STRUCTURAL DAMAGE DIAGNOSTICS VIA WAVE PROPAGATION-BASED FILTERING TECHNIQUES

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by

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To my wife, Jennifer, and loving children,

who, upon learning I was home, came running
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SUMMARY

Structural health monitoring (SHM) of aerospace components is a rapidly emerging field due in part to commercial and military transport vehicles remaining in operation beyond their designed life cycles. Damage detection strategies are sought that provide real-time information of the structure’s integrity. One approach that has shown promise to accurately identify and quantify structural defects is based on guided ultrasonic wave (GUW) inspections, where low amplitude attenuation properties allow for long range and large specimen evaluation. One drawback to GUWs is that they exhibit a complex multi-modal response, such that each frequency corresponds to at least two excited modes, and thus intelligent signal processing is required for even the simplest of structures. In addition, GUWs are dispersive, whereby the wave velocity is a function of frequency, and the shape of the wave packet changes over the spatial domain, requiring sophisticated detection algorithms. Moreover, existing damage quantification measures are typically formulated as a comparison of the damaged to undamaged response, which has proven to be highly sensitive to changes in environment, and therefore often unreliable.

As a response to these challenges inherent to GUW inspections, this research develops techniques to locate and estimate the severity of the damage. Specifically, a phase gradient based localization algorithm is introduced to identify the defect position independent of excitation frequency and damage size. Mode separation through the filtering technique is central in isolating and extracting single mode components, such as reflected, converted, and transmitted modes that may arise from the incident wave impacting a damage. Spatially-integrated single and multiple component mode
coefficients are also formulated with the intent to better characterize wave reflections and conversions and to increase the signal to noise ratios. The techniques are applied to damaged isotropic finite element plate models and experimental data obtained from Scanning Laser Doppler Vibrometry tests. Numerical and experimental parametric studies are conducted, and the current strengths and weaknesses of the proposed approaches are discussed. In particular, limitations to the damage profiling characterization are shown for low ultrasonic frequency regimes, whereas the multiple component mode conversion coefficients provide excellent noise mitigation. Multiple component estimation relies on an experimental technique developed for the estimation of Lamb wave polarization using a 1D Laser Vibrometer. Lastly, suggestions are made to apply the techniques to more structurally complex geometries.
CHAPTER I

INTRODUCTION

This chapter provides an overview of structural health monitoring (SHM), and the specific area in which the present damage diagnostic research is couched. In particular, the fundamentals of guided waves (GW) are qualitatively reviewed, and wave-based damage localization techniques are surveyed. Current methodologies of wave-based damage quantification are then described, with particular attention given to mode conversion phenomena. Full wavefield detection strategies are outlined, among which Scanning Laser Doppler Vibrometry (SLDV) is highlighted. The chapter concludes by describing how the present research satisfies several of the existing challenges in SHM, and by specifying the manner in which the research objectives are accomplished.

1.1 Structural Health Monitoring Overview

With the advent of an aging aircraft fleet, both commercially and militarily, there exists an increased need for efficient and effective SHM [104]. SHM has been widely defined as the process of implementing a damage detection strategy for engineering infrastructure in aerospace, civil, and mechanical engineering [19]. For clarity, damage is referred to as any material degradation, structural anomalies which adversely affect the component’s performance [36]. Within metallic structures, damage may manifest itself in the form of corrosion [97](Figure 1(a)), surface-indentation [117], and origination and development of cracks [103]. Typical forms of damage found in composites are delamination and fiber breakage, along with manufacturing defects such as matrix voids and fiber wrinkling (shown in Figure 1(b)), which often compromise the structural integrity [126, 41]. These defects, regardless of the material constituency, can lead to catastrophic results if not properly monitored and repaired [2].
Several review papers and seminal reports conducted by the Los Alamos National Laboratory group [29, 99, 118, 36] have attempted to provide universal nomenclature and axioms to the emerging distinctions within SHM. In particular, a subtlety between the existing non-destructive evaluation (NDE) field and SHM discipline is identified, where SHM is associated with online-global damage identification and NDE is primarily concerned with off-line localized damage assessment [36]. Regardless of this nuance, a tremendous overlap exists between the two communities in terms of data extraction techniques, and one common thread among both successful SHM and NDE techniques is an accurate sensing technology and a reliable signal analysis [59].

1.1.1 Diagnostic and prognostic SHM

SHM may be generally divided into two categories: 1) diagnostic SHM, and 2) prognostic SHM [3]. Diagnostic SHM is concerned with the specific damage identification, including damage location, damage type, and damage quantification, whereas prognostic SHM involves a further step aimed at the prediction of failure modes, remaining life cycles and future static stress allowables [30]. Both diagnostic and prognostic SHM fields have been further subclassified as either being passive or active [88]. Passive SHM involves measuring the operational parameters of the structure without any
external excitations [43], while active SHM deals with imposing excitations to the structure to intentionally interrogate its integrity [104]. Acoustic emission (AE) and strain/load monitoring methods are the most common forms of passive SHM [63, 53]. The most prominent active schemes utilize some form of dynamics-based monitoring, such as extraction of modal parameters or guided wave-based monitoring, whose greatest advantage over AE and other passive schemes is that a sensor/actuator pair may be positioned over large areas, and the coarseness of sensor density may be sufficient to extract health monitoring information [88]. Overall, both diagnostic and prognostic SHM methods consist of system state definition, data acquisition, data filtration, feature extraction, data reduction, pattern recognition and decision making [19].

The main motivation for damage prognosis is derived from the tremendous economic benefits and life safety, where the cost of maintenance is less than the high capital expenditure costs. For example, the U.S. Air Force plans to commit 63% of costs to sustainment, and only 32% to development and acquisition, which means great impetus in developing prognosis techniques [65]. More specifically, structural damage prognosis utilize assessment techniques that are classified as either physics or data-based, where the physics-based theoretically predicts the component’s behavior and data-based rely on previous measurements to assess the current state of damage [38]. These techniques predict a set of consequences for assumed future operational conditions, such that prognosis is effectively a prediction or extrapolation problem that necessitates a probabilistic approach. Probabilistic methods typically employ outlier and novelty detection schemes, sequential probability ratio tests, statistical process control, group clustering, and Bayesian decision theory [37]. These techniques require parametric or non-parametric characterization of feature distributions, and estimates of statistical properties of the assumed distribution. When these prognostic techniques have been applied, the resulting costs to monitor and update the component’s
remaining life have validated the need for reliable prognosis techniques [20].

One area that has generated the greatest amount of damage prognosis research is rotary machinery, primarily as a result of the large data sets available for developing regression modes for failure prediction [37]. Rotary machinery prognostics has demonstrated greater success when failure testing of critical components is allowed, which calibrates the predicted failure to actual failure, and ensures refinement of the predictions [116]. Successful rotary prognostics have also included the inherent statistical uncertainties of a component’s structure and assumed operational loads, which are addressed in the final stages of the prognosis decision-making process [38]. The transition from lessons learned in rotary machinery prognostics to fixed-wing aircraft prognostics has received great attention, and the reader is referred to the recent literature for greater details [53, 37].

However, prior to reaching the prognostic process, structural damage identification, localization, and quantification information are necessary. The most common non-destructive diagnostic methods have included, but are not restricted to, visual inspection, ultrasonic inspection [23], eddy current methods [98], AE methods [84], and radiography [58]. An overview treatment of each diagnostic method is provided in [104]. The pertinent damage information typically requires feature extraction methods, and digital signal-processing techniques are currently being developed and applied to SHM diagnosis. Signal processing methods can be classified into time-domain analysis, frequency domain analysis, and joint time-frequency analysis [99]. The following research will focus on frequency domain dynamics-based damage diagnostic analysis, and critically compares several methods for processing spectral damage signatures.
1.1.2 Dynamics-based SHM

Two of the most common forms of dynamics-based SHM may be broadly classified as either: 1) vibration-based or, 2) wave-based monitoring. A comprehensive review of vibration-based SHM techniques is provided by Doebling et al. [30] and then summarized by Farrar et al. [35]. The basic premise of vibration-based damage detection is that the damage will alter the stiffness, mass or energy dissipation properties of a system. The most typical vibration-based detection methods are the distinctions in basic modal properties, such as resonant frequencies and mode-shapes, where resonant frequencies may indicate a stiffness differential and consequent anomaly [44]. One limitation is that changes in frequencies may not provide spatial information about structural changes, except at higher modal frequencies [81]. However, mode shape vectors do provide spatial information and can be used to locate damage, although a large amount of measurements are required to accurately characterize the mode shape [62]. In addition, mode shape derivatives such as second derivative (curvatures) have shown extreme sensitivity to small perturbations in the system [96]. Vibration-based damage detection systems have the fundamental challenge that damage behavior is typically a local phenomena, and may not influence the low frequency response that is normally measured during vibration tests [35].

Conversely, wave-based damage detection techniques have been shown to be able to locate defects at scales which are the dictated by the excitation wavelength [118]. One type of ultrasonic inspection technique draws on the usage of guided waves (GW), which are stress waves that are confined to the material boundaries of the structure [88]. Historically, theoretical studies on GWs date back to Lord Rayleigh in 1887 for waves that propagate near a free surface of an elastic body [113]. Lamb in 1918 further developed on the concept of Rayleigh waves and imposed an additional parallel boundary, free of surface tractions, to describe the propagation of waves now denoted as “Lamb waves” [64]. Other classes of GWs involve travel at material
interfaces, such as Love waves (horizontally polarized waves at the interface of elastic solid and a vacuum), Stonely waves (interface of two dissimilar solid media), and Scholte waves (interface of solid and fluid) [1].

Worlton in 1961 [119] is attributed with demonstrating the application of guided waves for potential non-destructive evaluation (NDE), and a litany of publications over the last thirty years have been focused on GW-based NDE. Today, multiple texts are fully dedicated to the theoretical underpinnings of GWs [113, 1, 78, 46, 91, 43], and several review papers on GWs in connection with SHM and NDE already exist [89, 108, 88]. Particular attention is given to the well-known Imperial College NDE group, which has published exhaustively on basic GW research and their interactions with structural defects in plates and pipes since the early 1990s [5, 66, 69, 26]. In fact, much of the current research builds on the findings by the Imperial College group, and are frequently compared to throughout this thesis.

1.2 Objectives and Contributions

As a response to the challenges inherent to SHM using GW inspections, this research develops techniques that attempt to locate and estimate the severity of the damage, while mitigating noise susceptibility. Specifically, the objectives are:

1. Investigation of a phase-gradient approach that accurately identifies the damage location.

2. Estimation of spatially-integrated single component and multi-component mode conversion and reflection coefficients as tools for damage quantification.

Numerical and experimental parametric studies are conducted, and the current strengths and weaknesses of the approaches are discussed. In particular, the following contributions are developed within this thesis:
1. A novel phase gradient-based damage localization technique is found to be independent of excitation frequency, damage depth, and applicable to both reflected and converted modes in successfully locating the apparent damage.

2. Demonstrates the superior ability of spatially-integrated multi-component mode coefficients to mitigate noise compared to established single component/single point formulations.

3. Experimentally validates the multi-component extraction via Lamb wave polarization in comparison with analytical formulations.

The scope of this research is strictly diagnostic in nature, and analysis that is prognosis-centered is beyond the current development. The following section provides an overview of the published literature of how GWs have been used in damage detection inspection, and its relevancy to the scope of this research.

1.3 Guided Wave Based Inspection Techniques

1.3.1 Guided wave fundamentals

When an elastic wave is propagating along the structural geometry, a structure acts as a guide, and is therefore known as a “waveguide” [78]. Lamb waves belong to a subset of guided waves which ultrasonically propagate in thinned-wall structures that are free of surface traction [69]. Lamb waves exhibit more than one mode for a given frequency [46], and have the useful property of low attenuation over large distances [91]. Typically excited by surface-bonded or angled wedge piezoelectric transducers, Lamb waves initially propagate as a system of incident and reflected waves from the nearby side-walls [1]. Upon reaching a steady-state condition, a standing wave is produced through the thickness of the layer, which travels along the plane of the structure [43, 46] (Figure 2). The Lamb wave is a superposition of longitudinal and shear modes, and each Lamb mode can be either symmetric or anti-symmetric with respect to the mid-surface [108]. For a mathematical treatment of
Lamb modes and their respective properties, the reader is referred to Viktorov [113], and more recently to Graff [46].

![Wave transducer on plexiglass wedge](image)

**Figure 2:** Development of Lamb waves within a thin-bounded medium

One attractive feature of Lamb waves for SHM purposes is the creation of stresses within the entire thickness of the plate, allowing for interior as well as exterior interrogation [69]. The through-the-thickness stress profile varies with the excited mode [42], such that proper mode selection is a significant factor in damage detection schemes. One consideration for mode selection is based on most Lamb modes being highly dispersive, which means that the phase velocity is a function of frequency. Physically, dispersion manifests itself as a distortion of the injected signal in both time and space. Such distortion increases with distance from the source [27]. As an example, the dispersion and phase velocity curves of an aluminum plate of 1.4 mm thickness are shown in Figure 3. From Figure 3(a), it is clear that for each frequency, there exists at least two modes, while Figure 3(b), shows how the phase velocity of the symmetric, or $S_0$ mode, is nearly constant at frequencies below 1 MHz, and therefore behaves almost non-dispersively. The $S_0$ mode stress profile across the thickness for frequencies less than 1 MHz is straight and the displacement field resembles a simple axial wave [69, 42]. This dominant in-plane component of the displacement field makes the fundamental $S_0$ mode an excellent choice for inspecting full or part-depth defects [95], since any induced complexity may be attributed to the presence of defects and not from the dispersive nature.
Figure 3: Lamb wave dispersion curves of an aluminum plate: (a) Frequency-wavenumber, and (b) Phase velocity

1.3.2 Generation of guided waves

Guided waves can be generated by angled piezoelectric wedge transducers, comb transducers, electromagnetic acoustic transducers (EMATs), dry-coupled contact transducers, Hertzian-contact transducers, piezoelectric wafer actuators, and laser-induced impact. Specific examples of these applications are reviewed by Staszewski et al. [105] and Su et al. [108], among others.

By way of summary, comb transducers have been shown to produce an acceptable plane wave front, although it was concluded by Rose et al. [90] that these cannot generate just a single mode due to source influence effects. EMATs have shown promise in generating the shear horizontal mode, yet have exhibited difficulty in broadening the frequency range and require electrical conductivity of the object [2]. Laser-induced waves usually are generated by Nd: YAG lasers. The drawback of incidental damage on the specimen has relegated the practice to mostly laboratory experiments that admit minimal impact damage while desiring broad-band impulse without additional transducer mass [33]. Alleyne et al. [5] and Hayashi et al. [50] used
angled piezoelectric wedge transducers positioned on acrylic blocks to generate symmetric ($S_0$) and anti-symmetric Lamb modes ($A_0$ and $A_1$), whereas Lowe et al. [69] employed a wide-band transducer that was immersed in water and held at an angle by paraffin wax to produce $S_0$ modes. More recently, Diligent et al. [28] demonstrated that dry-coupled contact transducers placed on opposite sides of an aluminum plate may yield dominant $S_0$ modes, although the purity of the mode is subject to uncertainty of the transducer placement and impedance. Similar $S_0$ dominant modes have been generated by piezoelectric wafer actuators that are temporarily bonded to the structure through an epoxy couplant [10]. These surface-bonded piezoelectric wafer actuators operate by causing a tangential traction at the actuator edges on the structure surface, and have the attractive SHM traits of being removable, lightweight, and relatively inexpensive [42]. The experimental generation of GWs in the following research utilize piezoelectric actuators symmetrically located on opposite sides of aluminum plates to create dominant incident $S_0$ modes.

1.3.3 Guided waves and damage detection

Two approaches commonly used in GW damage detection schemes are the so-called pulse-echo (P/E) and pitch-catch (P/C) methods. The P/E method consists of one probe that acts both as actuator and as sensor [45], whereas the P/C approach employs two probes strategically placed to collect and transmit the acoustic-elastic waves [56, 76]. P/E approaches generally utilize the known wave speed of the inspected medium to eliminate boundary effects as the echo response is recorded, whereas P/C methods must typically incorporate an array of transducers that gather and analyze the time delay, amplitude, spatial arrangement, and frequency content of the captured response prior to assessing the damage location [88].

An insightful comparison of the two methods is recently provided by Croxford et al. [23], who compared the two approaches for damage detection by evaluating
the worst signal-to-noise ratio after subtraction of the reference signal. Croxford and co-workers concluded that, for the given configuration in Figure 4, the signal to noise ratios are identical, and inversely proportional to the sensor pitch location in reference to the damage. Croxford et al. [23] also suggested that practical SHM strategies which

![Figure 4: Schematic of required sensor locations: (a) General sensor layout on flat structure, (b) pulse-echo (P/E) approach, and (c) pitch-catch (P/C) approach (reproduced from [23]).](image)

seek to maintain high sensitivity to damage locations require at least one sensor per square meter. This general rule of one sensor per square meter may impose additional weight constraints, and alternative strategies have been developed. In particular, other investigators have sought strategies based on phased-array spatial mapping [80], statistical-based correlation analysis mapping [124], and passive structural neural systems [61].

In general, a significant advantage exists when no baseline data is required prior to detecting the position of the damage. Reference-free techniques based on Lamb wave
interrogation include the time reversal process (TRP) [100], and polarizing directionality of piezoelectric wafer transducers [82, 60, 83]. Using a pitch-catch approach with transducers on both sides of the plate, the specific technique in [60] draws upon the relative phase shift of fundamental symmetric and antisymmetric modes imposed by the presence of the damage, and after applying a statistical threshold on the amplitudes of decomposed modes, the damage is identified where the threshold is exceeded. A reported drawback to this approach is the sensitivity of the damage localization to the threshold parameter [60]. The present research draws on this concept of phase differential for its damage localization technique, and simultaneously being reference-free.

1.4 Damage Quantification using Lamb Waves

Several strategies have been formulated to utilize Lamb waves in the quantifying of damage. One common strategy has been to restrict the range of the excitation signal, such that either the fundamental $S_0$ mode and $A_0$ are generated as the incident mode [69, 50, 6, 93, 8, 14, 13]. The excitation of modes higher than the fundamental modes produce complex stress profiles, in addition to the highly dispersive nature of the modes. This requires greater sophistication in the signal processing algorithms where more than two modes are excited at each frequency. Hence, this frequency restriction greatly simplifies the signal processing techniques required to isolate the individual modes, thereby eliminating additional obfuscation with the damage signature response. Metrics based on mode amplitudes are then constructed that attempt to describe the damage extent.

Most of the damage defect studies investigate the variation of reflection, transmission or/and conversion coefficients with respect to defect size. Assuming operation below the $A_1$ cutoff, the reflection, transmission and conversion coefficients are defined as the ratios between the amplitudes of transmitted, reflected $S_0$ and/or converted $A_0$
modes and the amplitude of the incident $S_0$ or $A_0$ modes. As an overview, these ratios are defined in time [7, 82] or frequency domain [70], at one location [70, 125] or as an average over a finite/infinite spatial domain [15]. Finite element [70, 125], global-local or hybrid [125, 4], boundary element [123, 54], and modal expansion [15] methods, as well as higher order plate theory [115] and experimental results [67] are used to estimate the wave coefficients and to relate them with the damage size for different configurations. The literature shows that these coefficients are strongly dependent on the frequency-thickness product, on the mode type (symmetric or antisymmetric), the particular direction of the particle motion relative direction of the wave propagation (wave polarization), and on the ratio between the wavelength and a characteristic length of the defect [70, 4, 123, 115, 39, 15].

