RADOME DESIGN AND BORESIGHT ERROR MEASUREMENT SIMULATION

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**Abstract (Continue on reverse side if necessary and identify by block number)**
The design of an ablative radome for supersonic missile seeker applications at microwave frequencies is described. The results of a computer simulation of three boresight error measurement methods are also presented.
FOREWORD

This report was prepared by the School of Electrical Engineering, Georgia Institute of Technology, Atlanta, Georgia, under Delivery Order 0015 of BOA DAH001-83-D-A013. The report author is Gene K. Huddleston, Associate Professor, School of Electrical Engineering.

The work was performed for the U.S. Army Missile Command under the direction of M. M. Hallum (DRSMI-RDF), K. N. Letson (-RLA), and Steven P. Risner (-RLA).

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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>FOREWORD</td>
<td>i</td>
</tr>
<tr>
<td>1. INTRODUCTION AND SUMMARY</td>
<td>1</td>
</tr>
<tr>
<td>2. ABLATIVE AND CERAMIC RADOME DESIGN</td>
<td>3</td>
</tr>
<tr>
<td>2.1 Introduction</td>
<td>3</td>
</tr>
<tr>
<td>2.2 Flat Panel Analysis</td>
<td>5</td>
</tr>
<tr>
<td>2.3 Ablative Radome Analysis for Uniform Wall Thickness</td>
<td>6</td>
</tr>
<tr>
<td>2.4 Ablative Radome with Tapered Wall Thickness</td>
<td>6</td>
</tr>
<tr>
<td>2.5 Fused Silica Radome Design</td>
<td>8</td>
</tr>
<tr>
<td>3. BORESIGHT ERROR MEASUREMENT SIMULATION</td>
<td>9</td>
</tr>
<tr>
<td>3.1 Introduction</td>
<td>9</td>
</tr>
<tr>
<td>3.2 BSE Algorithms</td>
<td>11</td>
</tr>
<tr>
<td>3.3 Effects of Distance</td>
<td>14</td>
</tr>
<tr>
<td>3.4 Effects of Reflections</td>
<td>16</td>
</tr>
<tr>
<td>3.5 Effects of Frequency</td>
<td>16</td>
</tr>
<tr>
<td>4. CONCLUSIONS AND RECOMMENDATIONS</td>
<td>17</td>
</tr>
<tr>
<td>4.1 Ablative and Ceramic Radomes</td>
<td>17</td>
</tr>
<tr>
<td>4.2 BSE Measurement Simulation</td>
<td>17</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>19</td>
</tr>
<tr>
<td>APPENDIX A - FLAT PANEL ANALYSIS RESULTS</td>
<td>21</td>
</tr>
<tr>
<td>APPENDIX B - UNIFORM ABLATIVE RADOME PERFORMANCE</td>
<td>29</td>
</tr>
<tr>
<td>APPENDIX C - TAPERED ABLATIVE RADOME PERFORMANCE</td>
<td>37</td>
</tr>
<tr>
<td>APPENDIX D - TAPERED SUBSTRATE RADOME PERFORMANCE</td>
<td>45</td>
</tr>
<tr>
<td>APPENDIX E - FUSED SILICA RADOME PERFORMANCE</td>
<td>53</td>
</tr>
<tr>
<td>APPENDIX F - EFFECTS OF DISTANCE ON BSE ALGORITHMS</td>
<td>61</td>
</tr>
<tr>
<td>APPENDIX G - EFFECTS ON REFLECTIONS ON BSE ALGORITHMS</td>
<td>69</td>
</tr>
<tr>
<td>APPENDIX H - EFFECTS OF FREQUENCY ON BSE ALGORITHMS</td>
<td>77</td>
</tr>
</tbody>
</table>
CHAPTER 1
INTRODUCTION AND SUMMARY

This report describes the electrical design of an ablative and a ceramic radome for a supersonic microwave seeker application. In addition, the results of a computer simulation of the effects of the anechoic chamber environment on boresight error measurements are presented.

