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FINAL REPORT

PERFORMANCE OPTIMIZATION

OF

ON-LINE STATE ESTIMATION

AND

SECURITY ASSESSMENT

Prepared for

Bonneville Power Administration
Portland, Oregon

by

School of Electrical Engineering
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I. EXECUTIVE SUMMARY

1.1 General

This Final Report corresponds to the work conducted under Contract 14-03-6056N (Georgia Tech Project E21-681). The primary objective of the original proposal was to extend and refine the results of work under an earlier contract (No. 14-03-5098N) and an NSF sponsored project on parameter estimation whereby BPA was a "cooperating industry." The tasks established for this concentrated on three primary topics:

- Main grid network tuning
- On-line implementation procedures
- External equivalent validation

These tasks were written under the assumption that SCADA information would be readily available for actual testing. As it turned out, this was hardly the case except at the final days of the project. This, however, did not diminish the usefulness of the work, since several issues involved required a substantial amount of analytical work and simulation testing. And, since state estimation and security assessment are going through their evaluation test period starting October 1, 1977, the findings of our study should be quite relevant and timely.

1.2 Project Outputs

1.2.1 Main Grid Network Tuning

In the area of main grid network tuning considerable simulation
testing and program refinement were done in order to study on-line state estimation in a realistic environment. This environment consisted of errors present in line and transformer admittance values, anomalies in the status of some breakers, incomplete knowledge of the accuracy of measurements (i.e. errors in the values of the standard deviations of measurement errors), and others. Here, the parameter estimation program proved to be very valuable. The preliminary study conclusions, which, during the summer of 1977 were fully substantiated using actual data, were:

-- Performance of on-line state estimation using the Sequential State Estimator (SSE) or the Weighted Least Squares (WLS) estimator, without parameter estimation is unacceptable from the statistical or the practical points of view.

-- The Decoupled State/Parameter estimator (DSP), which is a combination of two WLS estimators, one for state and the other for parameter variables, is capable of improving parameter estimates and state estimates and state estimator performance to statistically and practically acceptable levels.

-- Data anomalies which were few but significant, consisted mainly of:

-- incorrect breaker status reporting because of lack of telemetry of disconnect switch status
improper calibration of certain measuring instruments

1.2.2 On-Line Implementation Procedures

Effort in this area consisted in the development of procedures for a rational evaluation of the frequency of updates of on-line state estimation, security monitoring and bus-load forecasting programs. In essence, the level of bus load forecast uncertainty is translated into corresponding uncertainties in line flows and bus voltages under normal and contingency conditions. On that basis, the system can adaptively decide whether state estimation or security monitoring are needed in the next cycle. In the mean time, when state estimation is not called for, actual raw measurements are monitored and compared with forecasted values. This provides another input into this adaptive decision process.

1.2.3 External Equivalent Validation

A substantial portion of the effort went into the study of external network equivalents for on-line contingency analysis purposes. Because of the complexity of BPA's external system, and the fact that external system topology and operating conditions are not available on a continuous basis, the conclusion was made to the effect that the best source of information on external systems should come from SCADA. Here SCADA information can be used in two respects:

1. To provide information on the validity of any postulated external equivalent representation by monitoring system power flows before and after any switching operation.
2. To construct an empirical equivalent based on pre- and post-switching information.

In our effort, program EQUIV was developed for the purpose of constructing the empirical equivalent. Given any postulated boundary bus interconnection, this program will compute admittance parameters of the connecting equivalent lines based on pre- and post-switching information using a weighted-least-squares approach, or, alternatively, a constrained optimization approach that guarantees upper limits on errors in contingency analysis. This effort culminated in the Ph.D. dissertation of G. Contaxis.

1.3 Recommendations

a. A thorough evaluation procedure should be developed to address the following issues:
   -- statistical and practical performance of state and parameter estimation over a reasonable operating period
   -- validity of contingency analysis in on-line security monitoring

b. Program EQUIV should be implemented in a study-mode to determine its effectiveness and applicability using real system data.

c. If EQUIV does not produce the desired results because of the complexity of the issue, some further work will be required which is based on some exchange of data
with neighboring systems, as well as, new developments in equivalencing techniques.
II. MAIN GRID NETWORK MODEL TUNING

2.1 Introduction

The theory of state estimation for power systems is based on a set of assumptions whose validity needs verification and, in some cases, is questionable. These assumptions are the following:

a. Network topology is known.
b. Line, transformer and shunt-capacitor admittances are exactly known.
c. Measured quantities (KV, MW, and MVAR's) are accurate within errors whose means are zero and whose standard deviations are known. Furthermore, the errors are Gaussian distributed.

Under these assumptions, one can formulate the state estimation problem as follows:

Given

\[ z = h(x,p) + v \]  \hspace{1cm} (2.1)

where

\[ z \triangleq \begin{bmatrix} z_1 \\ \vdots \\ z_m \end{bmatrix} \triangleq \text{vector of measured quantities} \]
state vector (i.e. voltage magnitudes and angles of all buses)

\[ \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \]

parameter vector of admittances

\[ \begin{bmatrix} p_1 \\ \vdots \\ p_k \end{bmatrix} \]

error vectors whose mean is zero and whose covariance matrix is \( R \) (\( R \) is diagonal, usually).

Assuming that \( p \) is known exactly (assumptions (a) and (b)) and that \( R \) is true (assumption (c)), the best estimate \( \hat{x} \) of \( x \) minimizes

\[
J = \sum_{i=1}^{m} \left( \frac{z_i - h_i(x,p)}{\sigma_i} \right)^2
\]

\[
= (z-h(x,p))^T R^{-1} (z-h(x,p))
\]

One way to obtain a solution to this problem is through the iterative algorithm

\[
\hat{x}^{k+1} = \hat{x}^k + \sum_{k} H_k R^{-1} (z-h(\hat{x}^k,p))
\]

where

\[
k = 1, 2, \ldots
\]

\[
H_k = \left. \frac{\partial h}{\partial x} \right|_{\hat{x}^k}
\]

(2.4)
\[ \sum_{k} = H_k^T R^{-1} H_k \quad . \quad (2.5) \]

It is known that \( J \) is essentially a random variable which is \( \chi^2 \) distributed with \((m-n)\) degrees of freedom, i.e.

\[ E(J) = m-n \quad (2.6) \]

Obviously,

\[ E\left[\frac{J}{m}\right] = \frac{m-n}{m} = 1 - \frac{n}{m} < 1 \quad (2.7) \]

Because of its simplicity, the so-called \( \chi^2 \)-test can be easily implemented by simply computing \( J/m \) and checking the level of confidence in the test from published tables. A successful \( \chi^2 \)-test verifies the statistical validity of the estimation process.

Now one can process raw data without close regard to the validity of assumptions (a), (b) and (c) and obtain a practically acceptable \( \chi^2 \)-test. Suppose, for example, in a given test

\[ \frac{J}{m} = 4 \quad . \]

This can be interpreted in the sense, that, on the average, calculated quantities are \( \sqrt{4} = 2 \) standard deviations from measured quantities. Now, since measured quantities are, on the average, one standard deviation from true values, one concludes that the calculated quantities are worse than the measurements themselves. However, if the standard deviations of
error is very small (i.e. measurements are very accurate), then the result with \( J/m = 4 \) may still be useful in on-line applications where excessive accuracy is not called for. As a result one can define a practical upper limit on the value of \( J/m \) based on desired accuracy of the results.

The reasons for large values of \( J/m \) are strongly tied to the validity of assumptions (a), (b) and (c) above. Violation of assumption (a) can cause severe estimator errors. Violations of (b) will have varying effects depending on the accuracy of \( p \)-values. Finally, violations of (c) can cause severe errors if bad-data were present, or of minimal effect if the errors in the \( \sigma \)'s are small.

The objective of main grid network tuning is to comprehensively study uncertainties arising from violations of assumptions (a), (b) and (c) and to provide necessary solutions. In the next section we provide the basic methodology for approaching this problem. The following sections contain additional details, as well as representative results.

2.2 Methodology

In this section the theoretical basis of the methodology is outlined. Basically, we are concerned with two basic effects: The first is that of small errors in the parameters and noise variances, and the second is that of large parameter (status) errors and bad-data errors.

2.2.1 Small Parameter Errors

In this case we assume that the parameters \( p \) are known within a certain level of accuracy. Published references estimate parameter errors in the order of 5-10%. Hence, one can write
\( p^o = p + \omega \) \hspace{1cm} (2.8)

where \( p^o \) is the vector of given parameters, \( p \) is the vector of true (but unknown) parameters and \( \omega \) is the error vector. Assume that \( \omega \) has a mean of zero and a diagonal covariance matrix \( M \).

The problem of state and parameter estimation is now posed as follows:

Given

\[ z = h(x,p) + v \] \hspace{1cm} (2.9)

\( p^o = p + \omega \) \hspace{1cm} (2.10)

find \( \hat{x} \) and \( \hat{p} \) which minimize

\[ J_p = (p^o - p)^T M^{-1} (p^o - p) + (z - h(x,p))^T R^{-1} (z - h(x,p)) \] \hspace{1cm} (2.11)

For this problem the algorithm of Equation (3) becomes

\[ \begin{bmatrix} x_{k+1}^T \\ p_{k+1}^T \end{bmatrix} = \begin{bmatrix} x_k^T \\ p_k^T \end{bmatrix} + \sum_{k} \begin{bmatrix} H_{k}^{-1} & 0 \\ 0 & G_{k}^{-1} \end{bmatrix} \begin{bmatrix} H_{k}^{-1}(z - h(x_k, p)) \\ G_{k}^{-1}(z - h(x_k, p)) \end{bmatrix} \] \hspace{1cm} (2.12)

where

\[ H_k = \frac{\partial h}{\partial x} \bigg|_{x_k, p_k} \] \hspace{1cm} (2.13)

\[ G_k = \frac{\partial h}{\partial p} \bigg|_{x_k, p_k} \] \hspace{1cm} (2.14)
Because of its added complexity, this algorithm was used only during the initial phases of the study on parameter estimation. Later on, a decoupled algorithm was used, whereby the off-diagonal blocks in $\tilde{Y}_k$ were artificially set to zero. The resulting algorithm is as follows:

**Step 1:** Given $p^k$, $k=0,1,\ldots$, compute $\hat{x}^k$ iteratively as follows:

\[
\hat{x}^{k,i+1} = \hat{x}^{k,i} + \hat{\Sigma}^{-1}_{k,i} (H_k^T R^{-1}(z-h(x^{k,i},p^k)),
\]

\[i = 0,1,\ldots\]

**Step 2:**

\[
p^{k+1} = p^k + \sum_{k} \hat{C}_k^T R^{-1}(z-h(x^{k},p^k)) + M^{-1}(p^0 - p^k)
\]

where

\[
H_{k,i} = \frac{\partial h}{\partial x}|_{x^{k,i},p^k}
\]

\[
\hat{\Sigma}_{k,i} = H_k^T R^{-1} H_k, i
\]

\[
\hat{\Sigma}_{pk} = G_k^T R^{-1} G_k + M^{-1}
\]

In simple words, the decoupled algorithm consists first of solving iteratively for the state estimate given an estimate of the parameters.
\(^k\) \(p\), and second, of solving for the parameters given the state estimate of the first step. This algorithm is convergent to the true minimum of \(J_p\) (or at least one of its local minima which is highly unlikely on basis of experience). It has the advantages of

- Reduced storage requirements, and
- Separation of state estimation from pure parameter estimation

The decoupled approach has been extended to cases where several snapshots of the system under varying operating conditions are used. Details of this are given in Ref. (1).

### 2.2.2 Detection of Large Measurement and Parameter Errors

The literature contains several works on the subject of bad-data detection. The primary concern there is in the detection of large measurement errors. In at least one work, \(^{13}\) the problem of detecting large parameter errors is considered. Invariably, however, the detection and identification of bad-data is based on the analysis of the so called residuals. The residual of measurement \(z_i\) is defined as

\[
 r_i = \frac{z_i - h_i(\hat{x}, \hat{p})}{\sigma_i} \tag{2.16}
\]

The contention here is that if \(z_i\) is a bad measurement then its residual is large. However, other residuals will also be large. In order to identify the truly bad measurement, several trials of removing, in turn, measurements with large residuals will have to be attempted until the performance index passes the statistical test of validity. The problem
becomes more complicated when parameter estimation is also attempted. In that case, the parameter residuals associated with parameter $p_i$ can be computed as

$$r_{p_i} = \frac{\hat{p}_i - p_i}{\sqrt{N_{ii}}} \tag{2.17}$$

And, consequently, parameters with large errors may conceivably be identified. Cause of these errors could be related to errors in status quantities leading to wrong network topology, or actual input data errors.

This method of analyzing residuals (as well as its variants in the literature), does not take into account the possibility of pre-estimation analysis of data.

There are two aspects of pre-estimation data analysis. The first consists of limit checking whereby obviously erroneous quantities are discarded. And the second is that of data consistency analysis. In all production programs, limit checking is most probably always utilized. There may also be some simple versions of data consistency analysis.

In our version of data consistency analysis, we strive at accomplishing the following:

- Classify input measurements as (a) valid, (b) suspect, or (c) raw. Valid measurements are those which pass a consistency test. Raw measurements are those for which a consistency test is not possible because of lack of local measurement redundancy. And suspected measurements are those which fail the consistency test.
• Adjust valid measurement error variances to more accurate values as a result of the consistency tests.

• Design the consistency tests to be insensitive to parameter errors in order to truly isolate the effects of bad measurements from those of parameter errors.

• Identify possible status errors from the values of measured quantities. An open line which is erroneously assumed to be closed will show zero, or very small power flow.

In Figure 1 we show an overall flow chart of the process of pre-estimation data consistency analysis. Examples of consistency sets actually used are

• Sum of real flows minus loss estimate on both ends of a line, or

• Sum of flows minus injection at a bus.

In each case the sum of measured quantities should add up to zero (in the absence of errors), or to the sum of the errors. The variance of the sum of errors is the sum of the corresponding variances, i.e.

\[ \sum_{i \in S_j} z_i = \sum_{i \in S_j} v_i \]

\[ E(\sum_{i \in S_j} v_i)^2 = \sum_{i \in S_j} \sigma_i^2 + \sigma_j^2 \]
FOR ZERO FLOWS
TEMPORARILY DISCARD
THEM AS WELL AS
BUS INJECTIONS ON
BOTH SIDES

CHECK FOR ZERO
FLOWS ON CLOSED
LINES OR TRANSFORMERS

SCAN THROUGH BUSES
TO IDENTIFY
CONSISTENCY SETS

SET
CONSISTENT?

YES
FLAG ALL SET MEMBERS AS VALID.
COMPUTE SET RESIDUAL.

NO

FLAG PREVIOUSLY RAW
MEASUREMENTS IN SET AS
SUSPECT. DO NOT FLAG
VALID DATA AS SUSPECT

COMPUTE VARIANCE
MULTIPLIER

RETURN

FIGURE 2.1 Flow Chart Indicating Major Functions in
Pre-Estimation Data Analysis
where $S_j$ is the $j$th consistency set. By knowing $(\sum_{i \in S_j} v_i)^2$, one can evaluate the accuracy of the measurements involved, or, effectively the accuracy of $\sigma_i^2$, $i \in S_j$. In the actual program real power and reactive power consistency sets are treated separately. Defining

$$S_P \triangleq \{ S_{P1}, \ldots, S_{P_k} \}$$

A set of real power consistent sets

and

$$S_Q \triangleq \{ S_{Q1}, \ldots, S_{Q_m} \}$$

A set of reactive power consistent sets

Then the real and reactive measurement variance multipliers are computed as follows:

$$\alpha_P = \frac{\sum_{j=1}^{\ell} \left( \sum_{i \in S_{Pj}} v_i \right)^2}{\sum_{j=1}^{\ell} \alpha_{Pj}^2}$$  \hspace{1cm} (2.18)$$

$$\alpha_Q = \frac{\sum_{j=1}^{p} \left( \sum_{i \in S_{Qj}} v_i \right)^2}{\sum_{j=1}^{p} \alpha_{Qj}^2}$$  \hspace{1cm} (2.19)$$

Consequently, all real (reactive) flow and injection measurement variances are multiplied by $\alpha_P(\alpha_Q)$. This should ensure that measurement variances are, at least, closer to their true values. For example, if
\[ \alpha_p = .5 \], then, on the average, the variances of error of real quantities are 1/2 the given values as evidenced by simple consistency analysis.

Having done pre-estimation consistency analysis, state estimation is performed, and the residuals are computed. A measurement is discarded if it is not classified as valid and if it has large residual. Finally, parameter estimation is conducted and parameter and measurement residuals are analyzed. A flow chart for this process is shown in Figure 2.

