameters for this test case were then estimated using the
OLS procedure with initial values of (1, 1, 1). The results obtained are summarized in Table 4.3. When only the first four frequencies are used, the estimates are (0.846, 0.960, 0.916). The inclusion of the first three mode shapes results in (0.845, 0.959, 0.916), which is very close to the values obtained with only the frequencies. The two sets of estimates are close because the mode shapes corresponding to the initial estimates are very close to the exact mode shapes. This is evident from the values of SSQ, which increases from $0.9103 \times 10^{-1}$ when only the frequencies are considered to $0.9111 \times 10^{-1}$ when the mode shapes are also included.

In several other test cases it was observed that the farther the "true" values were from the initial estimates (1, 1, 1), the better the estimates became when mode shapes were included. This is due to larger differences between the exact and the initial mode shapes. In some cases the final estimates were considerably different from the true values when only the frequencies were considered. This is a direct consequence of the fact that, when the total stiffness of the structure is modeled as in Equation 4.1, the frequencies are much more sensitive than the mode shapes to changes in $\theta$ and several combinations of the parameters could produce approximately the same frequencies. Thus the estimates obtained are not unique. Even when the mode shapes are included, the final estimates depend on the starting values to some extent. But if the minimum is sufficiently close to the starting values, the final estimate is a good approximation of the minimum. This implies that if the measured mode shapes are
Table 4.3. Results for the Test Case with $\theta_{11} = 0.85$, $\theta_{12} = 0.95$, $\theta_{13} = 0.90$

<table>
<thead>
<tr>
<th>Parameters</th>
<th>SSQ</th>
<th>No. of Function and Gradient Evaluations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core</td>
<td>Frame</td>
<td>Cladding</td>
</tr>
<tr>
<td>4 Freq. Only</td>
<td>0.846</td>
<td>0.960</td>
</tr>
<tr>
<td>4 Freq. +3 Mode Shapes</td>
<td>0.845</td>
<td>0.959</td>
</tr>
</tbody>
</table>
sufficiently close to the initial mode shapes corresponding to the initial estimates, good approximations to the minimum will be obtained.

In the rest of this chapter, the estimates of stiffness parameters for different components in different directions determined using various estimation schemes are examined. In all cases, no constraints were imposed on the parameters.

4.5.1 Braced Frame Direction

Table 4.4 lists the results of OLS estimation employed to determine the parameters for bending in the braced frame direction. The value of SSQ decreases from $0.108 \times 10^{-1}$ at the initial estimate $(1, 1, 1)$ to $0.33 \times 10^{-2}$ at the final estimate, for a reduction of about 69%, when the frequencies alone are considered. Inclusion of mode shapes causes SSQ to decrease from 0.197 to 0.191, a reduction of only 3%. Examination of the measured and the initial mode shapes reveals that the initial estimate $(1, 1, 1)$ yields mode shapes that are quite close to the experimental values and, as a result, no significant improvement is achieved by including the mode shapes. It is also observed from this table that the third natural frequency, which was approximately equal to the experimental value initially (2.35 Hz), has undergone considerable change after estimation and is no longer a good approximation of the experimental value. This makes it clear that the improvement in the match between the measured and the model values due to the parameter estimation procedures is only in an overall sense and not for any specific parameter.
Table 4.4. OLS Estimates for Bending in Braced Frame Direction

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Freq. Only (Hz)</th>
<th>4 Freq. + 3 Mode Shapes</th>
<th>( M_1 )</th>
<th>( M_2 )</th>
<th>( M_3 )</th>
<th>( M_4 )</th>
<th>( M_5 )</th>
<th>( M_6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.411</td>
<td>0.414</td>
<td>1.00</td>
<td>0.93</td>
<td>0.85</td>
<td>0.76</td>
<td>0.67</td>
<td>0.56</td>
</tr>
<tr>
<td>2</td>
<td>1.291</td>
<td>1.301</td>
<td>1.00</td>
<td>0.69</td>
<td>0.30</td>
<td>-0.11</td>
<td>-0.47</td>
<td>-0.73</td>
</tr>
<tr>
<td>3</td>
<td>2.402</td>
<td>2.419</td>
<td>1.00</td>
<td>0.30</td>
<td>-0.47</td>
<td>-0.97</td>
<td>-0.98</td>
<td>-0.46</td>
</tr>
<tr>
<td>4</td>
<td>3.347</td>
<td>3.371</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Freq.</th>
<th>( M_1 )</th>
<th>( M_2 )</th>
<th>( M_3 )</th>
<th>( M_4 )</th>
<th>( M_5 )</th>
<th>( M_6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core</td>
<td>1.051</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.061</td>
</tr>
<tr>
<td>Frame</td>
<td>0.995</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.041</td>
</tr>
<tr>
<td>Cladding</td>
<td>1.034</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.061</td>
</tr>
</tbody>
</table>