As discussed previously, mode conversion is defined as the phenomena where energy is transferred from one wave mode to another [43]. For example, a launched $S_0$ mode partially converts to an $A_0$ mode upon arrival at the defect location, or vice versa [14] (Figure 5). Several texts in the early 1970s by Miklowitz [78] and Achenbach [1] generally describe this mode conversion phenomena as tools to measure the extent of the damage.

![Figure 5: 1D waveguide with pure $S_0$ incident mode and consequent mode conversion ($A_0$). The superscripts $(i)$, $(r)$, and $(t)$ denote the incident, reflected, and transmitted modes. $x = x_D$ shows the damage location.](image)

Cawley et al. [6] illustrate the advantage of using Lamb waves over conventional ultrasonic methods, which employ elastic bulk waves in thin-structural components,
and the resolution of the bulk wave echo due to defects generated from top and bottom surfaces. Major results were: 1) a toneburst modified by Hanning window produces limits of mode excitation versus broadband toneburst, where higher modes are undesirably generated, and 2) when operating below the $A_1$ cut-off frequency, the presence of mode conversion occurs which may describe the extent of the defect. The authors presented this data from a more qualitative approach, with time histories for defects of 30%, 40% and 60% shown side-by side. A 2D FT is referenced as the procedure that could be used to determine the amplitude of the different propagating Lamb modes, although no coefficients of reflection or conversion were shown.

A follow-on paper by the same authors [7] explained that the key problem in Lamb wave testing is the measurement of the amplitudes of the individual modes present in a multi-mode, dispersive signal. Using a similar FEM package in [5], an aluminum plate having a surface breaking, straight sided notch of constant width running normal to the plate surface was numerically analyzed. Cawley et al. [7] imposed a plane strain condition on the FEM model to investigate incident modes of $A_0$, $A_1$ and $S_0$ and related reflection and transmission coefficients. These papers [7, 5] showed how the ratio of notch depth to plate thickness and frequency-depth are main parameters rather than the absolute notch depth and frequency. In addition, it was shown how it is possible to detect defects of dimensions greater than about 6% of the wavelength. Finally, provided the width is small compared to the wavelength, the transmission and reflection amplitudes are insensitive to the notch width so the ratio of notch depth to plate thickness is the driving parameter. Single point coefficients were used to quantify the mode conversion, where the ratio of the reflected/ transmitted mode is compared to the incident mode.

Later, a less cited, yet equally significant publication by Hayashi et al. [50] depicted a filtering technique where a single mode is extracted from multi-modal responses. Particular attention was paid to the $A_0$ mode and its non-dispersive behavior in the
higher frequency-thickness regime. Using a 2D Fast Fourier Transform (FFT), a filtering function retains the mode of interest and eliminates unwanted modes in the frequency-wavenumber domain. The authors also used FEM to model damage as a straight-side vertical notch, whose width is at least twice the size of the plate thickness. Experimental verification was performed by angle beam wedges with air-coupled transducers as a non-contact receiver running along a linear stage for spatial scanning measurements. The filtering technique explicitly demonstrated as an effective tool to eliminate undesired modes. In addition, notch width was estimated based on the arrival times from first and second reflected waves. Three notch widths were shown to compare with the actual prediction.

In terms of parametrization of the notch depths and resulting quantification, a fundamental paper from Lowe et al. [69] used a plane strain FEM from an in-house modeling package to simulate an $S_0$ mode interrogating an isotropic plate with a non-propagating, rectangular notch. The monitoring zone was 60 plate thicknesses away from the notch. In experiments, the excitation was produced by a piezoelectric transducer, two laser interferometers positioned at 30 degrees, and in-plane displacement was recorded. The authors found that a sinusoidal variation of the reflection coefficient occurs when displayed as a function as a notch width to wavelength. Also, the amplitude of the reflection coefficient monotonically increases with frequency and notch depth.

Subsequently, Lowe et al. [68] considered the $A_0$ mode launched by an angled transducer partially immersed in water. The $A_0$ reflection coefficient was displayed as a function of notch width to wavelength, which produced a cosinusoidal variation (Figure 6(a)), with maximum and minimum amplitude values related to the phase difference and consequent amplitude cancelation from the reflected $A_0$ and $S_0$ modes at the notch boundaries. The reflection coefficient begins to drastically vary when the frequency-thickness increases at crack depths greater than 70% the plate thickness.
(Figure 6(b)), and a contribution from the axial and shear stresses generate the reflection, as explained by an analytical S-parameter investigation coupled with ray theory. Experimentally, a pulse-echo set up was used to emit and receive the signal, while processing of the measurements was performed in the frequency domain.

![Figure 6](image.png)

Figure 6: Results from $A_0$ mode excitation imparted on an aluminum plate with different notch depths: (a) Cosinusoidal variation(b) Reflection coefficient as function of notch depth [68].

Aside from only notch geometries, Diligent et al. [28] used membrane elements in FEM to model a plate with a through-thickness circular hole. A $S_0$ wave was prescribed at one end as the interrogative mode, and the hole was centrally located,
with only half the plate modeled and thus symmetry conditions were applied. The scattering of the fundamental shear horizontal, or $SH_0$ mode, from the hole is monitored together with the reflected $S_0$ mode. The experimental setup employed dry transducers positioned at the top and bottom surfaces of the plate to generate $S_0$ mode, and the salient results were that single point reflection coefficients for both the $S_0$ and $SH_0$ modes increase with the diameter to wavelength ratio. The mode conversion coefficients are represented as ratio of $SH_0$ over incident $S_0$ mode.

Other investigations on damage location were provided in [14, 13], where the mode conversion of symmetric and asymmetric defects with a rectangular notch is analyzed. The specific findings were that the mode conversion does not occur for symmetric defects, whereas it occurs for asymmetric defects. In addition, the $A_0$ mode elicits greater mode conversion when launched as the incident mode than the $S_0$ mode, although the low frequency regime non-dispersive nature of the $S_0$ mode lends itself as the preferred excitation mode.

Castaings et al. [15] also looked at symmetric and asymmetric cracks. A modal decomposition method for simulating interaction of the $S_0$ and $A_0$ modes is used to model an internal symmetric crack and a single opening crack under plane strain assumptions. The experimental setup included an air-coupled receiver and imasonic transmitter, that produced a plane wave front. A 2D FT was applied to the experimental data, and the reflection and transmission coefficients were evaluated as ratios of the reflected and transmitted mode amplitudes as the incident one. A formulation of the mode coefficients is based on the 2D FT at a specified frequency. The in-plane and out-of-plane numerical results are presented for both the $S_0$ and $A_0$ mode coefficients, with the $S_0$ mode being more suitable than $A_0$ in detecting internal vertical cracks close to the mid-plane of the plate. Also, both the reflected and transmitted $A_0$ modes vary monotonically for opening cracks and are good indicators of defect size, whereas the converted $A_0$ mode coefficient has a parabolic shape.
(Figure 7), which peaks at approximately 70% of the crack height to plate thickness, which means the converted $A_0$ mode coefficient may not be appropriate for crack sizing. The investigators showed good agreement between analytical, numerical, and experimental results.

![Figure 7](image)

**Figure 7**: Predicted and measured reflection and transmission coefficients for $A_0$ incident on single opening cracks as a function of crack-height-to-plate thickness ratio: (a) In-plane surface displacement, and (b) Out-of-plane surface displacement [15].

Rather than using FEM, Cho and Rose [18] produced reflection and transmission factors that were numerically calculated from a boundary element modeling (BEM) formulation, which parameterized the incident mode, defect shape and depth. Cho et al. compared results of BEM and FEM from Alleyne and Cawley [7], which show excellent agreement for the transmitted mode coefficients. In addition, a sharp defect was modeled and shown to produce monotonic increase and decrease for reflection and transmission of the $S_0$ incident mode, whereas a sharp defect yields a minimal variation in the $A_0$ mode reflection and transmission. Later, Rose et al. [123] modeled half-elliptical shaped surface breaking defects of varying lengths and depths using the hybrid BEM normal mode expansion technique. The monotonic trends for increasing reflected $S_0$ mode and decreasing transmitted $S_0$ mode for an increase in the defect
depth were displayed, and these monotonic results do not exist at higher frequencies (greater than 300 kHz) due to multiple mode conversion effects, where the wave energies are redistributed.

From this brief review of significant work in Lamb wave damage quantification, it is clear that several trends among the published literature exist. In particular, incident $S_0$ and $A_0$ modes have shown promise in measuring damage for a range of defect profiles and damage width. Numerical models (based on FEM, BEM, modal decomposition, etc.) and semi-analytical models have been successfully verified to match experimental results using piezoelectric transducers, single point laser interferometers, and comb transducers. In addition, single point mode coefficients based on out-of-plane displacement components with some method of signal processing were experimentally tested.

However, several issues require additional consideration in terms of Lamb wave damage quantification using mode conversion. For example, the literature is sparse in regards to sensitivity assessments of the single point mode coefficients. Also, the out-of-plane component of an incident $S_0$ mode is relatively weak to the $A_0$ out-of-plane component, and it appears likely that using the dominant in-plane $S_0$ component would increase the signal to noise ratios in the estimation of conversion phenomena. In expanding on this concept of displacement components, the question arises how the researcher may empirically validate that the proper components are purely extracted without additional instrumentation, such as a 3D laser setup. Thus, the need for reliable signal processing techniques that utilize Lamb waves are still an ongoing research topic for damage quantification.

### 1.5 Analysis of wave polarization

Ultrasonic waves and their polarization properties in elastic media have been studied as a method to detect and quantify the presence of local stress fields, flaws, and
seismic shifts [48]. The term polarization generally defines the direction of wave oscillations relative to the direction of wave propagation [1]. This phenomena is significant for earthquake tracking in the geophysical community, where polarized seismic waves are filtered and statistically analyzed in determining their origin [92]. In addition, the polarization of surface and guided waves has been investigated by the NDE community since the early 1960s as a technique to measure the state of stress in a structure in conjunction with the well established acoustoelastic effect [110, 22, 52]. The acoustoelastic effect is defined as the effect on ultrasonic wave propagation of the state of stress. Multiple studies on the polarization of Rayleigh waves have demonstrated its potential to characterize the state of stress of a surface [51, 31, 25, 55, 109]. In particular, Junge et al. [55] further evaluated the ratio of the maximum out-of-plane displacement to maximum in-plane displacement, and found analytically that the relative polarization is more sensitive than wave speed to characterize a state of surface stress. A study by Tanuma et al. [109] independently concurred with the work of [55] by rigorously utilizing a first order perturbation formulation, and showed that the polarization ratio can be generalized for any given state of stress.

However, many aerospace applications are thin-walled structures, and are not conducive to the Rayleigh wave boundary condition assumptions, and more commonly exhibit Lamb wave phenomena [91]. There exists relatively sparse published literature in the analysis of Lamb wave polarization in comparison to Rayleigh polarization research. DelSanto et al. [25] applied the numerical Local Interaction Simulation Approach (LISA) to compare the sensitivity of the Rayleigh and Lamb waves to the acoustoelastic effect. Introducing a simulated residual stress field within a steel plate, DelSanto et al. [25] concluded that the Rayleigh wave showed an appreciable phase delay and distortion contributing to a fairly sensitive acoustoelastic effect. On the other hand, the same work reported that the phase delay for Lamb waves are negligible, and the acoustoelastic effects are rapidly smoothed out from the region
of the residual stress. Therefore, Lamb waves are considerably less sensitive than Rayleigh waves in detecting the residual stress field due to the inherent nature of the wave probing the entire depth of the plate [25].

Rather than using Lamb wave polarization to characterize the state of stress, the present research measures Lamb wave polarization to verify that multi-component extraction from a measured single component response can achieved. Wave polarization is thereby used as a validation method within the post-processing of the measured data. The single component formulation may produce significant noise inclusion if the out-of-plane component possesses a low-amplitude response. One aim of this research is to mitigate noise influence by involving both the in-plane and out-of-plane components in the analysis.

For extraction of the polarized Lamb modes, the following thesis describes in detail a frequency-wavenumber domain filtering technique that separates the individual Lamb modes. The current work provides additional modification to the procedure and develops a spatially-integrated multi-component mode coefficient. The successful capturing of the respective multi-displacement/velocity components is shown by comparing the wave polarization measured response to published analytical solutions [87].

1.6 Wavefield Detection and Related Damage Detection Techniques

The utility of an ultrasonic full wavefield damage detection technique is based on the detailed spatial information that is available for further analysis [108, 94]. Various wavefield damage detection techniques have incorporated the wave’s physical phenomena, such as wave attenuation [85], reflection [8], scattering [112, 39], and mode conversion [72, 101, 105]. Depending on the geometry of the structural component, boundary conditions, and damage configuration, the techniques have applied longitudinal or p-waves [71], polarizing shear waves such as Lamb and Love waves [45],
The ultrasonic wavefield is typically recorded by some form of ultrasonic probing, which is generally classified as either contacting or non-contacting. One conventional wavefield measuring method was demonstrated by Kazys et al [57], who immersed a steel plate in water under an axial loading, and using an equally immersed receiver transducer elevated above the plate, measured the full wavefield of $A_0$ and $S_0$ Lamb modes. The major disadvantage of this approach is the obvious requirement of removing the structure from its original configuration and the potential of damaging the part due to water absorption/corrosion. Non-contacting probes have been used by Michaels et al. [77], who employed an embedded array of transducers to excite an ultrasonic wave. Using a probe suspended above the test article, the incident and reflective waves from flaws are scanned using a process termed as Wavefield Imaging (WI).

More recently, Scanning Laser Doppler Vibrometers (SLDV) have demonstrated the ability of measuring and characterizing ultrasonic wavefields [106, 93]. A typical LDV is a non-contact velocity transducer that utilizes the principle of Doppler frequency shift of a laser beam scattered from a moving target by means of an interferometer to measure the surface velocity [34]. An advancement from the LDV is the SLDV, which employs two moving mirrors driven by galvanometric actuators, and allows for the laser beam to be directed at multiple points for data recording. Rather than a single point system, the SLDV has found wide usage in modal analysis [102], and vibration-based damage detection [114], due to its ability to rapidly scan large surface areas (Figure 8). Aside from being a non-intrusive method, the specific benefits of the SLDV over traditional non-contacting sensors are: 1) measurement may be rapidly and precisely positioned at the laser point via a scanning system of a highly spatially-dense grid, and 2) the sensing may occur at a distance, which is compatible with field testing [73, 16]. An example of an SLDV full wavefield scan for large
structures is shown in Figure 8, where a temporarily bonded piezoelectric actuator is shown to produce circular-crested Lamb waves propagating through an aircraft’s horizontal tail.

Figure 8: Full wavefield scan of an aluminum aircraft’s horizontal tail using a Polytec-400 SLDV: (a) Set-up, (b) Propagating wave, and (c) RMS of wavefield

1.7 Organization of the Work

The thesis is organized as follows: chapter 2 describes the methodologies used to both locate and quantify structural damage. Specifically, detail is given to the frequency-wavenumber domain filtering technique, the phase gradient and mode conversion
formulation. Chapter 3 illustrates the techniques applied to numerical results, where 1D FEM parametric studies are shown that demonstrate the diagnostic methods, including damage profile variation and investigate the influence of relevant parameters. Chapter 4 provides a discussion of Lamb wave polarization estimation, with theoretical and experimental validation. Chapter 5 presents the experimental results implementing the methods, while Chapter 6 overviews the conclusions of the research, and suggests ways to further amplify the scope of the work.
CHAPTER II

THEORETICAL BACKGROUND: PHASE GRADIENT AND MODE CONVERSION ESTIMATION FOR DAMAGE LOCALIZATION AND QUANTIFICATION

The following chapter provides the theoretical underpinning for how damage localization and quantification are addressed. Using a simplified 1D waveguide model, a frequency-wavenumber filtering technique of Lamb wave modes is presented as a tool for mode separation and incident wave removal. A damage localization technique that utilizes the change in phase of the reflected and converted modes is then explained. Lastly, the formulation for defect quantification based on mode conversion phenomena is described.

2.1 Overview

Multimodal wave propagation in a 1D waveguide is discussed as the platform to introduce the damage localization and quantification under investigation. The analysis is restricted to the frequency range of the fundamental symmetric and antisymmetric Lamb waves, $S_0$ and $A_0$, respectively. As illustrated in Fig. 9, it is assumed that a pure $S_0$ mode is generated to interrogate the structure. The interaction of the $S_0$ mode with a defect located at $x = x_D$ causes partial $S_0$ reflection and partial mode conversion into the $A_0$ mode, while simultaneously producing a partial $S_0$ and $A_0$ mode transmission. Analytical models of mode conversion, in combination with numerical techniques, have been previously developed by Cho and Rose [18], Grahn [47], Flores-Lopez et al. [40], and later adapted by Diligent et al. [28]. This mode conversion phenomena is evident from finite element and experimental results, although
at the present time, a completely analytical closed-form formulation of Lamb wave mode conversion is not currently published [18, 40].

2.2 Description of Wave Propagation in a 1D Waveguide

Consider the propagation of a multi-modal harmonic wave in a waveguide having a notch damage (Figure 9). The 3D local effects at the damage are neglected and a 1D waveguide is assumed. The following formulation suggests that we are considering the propagation of Lamb waves in a thin plate, even though the description may be considered general for multi-modal harmonic waves, and is not completely rigorous for the case of Lamb waves. It is however sufficient and simple enough to illustrate the filtering and damage characterization concepts under investigation.

Specifically, it is assumed that the incident pure $S_0$ mode has arrived at the damage location, and the reflected, converted and transmitted $S_0$ and $A_0$ modes are now propagating within the structure. The out-of-plane component, $w(x, t)$, at the surface of the plate ($z = h/2$) along a measured spatial length ($0 \leq x \leq L$), denoted as the monitoring length, may be expressed as:

$$w(x, t) = \left[ w_{S_0}^{(r)} e^{ik_{S_0}(x-2xD)} + w_{A_0}^{(r)} e^{ik_{A_0}(x-2xD)} + w_{S_0}^{(t)} e^{-ik_{S_0}x} + w_{A_0}^{(t)} e^{-ik_{A_0}x} \right] e^{i\omega t}$$

(1)

where $k_{S_0|A_0} = k(\omega_0)$ identifies the wavenumber of the $S_0$ and $A_0$ wave modes at
frequency $\omega$ and $w_{S_0|A_0}$ are the out-of-plane wave amplitudes, which follows the development in [43]. In Eq. (1), the superscripts $(i), (r),$ and $(t)$ denote the incident, reflected, and transmitted modes. Eq. (1) may be rewritten as the following:

$$w(x, t) = w_{S_0}^{(i)} \left[ R_{S_0} e^{ik_{S_0}(x-2xD)} + C_{A_0} e^{ik_{A_0}(x-2xD)} + T_{S_0} e^{-ik_{S_0}x} + T_{A_0} e^{-ik_{A_0}x} \right] e^{i\omega t}$$

The terms $R_{S_0}, C_{A_0}, T_{S|A_0}$ in Eq. (2) denote the reflection, mode conversion, and transmission coefficients, under the current assumption. It is noted that the monitored length is assumed shorter than the true length of the plate, and the time interval is windowed such that reflections from the boundaries are not present.