2.2.3 Discussion

In our assessment, one of the most significant steps in the above tuning process is that of consistency checking yielding adjusted values for measurement variance values. Here we have an independent means to evaluate the accuracy of incoming measurement data. The consistency sets are chosen to minimize the effect of parameter errors on the pre-estimation bad data detection process. By knowing the accuracy of incoming data, we achieve two primary goals:

(a) \( \chi^2 \)-tests will have their full statistical meaning. Thus with the adjusted variances the performance index \( J \) should be less than 1.0.*

b. The level of accuracy of parameter estimates can be properly assessed. If the measurements are very accurate, then accurate parameters will result. On the other hand, if the measurements are very inaccurate (large variances), then

*Note that one can easily "cheat" in \( \chi^2 \)-testing by claiming that actual measurement variances are larger than given values.
PERFORM PRE-ESTIMATION
CONSISTENCY ANALYSIS
AND IDENTIFY SUSPECTED
MEASUREMENTS

PERFORM STATE ESTIMATION
AND DISCARD SUSPECTED
MEASUREMENTS WITH VERY
LARGE RESIDUALS

PERFORM PARAMETER ESTIMATION
AND DISCARD ADDITIONAL
MEASUREMENTS WITH LARGE
RESIDUALS. IDENTIFY
SUSPECTED PARAMETERS
FOR OFF-LINE STUDY

START

STOP

FIGURE 2.2 Steps Involved in Overall Process
of Main Grid Network Tuning
parameter estimates may be poor although the performance index may be good.

Improvements in parameter estimates can be achieved by processing several snapshots of the system at different operating conditions. This will tend to filter out more of the measurement error. The main word of caution here is that if measurement errors are actually bias errors over a wide range of measurement values, then a single snapshot is as good as several snapshots for parameter estimation purposes.

2.2.4 Simulation Results

A. Generation of Simulated SCADA Data Base

Due to the incompleteness of the real-time data base at BPA, it was necessary to develop a simulator which generates adequate data for the functions required by state and parameter estimation programs. Specifications for that simulator include:

1. Ability to provide base-case power flow solutions associated with BPA's main-grid 91 (NET 1) bus system under varying operating conditions.

2. Ability to simulate values of noisy SCADA measurements with realistic values of the noise component.

3. Ability to simulate values of noisy network parameter values in accordance with anticipated error values.

4. Ability to introduce bad-data values to simulated SCADA measurements.

In developing this data-base, we followed the following steps:
a. Solve load-flow problem for the BPA 444-bus system for different sets of loads and generations obtained from a base-case plus random fluctuations.

b. Compute exact line flows, injections and voltage magnitudes for the 91-bus system for each case solved in (a).

c. Generate simulated SCADA measurements by adding Gaussian distributed random noise to quantities monitored by SCADA.

d. Corrupt network admittance parameters by adding Gaussian random errors to selected sets of parameters. Normally these sets consisted of
   1. Transmission line series susceptances.
   2. Transformer series susceptances.
   3. Transformer nominal primary voltage level (in p.u.).

The standard deviations of parameter errors are fractions of given values. These fractions are given as inputs to the program.

Three separate programs were added to generate the data base. The first is a standard load flow program. The second is a program to compute load-flow solutions of the 91-bus system. And the third is a program for generating noisy SCADA measurements and parameter values.

Normally, we obtain a set of 40 snapshots of the system under different load/generation conditions.
B. Testing of Parameter Estimation Program

At the outset, the following test objectives were formulated.

-- Verify state estimator statistical performance when no parameter errors are present. (Note the state estimator is integral to the parameter estimation program.)

-- Assess state estimator statistical performance in the presence of noisy parameter values. This should establish the need for performing parameter estimation.

-- Assess performance of parameter estimator in terms of improved state estimator performance.

-- Assess performance of parameter estimator in terms of improved values of estimated parameters.

-- Update and refine programs based on above assessments.

Efforts pertaining to these objectives are now discussed.

1. State Estimator Statistical Performance

In Table I we show results of state estimator performance for various tests. From this table the following conclusions can be drawn.

a. Statistical performance of the state estimator in the absence of parameter errors is in excellent agreement with theoretical predictions.

b. In interpreting these results, we say that estimates of measured quantities are, on the average, better than the measurements themselves.
TABLE I. Comparison of Actual Performance Index J with Its Theoretical Expected Value for 91-Bus System Using Simulated SCADA Measurements

<table>
<thead>
<tr>
<th>SNAPSHOT NUMBER</th>
<th>NUMBER OF MEASUREMENT</th>
<th>NUMBER OF STATES</th>
<th>PERFORMANCE INDEX J</th>
<th>EXPECTED VALUE OF J</th>
</tr>
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<tr>
<td>1</td>
<td>405</td>
<td>181</td>
<td>.639</td>
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<td>181</td>
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</table>
This improvement in accuracy will continually increase with increasing numbers of measurements.

In Table II, we show performance indices of the five measurement categories considered: real flows (T), reactive flows (U), real injections (P), reactive injections (Q), and voltage magnitudes (V). For each type, the corresponding performance index is computed on the basis of Equation (1) limited to measurements in each category. The main conclusion here is that all of these performance indices are less than 1.0 in value, which is to be expected.

2. State Estimator Performance with Noisy Parameters

In the presence of random errors in network parameter values, state estimator performance is degraded. Our interest here is in studying the extent of this degradation for reasonable assumptions on parameter error values. In this respect, six basic tests were performed. In each test a different combination of parameter errors was introduced. These combinations are summarized in Table III.

In Table IV, we show the various performance indices described earlier for each one of the given tests using a single snapshot of the system. From these tests we conclude the following:

a. State estimator statistical performance is degraded by factors ranging from 4.2 to 6.95. This is reflected in the factors shown on the last column of Table IV. What those factors indicate are the levels of degradation from the optimum standard
TABLE II. Performance Indices Associated With Individual Types of Measurements

<table>
<thead>
<tr>
<th>SNAPSHOT NUMBER</th>
<th>J_T  (143)</th>
<th>J_U  (139)</th>
<th>J_P  (31)</th>
<th>J_Q  (31)</th>
<th>J_V  (61)</th>
<th>J    (405)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.632</td>
<td>.700</td>
<td>.375</td>
<td>.409</td>
<td>.768</td>
<td>.639</td>
</tr>
<tr>
<td>2</td>
<td>.533</td>
<td>.630</td>
<td>.263</td>
<td>.347</td>
<td>.551</td>
<td>.534</td>
</tr>
<tr>
<td>3</td>
<td>.493</td>
<td>.694</td>
<td>.275</td>
<td>.278</td>
<td>.680</td>
<td>.557</td>
</tr>
<tr>
<td>4</td>
<td>.675</td>
<td>.774</td>
<td>.331</td>
<td>.287</td>
<td>.726</td>
<td>.661</td>
</tr>
<tr>
<td>5</td>
<td>.508</td>
<td>.636</td>
<td>.352</td>
<td>.324</td>
<td>.691</td>
<td>.554</td>
</tr>
<tr>
<td>6</td>
<td>.546</td>
<td>.655</td>
<td>.184</td>
<td>.262</td>
<td>.596</td>
<td>.542</td>
</tr>
<tr>
<td>7</td>
<td>.479</td>
<td>.689</td>
<td>.239</td>
<td>.225</td>
<td>.799</td>
<td>.561</td>
</tr>
<tr>
<td>8</td>
<td>.635</td>
<td>.622</td>
<td>.521</td>
<td>.273</td>
<td>.761</td>
<td>.613</td>
</tr>
<tr>
<td>9</td>
<td>.563</td>
<td>.757</td>
<td>.359</td>
<td>.280</td>
<td>.578</td>
<td>.595</td>
</tr>
</tbody>
</table>

LEGEND:

T - Real Flows
U - Reactive Flows
P - Real Injections
Q - Reactive Injections
V - Voltage Magnitudes

Numbers in parenthesis indicates total number of measurements in given category.
TABLE III. Errors Introduced in the Six Basic Tests. These Errors Correspond to Standard Deviations Leading Actually to Random Error Values.

<table>
<thead>
<tr>
<th>TEST NUMBER</th>
<th>TRANSMISSION LINE SUSCEPTANCE ERRORS</th>
<th>TRANSFORMER SUSCEPTANCE ERRORS</th>
<th>TRANSFORMER TAP SETTING ERRORS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5% of Value</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>2</td>
<td>0.0</td>
<td>5% of Value</td>
<td>0.0</td>
</tr>
<tr>
<td>3</td>
<td>0.0</td>
<td>0.0</td>
<td>2% of Value</td>
</tr>
<tr>
<td>4</td>
<td>5% of Value</td>
<td>5% of Value</td>
<td>0.0</td>
</tr>
<tr>
<td>5</td>
<td>5% of Value</td>
<td>0.0</td>
<td>2% of Value</td>
</tr>
<tr>
<td>6</td>
<td>5% of Value</td>
<td>5% of Value</td>
<td>2% of Value</td>
</tr>
</tbody>
</table>
TABLE IV. State Estimator Performance Indices in the Presence of Various Combinations of Parameter Errors.

<table>
<thead>
<tr>
<th>TEST NUMBER</th>
<th>( J_T )</th>
<th>( J_U )</th>
<th>( J_P )</th>
<th>( J_Q )</th>
<th>( J_V )</th>
<th>( J )</th>
<th>( \sqrt{J/J^*} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>23.01</td>
<td>8.4</td>
<td>4.9</td>
<td>3.9</td>
<td>1.98</td>
<td>11.98</td>
<td>4.65</td>
</tr>
<tr>
<td>2</td>
<td>17.77</td>
<td>6.44</td>
<td>3.12</td>
<td>8.46</td>
<td>1.04</td>
<td>9.53</td>
<td>4.15</td>
</tr>
<tr>
<td>3</td>
<td>2.95</td>
<td>21.1</td>
<td>2.26</td>
<td>9.46</td>
<td>4.244</td>
<td>9.832</td>
<td>4.21</td>
</tr>
<tr>
<td>4</td>
<td>41.55</td>
<td>14.12</td>
<td>6.70</td>
<td>12.96</td>
<td>2.35</td>
<td>21.374</td>
<td>6.22</td>
</tr>
<tr>
<td>5</td>
<td>25.22</td>
<td>27.58</td>
<td>7.87</td>
<td>14.86</td>
<td>4.89</td>
<td>20.85</td>
<td>6.14</td>
</tr>
<tr>
<td>6</td>
<td>37.67</td>
<td>28.76</td>
<td>9.50</td>
<td>26.98</td>
<td>5.15</td>
<td>26.74</td>
<td>6.95</td>
</tr>
</tbody>
</table>

\( J^* \) Theoretical expectation of \( J \) with no parameter errors. In this case, \( J^* = 0.552 \).
deviations of error produced by a state estimator with no parameter errors. In other words, the errors in the estimated values of the measurements are 4-6 times greater than the measurement errors themselves, on the average.

b. As expected, transformer tap-setting errors produced large errors in the estimates of voltage magnitudes and reactive flows and injections.

c. Errors in line and transformer susceptances produced large errors in estimates of real flows and injections.

d. In general, these simulated tests should provide a guide for the location of parameter errors in the real-life situation.

3. **Parameter Estimator Performance (I)**

Using the Decoupled State/Parameter Estimator, values of network parameters were changed so that a better state estimate is obtained. For each of the six test systems described in Table III, a single snapshot of the system was used as input data to the state/parameter estimation program. In Figure 2.3, we show the convergence characteristics of the performance index $J$. Typically, about 100 iterations or so were needed in all six test cases to bring $J$ to a value less than unity. Additional iterations further improved performance.

In Figures 2.4 and 2.5, the real flow and reactive flow performance indices $J_T$ and $J_U$ are plotted against iteration number, respectively. From all of these plots and the mass of data generated, the following
FIGURE 2.3 Convergence of Parameter Estimator for Various Assumptions on Parameter Errors
FIGURE 2.4 Real Flow Performance Index in Presence of Various Types of Parameter Errors

LEGEND: SEE FIG. 2.3

AVERAGE OF $J_T$ IN ABSENCE OF PARAMETER ERRORS

ITERATION NUMBER
LEGEND: SEE FIG. 2.3

FIGURE 2.5 Reactive Flow Performance Index in Presence of Various Types of Parameter Errors
conclusions are drawn:

a. Although convergence of algorithm is slow, performance steadily improves with additional iterations. Theoretically, in the limit, the exact optimum solution will be obtained.

b. After 100-120 iterations, state estimator performance is compatible with that of the parameter error-free situation. This implies that line-flow, injection, and voltage magnitude estimates are no far more reliable and accurate than those obtained with erroneous parameters.

c. Algorithm convergence rate seems to be strongly related to system size. With the smaller systems encountered in previous efforts, convergence was much faster. This raises two possibilities. The first is to decompose the system into a group of smaller systems and identify each one separately as an initial step and then identify the entire system in the usual way. The second is to incorporate the off-diagonal terms which are eliminated in the decoupling approach. The second approach was attempted and we are now in the final debugging stage. The first approach can be implemented with little effort. Personally, I would favor the first approach since it has a strong impact on bad-data selection.
d. Since parameter estimation is an off-line process convergence rates should not be of major concern as long as a reliable solution is obtained. Normally, the 120 iterations cited above took 10 minutes of execution time including all input/output requirements. This is reasonable for an off-line process which is not repeated very frequently.

4. **Parameter Estimator Performance (II)**

Here we discuss the important subject of ability of the parameter estimator to improve the accuracy of line and transformer parameter values. In a typical test case, we looked at the errors both in the given values and then estimated values of transmission line susceptances. The following was obtained.

a. On the average, the given errors were 3.97% of true values. Errors in parameter estimates were 2.17%. This is an improvement of a factor of 2.

b. In 63% of the individual line cases, the parameter estimates were better than the given values.

c. For the given values, 21 lines had errors between 5 and 10%. For the estimated values, only 11 lines had errors between 5 and 10%.

d. For the given values, 5 lines had errors greater
than 10%. No estimated value had an error
greater than 10%.

Thus, with one snapshot of the system, we obtained a substantial
improvement in the estimates of line parameters. The tendency here is
to greatly reduce errors whenever they are large. This is done at the
expense of increasing the error slightly for the more accurate values.
With several system snapshots at widely different operating conditions,
better improvements are to be expected.

5. Program Refinements

In our effort to arrive at the above encouraging results, several
program refinements were performed. Furthermore, substantial effort
went into some marginally successful efforts which were aimed at im-
proving convergence characteristics. In one case, a separate large
subroutine was written to estimate line or transformer parameters
strictly from measurements at both ends of the line or transformer. This
did produce some marginal results. In the present version of the program,
we have added the following features:

a. Improved output subroutines for both the state
   and parameter portions of the process.

b. Adequate incorporation of a-priori knowledge of
   the accuracy of various parameters.

c. Redimensioning the program to allow the use of
   all injection measurements. Normally, these
   measurements penalize storage requirements
greatly.
Obviously, in this whole process, a good deal of debugging was performed.

D. **Testing and Tuning of Sequential State Estimation Program**

As outlined in Ref. (1), tuning of the sequential state estimation program (SSE) depends on two factors. The first is the order in which measurements are processed, and the second is the weighting associated with each measurement variance, as well as, the "covariance" term $Q_E$ and $Q_F$.

In the tests that were conducted, the following objectives were sought:

- a. Improve statistical performance vis-a-vis the optimal case provided by state estimator used in conjunction with the parameter estimation program discussed earlier.
- b. Reduce solution time requirements.

These, and other aspects are now discussed.

1. **Sequential Ordering of Measurements**

Subroutine ORDER is designed to order buses in accordance with the ordering rules provided in Ref. (1).

Having ordered the buses, however, one is still faced with the order in which specific measurements are processed. The scheme that was in use in Ref. (1) consisted with a four-pass sequence as follows:

- **Pass No. 1**: Process all voltage measurements
- **Pass No. 2**: Process, at each bus, all near and far end line flow measurements
- **Pass No. 3**: Process injection measurements
- **Pass No. 4**: Process injection measurements
Obviously, in each pass all buses in the system are scanned in order. This scheme works very well whenever voltage and line flow measurements are sufficient to "observe" the entire system. In cases where those measurements cannot observe the system, the rules for ordering outlined in Ref. (1) are violated.

In the SCADA system, all but three buses in the system are observable by line and voltage measurements. However, no islanding occurs because of that. And, hence, no major difficulties were experienced.

From the computational point-of-view, the above scheme does not utilize the efficient code we have to the fullest extent. Basically, at each bus, we would like to process all the measurements associated with that bus and then proceed to the next bus. After considerable experimentation, a two-pass sequence turned out to be very successful. These passes are:

- **Pass No. 1**: Process all bus voltage measurements
- **Pass No. 2**: Process, at each bus, all the line flows for near and far end buses and then proceed to process injection measurements.

In effect, this reduced the computational time requirements per iteration,* to less than one-half of that of the four-pass sequence. However, more iterations were required to converge to a solution.

2. **Tuning Parameters**

In order to improve statistical performance of the estimator,

*An iteration consists of the multi-pass sequence in which all measurements are processed.
seven tuning parameters are employed. These are described in Ref. (1). The particular parameter values used will have an effect on solution accuracy provided that a fairly good solution is already available with the initial nominal parameter values.

The main difficulty in the tuning process is that it is manual requiring many trials and errors. Because of that, we initiated a procedure which should automate the whole process. In this procedure a tuning parameter is associated with every individual measurement. The automated procedure will modify the tuning parameter whenever a difference of two or more standard deviations of error exist between the measurement and corresponding calculated estimate. In this respect, we guarantee that large discrepancies between measured and calculated quantities will not take place.