Chapter 2 describes the electrical design of tangent ogive (L/D = 3) ablative radome consisting of a load-bearing substrate material ($\varepsilon_r = 4.50$, $\tan\delta = .008$) and a fibre-loaded Teflon ablator outer layer ($\varepsilon_r = 2.45$, $\tan\delta = .003$). The computed electrical performance of both flat dielectric panels and full scale radomes indicate that the optimum ablative radome design with respect to boresight error slope consists of a thick substrate and a thin ablator. The best performance would be obtained with a half-wave wall of the single substrate material; the optimum two-layer structure is a compromise between the half-wave wall and the ablation requirements.

Boresight error slopes (BSES) less than 5% and radome transmission loss less than 0.6 dB are predicted for the optimum ablative radome configuration consisting of .600" substrate and .060" (uniform) ablator thicknesses. To provide for ablation during flight so as to reach this optimum design at the terminal phase, a tapered ablator (.110" at the tip and .070" at the base of the radome) can be provided having initial BSES < 11% and loss less than 0.6 dB. Hence, a .05" tapered change in ablator thickness results in a change in maximum BSES from 11% to 5%.

The design of a comparable fused silica radome is also described in Chapter 2. Designs having uniform wall thickness and an asymmetrical wall thickness design are examined. The ceramic radome design work is not complete.
Chapter 3 describes the results of a computer simulation of a radome boresight error measurement facility. The simulation quantifies the effects of reflections from the anechoic chamber boundaries, the effects of frequency drifts during measurement, and the effects of separation distance between the source antenna and the radome/antenna combination under test (RAUT).

Measurement results are simulated for three BSE measurement techniques: null seeker, offset 1-point method, and a two-point method. All three methods are more sensitive to the distance of separation than to reflections. The two-point method gives approximately the same results as the null seeker. All three methods show significant sensitivity to even small (1%) frequency changes.

The simulation results also show that the offset 1-point method of BSE measurement yields widely varying results and is deemed unsuitable for use. A modified (non-offset) 1-point method of measurement does yield results comparable to the null seeker for large separation distance and no reflections; however, the performance of the modified 1-point method has not been fully studied.
2.1 Introduction

The geometry of the optimum ablative radome design is shown in Figure 2-1. The placement of the antenna, and its radiating characteristics, are also indicated. A metal rain cap is located at a distance of 45.42" from the radome base as indicated by the large tic mark extending above the horizontal axis. The location of the bulkhead is indicated by the other vertical tic mark at 5.32".

Several design constraints were imposed:

(1) The outer shape of the radome was specified as a tangent ogive with base diameter $D_{os} = 16.00"$ and radius of curvature $R_{os} = 148.03"$.

(2) The radome wall thickness could not exceed 0.75" lest the antenna would not gimbal.

(3) The minimum substrate thickness was 0.35" for structural rigidity and strength.

(4) The minimum ablator thickness must be compatible with expected ablation during flight ($\sim 0.045"$ maximum) and manufacturing techniques.

The ablative radome design of Figure 2-1 was arrived at by examining the plane wave transmission properties of several two-layer flat panel designs. From these data, the basic two-layer radome wall design was selected for further refinement using a three-dimensional computer-aided radome analysis [1]. The radome design parameters consisted of substrate and ablator thicknesses at a single frequency. The designs were compared on the basis of boresight error slope (BSES), assuming reasonable ($< 1.0$ dB) transmission loss. The effects of tapered ablator thickness were also studied.
Figure 2-1. Ablative Radome Geometry.
Preliminary performance data for three fused silica radome designs are present in the last section.

2.2 Flat Panel Analysis

The plane wave transmission properties of the two-layer flat panels are shown in Appendix A. Substrate ($\varepsilon_r = 4.50$, $\tan\delta = 0.008$) thicknesses range from 0.25" to 0.600". The ablator thicknesses are shown in each legend. The lefthand ordinate (goes with upper set of curves) is power transmittance for perpendicular polarization (always worse than for the incident electric-field parallel to the plane of incidence). The abscissa is angle of incidence. The righthand ordinate (lower set of curves) is delta insertion phase delay; i.e., $\Delta IPD = IPD_{\perp} - IPD_{\parallel}$.

The flat panel design criteria used were: (1) transmittance greater than 80% over the range of incidence angles expected in the radome; (2) $\Delta IPD = 0$ over the same range.