From a time history versus monitored length perspective, an alternative illustration of the multimodal wave phenomena is shown in Figure 10, where the given time history is generated from a numerical FEM plane strain model with a notch location at $x = 0.5$ m. Figure 10 provides a graphical representation of the time history at each location, where the incident $S_0$ mode denoted by the superscript $(i)$ initiates at $x = 0$. Upon arriving at the notch location at $x = 0.5$ m, mode conversion from the $S_0$ to $A_0$ occurs, with partial transmission and reflection of the $S_0$, denoted by the superscripts $(t)$ and $(r)$, respectively.

### 2.3 Frequency-Wavenumber Filtering Technique

Lamb waves exhibit a complex multi-modal behavior and temporal-spatial windowing of the response is typically not sufficient by itself to identify individual reflected and converted modes. The following section describes in detail a filtering technique that separates the individual Lamb wave modes in the frequency-wavenumber domain. The concept was introduced by Alleyne et al. [5, 7, 6] and Hayashi et al. [49, 50], and later refined by Ruzzene et al. [93, 9, 11]. Using a 2D FT, the frequency-wavenumber technique enables extraction of incident, reflected, transmitted, and converted modes which are central to the phase gradient damage localization and mode conversion.
Figure 10: Time history along monitored length of waveguide with pure $S_0$ incident mode and consequent mode conversion ($A_0$). The superscripts (i), (r), and (t) denote the incident, reflected, and transmitted modes.

damage quantification methodologies described later in Sections 2.4-2.5. An overview of the process is initially provided followed by a step-by-step analysis of the incident wave removal and mode separation.

2.3.1 Overview of the filtering process

Filtering of the incident wave, and the selection of one of the wave components may be performed through the application of windowing functions according to the procedure illustrated in [93], and further detailed herein. The following is a summary of the procedure:

- Time domain waveform data $w(x,t)$ from Eq. (1) is windowed over the monitoring spatial region indicated in Fig. 9, such that:

$$w_{h_1}(x,t) = w(x,t)h_1(x,t)$$

(3)

where $w_{h_1}(x,t)$ is the windowed time response, and $h_1$ is a spatial-temporal window. The window is selected to minimize leakage errors when the response
is represented in the frequency-wavenumber domain.

- The windowed response is then transformed to the frequency-wavenumber domain, such that:

\[ W(k, \omega) = \mathcal{F}_{2D}[w_{h_1}(x, t)] \]  

(4)

where \( W(k, \omega) \) is the response in the frequency-wavenumber domain, and \( \mathcal{F}_{2D} \) is the 2D FT.

- Recognizing that the reflected and incident modes appear in distinct quadrants of the frequency-wavenumber domain, apply a rectangular window that removes the incident wave from the response:

\[ W_{h_2}(k, \omega) = W(k, \omega)h_2(k, \omega) \]  

(5)

where \( W_{h_2}(k, \omega) \) is the filtered 2D FT response, and \( h_2 \) is a rectangular windowing function operating in the frequency-wavenumber domain that removes the incident wave response. Illustration of this filtering process and \( h_2 \) windowing is explained in Section 2.3.2.

- Filtering in the frequency-wavenumber domain to extract individual modes, which may be generally written as:

\[ W_{h_3}(k, \omega) = W_{h_2}(k, \omega)h_3(k, \omega) \]  

(6)

where \( W_{h_3}(k, \omega) \) is the filtered 2D FT response that excludes the incident wave, and \( h_3 \) is a 2D frequency-wavenumber windowing function designed to separate the various modes.

- Transform the filtered frequency-wavenumber response, \( W_{h_3}(k, \omega) \) back to the time domain:

\[ \tilde{w}(x, t) = \mathcal{F}_{2D}^{-1}[W_{h_3}(k, \omega)] \]  

(7)
where \( \tilde{w}(x, t) \) is the filtered spatial/ temporal response, and \( \mathcal{F}_{2D}^{-1} \) is the inverse 2D FT.

- Finally, perform a 1D FT along the time domain at each monitored position, such that:

\[
\tilde{w}(x, \omega) = \mathcal{F}_{1D}[\tilde{w}(x, t)]
\]  

(8)

where \( \tilde{w}(x, \omega) \) is the filtered response in the spatial-frequency domain of a single extracted mode.

### 2.3.2 Illustration of filtering for incident wave removal and mode separation

As an example of the filtering procedure, 1D experimental data taken by an SLDV of an aluminum plate with a partial rectangular notch is analyzed. The recorded response in the temporal-spatial domain (Figure 11(a)) is represented in the frequency-wavenumber domain in Figure 11(b) by performing a 2D FT. Four wave components \((S_0^{(t)} + S_0^{(r)}, S_0^{(r)}, A_0^{(t)} \text{ and } A_0^{(r)})\) appear uncoupled as they correspond to peaks centered at different wavenumbers and in different regions of the frequency/wavenumber spectrum. As verification of the observed Lamb wave phenomena, analytical dispersion curves developed from the algorithm provided in \([91]\), are superimposed to the contours representing the amplitude of the frequency-wavenumber spectrum in Figure 11(b). Figure 11(b) shows the four quadrants, denoted with \(Q_1-Q_4\) corresponding to positive and negative values for frequencies and wavenumbers. Both reflected waves, \(S_0^{(r)}\) and \(A_0^{(r)}\), appear in the second \((Q_2)\) and the fourth \((Q_4)\) quadrant while the first \((Q_1)\) and the third \((Q_3)\) quadrants contain the incident wave components. A rectangular window, \(h_2\), is applied to quadrants \(Q_2 \text{ and } Q_4\), which eliminates the incident components in quadrants \(Q_1\) and \(Q_3\). The result of this first filtering step is presented in Figure 12(a).
Figure 11: (a) Space-time variation of recorded response showing multi-modal wave propagation, mode conversion and reflection at crack location. The excitation frequency is 200 kHz; (b) Frequency-wavenumber variation of recorded response showing uncoupled waves and the analytical dispersion curves.

After separating the incident and the reflected waves, the next step is the separation of $S_0^{(r)}$ and $A_0^{(r)}$ modes. Based on analytical dispersion curves, four lines above and below each mode are computed (Figure 12(a)). These lines represent boundaries for regions occupied by each mode and they are used as limits for separating the modes. Applying filtering windows $h_3$ within each boundary as generally described in Eq. (6), each mode is extracted. The results are shown in Figures 12(b)-(c). Figure 12(b) shows the filtered $A_0^{(r)}$ mode whereas Figure 12(c) shows the filtered $S_0^{(r)}$ mode. After applying an inverse 2D-FT, the filtered signals are presented in space-time domain in Figure 13(a)-(b). These signals can be used to estimate the source of the converted signal in order to obtain a precise indication of the damage location, and the mode conversion and reflection coefficients which can be related to the extent of damage.
Figure 12: (a) Frequency-wavenumber representation of the reflected modes; (b) Filtered $A_0^{(r)}$ mode; (c) Filtered $S_0^{(r)}$ mode.
Figure 13: Filtered modes in time-space domain: (a) $S_0^{(i)} + S_0^{(l)}$ mode; (b) $A_0^{(r)}$ mode.
2.4 Phase Gradient Technique for Damage Localization

Upon mode separation in the frequency-wavenumber domain and transforming the filtered data back to time domain as explained in Equation (7), windowed 1D FT is applied such that the Lamb modes are expressed in the space-frequency domain. The converted $A_0$ wave originates at the damage location, so its mathematical expression should read:

$$\tilde{w}_{A_0}(x, \omega_0) = \bar{w}_{A_0} e^{ik_{A_0}(x-2x_D)} \quad x \leq x_D$$

(9)

where $\tilde{w}_{A_0}(x)$ is the filtered displacement response evaluated at the frequency of interest, $\omega_0$. The phase of the complex signal in (9), varies linearly in the $x < x_D$ region:

$$\angle \tilde{w}_{A_0}(x, \omega_0) = k_{A_0}(x-2x_D) \quad x \leq x_D$$

(10)

The linear variation of the phase is used to estimate the precise location of the damage. This can be achieved by estimating the phase of the recorded response at a number of measuring points, and then identifying the location at which the phase exhibits a discontinuity, such as a change in slope where the phase may become constant. A more robust technique consists in taking the second derivative of the phase in terms of the spatial coordinate, which gives:

$$\frac{\partial^2}{\partial x^2} [\angle \tilde{w}_{A_0}(x, \omega_0)] = \delta(x-2x_D)$$

(11)

where $\delta$ is the Dirac Delta function centered at the damage location. This technique, which can generally be applied to any propagating wave, defines the location of the damage as origin of the reflected signal. This approach is attractive for the following reasons:

1. The presence of a linearly varying phase originating at the source of a reflection occurs independently from the amplitude of the signal, and therefore provides the means to highlight the presence of small reflected waves, resulting from very small damages.
2. The linear variation of the phase may be used to reduce the number of measurement points while still enabling the precise localization of damage through linear regression of the phase values.

2.5 Reflection, Transmission and Mode Conversion Coefficients

When the $S_0$ mode of a propagating Lamb wave is incident upon a crack, the mode is transmitted, reflected and converted. Based on the previously described filtering procedure, these waves can be separated and used to estimate the extent of damage. Specifically, reflection and conversion coefficients, defined as the ratios between the amplitudes of reflected $S_0$ and/or converted $A_0$ modes and the amplitude of the incident $S_0$ can be used to quantify the damage severity [15]. Upon filtering and evaluation of the individual modes and their FT at the frequency of interest $\omega_0$, and following the simplified notation introduced in Eq. (1), one may express the filtered incident and reflected/converted modes in Eq. (1) within the $x < x_D$ spatial range in Fig. 9:

$$w_{A_0}^{(r)}(x, \omega) \cong w_{S_0}^{(i)} A_0 e^{ikA_0(x-2kA_0x_D)}, x \leq x_D$$

(12)

and,

$$w_{S_0}^{(i)}(x, \omega_0) \cong w_{S_0}^{(i)} S_0 e^{-ikS_0 x}, x \leq x_D$$

(13)

Similarly, the reflected and transmitted $S_0$ mode may be expressed as:

$$w_{S_0}^{(i)}(x, \omega_0) \cong w_{S_0}^{(i)} R_{S_0} e^{ikS_0(x-2xD)}, x \leq x_D$$

(14)

$$w_{S_0}^{(i)}(x, \omega_0) \cong w_{S_0}^{(i)} T_{S_0} e^{ikS_0(x+2xD)}, x \geq x_D$$

(15)
Taking the ratio of the magnitudes of the various separated mode components can lead to an estimation of the reflection and conversion coefficients. Specifically, the conversion coefficient $A_0$ can be estimated as:

$$C_{A_0} \approx \frac{|w^{(r)}_{A_0}(x, \omega_0)|}{|w^{(i)}_{S_0}(x, \omega_0)|}$$

(16)

while the reflection and transmission coefficients may be found from:

$$R_{S_0} \approx \frac{|w^{(r)}_{S_0}(x, \omega_0)|}{|w^{(i)}_{S_0}(x, \omega_0)|}$$

(17)

$$T_{S_0} \approx \frac{|w^{(t)}_{S_0}(x, \omega_0)|}{|w^{(i)}_{S_0}(x, \omega_0)|}$$

(18)

2.5.1 Coefficients estimation using a single displacement component

It is interesting to note how both Eqs. ((16)), ((17)) are in theory independent upon the location $x$ at which they are estimated. However, in a practical scenario, where measurement noise affects the quality of the data, or in general when considering propagation in a two–dimensional structure, where geometrical spreading occurs, such assumption cannot be imposed. Previous work has investigated mode coefficients and has provided recommendations in regards to the distance where they can be evaluated to avoid influence of near field phenomena developing near the damage [67, 115, 69]. The recommended distance is typically defined as a multiple of the wavelength of the incident wave $\lambda$, with a typical recommended value of $4\lambda$. Such recommendation clearly drives the positioning of the transducer to be used for the measurement. Wavefield data however, provide the opportunity to select various points over the measured distance, so that the value of the mode coefficients can be estimated at various locations. This allows the estimation of its spatial-independence, and more importantly of a spatially averaged value, which is less affected by amplitude fluctuations due to noise, or by general trends associated with geometrical spreading. The spatial averaging procedure can be mathematically described as an integral operation over a portion of the measurement domain defined by two coordinates $x_1, x_2$. 

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Considering the relation between incident, the converted, reflected, and transmitted amplitudes corresponding to Eqs. (16), (17):

\[ |w_{A_0}^{(r)}(x, \omega_0)| \approx C_{A_0} |w_{S_0}^{(i)}(x, \omega_0)| \]  

(19)

and

\[ |w_{S_0}^{(r)}(x, \omega_0)| \approx R_{S_0} |w_{S_0}^{(i)}(x, \omega_0)| \]  

(20)

\[ |w_{S_0}^{(t)}(x, \omega_0)| \approx T_{S_0} |w_{S_0}^{(i)}(x, \omega_0)| \]  

(21)

and integrating both sides of the equations between integration limits \( x_i \) gives:

\[ C_{A_0} \approx \frac{\int_{x_1}^{x_2} |w_{A_0}^{(r)}(x, \omega_0)| \, dx}{\int_{x_1}^{x_2} |w_{S_0}^{(i)}(x, \omega_0)| \, dx} \]  

(22)

\[ R_{S_0} \approx \frac{\int_{x_1}^{x_2} |w_{S_0}^{(r)}(x, \omega_0)| \, dx}{\int_{x_1}^{x_2} |w_{S_0}^{(i)}(x, \omega_0)| \, dx} \]  

(23)

\[ T_{S_0} \approx \frac{\int_{x_1}^{x_3} |w_{S_0}^{(t)}(x, \omega_0)| \, dx}{\int_{x_1}^{x_2} |w_{S_0}^{(i)}(x, \omega_0)| \, dx} \]  

(24)

It is important to note that this spatial integration method departs from existing literature, where the mode amplitudes are taken at a fixed location [125, 70]. The choice of the location appears critical, and at times arbitrary. The general requirement is that all evanescent waves are attenuated and therefore that the wavefield is not distorted by local effects due to the presence of the crack. The definitive advantage of spatially integrating the mode amplitude is more readily apparent for experimental data, where amplitude variation is susceptible to external factors outside of the physical phenomena.

2.5.2 Coefficients estimation using two displacement components

Previous mode coefficient formulations reported in [15, 69] utilized only the out-of-plane component due to experimental limitations in extracting the pure in-plane component. However, for an excited \( S_0 \) mode propagating through the plate, the
in-plane $S_0$ amplitude is larger than the out-of-plane $S_0$ amplitude. Researchers such as Castaings et al. [15] theoretically displayed in-plane component mode coefficient formulations as a ratio of only in-plane reflected and incident modes, while showing measurement verification to out-of-plane component mode coefficients. Others such as Lowe et al. [69] developed mode coefficients solely as a function of out-of-plane components. At the present time, no published literature has accounted for combining both the in-plane and out-of-plane components, which may allow for greater information into accurate damage estimation, and potentially achieve a relative higher signal to noise ratio (SNR). One proposed formulation of a spatially-integrated multi-component mode coefficient may be defined as:

$$R_{S_0}^{uw}(\omega) = \frac{\int_{x_1}^{x_2} ((\tilde{u}_{S_0}^{(r)})^2 + (\tilde{w}_{S_0}^{(r)})^2)^{\frac{1}{2}} dx}{\int_{x_1}^{x_2} ((\tilde{u}_{S_0}^{(i)})^2 + (\tilde{w}_{S_0}^{(i)})^2)^{\frac{1}{2}} dx} \quad (25)$$

$$T_{S_0}^{uw}(\omega) = \frac{\int_{x_3}^{x_4} ((\tilde{u}_{S_0}^{(r)})^2 + (\tilde{w}_{S_0}^{(r)})^2)^{\frac{1}{2}} dx}{\int_{x_1}^{x_2} ((\tilde{u}_{S_0}^{(i)})^2 + (\tilde{w}_{S_0}^{(i)})^2)^{\frac{1}{2}} dx} \quad (26)$$

$$C_{A_0}^{uw}(\omega) = \frac{\int_{x_1}^{x_2} ((\tilde{u}_{A_0}^{(r)})^2 + (\tilde{w}_{A_0}^{(r)})^2)^{\frac{1}{2}} dx}{\int_{x_1}^{x_2} ((\tilde{u}_{A_0}^{(i)})^2 + (\tilde{w}_{A_0}^{(i)})^2)^{\frac{1}{2}} dx} \quad (27)$$

where $R_{S_0}^{uw}$ and $T_{S_0}^{uw}$ are the multi-component reflected and the transmitted $S_0$ mode coefficients, and $C_{A_0}^{uw}$ are the converted $A_0$ modes emanating from the notch. The superscripts $uw$ denote the formulation is based on both the in-plane and out-plane components. This formulation may be seen as taking the resolved component at each location and then integrating between prescribed spatial limits outside of near-field effects. Note that the mode coefficients are evaluated at the frequency of maximum response, where by experience, the SNR is typically the largest.

2.5.3 Compact Matrix Notation

In addition to the out-of-plane single component and the multi-component mode coefficient, an in-plane single component mode coefficient may likewise be constructed.
In terms of compact matrix notation, the mode coefficients may be summarized by the following:

\[
\Pi_{ij}(\omega) = \frac{\int_{\Omega_{ij}} |\Psi_{ij}(x, \omega)| \, dx}{\int_{\Omega_{ij}} |\Phi_i(x, \omega)| \, dx}
\]  

(28)

where,

\[
\Pi_{ij}(\omega) = \begin{bmatrix}
R^u_{S_0} & T^u_{S_0} & C^u_{A_0} \\
R^w_{S_0} & T^w_{S_0} & C^w_{A_0} \\
R^{uw}_{S_0} & T^{uw}_{S_0} & C^{uw}_{A_0}
\end{bmatrix}
\]  

(29)

\[
\Psi_{ij} = \begin{bmatrix}
\tilde{u}_{S_0}^{(r)} & \tilde{u}_{S_0}^{(t)} & \tilde{u}_{A_0}^{(r)} \\
\tilde{w}_{S_0}^{(r)} & \tilde{w}_{S_0}^{(t)} & \tilde{w}_{A_0}^{(r)} \\
\sqrt{(\tilde{u}_{S_0}^{(r)})^2 + (\tilde{w}_{S_0}^{(r)})^2} & \sqrt{(\tilde{u}_{S_0}^{(t)})^2 + (\tilde{w}_{S_0}^{(t)})^2} & \sqrt{(\tilde{u}_{A_0}^{(r)})^2 + (\tilde{w}_{A_0}^{(r)})^2}
\end{bmatrix}
\]  

(30)

\[
\Phi_i = \begin{bmatrix}
\tilde{u}_{S_0}^{(i)} & \tilde{w}_{S_0}^{(i)} & \sqrt{(\tilde{u}_{S_0}^{(i)})^2 + (\tilde{w}_{S_0}^{(i)})^2}
\end{bmatrix}
\]  

(31)

and,

\[
\Omega_{ij} = \begin{bmatrix}x_1 & x_3 & x_1 \\
x_2 & x_4 & x_2
\end{bmatrix}
\]  

(32)

where the superscripts \(u\), \(w\), and \(uw\) denote the formulation is based on the in-plane, out-plane, or multi-components.
CHAPTER III

NUMERICAL ANALYSIS OF PHASE GRADIENT TECHNIQUE AND MODE COEFFICIENTS

An evaluation of the phase gradient technique and mode coefficients is conducted using numerically generated data from the FE model of a waveguide. The purpose of this numerical analysis is to verify the techniques’ proof of concepts with data that is insusceptible to noise outside of the physics of the problem. In addition, the FE model allows for expeditious parametrization runs, and these are discussed within the ensuing chapter. Such parameters include variation of the damage depth, excitation frequency, damage profile, influence of numerically generated noise, and the use of single and multiple displacement components for mode coefficient evaluation.