3. **Performance of SSE**

In Table V, we compare performance of SSE with that of the weighted-least-squares (WLS) estimator used in conjunction with the parameter estimator using the same input data. On the average SSE is 15% off from the theoretical average while WLS is only 4% off. With tuning refinements, this performance may be improved upon but probably is not warranted.

In terms of computational requirements, it turns out that both programs will use roughly the same execution times for a complete solution. Normally, WLS will converge in 4-5 iterations using .4-.5 sec. per iteration of CPU time. SSE will converge in 10-12 iterations using .2-.3 sec. of CPU time per iteration.*

*Computer used is Georgia Tech's CDC Cyber 74.
TABLE V. Comparison of SSE Performance with that of Optimal WLS Estimator

<table>
<thead>
<tr>
<th>ITEM</th>
<th>( J_T )</th>
<th>( J_U )</th>
<th>( J_P )</th>
<th>( J_Q )</th>
<th>( J_V )</th>
<th>( J_{(TOTAL)} )</th>
<th>( J^* )</th>
<th>( \sqrt{J/J^*} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Measurement</td>
<td>143</td>
<td>143</td>
<td>71</td>
<td>71</td>
<td>61</td>
<td>489</td>
<td>---</td>
<td>----</td>
</tr>
<tr>
<td>SSE Performance</td>
<td>.788</td>
<td>.808</td>
<td>.768</td>
<td>.608</td>
<td>1.14</td>
<td>.809</td>
<td>.63</td>
<td>1.157</td>
</tr>
<tr>
<td>Optimal Estimator</td>
<td>.741</td>
<td>.855</td>
<td>.308</td>
<td>.426</td>
<td>.881</td>
<td>.683</td>
<td>.63</td>
<td>1.04</td>
</tr>
</tbody>
</table>
2.2.5 Results with Actual Data

When using actual system data only line and transformer susceptances were estimated together with transformer tap setting at the fixed end. The a-priori standard deviation of error on those parameters was set at 5% of nominal values.

Because raw data is used, the full capability of the main grid tuning process was employed. In fact, with purely raw data, the performance index of the system ranged from 30 to 60 in values using either the WLS or the SSE programs. Our initial personal assessment of this poor performance was that it was due to

a. Some status errors resulting from lack of the reporting of disconnect switch status on power circuit breakers.

b. Meters reading zero for a variety of possible reasons.

c. Parameter errors.

d. Other bad data.

All items in (a) or (b) categories are easily detected and their bad influence nullified by means of pre-estimation consistency analysis. Following that, post-estimation bad-data analysis based on residuals seems to discard one or two measurements. Finally, only parameter estimation seems to provide the essential refinement needed to establish statistical validity of the results. Summary of system performance for a typical run following each one of the tuning steps is shown in Table VI. It is clear from that table that state estimator statistical
TABLE VI. Performance of System Following Various Main Grid Tuning Steps Using Actual Data

<table>
<thead>
<tr>
<th>ITEM</th>
<th>( J_T )</th>
<th>( J_U )</th>
<th>( J_P )</th>
<th>( J_Q )</th>
<th>( J_V )</th>
<th>( \overline{J} )  (TOTAL)</th>
<th>( J^* )</th>
<th>( \sqrt{J/J^*} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of Valid Measurements</td>
<td>119</td>
<td>103</td>
<td>55</td>
<td>52</td>
<td>47</td>
<td>376</td>
<td>.520</td>
<td>----</td>
</tr>
<tr>
<td>Performance After Consistency Analysis</td>
<td>27.2</td>
<td>4.6</td>
<td>2.2</td>
<td>0.9</td>
<td>2.9</td>
<td>10.4</td>
<td>.529</td>
<td>4.4</td>
</tr>
<tr>
<td>Performance After Bad Data Analysis</td>
<td>25.2</td>
<td>4.4</td>
<td>2.1</td>
<td>0.8</td>
<td>1.1</td>
<td>9.5</td>
<td>.529</td>
<td>4.25</td>
</tr>
<tr>
<td>Performance After Parameter Estimation</td>
<td>.653</td>
<td>.432</td>
<td>.455</td>
<td>.595</td>
<td>.989</td>
<td>0.6</td>
<td>.529</td>
<td>1.06</td>
</tr>
</tbody>
</table>
performance can be truly improved close to theoretical expectations.

In Figure 2.6, we show a histogram of the percent change from nominal for series impedances of all network elements due to parameter estimation. 92 percent of all changes are less than 5% of nominal values. There are 12 parameters where the errors ranged from 7% to 38%. From an analysis of the data, the largest parameter errors proved to be caused by status errors on series capacitor banks. These findings and results confirm the following conclusions.

a. Most parameter errors are within the previously suspected 5% range.

b. Large parameter errors were detected and corrected for with verification from inquiries with System Operation staff.

2.2.6 Conclusions

- Main grid network tuning, especially at the initial implementation phases, is necessary yielding statistically acceptable state estimator performance.

- Pre-estimation consistency analysis is essential in detecting various anomalies and tuning the values of measurement variances. It is also essential in validating a substantial number of analog data points.

- Parameter errors in the range of 0-10% can cause a severe deterioration of estimator performance.

- Parameter estimation can tune the values of line and transformer susceptances and provide a tool
FIGURE 2.6. Histogram of Percentage Deviation from Nominal in Line and Transformer Susceptances Caused by Parameter Estimation
for detecting large parameter errors.
III. ON-LINE IMPLEMENTATION

3.1 Summary

This chapter integrates the roles of near-term bus load forecasting, state estimation and contingency analysis.

In the first section, a forecasting scheme is outlined. In the second section, a fast stochastic contingency analysis is presented based on D-C analysis. These results are used in the third section to compute the security limits. The security limits indicate outages for which A-C contingency analysis needs to be performed. Finally, in the fourth section we provide a methodology for evaluating the frequency of state estimator updates and the frequency of contingency analysis in connection with on-line security assessment.

3.2 Bus-Injection Forecasting

Several forecasting schemes can be found in the literature. The proposed forecasting scheme is based on the work by Keyhani [34]. The problem is to forecast the load for each bus with lead time of one hour to be used in connection with the security assessment. The proposed scheme is as follows.

Step 1: Obtain estimates of bus loads
Step 2: Obtain estimate of losses
Step 3: Combine results from Steps 1 and 2 to obtain the final estimates of bus loads
The above three steps are described in detail in the following three subsections. The theory to be presented deals with real injections but the same theory can apply for reactive injections also.

3.2.1 Step 1: Forecasting of the Load of Each Bus

Let $Y(k)$ denote the bus load at the $k^{th}$ time and let us assume that the process

\[ Y(1), Y(2), \ldots, Y(k) \]  

is available.

Based on these past observations, we want to forecast the load at time $k+1$ which we denote by $\hat{Y}(k+1)$. The above process is a time variant process and it can be modeled by a time variant model:

\[ Y(k) = a^T(k)Z(k-1) + \omega(k) \]  

where:

\[ a^T(k) = (a_1, \ldots, a_{n+m}, \ldots, a_{n+m+l}) \]  

is the vector of unknown coefficients

\[ Z^T(k) = [Y(k), \ldots, Y(k-n+1); \omega(k), \ldots, \omega(k-m+1); \phi_1, \ldots, \phi_l] \]  

\(\omega(j)\) is the residual at time \(j\) defined as:
\[ \omega(j) \triangleq Y(j) - \hat{Y}(j) \]

and \( \phi_j \) are known functions. These functions can include periodic trends or weather information about the process. In summary, the model consists of \( n \) past observations, \( m \) past residuals and \( l \) known functions.

The integers \( n \) and \( m \) and the functions \( \phi_j, j=1, \ldots, l \) can be selected by using off-line studies. For full details see reference [ ].

The unknown coefficients are computed by minimizing the function:

\[ J_N(a) = \sum_{k=1}^{N} \hat{W}^2(k,a) \] (3.5)

This function is the sum of the squares of the past \( N \) prediction errors \( \hat{W}(k,a) \), where the prediction error is defined as:

\[ \hat{W}(k,a) = Y(k) - a^T(k)Z(k-1) \] (3.6)

The algorithm for the coefficients of the predictor is described by:

\[ S(k+1) = S(k) - S(k)Z(k)Z^T(k)S(k) / (1 + Z^T(k)S(k)Z(k)) \] (3.7)

\[ \hat{a}(k+1) = \hat{a}(t) + S(k+1)Z(k)(Y(k+1) - \hat{a}^T(k)Z(k)) \] (3.8)

\[ \hat{W}(k+1) = Y(k+1) - \hat{a}^T(k+1)Z(k) \] (3.9)

Predictor:
\[ Y(k+1) = a^T(k)Z(k) \]  
\[ (3.10) \]

Prediction error:
\[ e(k+1) = Y(k+1) - \hat{Y}(k+1) \]  
\[ (3.11) \]

The initial conditions are:
\[ S(0) = aI, \quad a - a \text{ large constant} \]
\[ \hat{a}_0 = 0 \]
\[ \hat{W}(k) = 0 \quad \text{for } k=0,-1, \ldots \]

The mean square value of the prediction error is given by:
\[ \sigma^2 = \frac{1}{N} \sum_{k=1}^{N} e^2(k) \]

The above algorithm gives for each bus in the system the forecasted load \( \hat{Y}_j(k+1) \) and the associated variance \( \sigma_j^2 \), \( j=1,\ldots,NTOT \) where \( NTOT \) is the total number of buses in the system.

3.2.2 Forecasting of the Losses in the System

If \( P_L(j) \) is the real loss of the system at time \( j \), then the process:
\[ P_L(1), \ldots ,P_L(k) \]

can be modeled as:
\[ P_L(k) = a^T(k)Z(k-1) + W(k) \]  \hspace{1cm} (3.12)

where

\[ Z^T(k) = P_L(k), P_L(k-1), \ldots, P_L(k-n+1) \]  \hspace{1cm} (3.13)

The unknown vector \( a^T(k) \) is computed using the recursive scheme we described in section 3.2.1. The result will be \( P_L(k+1) \) and an associated \( \sigma_L^2 \).

**3.2.3 Overall Forecasting Scheme**

In this section the results from sections 3.2.1 and 3.2.2 are combined together to give the final results for the forecasted loads and variances. If \( Y_j(k+1) \) denotes the true value of the load of the \( j^{th} \) bus at \( (k+1)^{th} \) time, then we can write

\[ \hat{Y}_1(k+1) = Y_1(k+1) + V_1 \]  \hspace{1cm} (3.14)

\[ \vdots \]

\[ \hat{Y}_{NTOT}(k+1) = Y_{NTOT}(k+1) + V_{NTOT} \]  \hspace{1cm} (3.15)

\[ \hat{P}_L(k+1) = Y_1(k+1) + \ldots + Y_{NTOT}(k+1) + V_L \]  \hspace{1cm} (3.16)

where

\[ E(V_j^2) = \sigma_j^2, \quad j = 1, \ldots, NTOT \]  \hspace{1cm} (3.17)

and

\[ E(V_L^2) = \sigma_L^2 \]  \hspace{1cm} (3.18)
The above set of equations can be written in the form:

\[ Z = Hx + v \]  

(3.19)

where:

\[ H = \begin{bmatrix} 1 & & & \cdot & & \cdot & & 1 \\ & 1 & \cdot & & \cdot & & \cdot & \cdot & 1 \\ & & \cdot & & \cdot & & \cdot & & \cdot \\ 1 & & 1 & & & & & & \end{bmatrix} \]

\[ x = \begin{bmatrix} Y_1 \\ \cdot \\ \cdot \\ Y_{\text{NTOT}} \end{bmatrix} \quad \text{and} \quad z = \begin{bmatrix} \gamma_1 \\ \cdot \\ \cdot \\ \gamma_{\text{NTOT}} \end{bmatrix} \]

\[ E\{v\} = 0 \quad E\{vv^T\} = \begin{bmatrix} \sigma_1^2 & \cdot & \cdot \\ \cdot & \sigma_1^2 & \cdot \\ \cdot & \cdot & \sigma_{\text{NTOT}}^2 \\ \cdot & \cdot & \cdot & \sigma_L^2 \end{bmatrix} = R \]

The final values for the forecasted load are given by:
The contingency analysis to be described in this section is not exact. It is based on DC analysis and it attempts in first approximation to include the uncertainties introduced by the forecasting method described in section 2.1.

Let us assume that the vector of the real injections $\underline{P}$ can be written as

$$\underline{P} = \hat{\underline{P}} + \Delta \underline{P}$$  \hspace{1cm} (3.21)

where $\hat{\underline{P}}$ is the vector of forecasted real injections and $\Delta \underline{P}$ is the vector of the uncertainties. It is also assumed that the decoupling between $(\underline{P}, \underline{\theta})$ and $(\underline{Q}, \underline{V})$ is valid.

The load flow equations become:

$$\underline{P} = \underline{F}(\underline{\theta})$$  \hspace{1cm} (3.22)

or

$$\hat{\underline{P}} + \Delta \underline{P} = \underline{F}(\hat{\underline{\theta}} + \Delta \underline{\theta})$$  \hspace{1cm} (3.23)
where

\[ \hat{P} = P(0) \]  \hspace{1cm} (3.24)

i.e

\[ \Delta P = B \Delta \theta \]  \hspace{1cm} (3.25)

or

\[ \Delta \theta = B^{-1} \Delta P \]  \hspace{1cm} (3.26)

If the \( i^{th} \) branch is between the \( i^{th} \) and \( j^{th} \) bus, we denote by \( e_{i,j} \) the vector:

\[
\begin{bmatrix}
0 \\
\vdots \\
1 \\
\vdots \\
-1 \\
0
\end{bmatrix}
\]

\( i^{th} \) entry

\( j^{th} \) entry

Then the voltage phase angle difference for the \( i^{th} \) line is given by

\[ \psi_i = \psi_{i,j} + \Delta \psi_i \]  \hspace{1cm} (3.28)

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where $\psi_k$ is the voltage phase angle difference for the $k$th line corresponding to the forecasted load conditions $\hat{P}$ and the correction $\Delta \psi_k$ is given by:

$$\Delta \psi_k = e_k^T B^{-1} \Delta P$$

(3.29)

Following the outage of the $k$th branch, the matrix is modified as:

$$B_k = B + \beta_k e_k e_k^T$$

(3.30)

where $\beta_k$ is the susceptance of the $k$th branch. Using the matrix inversion lemma:

$$(B_k)^{-1} = B^{-1} - \lambda_k B^{-1} e_k e_k^T B^{-1}$$

(3.31)

where $\lambda_k$ is a scalar given by

$$\lambda_k \triangleq \frac{1}{1 + \beta_k e_k^T B^{-1} e_k}$$

(3.32)

Following the outage of the $k$th branch, the voltage phase angle difference of the $k$th branch is given by:

$$\psi_k^k = \psi_k + \lambda_k \psi_k$$

(3.33)

where
\[
\psi_k = \hat{\psi}_k + \Delta\psi_k \quad (3.34)
\]

\[
\Delta\psi_k = e_k^B T^{-1} \Delta P \quad (3.35)
\]

and

\[
A_{\lambda,k} = e_k^B T^{-1} \quad (3.36)
\]

Let us assume for simplicity that

\[
\Delta P = \hat{P} \quad (3.37)
\]

where

\[
E\{a\} = 0 \quad \text{and} \quad \text{var}\{a\} = \sigma^2 \quad (3.38)
\]

Then:

\[
E\{\Delta\psi_k\} = 0 \quad (3.39)
\]

\[
E\{\hat{\psi}_k\} = \hat{\psi}_k \quad (3.40)
\]

If

\[
\hat{\mu}_{\lambda} = e_{\lambda}^B T^{-1} \hat{P} \quad (3.41)
\]

Then
(\sigma_\lambda^2)^2 = \text{var}\{\Delta_\lambda\} = \rho^2 \mu_\lambda^2 \quad (3.42)\\

\hat{\psi}_k \triangleq \mathbb{E}\{\psi_k\} = \hat{\psi}_k + A_{k,k} \hat{\psi}_k \quad (3.43)\\

and\\

(\sigma_\lambda^2)^2 \triangleq \text{var}\{\Delta_\lambda\} = \mathbb{E}\{\psi_k^2 - \hat{\psi}_k^2\} =
\rho^2 (\mu_\lambda + A_{k,k} \mu_\lambda)^2 \quad (3.44)\\

In the following \(\bar{\psi}_\lambda\) denotes the maximum allowable voltage phase angle difference (thermal limit).

If \(f(n)\) is the density function for the Normal Distribution, then the probability that the \(i\)th line will be overloaded when the \(k\)th branch goes out of operation is given by:

\[ P_r^k = \int_{-\bar{\psi}_\lambda}^{\psi_k - \hat{\psi}_k} f(n) \, dx \quad (3.45) \]

The above theory can be illustrated in the following figure.
In the above figure $\psi^k$ is the expected voltage phase angle difference of the $k$th branch, $\psi^k$ is the expected voltage phase angle difference of the $k$th branch when the $k$th branch goes out of operation and $\psi^k$ is the maximum allowable voltage phase angle difference. The shaded area gives that the $k$th branch will be overloaded when the $k$th branch goes out of operation. The $k$th line is called emergency line if:

$$p^k_e > \rho$$

where $\rho$ is a predetermined value.

The set of emergency lines is denoted by $S_e$.