| SSQ        | 0.330 \( \times 10^{-2} \) |       |       |       |       |       | 0.191 |

| No. of Funct. and Grad. Evaluations | 9 | 6 |
The estimates from the ML, MPD and WLS procedures are tabulated in Tables 4.5 - 4.7. Except for the exterior frame stiffness parameter, the results are almost identical for the three procedures when only the frequencies are used. Taking the mode shapes into account does not appreciably alter the parameters in ML and MPD estimation methods. The WLS procedure produces slightly different values, however. The value of SSQ is the same in all three procedures (0.189). The core stiffness parameter assumes approximately the same value in all three procedures with or without the mode shapes included.

4.5.2 Rigid Frame Direction

Estimation of the stiffness parameters in the rigid frame direction yields the results given in Tables 4.8 - 4.11. With only the frequencies included, SSQ is reduced from an initial value of $0.179 \times 10^{-1}$ to $0.472 \times 10^{-2}$ in OLS, $0.537 \times 10^{-2}$ in WLS, $0.553 \times 10^{-2}$ in ML and $0.662 \times 10^{-2}$ in MPD estimation, for reductions of 74, 70, 69 and 63 percent respectively. As in the braced frame direction, the ML and MPD estimates remain relatively unaffected compared to the OLS and WLS estimates when the first three mode shapes are included. The ordinary and weighted least squares procedures produce about 70 percent change in the parameters from the initial values of $(1, 1, 1)$. For this reason, the match obtained in these procedures is far better than that obtained in ML or MPD estimation. The reduction in SSQ is about 82 - 83% (from 0.590 to 0.102 in OLS and 0.106 in WLS) for the least squares procedures as opposed to a reduction of only 3% (from 0.59 to 0.57) in ML and MPD estimation. Among the two least squares methods, the WLS procedure converges after 15 function
Table 4.5. ML Estimates for Bending in Braced Frame Direction

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Freq. Only (Hz)</th>
<th>4 Freq. + 3 Mode Shapes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Freq. (Hz)</td>
<td>Floor No.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Roof 23 21 19 17 15 12 9 6</td>
</tr>
<tr>
<td>1</td>
<td>0.409</td>
<td>1.00 0.93 0.85 0.76 0.66 0.56 0.45 0.28 0.13</td>
</tr>
<tr>
<td>2</td>
<td>1.285</td>
<td>1.00 0.69 0.30 -0.11 -0.47 -0.73 -0.80 -0.69 -0.38</td>
</tr>
<tr>
<td>3</td>
<td>2.390</td>
<td>1.00 0.30 -0.47 -0.97 -0.97 -0.45 0.34 1.26 1.02</td>
</tr>
<tr>
<td>4</td>
<td>3.332</td>
<td>1.00 0.30 -0.47 -0.97 -0.97 -0.45 0.34 1.26 1.02</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Core</th>
<th>Frame</th>
<th>Cladding</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.042</td>
<td>0.959</td>
<td>1.022</td>
</tr>
</tbody>
</table>

| SSQ        | 0.334 X10^-2 | 0.189 |

| No. of Funct. and Grad. Evaluations | 10 | 10 |
Table 4.6. MPD Estimates for Bending in Braced Frame Direction

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Freq. Only (Hz)</th>
<th>4 Freq. + 3 Mode Shapes</th>
<th>Mode Shapes</th>
<th>Core Parameters</th>
<th>Frame Parameters</th>
<th>Cladding</th>
<th>SSQ</th>
<th>No. of Funct. and Grad. Evaluations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Freq. Only (Hz)</td>
<td></td>
<td>Floor No.</td>
<td>Core</td>
<td>Frame</td>
<td>Cladding</td>
<td>SSQ</td>
<td>X10</td>
</tr>
<tr>
<td>1</td>
<td>0.409</td>
<td>1.00 0.93 0.85 0.76 0.66 0.56 0.45 0.28 0.13</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1.285</td>
<td>1.00 0.69 0.30 -0.11 -0.47 -0.73 -0.80 -0.69 -0.38</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2.391</td>
<td>1.00 0.30 -0.47 -0.97 -0.97 -0.45 0.34 1.26 1.02</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>3.332</td>
<td>1.00</td>
<td>1.043</td>
<td>1.023</td>
<td>0.949</td>
<td>0.334</td>
<td>0.334 -2 0.189</td>
<td></td>
</tr>
</tbody>
</table>