For the tangent ogive shape of Figure 2-1, the largest incidence angle encountered is approximately 72° as determined by drawing a ray normal to the center of the aperture antenna to the tip of the radome. The angle between the ray and the normal to the inner wall is the incidence angle. The lowest angle of incidence is determined by gimbaling the aperture to the expected limit and measuring the normal ray incidence angle on the radome wall. This approximate estimating procedure yields a range of $30^\circ < \theta_i < 72^\circ$.

Examination of the data in Appendix A shows that the thicker substrate designs yield better transmittance and $\Delta IPD$ performance. The performance would be best for a 1/2-wave wall of substrate only (.622" @ $\theta_D = 72^\circ$). But it is not possible to conclude from the flat panel results if the 0.550" substrate design will yield better radome BSES performance than the 0.600" substrate design; hence, the need for the following 3-D radome analysis.
2.3 Ablative Radome Analysis for Uniform Wall Thickness

The computed radome performance for five combinations of substrate and ablator thicknesses are shown in Appendix B. Data for both pitch and yaw planes are shown. The ordinates of interest are boresight error slope (BSES), boresight error (BSE), and gain. The abscissa in every case is radome gimbal angle. The legend identifies the five different designs.

Examination of the data in Appendix B clearly reveals that the optimum ablative radome design has a substrate thickness of 0.600" and ablator thickness of 0.060".

2.4 Ablative Radome with Tapered Wall Thickness

The computed performance of six ablative radome designs having tapered wall thickness are presented in Appendix C. The optimum design (designated hereafter as Design 1) is also shown as the standard of comparison. The performance of a over-dimensioned prototype design (Design 0) from which the optimum design and any tapered designs will be machined is also shown.

The tapered designs considered are further identified in Table 2-1. The thickness taper is linear with respect to the axial radome coordinate. The first thickness given in the table is the thickness in the layer at the base of the radome; the second thickness is the thickness of the layer at the tip. Only one thickness is given for the uniform thickness. Table 2-1 also identifies the three fused silica radome designs to be discussed later.

The ablator of Design 6 is tapered to be thicker at the tip as anticipated at the initial point of flight. Design A shows the performance at a later time in flight -- and approaches the optimum design performance (Design 1). Design D shows what happens to the performance if the ablator thickness erodes to zero at the tip.
<table>
<thead>
<tr>
<th>Design No.</th>
<th>Base Diameter</th>
<th>Substrate Base/Tip</th>
<th>Ablator Base/Tip</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>16.10</td>
<td>.690</td>
<td>.110</td>
<td>First Delivered Prototype</td>
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<tr>
<td>1</td>
<td>16.00</td>
<td>.600</td>
<td>.060</td>
<td>Optimum Design</td>
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<td>6</td>
<td>16.02</td>
<td>.600</td>
<td>.070/.110</td>
<td>Tapered Ablator</td>
</tr>
<tr>
<td>A</td>
<td>16.004</td>
<td>.600</td>
<td>.062/.070</td>
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</tr>
<tr>
<td>D</td>
<td>16.00</td>
<td>.600</td>
<td>.060/.000</td>
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</tr>
<tr>
<td>E</td>
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<td>.600/.620</td>
<td>.060/.000</td>
<td>Tapered Ablator &amp; Substrate</td>
</tr>
<tr>
<td>F</td>
<td>16.00</td>
<td>.600/.640</td>
<td>.060/.000</td>
<td>Tapered Ablator &amp; Substrate</td>
</tr>
<tr>
<td>G</td>
<td>16.00</td>
<td>.600/.660</td>
<td>.060/.000</td>
<td>Tapered Ablator &amp; Substrate</td>
</tr>
<tr>
<td>H</td>
<td>16.00</td>
<td>.730</td>
<td>0.</td>
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<td>I</td>
<td>16.00</td>
<td>.710</td>
<td>0.</td>
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</tr>
<tr>
<td>J</td>
<td>16.00</td>
<td>Tailored</td>
<td>0.</td>
<td>Tailored SiO₂</td>
</tr>
</tbody>
</table>
Appendix D shows the effects on radome performance caused by tapering the substrate material for the case where the ablator has eroded to zero thickness at the tip. Designs 1 and D are shown for reference. These data merely show that substrate thickness taper can also be used as a degree of freedom in the design process.