3.4 Security Limits

$S_u$ is the set of critical lines. This set includes the outages for which contingency analysis needs to be performed.

In this section conditions are derived which indicate that if at time $t$ the system is in secure or insecure normal state. It is assumed that:

$$T_o < t < T_1$$

where $T_o$ is the time when forecasting has been made for time $T_1$.

In section 2.2, the set $S_e$ of emergency lines was defined. It was proven also that following the $k$th outage ($k \in S_e$) the voltage phase angle difference of the $k$th line ($k \in S_e$) becomes
Without loss of generality it is assumed that \( \psi \geq 0 \) and since the system at time \( t \) is in normal state, it is assumed also that \( \psi < \psi^* \).

The system is in secure normal state if:

\[
\psi^k < \psi^*
\]  

(3.48)

or if:

\[
\psi^k + A_{k,k} \psi^k < \psi^*
\]  

(3.49)

or if:

\[
A_{k,k} \psi^k < \psi^* - \psi^k
\]  

(3.50)

If \( A_{k,k} \psi^k < 0 \), the above inequality is satisfied. If \( A_{k,k} \psi^k > 0 \), the above inequality is satisfied if:

\[
|\psi_k| < \frac{\psi^* - \psi^k}{|A_{k,k}|}
\]  

(3.51)

The security limit \( \psi^*_k \) is defined as

\[
\psi^*_k = \min_{k \in S_k} \left\{ \frac{\psi^* - \psi^k}{|A_{k,k}|}, \quad A_{k,k} \psi^k > 0 \right\}
\]  

(3.52)

Then the system at time \( t \) is in secure normal state if:
Note that the security limits are dependent on the flows of the lines of the set $S_u$.

To examine if the system at time $t$ is in secure state, the algorithm described in Figure 3.1 needs to be followed. The function of this algorithm is called security testing (S.T.). In Figure 3.1, $t_k$ denotes the last time state estimation computation was performed.

3.5 On Line Implementation Procedures

In this section the frequency of state estimator updates (S.E.) and the frequency of contingency analysis (S.T.) in connection with the forecasting scheme is discussed. The minimum number of state estimator updates is determined by the requirements of the forecasting scheme outlined in Section 3.2. The forecasting scheme requires state estimation once every hour. During the period of one hour, state estimation needs to be performed so many times depending on the difference of the vectors $P(k)$ and $\hat{P}(k+1)$, where $P(k)$ is the vector of real injections at the $k^{th}$ time and $\hat{P}(k+1)$ is the forecasted vector of real injections for the $(k+1)^{th}$ time.

Two possible schemes are outlined below where F.A. stands for forecasting analysis, S.E. stands for state estimation and S.T. stands for security testing.
FIGURE 3.1 Security Testing
Define a set $S_c$ from outages in $S_u$ which are critical.

Are the flows of the critical lines ($S_u$) close or above the security limits $\Psi_k$?

Yes: Define a set $S_c$ from outages in $S_u$ which are critical.

No: Stop

Perform A-C contingency analysis for the lines in set $S_c$.

Is system secure?

Yes: Stop

No: Initiate preventive control action.

Stop

FIGURE 3.1 (Continued)
SCHEME #1

* 15 min * 15 min * 15 min * 15 min *

\[ t_0 \quad t_1 \quad t_2 \]

\[ T_k \quad T_{k+1} \]

F.A. F.A.

S.E. S.E. S.E.

S.T. S.T. S.T.

SCHEME #2

* 10 * 10 * 10 * 10 * 10 * 10 *

\[ t_0 \quad t_1 \quad t_2 \]

\[ T_k \]

F.A.

S.E. S.E. S.E.

S.T. S.T. S.T. S.T. S.T.
3.6 Conclusions

- By means of bus-injection forecasting and stochastic contingency analysis security assessment can be performed in a predictive mode.
- Information from stochastic contingency analysis will aid in minimizing the frequency of state estimator and security monitoring updates.
- Several schemes can be (and have been) developed for obtaining an adaptive process for scheduling on-line state estimation and security monitoring.
IV. EXTERNAL EQUIVALENT VALIDATION

4.1 General

This chapter addresses the problem of the static equivalent model of an electric power system connected to several external systems as it relates to the contingency evaluation problem in the process of security assessment. Its objective is the development of a systematic procedure which yields an equivalent representation of the external system with guaranteed accuracy in predicting the effects of postulated outages for on-line steady-state security assessment.

Over the last decade much importance has been given to the security assessment of electric power systems. Advanced techniques from different areas as control theory, pattern recognition, etc., have been introduced in power system analysis. All these techniques aim at helping the operator to assure electric power service under all conditions of operation. The system's operator is concerned with various inequality constraints (frequency drop, overloading of lines, etc.) and with equality constraints (generation meets the demand).

Based on these equality and inequality constraints, it is possible to classify the operating conditions of the system which might prevail into three basic states:

1. Normal State
2. Emergency State
3. Restorative State
In the Normal State all equality and inequality constraints are satisfied. In the Emergency State some of the inequality constraints are violated. In the Restorative State some of the equality constraints are violated.

Figure 4.1 shows the several operating states and the associated control strategies. A brief description of the control strategies follows.

If the system is in the emergency state, the operator should try to maintain the generation vs load balance without any further frequency drop. The control action in this case, referred to as emergency control action, consists of a set of strategies for dropping generation and/or load for every possible major fault. The result of the emergency control is to bring the system to the restorative state. Further control action is needed, known as restorative control action, to bring the system from the Restorative State to the Normal State.

The Normal State can be decomposed into two states:

1. Secure Normal State
2. Insecure Normal State

If the system is in the Secure Normal State, single system contingencies such as a loss of transmission line or a generator does not cause departure from the Normal State. If the system is in the Insecure Normal State, single system contingencies may cause departure from the Normal State to the Emergency State.
Figure 4.1. Power System Operating States with Associated State Transitions.
The primary concern of the system operator is to keep the operating condition of the power system in the Normal State to ensure service continuity at standard frequency and voltage. The operator should continuously test the capability of the power system to withstand postulated next contingencies. This testing is referred to as contingency analysis. The contingency analysis involves two steps.

(1) Computation of a load flow solution of the present operating condition of the system. Load-flow solution means solution of the power flow equations for the voltage magnitudes and voltage phase angles of the busses of the system. This requires application of the classical load-flow methods which utilize short-term bus load forecasting or application of more advanced techniques such as estimation techniques which utilize on-line information.

(2) Computation of the load-flow solution of the system for the various single line or generator outages.

Based on the contingency analysis, security indices are computed. The security indices show how "secure" the system is under the present operating conditions and indicates if the system is in Secure Normal State or in Insecure Normal State [19]. If the system is in the Secure Normal State, no control action is needed. If the system is in the Insecure Normal State, preventive control action should be taken to bring the system back in the Secure Normal State in the most economical way. Examples of preventive control action are:

(a) Shifting of generation schedules
(b) Switching operations
(c) Start-up of units
(d) Changing of the scheduled exchange of power with the neighboring companies.

The role of the static equivalent model in the security assessment of power systems is discussed next.

In recent years the number of interconnections between neighboring companies has been increased. Power companies do not operate independently of each other as it was the common practice in the past. Capital savings which are achieved by reducing spinning reserve requirements or by reducing capacity requirements force the individual companies to become parts of a power pool. Therefore, the operating conditions and performance of each company becomes dependent on the operating conditions of the neighboring companies.

The interconnections with the neighboring companies considerably influence the redistribution of network power flows and voltage levels after outages take place in the particular company. Therefore, in performing contingency analysis, the knowledge of the precontingency load flow solution of the entire area is required. This requires complete exchange of information between the neighboring companies. This is impractical and difficult to achieve at present because of storage and time limitations of today's computers. In order for a particular company to perform the contingency analysis an equivalent representation of the external (neighboring) systems is needed. An equivalent representation is a mathematical model which represents the unobservable part of the system in the process of contingency analysis. In some cases, this representation is exact. In the cases under study, this
equivalent representation is only an approximation.

The existing approaches to obtain the equivalent representation of the external system can be classified into two categories.

(1) **Norton-type equivalents**: To obtain a Norton-type equivalent, knowledge of the topology and network parameters of the external system is necessary. The model is obtained by linear reduction of the external network to the boundaries of the internal system.

(2) **On-line type equivalents**: On-line type equivalents assume no knowledge about the external system and they use information from the internal system only to obtain the equivalent representation of the external system.

In this work emphasis is placed on obtaining the equivalent model by utilizing information from the internal system only because of limited exchange of information between neighboring companies. The assumption that the topology and the parameter values of the external system are available is valid for planning purposes but in most cases is unrealistic for on-line operation.

It should be emphasized that equivalence techniques are applied also for planning purposes but for different reasons than for on-line security assessment. In planning the primary purpose of the network reduction is to avoid the computational burden of solving the load-flow for the entire area.

The next section reviews the available methods to obtain the equivalent representation of the external system.
4.2 Historical Background

In any network equivalencing problem the overall area is divided into an internal system and an external system as it is shown in Figure 4.2. In stricter terms, the internal system consists of the observable part of the overall system as obtained from on-line measurements and bus load forecasts and estimates. Some buses of the internal system are connected to the external system. These buses are called boundary buses.

In most of the approaches given in the literature, the following steps are taken:

(a) Define the boundaries of the internal system.

(b) Reduce by means of Norton equivalent the external system to the boundary.

(c) Classify the boundary buses as generation buses or as load buses.

In reference (22), the internal system is augmented by a buffer zone as it is shown in Figure 4.3. This buffer zone includes:

(1) Buses of the external system critical to the accuracy of the equivalent.

(2) Components of the external system of which the operational limitations may be violated due to disturbances in the internal system (weak links).

(3) Generation buses of the external system which control the operating conditions in the internal system (controlling buses).

The weak links are found by imposing extreme stressing conditions
Figure 4.2. System Decomposition with Associated Term Definitions.
Figure 4.3. Augmentation of Internal System by Means of a Buffer Zone [22].
in the internal system. Since an unobservable part of the external system is included in the equivalent, several simplifications and assumptions are needed which jeopardize the accuracy of the equivalent system.

In reference (23), some buses of the external system are included in order to preserve sparsity in the equivalent representation. Simulation studies on power systems, however, have shown that the problem of sparse structure is not so crucial. Even if the number of the equivalent branches is extremely large, a portion of them may be eliminated by using a technique proposed in reference (24) or by a simpler method as we will propose later without significant sacrifice of the accuracy.

In reference (24), the boundary buses are assumed to be load buses; therefore, many of the equivalent branches between the boundary buses are eliminated by using, as criterion, the ratio

$$\frac{Z_{E,ij}}{Z_{T,ij}}$$

where $Z_{E,ij}$ is the impedance of the equivalent branch $ij$ and $Z_{T,ij}$ is the corresponding transfer impedance given by the rest of the network. If

$$\frac{Z_{E,ij}}{Z_{T,ij}} > C \quad (4.1)$$

where $C$ is a predetermined value, the branch $ij$ is eliminated from the equivalent representation.
In references 25 and 26 two approaches are suggested. The first is a Norton-type equivalent where the boundary busses are treated as generation busses. The other is based on DC approximation of the external system. If $P_B$ is the vector of tie line flows, one can write

$$\begin{bmatrix} P_E \\ P_B \end{bmatrix} = \begin{bmatrix} K_{EE} & K_{EB} \\ K_{BE} & K_{BB} \end{bmatrix} \begin{bmatrix} \theta_E \\ \theta_B \end{bmatrix}$$

Elimination of the vector $\theta_E$ yields

$$P_E = K_{BE}K_{EE}^{-1}P_B + \left(K_{BB} - K_{EE}K_{EB}K_{EE}^{-1}\right)\theta_B$$

$$= \theta_B + H\theta_E$$

Since $P_B$ and $\theta_B$ are known, the vector

$$H\theta_E = P_B - \theta_B$$

is also known and it is assumed to be constant when a contingency occurs. If

$$V = \begin{bmatrix} V_I \\ V_B \end{bmatrix}, \quad \theta = \begin{bmatrix} \theta_I \\ \theta_B \end{bmatrix}$$
then the real and reactive injections in the internal system are:

\[
P_I = P_I(\theta, V) \quad (4.5)
\]

\[
Q_I = Q_I(\theta, V) \quad (4.6)
\]

and

\[
P_B = P_B(\theta, V) \quad (4.7)
\]

or by taking into consideration the linear approximation for the external system given by equation (3), equation (7) becomes:

\[
P_B(\theta, V) + HP_E + G\theta_B = 0
\]

or

\[
-HP_E = P_B(\theta, V) + G\theta_B \quad (4.8)
\]

Equations (5), (6), and (8) are the load flow equations. The boundary busses are assumed to be generation busses.

In reference 27 a model based on DC analysis is suggested. Deviations from the operating point are used to provide the necessary information for the equivalent representation. The statement of the method is:

The system between the boundary busses is modeled as
\[ Z = Hu + v \quad (4.9) \]

where

- \( Z \) is the vector of the phase angles of the boundary busses
- \( H \) is the unknown boundary impedance matrix
- \( u \) is the vector of real powers which depend upon the topology and real injections of the internal system
- \( v \) is the vector which depends upon the topology and real injections of the external system.

If

\[ Z(n) = Z(t_{n+1}) - Z(t_n) \quad (4.10) \]
\[ u(n) = u(t_{n+1}) - u(t_n) \quad (4.11) \]
\[ v(n) = v(t_{n+1}) - v(t_n) \quad (4.12) \]

then

\[ Z(n) = Hu(n) + v(n) \quad (4.13) \]

It is assumed that \( v(n) \) has zero expected value and covariance matrix \( E[v(n_1)v^T(n_2)] = R \cdot \delta(n_1 - n_2) \). Furthermore, \( u(n) \) and \( v(n) \) are uncorrelated. The problem is to estimate \( H \) and \( R \) by using \( Z(n) \), \( n = 1, \ldots, N \) where \( N \) is the number of observations.
Least squares estimation yields:

\[
\hat{H} = \left[ \sum_{n=1}^{N} Z(n)u_T(n) \right] \left[ \sum_{n=1}^{N} u(n)u_T(n) \right]^{-1} \tag{4.14}
\]

\[
\hat{R} = \frac{1}{N} \sum_{n=1}^{N} (Z(n) - Hu(n))(Z(n) - Hu(n))^T \tag{4.15}
\]

provided that the inverse of

\[
\sum_{n=1}^{N} u(n)u_T(n)
\]

exists.

Objections to this approach are:

1. Since the entire system for a time period is moving in the same direction, \( \bar{v}(n) \) has an expected value different than zero.
2. \( u(n), v(n) \) are not uncorrelated since both depend upon the power injections.
3. The accuracy of the DC model does not suffice for this problem.

In reference 28 information from outages in the internal system are used to obtain the equivalent system. If \( Z^1 \) and \( Z^2 \) are pre- and post-outage internal system measurement vectors, then

\[
Z^1 = h^1(x^1) + v^1 \tag{4.16}
\]

\[
Z^2 = h^2(x^2) + v^2 \tag{4.17}
\]
where $\mathbf{x}^1$ and $\mathbf{x}^2$ are the pre- and post outage state vectors, $\mathbf{v}^1$ and $\mathbf{v}^2$ are measurement error vectors with zero mean and covariances $R_1$, $R_2$. It is assumed that the boundary busses have been classified as load or as generation busses. For contingency analysis, the real power and voltage magnitudes at generation busses are assumed to be constant. All these quantities define the vector $\mathbf{C}$. If $\mathbf{C}^1$ and $\mathbf{C}^2$ denote the pre- and post-outage cases, then

$$\mathbf{C}^1 = \mathbf{C}^2 + \mathbf{v}^3 \tag{4.18}$$

where $\mathbf{v}^3$ is a random vector of zero mean and covariance $R_3$, and

$$\mathbf{C}^1 = F^1(\mathbf{x}^1, \mathbf{p}) \tag{4.19}$$
$$\mathbf{C}^2 = F^2(\mathbf{x}^2, \mathbf{p}) \tag{4.20}$$

$\mathbf{p}$ is the vector of external network equivalent parameters, with initial value $\mathbf{p}^0$ and a priori covariance error matrix $M_0$.

Equations (18), (19), and (20) are combined to give:

$$\mathbf{z}^3 = 0 = F^2(\mathbf{x}^2, \mathbf{p}) - F^1(\mathbf{x}^1, \mathbf{p}) + \mathbf{v}^3 \triangleq g(\mathbf{x}^1, \mathbf{x}^2, \mathbf{p}) + \mathbf{v}^3$$

The optimum $\hat{\mathbf{x}}^1$, $\hat{\mathbf{x}}^2$, and $\hat{\mathbf{p}}$ are those which minimize the index.
\[ J = (\varphi^0 - \varphi^0)T_0^{-1}(\varphi - \varphi^0) + (Z_1 - h(x^1))T_R^{-1}(Z_1 - h(x^1)) \]

\[ + (Z_2 - h(x^2))T_R^{-1}(Z_2 - h(x^2)) \]

\[ + (Z_3 - g(x^1, x^2, \varphi))T_R^{-1}(Z_3 - g(x^1, x^2, \varphi)) \]

In reference 35 the equivalent representation of the external system is obtained by using real-time information on the voltage magnitude and angle and the real and reactive power at boundary busses. The basis of the method is the decoupled form of the Jacobian equations for power systems. The following model between the boundary busses is proposed:

\[ \frac{\Delta P_T}{V_b} = B_{TT}^T \Delta \delta_b \quad (4.21) \]

where

\[ \Delta P_T \] A difference of the vector of the net tie line flows into the external system between a past time instant and the present.

\[ \Delta \delta_b \] A difference of the vector of the voltage phase angles of the boundary busses between a past time instant and the present.

\[ V_b \] A vector of voltage magnitudes of the boundary busses.

\[ B_{TT}^T \] An unknown admittance matrix.

