Project Title: Algebraic Methods Applied to Systems Containing Pure or Distributed Time Delays

Project No: E-21-619

Project Director: Dr. Edward W. Kamen

Sponsor: U.S. Army Research Office; Research Triangle Park, N.C. 27709

Date: 3/2/78

Agreement Period: From 2/1/78 Until 11/30/79

Type Agreement: Grant No. DAAG29-78-G-0063

Amount: $19,999 ARO
2,066 GIT (E-21-320)
$22,065 Total


Sponsor Contact Person(s):

Technical Matters
Dr. Jagdish Chandra, Director
Mathematics Division
U.S. Army Research Office
P.O. Box 12211
Research Triangle Park, N.C. 27709

Contractual Matters
(thru OCA)
Mr. Jack L. Harless, Chief
Procurement Office
U.S. Army Research Office
P.O. Box 12211
Research Triangle Park, N.C. 27709

Property Administration/Plant Clearanc and Closeout Administration

NOTE: FOLLOW-ON TO E-21-606

Defense Priority Rating: None

Assigned to: EE

(School/Laboratory)

COPIES TO:

Project Director
Division Chief (EES)
School/Laboratory Director
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EES Information Office
EES Reports & Procedures
Project Files (OCA)
Project Code (GTRI)
Other
GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF CONTRACT ADMINISTRATION

SPONSORED PROJECT TERMINATION

Date: September 25, 1980

Project Title: Algebraic Methods Applied to Systems Containing Pure or Distributed Time Delays

Project No: E-21-619

Project Director: Dr. Edward W. Kamen

Sponsor: U.S. Army Research Office; Research Triangle Park, NC 27709

Effective Termination Date: 5/31/80

Clearance of Accounting Charges: 5/31/80

Grant/Contract Closeout Actions Remaining:

- Final Invoice and Closing Documents
- Final Fiscal Report
- Final Report of Inventions
- Govt. Property Inventory & Related Certificate
- Classified Material Certificate
- Other Final Financial Status Report* and Closing Documents

*OCA has copy of transmittal letter, dated August 6, 1980, to ARO. Need copy of Final Financial Status Report which should have been submitted to OCA for closing documents.

NOTE: "Continued" by E-21-620

Assigned to: Electrical Engineering (School/Department)

COPIES TO:

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Project File (OCA)
Project Code (GTRI)
Other C. E. Smith

CA-4 (1/79)
PROGRESS REPORT
(TWENTY COPIES REQUIRED)

1. ARO PROPOSAL NUMBER: 13162-M

2. PERIOD COVERED BY REPORT: February 1, 1978 through June 30, 1978

3. TITLE OF PROPOSAL: Algebraic Methods Applied to Systems Containing Pure or Distributed Time Delays

4. CONTRACT OR GRANT NUMBER: DAAG29-78-G-0063

5. NAME OF INSTITUTION: Georgia Institute of Technology

6. AUTHOR(S) OF REPORT: Edward W. Kamen

7. LIST OF MANUSCRIPTS SUBMITTED OR PUBLISHED UNDER ARO SPONSORSHIP DURING THIS PERIOD, INCLUDING JOURNAL REFERENCES:

8. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED DURING THIS REPORTING PERIOD:
   E. W. Kamen, Principal Investigator

Dr. Edward W. Kamen
Georgia Institute of Technology
School of Electrical Engineering
Atlanta, GA 30332
The first part of the research has centered on a new algebraic approach to large (high-dimensional) discrete-time systems given by a first-order vector difference equation with coefficients belonging to a ring of shift operators. Via this representation, it is possible to study a given system in terms of some subset of the set of all possible state variables. In this way, a large system can be studied in terms of a low-order model defined over a ring of shift operators. Such models are ideally suited for the study of large systems consisting of an interconnection of subsystems separated by time lags (a common type of interconnection in applications). New results have been obtained on reaching (from zero) or controlling (to zero) states or finite sequences of states belonging to a preselected subspace of the space of all possible states. Results have also been obtained on the design of controllers that feed back a part of the total state vector. It has been shown that these results can be dualized, giving new procedures for state reconstruction.