3.1 Finite Element Model of 1D Damaged Waveguide

3.1.1 Geometry and mesh

The model of an isotropic, homogeneous waveguide with a rectangular notch defect is developed using the commercial software ABAQUS. The model corresponds to a cross section of a plate, which is assumed very long in the normal direction to the plane. Such conditions are enforced by imposing a state of plane strain throughout the structure. The plate dimensions are: thickness $h = 2 \times 10^{-3}$ m, width $b = 5 \times 10^{-1}$ m, and length $L = 1$ m. The material properties are: density $\rho = 2800$ kg/m$^3$, elastic modulus $E_x = 72$ GPa and Poisson’s ratio $\nu = 0.33$. The rectangular notch has a length, $\Delta l = 1 \times 10^{-3}$ m and depth, which is non-dimensionalized as $\epsilon = h_d / h \in [0, 1]$, where $h_d$ is the notch depth. This notch depth to plate thickness parameter, $\epsilon$, is a critical parameter in assessing the trends of the respective mode
coefficients. A schematic of the plate is shown in Figure 14.

![Figure 14: Magnified view of 2D plate: (a) Plate schematic with identified geometry parameters and applied uniform force](image)

The equations of motion are solved through an explicit central-difference time integration scheme. To ensure stability, the integration time step $\Delta t$ is chosen such that $\Delta t = 1/f_0/20$, where $f_0$ is the highest frequency of interest [122]. The element size ($\Delta l_e = \Delta t c_L$, where $c_L$ is the longitudinal wave speed) is selected on the basis of the minimum wavelength of elastic waves propagating in the plate [79]. Note that the element size is a critical parameter in ensuring that the wave propagation behavior is appropriately captured, and several iterations of the mesh size were performed to achieve a compromise between computational costs and adequately observing Lamb wave propagation through analytical dispersion comparisons (refer to Section 3.1.2). For example, an aluminum plate with the material properties already defined in this section requires a maximum element size of 2.5 mm for an excitation frequency of 100 kHz. Note that the driving parameter in mesh size selection was generally the preservation of the rectangular notch geometry, where widths of 1 mm required that the mesh size be on the order of 0.5 mm, which already satisfies the mesh requirement for capturing the minimal wavelength.

The specific element type called from the ABAQUS standard library is known as CPE4R, which is a 4-node bilinear, reduced integration plane strain element with an hourglass control module [24]. Due to the length of the plate and the necessary mesh
size required by the wavelength constraints, the reduced integration formulation is selected for computational efficiency. A representation of the meshed plate is shown in Figure 15. A longitudinal uniform pressure tip load is applied at one end of the plate whereas the opposite end is fixed. The plane strain assumption enforces that the strains do not vary in the direction perpendicular to the plane, which is an acceptable assumption for plane, straight wave loading applications. The symmetry of the applied in-plane load ensures that the $S_0$ mode is primarily excited [12], which helps simplify the reflected waves response from the notch since the given frequency range is designed to where the $S_0$ mode is non-dispersive.

**Figure 15:** Magnified view of 2D plate: FEM mesh using plane strain elements with applied load on the left end and fixed boundary conditions on the right end

### 3.1.2 Model validation through comparison with analytical dispersion curves

A broad-band square impulse load is imposed on the FE meshed structure, whose time trace is shown in Figure 16(a). The transverse displacements of the top surface of the plate, are selected as reference for future comparisons to experimental results where 1D SLDV measurements are out-of-plane. Upon applying a 2D FT to the spatial-time history, the data is transformed to the frequency-wavenumber domain (Figure 16(b)). In order to validate the FE model characteristics such as the mesh size and time step, Figure 16(b) shows the analytical Lamb waves dispersion curves as solid lines, which have been derived by following available algorithms in [91]. The analytical dispersion curves pass through the peak amplitudes of the 2D FT, and excellent comparison is achieved. Therefore it is concluded that the FE model properly captures the Lamb wave propagation. Note that the contoured $A_0$ mode in Figure 16(b) is a converted $A_0$ mode, and produced from the incident $S_0$ mode arriving at the presence
of an asymmetric damage. To ensure this claim, an additional numerical model was constructed that removed the notch geometry, and upon processing of the data, the converted $A_0$ mode was correspondingly eliminated from the frequency-wavenumber domain response.

![Graph](image)

**Figure 16:** (a) Broad-band square impulse tip load, (b) Analytical and FE dispersion relations for the transverse response.

### 3.2 Phase Gradient Technique

The following section explores the capability of the phase gradient technique to correctly identify a notch damage undergoing a parametrization study from numerically
generated data, such that its strengths and limitations as a damage localization technique are more widely understood.

3.2.1 Parametric study

For the given simulations, the excitation frequency and damage depth are parametrically varied, whereas the plate thickness, material properties, and damage width remain constant. This damage depth parametrization is consistent with a damage localization study by [121], and the frequency variation is consistent with the frequency sweep conducted in [75]. For the given parametric studies, $\epsilon$, the non-dimensional ratio of the damage depth ($h_d$) to plate thickness ($h$) ranges from 0 to 1 at increments of 0.1. The frequency range at which to excite the structure is selected by referring to the analytical dispersion curves, which are based on the plate’s material and geometric properties (Figure 17(a)). The desired frequency regime is selected prior to the frequency cutoff for the $A_1$ Lamb wave mode in order to eliminate additional complexities from higher multi-modal responses. For the given plate dimensions and material properties, the frequency range is restricted between 0 and 800 kHz. Upon selecting the applicable frequency range for the $S_0$ and $A_0$ modes, the resulting wavelength per mode for each corresponding frequency is found from Figure 17(b). A non-dimensional parameter $\Lambda$ defines the ratio of the wavelength corresponding to the excitation frequency and damage width:

$$\Lambda_{S_0|A_0} = \frac{\lambda_{S_0|A_0}}{\Delta l}$$  (33)

where $\lambda$ is the excitation wavelength, and $\Delta l$ is the damage width. For the considered parametric studies, $\Lambda_{S_0}$ ranges from 15 to 55 and $\Lambda_{A_0}$ is between 6 and 14. These non-dimensional $\Lambda$ ranges are consistent with previous low frequency Lamb wave reflection studies [69].
Figure 17: (a) Dispersion relations with identified frequency range from 0 to 800 kHz, and (b) Wavelength as a function of frequency.
3.2.2 Phase gradient results

After verifying that Lamb wave behavior is adequately captured by the FEM using a broad-band pulse, the parametrization runs utilize a 5-cycle in-plane tone burst modulated by a Hann window for narrow-band frequency response. This narrow-band response is desired since the signal post-processing from the filtering techniques described in Section 2.3.1 in conjunction with Equation (9) seek the largest SNR possible, which generally occurs at the excitation frequency. As an example, an in-plane 200 kHz tone burst is applied to the meshed plate in Figure 15, and the FE data is then recorded and post-processed using the procedure in Section 2.3.1 to separate the different modes. Figure 18(a) displays the spatial-temporal response prior to the filtering process, where the boundary reflections are also present for the given time window. The filtered reflected $S_0$ and converted $A_0$ modes are shown in Figures 18(b)-(c), where the mode separation is evident from the filtering techniques.

From Equation (8), the separated data is transformed back to the frequency-spatial domain by a 1D FT, and Figure 19(a) illustrates the transformed filtered $S_0$ mode with a frequency axis that includes both the negative and positive frequency range. Figure 19(a) demonstrates that two maximum peaks occur at the excitation frequency at 200 kHz, such that $f_{\text{max}} = \pm 200$ kHz. A cross-section of the spatial-frequency response at the maximum frequency is used to evaluate amplitude and phase variations along the spatial coordinate. The amplitude of the reflected $S_0$ and converted $A_0$ modes at $f_{\text{max}}$ are shown in Figure 20, as evaluated in accordance to Equation (10). The corresponding phase variation is shown in Figure 21(a). The phase needs to be “unwrapped” to obtain the linear variation predicted by theory. The unwrapped phase is shown in Figure 21(b). The second derivative of the corrected phase is numerically calculated (Figure 22), which shows peak locations corresponding to discontinuities or phase changes. The phase change in the response represent boundary reflections, such as the presence of the notch damage.
Figure 18: FEM results at 200 kHz excitation: (a) Unfiltered spatial-time history, (b) Filtered reflected $S_0$ mode, and (c) Filtered converted $A_0$ mode
Figure 19: Frequency-spatial data: (a) Reflected $S_0$ mode, and (b) Converted $A_0$ mode.
Figure 20: Amplitude of the reflected $S_0$ and converted $A_0$ modes at excitation frequency of 200, $\epsilon = 0.5$, and notch location at $x_D = 0.5$ m.

The results at $\Lambda = 33.87$ for three depths ($\epsilon = 0.1, 0.5, 0.9$) of the converted $A_0$ mode are presented in Figure 23, where the phase variations are identical for three notch depths at a given wavelength. Upon taking the second derivative, the damage location is successfully identified in Figure 23 at $x_D = 0.5$ by virtue of the peaks at the discontinuity. Similarly, Figures 24(a)-(b) demonstrate the phase and phase gradient for the reflected $S_0$ mode. It is interesting to note that the magnitude and slope of the phases are different for the $A_0$ and $S_0$ modes, which is an expected result since this variation indicates waves have different phase velocities (refer to Figure 3).

Conversely, for varying wavelengths at a constant notch depth, the results in Figure 25(a) depict the phase velocity differences of an $A_0$ mode for each wavelength with a similar discontinuity at $x_D = 0.5$, from which the converted mode originates, denoted by the peak in Figure 25(b). Similarly, Figures 26(a)-(b) show the phase and the second derivative for the reflected $S_0$ mode.

Thus, the damage localization method successfully locates the damage presence independently of both damage size and excitation wavelength, in addition to being
Figure 21: Calculated phase of the converted $A_0$ mode: (a) Initial results from Matlab angle command (b) Corrected phase results from Matlab unwrap command.
irrespective of the selected fundamental mode. This independence of such input parameters makes this technique attractive for blind damage localization tests.

### 3.3 Single Component Mode Estimation

Similar to the phase gradient technique, the mode coefficients are calculated after the modes are filtered following the procedure in Section 2.3.1. The separated modes in the temporal-spatial domain are transformed to the frequency-wavenumber domain via a 1D FT along the temporal dimension, where Figures 19(a)-(b) are representative of the transformed data. In order to ensure the maximum SNR of the response, the 1D data along the span of the plate that corresponds to the frequency of the peak amplitude is extracted.

As an example of the mode amplitude along the span of the plate, Figure 27(a) displays the amplitude of the filtered incident and transmitted $S_0$ mode for different damage depths at 200 kHz excitation frequency. The marginal decay in mode amplitude from the source in Figures 27(a-c) is attributed to the windowing process described by Equation (6). From several iterations of modifying the window size and type, the response was optimized to yield the least amount of decay for the given
Figure 23: Variation of (a) phase of $\tilde{w}_{A_0}^e$ and (b) phase gradient versus notch depth. Damage location is $x_D = 0.5$ mm
Figure 24: Variation of (a) phase of $\tilde{w}_{S_0(r)}$ and (b) phase gradient versus notch depth. Notch location is $x_D = 0.5$ mm
Figure 25: Variation of (a) phase of $\tilde{w}_{A_0(r)}$ and (b) phase gradient versus excitation frequency/wavelength.
Figure 26: Variation of (a) phase of $\tilde{w}_{S_0}$ and (b) phase gradient versus excitation frequency/wavelength.
frequency. This optimization is observed in Figure 27(a), where the incident $S_0$ mode has the same amplitude for all frequencies prior to the notch location ($x < x_D = 0.5$ m). The incident $S_0$ mode distinctly separates in amplitude after propagating past the notch ($x > x_D = 0.5$ m), which denotes that the mode amplitude of the transmitted wave is a function of the damage size.

The reflected $S_0$ mode in Figure 27(b) conveys the expected trend of an increase in damage depth produces an increase in the amplitude of the reflected $S_0$ mode. In the case of the converted $A_0$ mode, Figure 27(c), the peak amplitude occurs when $\epsilon \approx 0.7$ which is consistent with published results [39, 69]. Further, these $A_0$ mode results displaying a peak amplitude prior to the full depth of the notch indicates that for blind testing for damage based solely on the converted $A_0$, aliasing of the defect severity depth can occur, and further information is required.

### 3.3.1 Single point coefficients

The formulation of out-of-plane single point mode coefficients have been extensively studied and published results are found in Lowe et al [69], among others. The single point-based results from the current FEM generated data are now shown to provide background to the spatially-integrated mode coefficients. Using an excitation frequency of 200 kHz, the corresponding non-dimensional wavelengths per mode are $\Lambda_{S_0} = 33.87$ mm and $\Lambda_{A_0} = 10.86$ mm. Using the filtered mode amplitudes in Figures 27(a)-(c), out-of-plane single point mode coefficients based on Equations (19)-(21) are determined for 1 to 4 wavelengths from the notch location and excitation source. Table 1 provides a summary of the single point mode coefficients for the given excitation and notch depth, $\epsilon = 0.5$. Qualitatively, the mode coefficient values begin to stabilize as the local effects of the notch are reduced, particularly at $4\Lambda$. Figure 28 shows the respective out-of-plane single point mode coefficients as a function of $\epsilon$. 

56
Figure 27: Filtered modes along the length of the plate at 200 kHz excitation frequency, for a plate whose notch width of 1 mm is centered at $x = 0.5$, and where $\epsilon$ is a ratio of notch depth to beam thickness: (a) Incident and transmitted $S_0$ mode, (b) Reflected $S_0$ mode, and (c) Converted $A_0$ mode.
Table 1: Single point mode coefficients as a function of wavelength [mm], $\lambda$, for an excitation frequency of 200 kHz given a notch depth of $\epsilon = 0.5$

<table>
<thead>
<tr>
<th>Mode</th>
<th>$1 \lambda$</th>
<th>$2 \lambda$</th>
<th>$3 \lambda$</th>
<th>$4 \lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{So}$</td>
<td>0.416</td>
<td>0.354</td>
<td>0.358</td>
<td>0.356</td>
</tr>
<tr>
<td>$T_{So}$</td>
<td>0.800</td>
<td>0.737</td>
<td>0.744</td>
<td>0.745</td>
</tr>
<tr>
<td>$C_{Ao}$</td>
<td>3.926</td>
<td>3.757</td>
<td>3.723</td>
<td>3.681</td>
</tr>
</tbody>
</table>

Figure 28: FEM out-of-plane single point mode coefficients at 200 kHz excitation frequency as a function of $\epsilon$, the notch depth to plate thickness ratio.
3.3.2 Spatially averaged coefficients

The previous single point mode coefficient formulations are susceptible to noise since the formulation is the ratio of two individual values, particularly when near the source. For this reason, the spatially averaged coefficients are developed in Section 2.5.1 for a single component. Using the filtered data presented previously, the mode amplitudes are integrated along the spatial coordinates using spatial integration limits that are determined by 3 wavelengths from any source producing local effects, such as a piezoelectric actuator or defect. This rule of thumb is consistent with the single point results in Section 3.3.1, and with published results of the circular crested waves having a shape that varies rapidly with radial distance, where beyond 3 wavelengths, the behavior has stabilized, and asymptotically approached the behavior of the straight Lamb waves [43].

Figures 29, 31, and 32 provide the integrated mode coefficients from Equations (25)-(26) as a function of damage depth to plate thickness for different wavelengths to notch widths, \( \Lambda \). Qualitatively, the reflected \( S_0 \) mode coefficients, shown in Figure 29(a), increase almost linearly with increasing notch depth, which is consistent with previous single point studies by [69]. In addition, Figure 29(b) demonstrates a relative linear variation in the \( R_{S_0} \) mode coefficient over the given wavelength range per damage depth. The importance of these findings is that \( R_{S_0} \) is sensitive to the applied excitation wavelength holding the notch depth constant, and the practitioner in the field cannot accurately assume that the measured \( R_{S_0} \) mode coefficient is universally applicable to other wavelengths. Recommendations for utilizing these damage estimation curves similar to those in Figures 29-32 are provided in Section 6.4.

Figure 31(a) depicts the transmitted \( S_0 \) mode coefficient which decreases with increasing notch depth. This result is compatible with the physics of the problem, where less energy is transmitted as the cross-sectional area of the plate is reduced. In terms of the energy being reflected, transmitted, and converted, the following is a brief
Figure 29: Integrated reflected $S_0$ mode coefficients as function of: (a) $\epsilon$, damage depth to total plate height, and (b) $\Lambda$, the ratio of excitation wavelength to notch width.

Discussion as to how an energy balance is carried out for the given wave propagation problem. In general, the power per unit area, $P$ is:

$$P(x, z, t) = \mathbf{t}(x, z, t) \cdot \dot{\mathbf{u}}(x, z, t)$$  \hspace{1cm} (34)

where $\mathbf{t}$ is the traction vector, and $\dot{\mathbf{u}}$ is the velocity vector [1]. In terms of flux of energy in time-harmonic waves, the rate at which energy is communicated per unit area is equal to the power per unit area, or Equation (34). Averaged over time, the energy flux is expressed as:

$$W(x, z, \omega) = \frac{1}{T} \int_t^{t+T} P(x, z, t) \, dt = \frac{1}{T} \int_t^{t+T} \mathbf{t}(x, z, t) \cdot \dot{\mathbf{u}}(x, z, t) \, dt$$  \hspace{1cm} (35)

where $T = 2\pi/\omega$, where $\omega$ is the harmonic wave frequency, and $T$ is the period over which is the range of integration. For a Lamb wave of order $i$, where $i = 0, 1, 2...$, the resulting energy flux is:

$$W_i(x, z, \omega) = \frac{1}{T} \int_t^{t+T} \int_{-h}^{h} \left[ \text{Re}(\sigma_{xx})\text{Re}(\dot{u}_x) + \text{Re}(\sigma_{zz})\text{Re}(\dot{u}_z) \right] \, dz \, dt$$  \hspace{1cm} (36)

where $\text{Re}(.)$ represents the real component, and $\sigma$ and $\dot{u}$ are the stress and velocity components in the designated $x$ and $z$ directions, respectively [86]. A representative
schematic of the FEM configuration is provided in Figure 30. The energy balance is verified when the energy flux from incident mode is equivalent to the summation of the energy fluxes from the reflected and transmitted modes, or simply:

\[
W_{S_0}(\omega) + W_{S_0}(\omega) + W_{A_0}(\omega) + W_{A_0}(\omega) = W_{S_0}(\omega) \quad (37)
\]

or, in terms of energy flux coefficients [120]:

\[
R_{S_0} + R_{A_0} + T_{S_0} + T_{A_0} = 1 \quad (38)
\]

where \( R_{S_0|A_0} \) and \( T_{S_0|A_0} \) are taken as the ratio of energy fluxes from the reflection and transmission modes for the \( S_0 \) and \( A_0 \) modes, respectively, with the energy flux from the incident \( S_0 \) mode in the denominator. Experimentally, this energy balance procedure is a difficult task since the monitoring system can only scan the surface of the plate, and the formulation requires through-the-thickness traction components. The FEM formulation itself is based on an energy method known as the principle of virtual work, and thus the energy balance is inherently preserved. However, it is standard practice for methodologies who want to compare with FEM results (such as modal decomposition methods [15]) by ensuring an energy balance process similar to the one outlined.

\[
\text{Figure 30: FEM configuration with identified measurement regions.}
\]
Figure 31(b) also shows the relative $T_{S_0}$ amplitude variation with excitation wavelength, which is even more pronounced than the $R_{S_0}$ mode coefficient. It should be noted that the amplitude for both the $R_{S_0}$ and $T_{S_0}$ mode coefficients range between 0-1 as a result of the denominator in Equations (25)-(26), where the divisor is the incident out-of-plane $S_0$ mode, which is clearly larger in amplitude than the reflected and transmitted out-of-plane $S_0$ modes.