The unknown matrix \( B_{TT}^T \) is computed from a given sequence of \( r \).
measurements:

\[ \{ \Delta P_T(1), \Delta P_T(2), \ldots, \Delta P_T(r) \} \]

so that the objective function

\[
J = \sum_{i=1}^{r} \left[ (\Delta P_T(i) - B'_T \Delta \delta_T(i)) \right]^T \left[ (\Delta P_T(i) - B'_T \Delta \delta_T(i)) \right]
\]

is minimized.

In summary, the available methods do not optimize the equivalent representation of the external system; therefore, the available methods do not guarantee accuracy in predicting the effects of postulated outages for on-line steady-state security assessment.

4.3 Mathematical Formulation of the Problem

4.3.1 Objectives

The objective of this work is the estimation of the equivalent representation of a power system which is connected to a number of external power systems. The equivalent model should satisfy the following requirements:

1. It should be accurate, in the sense, that it can yield voltage levels and power flows which are very close to the actual values for a set of postulated conditions.

2. Changes in the external system should be easily handled.

Two kinds of changes in the external system may take place:
(a) Transmission line outages
(b) Generator outages

With regard to the second requirement, three cases may be distinguished.

(1) The equivalent model is insensitive to the change, whereby, it is not necessary to modify the equivalent model.

(2) The equivalent model is sensitive to a known change. In this case, the equivalent model should be updated.

(3) The equivalent is sensitive to an unknown change whereby the change must first be detected and then the equivalent model should be updated.

4.3.2 Basic Assumption

This research assumes that the topology and the parameter values of the internal system are known. Also, it is assumed that the internal system is observable to a state estimator. This insures that the present operating condition of the internal system is always available.

The equivalent model is designed so that the boundary buses behave as load buses, i.e. the real and reactive injections at the boundary buses should remain constant before and after each of the N postulated outages. Real error at each bus is defined as the absolute value of the difference between the pre- and post-outage real injections, if the true pre- and post-outage conditions were used to compute these real injections. Similarly, reactive error at each bus is defined as the absolute value of the difference between the pre-and post-outage reactive injections if the true pre- and post-outage conditions were used to compute these reactive injections.
In the ideal case the equivalent model should give zero values for all these errors. In practice, however, this is not feasible. The equivalent model will be determined to satisfy the following inequalities.

\[ \text{Total error} < S_1 \quad (4.22) \]
\[ \text{Maximum real error} < S_2 \quad (4.23) \]
\[ \text{Maximum reactive error} < S_3 \quad (4.24) \]

\( S_1, S_2 \) and \( S_3 \) are specified tolerances.

This research describes a procedure which determines the optimal equivalent representation of the external system for the following cases.

(a) The topology and the parameter values of the external network are well defined.

(b) Information about the external system is not available.

In some cases an equivalent model, such as the Norton equivalent model, is available. The model should be tested if it satisfies the inequalities (4.22), (4.23), (4.24). If these inequalities are satisfied, the model is sufficient. If the inequalities are not satisfied or such equivalent model is not available, the problem described by the inequalities (4.22), (4.23), (4.24) is relaxed by the inequality (4.22) and the model is obtained by solving the following optimization problem.

\[ \text{Minimize: Total error} \quad (4.25) \]

subject to the constraints:

\[ \text{Maximum real error} < S_2 \quad (4.26) \]
\[ \text{Maximum reactive error} < S_3 \quad (4.27) \]
In the following sections, the problem described by the relations (4.25), (4.26) and (4.27) is formulated as an optimization problem.

Minimize:

\[ J = q^T(u)q(u) \]

subject to the constraints:

\[ \mathcal{P}_N(u) \leq 0 \]

\( u \) is the vector of the decision variables. The following sections are devoted to the description and interpretation of the various elements of the optimization problem.

4.3.3 Decision Variables

The equivalent model of the external system consists of fictitious network branches. The conductances and susceptances of these fictitious branches are the unknown variables. The fictitious branches may be lines between the boundary buses, and capacitors or reactors at the boundary buses. The vector of the decision variables is denoted by \( u \). This unknown vector \( u \) will be determined from the solution of the problem.

4.3.4 The Objective Function

The equivalencing technique of this research assumes the boundary buses behave as load buses, i.e. the equivalent real and reactive injections on the boundary buses remain constant before and after the \( k^{th} \) outage.

We denote by \( x \) the vector of the complex bus voltages of the internal system. \( x \) is a known vector because it has been assumed that
the internal system is observable to a state estimator. Let the vectors \( \mathbf{x}_{lk} \) and \( \mathbf{x}_{2k} \) denote the pre- and post-outage vector of the complex bus voltages, respectively. Let \( \mathbf{I}_{lk} \), \( \mathbf{I}_{2k} \) be the vectors of the real and reactive injections on the boundary busses before and after the \( k \)th outage.

\[
\mathbf{I}_{lk} = \mathbf{I}_{lk}(\mathbf{x}_{lk}, \mathbf{u}) \tag{4.28}
\]

\[
\mathbf{I}_{2k} = \mathbf{I}_{2k}(\mathbf{x}_{lk}, \mathbf{u}) \tag{4.29}
\]

If \( N \) is the number of postulated outages in the internal system, the objective of the problem is to find a vector \( \mathbf{u} \) such that:

\[
\mathbf{I}_{lk} = \mathbf{I}_{2k}; \quad k = 1, \ldots , N
\]

This is equivalent to:

Find \( \mathbf{u} \) such that:

\[
\mathbf{g}_k(\mathbf{x}_{lk}, \mathbf{x}_{2k}, \mathbf{u}) = \mathbf{I}_{lk}(\mathbf{x}_{lk}, \mathbf{u}) - \mathbf{I}_{2k}(\mathbf{x}_{lk}, \mathbf{u}) = \mathbf{0} \tag{4.30}
\]

for \( K = 1, \ldots , N \)

If:
The objective of the problem is to find \( u \) such that:

\[
g(x^1, x^2, u) = 0 \quad (4.31)
\]

The dimension of \( g \) is \( 2 \times N \times b \) if \( b \) is the number of the boundary busses. The dimension of the vector \( u \) determines the sparsity of the admittance matrix of the equivalent model. It is desirable to keep the dimension of the vector \( u \) as small as possible so that the equivalent admittance matrix is a sparse matrix. Therefore, equation (31) is an over-determined set of equations and, in general, there is not a solution which satisfies these equations. Hence, we seek a solution which will minimize the following defined error:

\[
J = g^T(x^1, x^2, u)g(x^1, x^2, u) \quad (4.32)
\]

This is a measure of the total error as a function of the decision variables and defines the performance of the equivalent model. If
\( P_{jk}^{1k}, P_{jk}^{2k} \) real injected power at the \( j^{th} \) bus before and after the \( k^{th} \) outage

\( Q_{jk}^{1k}, Q_{jk}^{2k} \) reactive injected power at the \( j^{th} \) bus before and after the \( k^{th} \) outage

then, we define by \( S \):

\[
S = \frac{1}{N \times b} J = \frac{1}{N \times b} \sum_{k=1}^{N} \left( I^{1k} (x^{1k}, u) - I^{2k} (x^{1k}, u) \right)^T \left( I^{1k} (x^{1k}, u) - I^{2k} (x^{2k}, u) \right)
\]

\[
= \frac{1}{N \times b} \sum_{k=1}^{N} \sum_{j=1}^{b} \left( (P_{jk}^{1k} - P_{jk}^{2k})^2 + (Q_{jk}^{1k} - Q_{jk}^{2k})^2 \right)
\]

If an equivalent model was available, this model would be satisfactory if:

\[
S < S
\]

4.3.5 Constraints

The objective function measures the total error of the real and reactive injections at the boundary buses over the set of \( N \) postulated outages. However, the maximum observed local error is, also, of great importance. The constraints which will be discussed in this section deal with the maximum real and reactive error.

Random changes take place in the external system. Depending on the location and the size of this change, it may or may not have an effect on the equivalent representation. It is expected that outages
of transmission lines far away from the boundaries do not affect considerably the equivalent model.

It is assumed that the set of $N$ postulated outages is consisted of $N_1$ outages with a nominal topology of the external system and $N_2$ outages while single outages in the external system took place. Furthermore,

$$N_2 = N_{2,1} + N_{2,2} + \ldots + N_{2,k} + \ldots + N_{2,L}$$

(4.34)

where $N_{2,\ell}$ is the number of switching operations in the internal system with the $\ell^{th}$ branch of the external system out of operation.

Therefore, the vector $\mathbf{g}$, defined by equation (4.31) can be decomposed as:

$$\mathbf{g} = \begin{bmatrix} g_{N_1} \\ g_{N_2,1} \\ \vdots \\ g_{N_2,k} \\ \vdots \\ g_{N_2,L} \end{bmatrix}$$

(4.35)

Next, we define the following accuracy indices:
\[ MP = \text{Maximum} \left| p_{jk}^{1k} - p_{jk}^{2k} \right| \]
\[ k = 1, \ldots, N_1 \]
\[ j = 1, \ldots, b \]
\[ (4.36) \]

\[MQ = \text{Maximum} \left| Q_{jk}^{1k} - Q_{jk}^{2k} \right| \]
\[ k = 1, \ldots, N_1 \]
\[ j = 1, \ldots, b \]
\[ (4.37) \]

and

\[ (MP)_l = \text{Maximum} \left| p_{jk}^{1k} - p_{jk}^{2k} \right| \]
\[ k = 1, \ldots, N_2, l \]
\[ j = 1, \ldots, b \]
\[ (4.38) \]

\[ (MQ)_l = \text{Maximum} \left| Q_{jk}^{1k} - Q_{jk}^{2k} \right| \]
\[ k = 1, \ldots, N_2, l \]
\[ j = 1, \ldots, b \]
\[ (4.39) \]

for \( l = 1, \ldots, L \).

The equivalent model is further constrained to the following inequalities:

\[ MP \leq \overline{MP} \] \hspace{1cm} (4.40)

\[ MQ \leq \overline{MQ} \] \hspace{1cm} (4.41)
(MP)_l \leq c_2 \overline{MP} \quad (4.42)
(MQ)_l \leq c_3 \overline{MQ} \quad (4.43)

for \ l = 1, \ldots, L.

The values of \overline{MP}, \overline{MQ} are predetermined maximum allowable local errors according to the requirements and applications of the equivalent model. Generally, better accuracy is needed for on-line operations than for planning purposes. \( c_2, c_3 \) are positive numbers which correspond to the specified accuracy tolerance.

The differences \((P^1_k - P^2_k), (Q^1_k - Q^2_k)\) for \( k = 1, \ldots, N_1 \) are elements of the vector \( g_{N_1} \). Therefore, the inequalities (4.40) and (4.41) are satisfied if

\[ |g_i| < \overline{MP} \]

for all the rows of \( g_{N_1} \) which correspond to the real power error and

\[ |g_i| < \overline{MQ} \]

for all the rows of \( g_{N_1} \) which correspond to the reactive error.

The inequalities (4.40) and (4.41) are equivalent to

\[ F^1_{N_1}(u) = |g_{N_1}(x^1, x^2, u)| - \overline{g}_{N_1} \leq 0 \]

(4.44)

where
$g_{N_1,1,i} = \begin{cases} 
  \text{MP} & \text{if the } i^{th} \text{ row of } g_{N_1} \text{ corresponds to } \\
  \text{real power error} \\
  \text{MQ} & \text{if the } i^{th} \text{ row of } g_{N_1} \text{ corresponds to } \\
  \text{reactive power error} 
\end{cases}$

Following the above thinking, inequalities (4.42) and (4.43) are equivalent to:

$$F_{N_2,1}(u) = |g_{N_2,1}(x^1, x^2, u)| - g_2 \leq 0$$ (4.45)

for $l = 1, \ldots, L$

where

$$g_{l,1,i} = \begin{cases} 
  C_2 \text{MP} & \text{if the } i^{th} \text{ row of } g_{N_2,1} \text{ corresponds to } \\
  \text{real power} \\
  C_2 \text{MQ} & \text{if the } i^{th} \text{ row of } g_{N_2,1} \text{ corresponds to } \\
  \text{reactive power error} 
\end{cases}$$

The inequalities (4.44) and (4.45) are the constraints of the problem. These can be combined as:

$$F_N(u) = |g(x^1, x^2, u)| - \underline{g}_N \leq 0$$ (4.46)
4.3.6 General Solution to the Problem

The equivalent problem has been formulated as an optimization problem. Minimize:

\[ J = \bar{g}^T (u) \bar{g}(u) \]  

subject to the constraints:

\[ P_N (u) \leq 0 \]  

The dimensionality of the problem is huge. It has been observed that in many cases it is not necessary to solve this large optimization problem. A simpler model may yield a solution which satisfies the constraints (4.49). Therefore, it pays back if prior to the solution of the general problem, simpler models are tested. In this line of thinking
we have developed a procedure which is depicted in Figure 4.4. This procedure yields the equivalent model which satisfies inequalities (4.49) assuming that there is such a model.

A brief description of the procedure follows. If the topology and the parameter values of the external system are available, the Norton equivalent model can be calculated. This model should be tested if it satisfies inequalities (4.49). The performance index $S$ should be calculated also. If this performance index $S$ is less than a predetermined value $\bar{S}$, the Norton equivalent model is used as the equivalent representation of the external system and the solution of the problem is avoided. If the Norton equivalent model does not satisfy the above requirements, the problem described by the relations (4.48) and (4.49) should be solved. The following applies, also, to the case where the topology and the parameter values of the external system are not available. Before one has to solve the constrained problem, the unconstrained problem should be considered. The model obtained from the solution of the unconstrained problem should be tested if it satisfies the inequalities (4.49). If these inequalities are satisfied, this model is used as the equivalent representation of the external system. If the model does not satisfy the inequalities (4.49), the entire problem should be considered. Solution to the problem exists assuming that the solution space described by the inequalities (4.49) is feasible.
Find \( u \) s.t \( S^{<S} \) 
\[ F_{N}(u) \leq 0 \]

is topology and parameter values of the External System Available?

NO

Find the Norton-Type Equivalent

is \( S^{<S} \) 
\[ F_{N}(u) \leq 0? \]

YES

Find \( u \) by Minimizing 
\[ j = s^{T}(u)g(u) \]

is \( F_{N}(u) \leq 0? \)

YES

STOP

NO

Find \( u \) by Minimizing 
\[ j = s^{T}(u)g(u) \]
\[ s.t. \ F_{N}(u) \leq 0 \]

STOP

Figure 4.4. General Solution to the Equivalencing Problem.
4.4. Equivalence by Reduction to the Norton Equivalent

4.4.1 Introduction

The Norton equivalent has a long history of application as the model to represent the external network. It was introduced, by Ward in 1949, because of limitations imposed by the number of analyzer circuits. Today the analyzer power-flow studies have been substituted by digital computer power flow studies. However, the difficulties in performing the contingency analysis remain the same.

The Norton type equivalent assumes that the topology and the parameter values of the external system are known. The loading conditions of the external system are not available and the internal system is assumed to be observable to a state estimator. The next section is a review of the Norton equivalent theory.

4.4.2 General Norton Equivalencing Theory

The entire system nodal matrix equation is:

$$YV = S$$  \hspace{1cm} (4.50)

where $S$, $V$ are the vectors of injected complex bus currents and complex bus voltages, respectively, and $Y$ is the nodal admittance matrix. The
transmission line model is assumed to be the pi equivalent circuit as it is shown in Figure 4.5.

\( y_{kl} \) is the complex admittance of the transmission line which connects the \( k^{th} \) bus with the \( l^{th} \) bus and it is defined as:

\[
y_{kl} = G_{kl} + jB_{kl}
\]  

(4.51)

where \( G_{kl} \) is the conductance of the line and \( B_{kl} \) is the susceptance of the line.

\( y_{SH,kl} \) is the complex shunt admittance of the line. According to the above notation, the nodal admittance matrix is defined as:

\[
y_{kl} = \begin{cases} 
  -y_{kl} & \text{if } k \neq l \\
  \sum_{l \in a(k)} y_{kl} + \sum_{l \in a(k)} y_{SH,kl} & \text{if } k = l 
\end{cases}
\]

(4.52)

\( a(k) \) is the set of busses connected to the \( k^{th} \) bus.

The vectors \( S, V \) and the matrix \( Y \) are decomposed as follows:

\[
S = \begin{bmatrix} S_E \\ S_B \\ S_I \end{bmatrix}, \quad V = \begin{bmatrix} V_E \\ V_B \\ V_I \end{bmatrix}, \quad Y = \begin{bmatrix} Y_{EE} & Y_{EB} & 0 \\ Y_{BE} & Y_{BB} & Y_{BI} \\ 0 & Y_{IB} & Y_{II} \end{bmatrix}
\]

where the subscripts \( E, B \) and \( I \) refer to external, boundary, and
Figure 4.5. Transmission Line Model.
internal systems, respectively. The two zero submatrices indicate
that internal and external busses are not connected to one another.