2.5 Fused Silica Radome Design

Computed performance data for three fused silica radome designs are shown in Appendix E. Ablative Designs 0 and 1 are also shown for reference.

Design H is the fused silica "blank" as received from the manufacturer. It would be rough machined to a wall thickness of 0.73" ± 0.010". Its performance is closer to the optimum ablator performance than the other two SiO₂ designs.

Making the SiO₂ wall thickness thinner (Design I) does not help.

An asymmetrical wall thickness prescription (Design J) does not help the performance of the fused silica radome, where the average wall thickness is approximately 0.67".

The design of the fused silica radome is not complete.
3.1 Introduction

A computer-aided simulation of the boresight error measurement procedure and facility was carried out to quantify the effects of separation distance, wave reflections from anechoic chamber boundaries, and frequency drifts.

The 3-D radome analysis program used earlier to design the ablative and ceramic radomes was modified to include the near-field and reflection effects as illustrated in Figure 3-1(b). Waves emanate from the source antenna in the directions indicated by the rays (one arrowhead). These direct rays impinge on the radome as shown. Note that the angles of incidence on the radome wall for these rays are different than those of a true plane wave (horizontal rays).

Some of the rays emanating from the source antenna strike the walls, floor, and ceiling of the chamber and are reflected onto the radome. These reflections can be conveniently included using image sources, one for each boundary of the chamber (4 total). Each image is mirrored into the associated boundary and is given strength \( E' \) with respect to the actual source strength \( E_0 \) according to

\[
\frac{E'}{E_0} = 10^{-\frac{R_{\text{dB}}}{20.}} \quad (3-1)
\]

where \( R_{\text{dB}} \) is the reflectivity of the chamber wall in decibels. The reflectivity is assumed to be independent of incidence angle.
(a) View of source antenna at far end of chamber.

(b) Top view of chamber showing one image source.

Figure 3-1 Geometry of boresight error measurement simulation.
3.2 BSE Algorithms

Three BSE measurement procedures or algorithms were simulated: null seeker, offset 1-point method, and 2-point method. In the null seeker method, the computation is done such that the source is moved around until nulls are obtained in each Δ/c signal channel of the monopulse antenna. The direction to the source when it is in the null position is defined as the boresight error.

Figure 3-2 shows tracking functions computed for the radome/antenna combination under test (RAUT), where the tracking functions in elevation and azimuth are defined by

\[ f_{EL} = \Im \left( \frac{\Delta EL}{\sum} \right) \]  

(3-2a)

\[ f_{AZ} = \Im \left( \frac{\Delta AZ}{\sum} \right) \]  

(3-2b)

Four computed tracking functions are shown in Figure 3-2 as indicated on each graph. The tracking functions are graphed versus the angle θ from boresight in a diagonal plane defined by \( x_A = y_A \) in antenna coordinates. Without the radome, \( f_{EL} \) and \( f_{AZ} \) are almost identical so that only one solid graph is shown for both functions. When the radome is placed over the antenna and aligned with the true antenna boresight (Pitch = 0°, Yaw = 0°), the tracking functions are slightly different as indicated by the AZ(0°,0°) and EL(0°,0°) graphs. Note also that the slopes of these functions (monopulse error slope MES) are different but are approximately equal to the MES of the antenna without the radome. Finally, the offset dash graph EL(6°,0°) of Figure 3-2 shows the elevation tracking function when the radome is pitched up by 6°; \( f_{AZ} \) is essentially the same as for the (0°,0°) case.
FIGURE 3-2: TRACKING FUNCTIONS IN ELEVATION (---) AND AZIMUTH (-----) PLANES WITH AND WITHOUT (-----) RADOME FOR (0,0) ORIENTATION.
The BSE algorithms can be explained using the EL(6°,0°) graph of Figure 3-2. The null seeker algorithm finds the zero-crossing of the tracking functions f_{EL} and f_{AZ}: f_{EL} = 0 at -11.5 mrad; f_{AZ} = 0 at 0 mrad in Figure 3-2. The 2-point method uses the values of each tracking function computed at only two points at ±20 mrad to generate a linear estimate of each tracking function, and, hence, an estimate of where the zero crossings occur.