10
Table 4.7. WLS Estimates for Bending in Braced Frame Direction

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Freq. Only (Hz)</th>
<th>4 Freq. + 3 Mode Shapes</th>
<th>Mode Shapes</th>
<th>Floor No.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Freq. (Hz)</td>
<td></td>
<td>23 21 19 17 15 12 9 6</td>
</tr>
<tr>
<td>1</td>
<td>0.409</td>
<td>0.411</td>
<td>1.00 0.93 0.85 0.76 0.67 0.56 0.45 0.28 0.13</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1.285</td>
<td>1.290</td>
<td>1.00 0.69 0.30 -0.11 -0.47 -0.73 -0.80 -0.69 -0.38</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2.392</td>
<td>2.399</td>
<td>1.00 0.30 -0.47 -0.97 -0.98 -0.46 0.33 1.26 1.02</td>
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</tr>
<tr>
<td>4</td>
<td>3.333</td>
<td>3.343</td>
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<td></td>
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</tbody>
</table>

Parameters:

- Core: 1.043
- Frame: 0.940
- Cladding: 1.059

SSQ:

- X10: 0.332
- X10: 0.189

No. of Funct. and Grad. Evaluations: 10
Table 4.8. OLS Estimates for Bending in Rigid Frame Direction

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Freq. Only (Hz)</th>
<th>4 Freq.+ 3 Mode Shapes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Freq. (Hz)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.333</td>
<td>0.331</td>
</tr>
<tr>
<td>2</td>
<td>0.963</td>
<td>0.975</td>
</tr>
<tr>
<td>3</td>
<td>1.720</td>
<td>1.665</td>
</tr>
<tr>
<td>4</td>
<td>2.290</td>
<td>2.306</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Freq. (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core</td>
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<tr>
<td>Frame</td>
<td>1.026</td>
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<tr>
<td>Cladding</td>
<td>0.944</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SSQ</th>
<th>-2 x 10^4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.472</td>
<td>0.102</td>
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</tbody>
</table>

| No. of Funct. and Grad. Evaluations | 9 | 7 |
Table 4.9. ML Estimates for Bending in Rigid Frame Direction

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Freq. Only (Hz)</th>
<th>4 Freq. + 3 Mode Shapes</th>
<th>Freq. (Hz)</th>
<th>Mode Shapes Floor No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.329</td>
<td>0.327</td>
<td>1.00</td>
<td>0.95 0.87 0.78 0.67 0.56 0.45 0.28 0.13</td>
</tr>
<tr>
<td>2</td>
<td>0.949</td>
<td>0.944</td>
<td>1.00</td>
<td>0.69 0.27 -0.16 -0.52 -0.72 -0.75 -0.60 -0.31</td>
</tr>
<tr>
<td>3</td>
<td>1.695</td>
<td>1.687</td>
<td>1.00</td>
<td>0.16 -0.67 -1.00 -0.77 -0.18 0.52 1.04 0.72</td>
</tr>
<tr>
<td>4</td>
<td>2.256</td>
<td>2.245</td>
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<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Core</td>
<td>0.902</td>
<td>0.894</td>
</tr>
<tr>
<td>Frame</td>
<td>0.948</td>
<td>0.929</td>
</tr>
<tr>
<td>Cladding</td>
<td>0.914</td>
<td>0.905</td>
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</tbody>
</table>

SSQ 0.553 X 10^{-2} 0.570

No. of Funct. and Grad. Evaluations 9 8
<table>
<thead>
<tr>
<th>Mode No.</th>
<th>4 Freq. Only (Hz)</th>
<th>4 Freq. + 3 Mode Shapes</th>
<th>3 Mode Shapes</th>
<th>Floor No.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Roof</td>
</tr>
<tr>
<td>1</td>
<td>0.327</td>
<td>1.00</td>
<td>0.95</td>
<td>23</td>
</tr>
<tr>
<td>2</td>
<td>0.944</td>
<td>1.00</td>
<td>0.69</td>
<td>21</td>
</tr>
<tr>
<td>3</td>
<td>1.687</td>
<td>1.00</td>
<td>0.16</td>
<td>19</td>
</tr>
<tr>
<td>4</td>
<td>2.245</td>
<td>1.00</td>
<td>-0.67</td>
<td>17</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th>4 Freq. Only (Hz)</th>
<th>4 Freq. + 3 Mode Shapes</th>
<th>3 Mode Shapes</th>
<th>Floor No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core</td>
<td>0.895</td>
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<td>0.894</td>
<td></td>
</tr>
<tr>
<td>Frame</td>
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<td></td>
<td>0.939</td>
<td></td>
</tr>
<tr>
<td>Cladding</td>
<td>0.903</td>
<td></td>
<td>0.905</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SSQ</th>
<th>0.662 X10^-2</th>
<th>0.569</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Funct. and Grad. Evaluations</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>
Table 4.11. WLS Estimates for Bending in Rigid Frame Direction