In contrast, the converted $C_{A_0}$ mode coefficients shown in Fig. 32(a), whose peak amplitude at $\epsilon \approx 0.7$, result in mode coefficient amplitudes greater than unity. This may be explained by Equation (25), where the converted $A_0$ mode has a larger out-of-plane amplitude than the out-of-plane amplitude of the incident $S_0$ mode. Also, the slope of the parabolic profile in Fig. 32(a) increases with $\Lambda$, and Fig. 32(b) further accentuates this wavelength dependency, particularly at $\epsilon = 0.7$ and $\epsilon = 0.9$.

In order to investigate the damage depth for which the $C_{A_0}$ mode coefficient peaks, the resolution of $\epsilon$ is refined, varying from 0.55 to 0.75 at increments of 0.01 for a selected wavelength, $\Lambda = 33.2$. The results are shown in Figure 33 for both the integrated formulation and at a single point 4 wavelengths away from the damage.
Figure 32: Integrated converted $A_0$ mode coefficients as function of: (a) $\epsilon$, damage depth to total plate height, and (b) $\Lambda$, the ratio of excitation wavelength to notch width.
It is evident that the peak amplitude occurs at $\epsilon = 2/3$, for both the integrated and single point mode coefficients at this excitation wavelength. Another observation from Fig. 33 is that the integrated and single point coefficients have equivalent magnitude, which is expected in the absence of noise (Equation (25)).

![Figure 33: Magnified view of FEM $C_{A_0}$ mode coefficient as a function of $\epsilon$ for both integrated and single point formulation at excitation wavelength of $\Lambda = 33.2$.](image)

### 3.4 Multi-component Mode Coefficient Estimation

Similar to the single component mode coefficients results, this section provides the amplitudes of the in-plane and out-of-plane displacements at a given excitation frequency. These results are provided in order to demonstrate the relative amplitude differences between the multi-components, and the consequent effect on the mode coefficient calculation.

#### 3.4.1 Non-integrated mode amplitude comparison

The mode amplitudes in Figures 34(a)-(b) correspond to the in-plane and out-of-plane amplitudes along the spatial coordinate at the maximum frequency. The in-plane response is approximately an order of magnitude greater than the out-of-plane
response per damage depth along the span of the plate. This difference is expected since an in-plane load is applied at the plate tip. Likewise, Figures 35 show the relative magnitude difference for the reflected $S_0$ mode. For the $S_0$ mode, the in-plane component is consistently one order of greater than the out-of-plane component, with the amplitude initiating at the crack location, $x_D = 0.5$ mm. In contrast, the out-of-plane converted $A_0$ mode in Figure 36(b) is 5/3 larger than the in-plane component (Figure 36(a), which in summary, means that the $S_0$ mode has a nearly 10 to 1 ratio from the in-plane to out-of-plane amplitude, and the $A_0$ mode is less than 2 to 1.

![Graph](image)

**Figure 34:** $S_0$ mode amplitude along the length of the plate at 200 kHz for a plate of 1 mm width at $x_D = 0.5$, and: (a) In-plane displacement, and (b) Out-of-plane displacement

### 3.4.2 Spatially-integrated mode coefficient comparison

In terms of the spatially-integrated mode coefficients, the $R_{S_0}$, $T_{S_0}$, and $C_{A_0}$ multicomponent modes are provided in Figures 37(a)-(c). One interesting outcome from the Figures 37(a)-(b), is that the mode coefficients of the in-plane and out-of-plane are almost identical despite the appreciable amplitude difference shown in Section 3.4.1. This is due to the coefficients being formulated as a ratio, where an incremental order of magnitude difference per each mode in the numerator (or amplitude difference) is
Figure 35: Reflected $S_0$ mode amplitude along the length of the plate at 200 kHz for a plate of 1 mm width at $x_D = 0.5$, and: (a) In-plane displacement, and (b) Out-of-plane displacement.

Figure 36: Converted $A_0$ mode amplitude along the length of the plate at 200 kHz for a plate of 1 mm width at $x_D = 0.5$, and: (a) In-plane displacement, and (b) Out-of-plane displacement.
effectively countered by the corresponding amplitude of the incident $S_0$ mode. Note that in Figure 37, the multi-component converted $A_0$ mode is more closely aligned to the in-plane converted $A_0$ mode coefficient as a result of the dominant $S_0$ excitation.

Figure 37: Comparison of integrated mode coefficients as function of damage depth to total plate height for an excitation frequency of 200 kHz and whose notch width of 1 mm is centered at $x = 0.5$: (a) Reflected $S_0$ mode coefficient, (b) Transmitted $S_0$ mode coefficient, and (c) Converted $A_0$ mode coefficient. The superscripts $u$, $w$, and $uw$ indicate formulations based on in-plane, out-of-plane, and multi-component displacements, respectively.
3.5 Analysis of Influence to Noise

3.5.1 Phase gradient results

One question that arises from a damage localization method, which implements the second derivative of the converted $A_0$ phase to accurately identify the notch location, is its sensitivity to noise. As a case study, non-correlated noise is simulated by adding a random sequence to the data. The noise is calculated with a variance which is a percentage of the root mean square (RMS) of the simulated response at a given nodal location. The considered percentage of the RMS ranges from 0 − 50%. The randomized set of data has a mean of 0 with a standard deviation of 1. The resulting noise vector is summed to the original displacement data at each node for every time sample. The final result is a set of FEM displacements with simulated randomized noise. Figures 38(a)-(c) show time histories with added noise at varying percentages of the RMS level. Note that the filtering procedure was followed as in previous processing of the data in order to evaluate the effect of noise on the damage localization and quantification methods without introducing additional windows or filtering techniques.

Figures 39(a)-(b) shows the phase and second derivative for a notch size of $\epsilon = 0.5$ and excitation wavelength of $\Lambda = 55.0$ at varying noise percentage levels of RMS. From Figures 39(a)-(b), it is evident that the damage location does not vary with the increasing noise levels. Thus, the damage localization technique is found to be robust to the noise level, excitation wavelength, and the damage size.

3.5.2 Single component mode coefficient results

A comparison of single point and the spatially integrated formulations for $R_{S_0}$ and $C_{A_0}$ mode coefficients, shown in Figures 29-32, is provided in Figures 40-41. In order to highlight the benefits and limitations of the spatially-integrated formulation,
Figure 38: Time traces at $x = 100$ mm for varying percentages of RMS levels: (a) 0 % Noise, (b) 30 % Noise, and (c) 50% Noise
Figure 39: Variation of (a) phase of $\tilde{w}_{A(r)}$ and (b) phase gradient at varying noise percentage RMS levels for constant $\Lambda = 55.0$ and $\epsilon = 0.5$. 
numerical noise is introduced to the response, as described in Section 3.5.1. Qualitatively for each excitation wavelength as a function of damage depth, Figure 40(a) demonstrates the discrepancy produced by the single point formulation for the increasing percentage RMS levels throughout the damage depth range, whereas the integrated mode coefficients in Figure 40(b) display a slight variation at the latter notch levels for $\epsilon$ greater than 0.7. Quantitatively, Figure 42(a) provides the infinite norm of the error percentage for the $R_{S_0}$ mode coefficient, where the reference value is the response with zero noise. These results demonstrate the superiority of the integrated formulation versus the single point in mitigating the influence of the rising levels of noise for each damage depth. In particular, the average percentage of the maximum error, known as the average infinite norm of error percentage, for the single point $R_{S_0}$ is 20.1 compared to 5.8 for the integrated $R_{S_0}$. Conversely, the single point and integrated $C_{A_0}$ coefficients are surprisingly similar in mitigating the numerical noise levels, as shown side-by-side in Fig. 41(a)-(b), and the error is quantified in Fig. 42(b). The average infinite norm of error percentage for the single point $C_{A_0}$ is 6.4 compared to 7.1 for the integrated $C_{A_0}$. The primary difference between the noise level averages for the $R_{S_0}$ and $C_{A_0}$ coefficients exist in the in-plane versus out-of-plane nature of the modes, where the $S_0$ mode is pre-dominantly in-plane and the $A_0$ mode is primarily out-of-plane. By introducing the numerical noise as a percentage of the RMS of the out-of-plane amplitude, the $A_0$ mode has a significant increase in noise and thus the corresponding data is more susceptible to variation, regardless of the mode coefficient formulation.

### 3.5.3 Multi-component mode coefficient comparison

It is of interest to investigate how the multi-component formulation handles the increasing noise as percentage levels of RMS in order to assess noise mitigation capability. Qualitatively, Figures 43(a)-(b) illustrate that the noise precipitated by the 50%
Figure 40: $R_{S_0}$ mode coefficient comparison with numerical noise at varying RMS levels for: (a) Single point, and (b)Spatially integrated formulations.
Figure 41: $C_{A_0}$ mode coefficient comparison with numerical noise at varying RMS levels for: (a) Single point, and (b) Spatially integrated formulations.
Figure 42: Error comparison of integrated and single point formulations as a function of $\epsilon$: (a) $R_{S_0}$ mode coefficient (b) $C_{A_0}$ mode coefficient.
RMS levels in the out-of-plane single component, which are not observed in the multi-component $R_{S_0}$ formulation. In addition, the maximum variation in the $C_{A_0}$ mode coefficient in Figure 43(a) at $\epsilon = 0.7$ compared to the $C_{A_0}$ mode coefficient in Figure 43(b) is reduced by an order of magnitude for the multi-component formulation. Quantitatively, Figure 45(a) displays the infinite norm of error percentage for the $C_{A_0}$ mode coefficient. The multi-component formulation exhibited an average error of 4.0 compared to 4.6 for the single component $C_{A_0}$. The primary difference between the noise level exists in the $S_0$ mode, as seen in Figure 45(b), where the multi-component yields an average error of 3.7 compared to 6.5 for the out-of-plane single component formulation. This is due to the multi-component formulation accounting for the pre-dominant in-plane component that is obviously not accounted for in the out-of-plane single component. Thus, the spatially integrated multi-component formulation has demonstrated an ability to mitigate the noise amplitude that spatially-integrated single components cannot achieve.

### 3.6 Influence of Damage Profile

A common trend among these formulations is the usage of a non-propagating, surface-breaking asymmetric rectangular notch as the selected damage geometry. In terms of ballistic-impacts and the residual damage shape, it has been shown by Hosur et al. [111] and Woodward [117] that ballistic projectiles may represent three simplistic and generalized profiles in the target as a function of the penetration: rectangular, triangular, and semi-circular punches. Therefore, it is of particular interest and the primary objective to investigate whether mode coefficients yield distinctive results for varying damage profiles, and their sensitivities to the damage parameters, such as damage depth and width, symmetry effects, and equivalent damage area in the considered ranges of frequency.

In order to examine the effects of varying geometries, the simplified plane strain
Figure 43: Spatially integrated $R_{S_0}$ mode coefficient comparison with numerical noise at 200 kHz excitation frequency for varying RMS levels: (a) Single out-of-plane component, and (b) Multi-component formulation
Figure 44: Spatially integrated $C_{A_0}$ mode coefficient comparison with numerical noise at 200 kHz excitation frequency for varying RMS levels: (a) Single out-of-plane component, and (b) Multi-component formulation
**Figure 45:** Error comparison of multi-component and out-of-plane single component formulations as a function of $\epsilon$: (a) $R_{S_0}$ mode coefficient (b) $C_{A_0}$ mode coefficient.
finite element model is modified to include different notch types. Figure 46 illustrates the representative plane strain meshing of five selected ballistic geometry variations that vary the notch geometry, and symmetry about the mid-plane.

Figure 46: Representative 2D plane strain FEM mesh of simplified damage geometry configurations. Asymmetric surface-breaking: (a) rectangular notch, (b) triangular notch, (c) semi-circular notch. Interior to plate: (d) symmetric rectangular notch, (e) asymmetric rectangular notch about mid-plane.

The in-plane displacements around each damage profile are provided in Figures 47-51.

The surface wavefield around the damage for profiles of rectangular, triangular and semi-circular punches are provided in Figures 52-54 for varying times when the wave arrives at the notch damage for constant notch depth, width, and frequency. These results demonstrate a marginal variation of the response as a result of the profile influence.

Several aspects of the damage profile characterization are presented: (1) the influence of damage mid-plane symmetry of the response (based on mode conversion ratio), (2) the equivalent damage width to excitation frequency, and (3) the equivalent notch area penetration for varying damage geometry. A numerical test matrix was constructed, where excitation frequencies ranging from 0.1 MHz and 0.4 MHz are applied to 5 structures at 7 notch depths. The corresponding notch area is calculated for comparison purposes in determining if the mode coefficients are a function
Figure 47: FE snapshots of surface breaking rectangular notch at 400 kHz: (a) Time = 100 µs, (b) Time = 102 µs, and (c) Time = 104 µs
Figure 48: FE snapshots of interior asymmetric rectangular notch at 400 kHz: (a) Time = 100 µs, (b) Time = 102 µs, and (c) Time = 104 µs
<table>
<thead>
<tr>
<th>Step</th>
<th>Increment</th>
<th>Time</th>
</tr>
</thead>
<tbody>
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<td>2699</td>
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</tr>
<tr>
<td>2754</td>
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</tr>
<tr>
<td>2809</td>
<td></td>
<td>1.0459E-04</td>
</tr>
</tbody>
</table>

Figure 49: FE snapshots of interior symmetric rectangular notch at 400 kHz: (a) Time = 100 µs, (b) Time = 102 µs, and (c) Time = 104 µs
Figure 50: FE snapshots of triangular notch at 400 kHz: (a) Time = 100 µs, (b) Time = 102 µs, and (c) Time = 104 µs
Figure 51: FE snapshots of semi-circular notch at 400 kHz: (a) Time = 100 µs, (b) Time = 102 µs, and (c) Time = 104 µs
Figure 52: Wavefield response for varying notch profiles at time = 100 $\mu$s, where $x_D = 0.5$, $\epsilon = 0.5$, at 400 kHz: (a) In-plane displacement, and (b) Out-of-plane displacement

Figure 53: Wavefield response for varying notch profiles at time = 105 $\mu$s, where $x_D = 0.5$, $\epsilon = 0.5$, at 400 kHz: (a) In-plane displacement, and (b) Out-of-plane displacement
Figure 54: Wavefield response for varying notch profiles at time = 110 µs, where $x_D = 0.5$, $\epsilon = 0.5$, at 400 kHz: (a) In-plane displacement, and (b) Out-of-plane displacement

of damage area. A selection of the results is shown in Figures 55-58.

From Figures 55(a)-(c), it is clear that several trends appear. The internal symmetry and asymmetry profiles are effectively identical at lower frequencies (0.1 and 0.2 MHz). As $\epsilon$ exceeds 0.5, where $h_d$ is the damage depth, the transmission coefficients of the surface-breaking asymmetric rectangular damage profile in Figures 56(a)-(c) significantly differ from the internal damage profiles, such that the boundary condition effects are clearly seen.

From Figures 57(a)-(c), the mode coefficient of the semi-circular damage profile is lower the other damage profiles for frequencies greater than 0.2 MHz. This discrepancy is attributed to the notch depth $\frac{1}{3}$ to $\frac{1}{2}$ of the rectangular and triangular notch runs. Holding the notch width constant, and varying the frequency and equivalent damage depth is shown in Figure 58(a)-(c), where it is qualitatively evident that the variation of the $R_{S_0}$ mode coefficient increases with increasing frequency and increasing damage depth.
Figure 55: Out-of-plane reflected $S_0$ mode coefficients at varying frequencies: (a) 0.1 MHz, (b) 0.2 MHz, and (c) 0.4 MHz.
Figure 56: Out-of-plane reflected $S_0$ mode coefficients at varying frequencies: (a) 0.1 MHz, (b) 0.2 MHz, and (c) 0.4 MHz.
Figure 57: Out-of-plane reflection mode coefficients as a function of frequency for the five selected notch profiles having an equivalent removed damage area of: (a) 0.4 mm$^2$, (b) 0.8 mm$^2$, and (c) 1.2 mm$^2$. 
Figure 58: Out-of-plane reflected $S_0$ mode coefficients as a function of frequency for the five selected notch profiles having an equivalent depth of: (a) $\epsilon = 0.2$, (b) $\epsilon = 0.4$, and (c) $\epsilon = 0.6$. 
CHAPTER IV

ESTIMATION OF WAVE POLARIZATION USING A 1D LASER VIBROMETER

4.1 Overview

The aim of this chapter is to illustrate the experimental estimation of multi-component displacement associated with the propagation of Lamb waves. Such components will be used for the evaluation of mode coefficients and for damage quantification. The approach is validated by comparing experimental results with analytically derived 3D-elasticity Lamb wave solutions. Specifically, an analytical formulation of Lamb waves generated by a circular piezoelectric disc is used, along with descriptions of bi-modal and single mode polarization characteristics. This analytical formulation is compared to experimental Lamb wave polarization results, which are similarly estimated through the proposed technique. The technique is demonstrated for a single point location and then shown for multiple points along a radial distance from the disc.

Polarization defines the phase and amplitude relationships between the various components of wave motion, and is significant in all technological applications based on wave propagation, such as optics, seismology, telecommunications and radar science. As opposed to other fields, wave polarization in mechanics has received relatively little attention, due to the general difficulty in evaluating it experimentally. It is however a well-recognized fact that the ability to measure and characterize the polarization of ultrasonic waves could lead to the development of novel diagnostic tools, which could rely on the sensitivity of polarization to surface roughness, cracks, temperature or residual stresses, among others. The theoretical study of the polarization
of Rayleigh surface waves presented in [78], suggests its estimation as an alternative method for surface stress estimation. A polarization parameter can be defined as the ratio between the maximum in-plane and out-of-plane displacement components, and can be directly related to the state of surface stress, and thus be used as an absolute measurement of pre-stress. Studies on the acousto-elastic effect on Rayleigh waves in a homogeneous material include the work of Hirao et al. [51] and Duquennoy et al. [31, 32]. The analysis of polarization of ultrasonic guided waves is even more limited, and has mostly focused on theoretical aspects related to the description of the wavefield, and to tuning criteria for the excitation of specific wave modes through surface mounted transducers [43, 88, 69, 123].