Another part of the research has been concerned with the realization of linear time-varying continuous-time systems or networks. A new approach has been developed which is based on a "time-varying" version of rationality. This has led to realizability criteria given in terms of a Hankel matrix constructed from a formal power series associated with the kernel of a given integral operator. The theory is remarkably similar to the realization theory of linear time-invariant systems defined over a ring. In the time-varying case, the underlying ring is a ring of time functions. This approach should yield procedures for constructing partial realizations of integral operators (which often appear in applications).
PROGRESS REPORT
(TWENTY COPIES REQUIRED)

1. ARO PROPOSAL NUMBER: 13162-M

2. PERIOD COVERED BY REPORT: July 1, 1978 through December 31, 1978

3. TITLE OF PROPOSAL: Algebraic Methods Applied to Systems Containing Pure or Distributed Time Delays

4. CONTRACT OR GRANT NUMBER: DAAG29-78-G-0063

5. NAME OF INSTITUTION: Georgia Institute of Technology

6. AUTHOR(S) OF REPORT: Edward W. Kamen

7. LIST OF MANUSCRIPTS SUBMITTED OR PUBLISHED UNDER ARO SPONSORSHIP DURING THIS PERIOD, INCLUDING JOURNAL REFERENCES:

See attached page

8. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED DURING THIS REPORTING PERIOD:

E. W. Kamen, Principal Investigator

Dr. Edward W. Kamen
Georgia Institute of Technology
School of Electrical Engineering
Atlanta, GA 30332
7. LIST OF MANUSCRIPTS


BRIEF OUTLINE OF RESEARCH FINDINGS

Further progress has been made on the operator approach to large (high-dimensional) linear discrete-time systems developed in the previous work period. In this framework, large systems are given by low-order vector difference equations with coefficient matrices defined over a ring of shift operators. A "fast" algorithm based on a generalization of Faddeeva's algorithm has been developed for determining functional reachability and controllability using criteria derived in the previous work period. Results have also been obtained on a generalized notion of coefficient (or pole) assignability using state feedback. A simple and easily computable algebraic criterion has been developed for determining assignability. These results can be utilized to design "dead-beat controllers" and state estimators for large systems.

An extensive collection of new results has been obtained on the realization of linear time-varying integral operators by systems described by vector differential equations with analytic coefficients. The approach is quite different from previous methods in that it is based on modules defined over the ring of real analytic functions. In particular, it has been shown that realizability is equivalent to the finiteness of modules generated from the kernel of the integral operator. This framework has resulted in the (apparently first) general procedure for computing minimal realizations. The procedure is based on a method for determining bases for modules defined over a Bezout domain. These results have also been applied to the important problem of system reduction, which can be viewed as the problem of constructing a minimal realization from a given nonminimal realization.
PROGRESS REPORT
(TWENTY COPIES REQUIRED)

1. ARO PROPOSAL NUMBER: 13162-M

2. PERIOD COVERED BY REPORT: January 1, 1979 through June 30, 1979

3. TITLE OF PROPOSAL: Algebraic Methods Applied to Systems Containing Pure or Distributed Time Delays

4. CONTRACT OR GRANT NUMBER: DAAG29-78-G-0063

5. NAME OF INSTITUTION: Georgia Institute of Technology

6. AUTHOR(S) OF REPORT: Edward W. Kamen

7. LIST OF MANUSCRIPTS SUBMITTED OR PUBLISHED UNDER ARO SPONSORSHIP DURING THIS PERIOD, INCLUDING JOURNAL REFERENCES:

See attached page

8. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED DURING THIS REPORTING PERIOD:

E. W. Kamen, Principal Investigator

Dr. Edward W. Kamen
School of Electrical Engineering
Georgia Institute of Technology
Atlanta, GA 30332
7. LIST OF MANUSCRIPTS

E.W. Kamen, "New results in realization theory for linear time-varying analytic systems," revised version (July 1979) accepted for publication in the IEEE Transactions on Automatic Control.


BRIEF OUTLINE OF RESEARCH FINDINGS

Further progress has been made in the area of linear systems with time delays. In an effort to construct a "unified theory," part of the research has centered on the relationship between systems with time delays, lumped-distributed networks, and two-dimensional systems. In particular, a general representation theory has been developed which includes these three types of network-systems as special cases. The correspondence between systems with time delays and two-dimensional systems has resulted in new stability criteria for systems with time delays. Preliminary results include new sufficient conditions for the zeros of an exponential polynomial \( p(s, e^{-s}) \) to be within the open left-half-plane (a necessary and sufficient condition for stability). In many cases, the criteria can be checked using a finite number of algebraic operations. A preliminary version of this work will be presented at the Canadian Mathematical Society Annual Seminar, in Toronto, on July 30, 1979.
PROGRESS REPORT

(TWENTY COPIES REQUIRED)

1. ARO PROPOSAL NUMBER: 13162-M

2. PERIOD COVERED BY REPORT: July 1, 1979 through December 31, 1979

3. TITLE OF PROPOSAL: Algebraic Methods Applied to Systems Containing Pure or Distributed Time Delays

4. CONTRACT OR GRANT NUMBER: DAAG29-78-G-0063

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6. AUTHOR(S) OF REPORT: Edward W. Kamen

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See attached page.

8. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED DURING THIS REPORTING PERIOD:

E.W. Kamen, Principal Investigator

EDWARD W. KAMEN
GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF ELECTRICAL ENGINEERING
ATLANTA, GA 30332
7. LIST OF MANUSCRIPTS


Work has continued on the development of algebraic criteria for asymptotic stability of delay differential equations. We now have an algebraic test for determining when a system with commensurate time delays is asymptotically stable for any positive value of delay. Additional tests are currently being developed using recent results on the stability of linear shift-invariant two-dimensional digital filters.

A major part of the work has centered on the study of a large class of linear infinite-dimensional discrete-time systems given by a state model with coefficients in a commutative Banach algebra. A major application of this theory is in the study of linear two-dimensional systems and discretized partial differential equations. The initial part of this effort has centered on the study of linear quadratic optimal control and optimal linear recursive filtering. It has been discovered that these problems can be approached in terms of a (generalized) Riccati equation defined over a commutative Banach algebra. A major new result is that "steady-state gains" for controllers and filters can be shown to exist by considering limiting solutions to the generalized Riccati equation. This framework yields a new approach to the optimal filtering problem for linear two-dimensional systems.
August 6, 1980

Mr. Jack L. Harless  
Chief, Procurement Office  
U.S. Army Research Office  
P.O. Box 12211  
Research Triangle Park, N.C.  27709

Dear Mr. Harless:

Enclosed is the Final Financial Status Report (Form SF-269) for Grant No. DAAG29-78-G-0063 covering the period 2/1/78 - 5/30/80. Also enclosed is Georgia Institute of Technology check No. 124506 for $3.69 to return unexpended funds.

If you have any questions or require additional information, please let us know.

Sincerely,

David V. Welch, Manager  
Grants and Contracts Accounting

Enclosure

cc: Dr. W. W. Kamen  
    Dr. D. T. Paris  
    Mr. E. E. Renfro  
    Mr. O. H. Rodgers  
    File E-21-619
Pay to the order of

US Army Research Office
PO Box 12211
Research Triangle Pk NC 27709

Georgia Institute of Technology
Atlanta, GA 30332

Pay Exactly

Net Amount

Check Void if Not Cashed Within 120 Days

Operating Account

Vice President for Business and Finance
**FEDERAL CASH TRANSACTIONS REPORT**

*See instructions on the back. If report is for more than one grant or assistance agreement, attach completed Standard Form 272-A.*

### RECIPIENT ORGANIZATION
- Georgia Institute of Technology
- Atlanta, Georgia 30332

### FEDERAL EMPLOYER IDENTIFICATION NO.
- 58-6002023

### STATUS OF FEDERAL CASH

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
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<tbody>
<tr>
<td>a. Cash on hand beginning of reporting period</td>
<td>$59.44</td>
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<td>b. Letter of credit withdrawals</td>
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<tr>
<td>c. Treasury check payments</td>
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<tr>
<td>d. Total receipts (Sum of lines b and c)</td>
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<td>e. Total cash available (Sum of lines a and d)</td>
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<td>f. Gross disbursements</td>
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<td>g. Federal share of program income</td>
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<tr>
<td>h. Net disbursements (Line f minus line g)</td>
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<tr>
<td>i. Adjustments of prior periods</td>
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<tr>
<td>j. Cash on hand end of period</td>
<td>3.69</td>
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### THE AMOUNT SHOWN ON LINE 11J, ABOVE, REPRESENTS CASH REQUIREMENTS FOR THE ENSUING

-0- Days

### OTHER INFORMATION

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
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<tbody>
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<td>a. Interest income</td>
<td>$0</td>
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<tr>
<td>b. Advances to subgrantees or subcontractors</td>
<td>$0</td>
</tr>
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</table>

### REMARKS (Attach additional sheets of plain paper, if more space is required)

* Represents Reporting Period and not time charges were incurred.