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td>Floor No.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Roof 23 21 19 17 15 12 9 6</td>
</tr>
<tr>
<td>1</td>
<td>0.329</td>
<td>0.327</td>
<td></td>
<td></td>
<td></td>
<td>1.00 0.97 0.92 0.85 0.76 0.66 0.55 0.36 0.17</td>
</tr>
<tr>
<td>2</td>
<td>0.950</td>
<td>0.965</td>
<td></td>
<td></td>
<td></td>
<td>1.00 0.78 0.42 0.00 -0.41 -0.70 -0.81 -0.72 -0.40</td>
</tr>
<tr>
<td>3</td>
<td>1.697</td>
<td>1.640</td>
<td></td>
<td></td>
<td></td>
<td>1.00 0.40 -0.37 -0.94 -1.02 -0.56 0.33 1.24 1.00</td>
</tr>
<tr>
<td>4</td>
<td>2.258</td>
<td>2.277</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Freq. Only (Hz) | Mode Shapes
--- | ---
Freq. (Hz) | Floor No.
Roof 23 21 19 17 15 12 9 6

Table 4.11. WLS Estimates for Bending in Rigid Frame Direction

<table>
<thead>
<tr>
<th></th>
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<td>Floor No.</td>
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<td></td>
<td></td>
<td>Roof 23 21 19 17 15 12 9 6</td>
</tr>
<tr>
<td>1</td>
<td>0.329</td>
<td>0.327</td>
<td></td>
<td></td>
<td></td>
<td>1.00 0.97 0.92 0.85 0.76 0.66 0.55 0.36 0.17</td>
</tr>
<tr>
<td>2</td>
<td>0.950</td>
<td>0.965</td>
<td></td>
<td></td>
<td></td>
<td>1.00 0.78 0.42 0.00 -0.41 -0.70 -0.81 -0.72 -0.40</td>
</tr>
<tr>
<td>3</td>
<td>1.697</td>
<td>1.640</td>
<td></td>
<td></td>
<td></td>
<td>1.00 0.40 -0.37 -0.94 -1.02 -0.56 0.33 1.24 1.00</td>
</tr>
<tr>
<td>4</td>
<td>2.258</td>
<td>2.277</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Freq. Only (Hz) | Mode Shapes
--- | ---
Freq. (Hz) | Floor No.
Roof 23 21 19 17 15 12 9 6

Table 4.11. WLS Estimates for Bending in Rigid Frame Direction

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
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<td></td>
<td>Floor No.</td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
<td>Roof 23 21 19 17 15 12 9 6</td>
</tr>
<tr>
<td>1</td>
<td>0.329</td>
<td>0.327</td>
<td></td>
<td></td>
<td></td>
<td>1.00 0.97 0.92 0.85 0.76 0.66 0.55 0.36 0.17</td>
</tr>
<tr>
<td>2</td>
<td>0.950</td>
<td>0.965</td>
<td></td>
<td></td>
<td></td>
<td>1.00 0.78 0.42 0.00 -0.41 -0.70 -0.81 -0.72 -0.40</td>
</tr>
<tr>
<td>3</td>
<td>1.697</td>
<td>1.640</td>
<td></td>
<td></td>
<td></td>
<td>1.00 0.40 -0.37 -0.94 -1.02 -0.56 0.33 1.24 1.00</td>
</tr>
<tr>
<td>4</td>
<td>2.258</td>
<td>2.277</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Freq. Only (Hz) | Mode Shapes
--- | ---
Freq. (Hz) | Floor No.
Roof 23 21 19 17 15 12 9 6
and gradient evaluations which is more than twice the number required for the OLS method.

4.5.3 Torsion

The results of the various estimation techniques for the torsional response of the building are listed in Tables 4.12 - 4.15. The decrease in SSQ is the highest for the OLS method, from $0.322 \times 10^{-1}$ to $0.468 \times 10^{-3}$ or about 99%, which indicates an excellent match. The reduction in the other methods is about 87 - 88%.

4.5.4 Comparison

It is seen from Tables 4.4 - 4.15 that, in most cases, the OLS method requires the lowest number of function and gradient evaluations and also produces the best match by yielding the lowest value for SSQ. The different estimation methods do not yield significantly different values in the braced frame direction as in the rigid frame direction when mode shapes are included. This is mainly due to the remarkable agreement between the experimental and the prior analytical braced frame mode shapes. All four methods converge to values that do not differ much from the initial estimates (less than 7 percent change in all $\theta_{ij}$).