4.2 Analytical Study of Lamb Wave Polarization

This section summarizes the basic steps of the derivations and provides the solution for both the in-plane and out-of-plane displacement components of Lamb waves generated by circular piezoelectric discs. The derivations are described in detail in [87].

4.2.1 Lamb wave generated by a circular piezoelectric disc

The motion of an infinite isotropic plate of thickness $2h$ is described in terms of the reference system depicted in Figure 59, with origin located at the plate mid-thickness. The governing equation of the plate in the absence of body forces has the well-known form:

$$\mu \nabla^2 \mathbf{u} + (\lambda + \mu) \nabla \nabla \cdot \mathbf{u} = \rho \ddot{\mathbf{u}}$$

(39)

where $\lambda$ and $\mu$ are the Lamé constants, and where $\mathbf{u} = \{u_1, u_2, u_3\}^T$. A bonded piezoelectric disk of radius $a$ is modeled as an externally applied distribution of surface
traction, which corresponds to the following surface stress components:

\[
\begin{align*}
\sigma_{31}(r, \theta, t) &= \tau(t) \delta(r - a) \cos \theta \\
\sigma_{32}(r, \theta, t) &= \tau(t) \delta(r - a) \sin \theta \\
\sigma_{33}(r, \theta, t) &= 0
\end{align*}
\] (40)

In Equation (40), \( r \) and \( \theta \) are polar coordinates with origin at the center of the piezoelectric disk (Figure 59), and \( \tau(t) \) defines the time history of the surface shear stress. It is convenient to consider a harmonic input \( \tau(t) = \tau_0(\omega)e^{i\omega t} \), and formulate the solution in the wavenumber domain. Following the approach presented in [87], polar coordinates \( k, \gamma \) in the wavenumber domain \( (k_1 = k \cos \gamma, k_2 = k \sin \gamma) \) are conveniently used to express the radial and out-of-plane displacement components, with the tangential displacement being equal to zero due to the axi-symmetric configuration of the actuator. Application of the residue theorem from complex analysis, and imposing wave propagation conditions, leads to the expressions for the in-plane and
out-of-plane displacement components generated by the piezoelectric disk. Specifically, the in-plane radial displacement is given by:

\[
 u_r(r, \omega) = -\pi i \frac{\tau_0 a}{\mu} \left[ \sum_{k_S} J_1(ak_S) \frac{N_S(k_S)}{D_S(k_S)} H_1^{(2)}(k_S r) + \sum_{k_A} J_1(ak_A) \frac{N_A(k_A)}{D_A(k_A)} H_1^{(2)}(k_A r) \right]
\]

where:

\[
 N_S(k) = k \beta (k^2 + \beta^2) \cos \alpha h \cos \beta h 
\]

\[
 D_S(k) = (k^2 - \beta^2)^2 \cos \alpha h \sin \beta h + 4k^2 \alpha \beta \sin \alpha h \cos \beta h 
\]

and

\[
 N_A(k) = k \beta (k^2 + \beta^2) \sin \alpha h \sin \beta h 
\]

\[
 D_A(k) = (k^2 - \beta^2)^2 \sin \alpha h \cos \beta h + 4k^2 \alpha \beta \cos \alpha h \sin \beta h 
\]

In Equations (42)-(45) \( \alpha^2 = \omega^2/c_l^2 - k^2 \), \( \beta^2 = \omega^2/c_t^2 - k^2 \), where \( c_l \) and \( c_t \) are the phase velocities of longitudinal and shear waves, Also in Equation (41), \( H_1^{(2)}(\cdot) \) is the complex Hankel function of the second type and order 1, and \( J_1(\cdot) \) is the Bessel Function of the first kind and order 1. The out-of-plane component is obtained through a similar procedure and it is expressed as:

\[
 u_3(r, \omega) = -\pi i \frac{\tau_0 a}{\mu} \left[ \sum_{k_S} J_1(ak_S) k_S^2 \frac{N_S^*(k_S)}{D_S(k_S)} H_0^{(2)}(k_S r) + \sum_{k_A} J_1(ak_A) k_A^2 \frac{N_A^*(k_A)}{D_A(k_A)} H_0^{(2)}(k_A r) \right]
\]

where:

\[
 N_S^*(k) = 2\alpha \beta \sin \alpha h \cos \beta h + (k^2 - \beta^2) \sin \alpha h \cos \beta h 
\]

\[
 N_A^*(k) = 2\alpha \beta \cos \alpha h \sin \beta h + (k^2 - \beta^2) \cos \alpha h \sin \beta h 
\]

and \( H_0^{(2)}(\cdot) \) is the complex Hankel function of the second type and order 0.
For the given expressions, the far field occurs rapidly due to the nature of the Hankel function approaching its asymptotic expression after four or five spatial wavelengths. Hence, the complex Hankel functions of the second type can be approximated at the far field as:

\[
H_1^{(2)}(kr) \approx \sqrt{\frac{2}{\pi kr}} e^{-i(kr - \frac{\pi}{4})} \\
H_0^{(2)}(kr) \approx \sqrt{\frac{2}{\pi kr}} e^{-i(kr - \frac{\pi}{4} - \frac{\pi}{2})}
\]  

(47)  

(48)

4.2.2 Bi-modal wave polarization

In order to compare the experimental and the analytical data, the particle trajectory at a given location is calculated based on the analytical model. As for the two experiments presented below, two cases are considered: (a) the displacement is a superposition of the first symmetric \((S_0)\) and the first asymmetric \((A_0)\) Lamb wave modes, at a frequency lower than the \(A_1\) mode cut-off; (b) the displacement contains only a single mode \((S_0\) or \(A_0\)). In both cases the far field approximations (47)-(48) are used and the Equations (41), (46) are written in the following compact form:

\[
u_1(r, \omega) = A_1^S e^{i(\varphi_S - \pi/2)} + A_1^A e^{i(\varphi_A - \pi/2)}
\]

(49)

\[
u_3(r, \omega) = A_3^S e^{i\varphi_S} + A_3^A e^{i\varphi_A}
\]

(50)

where:

\[
\varphi_S = -k_S r + \frac{\pi}{4}
\]

(51)

\[
\varphi_A = -k_A r + \frac{\pi}{4}
\]

(52)
and

\begin{align}
A_1^S &= -\pi i \sqrt{\frac{2}{\pi k Sr}} \frac{\tau_0 a}{\mu} J_1(ak_S) \frac{N_S(k_S)}{D_S(k_S)} \\
A_1^A &= -\pi i \sqrt{\frac{2}{\pi k Ar}} \frac{\tau_0 a}{\mu} J_1(ak_A) \frac{N_A(k_A)}{D_A(k_A)} \\
A_3^S &= -\pi i \sqrt{\frac{2}{\pi k Sr}} \frac{\tau_0 a}{\mu} J_1(ak_S) k_S^2 \frac{N_S(k_S)}{D_S(k_S)} \\
A_3^A &= -\pi i \sqrt{\frac{2}{\pi k Ar}} \frac{\tau_0 a}{\mu} J_1(ak_A) k_A^2 \frac{N_A(k_A)}{D_A(k_A)}
\end{align}

The phases of \( S_0 \) or \( A_0 \), denoted by \( \varphi_S \) and \( \varphi_A \), are functions of the distance from the source, \( r \), and of the wavenumber \( k_S \) and \( k_A \). For comparison with the experimental results, both components are normalized with respect to the amplitude of the out-of-plane displacement, according to the following notation:

\[ \bar{u}_j = \frac{\text{Real}(u_j)}{|u_3|} \quad (j = 1, 3) \]

Physically, it can be shown that the representation of the polarized components of the guided waves, \( \bar{u}_1 \) and \( \bar{u}_3 \), yield an elliptical, rotated profile of the particle trajectory [78].

From Equations (49)-(50) and (53)-(56), the normalized displacements can be written as:

\begin{align}
\bar{u}_1 &= \frac{A_1^S \sin(\varphi_S) + A_1^A \sin(\varphi_A)}{\sqrt{(A_3^S)^2 + (A_3^A)^2 + 2A_3^S A_3^A \cos \varphi}} \\
\bar{u}_3 &= \frac{A_3^S \cos(\varphi_S) + A_3^A \cos(\varphi_A)}{\sqrt{(A_3^S)^2 + (A_3^A)^2 + 2A_3^S A_3^A \cos \varphi}}
\end{align}

where \( \varphi = \varphi_A - \varphi_S \). For brevity, we denote the denominator in Equations (58)-(59) as:

\[ \Gamma = \sqrt{(A_3^S)^2 + (A_3^A)^2 + 2A_3^S A_3^A \cos \varphi} \]

Using the trigonometric identities,

\begin{align}
\sin(\varphi_S + \varphi) &= \sin \varphi_S \cos \varphi + \cos \varphi_S \sin \varphi \\
\cos \varphi_A &= \cos \varphi \cos \varphi_S - \sin \varphi \sin \varphi_S
\end{align}
then Eq. (58)-(59) may be rewritten as:

\[
\bar{u}_1 = \frac{1}{\Gamma}(A_1^S \sin \varphi_S + A_1^A \sin \varphi_S \cos \varphi + A_1^A \cos \varphi_S \sin \varphi) \quad (63)
\]
\[
\bar{u}_3 = \frac{1}{\Gamma}(A_3^S \cos \varphi_S + A_3^A \cos \varphi_S \cos \varphi - A_3^A \sin \varphi_S \sin \varphi) \quad (64)
\]

and in compact matrix notation:

\[
\begin{pmatrix}
\bar{u}_1 \\
\bar{u}_2 \\
\end{pmatrix} =
\begin{bmatrix}
B_{11} & B_{12} \\
B_{21} & B_{22} \\
\end{bmatrix}
\begin{pmatrix}
\sin \varphi_S \\
\cos \varphi_S \\
\end{pmatrix} \quad (65)
\]

where:

\[
B_{11} = \frac{1}{\Gamma}(A_1^S + A_1^A \cos \varphi) \quad (66)
\]
\[
B_{12} = \frac{1}{\Gamma}A_1^A \sin \varphi \quad (67)
\]
\[
B_{21} = -\frac{1}{\Gamma}A_3^A \sin \varphi \quad (68)
\]
\[
B_{22} = \frac{1}{\Gamma}(A_3^S + A_3^A \cos \varphi) \quad (69)
\]

Solutions for \(\sin \varphi_S\) and \(\cos \varphi_S\) gives:

\[
\sin \varphi_S = \frac{1}{B}(\bar{u}_1 B_{22} - \bar{u}_3 B_{12}) \quad (70)
\]
\[
\cos \varphi_S = \frac{1}{B}(-\bar{u}_1 B_{21} + \bar{u}_3 B_{11}) \quad (71)
\]

where \(B = B_{11} B_{22} - B_{12} B_{21}\). Squaring and summing Eqs. (70)-(71) yields:

\[
\frac{1}{B^2}[(\bar{u}_1 B_{22} - \bar{u}_3 B_{12})^2 + (-\bar{u}_1 B_{21} + \bar{u}_3 B_{11})^2] = 1 \quad (72)
\]

Letting:

\[
a = \frac{B}{\sqrt{B_{22}^2 + B_{12}^2}} \quad (73)
\]
\[
b = \frac{B}{\sqrt{B_{11}^2 + B_{21}^2}} \quad (74)
\]

and:

\[
\cos(\Omega) = \frac{B_{22}}{\sqrt{B_{22}^2 + B_{12}^2}} \quad (75)
\]
gives:
\[
\left( \frac{\bar{u}_1 \cos \Omega}{a} - \frac{\bar{u}_3 \sin \Omega}{a} \right)^2 + \left( \frac{\bar{u}_1 \sin \Omega}{b} + \frac{\bar{u}_3 \cos \Omega}{b} \right)^2 = 1 \quad (76)
\]
which represents a rotated ellipse, which is represented by Figure 60. The rotated

\[2a\]
\[2b\]

**Figure 60:** Rotated ellipse at the \(\xi-\eta\) coordinate system, where \(a\) and \(b\) denote the maximum amplitude, and \(\Omega\) is the orientation angle [21].

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ellipse presented in (76) has a complicated polarization pattern, whereby at each spatial point, the ellipse has variation in the semi-axes (Figure 61(a)), in addition to a different orientation ((Figure 61(b)). One reason for this diverse inclination is seen from Equations (49) and (49), where the phase is the product of the wavenumber and radial distance, such that each position yields a phase change.

### 4.2.3 Single mode polarization: \(S_0\) and \(A_0\) modes

Since the experimental data can be separated, the polarization for a single mode is considered. The surface displacement components of a single mode (\(S_0\) or \(A_0\)) at a given location and frequency is given by:

\[u_1(r, \omega) = A_1 e^{i(\varphi - \pi/2)} \quad (77)\]

\[u_3(r, \omega) = A_3 e^{i\varphi} \quad (78)\]

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Figure 61: Polarization of in-plane, $\bar{u}_1$, to out-of-plane, $\bar{u}_3$, displacement components: (a) Function of radial distance from the source, and (b) planform view.
where $\varphi$ is a phase angle in the $[0, 2\pi]$ interval and $A_{1,3}$ are the components amplitudes. The components normalized with respect to the amplitude of the out-of-plane displacement are defined as:

$$
\bar{u}_1 = \frac{\text{Real}(u_1)}{|u_3|} = \frac{A_1}{A_3} \cos(\varphi - \frac{\pi}{2}) 
$$

(79)

$$
\bar{u}_3 = \frac{\text{Real}(u_2)}{|u_3|} = \cos \varphi
$$

(80)

Then, the particle trajectory at a given location is determined by squaring and summing Equations (79)-(80), such that an ellipse is given by:

$$
\left( \frac{\bar{u}_1}{A_1 A_3} \right)^2 + \bar{u}_3^2 = 1
$$

(81)

As the variation of both amplitudes $A_1$, $A_3$ with location $(r)$ is of the form $\sqrt{\frac{1}{r}}$, the ellipse will be theoretically the same for different locations, where Figures (62)-(63) show the constant elliptical axes for the $A_0$ and $S_0$ modes, respectively. The vertical axis of the ellipse, the normalized $\bar{u}_3$ component, is always unity, whereas the horizontal axis, $\bar{u}_1$, is larger than unity for $S_0$ mode and smaller than unity for $A_0$ mode.

Figure 64 presents the analytical results separately for $S_0$ (solid line) and $A_0$ (broken line). The ratio of major axis to the minor axis for $S_0$ mode is approximately $12/1$ whereas the same ratio for the $A_0$ mode is approximately $1/2$.

4.3 Wave Polarization estimation using 1D SLDV

The measurement of multi-component Lamb waves is now discussed, where the polarization phenomena is used to verify the experimental techniques of extracting single components from originally a multi-component response. The general experimental procedure is to measure the signal at two pre-determined angles by rotating the aluminum plate on the rotation stage, and then through additional signal processing and trigonometric calculations, resolve the multi-component multi-modal signal.
Figure 62: Elliptical axes for the $A_0$ mode at radial distances from the piezoelectric source.

Figure 63: Elliptical axes for the $S_0$ mode at radial distances from the piezoelectric source.
Figure 64: Normalized analytical trajectories for $S_0$ (solid line) and $A_0$ (broken line) into both in-plane and out-of-plane single modal responses. This process is initially demonstrated for a single point on the plate, and then extended to a 1D scanned line.

4.3.1 Two-angle technique

The 1D SLDV measures velocity and displacements of the surface under consideration in the direction aligned with the Laser beam. Consider the configuration shown in Figure 65, which presents a schematic of the experimental set-up used for this study. Let the orientation of the specimen with respect to the Laser beam be described by the angle $\theta$, which is formed between the unit vector $i_1$ along the surface of the specimen, and the unit vector $i_L$ aligned with the Laser beam. The response of the specimen to an excitation provided by the source $S$ is measured at a number of points located along the scan line, which is defined by an initial point $I$ and a final point $F$.

For simplicity, we assume that the origin of the axis $x_1$ is located at point $O$ which corresponds to the point illuminated by a Laser beam parallel to the $X$ axis fixed to the Laser head. In Figure 65, $d$ defines the distance between the Laser head and the specimen measured along the $X$ axis. Knowledge of such distance is needed for the estimation procedure described in what follows.
First, the estimation of in-plane and out-of-plane components is illustrated for point $O$ and it is then extended for a generic point $K$ located at $x_{1k} \in [x_{1I}, x_{1F}]$. The description is here based on velocity components, with the understanding that a similar procedure generates the displacement components should displacement data be provided as a result of the experiments. Let the velocity of point $O$ be expressed as:

$$\mathbf{v}(x_{1O}) = u_1(x_{1O})\mathbf{i}_1 + u_3(x_{1O})\mathbf{i}_3$$

(82)

where $u_1, u_3$ denote the in-plane and out-of-plane components of the velocity at the point $O$, located at position $x_{1o}$ along the $x_1$ coordinate, per Figure 65.

The SLDV measures the component of the velocity parallel to the beam, which is denoted as $u_L$, and identified by the unit vector $\mathbf{i}_L$. The component can be expressed in terms of $u_1, u_2$ by:

$$u_L(x_{1O}) = \mathbf{v}(x_{1O}) \cdot \mathbf{i}_L$$

(83)
which, given that $i_L = \cos \theta i_1 + \sin \theta i_2$, yields

$$u_L(x_{1o}) = u_1(x_{1o}) \cos \theta + u_3(x_{1o}) \sin \theta$$  \hspace{1cm} (84)$$

Given that $u_L$ is known as provided by the SLDV, while $u_1, u_3$ need to be determined, two measurements at two orientation angles $\theta_a, \theta_b$ need to be performed in order to obtain two equations of the kind of Eq. (84). This leads to the following system:

$$\begin{bmatrix}
    u^{(a)}_L(x_{1o}) \\
    u^{(b)}_L(x_{1o})
\end{bmatrix} = 
\begin{bmatrix}
    \cos \theta_a & \sin \theta_a \\
    \cos \theta_b & \sin \theta_b
\end{bmatrix} 
\begin{bmatrix}
    u_1(x_{1o}) \\
    u_3(x_{1o})
\end{bmatrix}$$  \hspace{1cm} (85)$$

and:

$$\begin{bmatrix}
    u_{1o} \\
    u_{3o}
\end{bmatrix} = \frac{1}{\sin(\theta_a - \theta_b)} 
\begin{bmatrix}
    -\sin \theta_b & \sin \theta_a \\
    \cos \theta_b & -\cos \theta_a
\end{bmatrix} 
\begin{bmatrix}
    u^{(a)}_{LO} \\
    u^{(b)}_{LO}
\end{bmatrix}$$  \hspace{1cm} (86)$$

where $u_{1o}$ is a change in notation to describe $u_{1o} = u_1(x_{1o})$. Equation (86) suggests the importance of the proper selection of the orientation angles $\theta_a, \theta_b$, that need to be different enough to avoid an indetermination in the evaluation of the two velocity components. Their value must also consider the limited sensitivity of the SLDV which is optimal at normal angular incidence ($\theta = 90^\circ$), and in general, it was found that acceptable signal to noise ratios were found for angles up to $\pm 45$ degrees from the normal. In addition, the value of the angles to be used in Equation (86) needs to be corrected when the velocity components are evaluated at locations other than $O$ (Figure 65). The measured velocity at point $K$ is given by:

$$u_L(x_{1K}) = \mathbf{v}(x_{1K}) \cdot i_{L_k}$$  \hspace{1cm} (87)$$

which is expressed in terms of the corresponding in-plane and out-of-plane components through the following relation:

$$u_{LK} = u_1(x_{1K}) \cos \theta_k + u_3(x_{1K}) \sin \theta_k$$  \hspace{1cm} (88)$$

Lastly, the angle $\theta_k$ may be determined from the angle provided by the Polytec DAQ system, which is measured from the optic axis of the laser. Only the angles of the
extremum points along the line are required, such that the angles per grid point may be calculated using the distance from the SLDV laser head to the plate. The Polytec DAQ also provides unresolved spatial coordinates of the grid. The measured coordinates must be resolved relative to the orientation of the plate to the laser head in order to yield the true coordinates of the grid point on the plate with respect to the source. The corresponding distances between each grid point, at location $k$, are used to calculate the corresponding $\theta_k$ through the use of a simple algorithm that uses the Law of Sines and Cosines, where $\theta_k$ is ultimately given by:

$$\theta_k = \sin^{-1}(\frac{u_{Lk-1} \sin \theta_{k-1}}{u_{Lk}})$$

(89)

The results are ensured by comparing the summation of the two recorded angles from the DAQ to the summation of the calculated angles. The average error from measured to calculated angles was less than 1% for all tests, which is acceptable for the purposes of these experiments.