The offset 1-point method uses the value of each tracking function as measured at the angle-off-boresight of -25 mrad. This single value (Point A in Figure 3-2), combined with the MES yields the following linear tracking model

\[ f_{EL} = MES_{EL} \theta_{EL} + B_{EL} \]  

(3-3)

where the ordinate intercept B_{EL} is given in terms of the measured tracking function at the known angle \( \theta = -25 \) mrad by

\[ B_{EL} = f_{EL}(-25 \text{ mrad}) - MES_{EL}(-25 \text{ mrad}) \]  

(3-4)

The zero-crossing, or BSE_{EL}, is then obtained by setting Eqn. (3-3) equal to zero and solving for \( \theta_{EL} \); i.e.,

\[ \theta_{EL} = \frac{f_{EL}(-25 \text{ mrad})}{MES_{EL}} = \text{BSE}_{EL} \]  

(3-4)

A similar treatment holds for the azimuth tracking function.

The on-axis 1-point method of BSE measurement or computation uses the single value of the tracking function obtained when the target (source) is located on the true boresight of the antenna. The value of f_{EL} is indicated by Point B in Figure 3-2. The boresight error is then given by Eqn. (3-4).
In both 1-point methods, the monopulse error slope that should correctly be used is the slope of the tracking function for that particular radome orientation. In practice, the true slope is not used; instead, the MES of the antenna without the radome is used in Eqn. (3-4). The significance of this source of error is investigated in the following presentation of the BSE measurement simulation.

A comparison of the results obtained in the simulation of the three BSE algorithms is shown in Figure 3-3 for scan of the radome in the pitch plane. A true plane wave (source at \( R = \infty \)) was incident on the radome, and no reflections from the chamber boundaries were allowed \( (R_{dB} > 100 \text{ dB}) \). The graphs show excellent agreement between the null seeker and 2-point methods. Discrepancies are noted for the 1-point method. In what follows, the null seeker results for \( R = \infty \) and no reflections are considered to be the true data.

### 3.3 Effects of Distance

The effects of the distance \( R \) of separation between the source antenna and the monopulse seeker AUT are presented in Appendix F for each BSE algorithm. Distances of \( R = 20', 30', 40', \) and \( R = \infty \) are used. No reflections are included.

The simulation results of Appendix F indicate that the distance of separation is a significant source of error in BSE measurements. For example, for a 20' separation, a maximum error of 5 mrad is observed using the null seeker or 2-point method. Oscillatory errors are observed using the offset 1-point method. The errors in gain (radome loss) are minor for all three methods.

The antenna used in the simulation has a value of \( D^2/\lambda = 71.6 \). The 20' separation corresponds to 3.35 \( D^2/\lambda \). The radome value of \( L_1^2/\lambda = 558 \) yields only 0.43 \( L_1^2/\lambda \) for the 20' separation. (The radome length \( L_1 \) used is the radome length from the gimbal point to the tip.) These considerations indi-
Figure 3-3. Comparisons of Boresight Errors Computed Using Three Different Algorithms.
cate that any rules of thumb for separation distance in BSE measurements should utilize radome length rather than antenna diameter.

3.4 Effects of Reflections

The effects of wave reflections from the anechoic chamber walls, floor, and ceiling are presented in graphical form in Appendix G. Only the 20' separation distance was considered since it corresponds roughly to the length of the chamber of interest. Chamber reflectivities of 20 dB, 30 dB, and 100 dB were considered.

The BSE data in Appendix G shows that the 2-point method is least sensitive to reflections, and that the offset 1-point method is the most sensitive. And in some instances the reflections tend to compensate for the 20' separation distance.

The gain data in Appendix F shows that chamber reflections have a significant effect on this parameter.

3.5 Effects of Frequency

The simulation was performed to determine the effects of small (±1%) frequency drifts on measured BSE for the case of R = 30' separation and reflectivity of 30 dB. The computed results are presented in Appendix H for all three BSE algorithms.