The large changes for the parameters given by the OLS and WLS methods in the rigid frame direction would normally be considered unacceptable. These changes are mainly attributable to the considerable difference between the measured and the prior analytical rigid frame mode shapes. Due to the relative insensitivity of the mode shapes to changes in $\{\theta\}$, the measured and the analytical mode shapes can be reconciled only by producing large changes in the parameters. This is easily accomplished in the OLS procedure since it does not impose
Table 4.12. OLS Estimates for Torsional Response

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Frequencies (Hz)</th>
<th>Mode No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core</td>
<td>Frame Cladding</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Core Frame Cladding</td>
<td></td>
</tr>
<tr>
<td>1.091</td>
<td>1.079 1.102</td>
<td>0.427</td>
</tr>
<tr>
<td>2.380</td>
<td>2.093 2.907</td>
<td></td>
</tr>
<tr>
<td>SSQ</td>
<td>0.468 X 10</td>
<td>-3</td>
</tr>
<tr>
<td>No of Funct. and Grad. Evaluations</td>
<td>9</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.13. ML Estimates for Torsional Response

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Frequencies (Hz)</th>
<th>Mode No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core</td>
<td>Frame Cladding</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Core Frame Cladding</td>
<td></td>
</tr>
<tr>
<td>1.051</td>
<td>1.022 1.081</td>
<td>0.421</td>
</tr>
<tr>
<td>2.363</td>
<td>2.063 2.966</td>
<td></td>
</tr>
<tr>
<td>SSQ</td>
<td>0.401 X 10</td>
<td>-2</td>
</tr>
<tr>
<td>No of Funct. and Grad. Evaluations</td>
<td>9</td>
<td></td>
</tr>
</tbody>
</table>
Table 4.14. MPD Estimates for Torsional Response

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Frequencies (Hz)</th>
<th>Mode No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core</td>
<td>Frame</td>
<td>Cladding</td>
</tr>
<tr>
<td>1.049</td>
<td>1.015</td>
<td>1.081</td>
</tr>
<tr>
<td>SSQ</td>
<td>-2</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.15. WLS Estimates for Torsional Response

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Frequencies (Hz)</th>
<th>Mode No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core</td>
<td>Frame</td>
<td>Cladding</td>
</tr>
<tr>
<td>1.051</td>
<td>1.023</td>
<td>1.081</td>
</tr>
<tr>
<td>SSQ</td>
<td>-2</td>
<td></td>
</tr>
</tbody>
</table>
any constraints on the parameters nor does it make use of a weighting matrix by which the frequencies are given more importance than the mode shapes. Therefore, the OLS method generates final estimates that differ by large amounts from (1, 1, 1).

In maximum likelihood estimation, the mode shape terms contribute a relatively small amount to the objective function on account of the relatively high values of variance assumed for the mode shapes. As a result, the match between the analytical and the experimental mode shapes is given much less consideration than the match in the frequencies. Hence, this procedure yields estimates reasonably close to the prior values so that the improved analytical model reproduces the experimental frequencies with a greater degree of accuracy than it does for the mode shapes.

The same is also true of Bayesian estimation in which, in addition to the smaller weights used for the mode shapes, the parameters are also constrained by the inclusion of the prior distribution term in \( \psi \). The effect of this term is such that the objective function increases as the parameters deviate more from their prior mean values, which in this case are all equal to one. Consequently, this procedure also produces estimates that lie near the initial values. However, the WLS method converges to values that differ as much from the initial values as the OLS estimates. This could be ascribed to the following. Since the mode shapes are much less sensitive to \( \theta \) than the frequencies, they also possess much smaller variance for a given parameter covariance \( [V_\theta] \). When these values are added to the
experimental covariance \([V_y]\), they tend to reduce the differences between the low values of variance for the frequencies and the high values for the mode shapes. The weights generated by the inverse of \([V_e]\) may then be such that the frequencies and the mode shapes are given about the same weightage. It may even be true in some cases that some of the mode shape coefficients are given more weight than any of the frequencies. The weighted least squares method will, in this case, function like the ordinary least squares method and the estimates will tend to differ by large amounts as in the OLS method.

Comparison of the two methods that utilize the prior covariance of the parameters, namely the WLS and the MPD procedures, shows that in all cases the WLS method gives a better match with lower values of SSQ. The effect of changing \([V_e]\) in these two methods was examined by using various values for \(\sigma(\theta_{ij})\). In addition to the 3, 4 and 5% standard deviations for the core, frame and cladding parameters, two cases with 3, 5 and 10 and 5, 10 and 15% standard deviations were considered. The results obtained are summarized in Table 4.16 for bending in the rigid frame direction considering the frequencies alone. This table suggests that the value of SSQ is higher for increasing variance of the parameters in both of the methods. The core and the cladding parameters remain relatively unaltered for the different cases.