4.4 Experimental set-up

The experimental data are obtained using piezoceramic discs as actuators, and a SLDV as a sensor. The SLDV (Polytec PI, Model PSV400M2) allows frequency sampling up to 1 MHz, which enables ultrasonic waves detection and visualization. In the wave propagation tests, the piezoelectric discs are driven (Trek PZD350 Piezo-Driver) by a sinusoidal burst generated by a signal wave generator (Agilent 33220A). The resulting elastic waves are recorded at the measurement grid points defined on the scanning system. The operation of the SLDV requires the generation of a pulse at each grid point in order to record the corresponding response. Phase information is retained by triggering the excitation signal through a low frequency signal (10 Hz), which also defines the scanning rate.

Experiments are performed on an aluminum (Young’s modulus $E = 70$ GPa, Poisson’s ratio $\nu = 0.33$, density $\rho = 2700$ kg/m$^3$) plate of thickness $2h = 1.55$ mm.
A circular piezoelectric disc (SMD1000D15T21-RS from STEMINC) of diameter $2a = 15$ mm and thickness $h_p = 2.1$ mm is used as excitation source. The disc is bonded to the plate through room temperature curing epoxy-gel and is actuated by a voltage signal controlled by a signal generator through a voltage amplifier. The plate itself is mounted to a 360 degree rotational stage, and aluminum angle brackets are fastened to the rotational stage to support the aluminum plate upright. Figures 66(a)-(c) identify the primary specimen features, with specimen oriented at 90 and $\pm 30$ degrees from the plate normal to the optic axis of the laser.

4.5 Results

The chief objective of the two-angle technique and experimental testing is to resolve the multi-component multi-modal measurements into single component individual modes. The general steps in processing the results are as follows. The measured multi-component and multi-modal response from the 1D SLDV requires initial filtering of the data to separate the modes. The frequency-wavenumber filtering technique described in Section 2.3.1 is an effective tool that enables mode separation. The individual modes are then resolved using Equation (86). Once resolved, single point measurements are compared to both analytical responses and additional experimental testing at varying orientations to ensure that the in-plane and out-of-plane components are in good agreement. The scanned 1D line is then post-processed to extract the single components along the spatial coordinate. A visualization of the process is seen using the elliptical polarization plots to compare analytical behavior to experimental measurements. The following section provides in detail the specific tests, processing of the data, and the consequent elliptical polarization plots.

4.5.1 Data processing for mode separation

Before estimating the in-plane and out-of-plane components, the data recorded by the Laser system is initially filtered to ensure that individual modes are separated. The
Figure 66: Experimental setup: (a) Aluminum plate with identified testing hardware and oriented at 90 degrees to the laser’s optic axis, (b) oriented at 120 degrees, and (c) oriented at 60 degrees
recorded wave $u_L(x_j, t)$ along a line (Figure 67) is first windowed in the temporal-spatial domain to eliminate additional reflections, and can be represented in the frequency-wavenumber domain by performing a 2D-FT as presented in Equation (4). In this domain, the wave $u_L(k, \omega)$ has two components, $u_L^{S_0}$ and $u_L^{A_0}$ which appear decoupled as they correspond to peaks centered at different wavenumbers (Figure 68(a)).

Analytical frequency-wavenumber dispersion curves [43], plotted in Figure 68(a), are used as a basis for the separation of the different modes. Based on the analytical dispersion curves, four lines above and below each mode are computed (Figure 68(b)). These lines represent boundaries for regions occupied by each mode and they are used as limits for separating the modes.

![Space-time variation of recorded response showing multi-modal wave propagation and boundary reflection. The excitation frequency is 200 kHz.](image)

**Figure 67:** Space-time variation of recorded response showing multi-modal wave propagation and boundary reflection. The excitation frequency is 200 kHz.

Selection of one of the wave components is achieved very conveniently through the application of windowing functions as illustrated in [93]. Specifically, the $S_0$ or $A_0$ mode waves are obtained based on:

$$u_L^{S_0|A_0}(k, \omega) = u_L(k, \omega) h_1(k - k_{S_0|A_0}, \omega)$$  (90)

where $h_1(k - k_{S_0|A_0}, \omega)$ denotes a window centered at $k = k_{S_0|A_0}$. A Hann window is
Figure 68: Frequency-wavenumber variation of recorded response showing uncoupled waves and (a) the analytical dispersion curves; (b) boundaries used for the filtering.
used in this work, whose expression is:

\[
\begin{align*}
    h_1(k - k_{S_0|A_0}, \omega) &= \begin{cases} 
        0         & \text{for } |k - k_{S_0|A_0}(\omega)| > 2w(\omega) \\
        0.5 + 0.5 \cos \left[ \frac{\pi(k - k_{S_0|A_0}(\omega))}{w(\omega)} \right] & \text{for } |k - k_{S_0|A_0}(\omega)| < 2w(\omega)
    \end{cases}
\end{align*}
\]

(91)

where \(2w\) is the width of the window, which can vary as a function of frequency. At a frequency \(\omega\), the center of the window is estimated analytically from known dispersion relations (Figure 68(a)-(b)).

Figure 69(a) shows the filtered \(S_0\) mode whereas Figure 69(b) shows the filtered \(A_0\) mode. After applying an inverse 2D-FT, the filtered signals are presented in space-time domain in Figure 70(a)-(b). Figure 70(a) shows the incident \(S_0\) mode whereas Figure 70(b) presents the incident \(A_0\) mode.

### 4.5.2 Comparison of analytical and experimental trajectories

As schematically shown in Figure 65, the first experiment is performed in order to measure the components of the velocities at only one point due to a harmonic excitation of 200 kHz. The point, denoted \(O\) in the figure, corresponds to the point recorded by a Laser beam parallel with the \(X\) axis fixed to the beam. The distance between the source and point \(O\) is 360 mm. The plate is rotated at different inclination angles with respect to the laser beam in order to apply the previously described two-angle technique. These rotations correspond to angles \(\theta_a = 90^\circ, \theta_b = 70^\circ, \text{ and } \theta_c = 110^\circ\).

#### 4.5.2.1 Single point

The measurements of the single point are provided in Figures 71(a)-(c), where Figure 71(a) presents the detail of the raw data recorded at the point for different plate orientations. When the plate is oriented at 90 to the optic axis, only the out-of-plane component is recorded. Based on data recorded at several angles pairs, the in-plane and the out-of-plane components of the velocity are computed and presented in Figures 71(b)-(c). Figures 71(b)-(c) present the in-plane and out-of-plane measurements.
Figure 69: Frequency-wavenumber variation of recorded response showing filtered waves: (a) $S_0$ mode; (b) $A_0$ mode.
Figure 70: Filtered modes in time-space domain: (a) $S_0$ incident mode; (b) $A_0$ transmitted mode.
in comparison to the measured out-of-plane response in order to validate that the single components are accurately resolved. As expected different sets of data produce the same in-plane and out-of-plane components, which ensures that the process is acceptable.

Figure 71: (a) Detail of the data recorded at one point under different plate orientations given by: $\theta_a = 90^\circ$ (solid line), $\theta_b = 70^\circ$ (line with circle markers), $\theta_c = 110^\circ$ (dashed line) (b) The in-plane, and (c) The out-of-plane velocity components computed based on different experimental data sets.

For validation of the experimental data to analytical results, Figure 72 plots the experimental and analytical trajectories as polarization ellipses, obtained based on Equation (76). The inclination of the ellipse is due to the combination of $S_0$ and $A_0$. 

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Lamb wave modes which have different in-plane and out-of-plane polarizations. The smallest amplitude is recorded when the Laser beam is perpendicular to the plate because in this case the $S_0$ mode has the smallest contribution. As the two Lamb waves propagate with different velocities, the rotation as well as the shape of the ellipse presented in Figure 72 is a function of location of the point with respect to the source.

![Normalized trajectories and comparison with analytical predictions at $r = 360$ mm from the source (analytical - solid line, experimental - circles)](figure)

**Figure 72:** Normalized trajectories and comparison with analytical predictions at $r = 360$ mm from the source (analytical - solid line, experimental - circles)

### 4.5.2.2 Multiple point

After the single-point experiment was completed and validated, a second experiment is used to measure the plate multi-component response along a line. The major advantage of this second experiment is that it allows the separation of different Lamb wave modes, thus permitting polarization measurements for each component. This separation, presented in the previous section, is performed in the frequency-wavenumber domain. To obtain accuracy of the results in this domain, the data need to be recorded along a line.

Based on the known distance between the Laser head and the plate (denoted
d in the Figure 65) and on the angle $\theta$, the angle $\theta_k$ between the laser beam and the plate at a point $K$ is computed. Two rotations given by $\theta_a = 60^\circ$, $\theta_b = 120^\circ$ and the perpendicular case ($\theta_c = 90^\circ$) are considered. The two Lamb modes are separated using the procedure described in Section 4.5.1. The in-plane and out-of-plane components of the velocities of the point $K$ are then calculated based on Equation (86). In order to validate the results, the analytical model presented in [87] and summarized in Section 4.2.2 is used.

Figure 73 compares analytical, shown as the solid lines, and experimental trajectories, represented as a sequence of symbols. The experimental results corresponds to the responses recorded at various instants of time at 3 different locations at 360, 380, 400 mm from the source (center of the piezoelectric disc). Figure 73(a) shows the $S_0$ trajectories whereas Figure 73(b) shows the $A_0$ trajectories. The comparison shows an excellent agreement between experimental evaluations and the theoretical predictions which suggests that the estimation approach based on the two-angle measurement is accurate and provides the correct velocity components.

Because of the multi-modal nature of the Lamb waves, the particle trajectories have complex patterns. If the wave is a superposition of $S_0$ and $A_0$ modes, the polarization is a rotated ellipse. As these two modes travel with different velocities and attenuations, the size, shape and rotation of the polarization ellipse are a function of location. If the wave separation is used to extract different wave components a better understanding of their polarizations can be presented. As the in-plane and out-of-plane components of the same mode attenuate the same, these polarization ellipses are the same for any location. The measurement of multi-component wavefield data presented in this chapter now provides the experimental basis for the evaluation of a multi-component mode coefficient that has the potential for accurate damage estimation.
Figure 73: Comparison between experimental measurements and numerical predictions for wave polarization for (a) $S_0$ and (b) $A_0$ mode at 200 kHz (Solid line represents the analytical result whereas the symbols represent the experimental data at 360, 380, 400 mm from the source).
The experimental validation of the phase gradient technique and estimation of damage based on spatially-integrated mode coefficients are presented in this chapter. The setup consists of an SLDV and a set of aluminum plates with a rectangular notch of varying depth. The plates are mounted on a rotational stage, which allows the orientation of the test specimens at different angles relative to SLDV laser head. The phase gradient technique is shown to locate the notch presence independent of the damage depth and excitation frequency for both the converted $A_0$ and reflected $S_0$ modes. Likewise, single component mode coefficients based on single point and spatial-integration formulations are compared, and the resulting convergence of the results at distances 4 wavelengths from the source are demonstrated. Finally, utilizing the two-angle technique described in Section 4.3, results of spatially-integrated multi-component mode coefficients are examined.

5.1 Experimental Set-Up and Approach

An aluminum (Al 6061-T6) plate of 1.4 mm thickness with a notch of 12.65 mm length and 0.8 mm width is considered for the experiments. A Polytec PSV-400 SLDV is used to control the experiment and acquire the response data. The plate schematic and experimental setup are shown in Figure 74. In order to excite an enhanced $S_0$ mode, two actuators are placed on the opposite faces of the plate, and synchronized to impart a dominant symmetrical loading into the plate. The plate is excited by a
7-cycle tone burst modulated by a Hann window over a range of frequencies from 100 kHz up to 300 kHz (Figure 75). The dynamic response of the plate is recorded along a line covering a distance of 140 mm on either side of the notch, with a maximum total of 329 points measured at a minimum spatial resolution of 1.18 mm. The recorded waves are separated using the frequency-wavenumber domain described in Section 2.3.1.

5.2 Phase Gradient Results

The following section explores the capability of the phase gradient technique to experimentally identify a notch damage in a flat aluminum plate for varying frequencies and notch depths.

5.2.1 Notch depth variation

For the given phase gradient analysis, six cases of varying notch depths to plate thicknesses are considered: \( \epsilon = 0.3 - 0.9 \). After applying the filtering techniques discussed in Section 2.3.1, an example of the measured time history of the reflected \( S_0 \) and \( A_0 \) modes are transformed to the frequency-spatial domain (Figure 76) following Equation (8). From Figure 76, it is evident that the maximum amplitudes occur at the excitation frequency of 160 kHz, and the amplitude along the spatial coordinate of the maximum frequency is used in calculating phase gradient results. Figure 77 provides the amplitude of the \( S_0 \) and \( A_0 \) modes along the spatial coordinate at the 160 kHz, where the \( A_0 \) amplitude is more than 3 times larger than the \( S_0 \) mode.

The phase of the converted \( A_0 \) modes of the experimental data are calculated and plotted in Figure 78. Figure 78(a) presents the phase variation of the converted \( A_0 \) for all the cases along the scan line and shows that the phase mode varies linearly with the spatial coordinate and is constant in the \( x > x_D \) region \( (x_D = 142\text{mm}) \). Figure 78(b) presents the second derivative of the phase in terms of the spatial coordinate (Equation (11)) which highlights the location of the notch, and is not a function of
Figure 74: (a) Experimental setup to measure interaction of $S_0$ waves with notch type defect, (b) Schematic of the test plate, and (c) Rotational stage with aluminum mount.
Figure 75: Excitation of modulated 7-cycle sinusoidal tone burst at 200 kHz: (a) Time domain, and (b) Frequency domain.
Figure 76: Frequency-spatial domain of filtered modes for an excitation of 160 kHz at a notch depth of $\epsilon = 0.8$: (a) $\tilde{w}_{S_0(r)}$ mode, and (b) $\tilde{w}_{A_0(r)}$ mode. The location of the notch is $x_D \approx 140$ mm.
**Figure 77:** Spatial detail of amplitude of $|\tilde{w}_{A_0}(x, \omega_{\text{max}})|$ in terms of the spatial coordinate. The location of the notch is $x_D \approx 142$ mm.

Similarly, Figure 79(a) displays the phase variation for the reflected $A_0$ for different notch depths, and Figure 79(b) shows the second derivative. Since the amplitude of the reflected out-of-plane $S_0$ mode is less than the amplitude of the out-of-plane $A_0$ mode (Figure 77), the consequent SNR is lower and the some non-linearity in the phase is observed. However, the application of the second derivative in Figure 79(b) identifies the presence of the damage at $x_D = 142$ mm.

### 5.2.2 Excitation frequency variation

The phase gradient technique is also evaluated for three excitation frequencies (100-300 kHz) at a constant notch depth of $\epsilon = 0.6$ for both the reflected $A_0$ and $S_0$ modes Figures 80-81. Similar to the experimental trend in the notch variation, the reflected $A_0$ mode clearly exhibits a variation in the phase per excitation wavelength, $\Lambda$ (Figure 80(a)), and identifies the notch location without any additional inspection of the response (Figures 80(b)) at $x_D \approx 142$ mm. The phase of the reflected out-of-plane $S_0$ mode for varying wavelengths, $\Lambda$, is shown in Figure 81, and correctly identifies the notch location. Similar to the trends observed in the FEM simulations, the phase
Figure 78: (a) Spatial detail of the phases of $\tilde{w}_{A(r)}(x, \omega_{\text{max}})$ (for four notch depths) in terms of the spatial coordinate; (b) Spatial detail of the second derivative of the phase of $\tilde{w}_{A(r)}(x, \omega_{\text{max}})$ in terms of the spatial coordinate. The location of the notch is $x_D \approx 142$ mm.
Figure 79: (a) Spatial detail of the phases of $\tilde{w}_{S_0}(x, \omega_{max})$ (for four notch depths) in terms of the spatial coordinate; (b) Spatial detail of the second derivative of the phase of $\tilde{w}_{S_0}(x, \omega_{max})$ in terms of the spatial coordinate. The location of the notch is $x_D \approx 142$ mm.
velocity differences between the excitation frequencies produces the variation in the phase amplitude.

![Graph](image)

**Figure 80**: (a) Spatial detail of the phases of $\hat{w}_{A_0}^{(r)}(x, \omega_{\text{max}})$ (for three excitation wavelengths) in terms of the spatial coordinate; (b) Spatial detail of the second derivative of the phase of $\hat{w}_{A_0}^{(r)}(x, \omega_{\text{max}})$ in terms of the spatial coordinate. The location of the notch is $x_D \approx 142$ mm.

### 5.3 Single Component Mode Coefficient Estimation

The single component mode coefficients are determined by scanning along the aluminum plates with varying notch depths, filtering the data, and then applying the
Figure 81: (a) Spatial detail of the phases of $\tilde{w}_{S_0}(r, x, \omega_{max})$ (for three excitation wavelengths) in terms of the spatial coordinate; (b) Spatial detail of the second derivative of the phase of $\tilde{w}_{S_0}(r, x, \omega_{max})$ in terms of the spatial coordinate. The location of the notch is $x_D \approx 142$ mm.
single out-of-plane component formulation from Equations (22)-(24). As an intermediate result, Figure 82(a) displays the amplitudes of the filtered converted $A_0$ mode for given damage depths ($\epsilon = 0.3, 0.5, 0.7$) along the scanned line of the plate. The considered excitation signal is a 160 kHz sinusoidal modulated with a Hann window. In Figure 82(a), it is observed that the response approximately attenuates from the notch, located at $x_D = 142$ mm at a decay rate proportional to $1/\sqrt{r}$, where $r$ is the distance from the notch due to geometric spreading of the circular crested waves [43]. These generated crested waves differ from the plane strain FEM simulation plane wave $S_0$ loading, where no attenuation from the source exists. The integration limits for the single component mode coefficient are demonstrated in Figure 82(b), which are selected as 3 wavelengths away from the notch presence. For a 160 kHz $A_0$ mode, the corresponding wavelength is 10.7 mm, which means that the corresponding integration limit from the source, $x_2$, is located at the spatial coordinate 110 mm, as indicated in Figure 82(b). The integration limit $x_1$ is selected as the coordinate prior to the effects of the windowing process since as no boundary exists from another source, which is identified as the spatial coordinate of 15 mm.