According to the above decomposition, equation (4.50) becomes:

\[
\begin{bmatrix}
Y_{EE} & Y_{EB} & 0 \\
Y_{BE} & Y_{BB} & Y_{BI} \\
0 & Y_{IB} & Y_{II}
\end{bmatrix}
\begin{bmatrix}
V_E \\
V_B \\
V_I
\end{bmatrix}
= 
\begin{bmatrix}
S_E \\
S_B \\
S_I
\end{bmatrix}
\] (4.53)

Elimination of the vector \( V_E \) from equation (4.53) yields:

\[
\begin{bmatrix}
Y_{BB} & -Y_{BE} Y_{EE} Y_{EB} & Y_{BI} \\
0 & Y_{IB} & Y_{II}
\end{bmatrix}
\begin{bmatrix}
V_B \\
V_I
\end{bmatrix}
= 
\begin{bmatrix}
S_B - Y_{BE} Y_{EE} Y_{EB} S_E \\
S_I
\end{bmatrix}
\] (4.54)

The matrix \(-Y_{BE} Y_{EE} Y_{EB}\) represents equivalent network intercon-
nections between the boundary busses because of the linear reduction
of the external system to the boundaries of the internal system. The
vector \(-Y_{BE} Y_{EE} Y_{EB}\) represents equivalent current injections at the
boundary busses. The elements of the matrix \(Y_{BB} - Y_{BE} Y_{EE} Y_{EB}\)
correspond to transmission lines connecting the boundary busses. As such, the
individual line constants can be represented as a vector \( u \). The
matrix

\[
\begin{bmatrix}
Y_{BB} & -Y_{BE} Y_{EE} Y_{EB} & Y_{BI} \\
0 & Y_{IB} & Y_{II}
\end{bmatrix}
\]
is known as the equivalent matrix and it is denoted by $Y_{eq}$.

If the first row of the matrix $Y$ is premultiplied by the matrix $Y^{-1}_{EE}$, then the matrix $Y$ becomes:

$$Y_1 = \begin{bmatrix}
I & Y^{-1}_{EE}Y_{EB} & 0 \\
Y_{BE} & Y_{BB} & Y_{BI} \\
0 & Y_{IB} & Y_{II}
\end{bmatrix}$$  \hspace{1cm} (4.55)

where $I$ is the identity matrix.

If the first row of the matrix $Y_1$ is premultiplied by the matrix $Y_{BE}$ and the resultant row is subtracted from the second row, then $Y_1$ becomes:

$$Y_2 = \begin{bmatrix}
I & Y^{-1}_{EE}Y_{EB} & 0 \\
Y_{BE} & Y_{BB} & Y_{BI} \\
0 & Y_{IB} & Y_{II}
\end{bmatrix} - \begin{bmatrix}
I & Y^{-1}_{EE}Y_{EB} & 0 \\
0 & Y_{BB} - Y_{BE}Y^{-1}_{EE}Y_{EB} & Y_{BI} \\
0 & Y_{IB} & Y_{II}
\end{bmatrix}$$  \hspace{1cm} (4.56)

Note that the lower part of the matrix $Y_2$ is the equivalent admittance matrix $Y_{eq}$. Therefore, the equivalent admittance matrix is obtained by gaussian elimination of the rows of the admittance matrix $Y$ which corresponds to the busses of the external system. Direct inversion of the matrix $Y^{-1}_{EE}$ is thus avoided.

Since the internal system is observable, by definition, the
complex voltages of the busses of the internal system are known. Using equation (4.54), the equivalent complex current injections can be evaluated. If these injections were constant before and after an outage in the internal system, then contingency analysis could be performed exactly. However, these current injections do not remain constant and it is common practice in performing contingency analysis to classify the busses as generation busses denoted as (P,V) busses, or as load busses denoted as (P,Q) busses. Generation busses are defined as those for which the real power injection P and the voltage magnitude V remain constant before and after outages take place in the system. Load busses are defined as those for which the real power injection P and the reactive power injection Q remain constant before and after outages take place. One of the busses in the system is classified as slack bus and for this bus, arbitrarily, the voltage phase angle is set to be zero.

Therefore, the load flow equations include:

(1) Two equations for each load bus; one for the real injection and one for the reactive injection.

(2) One equation for each generation bus for the real injection.

From the references cited in Section 4.2, it is clear that the classification of the boundary busses as (P,V) busses or as (P,Q) busses is dependent on the particular system and the set of postulated outages. In our research, both assumptions were investigated and for our test systems both assumptions gave similar results.

In our research, the equivalent is designed so that the boundary busses behave as (P,Q) busses. To calculate the Norton equivalent
sparsity techniques have been implemented. The conductances 
and susceptances of the fictitious lines created between the boundary 
busses are the elements of the vector $u$.

After the equivalent model is found by computing the matrix 
$Y_{eq}$, this model is tested if it satisfies the requirements of the 
problem. For the set of $N$ postulated outages, the performance index $S$ 
is computed.

If

$$S < \bar{S}$$  \hspace{1cm} (4.57)  

and

$$F_N(u) \leq 0$$  \hspace{1cm} (4.58)  

the Norton-type equivalent is sufficient. If this set of inequalities 
is not satisfied, the Norton-type equivalent is rejected and another 
equivalent model needs to be derived by following the procedure to be 
presented in the next chapter. The $N$ postulated outages are either 
information from real switching operations or information from con-
tingency analysis simulation using the entire area.

4.4.3 Sparsity of the Equivalent Admittance Matrix

The reduction of the external system to the boundaries of the 
internal system creates equivalent branches between the boundary busses.
The admittance matrix of the equivalent network (between the boundary busses) is given by:

\[
Y_{BB} - Y_{BE}Y^{-1}_{EE}Y_{EB}
\]

The matrix \( Y_{EE} \) is a sparse matrix. However, its inverse is in general a full matrix. Therefore, reduction of the external system to the boundaries will create a large number of equivalent branches. The number of these branches will depend on the number of connections between the internal and external system. Three cases may be distinguished:

(a) All the busses of the external system are part of one area. If \( b \) is the number of the boundary busses, then the number of equivalent branches are:

\[
\frac{b(b-1)}{2}
\]

(b) The busses of the external system form \( m \) isolated areas. Each one of these \( m \) areas is connected to \( b_i \) boundary busses of the internal system. Then

\[
\sum_{i=1}^{m} \frac{b_i(b_i-1)}{2}
\]

equivalent branches are created, where:
\[ \sum_{i=1}^{m} b_i = b \]

(c) One or more busses of the external system form an isolated area and this area is connected to one boundary bus. In this case only the shunt admittance of this bus changes.

In general, if the internal system is highly interconnected to the external system, the number of the equivalent branches is large and the sparsity of the admittance matrix of the equivalent model is destroyed. This is undesirable for contingency analysis because of time and storage limitations. Some compromise between accuracy and sparsity is necessary. It has been suggested that some busses of the external system should be included in the equivalent model so the admittance matrix of the equivalent system will preserve its sparse structure.

The method is based on the ordering schemes for sparse matrices developed by Tinney. In order to include busses of the external system in the study area, systematic exchange of information between neighboring companies is required. If this information is not available, assumptions should be made about the loading conditions of the external system. These assumptions usually jeopardize the accuracy of the equivalent model.

This thesis reports another method. A large number of the equivalent branches are eliminated according to some criterion. This practice preserves both the sparse structure of the equivalent admittance matrix and the accuracy of the model. Two elimination schemes of branches of the equivalent model were examined; the two schemes are discussed next.
First Elimination Scheme

For the set of $N$ postulated outages, the indices

$$\Gamma_P^J = \frac{1}{N} \sum_{k=1}^{N} |P_{2k}^j|$$

(4.59)

$$\Gamma_Q^Q = \frac{1}{N} \sum_{k=1}^{N} |Q_{2k}^j|$$

(4.60)

are computed for every equivalent branch. $P_{2k}^j$, $Q_{2k}^j$ are the real and reactive flows in the $j$th branch after the $k$th outage. If for the $j$th branch,

$$\Gamma_P^J < \Gamma_P$$

(4.61)

$$\Gamma_Q^Q < \Gamma_Q$$

(4.62)

then, the $j$th branch is eliminated from the equivalent representation.

For various values of $\Gamma_P$, $\Gamma_Q$ the accuracy indices $S$, $M_P$, $M_Q$ are calculated. The optimum values of $\Gamma_P$, $\Gamma_Q$ are selected so that sparsity requirements and accuracy specifications are satisfied.

Second Elimination Scheme

If for the $j$th branch the conductance and susceptance satisfy the inequalities

$$|G_j| < \bar{G}$$

(4.63)

$$|B_j| < \bar{B}$$

(4.64)
the jth branch is eliminated. For various values of \( G, B \) the accuracy indices \( S, MP, MQ \) are calculated. The optimum values of \( G, B \) are selected so that sparsity requirements and accuracy specifications are satisfied.

4.5 On-Line Equivalents

4.5.1 General

AC power flow equations used in contingency analysis are non-linear. Switching and/or transformer tap-changing operations complicate; furthermore, the analysis of power systems. These nonlinearities should be accounted for by the equivalent model. The Norton-type equivalents are based on engineering insight; whereby, linear reduction is used to obtain equivalents for a non-linear set of equations. Normally, these equivalents give good results in most cases. But there are cases of serious discrepancies ranging from failure of the load flow algorithm to converge to cases of highly erroneous answers. If the external system consists of many buses, the Norton-type equivalent model requires a large amount of data to be processed. Another disadvantage of the Norton type equivalent model is that the classification of the boundary buses as generation buses or as load buses is system dependent. These shortcomings, together with the uncertainty with regard to the topology and the parameter values of the external system because of limited exchange of information between neighboring companies, have increased in recent years the interest for on-line type equivalents.

In this section, on-line type equivalents are derived by using information from the internal system only. Switching operations, together with on-line state estimation, are the main sources of information.
to obtain the equivalent model. In the next section the formulation of
the problem for on-line equivalents is presented.

4.5.2 Formulation of the Problem for On-Line Equivalents

In this section the mathematical formulation presented in Chapter
II is stated as it is applied for on-line equivalents.

For on-line equivalents it is assumed that no information from
the external system is available. An equivalent representation of the
external system needs to be obtained by using information from the
internal system only.

Figure 4.6 shows a set of boundary busses. The dotted lines
indicate equivalent branches between the boundary busses.

Let \( a(i) \) be the set of busses of the internal system connected to
the \( i \)th boundary bus and \( b(i) \) the set of the boundary busses connected
to the \( i \)th boundary bus. The real and reactive injection at the \( i \)th
boundary bus is given by:

\[
P_i = V_i^2 \left\{ \sum_{j \in b(i)} G_{ij} \right\} - V_i \sum_{j \in b(i)} V_j \left\{ G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij} \right\}
+ \sum_{j \in a(i)} P_{ij}
\]

\[
Q_i = -V_i^2 B_{SHUNT,i} + \sum_{j \in b(i)} B_{ij} - V_i \sum_{j \in b(i)} V_j \left\{ G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij} \right\}
- \sum_{j \in a(i)} Q_{ij}
\]

where:

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Figure 4.6. Boundary Bus Interconnections.
\[ V_i \quad = \quad \text{voltage magnitude of the } i^{\text{th}} \text{ bus} \]
\[ \theta_{ij} \quad = \quad \text{phase voltage angle difference across the line } ij \]
\[ G_{ij} \quad = \quad \text{conductance of line } ij \]
\[ B_{ij} \quad = \quad \text{susceptance of line } ij \]
\[ B_{\text{SHUNT},i} \quad = \quad \text{shunt susceptance at the } i^{\text{th}} \text{ bus} \]
\[ P_{ij} \quad = \quad \text{real flow from the } i^{\text{th}} \text{ bus to the } j^{\text{th}} \text{ bus} \]
\[ Q_{ij} \quad = \quad \text{reactive flow from the } i^{\text{th}} \text{ bus to the } j^{\text{th}} \text{ bus}. \]

\( G_{ij}, B_{ij}, B_{\text{SHUNT},i} \) are components of the vector \( u \).

Using equations (4.95) (4.96), the vectors of real injections at the boundary busses before and after the \( k^{\text{th}} \) outage can be expressed as:

\[
\begin{align*}
I_{1k} &= A(\bar{x}^{1k})u + T_{1k} \\
I_{2k} &= A(\bar{x}^{2k})u + T_{2k}
\end{align*}
\]

where:

\[ T_{1k}, T_{2k} \] vectors of pre- and post-outage power flows from the boundary busses to the internal system. These are known quantities.

\[ A(\bar{x}^{1k}), A(\bar{x}^{2k}) \] matrices which are strictly dependent on \( \bar{x}^{1k} \) and \( \bar{x}^{2k} \).

The difference between the pre- and post-outage injections at the
boundary busses becomes

\[ g_k = I_1^{1k} - I_2^{2k} \]

\[ = [A(x_1^{1k}) - A(x_2^{2k})]u + T_1^{1k} - T_2^{2k} \]

\[ = H_k^k u + M_k^k ; \quad k=1, \ldots, N \]  \hspace{1cm} (4.99)

Since the internal system is observable to a state estimator, the matrix \( H_k^k \) and the vector \( M_k^k \) are known quantities.

The objective function of the equivalent problem becomes:

\[ J = \sum_{k=1}^{N} g_k^T g_k \]

\[ = \sum_{k=1}^{N} (H_k^k u + M_k^k)^T (H_k^k u + M_k^k) \]  \hspace{1cm} (4.100)

Next the constraints of the problem are presented. Let us define as:

\[ g_{N_1} = H_{N_1} u + M_{N_1} \]  \hspace{1cm} (4.101)

\[ g_{N_2, \ell} = H_{N_2, \ell} u + M_{N_2, \ell} ; \quad \ell=1, \ldots, L \]  \hspace{1cm} (4.102)

where:
\[
\begin{align*}
\mathbf{H}_{N_1} &= \begin{bmatrix}
H^1 \\
\vdots \\
\vdots \\
H^k \\
N_1 \\
H
\end{bmatrix}, \\
\mathbf{M}_{N_1} &= \begin{bmatrix}
M^1 \\
\vdots \\
\vdots \\
M^k \\
N_1 \\
M
\end{bmatrix} \\
\mathbf{H}_{N_2, \ell} &= \begin{bmatrix}
H^1 \\
\vdots \\
\vdots \\
H^k \\
N_2, \ell \\
H
\end{bmatrix}, \\
\mathbf{M}_{N_2, \ell} &= \begin{bmatrix}
M^1 \\
\vdots \\
\vdots \\
M^k \\
N_2, \ell \\
M
\end{bmatrix}
\end{align*}
\] (4.103)

and

\[
\begin{align*}
\mathbf{H}_{N_1} &= \begin{bmatrix}
H^1 \\
\vdots \\
\vdots \\
H^k \\
N_1 \\
H
\end{bmatrix}, \\
\mathbf{M}_{N_1} &= \begin{bmatrix}
M^1 \\
\vdots \\
\vdots \\
M^k \\
N_1 \\
M
\end{bmatrix} \\
\mathbf{H}_{N_2, \ell} &= \begin{bmatrix}
H^1 \\
\vdots \\
\vdots \\
H^k \\
N_2, \ell \\
H
\end{bmatrix}, \\
\mathbf{M}_{N_2, \ell} &= \begin{bmatrix}
M^1 \\
\vdots \\
\vdots \\
M^k \\
N_2, \ell \\
M
\end{bmatrix}
\end{align*}
\] (4.104)

Then the constraints of the problem

\[
F_N(u) = |\mathbf{g}(x^1, x^2, u)| - \bar{g}_N \leq 0
\]

becomes

\[
F_N(u) = |\mathbf{H}_N u + \mathbf{M}_N| - \bar{g}_N \leq 0
\] (4.105)
where:

\[
H_N = \begin{bmatrix}
H_{N_1} \\
H_{N_2,1} \\
\vdots \\
H_{N_2,L}
\end{bmatrix} \quad \quad M_N = \begin{bmatrix}
M_{N_1} \\
M_{N_2,1} \\
\vdots \\
M_{N_2,L}
\end{bmatrix}
\]

In summary, the problem is:

Minimize:

\[
J = \sum_{k=1}^{N} (H_k^T u + M_k)^T (H_k^T u + M_k)
\]

subject to the constraints:

\[
F_N(u) = \left| H_N u + M_N \right| - \bar{g}_N \leq 0
\]

For each outage there are 2b inequality constraints; therefore, there are 2xbxN inequalities to be satisfied. The method to solve the optimization problem described by the equations (4.107) and (4.108) is discussed in the next sections.
4.5.3 Unconstrained Problem

If the internal system is highly interconnected to the external system and the number of switching operations under consideration is large, the number of the inequality constraints to be satisfied is high. Therefore, before the problem is solved in its entirety, the model obtained by solving the unconstrained problem should be tested. In this section the method of solving the unconstrained problem is derived and the various aspects of this model are discussed.