The final parameter estimates for the four procedures taken from Tables 4.4 - 4.15 are summarized in Table 4.17. Though the OLS
Table 4.16. Comparison of Estimates from MPD and WLS Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Maximum Posterior Density</th>
<th>Weighted Least Squares</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Core Frame Cladding SSQ</td>
<td>Core Frame Cladding SSQ</td>
</tr>
<tr>
<td>Std.dev. (Core,Frame, Cladding)</td>
<td>3,4,5 % 0.895 0.938 0.903 0.662 X -2 0.904 0.952 0.916 0.537 X -2</td>
<td>3,5,10 % 0.895 0.935 0.903 0.666 X -2 0.903 0.949 0.915 0.547 X -2</td>
</tr>
<tr>
<td></td>
<td>5,10,15 % 0.895 0.930 0.903 0.672 X -2 0.903 0.950 0.915 0.548 X -2</td>
<td>10</td>
</tr>
</tbody>
</table>
Table 4.17. Comparison of Estimates from Different Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Freq. Only</th>
<th>Freq. + Mode Shapes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Core</td>
<td>Frame Cladding</td>
</tr>
<tr>
<td><strong>Freq. Only</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OLS</td>
<td>1.051</td>
<td>0.995</td>
</tr>
<tr>
<td>ML</td>
<td>1.042</td>
<td>0.959</td>
</tr>
<tr>
<td>MPD</td>
<td>1.043</td>
<td>0.949</td>
</tr>
<tr>
<td>WLS</td>
<td>1.043</td>
<td>0.963</td>
</tr>
<tr>
<td><strong>Braced Frame</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Bending</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OLS</td>
<td>0.923</td>
<td>1.026</td>
</tr>
<tr>
<td>ML</td>
<td>0.902</td>
<td>0.948</td>
</tr>
<tr>
<td>MPD</td>
<td>0.895</td>
<td>0.938</td>
</tr>
<tr>
<td>WLS</td>
<td>0.904</td>
<td>0.952</td>
</tr>
<tr>
<td><strong>Rigid Frame</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Bending</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OLS</td>
<td>1.091</td>
<td>1.079</td>
</tr>
<tr>
<td>ML</td>
<td>1.051</td>
<td>1.022</td>
</tr>
<tr>
<td>MPD</td>
<td>1.049</td>
<td>1.015</td>
</tr>
<tr>
<td>WLS</td>
<td>1.051</td>
<td>1.023</td>
</tr>
<tr>
<td><strong>Torsion</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
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</tr>
</tbody>
</table>
procedure understandably yields the least error in most cases, the estimates obtained may be highly unreliable. The other three methods produce values that are approximately equal when only the frequencies are included. The estimates obtained with the mode shapes taken into account are not to be relied upon since the form of the stiffness model used is such that large changes in \( \theta \) are called for to cause considerable changes in the mode shapes. Furthermore, as discussed earlier, the OLS and ML estimates are not unique, especially when the mode shapes are excluded. The question of uniqueness does not arise in MPD and WLS estimation since a prior distribution is assigned to the parameters in each case.

Taking the above factors into account, the estimates obtained from the WLS method without the mode shapes are considered acceptable here. These values, for core, frame and cladding respectively, are 0.904, 0.952 and 0.916 in the rigid frame direction, 1.043, 0.963 and 1.023 in the braced frame direction and 1.051, 1.023 and 1.081 in torsion. The ML and MPD estimates also exhibit the same trend as the above values.

The parameter estimates for the cladding stiffness suggest that the interstory shear stiffness is greater than the assumed value of 625 Kips/inch in the braced frame direction and torsion and less in the rigid frame direction. All the estimates are not too far from one which implies that the initial value is a good approximation in all three directions. The final estimate in torsion is the highest
(about 8 percent more than the assumed value), which could mean that the cladding affects the torsional response of the structure slightly more than the bending response, or the contribution to stiffness is greater in torsion. This is consistent with the analytical finite element model studies of Reference 10, which show that the torsional frequencies are most altered by cladding effects. However, the above values are too close to make any decisive conclusions at this point and further investigations are needed to consolidate these findings.
5.0 SUMMARY AND CONCLUSIONS

Two highrise buildings were studied in an effort to investigate the effects of the exterior cladding on the dynamic behavior of the main structure. The role of cladding was studied experimentally by conducting vibration tests and employing parameter estimation techniques to determine the dynamic properties from test data. Parameter estimation techniques were also employed to evaluate certain parameters in the structural stiffness matrix which included the effects of cladding as an added interstory shear stiffness.