### CERTIFICATION

<table>
<thead>
<tr>
<th>AUTHORIZED</th>
<th>CERTIFYING OFFICIAL</th>
<th>SIGNATURE</th>
<th>DATE REPORT SUBMITTED</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>David V. Welch, Manager, Grants &amp; Contracts Accounting</td>
<td></td>
<td>July 21, 1980</td>
</tr>
</tbody>
</table>

### TELEPHONE
- Area Code: 404
- Number: 894-4624
- Extension:
FINANCIAL STATUS REPORT

1. FEDERAL AGENCY AND ORGANIZATIONAL ELEMENT TO WHICH REPORT IS SUBMITTED
   Army Research Office

2. FEDERAL GRANT OR OTHER IDENTIFYING NUMBER
   DAAG29-78-G-0063

3. RECIPIENT ACCOUNT NUMBER OR IDENTIFYING NUMBER
   E-21-619

4. EMPLOYER IDENTIFICATION NUMBER
   58-6002023

5. OMB Approved No. 80-R0180

6. PERIOD COVERED BY THIS REPORT
   FROM (Month, day, year) 2/1/78 TO (Month, day, year) 5/30/80

7. BASIS
   YES [ ] NO [ ]

8. PROJECT/GRANT PERIOD (see instructions)
   FROM (Month, day, year) 2/1/78 TO (Month, day, year) 5/30/80

9. STATUS OF FUNDS
   (TOTAL)
   (a) $ (b) $ (c) $ (d) $ (e) $ (f) $

   a. Net outlays previously reported
   b. Total outlays this report period
   c. Less: Program income credits
   d. Net outlays this report period
   e. Net outlays to date
   f. Less: Non-Federal share of outlays
   g. Total Federal share of outlays
   h. Total unliquidated obligations
   i. Less: Non-Federal share of unliquidated obligations shown on line h
   j. Federal share of unliquidated obligations
   k. Total Federal share of outlays and unliquidated obligations
   l. Total cumulative amount of Federal funds authorized
   m. Unobligated balance of Federal funds

10. INDIRECT EXPENSE
   Date Report Submitted
   TYPE OF RATE
   Provisional [ ] Predetermined [ ] Final [ ] Fixed [ ]
   PLACE "X" in appropriate box
   a. Net outlays previously reported
   b. Total outlays this report period
   c. Less: Program income credits
   d. Net outlays this report period
   e. Net outlays to date
   f. Less: Non-Federal share of outlays
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11. INDIRECT EXPENSE
   Date Report Submitted
   TYPE OF RATE
   Provisional [ ] Predetermined [ ] Final [ ] Fixed [ ]
   PLACE "X" in appropriate box
   a. Net outlays previously reported
   b. Total outlays this report period
   c. Less: Program income credits
   d. Net outlays this report period
   e. Net outlays to date
   f. Less: Non-Federal share of outlays
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12. INDIRECT EXPENSE
   Date Report Submitted
   TYPE OF RATE
   Provisional [ ] Predetermined [ ] Final [ ] Fixed [ ]
   PLACE "X" in appropriate box
   a. Net outlays previously reported
   b. Total outlays this report period
   c. Less: Program income credits
   d. Net outlays this report period
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   j. Federal share of unliquidated obligations
   k. Total Federal share of outlays and unliquidated obligations
   l. Total cumulative amount of Federal funds authorized
   m. Unobligated balance of Federal funds

13. CERTIFICATION
   I certify to the best of my knowledge and belief that this report is correct and complete and that all outlays and unliquidated obligations are for the purposes set forth in the award documents.

   TYPE OF RATE
   Provisional [ ] Predetermined [ ] Final [ ] Fixed [ ]
   PLACE "X" in appropriate box
   a. Net outlays previously reported
   b. Total outlays this report period
   c. Less: Program income credits
   d. Net outlays this report period
   e. Net outlays to date
   f. Less: Non-Federal share of outlays
   g. Total Federal share of outlays
   h. Total unliquidated obligations
   i. Less: Non-Federal share of unliquidated obligations shown on line h
   j. Federal share of unliquidated obligations
   k. Total Federal share of outlays and unliquidated obligations
   l. Total cumulative amount of Federal funds authorized
   m. Unobligated balance of Federal funds

   SIGNATURE OF AUTHORIZED CERTIFYING OFFICIAL
   David V. Welch
   Manager
   Grants & Contracts Acctg.

   DATE REPORT SUBMITTED
   July 21, 1980

   TELEPHONE (Area code, number and extension)
   404-894-4624