The data in Appendix H show that small frequency drifts are a significant source of error in BSE measurements. Therefore, frequency-stabilized (phase locked) sources should be used.
4.1 Ablative and Ceramic Radomes

Ablative radomes consisting of a load-bearing fibreglas substrate and a thin fibre-loaded Teflon ablator layer can be designed to have electrical performance comparable to what can be expected for ceramic radomes; however, the change in boresight error slope caused by ablation can be significant, and this effect must be taken into account in the radome design.

The investigation of the ceramic radome is not complete, and no conclusions can be drawn concerning the advantages of asymmetrical wall thickness. It is recommended that this work be completed, including an investigation of the effects of asymmetrical aerodynamic heating on BSE.

4.2 BSE Measurement Simulation

The computer-aided simulation shows that separation distance, reflections, and frequency drifts are all significant sources of error in BSE measurements. Also, the results obtained depend on the algorithm or procedure used to compute or measure BSE. The offset 1-point method yields the most variable results and is deemed unsuitable for use. The on-axis 1-point method has not been fully studied, and it is recommended that this be done using the simulation already developed.
REFERENCES

APPENDIX A

FLAT PANEL ANALYSIS RESULTS
Figure A-1. Substrate Thickness = 0.25".
Figure A-2. Substrate Thickness = 0.30".
Figure A-3. Substrate Thickness = 0.35".
Figure A-4. Substrate Thickness = 0.40".
Figure A-5. Substrate Thickness = 0.50".
Figure A-6. Substrate Thickness = 0.55".
Figure A-7. Substrate Thickness = 0.60".
APPENDIX B

UNIFORM ABLATIVE RADOME PERFORMANCE
LEGEND

1. 350°  .400°
2. 550°  .150°
3. 550°  .200°
4. 600°  .060°
5. 560°  .250°
GIMBAL ANGLE (DEGREES)

YAW ERROR (MILLIRADIANS)

LEGEND

1. 350°, 400°
2. 550°, 150°
3. 550°, 200°
4. 600°, 060°
5. 500°, 120°
APPENDIX C

TAPERED ABLATOR RADOME PERFORMANCE
APPENDIX D

TAPERED SUBSTRATE RADOME PERFORMANCE
APPENDIX E

FUSED SILICA RADOME PERFORMANCE
LEGEND

\( I \)

\( H \)

\( I \)

\( J \)

\( O \)

---

YAW ERROR SLOPE (PERCENT)

GIMBAL ANGLE (DEGREES)
LEGEND

- NULL SKR, R = ∞
- " R = 40', 100 dB
- " R = 30', 100 dB
- " R = 20', 100 dB

PITCH ERROR (MILLIRADIANs)

GIMBAL ANGLE (DEGREES)

PITCH FGHZ, LET2, 20 FT, 100 DB REF, 16X16 AUT, .500 SUB, .250 ABL. 7-17-83
LEGEND

1. Point, \( R = \infty \)

- \( R = 40', 100\text{dB} \)
- \( R = 30', 100\text{dB} \)
- \( R = 20', 100\text{dB} \)

**Diagram:**

- **GIMBAL ANGLE (DEGREES):** 0, 5, 10, 15, 20, 25, 30
- **PITCH GAIN (DECIBELS):** -3.0, -2.8, -2.6, -2.4, -2.2, -2.0, -1.8, -1.6, -1.4, -1.2, -1.0, -0.8, -0.6, -0.4, -0.2, 0.0

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67
APPENDIX G

EFFECTS OF REFLECTIONS ON BSE ALGORITHMS
LEGEND

- NUL SKR, R=∞
- R=20', 100dB
- R=20', 50dB
- R=20', 20dB

PITCH GAIN (DECIBELS)

GIMBAL ANGLE (DEGREES)
LEGEND

Z-Point, R=∞

R=20', 100dB

± 3dB

± 20dB

PITCH GAIN (DECIBELS)

GIMBAL ANGLE (DEGREES)
LEGEND

1 Point, R=∞

R=20', 10dB

R=20', 20dB
LEGEND

2-Point, $R = 0, f_o$

$R = 20, 30 \text{ ft}$

$1.0f_o$

$.99f_o$
LEGEND

2-Point, \( r = \infty, f_0 \)

" \( r = 20', 36\& 38, f_0 \)

" \( 1.04 f_0 \)

" \( 0.97 f_0 \)