Equivalent Model

The objective function is a quadratic function of the unknown vector $u$. This quadratic function takes its minimum when:

$$\frac{\partial J}{\partial u} = 0$$ (4.109)

The optimality condition becomes:

$$\sum_{k=1}^{N} (H^k)^T (H^k u + M^k) = 0$$ (4.110)

and the optimal solution is given by

$$\hat{u} = -\left[ \sum_{k=1}^{N} (H^k)^T H^k \right]^{-1} \left[ \sum_{k=1}^{N} (H^k)^T M^k \right]$$ (4.111)

Therefore, the solution is obtained in one iteration and the solution exists assuming that the matrix
\[ \sum_{k=1}^{N} (H^k)^T X^k \]

is a nonsingular matrix.

If \( l_1 \) is the number of the equivalent branches and \( l_2 \) is the number of the fictitious capacitors or reactors at the boundary busses, the solution of the unconstrained problem requires the inversion of a matrix of dimension \((2x l_1 + l_2) \times (2x l_1 + l_2)\). The computer storage requirements and the computational time are dependent upon the values of \( l_1, l_2 \). In general, using sparsity techniques both the storage requirements and the computational time are moderate and the method is suitable for on-line operation. The computational aspects to derive the solution of the unconstrained problem are given in Section ( ).

The value of the quadratic function at the optimum is given by

\[ J = N x b x S \]

where the index \( S \) has been defined.

The equivalent model obtained from equation (4.111) should be tested if it satisfies the inequality constraints:

\[ F_N(u) \leq 0 \]
then there are:

\[
\begin{pmatrix}
\frac{b(b-1)}{2} \\
\ell
\end{pmatrix}
\]

possible connectivities. It is impractical to examine all these cases to determine the optimal connectivity.

A systematic procedure to define the connectivity of the equivalent model so that it will compromise between the sparsity of the admittance matrix and the performance index is as follows:

**First Step:** An initial connectivity between the boundary buses is assumed. This is a-priori information which can be based either on equivalencing techniques using the Norton-type equivalent or on past experience of the particular system. This connectivity should contain sufficient number of equivalent branches. The vector \( \underline{u} \) which corresponds to the initial connectivity is denoted by \( \underline{u}_0 \). The dimension of the vector \( \underline{u} \) is restricted by the computer storage requirements to invert the matrix

\[
\sum_{k=1}^{N} (H^k)^T H^k
\]

**Second Step:** Using the connectivity defined in the first step, equation (111) is applied to determine the values of the conductances and susceptances of the equivalent branches. The optimum vector \( \underline{u} \) which corresponds to this connectivity is denoted by \( \underline{u}_o \).
Third Step: The vector $\mathbf{u}_0$ is used to compute for each equivalent branch the indices

$$
(\Gamma_P)_j = \frac{1}{N} \sum_{k=1}^{N} |P_j^{2k}|
$$

$$
(\Gamma_Q)_j = \frac{1}{N} \sum_{k=1}^{N} |Q_j^{2k}|
$$

(4.114) (4.115)

where $P_j^{2k}, Q_j^{2k}$ have been defined. If for the $j^{th}$ branch:

$$(\Gamma_P)_j < \Gamma_P$$

$$(\Gamma_Q)_j < \Gamma_Q$$

then the $j^{th}$ branch is eliminated from the equivalent representation.

For various selections of $\Gamma_P$, $\Gamma_Q$ the indices $S$, $MP$, $MQ$ are calculated. The connectivity which satisfies the accuracy requirements and preserves the sparse structure of the admittance matrix is selected. The vector $\mathbf{u}$ which corresponds to this connectivity is denoted by $\mathbf{u}_1$.

Fourth Step: Shunt terms are included at the boundary busses.

For the connectivity selected from the third step, the conductances and susceptances of the equivalent branches are computed by using equation (4.111).

Step four gives the equivalent representation of the external system. Note that by including shunt terms in the fourth step at the boundary busses the sparsity of the admittance matrix is not affected.
4.5.4 Constrained Problem

If all the equivalent models examined so far are unable to satisfy the requirements of the problem, one has to solve the problem in its entirety. In this section the method to solve the constrained problem is discussed.

In general, there are two approaches to solve the constrained optimization problem.

(1) Using penalty function methods
(2) Using quadratic programming

The penalty function methods transform the constrained problem into an unconstrained problem. A number of methods can be applied to solve the unconstrained problem. Convergence of these methods becomes dependent on the selection of the penalty factors. Early attempts to solve the constrained problem using penalty function methods were unsuccessful because of convergence difficulties.

Quadratic programming was chosen as the method to solve the problem. It guarantees that the optimal solution is obtained in a finite number of steps assuming that such solution exists.

Equivalent Model

The general statement of a quadratic problem is:

Minimize the quadratic function:

$$\mathbf{x}_0 = \mathbf{c}^T \mathbf{u} + \mathbf{u}^T \mathbf{D} \mathbf{u}$$

(4.116)

subject to a linear system of constraints
Since

$$\sum_{k=1}^{N} (M_k^k)^T M_k$$

is a constant, the equivalent problem described by equations (107) and (108) can be restated as:

Minimize:

$$x_0 = 2\left( \sum_{k=1}^{N} (H^k)^T M_k^k \right) u + u^T \left( \sum_{k=1}^{N} (H^k)^T H_k \right) u$$

subject to the constraints:

$$H_{\text{N}} u \leq \overline{g}_N - M_{\text{-N}}$$

$$-H_{\text{N}} u \leq \overline{g}_N + M_{\text{-N}}$$

The problem described by equations (4.119), (4.120) and (4.121) is in the form:

Minimize:

$$x_0 = c^T u + u^T D u$$

subject to:

$$A u \leq P_0$$

where:
The vector \( \mathbf{u} \) is of dimension \( n = 2l_1 + l_2 \) where \( l_1, l_2 \) as defined earlier. The vector \( \mathbf{c} \) is of dimension \( m = 4xNxb \). The defined matrix \( D \) is a positive semi-definite matrix.

The equivalent model \( \mathbf{u} \) is obtained by solving the quadratic programming problem described by the equations (119), (120), and (121).

The quadratic programming problem is solved by direct application of the Kuhn-Tucker conditions. The Kuhn-Tucker conditions reduce the quadratic problem to a linear programming problem. The problem becomes:

Find an \( n \)-vector, \( \mathbf{u} \), and \( n \)-vector, \( \mathbf{v} \), an \( m \)-vector, \( \mathbf{\lambda} \), and an \( m \)-vector, \( \mathbf{S} \), such that:

\[
-2D\mathbf{u} - \mathbf{A}^T\mathbf{\lambda} + \mathbf{v} = \mathbf{c} \quad (4.126)
\]

\[
\mathbf{A}\mathbf{u} + \mathbf{S} = \mathbf{p}_0 \quad (4.127)
\]
outages, the inequality constraints $F_N(u)$ are formed. First, the existing equivalent model should be investigated if it satisfies the specifications of the problem. The performance index $S$ is computed. If

$$S < \bar{S}$$

and

$$F_N(u) < 0$$

there is no need to update the equivalent model. If the above requirements are not satisfied, the equivalent model should be updated. If the model obtained by solving the unconstrained problem satisfies the inequality constraints, the model is sufficient. If not, one has to solve the constrained problem. The connectivity of the existing equivalent model is used when the updated values of the conductances and susceptances of the equivalent branches are computed. The connectivity of the equivalent model should be investigated again if major outages have been taking place in the external system.

4.5.5 Summary

In this section the solution to the optimization method has been presented. First, the model obtained by solving the unconstrained problem is presented. A systematic procedure to define the connectivity of the equivalent model is developed. Next, the optimization problem is formulated as a quadratic programming problem. Finally, the method to solve the quadratic programming problem is presented.
4.6 Test Cases

Five examples are presented in this chapter. The first deals with the Norton Equivalent. The second is an application of the elimination schemes derived in Section 4.4.3. The third deals with the unconstrained problem. The fourth deals with the constrained problem.

4.6.1 Example for the Norton Equivalent

This example deals with the Norton equivalent. The 30 bus system shown in Figure 4.7 is the entire system. This is an IEEE test system. The dotted line separates the internal system from the external system. The buses 8, 25 and 30 are the boundary buses. The Norton Equivalent is shown in Figure 4.8. Three equivalent branches are created between the three boundary buses and these branches are denoted by the dotted lines. Five outages were considered in the internal system. The post-outage conditions were obtained by performing the load-flow analysis with the entire area. The Norton Equivalent was tested for the set of these five outages. The indices $S$, $MP$, $MQ$ were computed. These indices are given below.

\[ S = 7.24 \text{ (MVA)}^2 \]
\[ MP = 0.883 \text{ (MW)} \]
\[ MQ = 8.541 \text{ (MVAR)} \]
FIGURE 4.7 Topology of the Internal and External System (Example 4.6.1)
FIGURE 4.8 Topology of the Norton Equivalent Model of the Internal System (Example 4.6.1)
If these values of $S$, $MP$, $MQ$ are less than the specified toler-
ances of the problem, the Norton equivalent model is satisfactory. The
index $S$ gives the total error. The average difference of the real and
reactive injections before and after the five outages is 2.69 (MVA).
($=\sqrt{S}$). From the values of $MP$ and $MQ$ it is concluded, for this particu-
lar example, that the assumption that $(p,v)$ assumption for bound
buses is more valid than the $(p,q)$ assumption. This was true for
all other systems and sets of outages we consider in this
research.

4.6.2. Example on the Sparsity of the Admittance Matrix of the Norton
Equivalent

The two elimination schemes presented in Section 3.3 were tested
with a 444 bus system. This is part of the Bonneville Power Administra-
tion (BPA) system. The internal system includes 87 buses with 31
boundary buses. The set of postulated outages includes 29 outages in
the internal system. The results of these outages were obtained by
performing load-flow analysis with the 444 bus system. The Norton
equivalent model was computed and 437 equivalent branches were created
between the boundary buses. Obviously, the admittance matrix of the
equivalent model is not a sparse matrix. The two elimination schemes
derived in Section 4.4.3 were applied to define a connectivity which
satisfies both the sparsity requirements and accuracy tolerances. For
various selections of $\Gamma_F$, $\Gamma_Q$ and $G$, $B$ the accuracy indices $S$, $MP$, $MQ$
were computed and the results are summarized in Tables VII and VIII. The
results are shown also in Figures 4.9, 4.10 and 4.11. By examination of
the Tables VII and VIII, we conclude that:
<table>
<thead>
<tr>
<th>IP (MW)</th>
<th>IQ (MVAR)</th>
<th>No. of Branches Between Boundary Busses</th>
<th>S (MVA)</th>
<th>MP (MW)</th>
<th>MQ (MVAR)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Initial Connectivity</td>
<td>437</td>
<td>753.34</td>
<td>114.95</td>
</tr>
<tr>
<td>1.</td>
<td>1.</td>
<td>130</td>
<td>726.31</td>
<td>113.21</td>
<td>328.63</td>
</tr>
<tr>
<td>2.</td>
<td>2.</td>
<td>109</td>
<td>716.69</td>
<td>111.54</td>
<td>322.84</td>
</tr>
<tr>
<td>3.</td>
<td>2.</td>
<td>97</td>
<td>704.55</td>
<td>111.54</td>
<td>306.23</td>
</tr>
<tr>
<td>3.</td>
<td>3.</td>
<td>92</td>
<td>675.44</td>
<td>111.54</td>
<td>246.28</td>
</tr>
<tr>
<td>4.</td>
<td>3.</td>
<td>83</td>
<td>674.09</td>
<td>111.54</td>
<td>246.28</td>
</tr>
<tr>
<td>5.</td>
<td>5.</td>
<td>74</td>
<td>665.51</td>
<td>105.41</td>
<td>246.28</td>
</tr>
<tr>
<td>8.</td>
<td>8.</td>
<td>72</td>
<td>665.97</td>
<td>105.41</td>
<td>246.28</td>
</tr>
<tr>
<td>10.</td>
<td>10.</td>
<td>66</td>
<td>653.12</td>
<td>151.78</td>
<td>185.37</td>
</tr>
<tr>
<td>12.</td>
<td>12.</td>
<td>63</td>
<td>650.59</td>
<td>190.93</td>
<td>184.37</td>
</tr>
<tr>
<td>15.</td>
<td>15.</td>
<td>59</td>
<td>648.68</td>
<td>190.93</td>
<td>186.37</td>
</tr>
<tr>
<td>18.</td>
<td>18.</td>
<td>57</td>
<td>650.31</td>
<td>190.93</td>
<td>186.37</td>
</tr>
<tr>
<td>20.</td>
<td>20.</td>
<td>54</td>
<td>639.37</td>
<td>190.93</td>
<td>194.80</td>
</tr>
<tr>
<td>23.</td>
<td>23.</td>
<td>48</td>
<td>886.53</td>
<td>230.28</td>
<td>186.63</td>
</tr>
<tr>
<td>25.</td>
<td>25.</td>
<td>47</td>
<td>1046.88</td>
<td>365.25</td>
<td>186.63</td>
</tr>
</tbody>
</table>
TABLE VIII. Performance of the Second Elimination Scheme (Example 4.6.2)

<table>
<thead>
<tr>
<th>G (p.u)</th>
<th>B (p.u)</th>
<th>No. of Branches Between Boundary Busses</th>
<th>S (MVA)^2</th>
<th>MP (MW)</th>
<th>MQ (MVAR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>__</td>
<td>__</td>
<td>Initial Connectivity</td>
<td>437</td>
<td>753.34</td>
<td>114.95</td>
</tr>
<tr>
<td>.05</td>
<td>.05</td>
<td>135</td>
<td>740.32</td>
<td>113.21</td>
<td>345.39</td>
</tr>
<tr>
<td>.1</td>
<td>.1</td>
<td>119</td>
<td>720.14</td>
<td>111.54</td>
<td>322.84</td>
</tr>
<tr>
<td>.15</td>
<td>.15</td>
<td>114</td>
<td>719.51</td>
<td>111.54</td>
<td>322.84</td>
</tr>
<tr>
<td>.1</td>
<td>.2</td>
<td>104</td>
<td>707.52</td>
<td>111.54</td>
<td>306.23</td>
</tr>
<tr>
<td>.2</td>
<td>.2</td>
<td>101</td>
<td>706.06</td>
<td>111.54</td>
<td>306.23</td>
</tr>
<tr>
<td>.2</td>
<td>.3</td>
<td>90</td>
<td>688.13</td>
<td>111.54</td>
<td>279.78</td>
</tr>
<tr>
<td>.35</td>
<td>.35</td>
<td>88</td>
<td>688.48</td>
<td>111.54</td>
<td>279.78</td>
</tr>
<tr>
<td>.4</td>
<td>.4</td>
<td>85</td>
<td>673.86</td>
<td>111.54</td>
<td>246.28</td>
</tr>
<tr>
<td>.4</td>
<td>.5</td>
<td>82</td>
<td>669.28</td>
<td>111.54</td>
<td>246.28</td>
</tr>
<tr>
<td>.5</td>
<td>.5</td>
<td>81</td>
<td>601.18</td>
<td>105.41</td>
<td>246.28</td>
</tr>
<tr>
<td>.6</td>
<td>.6</td>
<td>76</td>
<td>637.41</td>
<td>125.54</td>
<td>195.92</td>
</tr>
<tr>
<td>.7</td>
<td>.7</td>
<td>71</td>
<td>642.84</td>
<td>125.54</td>
<td>192.87</td>
</tr>
<tr>
<td>.9</td>
<td>.9</td>
<td>68</td>
<td>640.96</td>
<td>190.93</td>
<td>192.87</td>
</tr>
<tr>
<td>1.1</td>
<td>1.</td>
<td>66</td>
<td>1118.12</td>
<td>316.47</td>
<td>229.33</td>
</tr>
<tr>
<td>.01</td>
<td>.01</td>
<td>184</td>
<td>751.71</td>
<td>114.68</td>
<td>354.15</td>
</tr>
</tbody>
</table>
FIGURE 4.9. The Performance Index S as a Function of the Retained Equivalent Branches with the 1st and 2nd Elimination Scheme.
FIGURE 4.10. The Performance Index MP as a Function of the Retained Equivalent Branches with the 1st and 2nd Elimination Scheme.
FIGURE 4.11. The Performance Index MQ as a Function of the Retained Equivalent Branches with the 1st and 2nd Elimination Scheme.

First Elimination Scheme
Second Elimination Scheme
(1) The performance of the Norton equivalent model is improved by eliminating some of the equivalent branches.

(2) The first elimination scheme is more effective than the second elimination scheme. Using the first elimination scheme less equivalent branches between the boundary buses are required to succeed the minimum values of the indices $S$, $MP$, $MQ$. This was expected since the first elimination scheme takes into consideration not only the magnitudes of the conductances and susceptances of the equivalent branches, but, also, the operating conditions of the system.

(3) The index $MP$ is less sensitive to the number of retained equivalent branches than the index $MQ$.

The above conclusions cannot be generalized for every power system. Similar investigations should be performed with the particular system and set of postulated outages to define the optimum values of $FP$, $FQ$ or $G$. These values will be dependent on the loading condition of the system and the postulated outages.