5.3.1 Single and spatially averaged mode coefficients

For the given length of 140 mm prior to the notch location, the integration limits of the $C_{A_0}$ mode coefficients are selected at a prescribed distance away from the local effects of the notch. For single point mode coefficient formulations of straight crested wave excitation, this distance was previously selected as 60 plate thicknesses [69]. In practice, a 60 plate thickness region may be unattainable for structures of complex geometry, and thus it is of interest to investigate the ability of the spatially integrated mode coefficient to accurately estimate the mode conversion. For circular crested waves, it has been shown analytically that three to four wavelengths of the circular
Figure 82: Filtered converted $\tilde{w}_{A_0}(r)$ mode mode along the scanned length of an aluminum plate whose notch width of 0.8 mm is centered at $x_D \approx 142$ mm, and is excited at 160 kHz: (a) As a function of $\epsilon$, and (b) Limits of integration, $x_1$ and $x_2$. 

(a) 

(b)
crested excitation is within 0.1% of the straight crested behavior [43]. Figures 83(a)-(b) demonstrate this gradual convergence to unity with increasing wavelengths away from the notch, where the ratio is the integrated coefficient $C_{A_0}$ to the amplitude of the single point converted mode, $C_{P_0}^{P_t}$, whose superscripts $P_t$ denote the data is for a single point. Thus, it may be interpreted that the integrated formulation provides greater accuracy within the scanning region where the longitudinal coordinate, $x$, is less than four wavelengths, as a result of the single point more susceptible to the amplitude variation from the local notch effects.

The reflected and converted integrated mode coefficients are shown as a function of damage depth to plate thickness for different wavelengths to notch widths, $\Lambda$ in Figures 84-85. Figures 84(a)-85(a) are the single point results of the reflection $S_0$ and converted $A_0$ mode coefficients, respectively, taken at 4 wavelengths, or 4 $\lambda$, from the identified notch location at 140 mm. In terms of the $R_{S_0}$ mode coefficient, the discrepancy between the formulated single point and spatially-integrated mode coefficients is evident per each wavelength excitation, and highlights the advantage of integrating over a spatial domain versus relying upon a single measurement point. However, the $C_{A_0}$ mode coefficient for both the single point and integrated formulations are almost identical in both amplitude and trend for each wavelength, and this is attributed to the $4\lambda$ distance away from the point source. The monotonic trend of the $R_{S_0}$ mode coefficient and parabolic $C_{A_0}$ trend correspond to the FEM results presented in Section 3.3.

Although the trends from Figures 84(a)-(b) of the $R_{S_0}$ and $C_{A_0}$ modes compare favorably with the FEM trend shown in Figures 29, the apparent maximum amplitude discrepancy from 0.8 to 0.2 may be attributed to the FEM utilizing straight waves compared to the circular crested waves in the experiments, whereby the energy is loss at the wavefront due to geometric spreading.
Figure 83: Comparison of single point mode coefficients as a function of wavelength, $\lambda$, and notch depth, $\epsilon$, from the notch location, $x_D$ at an excitation of 160 kHz: (a) $R_{S_0}$ mode coefficient, and (b) $C_{A_0}$ mode coefficient.
Figure 84: Reflected $S_0$ mode coefficients, where $\Lambda$ is a ratio of wavelength to notch width: (a) results at $x = 4\lambda x_D$, (b) results for the spatially-integrated results value.
Figure 85: Converted $A_0$ mode coefficients, where $\Lambda$ is a ratio of wavelength to notch width: (a) results at a $x = 4\lambda x_D$, (b) results for the spatially-integrated results value
5.4 Multi-component Mode Coefficient Estimation

The multi-component mode coefficient is based on the development in Section 2.5.2, where the formulation is equivalent to the resolved component of the in-plane and out-of-plane components at each location. The resolved component is then spatially-integrated between prescribed limits outside of local effects, which is considered 4 wavelengths away from the source [43]. A reduced test matrix is determined for the multi-component testing, that consists of the identical aluminum plates with 7 varying notch depths ranging from $\epsilon = [0.3, 0.9]$, as described in Section 5.1. The plates are tested at three given frequencies: 100, 200, and 300 kHz for 2 plate orientations: 90 and 120 degrees normal to the optical axis of the SLDV laser head. These orientations are previously determined from the polarization testing validation in Section 4.3 to provide sufficient angle difference per Equation (86) in capturing the in-plane and out-of-plane components. At these frequencies, the corresponding wavelengths are found from the dispersion curves in Figure 86, and displayed in Table 2.

![Figure 86: Wavelength of $S_0$ and $A_0$ modes as a function of frequency.](image)

Upon filtering experimental data using the procedure outlined in Section 2.3.1, and applying the component extraction process in Section 4.3.1, the data is separated per
mode and displacement component, such as $u_{S_0}$ and $w_{S_0}$ indicate the reflected in-plane and out-of-plane $S_0$ mode that is produced from the notch damage presence, respectively. Figures 87(a)-(b) illustrate the amplitude variation in component amplitude after completing the filtering process, and performing a 1D FT to yield results in the spatial-frequency domain. The frequency from which the data is selected corresponds to the frequency of the maximum amplitude, which typically is the excitation frequency. Figures 87(a)-(b) show how the in-plane $S_0$ mode is more than an order of magnitude larger than the out-of-plane component across the monitored length of 300 mm, and empirically substantiates the need to incorporate the in-plane component in the mode coefficient formulation.

Table 2: Wavelengths [mm] of frequencies [kHz] for $S_0$ and $A_0$ modes used in experimental tests.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency [kHz]</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>100</td>
</tr>
<tr>
<td>$S_0$</td>
<td>55.1</td>
</tr>
<tr>
<td>$A_0$</td>
<td>11.3</td>
</tr>
</tbody>
</table>

5.4.1 Reflection, transmission, and conversion mode coefficients

Similar to the single component mode coefficients, the multi-component coefficients are initially presented as a function of damage depth to plate thickness. Figures 88(a)-(c) show the $R_{S_0}$, $T_{S_0}$, and $C_{A_0}$ in terms of the normalized wavelength, $\Lambda$. The general trend of the amplitudes $R_{S_0}$ is a monotonic increase from $\epsilon = 0.3$ to $\epsilon = 0.9$, whereas the amplitudes of $T_{S_0}$ monotonically decrease. These trends are consistent with the numerical results from Section 3.4. In addition, the $C_{A_0}$ modes follow the trend of a downward parabolic curve, with its maximum peak at $\epsilon = 0.6$ for $\Lambda = 9.5$ and $\epsilon = 0.7$ for $\Lambda = 14.2$. This parabolic trend and corresponding peak location are consistent with the single component and numerical results.

If the individual amplitudes of the $R_{S_0}$ for each wavelength are scaled by the product of $\Lambda$ (Figure 89), then the result is a collapsing of the three wavelengths for
Figure 87: The component amplitudes of the $S_0$ modes in the spatial-frequency domain at 200 kHz for damage notch depth of $\epsilon = 0.8$: (a) Incident $S_0$ mode, (b) Reflected $S_0$ mode
\( \epsilon = 0.6 \). This is significant in the inverse damage characterization problem, where the goal is to determine the damage depth given the mode coefficient. A reduced dependence on excitation wavelength is helpful to a field technician, who may not have to conduct a frequency sweep of a structure, and thereby reduces the complexity of applying these techniques to industry-like conditions.

The relative variability of the multi-component mode coefficients is determined by repeating the two-angle measurements at a given damage depth for varying frequencies, filtering the data, and calculating the mode coefficients. Figure 90 provides the results at frequencies 100 to 300 kHz for a constant notch depth of \( \epsilon = 0.7 \), and for orientations of 90 and 120 degrees normal to the optical axis. The error bars indicate the upper and lower limits of the mode coefficient calculations for 5 repeated measurements. Figures 90(a)-(c) demonstrate the general trend of an increase of variability as the frequency increased. This relationship may be attributed to the signal attenuation produced by the Trek amplifier at frequencies larger than 150 kHz.

5.4.2 Comparison of multi-component to single component mode coefficients

In terms of evaluating the multi-component relative to the single component mode coefficients, Figures 91-93 provide results based on the mode coefficient formulation for the three excitation frequencies, 100, 200 and 300 kHz. Recall that the mode coefficients denoted by a \((\cdot)^u\) and \((\cdot)^w\) represent the mode coefficients based solely on the in-plane and out-of-plane components, respectively per Section 2.5.3. Specifically, Figures 91(a)-(c) demonstrate that the \( R_{S_0}^{uw} \) mode coefficient closely aligns with the in-plane \( R_{S_0}^u \) mode coefficient for all three frequencies, which is expected since the in-plane reflected \( S_0 \) mode is at least an order of magnitude greater than the out-of-plane reflected \( S_0 \) mode (refer to Figure 87(b)). This trend of the multi-component mode coefficient having a larger in-plane component than out-of-plane component,
Figure 88: Multi-component mode coefficient as a function of $\epsilon$ for given $\Lambda$, where $\epsilon$ is the ratio of damage depth to plate thickness and $\Lambda$ is the ratio of wavelength to damage width: (a) $R_{S_0}^{uw}$, (b) $T_{S_0}^{uw}$, and (c) $C_{A_0}^{uw}$.
and consequently, an approximate equivalency to the single in-plane mode coefficient formulation is found for the \( T_{S_0} \) and \( C_{A_0} \) modes (Figures 92-93). The relative error between the single in-plane mode coefficients and the multi-component mode coefficients is shown in Table 3. The minimal error occurs at the 100 and 200 excitation frequencies for the reflected and transmitted \( S_0 \) modes, with the transmitted \( S_0 \) mode coefficient having the lowest variance and least amount of error. This may be attributed to the inclusion of the in-plane component, whereas the out-of-plane converted \( A_0 \) mode coefficient has the greatest amount of error at the 100 and 200 frequencies. For all three mode coefficients, the error rises for the 300 kHz case, which gives an indication that additional noise may have been induced due to the frequency range itself, where attenuation of the signal by the amplifier is present.

It is noteworthy that these experimental trends are consistent with the numerical simulations provided in Section 3.4, where the incident in-plane \( S_0 \) mode is clearly dominant. This is most evident in Figures 93(a)-(c), where the single component \( C_{A_0}^{w} \) is larger in amplitude than the in-plane \( C_{A_0}^{u} \) coefficient for the entire damage depth range, \( \epsilon \). Hence, it could be expected that the vectorial sum formulation used in the multi-component development would suggest that \( C_{A_0}^{uw} \) would more likely approach
Figure 90: Repeatability of multi-component mode coefficient calculations at $\epsilon = 0.7$ for varying frequencies: (a) Reflected $S_0$, (b) Transmitted $S_0$, and (c) Converted $A_0$. 
the out-of-plane $C_{A0}^w$. However, the inclusion of the incident in-plane $S_0$ mode in the denominator of the coefficient ratio actually drives the multi-component $C_{A0}^{uw}$ to resemble the in-plane $C_{A0}^u$. This is shown empirically for all three frequencies, and concurs with the numerical results in Figure 37(b).

Figure 91: Reflected $S_0$ mode coefficients as function of $\epsilon$ for given frequencies, $f$, where superscripts $u$ and $w$ denote in-plane and out-plane components, respectively: (a) $f = 100$ kHz, (b) $f = 200$ kHz, and (c) $f = 300$ kHz.
Figure 92: Transmitted $S_0$ mode coefficients as function of $\epsilon$ for given frequencies, $f$, where superscripts $u$ and $w$ denote in-plane and out-plane components, respectively: (a) $f = 100$ kHz, (b) $f = 200$ kHz, and (c) $f = 300$ kHz.

Table 3: Maximum relative percentage error between in-plane single mode coefficients, ($\cdot)^u$, and multi-component mode coefficients, ($\cdot)^{uw}$, per frequency.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency [kHz]</th>
</tr>
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<tbody>
<tr>
<td>$R_{S_0}$</td>
<td>0.4</td>
</tr>
<tr>
<td>$T_{S_0}$</td>
<td>0.2</td>
</tr>
<tr>
<td>$C_{A_0}$</td>
<td>10.2</td>
</tr>
</tbody>
</table>
Figure 93: Converted $A_0$ mode coefficients as function of $\epsilon$ for given frequencies, $f$, where superscripts $u$ and $w$ denote in-plane and out-plane components, respectively: (a) $f = 100$ kHz, (b) $f = 200$ kHz, and (c) $f = 300$ kHz.
CHAPTER VI

CONCLUSIONS AND FUTURE WORK

6.1 Summary

Wave propagation-based interrogative methods have shown promise to accurately identify and quantify structural defects. This research specifically provides a numerical and experimental foundation to a phase gradient-based damage localization approach. The technique successfully identifies damage location independently of the excitation frequency, damage depth, and fundamental mode selection. In addition, this research evaluates reflection, transmission, and mode conversion coefficients corresponding to the interaction of the injected wave with damage. Spatially-integrated mode coefficients are based on in-plane and out-of-plane displacement components and are proposed as measures of the degree of damage depth and profile. Both the phase gradient technique and mode coefficient estimation utilize a frequency-wavenumber filtering technique, which enables Lamb wave mode separation and the analysis of individual modes and their respective behavior.

The techniques are applied to the simulated response of damaged plates, and to experimental data measured using a Scanning Laser Doppler Vibrometry. Numerical and experimental parametric studies are conducted to evaluate the effects of damage depth and shape, and to estimate the influence of the choice of excitation frequency. Limitations to the damage profiling characterization are shown for lower frequency regimes. Mode coefficients based on multiple components are also evaluated as means to provide more detailed information regarding wave interaction with damage, and to mitigate noise effects. In-plane and out-of-plane displacements are recovered through an experimental procedure based on Laser detection at multiple
angles. The procedure is validated through comparison of experimental polarization curves with analytical ones.

6.2 Research Contributions

This research provided the following contributions to the state-of-the-art:

1. Introduced novel phase gradient-based damage localization technique, that was found to be independent of excitation frequency, damage depth, and applicable to both reflected and converted modes in successfully locating the damage.

2. Introduced single and multi-component spatially-integrated mode coefficient for damage quantification, and applied the formulations to numerical and experimental data produced by an SLDV.

3. Experimentally validated the multi-component extraction via Lamb wave polarization in comparison with analytical formulations.

6.3 Recommendations for Future Research

The following recommendations are provided for future development of the aforementioned damage localization and damage quantification methodologies.

6.3.1 Damage localization

1. Investigate the higher frequency-thickness regime (greater than 1 MHz-mm) and evaluate the relationship between excitation wavelength to damage width. In addition, it would be of interest to modify the interrogative mode to a fundamental $A_0$ mode, which despite its dispersive nature, has shown great sensitivity to asymmetric surface damages [68].

2. Extend the study to damage types other than the ones considered in this research which have abrupt, sharp edges. Corrosive build-up is typically beveled
and the transition is more gradual than abrupt, which may affect the ability of the mode to convert or reflect. An open research question exists in determining how the damage localization technique performs in detecting damages of gradual transitions.

3. Apply the phase gradient technique to a 2D problem, where multiple excitation sources and consequent signal processing on a plate yield the damage location and its size. One challenge associated with a 2D analysis is the development of a function that allows proper “unwrapping” of the phase of the spatial response. Such analysis will require additional signal processing tools not currently developed. A 3D numerical model that fully captures the circular-crested waves may prove instrumental in the validation of this concept.

4. Apply the localization technique to anisotropic materials and broaden the scope to more complex, industry-like structures beyond the plate-like structures examined in this research. For example, rather than a direct propagating path from the source to the damage presented in this research, it is recommended that additional stiffener-like components are mounted to the plate-like waveguide, which obstructs the direct path from the source to damage.

6.3.2 Damage quantification

1. Investigate the higher frequency-thickness regime (greater than 1 MHz-mm for typical aluminum plates of approximately 1 mm thickness) and evaluate the relationship between excitation wavelength to damage width, including variation of damage profile.

2. Evaluate mode coefficients in the presence of damage which cause gradual impedance changes as opposed to the abrupt ones considered in this research.

3. Investigate the ability to extract mode coefficient information for non-direct
wave guide paths. For example, it would be of interest to examine what occurs to the mode conversion and filtering process when stiffener-like panels obstruct the wave path.

6.4 Concluding Remarks

The primary lessons learned in conducting this research are three-fold. First, this research demonstrated that preliminary numerical modeling is a critical step in understanding the basic fundamentals of the problem without the associated time expense in producing an accurate and reliable laboratory setup. Also, the numerical analysis helps eliminate any outside noise that may distort the results, yet a word of caution exists in developing a complete reliance on the numerics for the scope of the problem. It is possible that the model either does not capture all the physics, or some erroneous assumptions are built into the model itself. For example, the attenuation associated with the experimental results is not conveyed in the 1D plane strain FE representation. However, despite this limitation, the mode coefficient processing algorithm was refined at various stages during the research period due to simple comparisons with the results from the numerical mode coefficient estimations. As an extension of this idea, if one were to pursue the 2D damage localization technique, it is recommended that an accurate 2D/3D model be initially constructed to replicate noise-free results that allow for proof-of-concept.

Second, in applying the mode coefficients to industry applications beyond a flat, aluminum plate-like structure, it is recommended that the reflected $S_0$ mode be the preferred mode coefficient estimation at the beginning stages of the damage assessment. This preference is based on its monotonic behavior, which appears almost linear. This is in contrast to the converted $A_0$ mode, which exhibits a parabolic shape for both single and multi-component when shown as a function of the damage depth to plate thickness, and may lead to false-damage evaluation for the inverse
Furthermore, spatial integration greatly reduces the potential for noise-induced error by the inherent nature of relying on more than one measured point. Also, the vectorial sum approach of the multi-component coefficient includes both modes in the formulation, such that the presence of a dominant component does not introduce errors or excessive noise.

Finally, Lamb wave polarization research remains an open research topic and the literature is relatively sparse in terms of structural health monitoring. The present research focused primarily on the elliptical motion of the Lamb wave particle to verify the appropriate extraction of the multi-components for damage quantification. However, one questions how to incorporate the elliptical motion as a damage identification parameter, where perhaps the specific elliptical orientation may show patterns related to damage can be of great interest. Such a method would prove valuable since the polarization behavior is already measured for the multi-component formulation, and could now be applied for improved damage characterization.
REFERENCES


[34] ESPOSITO, E., “Report on measurement sessions by scanning laser doppler vibrometry (sldv), ground penetrating radar (gpr), and infrared thermograph (ir-t),” tech. rep., Polytechnic University of Marche, 2005.