The first building was studied during its construction as the cladding was installed in order to directly assess its effects on the dynamic response. Ambient tests were carried out over a period of several months at different stages of construction, starting after the erection of the steel frame and the commencement of the installation of cladding and continuing at regular intervals until the building was completely clad. The time domain measurements were used to compute the spectral density functions. The modal parameters were obtained using nonlinear least squares curve fitting techniques. The analytical form of the magnitude of the frequency response function was fitted to the magnitude of each of the computed linear spectra. The Levenberg-Marquardt method was applied to minimize the sum of squares objective function. The modal parameters
were determined from the best possible fit. It was found that the frequencies (a) show an initial decreasing trend and, except for the fundamental frequencies in braced frame bending and torsion, (b) show a subsequent increasing trend throughout the construction phase.

The initial decrease in all the frequencies was attributed to the overall increase in the mass of the structure, which could more than offset any possible increase in the stiffness due to the cladding or other elements. The subsequent increase in the frequencies was ascribed to the stiffness effects being more predominant than the mass effects during this period. Comparing the amount of cladding that had been installed on different test dates with the frequencies, it was observed that the cladding levels had gone up considerably in the same period, suggesting that the cladding could play a significant role in altering the total stiffness of the structure. The fact that only the upper frequencies show an increasing trend is also in accordance with the anticipated behavior of cladding, which can be expected to interact with the higher modes to a greater degree than the lower modes, due to more curvature associated with the higher modes. The above considerations indicate that at least part of the increase in the frequencies could be ascribed to the exterior curtain wall. It should be noted that some of this increase could also be due to other effects or elements such as the interior partitions.

The second building was employed to evaluate the cladding performance from an analytical viewpoint, making use of dynamic test results. Full scale forced vibration tests were carried out with the aid of an electrohydraulic shaker. The ability of this shaker to
produce arbitrary waveforms for the input function was utilized and rapid sine sweep techniques were employed, resulting in a testing time that is only a fraction of the time required for steady-state testing methods. Transfer functions were computed and fitted to determine the modal parameter estimates for the building. These estimates were also compared with the ambient test results obtained earlier. The frequencies and the damping ratios from the ambient tests were found to be higher than the corresponding values from the forced vibration tests.

The forced vibration test results were used to modify an a priori stiffness matrix model of the building so that the match between the analytical and the experimental modal parameters as improved and also a measure of the cladding stiffness was obtained. A weighted least squares method was employed to estimate the stiffness parameters associated with the stiffness matrices for the core, the exterior frame and the cladding in each of the three directions, namely bending in the braced and rigid frame directions and torsion. The parameters were also determined using the ordinary least squares, maximum likelihood and maximum posterior density (Bayesian) estimation procedures and the results from the various methods were compared.

With only the frequencies taken into account in estimation, all the methods yielded reasonable parameter values that were close to unity. But when the mode shapes were also included, it was found that if the initial mode shapes were not sufficiently close to the experimental values, large and unacceptable changes in the parameters were necessary to significantly improve the mode shapes predicted by
the analytical model. The final parameter values were chosen
as those that gave an acceptable match for the frequencies alone.

The interstory shear stiffness parameter utilized in constructing
the cladding stiffness matrix was found to be somewhat higher in
torsion than in either of the two bending directions. This could
imply that, for the highrise building considered, the stiffness
effects of the curtain wall are slightly more in torsion, which
is consistent with the results of Reference 10.

The preliminary findings reported in this research work are
only a first step in understanding the role of cladding as it affects
the dynamic response of the main structure. More investigations,
both experimental and analytical, are needed to bring out fully the
cladding influence and the cladding-structure interaction effects.
Experimental studies during the construction of a building with
heavyweight cladding are necessary to determine the role of cladding
more effectively. In addition, further studies, in which the
experimental results are used with analytical models for each of the
different stages of construction with different levels of cladding,
would be valuable in evaluating the cladding performance. Analytical
studies are also needed in which the parametric model used is such
that the inclusion of the mode shapes would not cause excessive and
unacceptable changes in the prior values of the structural parameters.
This would make it possible to include the mode shapes in choosing
the final values for the parameters.
APPENDIX A

THE IROC METHOD

The inverse rank one correction (IROC) method belongs to the class of methods known as variable metric or quasi-Newton methods. These methods are applicable to any general objective function as opposed to the Gauss-Newton and the Levenberg-Marquardt methods which are applicable to sum of squares type functions only. In all these methods, the inverse to the Hessian \( S^k \) appearing in Equation (2.35), or the Hessian itself as in the IROC method, is approximated initially by a symmetric matrix which is then updated in the subsequent iterations. The various variable metric methods differ in the updating formula used. The sequence of matrices generated is expected to converge to the Hessian or the inverse of the Hessian as the minimum is approached. If the objective function is quadratic, this takes place in \( p \) iterations where \( p \) is the number of unknown parameters.