4.6.3 Example for the Unconstrained Problem

The model obtained by solving the unconstrained problem was tested with the 444 bus system described in Section 4.6.2. The set of postulated outages includes 29 outages in the internal system ($N=29$).

The equivalent model was obtained by following the procedure outlined earlier.

**Step One:** The initial connectivity between the boundary buses is obtained by the first elimination scheme developed for the Norton equivalent in Section 4.4.3. This connectivity consists of 92 branches between the 31 boundary buses. In section 4.6.2 the values of the
performance indices $S$, $MP$, $MQ$ for the set of 29 outages were given and are cited again:

\begin{align*}
S &= 675.44 \text{ (MVA)}^2 \\
MP &= 111.54 \text{ (MW)} \\
MQ &= 246.28 \text{ (MVAR)}
\end{align*}

**Step Two:** For the connectivity obtained in Step One, equation (4.111) was applied to determine the conductances and susceptances of the 92 equivalent branches ($l_1 = 92$, $l_2 = 0$). This model was tested for the set of 29 outages and the results are given below:

\begin{align*}
S &= 231.53 \text{ (MVA)}^2 \\
MP &= 67.59 \text{ (MW)} \\
MQ &= 109.49 \text{ (MVAR)}
\end{align*}

**Step Three:** The model obtained in Step Two was used to compute for each equivalent branch the indices $(FP)_j$ and $(FQ)_j$ defined by equations (4.114) and (4.115), respectively. For various selections of $(FP)$, $(FQ)$ the performance indices $S$, $MP$, $MQ$ were computed and the results are summarized in Table IX. From Table 3 it can be seen that all the selections of $(FP)$, $(FQ)$ except the last one gave equivalent models whose performances are almost the same with the performance of the model obtained by solving the unconstrained problem. Based on this elimination procedure, the connectivity consisted of 85 branches between the boundary buses was selected as the connectivity for the equivalent model.
Step Four: For the connectivity obtained in Step Three, the model by solving the unconstrained problem was computed. Using equation (111), the conductances and susceptances of the 85 equivalent branches were computed ($\ell_1 = 85$, $\ell_2 = 0$). This model was tested for the set of 29 outages and the results are given below.

$$S = 234.176 \text{ (MVA)}^2$$
$$MP = 67.70 \text{ (MW)}$$
$$MQ = 109.5 \text{ (MVAR)}$$

Finally, shunt terms were included on 17 boundary busses and equation (111) was applied to determine the conductances and susceptances of the 85 equivalent branches and the susceptances of the 17 shunt terms ($\ell_1 = 85$, $\ell_2 = 0$). This final equivalent model was tested and the results for the 29 outages are given below.

$$S = 184.12 \text{ (MVA)}^2$$
$$MP = 65.82 \text{ (MW)}$$
$$MQ = 106.22 \text{ (MVAR)}$$

The results of this example are summarized in Table IX.

From Table X, we can conclude that:

(1) The performance of the models obtained by solving the unconstrained problem is superior to the performance of the Norton-type equivalent.

(2) By increasing the number of the equivalent branches, the
TABLE IX. Results of Elimination Procedure for Example 4.6.3

<table>
<thead>
<tr>
<th>$\Gamma P$ (MW)</th>
<th>$\Gamma Q$ (MVAR)</th>
<th>No. of Branches Between Boundary Busses</th>
<th>$S^2$ (MVA)</th>
<th>$MP$ (MW)</th>
<th>$MQ$ (MVAR)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial</td>
<td></td>
<td>92</td>
<td>231.53</td>
<td>67.59</td>
<td>109.49</td>
</tr>
<tr>
<td>Connectivity</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>5.</td>
<td>88</td>
<td>235.8</td>
<td>67.59</td>
<td>109.49</td>
</tr>
<tr>
<td>7.</td>
<td>5.</td>
<td>87</td>
<td>236.33</td>
<td>67.59</td>
<td>109.49</td>
</tr>
<tr>
<td>8.</td>
<td>8.</td>
<td>85</td>
<td>237.06</td>
<td>67.59</td>
<td>109.49</td>
</tr>
<tr>
<td>10.</td>
<td>9.</td>
<td>83</td>
<td>259.10</td>
<td>164.73</td>
<td>109.49</td>
</tr>
</tbody>
</table>
TABLE X. Simulation Results for Example 4.6.3

<table>
<thead>
<tr>
<th>Equivalent Model</th>
<th>Performance Indices</th>
<th>Model by Solving the Unconstrained Problem</th>
<th>Model by Solving the Unconstrained Problem</th>
<th>Model by Solving the Unconstrained Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Norton Equivalent 92 Branches</td>
<td>$S$ (MVA)$^2$</td>
<td>675.11</td>
<td>231.53</td>
<td>234.17</td>
</tr>
<tr>
<td>92 Branches No Shunt Terms</td>
<td>$MP$ (MW)</td>
<td>111.54</td>
<td>67.59</td>
<td>67.70</td>
</tr>
<tr>
<td>85 Branches No Shunt Terms</td>
<td>$MQ$ (MVAR)</td>
<td>246.28</td>
<td>109.49</td>
<td>109.50</td>
</tr>
</tbody>
</table>
performance index $S$ is improved. This was expected since by increasing the number of equivalent branches the number of independent variables is increased.

(3) By including shunt terms in the equivalent model, the performance index $S$ is improved. Note that these shunt terms do not affect the sparse structure of the equivalent matrix.

4.6.4 Example for the Constrained Problem

This example demonstrates the feasibility of obtaining an equivalent model by solving a quadratic programming problem as it was presented in Section 4.4.4. This example will serve also as a comparison between the Norton-type equivalent model, the model obtained by solving the unconstrained problem, and the model obtained by solving the constrained problem. The entire system consists of 30 buses and is shown in Figure 4.7. The dotted line separates the internal system from the external system. The buses 8, 25, 30 are the boundary buses ($b=3$). The set of postulated outages consists of five outages in the internal system ($N=5$).

The Norton-type equivalent model was computed in Section 4.5.1. Three equivalent branches are created between the boundary buses. In Section 4.5.1, the performance indices $S$, $MP$, $MQ$ for the set of five outages were computed and are cited again:

\[
S = 7.24 \text{ (MVA)}^2
\]

\[
MP = .833 \text{ (MW)}
\]

\[
MQ = 8.541 \text{ (MVAR)}
\]

The on-line equivalent models were obtained by solving the
unconstrained and the constrained problem and assuming only two lines between the boundary busses, one line between the busses 25 and 30 and one line between busses 8 and 30 ($l_1 = 2, l_2 = 0$).

The model obtained by solving the unconstrained problem (Section 4.43) was tested for the set of five postulated outages. The performance indices $S$, $MP$, $MQ$ are given below.

\[
S = 4.43 \text{ (MVA)}^2 \\
MP = .624 \text{ (MW)} \\
MQ = 4.595 \text{ (MVAR)}
\]

The constrained problem (Section 4.4) was solved under two conditions of constraints:

(a) Maximum allowable real error
\[
\overline{MP} = 4.5 \text{ (MW)}
\]
Maximum allowable reactive error
\[
\overline{MQ} = 4.5 \text{ (MVAR)}
\]

This model was tested for the set of the five postulated outages and the results are given below.

\[
S = 4.45 \text{ (MVA)}^2 \\
MP = .603 \text{ (MW)} \\
MQ = 4.5 \text{ (MVAR)}
\]
The obtained model was tested for the same set of five postulated outages. The performance indices are given below.

\[ S = 6.19 \ (\text{MVA})^2 \]
\[ MP = 3.45 \ (\text{MW}) \]
\[ MQ = 4.25 \ (\text{MVAR}) \]

The results of this example are summarized in Table XI.

Table 8 leads to several conclusions:

1. The performances of the models obtained by solving the unconstrained and the constrained problem are superior to the performance of the Norton-type equivalent model.

2. The performances of the models obtained by solving the unconstrained and the constrained problem can be improved by including more fictitious branches in the equivalent model \((k_1 = 3, k_2 = 3)\).

3. As it was expected, the performance index \(S\) takes its minimum value for the model obtained from the solution of the unconstrained problem. As the constraints of the problem become tighter, the value of the performance index \(S\) becomes larger.
TABLE XI. Simulation Results for Example 4.6.4

<table>
<thead>
<tr>
<th>Performance Indices</th>
<th>Equivalent Model</th>
<th>Norton Equivalent 3 Branches</th>
<th>Model by Solving the Unconstrained Problem</th>
<th>Model by Solving the Constrained Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Model by Solving</td>
<td></td>
<td>( MP = 4.5 ) (MW)</td>
<td>( MP = 4.25 ) (MW)</td>
</tr>
<tr>
<td></td>
<td>2 Branches</td>
<td></td>
<td>( MQ = 4.5 ) (MVAR)</td>
<td>( MQ = 4.25 ) (MVAR)</td>
</tr>
<tr>
<td>( S ) (MVA)^2</td>
<td>7.24</td>
<td>4.43</td>
<td>4.45</td>
<td>6.19</td>
</tr>
<tr>
<td>( MP ) (MW)</td>
<td>.833</td>
<td>.624</td>
<td>.603</td>
<td>3.45</td>
</tr>
<tr>
<td>( MQ ) (MVAR)</td>
<td>8.541</td>
<td>4.595</td>
<td>4.5</td>
<td>4.25</td>
</tr>
</tbody>
</table>
V. REFERENCES


7. A. S. Debs, "Estimation of Model Parameters for Power System Monitoring and Control," proposal submitted to National Science Foundation (Engineering Division), and resulted in Grant GK-37474, December 1972.


APPENDIX A

DOCUMENTATION OF ON-LINE EQUIVALENTS PROGRAM

This Appendix includes a detailed presentation of the programs written to test the validity of the equivalencing techniques presented in the previous chapters. Ten programs are discussed. Two of them, namely the programs ELLAS and ELLASO solve the load flow and the contingency analysis problems. The outputs of these two programs are combined together to create the file BPATEST. This file contains the set of postulated switching operations. In a real environment, this information is to be obtained from on-line measurements. The above two programs require complete knowledge of the topology and the parameters of the entire area. Also the operating conditions of the system are required. All this information is contained in a file named BPA2000. Complete description of this file is given in section A.1.

Program EQUIV uses file BPA2000 as input also. This program computes the admittance matrix of the Norton equivalent model.

Program REDUVA uses the output file of the program EQUIV to compute the equivalent model based on the 2nd elimination scheme presented in Chapter III.

Program GEOR computes the average real and reactive flows \((I_P)_j\) and \((I_Q)_j\), respectively.

Program REDUVA uses the output files of REDUVA and GEOR to compute the equivalent model based on the 1st elimination scheme discussed in Chapter IV.
Program IDENT computes the equivalent model by solving the unconstrained optimization problem.

Program QUAD1 computes the equivalent model by solving the constrained optimization problem.

Each equivalent model obtained through REDUVA or REDUFL or IDENT or QUAD1 is tested with program INDIC.

The ten programs are discussed in the following sections.

A.1 Program ELLAS

Program ELLAS is an algorithm which solves the load flow problem using the Newton-Raphson method. The input to the program is information about the configuration and the parameter values of the entire area, classification of buses as generation, load or slack buses, the loading conditions, the buses forming the internal system and the boundary buses.

The output of the program contains the voltage magnitudes and the voltage phase angles of the boundary buses and also the real and reactive flows from the boundary buses to the buses of the internal system.

The program ELLAS has a main program to which six subroutines are linked. (See flow chart for program ELLAS in Figure A.1.)

Input Data

The program requires certain necessary information about the system. This information is given through three files. There are three input files. TAPE2, TAPE3, TAPE5. Each file's content and format is described below in detail.

Tape 5

The content of Tape 5 will be referred to as BPA2000 file for the
rest of this report. The cards for the BPA file are described below.

Card 1

This card has entries for:

1. No. of buses in the entire system (NTOT),

2. Twice the no. of branches in the entire system (IOFFD),

3. No. of branches in the entire system (JBR).

Recommended format (I6, 1X, I6, 1X, I6)

Card 2 through M

This set of cards has total cards equal to the no. of buses in the entire system. Each card contains information, on the bus being read, which is:

1. Bus no. ordered sequentially from 1 through NTOT (JCK),

2. Bus name alphanumeric (BUS),

3. Voltage magnitude of this bus (VI). If the bus is load bus, VI=1.0.

4. Voltage phase angle of this bus (DI). Recommended value of DI, zero.

5. Real power injection of this bus 'p.u.' (PI).

6. Reactive power injection of this bus 'p.u.' (QI). If the bus is generation bus, recommended value of QI, zero.

7. Self conductance of this bus 'p.u.' (GKKTI).

8. Self susceptance of this bus 'p.u.' (BKKTI).

9. Type of bus (IKI).

   IKI = 1 for load bus
   IKI = -1 for generation bus.

10. Total no. of branches encountered up to the previous bus incremented by one (ITKTI).

Recommended format (I4,1X,A6,2F7.3,2F8.3,1X,
    F12.5,1X,F12.5,I3,I5).
Card M+1

This supplies information.

1. Total no. of buses in the entire system incremented by one (JCK).

2. Twice the no. of branches in the entire system incremented by one (KK).

Recommended format (I4,66X,I5).

Card M+2 through N

This set of cards has two forms of card sets. The first set of cards M+2 through L is elucidated in form 1 and the second set of cards L+1 through N is elucidated in form 2. Each set of these cards has total no. of cards equal to the no. of branches in the system. Each card contains information on the branch being read, which is:

**FORM 1**

1. identifying no. which is sequential from 1 through the total no. of branches in the entire system (JCK)

2. sending end bus number (JCC)

3. receiving end bus number (KM)

4. branch no. (mutual branch no.) (KLI)

5. transfer conductance of the branch of which the mutual branch no. is JCK 'p.u.' (GKMTI)

6. transfer susceptance of the branch of which the mutual branch no. is JCK 'p.u.' (BKMTI)

7. shunt conductance associated with the identifying no. pointer 'p.u.' (GSHUNI)

8. shunt susceptance associated with the identifying no. pointer 'p.u.' (BSHUNI)

Recommended format (3I4,16X,I3,4(F12.5)).
FORM 2

1. identifying no. which is sequential from total no. of branches in the entire system incremented by one through the twice the total no. of branches in the entire system (JCK)

2. sending end bus number (JCC)

3. receiving end bus number (KM)

4. branch no. (mutual branch no.) (KLI)

5. shunt conductance associated with the identifying no. pointer 'p.u.' (GSHUNI)

6. shunt susceptance associated with the identifying no. pointer 'p.u.' (BSHUNI)

Recommended format (3I4,16X,I3,24X,2(F12.5))

Tape 2

This file informs the program the bus no. of the buses of the internal system, written in free format. This file consists of six cards and each card should have twenty entries. (BPAINT)

Tape 3

This file informs the program the bus no. of the boundary buses, written in free format. This file consists of six cards and each card should have twenty entries. (BPABOUN)

Output Data

Line 1 through M This set of lines have total lines equal to the no. of the boundary buses. Each line contains information on the boundary bus being written; which is:

1. Bus no. of the boundary bus (J)

2. Bus name alphanumeric (BUS)

3. Voltage magnitude of this bus (V)
4. Voltage phase angle of this bus (D)
5. Real power from this bus to the buses of the internal system (MW) (PIN)
6. Reactive power from this bus to the buses of the internal system (MVAR) (QIN)

Recommended format: (I5,1X,A6,2F10.3,2F12.5)

A flow chart for program ELLAS follows.

A.2 Program ELLASO

Program ELLASO is an algorithm which performs outage analysis using the Newton-Raphson method. The input to the program is information about the configuration and the parameter values of the entire area, classification of buses as generation, load or slack buses, the loading conditions, the buses forming the internal system, the boundary buses and the lines for which outage analysis is to be performed.

The output of the program contains the voltage magnitude and the voltage phase angle of the boundary buses as also the real and reactive flows from the boundary buses to the buses of the internal system for each outage.

The program ELLASO has a main program to which six subroutines are linked.

Input Data

This program requires certain necessary information about the system and lines which are considered for outage analysis. This information is given through four files, TAPE2, TAPE3, TAPE4, TAPE5.

The contents of TAPE2, TAPE3, TAPE5 were discussed in detail in section A.1 for program ELLAS. The additional required file TAPE 4 is discussed next.
Read Input Data

Set ITIMAX = 10
    PLIMIT = .01
    QLIMIT = .01

Reorder the Buses
According to the No.
of Branches Connected
to Each Bus
(ORDER)

Create the Pointers for
the Table of Factors for
the Jacobian Matrix
(LUPNTR)

ICITER = 0

ICITER = ICITER + 1

ICITER>ITIMAX

STOP

FIGURE A.1 Flow Chart for Program ELLAS
Compute Maximum Real (Reactive) Mismatch \( PMAX (QMAX) \)

\( PMAX < PLIMIT \) and \( QMAX < QLIMIT \)?

- YES
  - Print Results
  - STOP

- NO
  - Using the Results of the Last Iteration Compute the Jacobian Matrix \( (JACOB) \)
  - Compute the Elements of the Table of Factors for the Jacobian Matrix \( (LUFCTR) \)
  - Update Voltage Magnitudes and Voltage Phase Angles \( (SOLVE) \)

FIGURE A.1 (Continued)