In the well known Davidon-Fletcher-Powell (DFP) method (77, 78), the approximation to the inverse of the Hessian is updated by adding a matrix of rank two in each iteration. This method and its other modifications require a unidirectional search along the descent directions generated to locate the optimum value for the step size \( \xi \); that is, \( \psi \) is minimized with respect to \( \xi \) in every iteration. Rewriting Equation (2.35) with the step size parameter \( \xi \),
\[
\{e^{k+1}\} = \{e^k\} - \xi \left[ S^k \right]^{-1} \{g^k\} \tag{A.1}
\]

from which

\[
\{e^{k+1}\} = \{e^k\} - \xi^* \left[ B^k \right] \{g^k\} \tag{A.2}
\]

where \([B^k]\) is the current approximation to \([S^k]^{-1}\) and \(\xi^*\) is the value of \(\xi\) such that \(\psi(\xi^*)\) is a minimum as a function of \(\xi\).

Bard, in his survey (79), found that another group of methods that use correction matrices of rank one and that do not require unidirectional minimization performed considerably better than the DFP methods. The IROC method belongs to this group, but the matrix that is approximated is the Hessian itself (34, 79). The development of the IROC method is as follows.

Since \([S^k]\) is the matrix of the second derivatives, it can be approximated as

\[
[S^k] = \left[ \frac{\Delta g}{\Delta e} \right] \tag{A.3}
\]

where

\[
\{\Delta g\} = \{g^{k+1}\} - \{g^k\}
\]

and

\[
\{\Delta e\} = \{e^{k+1}\} - \{e^k\}
\]

Let \([A^k]\) denote the approximation to \([S^k]\) in the kth iteration and let \([A^{k+1}]\) be the updated version of \([A^k]\) such that

\[
[A^{k+1}] = [A^k] + [\Delta A^k] \tag{A.4}
\]
where $[\Delta A^k]$ is the correction matrix added to $[A^k]$. Substituting $[A^{k+1}]$ for $[S^k]$ in Equation (A.3)

$$[\Delta A^k] \{\Delta \theta\} = \{\Delta g\} - [A^k] \{\Delta \theta\}$$  \hspace{1cm} (A.5)

Let $[\Delta A^k]$ be of rank one. Therefore it can be represented as

$$[\Delta A^k] = \{b^k\} \{b^k\}^T$$  \hspace{1cm} (A.6)

where $\{b^k\}$ is an arbitrary vector. Equation (A.6) in (A.5) gives

$$\{b^k\} \{b^k\}^T \{\Delta \theta\} = \{a^k\}$$  \hspace{1cm} (A.7)

in which $\{a^k\}$ is defined as the right hand side of Equation (A.5). From Equation (A.7)

$$\{b^k\} = \{a^k\} / \{b^k\} \{\Delta \theta\}$$  \hspace{1cm} (A.8)

Substituting Equation (A.8) into (A.7), it can be shown that

$$\{b^k\}^T \{\Delta \theta\} = ([a^k] \{\Delta \theta\}]^T_{\text{12}}$$  \hspace{1cm} (A.9)

Equation (A.6) now becomes, in view of Equation (A.8) and (A.9),

$$[\Delta A^k] = \{a^k\} \{a^k\}^T / \{a^k\} \{\Delta \theta\}$$  \hspace{1cm} (A.10)
which is the required correction matrix. To make certain that all
the matrices generated remain positive definite, Marquardt type
corrections can be added to the diagonal elements of \([A^k]\). The
parameters \(\{\theta\}\) are calculated using

\[
\{\theta^{k+1}\} = \{\theta^k\} - \xi \left([A^k] + \mu^k[I]\right)^{-1} \{g^k\} \tag{A.11}
\]

where \(\mu^k\) is the Marquardt parameter. The procedure for choosing \(\mu^k\)
was discussed in detail in Chapter II. The value of \(\xi\) is chosen
so that \(\psi^{k+1} < \psi^k\). It is not necessary to determine the optimum
\(\xi\) as for the DFP methods.

The initial approximation \([A^1]\) for the first iteration can
be computed following the recommendations given in Reference 34.
The diagonal elements of \([A^1]\) are taken as

\[
A^1_{ii} = -\frac{1}{\theta^1_i} \tag{A.12}
\]

and the off-diagonal elements are all set equal to zero. When
the iterative procedure is terminated after locating the
approximate minimum \(\{\theta^*\}\), the matrix \([A^*]\) will be the approximate
Hessian evaluated at \(\{\theta^*\}\).
REFERENCES


